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MISSION ANALYSIS FLOW SEQUENCING OPTIMIZATION

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ABSTRACT

This investigation is an extension of last year's project dealing with the problem of optimal use of ground resources for future space missions. This problem was formulated as a linear programming problem using an indirect approach. Instead of minimizing the inventory level of needed ground resources, we minimize the overlapping periods during which the same types of resources are used by various flights. The model was built upon the assumption that during the time interval under consideration, the costs of various needed resources remain constant. Under other assumptions concerning costs of resources, the objective function, in general, assumes a non-linear form. In this study, one case where the form of the objective function turns out to be quadratic is considered. Also, disadvantages and limitations of the approach used are briefly discussed.

1. Introduction

The problem of optimal utilization of ground resources for scheduling future space missions has been one of continuing interest and concern in the Program Development Division of NASA. Currently, planning for efficient use of ground resources is carried out using "GROPE" (Ground Resources Operations Program Executive). This is a series of computer programs that works with a traffic model and ground processing time lines as basic inputs. The traffic model is the specific Shuttle flights in a given year. Requirements for different ground resources and equipment are determined by the specific type of each flight, e.g., a Spacelab (pallet or module), deployed satellites with or without upper stages or Department of Defense flight. All Shuttle flights require: an Obiter Processing Facility where Orbiter refurbishment is done and also where horizontally installed payloads are integrated; and a Vertical Assembly Building where the solid rockets and external tank are stacked on the mobile launcher platform, then the Orbiter is attached to this stack, and a launch pad. Many flights also use the Vertical Processing Facility for vertically installed payload processing.

GROPE can be constrained or unconstrained. Constrained means limited resources are available on limited dates. Here it may not be possible for all flights to be scheduled within a given year. If this is the case, those flights that are not placed in the schedule are moved to the following year and scheduled first. In the unconstrained case, the entire traffic model is scheduled in the proper year and it includes a complete specification of the various resource requirements in terms of "quantity" and "need" dates. Our interest in this project corresponds to the unconstrained case of GROPE. A precise description of the actual development of the mechanics of GROPE is, unfortunately, unavailable. However, it is known that its development is based mostly on heuristic grounds and lacks complete mathematical justification, rigor and formality. Due to the presence of a large number of variables, a complete mathematical formulation of this problem is no doubt very complex. The problem is further complicated by the fact that some of these variables are stochastic in nature. In our last year's report [5], we proposed an approximate mathematical model to formulate the problem. This formulation was based on assumptions which describe the actual situation fairly closely. It must, however, be pointed out that we have ignored a number of variables which have no direct bearing on the problem and all variables considered are assumed to be non-stochastic. Under the assumption that the costs of resources remain constant during the period under consideration, using an indirect approach, the problem was formulated as a linear programming problem. Under other assumptions for the costs of the resources, in general, the objective function is non-linear. In this study, one case where the form of the objective function turns out to be quadratic is considered. Also, in general, disadvantages and limitations of the approach used in this study are discussed.

2. The Model

The model was built and dealt with in an indirect manner.

Instead of analyzing the problem by minimizing the inventory level of needed resources, we use an approach whereby the flights are scheduled in such a way so as to minimize the overlapping periods during which the same types of resources are used by the various flights. Associated with each overlap for the use of the same type of resource by any pair of flights is a penalty cost which depends upon the number of units of that resource needed by these flights. Assuming that costs of resources remain unchanged during the whole period of the traffic model under consideration and that penalty costs are directly proportional to the lengths of the corresponding overlaps, the over-all objective function, which is the sum of such costs, is linear. Associated with the objective function is a number of sequencing and resource constraints which are also linear in form. Thus the problem under consideration falls within the domain of linear programming.

One program involvig use of linear programming for scheduling flights concerning space mission was prepared by Lockheed Electronics Co. in 1976 in the form of a technical report [1]. However, use of this program is basically restricted to cases dealing only with a fixed set of resources and the problem is one of selecting a traffic model from among various flight candidates which satisfy certain objectives. In our investigation, we have elaborated and refined the approach proposed in [6]. Stated below are the basic assumptions that concern our investigation.

(1) There are n flights to be launched during some given interval of time, [O, T].

(2) The order in which these flights are to be launched is predetermined.

(3) Of the n flights, there are p ($\leq n$) specific flights with fixed launch dates. Each such flight may, however, have a launch window of a certain specified length.

(4) All flights utilize at least one type of ground resource from a collection of M different types.

(5) Any flight that utilizes say, a type 'k' ground resource may require n_k (≥ 1) units of that resource.

3. Notation and Formulation

Let, t_i, ($i = 1,2,...,n$) be a variable denoting the launch time of the ith flight.

For any flight i that utilizes a type 'k' resource before its launch time, s_i^{k1} denotes the length of time in which this resource is seized before t_i and d_i^{k1} denotes the corresponding duration for its use.

For $1 \leq i < j \leq n$, the non-negative overlap variables are denoted by O_{ij}^{kl}, ($k = 1,2,...,M$; $r = 1,2$ $3,4$).

The variable O_{ij}^{k1} measures the amount of the overlapping period for use of a type 'k' resource by the ith and jth flights when the seize times for the resource occur before their launch times. The variable O_{ij}^{k2} measures the amount of the overlapping period for use of a type 'k' resource by the ith and jth flights when the seize times for the resource occur before t_i for the ith flight and after t_j for the jth flight. The variables O_{ij}^{k3} and O_{ij}^{k4} are similarly interpreted. In the case of O_{ij}^{k3}, the seize times for the resource occurs after t_i for the ith flight and before t_j for the jth flight. For O_{ij}^{k4}, the seize times for the resource by both flights occur after their launch times.

Clearly, for a traffic model consisting of n flights and M types of resources, the launch time variables $t_1, t_2, \ldots, t_n$ generate $2Mn(n-1)$ non-negative overlap variables.

The assumption that no two flights are to be scheduled at the same instant of time leads to constraints of the type

$$t_{i+1} - t_i \geqslant d > 0, \quad (i = 1,2,\ldots,n). \tag{1}$$

The provision that there are p specific flights with fixed launch dates at times, say, t_{n_i}, $(i = 1,2,\ldots,n)$ may have launch windows yields constraints of the type

$$t_{n_i} \geqslant \underline{l}_i, \; t_n \leqslant \bar{l}_i, \quad (i = 1,2,\ldots,p). \tag{2}$$

Also note that $t_n \leqslant T$. (3)

The relationships between the variables O_{ij}^{kr} and t_i, $(1 \leqslant i < j \leqslant n, \; r = 1,2,3,4; \; k = 1,2,\ldots,M)$ are given by the equality constraints

$$-t_i + t_j + O_{ij}^{kr} = d_{ij}^{kr}, \tag{4}$$

where the constants d_{ij}^{kr} are defined by

$$d_{ij}^{k1} = d_i^{k1} - s_i^{k1} + s_j^{k1},$$

$$d_{ij}^{k2} = d_i^{k1} - s_i^{k1} - s_j^{k2},$$

$$d_{ij}^{k3} = d_i^{k2} + s_i^{k2} + s_j^{k1},$$

$$d_{ij}^{k4} = d_i^{k2} + s_i^{k2} - s_j^{k2}.$$

Let n_i^{k1} and n_i^{k2} denote, respectively, the number of units of a type 'k' resource utilized by the ith flight when the seize times for this resource occur before and after t_i, n_j^{k1}

and n_j^{k2} for the jth flight are similarly defined. These quantities are used in the formulation of the cost functions. Under the assumption that costs of resources remain constant during the period under consideration, we consider the case where the cost, C_{ij}^{kr} associated with the overlap variable O_{ij}^{kr} is given by

$$C_{ij}^{kr} = n_{ij}^{kr}\ O_{ij}^{kr}\ C^{(k)},$$

$(1 \leq i < j \leq n;\ r = 1,2,3,4;\ k = 1,2,\ldots,M)$.

Here,

$$n_{ij}^{k1} = \mathrm{Min}(n_i^{k1},\ n_j^{k1}),$$

$$n_{ij}^{k2} = \mathrm{Min}(n_i^{k1},\ n_j^{k2}),$$

and $C^{(k)}$ is the weight given to a type 'k' resource based on cost considerations relative to other types of resources in the collection. The over-all objective function T, say, is the sum of costs C^{kr} over all values of k, r, i and j, $(i < j)$.

After a considerable amount of straight forward algebra, it can be shown that mininizing T is equivalent to minimizing T_1 given by

$$T_1 = \sum_{i=1}^{n} c_i t_i,$$

where

$$c_i = \sum_{k=1}^{M} C^{(k)} \left(\sum_{i < r \leq n} m_{ir}^{k} - \sum_{1 \leq s < i} m_{si}^{k} \right), \quad (i = 1,2,\ldots,n)$$

$$m_{ij}^{k} = \sum_{r=1}^{4} n_{ij}^{kr}, \quad (1 \leq i < j \leq n).$$

Recall that T_1 which is linear in t_i's is to be minimized subject to linear constraints given by (1), (2), (3), and (4).

4. Some Remarks

In the model developed above, we have assumed that there are p specific flights out of n with launch dates fixed at times t_{n_i}, $(i = 1,2,3,\ldots,p)$; each of which has a launch window of a certain specified length. Since the order of the flights for any given traffic model is predetermined, the launch times t_i, $(i = 1,2,\ldots,n)$ satisfy

$$0 \leq t_1 < t_2 < \ldots < t_{n_1} < t_{n_1+1} \ldots\ldots < t_{n_2-1} <$$
$$t_{n_2} < t_{n_2+1} \ldots\ldots < t_{n_p-1} < t_{n_p} < t_{n_p+1}$$
$$\ldots < t_{n-1} < t_n.$$

In the case where $n_p = n$, that is, the last flight is one with a fixed launch date, the constraint $t_n \leq T$ will be replaced by one of the form

$$t_n \leq \bar{l}_p.$$

Since all flights are to take place during the interval $[0,T]$, $\bar{l}_p$ should satisfy the condition

$$\bar{l}_p \leq T.$$

Similarly, in the case where $n_1 = 1$, we have the condition

$$t_1 \geq \underline{l}_1$$

with $\underline{l}_1 \geq 0$.

5. Other Cost Functions

In the model developed above, due to the assumption that costs of various resources remain constant, the objective function turned out to be linear. If this assumption is relaxed, in general, the objective function is non-linear and this would considerably increase the degree of difficulty of the problem. Below we consider the case where the objective function assumes a quadratic form.

Here, we assume that the cost of each type of resource increases linearly with increasing time. Specifically, we assume that the cost $C^{(k)}$ for the kth resource at time t_i is

$C^{(k)} = a_k + b_k t_i$,

and the penalty cost C_{ij}^{kr} associated with the overlap variable O_{ij}^{kr} is given by

$$C^{kr} = (C_j^{(k)} - C_i^{(k)})\, n_{ij}^{kr}\, O_{ij}^{kr}$$
$$= b_k\,(t_j - t_i)\, n_{ij}^{kr}\, O_{ij}^{kr},$$

$(r = 1,2,3,4;\ k = 1,2,\ldots,M;\ 1 \leq i < j \leq n)$.

The overall objective function is thus

$$T = \sum C_{ij}^{kr},$$

where the sum is to be performed over all values of r, k, i and j, $(1 \leq i < j \leq n)$.

After a lengthy but straight forward algebra, it can be shown that

$$T = \sum l_{ij}(t_i - t_j)^2 + \sum r_{ij}(t_i - t_j)$$

where each sum is to be performed over the set

$\{(i,j)/1 \leq i < j \leq n\}$, and

$$l_{ij} = - \sum_{k=1}^{M} \sum_{r=1}^{4} n_{ij}^{kr}\, b_k,$$

$$r_{ij} = - \sum_{k=1}^{M} \sum_{r=1}^{4} n_{ij}^{kr}\, d_{ij}^{kr}\, b_k.$$

Further simplification reduces the objective function to the form

$$T = \sum_{i=1}^{n} p_i t_i + \sum_{1 \le i < j \le n} 2(l_{ij} t_i t_j) + \sum_{i=1}^{n} l_{ii} t^2,$$

where

$$p_i = \sum_{1 \le j < i} r_{ji} - \sum_{i < j \le n} r_{ij},$$

$$l_{ii} = - \sum_{1 \le r < i} l_{ri} - \sum_{i < r \le n} l_{ir}, \quad (i = 1,2,\ldots,n).$$

The form of the objective function T does not in any way alter the form of the constraints and hence T is to be minimized subject to the same constraints given by (1), (2), (3), and (4). In terms of matrix notation, the objective function can be written as

$T = \underline{p}t' + \underline{t}Lt'$,

where

$L = (l_{ij})$ is a symmetric n X n matrix,

$\underline{t} = (t_1, t_2, \ldots, t_n)$,

$\underline{p} = (p_1, p_2, \ldots, p_n)$,

and t' and p' are column vectors corresponding to $\underline{t}$ and $\underline{p}$.

6. Discussion and Conclusion

At the present time, there are some convincing arguments that if we are dealing with a relatively short interval of time, (e.g., one year), the costs of resources could very well remain constant. Thus, in such a case, the formulation of the problem as a linear programming problem is well justified. Furthermore, this assumption does lend itself to easier treatment both from the mathematical and computational aspects of the problem. Other assumptions for the cost function, in general, considerably increase the complexity of the problem especially from the computational point of view.

As mentioned earlier, for a traffic model with n flights, in addition to the n launch times, variables, in general, there are 2Mn(n-1) overlap variables and the number of constraints is n + 2p + 2Mn(n-1). Thus, if there are no flights with fixed launch dates, the number of variables is the same as the number of constraints. An increase in the number of resources by one may cause an increase of as many as 2n(n-1) variables. In actual practice, the number of variables actually needed may be reduced if, for example, we know that the duration for use of any resource is short relative to the interval under consideration. On the other hand, for any flight type, there would be a minimum of about 15 ground resources where cost considerations should not be ignored. Thus, even if we have as few as 20 flights per year, the number of variables involved may be as many as 20 + 2(15)(20)(19) = 11420, a very large number indeed. The recent findings of Karmarkar [2] concerning an algorithm for large scale linear programming problems when available will, hopefully, prove useful to our problem. So far his approach exists only in what has been described as rougher computer code and, perhaps, it has not been tested on a wide range of problems.

In the analysis of the problem, we have used an indirect approach to optimize the use of needed ground resources. This approach provides us with launch times for scheduling flights of any given traffic model. However, note that it does not directly lend itself to the computation of the total cost of resources needed. In order to find the total cost corresponding to any solution $(t_1, t_2, \ldots, t_n)$ of the launch times, the peak requirement for each resource will have to be determined in each case.

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