GENERALIZED PARITY RELATIONS FOR LARGE SPACE STRUCTURES WITH UNCERTAIN PARAMETERS

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by

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Generalized Parity Relations for Large Space Structures with Uncertain Parameters

by

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Abstract

The method of generalized parity relations is one of the techniques that can be used to detect sensor and actuator failures on a large space structure. In this thesis, a model of a grid structure is used to evaluate the performance of these relations. It shows their relative sensitivity to modeling errors.

As no accurate model will be available before the structure is built in space, a method using sensor outputs and actuator inputs is required for the design of these relations. Three different estimators are studied. The second is the most interesting when computer memory is limited, while the third is the most accurate.

With a few modifications, the last estimator can also generate relations optimized for the detection of a particular failure. They are especially interesting when the level of sensor noise is high.

Thesis Supervisor: Wallace E. Vander Velde
Title: Professor of Aeronautics and Astronautics

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Chapter 1
Introduction

1.1 Space Structures and Generalized Parity Relations

Future space activities include many missions which will require the use of large space structures. Such structures will be lightweight and have small structural damping. This will allow small disturbances, in time, to produce large structural vibrations. For antennas and solar arrays, shape is critical, and will have to be controlled. Structural stress itself might be such that active shape control of the whole structure is required.

Shape control will require the use of many sensors and actuators. A large space structure will have hundreds of these components. As they will not be 100% reliable, many failures can be expected during the lifetime of the structure or even in between periodic repairs. For example, with 200 components, each having a mean time between failures of 100 000 hours, we expect 17 failures in a year [3]. To still be able to control the vibrations, redundancy, that is more components than are minimally required, will be introduced.

It is clear that the controller would have better performance if it knew which components have failed and stopped using them. A redundancy management system is designed to provide this information. It is generally made of two parts. First the failure detection system decides which component has failed. Then the reconfiguration system designs a new closed-loop configuration based on the remaining components. Figure 1-1 is a block diagram of the controlled structure

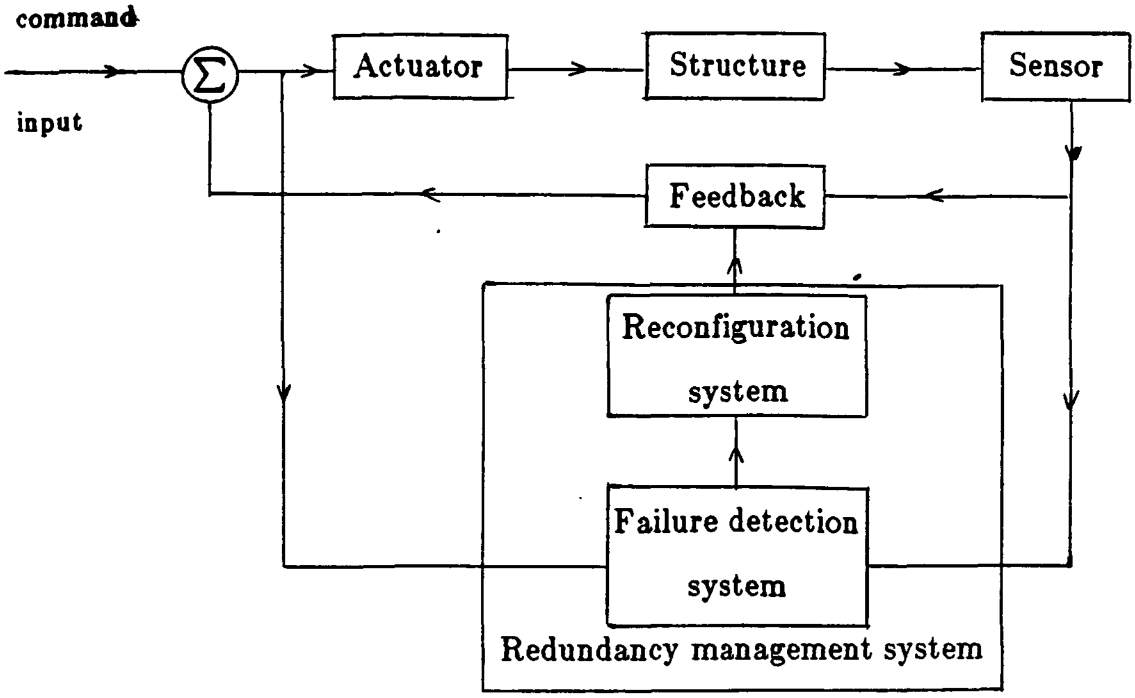


Figure 1-1: Redundancy management system of a controlled space structure with its redundancy management system.

Generally, failures are made detectable with the aid of a model representing structural vibrations. The method of generalized parity relations is one of such techniques. In [3] these relations were used to detect component failures on a free-free beam. They proved to be relatively insensitive to disturbances and sensor noise, but very sensitive to modeling errors.

As these space structures are designed for a zero gee environment they need not support their own weight. Thus they cannot be built and tested on earth. The model of the vibrations will have to be based only on theoretical knowledge. This can give errors as high as 20% and the resulting parity relations will certainly be useless. This is why a technique is required that can design relations once the structure is in space.

1.2 Thesis Goal

In this thesis, parity relations are designed to detect failures of components on a grid of aluminum beams. Chapter 2 presents the modeling of the grid and the performance of parity relations when sensor noise or modeling errors are introduced. Chapter 3 presents three different techniques that can build a parity relation using only actuator inputs and sensor outputs. In chapter 4, based on the same data we try to build relations with a better detection effectiveness for a particular failure than parity relations.

Chapter 2
Parity Relations and the Space Structure

2.1 The Parity Relation Method

A parity relation is some linear combination of the present and passed inputs and outputs of a system which should be small (ideally zero) when the system is operating normally.

Consider a system of L sensors and M actuators. Its linear discretized model based on an N dimensional state vector is

$$\begin{cases} X(i+1) = \Phi X(i) + \Gamma U(i) \\ Y(i) = C X(i) \end{cases} \tag{2.1}$$

where i and $i+1$ represent consecutive sampling times.
Φ is a N×N matrix
Γ is a N×M matrix
C is a L×N matrix

The relations between the inputs and outputs over consecutive time steps and the state at time i are

$$\begin{aligned} Y(i) &= C X(i) \\ Y(i+1) &= C X(i+1) = C \Phi X(i) + C \Gamma U(i) \end{aligned}$$

...and so on.

Combining these relations over S time steps into a matrix equation, we get

$$\begin{bmatrix} Y(i) \\ Y(i+1) \\ Y(i+2) \\ \cdots \\ Y(i+S-1) \end{bmatrix} = \begin{bmatrix} C \\ C\Phi \\ C\Phi^2 \\ \cdots \\ C\Phi^{S-1} \end{bmatrix} X(i) + \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C\Gamma & 0 & \cdots & 0 \\ C\Phi\Gamma & C\Gamma & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ C\Phi^{S-2}\Gamma & C\Phi^{S-3}\Gamma & \cdots & C\Gamma \end{bmatrix} \begin{bmatrix} U(i) \\ U(i+1) \\ U(i+2) \\ \cdots \\ U(i+S-2) \end{bmatrix}$$

which will be noted

$$Y^*(i) = C^* X(i) + D^* U^*(i) \tag{2.2}$$

where C^* is a $(L\ S)\times N$ matrix
D^* is a $(L\ S)\times(M\ (S-1))$ matrix

To form a parity relation we want to eliminate the unknown state X . This is done by multiplying the above relation by a vector W satisfying $W^T C^* = 0$. In other words, W^T is an element of the left null space of C^*. Such a vector will exist only if S is greater than the rank of C^*.

The resulting equation is

$$r(i) = W^T Y^*(i) - W^T D^* U^*(i) = W^T C^* X(i) = 0 \tag{2.3}$$

r(i) is called the parity residual

$\begin{bmatrix} W \\ (W^T D^*)^T \end{bmatrix}$ is the parity vector

$\begin{bmatrix} Y^*(i) \\ -U^*(i) \end{bmatrix}$ will be called the parity information vector at time i

Equation (2.3) will hold as long as the system operates according to the linear model (2.1). If a sensor or an actuator fails, the relations between inputs and outputs are modified. Consequently the residuals based on the failed component will be non zero. With an appropriate set of residuals the identity of the failed component can be found. Of course any other perturbation such as unmodeled modes, uncertan parameters or noise will also produce a non zero value. The effectiveness of the relation will depend on the magnitudes of these different values.

2.2 Single Sensor and Single Actuator Relations

The possible sets of parity relations that can be used to detect and isolate failures have been studied in [3]. One of particular interest is the set made of single sensor and single actuator parity relations. A single sensor parity relation is based on all actuator inputs but only on one sensor output. Similarily a single actuator parity relation will depend on all sensors but on only one actuator. A sensor failure will then have an effect on all the single actuator residuals but on only one single sensor residual. And an actuator failure will affect all sensor residuals but only one actuator residual.

Let us consider here a system of 6 sensors and 6 actuators. Let R1 through R6 denote the single sensor residuals and R7 to R12 the single actuator residuals. The effect of a sensor or actuator failure on the residuals is represented in figure 2-0, where 0 denotes a zero value and 1 a non zero value.

Such relations can be generated as suggested in [3] by performing linear combinations on all the possible parity relations. If we assume C^* has full rank N, then taking S=N+1 in equation (2.2) will give us a L(N+1)-N dimensional null space of C^*. The W vectors spanning this null space generate a corresponding number of independent parity relations, each made of L(N+1) sensor coefficients and MN actuator coefficients. By combination of these relations we can eliminate the (L-1)(N+1) sensor coefficients for the single sensor relations and if M is equal or less than N the (M-1)N actuator coefficients for the single actuator relations.

The main drawback of this technique is the amount of computation required.

Failure of	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	R12
Sensor 1	1	0	0	0	0	0	1	1	1	1	1	1
Sensor 2	0	1	0	0	0	0	1	1	1	1	1	1
Sensor 3	0	0	1	0	0	0	1	1	1	1	1	1
Sensor 4	0	0	0	1	0	0	1	1	1	1	1	1
Sensor 5	0	0	0	0	1	0	1	1	1	1	1	1
Sensor 6	0	0	0	0	0	1	1	1	1	1	1	1
Actuator 1	1	1	1	1	1	1	1	0	0	0	0	0
Actuator 2	1	1	1	1	1	1	0	1	0	0	0	0
Actuator 3	1	1	1	1	1	1	0	0	1	0	0	0
Actuator 4	1	1	1	1	1	1	0	0	0	1	0	0
Actuator 5	1	1	1	1	1	1	0	0	0	0	1	0
Actuator 6	1	1	1	1	1	1	0	0	0	0	0	1

Figure 2-1: Value of the residuals when a failure is present

The number of operations needed to generate one set of relations is of the order of L^4N^3. For the small space structure that will be studied later in this thesis, where we have 6 sensors , 6 actuators and 20 states, this gives about 10 million operations.

In fact, single sensor parity relations can be generated very easily. Starting with equation (2.2) we can select the N+1 lines involving the sensor 1 output to get the following :

$$\begin{bmatrix} y_1(i) \\ y_1(i+1) \\ y_1(i+2) \\ \cdots \\ y_1(i+S-1) \end{bmatrix} = \begin{bmatrix} C_1 \\ C_1\Phi \\ C_1\Phi^2 \\ \cdots \\ C_1\Phi^{S-1} \end{bmatrix} X(i) + \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C_1\Gamma & 0 & \cdots & 0 \\ C_1\Phi\Gamma & C_1\Gamma & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ C_1\Phi^{S-2}\Gamma & C_1\Phi^{S-3}\Gamma & \cdots & C_1\Gamma \end{bmatrix} \begin{bmatrix} U(i) \\ U(i+1) \\ U(i+2) \\ \cdots \\ U(i+S-2) \end{bmatrix}$$

where C_1 is the first row of matrix C which will be noted

$$Y_1^*(i) = C_1^* X(i) + D_1^* U^*(i)$$

If we assume C_1^* has full rank, then taking S=N+1 will give us one null space vector W_1. The single sensor 1 relation will then be

$$r_1(i) = W_1^T Y_1^*(i) - W_1^T D_1^* U^*(i)$$

The same can be done for the other sensors.

It would be interesting to be able to use the same technique to generate single actuator relations. In fact this can be done after performing a few operations on equation (2.2).

Let D_r^* be D^* without the first row of null matrices. The relation involving D_r^* is then

$$\begin{bmatrix} Y(i+1) \\ Y(i+2) \\ \dots \\ Y(i+S-1) \end{bmatrix} = \begin{bmatrix} C\Phi \\ C\Phi^2 \\ \dots \\ C\Phi^{S-1} \end{bmatrix} X(i) + \begin{bmatrix} C\Gamma & 0 & \dots & 0 \\ C\Phi\Gamma & C\Gamma & \dots & 0 \\ \dots & \dots & \dots & \dots \\ C\Phi^{S-2}\Gamma & C\Phi^{S-3}\Gamma & \dots & C\Gamma \end{bmatrix} \begin{bmatrix} U(i) \\ U(i+1) \\ \dots \\ U(i+S-2) \end{bmatrix}$$

which will be noted

$$Y_r^*(i) = C_r^* X(i) + D_r^* U^*(i) \tag{2.4}$$

If we have L=M, that is if we consider only the outputs of as many sensors as actuators, then D_r^* is a square $L(S-1) \times L(S-1)$ matrix. Provided that $\det(D_r^*) \neq 0$, D_r^{-1} exists. Multiplying relation (2.4) by $D^{-1}{}_r$ gives

$$U_r^*(i) = -D_r^{-1} C^*{}_r X(i) + D_r^{-1} Y_r^*(i) \tag{2.5}$$

Equation (2.5) is the equivalent of (2.2) with U_r^* taking the place of Y^* and Y_r^* of

U^*. We can then select the S-1 lines involving actuator 1 input, and by taking S=N+2 generate a single actuator 1 parity relation.

The problem then reduces to the computation of D_r^{-1} and $D_r^{-1} C_r^*$. This would require an important amount of computation if D_r^* were just any ordinary matrix. However its pseudo triangular form will be helpful here.

First lets find when D_r^* can be inverted. Using the relation

$$\det \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \det(A).\det(D - C\,A^{-1}\,B)$$

gives $\det(D_r^*) = (\det(C\,\Gamma))^{S-1}$. Then D_r^{-1} exists if $\det(C\,\Gamma) \neq 0$.

Now find an expression for D_r^{-1}. Assuming it is pseudo triangular like D_r^* we have

$$D_r^{-1} = \begin{bmatrix} X_0 & 0 & \cdots & 0 \\ X_1 & X_0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ X_{S-2} & X_{S-3} & \cdots & X_0 \end{bmatrix}$$

Solving $D_r^{-1}\,D_r^* = I_{M(S-1)}$ gives us the relations

$$X_0\,(\,C\,\Gamma\,) = I_M$$

$$X_1\,(\,C\,\Gamma\,) + X_0\,(\,C\,\Phi\,\Gamma\,) = 0$$

$$X_2\,(\,C\,\Gamma\,) + X_1\,(\,C\,\Phi\,\Gamma\,) + X_0\,(\,C\,\Phi^2\,\Gamma\,) = 0$$

... and so on.

Which leads to the solution

$$X_0 = (C \Gamma)^{-1}$$

$$X_1 = - X_0 C \Phi \Gamma (C \Gamma)^{-1}$$

$$X_2 = - (X_1 C \Phi \Gamma + X_0 C \Phi^2 \Gamma) (C \Gamma)^{-1}$$

$$X_3 = - (X_2 C \Phi \Gamma + X_1 C \Phi^2 \Gamma + X_0 C \Phi^3 \Gamma) (C \Gamma)^{-1}$$

... and so on

For implementation on a computer, the generation of the matrices X_n can be programmed recursively. Starting with

$$X_0 = (C \Gamma)^{-1} \ , \ XX_0 = (C \Gamma)^{-1} C \Phi \quad \text{and} \quad G = \Gamma (C \Gamma)^{-1}$$

we get from step n-1 to step n with the relations

$$\begin{cases} X_n & = - (XX_{n-1}) G \\ XX_n & = (X_n C + XX_{n-1}) \Phi \end{cases}$$

This also gives us the result of $D_r^{-1} C_r^*$ as we have

$$D^{-1}_r C^*_r = \begin{bmatrix} X_0 C \Phi \\ X_1 C \Phi + X_0 C \Phi^2 \\ X_2 C \Phi + X_1 C \Phi^2 + X_0 C \Phi^3 \\ \cdots \end{bmatrix} = \begin{bmatrix} XX_0 \\ XX_1 \\ XX_2 \\ \cdots \end{bmatrix}$$

The number of operations required by this technique is approximately $4M^2N^2+2MN^3$. Consequently this represents a dramatic improvement for a large space structure where M and N are large. In the case of our small structure the number of operations required to generate a set of actuator relations now reduces to about 150000 from 10 million.

Another advantage is that now the single sensor and actuator relations are generated from an identical matrix relation. In each case the W vector is made of the coefficients of the single element. As we will see this is very useful when we want a relation to be more sensitive to a particular failure.

2.3 Modeling of the Space Structure

A model representing a space structure was created to study the effectiveness of parity relations. This model represents an experimental apparatus used at NASA Langley Research Center to demonstrate control techniques for large space structures. The structure is a grid of aluminium beams controlled by 6 inertia wheel actuators. Position sensors and rate gyros measure the vibrations of the structure in the direction perpendicular to the grid.

The model is a state space representation based on the 10 lowest frequency modes and coresponding mode shapes and damping coefficients.

Freq. ω in rd/s	2.33	4.05	9.04	13.9	19.5	30.0	36.0	37.2	46.7	65.1
Damping coef. ξ	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01

Table 2-I: Frequencies and damping coefficients of the model

The state vector is made of the 10 modal amplitudes and 10 modal velocities.

$$X^T = [\, \psi_1 \;\; \psi_1 \;\; \psi_2 \;\; \psi_2 \;\; \cdots \;\; \psi_{10} \;\; \psi_{10} \,]$$

The input vector is made of the torques of the 6 actuators.

$$U^T = [\, u_1 \;\; u_2 \;\; u_3 \;\; u_4 \;\; u_5 \;\; u_6 \,]$$

The model takes into account the output of 3 position sensors and 3 rate gyros.

$$Y^T = [\, y_1 \;\; y_2 \;\; y_3 \;\; y_4 \;\; y_5 \;\; y_6 \,]$$

The matrices relating X,U and Y in the continuous case will be noted

$$\begin{cases} \dot{X}(t) = A\,X(t) + B\,U(t) \\ Y(t) = C\,X(t) \end{cases}$$

Let ω_i be the frequency of mode i and ξ_i its damping coefficient. Let p_{ij}, g_{ik} and a_{il} be the modal coefficients of mode i at position sensor location j, gyro location k and actuator location l. Then as shown in [3] we have

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ -\omega^2{}_1 & -2\xi_1\omega_1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & -\omega^2{}_2 & -2\xi_2\omega_2 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & 0 & \dots & -\omega^2{}_{10} & -2\xi_{10}\omega_{10} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & \dots & 0 \\ a_{1.1} & a_{1.2} & \dots & a_{1.6} \\ 0 & 0 & \dots & 0 \\ a_{2.1} & a_{2.2} & \dots & a_{2.6} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \\ a_{10.1} & a_{10.2} & \dots & a_{10.6} \end{bmatrix}$$

And

$$C = \begin{bmatrix} p_{1.1} & 0 & p_{2.1} & 0 & \dots & p_{10.1} & 0 \\ p_{1.2} & 0 & p_{2.2} & 0 & \dots & p_{10.2} & 0 \\ p_{1.3} & 0 & p_{2.3} & 0 & \dots & p_{10.3} & 0 \\ 0 & g_{1.1} & 0 & g_{2.1} & \dots & 0 & g_{10.1} \\ 0 & g_{1.2} & 0 & g_{2.2} & \dots & 0 & g_{10.2} \\ 0 & g_{1.3} & 0 & g_{2.3} & \dots & 0 & g_{10.3} \end{bmatrix}$$

The use of parity relations requires that a discrete model of the structure be obtained. Let T be the sampling time, then our model becomes

$$\begin{aligned} X(i+1) &= \Phi X(i) + \Gamma U(i) \\ Y(i) &= C X(i) \end{aligned}$$

with $\Phi = e^{A T} = I + A T + A^2 T^2/2 + \ldots$

and $\Gamma = [\int_0^T \Phi(t)\, dt] B = I T + A T^2/2 + A^2 T^3/3! + \ldots$

2.4 Parameters for the Simulations

The results of simulations involving parity relations will be highly dependent on the level of input and output signals and sensor noise. The ranges allowed on the experimental apparatus are 20 ounce-feet for the actuators and 1 inch for the position sensors. To simulate the constant movement of the structure we want to apply a random input to our model. But has the structure as very small damping, the level of torques we can apply without exceeding the range of 1 inch is very small. To be able to apply torques of higher level a state feedback controller was introduced. The input signal was then the response of the controlled system plus a small random signal.

Sensor noise can be expected to produce an error of 10^{-4} to 10^{-3} feet on the position sensors and 10^{-4} to 10^{-3} rd/s on the rate gyros. The two extremes were simulated by adding uniformly distributed errors between 10^{-4} feet and 10^{-4} rd/s or 10^{-3} feet and 10^{-3} rd/s.

Finally, we want to have a constant level of output in the simulations. To do this, state vectors were generated over 1000 time steps starting from X=0 . The last vector was then stored and used as the initial state of the simulation.

2.5 Simulation of Parity Relations and the Effect of Noise

Single sensor and single actuator parity relations were generated for the model of the grid using the techniques described previously. They are composed respectively of 146 and 147 coefficients. The sampling time was taken to be 0.10 sec.

But we still have one degree of freedom in the choice of our relations which corresponds to the magnitude of the null space vector W. In order to get comparable relations we will require that the norm of W be such that the effect of sensor noise on the residuals is the same for all relations. This effect is measured through the covariance of the residuals when no failure is present. Let n_i be the noise signal of sensor i and N the noise vector coresponding to Y^*.

$$N^T(i) = [\ n_1(i) \dots n_6(i)\ \ n_1(i+1) \dots n_6(i+1) \dots n_1(i+S-1) \dots n_6(i+S-1)\ ]$$

Our relation (2.3) becomes with noise

$$r(i) \ = \ W^T(Y^*(i) + N(i)) \ - \ W^T D^* \ U^*(i) \ = \ W^T N(i) \tag{2.6}$$

Then the covariance of the residuals is

$$E(r^2(i)) \ = \ W^T E(\ N(i)\, N^T(i)\ )\ W \ = \ \textstyle\sum_j \ w_j^{\,2}\, E(n_j^{\,2}(i))$$

Here w_j and $n_j(i)$ are the j^{th} components of W and N(i).

Let $||W||_N = [\ \sum_j \ E(n_j^{\,2}(i))\ w_j^{\,2}\ ]^{1/2}$ be the N norm of W. Then choosing $||W||_N = 1$ or any other constant will give us the same level of residuals in the no-fail case.

As in our case all the $E(n_j^{\,2}(i))$ have the same value q, $||\ ||_N$ is the euclidian norm. Then choosing $||W|| = 1$ for all the relations will give

$$E(r^2(i)) = \|W\|^2 q = q \qquad (2.7)$$

Simulations involving these relations with the model of the structure were carried out over 150 time steps. Two failures were introduced :

A "zero" failure of sensor 1 ($y_1(i)=0$) from time step 40 to 80
A "zero" failure of actuator 1 ($u_1(i)=0$) from time step 110 to 150

The first simulation is the ideal case where the sensor outputs are not affected by noise. Thus the residuals are zero when no failure is present or if the failure does not affect the residual, as is indicated in figure 2-0. Four of the residuals are presented in figure 2-2 to 2-5. R1 and R7 are the two residuals affected by both failures. R2 is one of the single sensor residuals that should not be affected by sensor 1 failure and R8 is its equivalent for actuator 1 failure. The two important failure signatures are the effect of sensor 1 failure on R1 and actuator 1 failure on R7. These are the ones that enable us to isolate the failed component. We can already see that sensor failures are easier to detect as their signature is much larger than the actuator one. The residuals tend to grow at the end of the simulation because the system response level grows after failure of actuator 1 due to poorer performance of the control system.

In the second simulation, sensor noise of 10^{-4} feet and 10^{-4} rd/s was introduced. The effect on residual R1 and R7 is shown in figures 2-6 and 2-7. The perturbation due to this level of noise is small.

Finally sensor noise of 10^{-3} feet and 10^{-3} rd/s was introduced. This corresponds to figure 2-8 and 2-9. R1 still allows us to detect the sensor failure. However for R7 the noise on the residuals covers the actuator failure signature and detection is impossible.

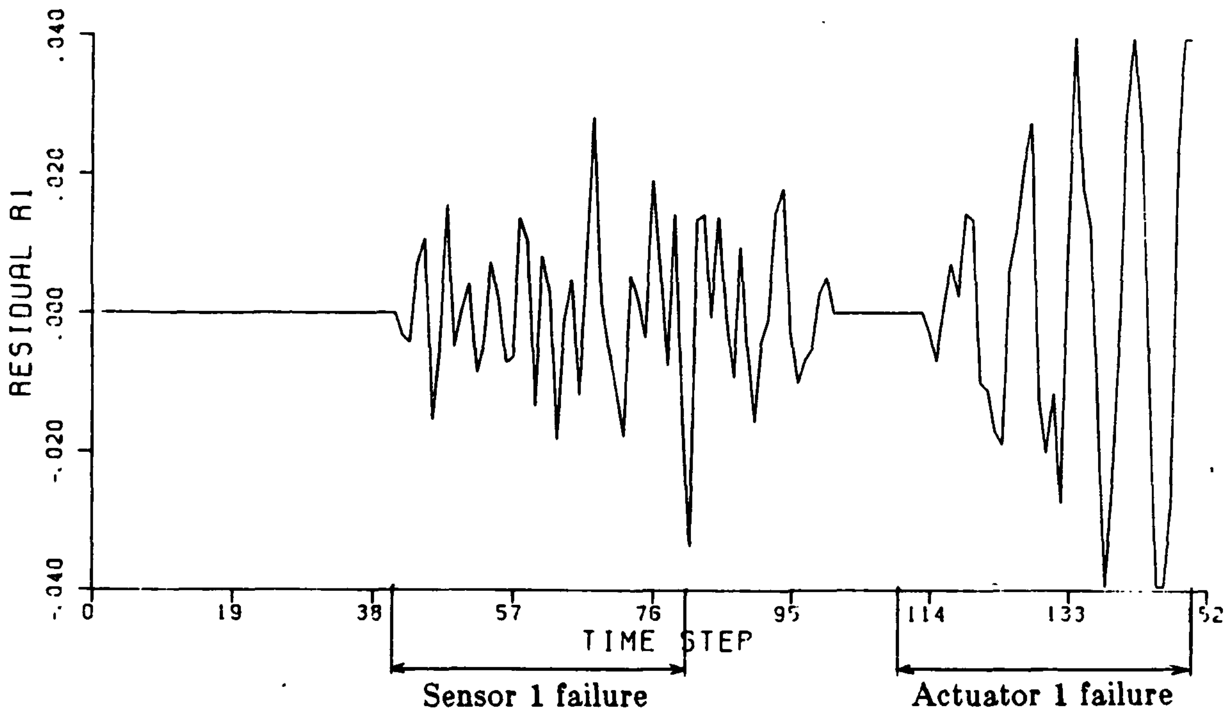


Figure 2-2: Residual R1 without sensor noise

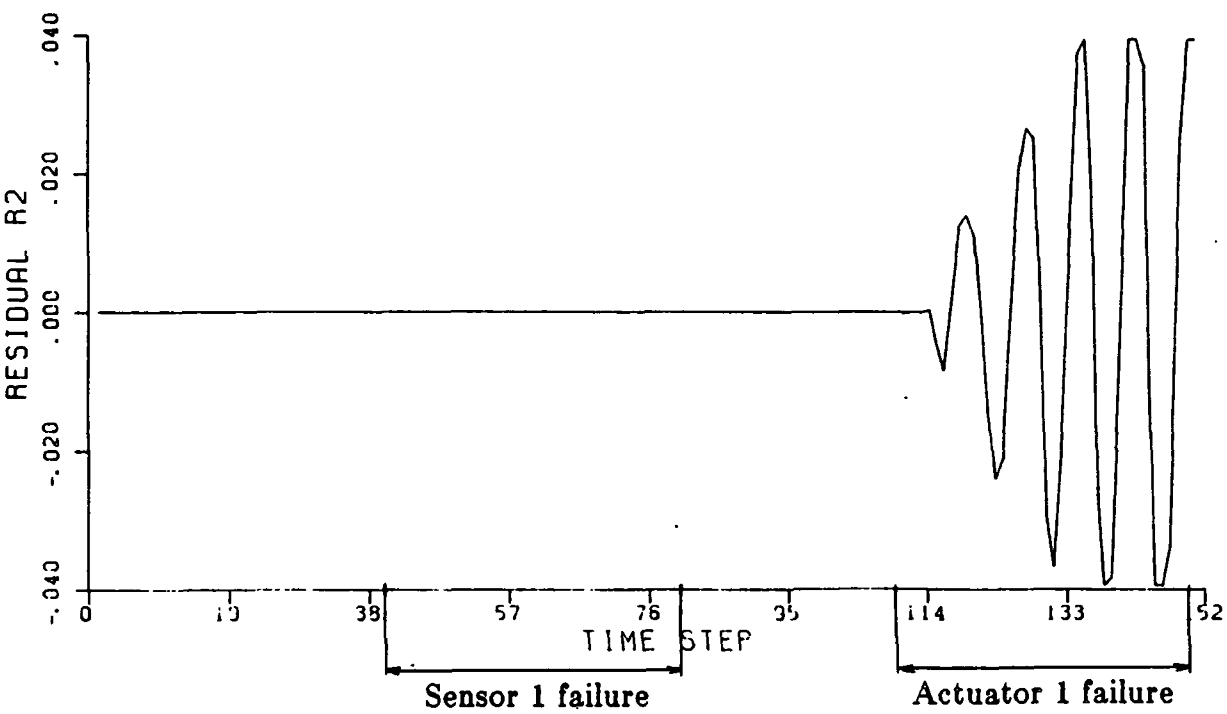


Figure 2-3: Residual R2 without sensor noise

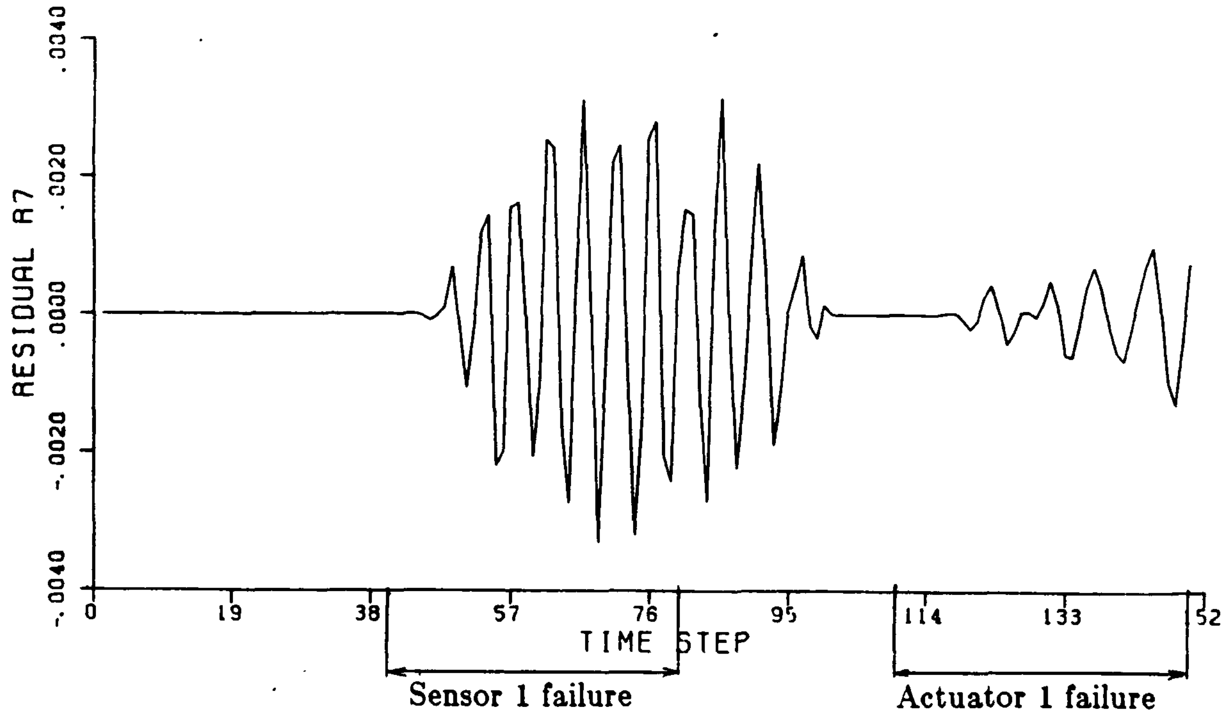


Figure 2-4: Residual R7 without sensor noise

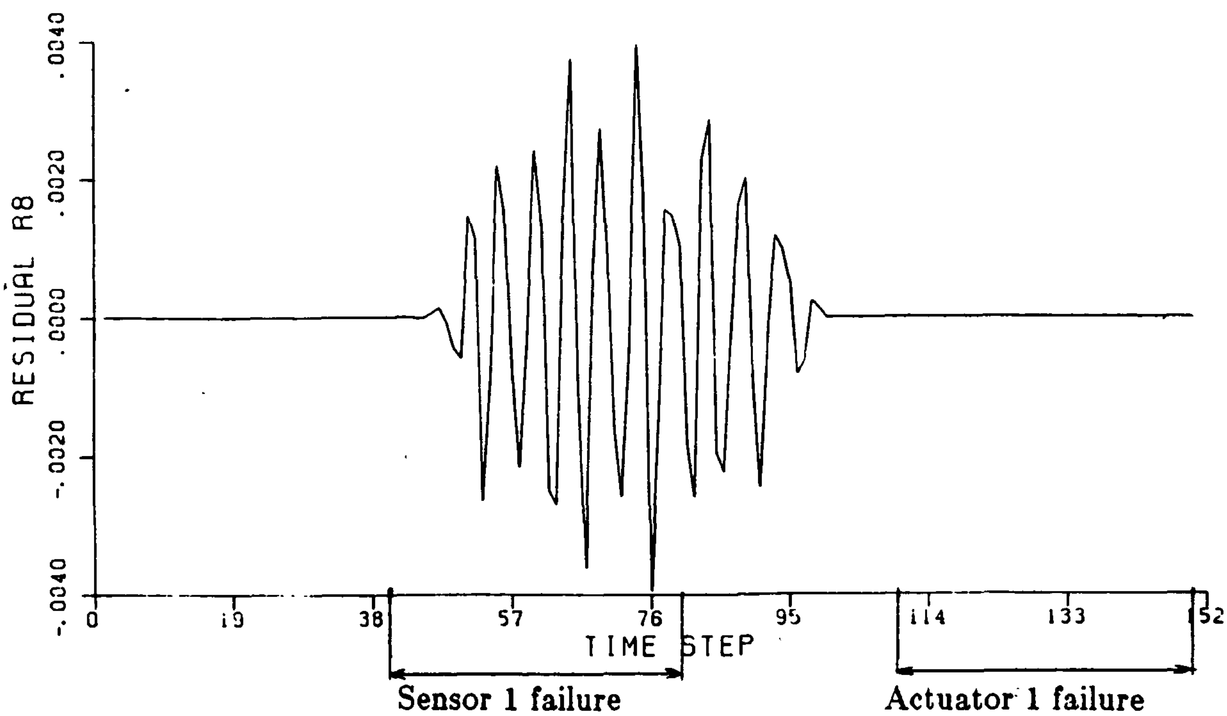


Figure 2-5: Residual R8 without sensor noise

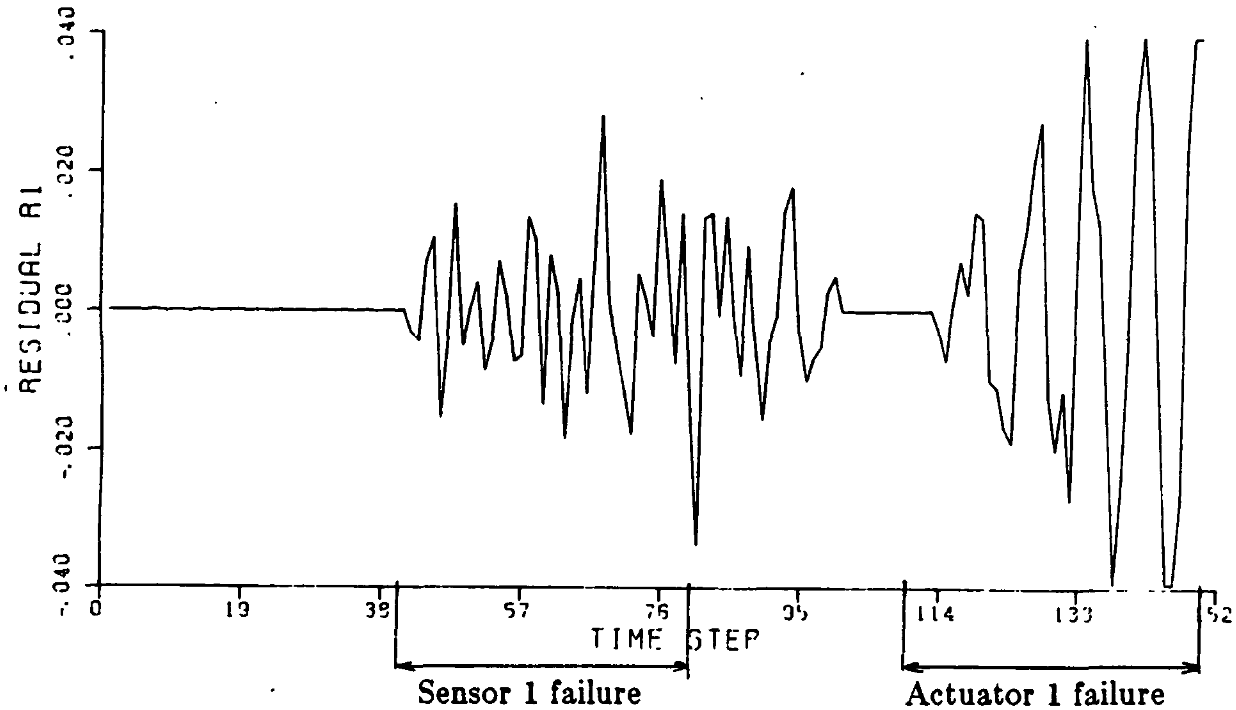


Figure 2-6: Residual R1 with sensor noise of 10^{-4} ft and 10^{-4} rd/s

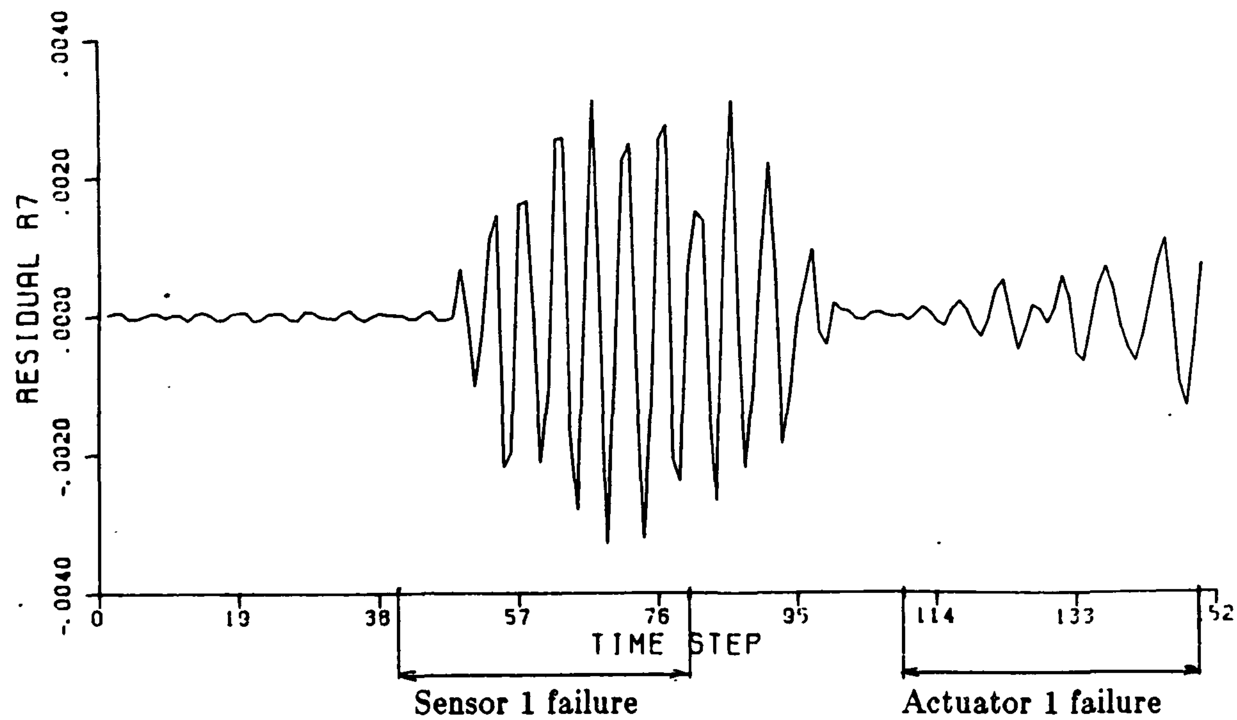


Figure 2-7: Residual R7 with sensor noise of 10^{-4} ft and 10^{-4} rd/s

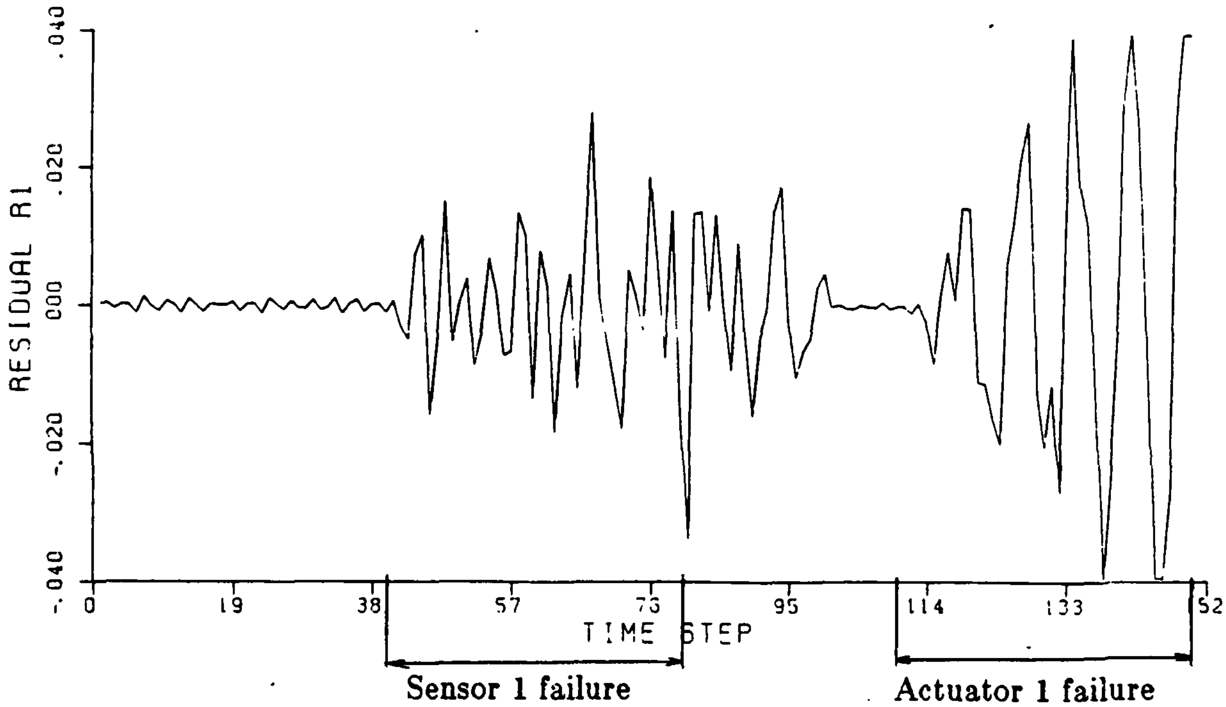


Figure 2-8: Residual R1 with sensor noise of 10^{-3} ft and 10^{-3} rd/s

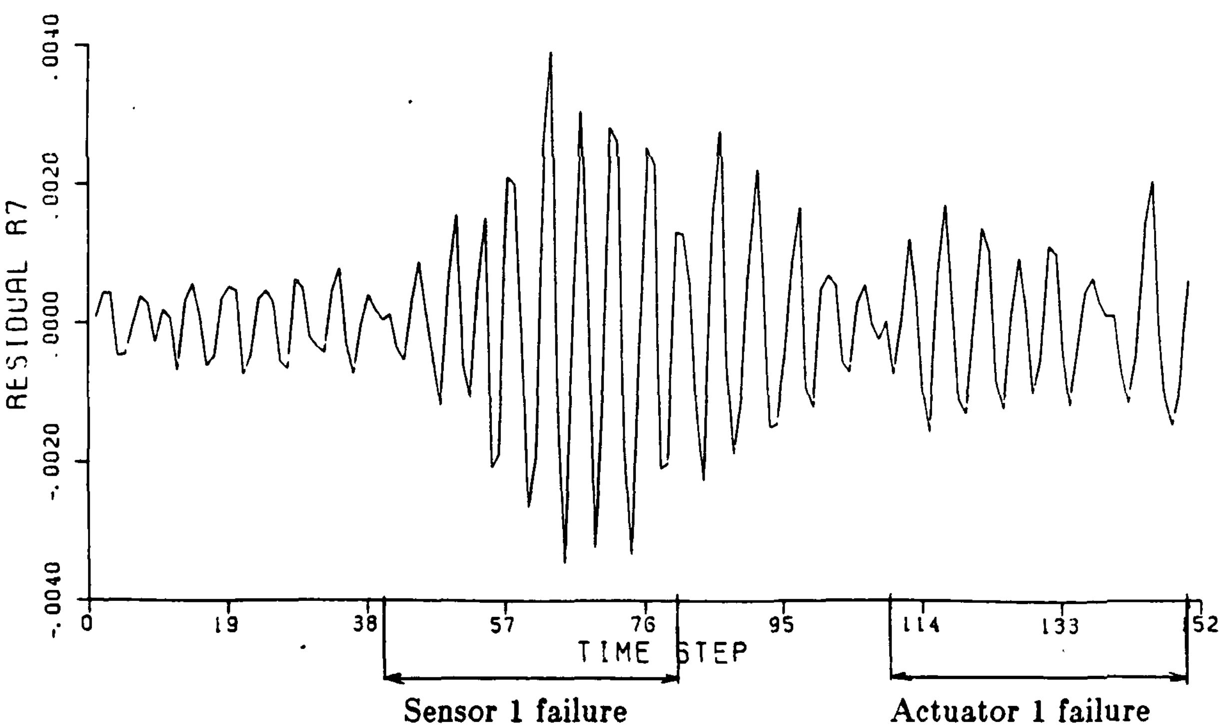


Figure 2-9: Residual R7 with sensor noise of 10^{-3} ft and 10^{-3} rd/s

2.6 Simulation with Parameter Mismatch

Having seen the performance of parity relations generated using the accurate parameters of the structure, it is now interesting to see how uncertainties in the knowledge of these parameters can affect the residuals.

Uncertainties were created using one model to generate the relations and another slighty different for the simulation. The mismatches were created using the law

$$p_m = p\left(1 + \frac{\Delta}{100} r\right)$$

where p_m is the mismatched parameter
p is the original parameter
Δ is the percentage of mismatch
r is a random number uniformely distributed between -1 and +1

This law was applied to the frequencies and mode shape coefficients in the continuous case. The model was then discretized.

The easiest way to carry out these simulations would have been to use the mismatched models with the original set of parity relations. However the result would biased by the fact that the mismatch also affects the controller. To avoid this a set of relations was generated for each mismatched model and used with the true model of the structure.

Four models were generated. Two corresponding to a 5% mismatch and two for 10% . The worst case for 5% is presented in figures 2-10 to 2-13. We can see that with R1 the sensor 1 failure can be detected. The actuator failure however cannot be detected as its effect on R7 is covered by the effect of the mismatch. As for the worst 10% case presented in figures 2-14 to 2-17, no failure can be detected.

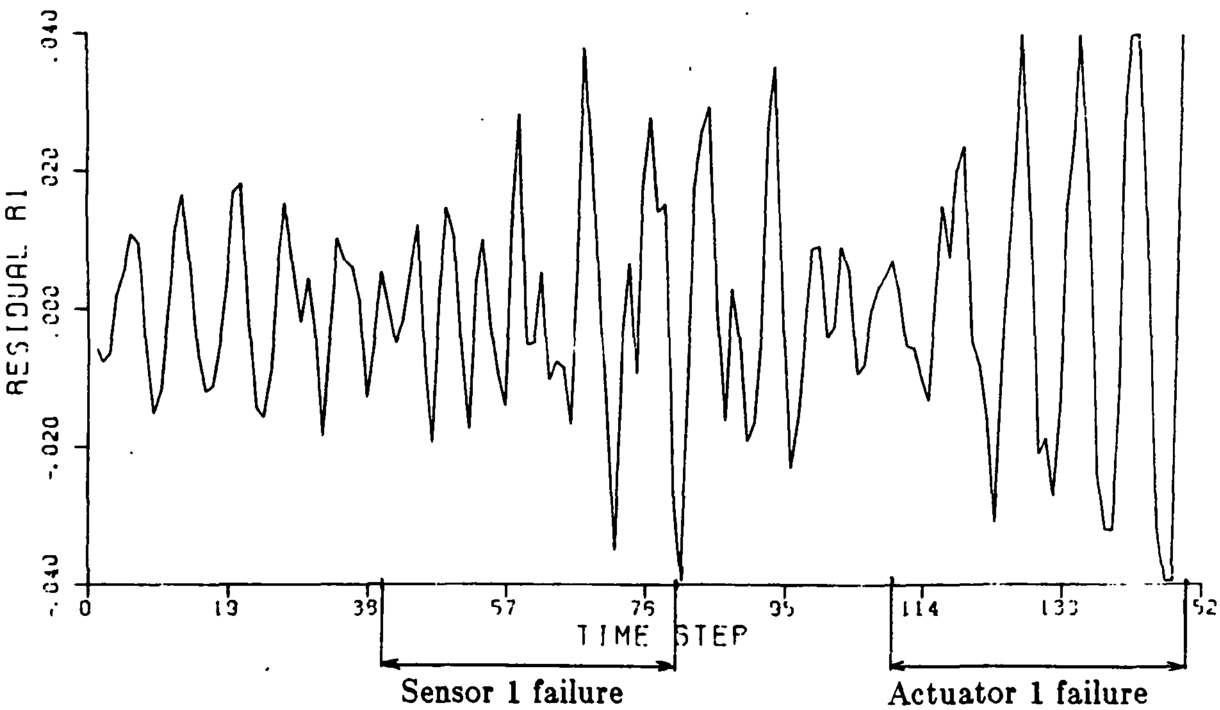


Figure 2-10: Residual R1 with 5% parameter mismatch

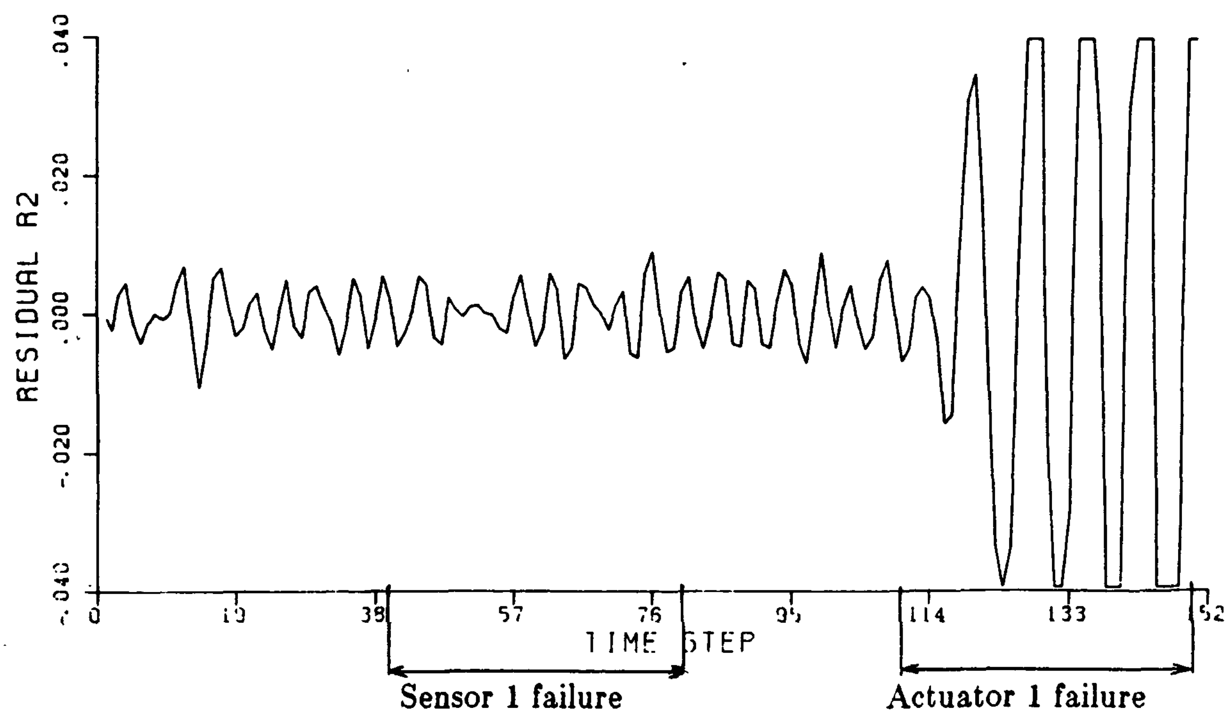


Figure 2-11: Residual R2 with 5% parameter mismatch

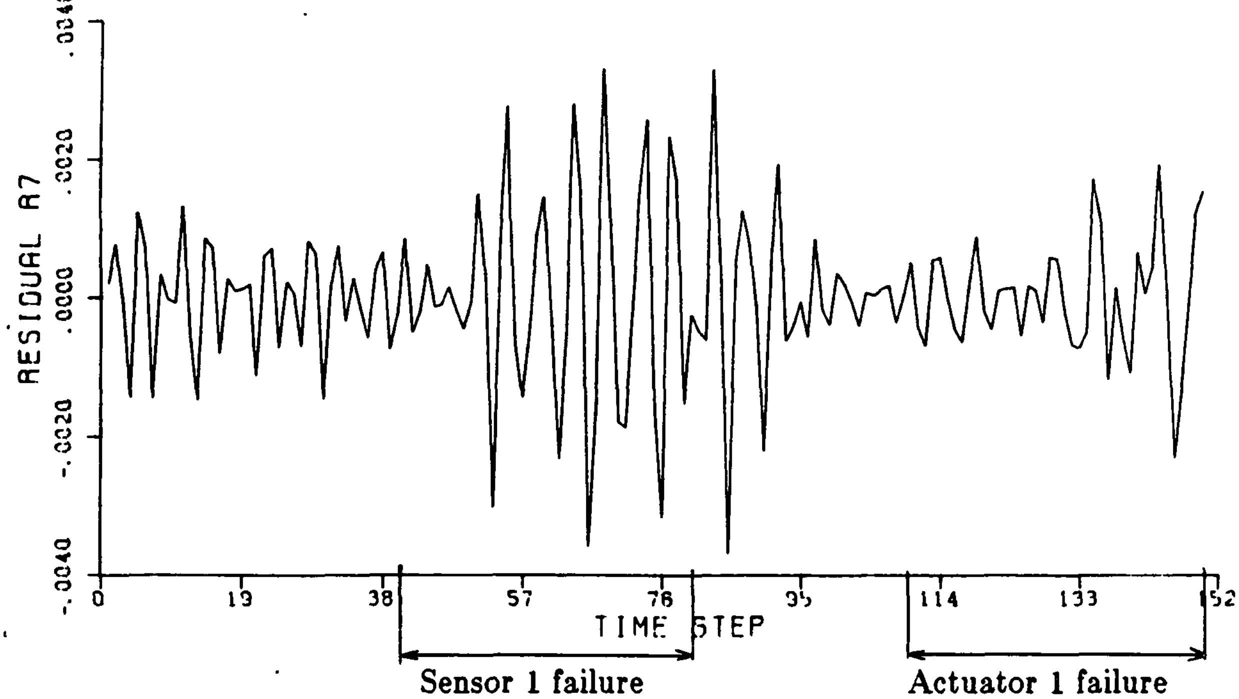


Figure 2-12: Residual R7 with 5% parameter mismatch

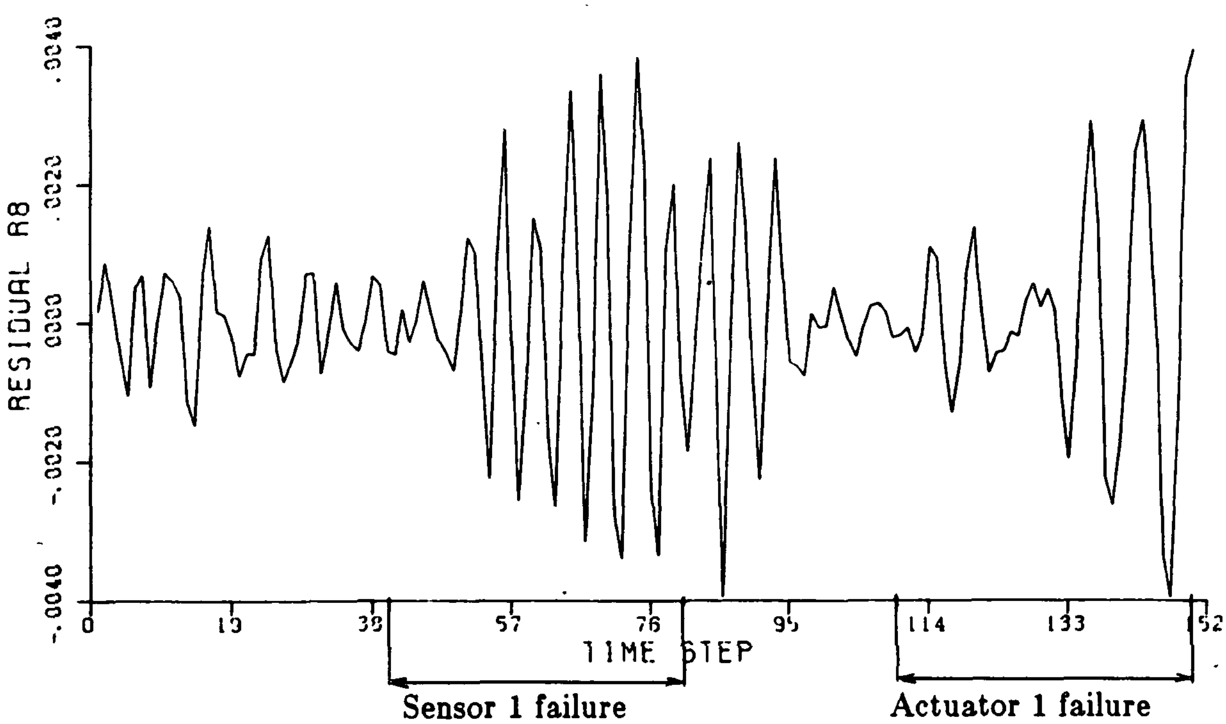


Figure 2-13: Residual R8 with 5% parameter mismatch

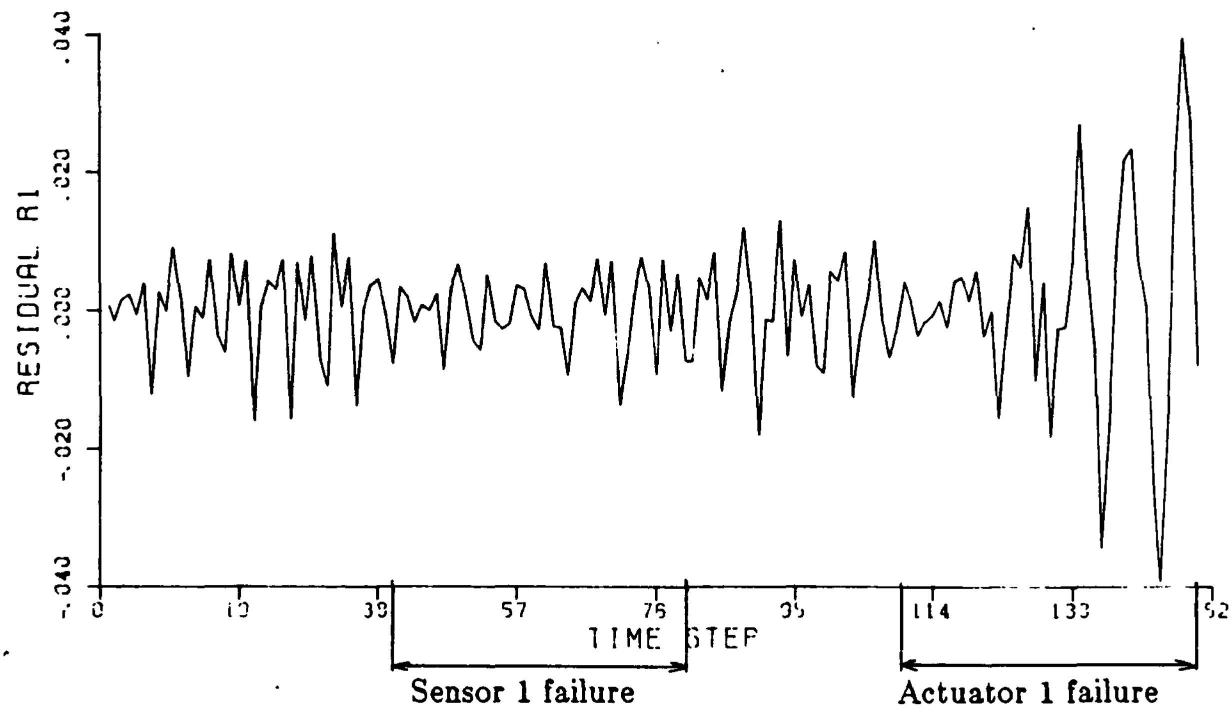


Figure 2-14: Residual R1 with 10% parameter mismatch

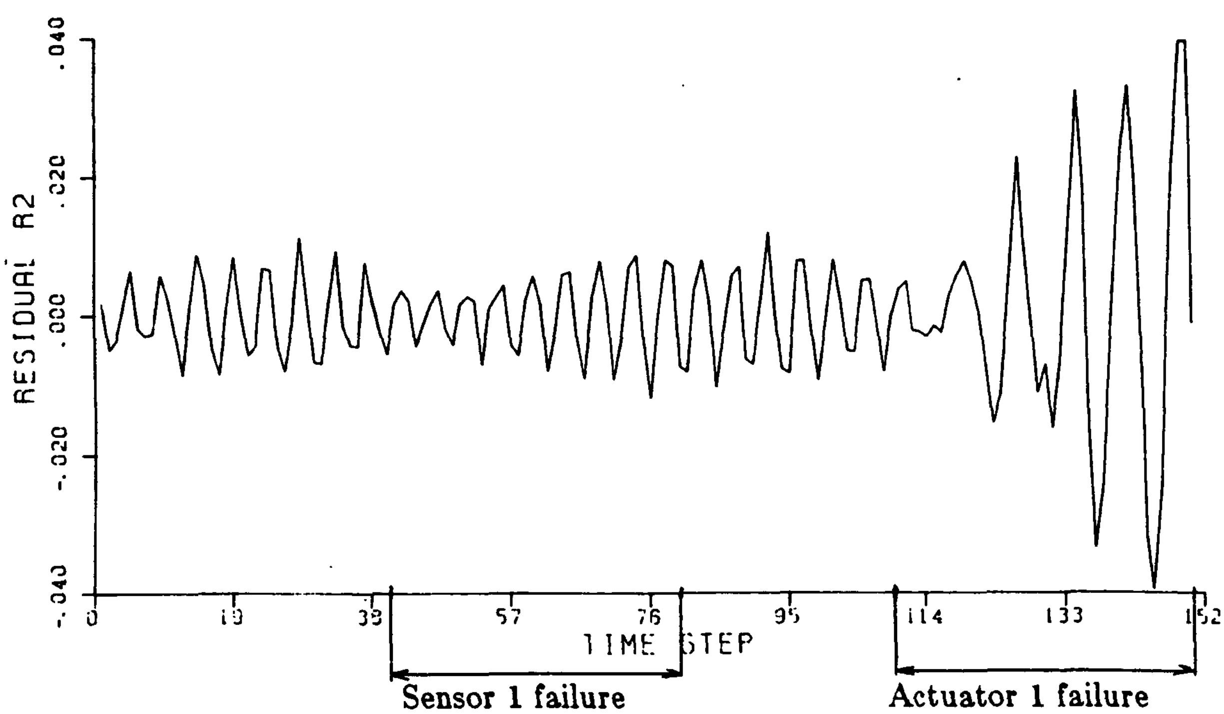


Figure 2-15: Residual R2 with 10% parameter mismatch

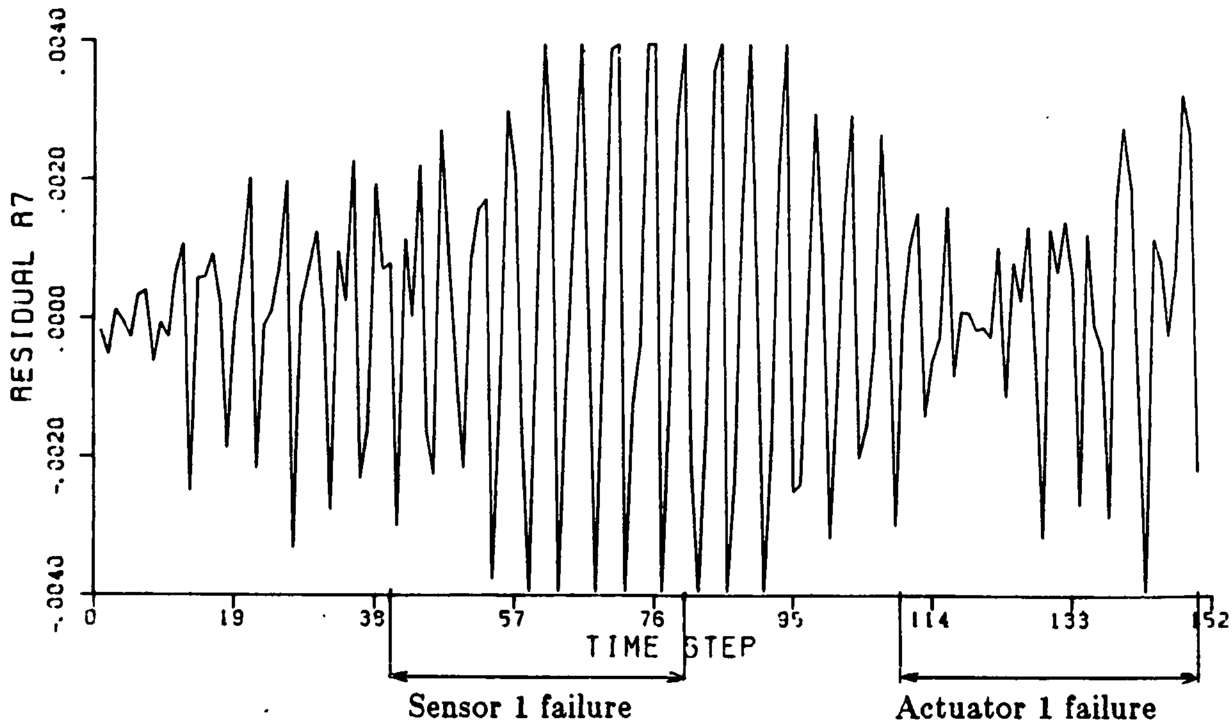


Figure 2-16: Residual R7 with 10% parameter mismatch

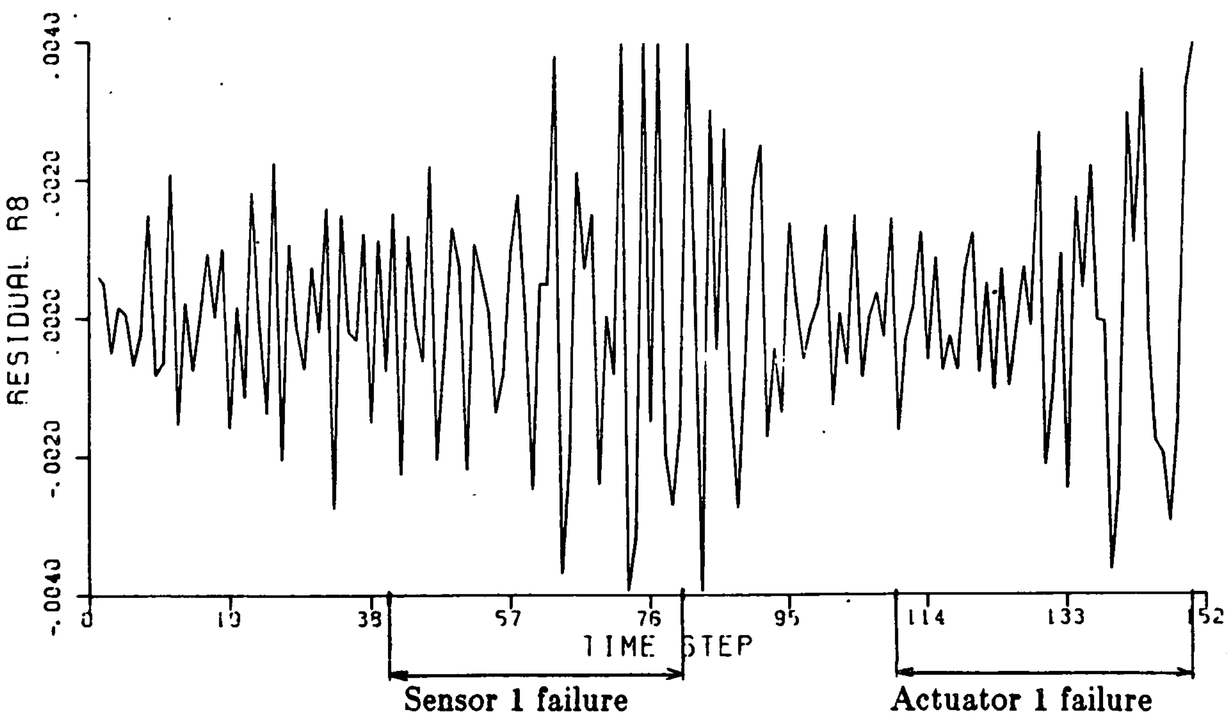


Figure 2-17: Residual R8 with 10% parameter mismatch

Chapter 3
Estimation of Parity Relations

3.1 Discussion of the Estimation Algorithm

We have seen that an error of 5% in the knowledge of the parameters of the structure will give us useless parity relations for the detection and isolation of actuator failures. Much greater uncertainties can be expected in the a priori knowledge of the parameters. Consequently a reestimation of the coefficients of each relation will have to be performed once the structure is built in space.

This could be done by running an on line identification of the parameters of the structure and using this data to generate parity relations. Another solution is to directly reestimate the coefficients of the relations through an on line identification. Assuming that the structure will not suffer any failure at the beginning of its life, we will then have available the inputs and outputs in the no-fail case. Based on this data the estimation algorithm will try to find the relation that gives us the smallest residual.

As R7, and more generally, the single actuator residuals are the most affected by mismatch and noise, only the estimation of the single actuator 1 parity relation will be considered in this chapter. The algorithms are however valid for any parity relation.

3.2 Kalman Filter Estimation

A parity relation can be estimated using the Kalman filter algorithm. Let the coefficients of the desired parity relation P_r be the state of the noise free stationary process

$$P_r(i+1) = P_r(i) \tag{3.1}$$

i being the time step of the estimation.

Under the no-failure hypothesis this relation should give us a zero residual in the noise-free case, and a noisy residual if some sensor noise is considered. This residual can be considered as a measurement of the process (3.1). Let n be the noise on the residual, n = W N according to (2.6). A measurement equation can be written as

$$y(i) = P_r^T P_i(i) - n = r_r(i) - n$$

where y is equal to 0 and n has covariance q as shown in (2.7).

Let P_e be our estimate of the desired parity relation P_r. The Kalman filter equations are

$$\text{Measurement incorporation} \left\{ \begin{array}{ll} K(i) & = P^-(i)\, P_i(i)\, [\, P_i^T(i)\, P^-(i)\, P_i(i) + q\,]^{-1} \\ P_e^+(i) & = P_e^-(i) + K(i)\, [\, 0 - P_e^{T-}(i)\, P_i(i)\,] \\ P^+(i) & = P^-(i) - K(i)\, P_i^T(i)\, P^-(i) \end{array} \right.$$

$$\text{Time update} \left\{ \begin{array}{ll} P_e^-(i+1) & = P_e^+(i) \\ P^-(i+1) & = P^+(i) \end{array} \right.$$

where P is the estimation error covariance and K the Kalman filter gain.

However these equations could lead to the trivial solution $P_e = 0$. To avoid this we will require that the norm of the sensor coefficients of P_e be kept equal to 1 as in (2.7). This will be done by changing our process equation to a norm taking equation

$$P_r(i+1) = P_r(i) \;/\; \mathrm{norm}(i)$$

which gives the corresponding time update equations

$$\begin{array}{ll} \text{Time} & \\ \text{update} & \end{array} \left\{ \begin{array}{lcl} P_e^-(i+1) & = & P_e^+(i) \;/\; \mathrm{norm}(i) \\ P^-(i+1) & = & P^+(i) \;/\; \mathrm{norm}^2(i) \end{array} \right.$$

where norm(i) is the norm of the sensor coefficients of $P_e(i)$.

An estimation using this algorithm was carried out over 200 time steps. The initial parity coefficients are those of the 5% missmatch case presented in figure 2-12. No sensor noise was introduced. However q was set to .33 10^{-8} which corresponds to sensor noise of the 10^{-4} case. Figure 3-1 shows the estimated residual $r_e = P_e^T(i)\, P_i(i)$ over the estimation sequence. Figure 3-2 shows the residuals given by the initial coefficients and the same information vector. The comparison of both allows us to visualize the effect of the filter. In fact it shows that the filter reduces the residual due to model mismatch, but makes no more progress after 50 time steps.

The last estimated relation was then used in a simulation identical to the ones performed in chapter 2. The result is given in figure 3-3. As far as the Kalman filter is concerned the results are what we expected. The residuals in the no-fail case were reduced to a level slightly higher than the .33 10^{-8} level given by the parameter q. The problem is that the resulting parity relation is useless as the actuator failure signature has almost disappeared. To get a relation that will still

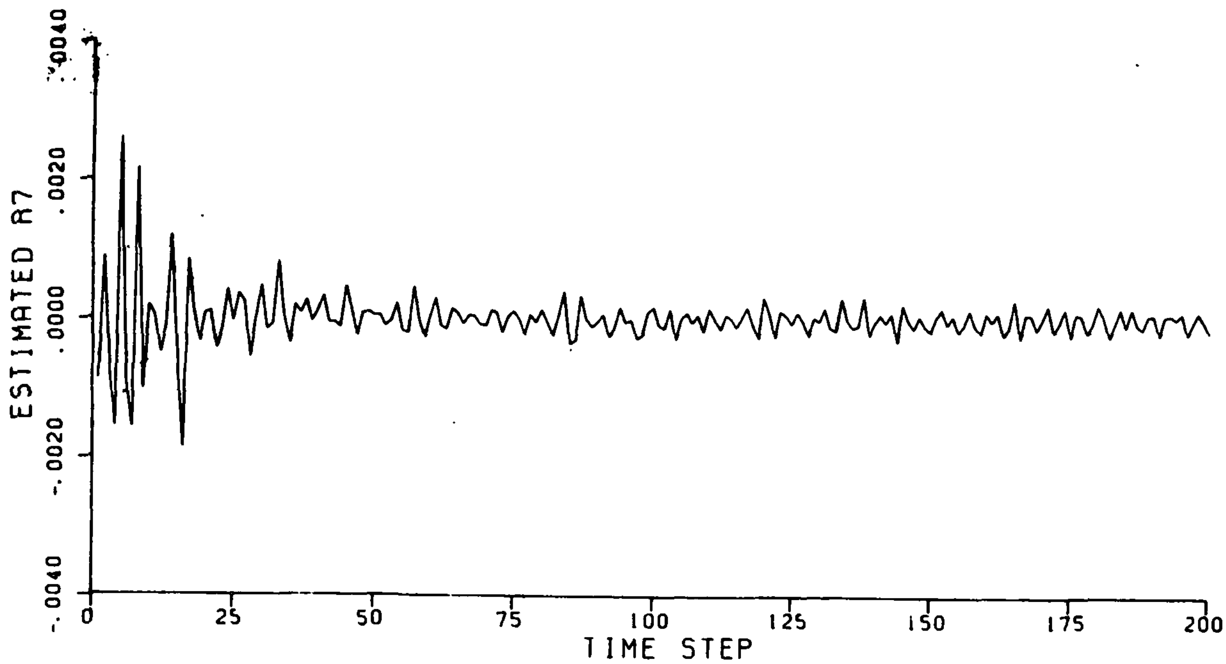


Figure 3-1: Residual estimated by the Kalman filter

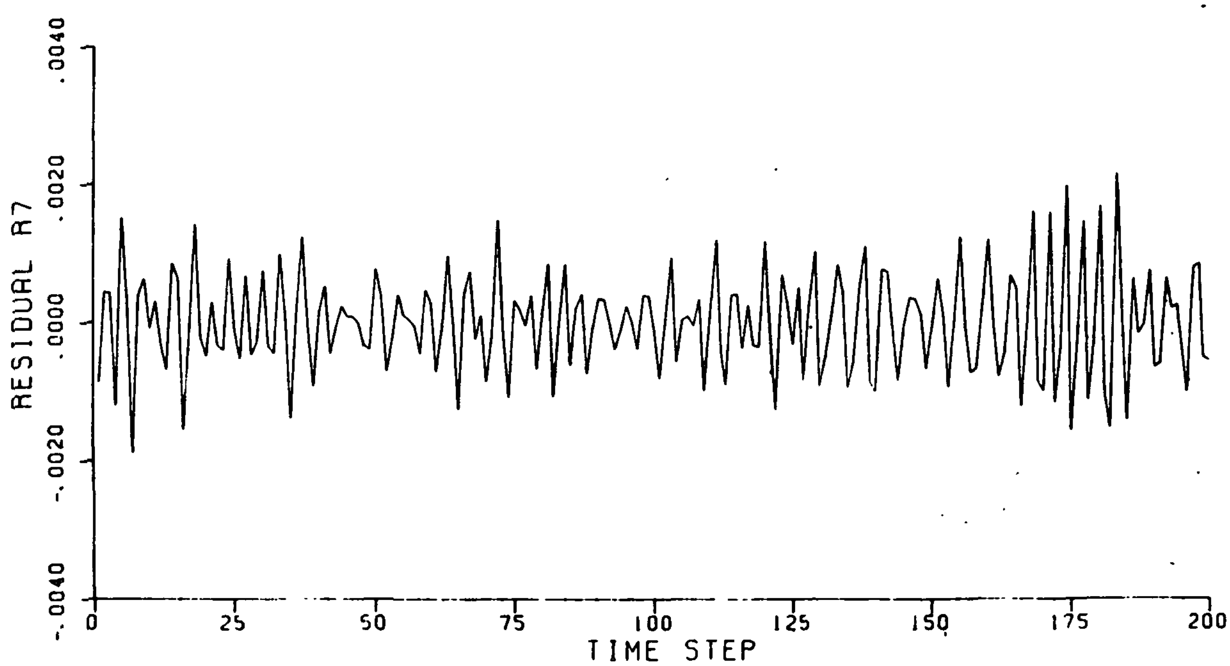


Figure 3-2: Residual of the 5% mismatch case

show the effect of the actuator failure we must estimate the coefficients more accurately. But this can only be done if the sensor noise level is smaller than that considered here.

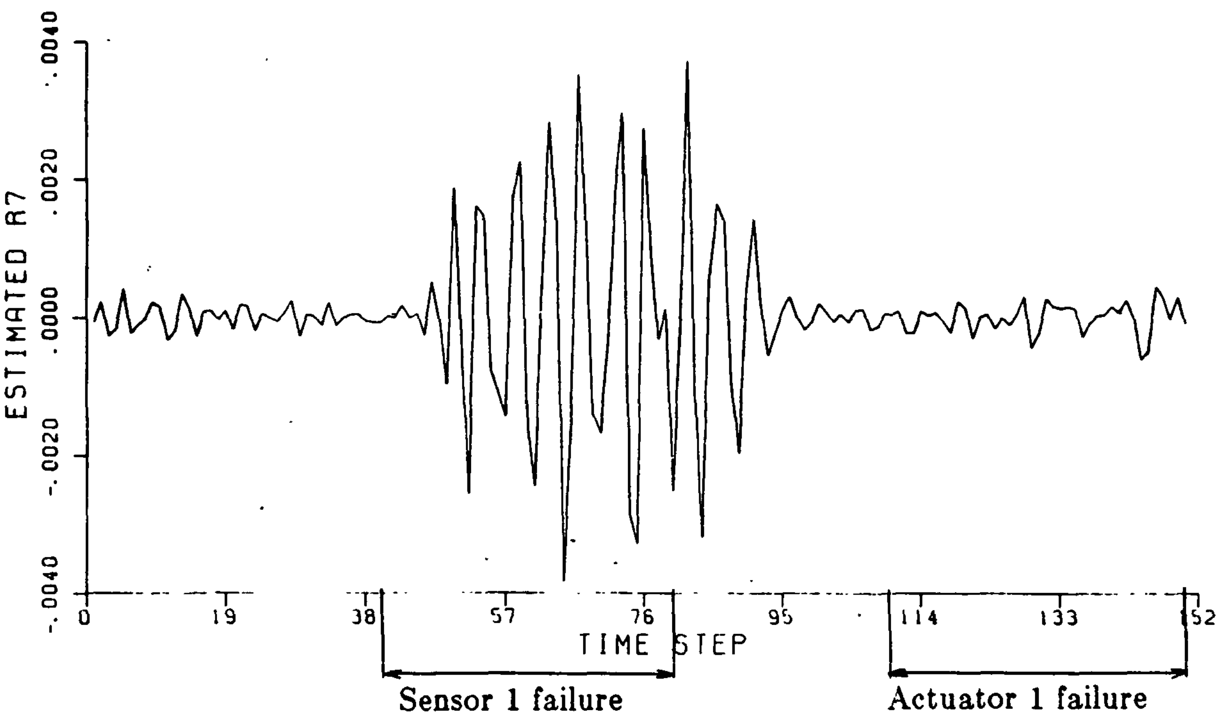


Figure 3-3: Relation estimated with the Kalman filter

Two other drawbacks of this technique are the computation time and amount of memory required by this algorithm. The number of operations per time step is $3R^2$ where R is the number of coefficients of the parity relation. In our case with 147 coefficient this gives around 65000 operations. This is very likely to be above the capacity of a space based computer. Hopefully we need not use consecutive time steps in our estimation. We could for example take only one parity information vector every 10 time step thus reducing by 10 the computation power required. The algorithm also requires the storage of ${}^1/_2N^2$ numbers for the covariance matrix P. This might be a major problem as parity relations for large space structures will be made of hundreds of coefficients.

3.3 Estimation through Minimization of a Structural Distance

We have seen that the Kalman filter algorithm requires an important amount of memory which might not be available on a space based computer. This is why developing an algorithm requiring less storage capacity would be of some interest. The minimization of a structural distance is one such algorithm. This technique is described in [4]. The basic points will be presented here.

Let P_e be our estimated parity relation and r_e the resulting residual. Let P_r and r_r be the desired relation and residual. We have

$$r_e(i) = P_e^T(i)\, P_i(i)$$

$$r_r(i) = P_r^T(i)\, P_i(i)$$

where $r_r(i) = 0$ in the no-fail case.

We want to minimize the distance $D(i)$

$$D(i) \;=\; [P_e(i) - P_r(i)]^T\, P\, [P_e(i) - P_r(i)]$$

where P is a symmetric weighing matrix.

Let $A(i) = P_e(i) - P_r(i)$. We have

$$D(i+1) - D(i) = [A(i+1) - A(i)]^T P\, [A(i+1) - A(i)] + 2\, A^T(i)\, P\, [A(i+1) - A(i)]$$

As $A(i)$ is unknown, we impose that it follows

$$A(i+1) - A(i) \;=\; h\, P^{-1}\, P_i(i)$$

where h is a scalar.

Then we have

$$D(i+1) - D(i) = h\,[\,h\,P_i^T(i)\,P^{-1}\,P_i(i) + 2\,P_e^T(i)\,P_i(i)\,]$$

Let

$$h = -\frac{\lambda\,P_e^T(i)\,P_i(i)}{P_i^T(i)\,P^{-1}\,P_i(i)}$$

our distance will be decreasing for $0 < \lambda < 2$, the optimum being $\lambda=1$. The identification algorithm will be

$$P_e(i+1) = P_e(i) - \frac{\lambda\,r_e(i)\,P^{-1}\,P_i(i)}{P_i^T(i)\,P^{-1}\,P_i(i)}$$

λ is used to reduce the effect of noise on the estimation. For $0 < \lambda < 1$, reducing the value of λ will reduce the effect of noise but will also increase the time required for the estimation. The P matrix is used to modify the importance of the different coefficients of P_e in the distance. The more importance the coefficient has, the more quickly it will be identified.

As in the case of the Kalman filter this estimation process would ultimately give us the trivial solution $P_e = 0$. To avoid this we will restore the norm of the sensor coefficients to 1 after each identification step.

$$P_e(i+1) = P_e(i+1) \,/\, \mathrm{norm}(i+1)$$

The algorithm was first tried with $\lambda=1$ and P the identity matrix. In this case a geometric interpretation of the algorithm can be given. Consider a parity relation made of 3 coefficients [0,0,1], and the corresponding information vectors [a,b,0] where a and b can take any value. Lets assume that the first 2 coefficients correspond to the X and Y directions of figure 3-4 and the third to the Z direction orthogonal to the figure. If our initial estimate of the relation is $P_e^T(0)=[1,1,1]$ and we have the information vectors $P_i^T(1)=[0,1,0]$ and $P_i^T(2)=[1,1,0]$ then our two successive estimates of P_e will be $P_e^T(1)=[1,0,1]$ and $P_e^T(2)=[1/2,-1/2,1]$. The x,y component of these vectors are represented on figure 3-4. As shown by the figure, the algorithm is equivalent to an orthogonal projection of $P_e(i)$ in the direction $P_i(i+1)$.

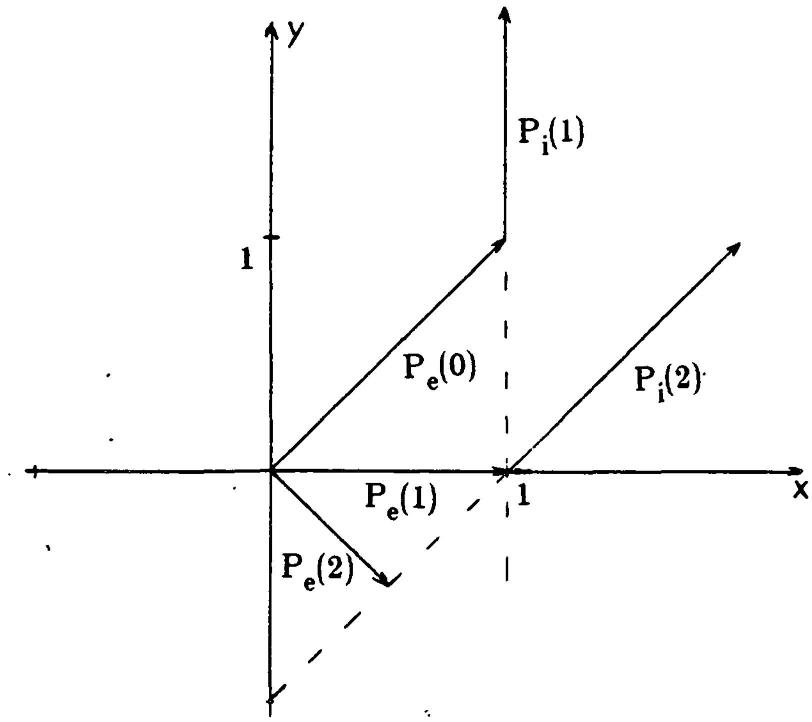


Figure 3-4: Successive estimates given by the structural distance

The estimation with $\lambda=1$ and P=I was carried out over 3050 time steps without introducing any sensor noise. The initial relation is the 5% case of figure 2-12. Figure 3-5 shows the estimated residual over periods of 50 time steps covering the estimation time. Figure 3-6 shows the residuals given by the initial relation. The algorithm is not too succesful in reducing the covariance of the residuals. This can be blamed on the bad weighting of the gains. Actuator coefficients are much larger than their sensor counterparts and the algorithm puts all the weight on them. Consequently the sensor coefficients are almost kept

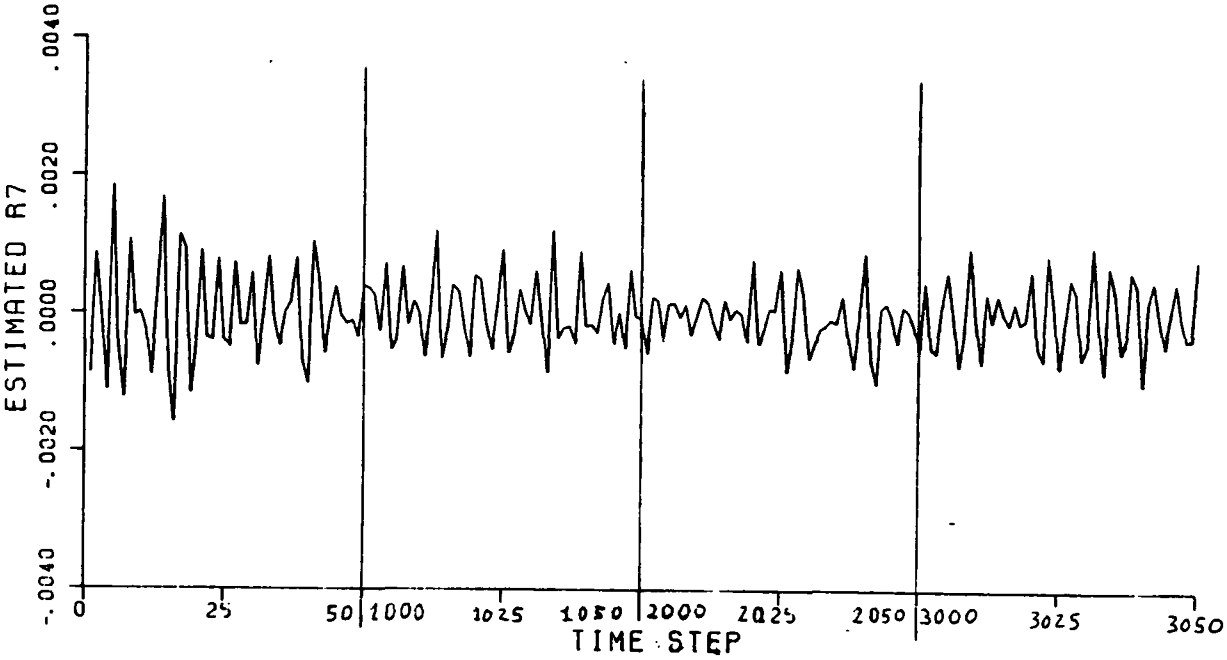


Figure 3-5: Residual of the structural distance with P=I and λ=1

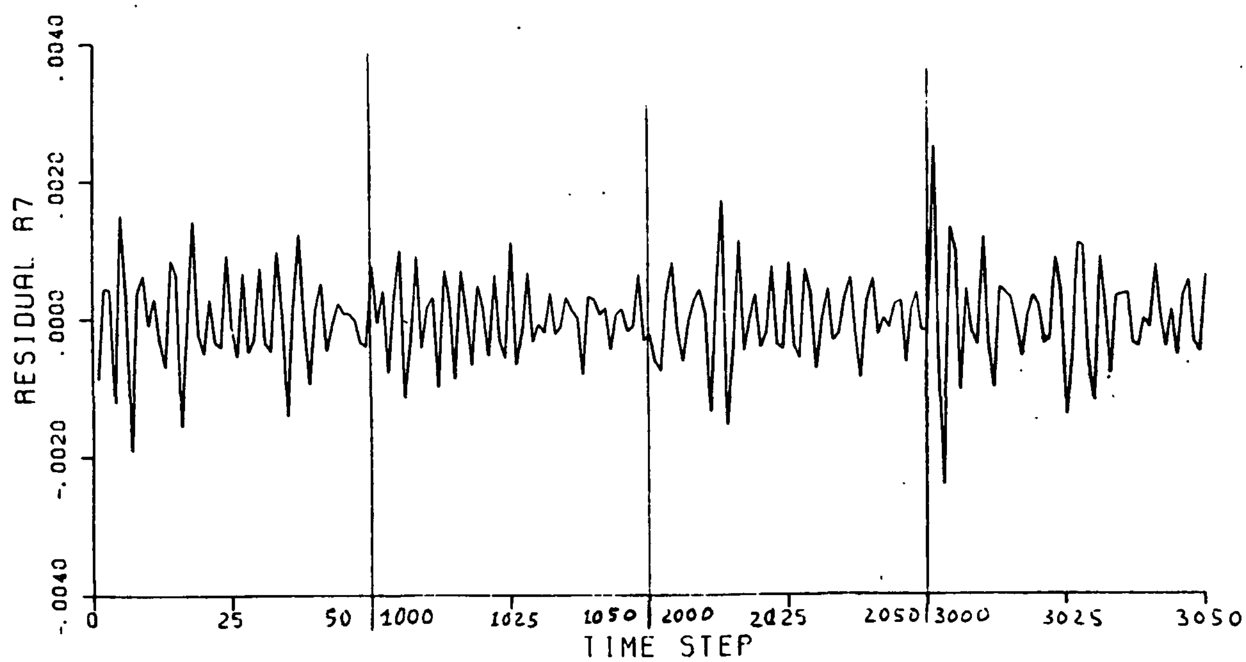


Figure 3-6: Residual of the 5% mismatch case

constant and the possible reduction of covariance is limited.

We have to introduce some weighting P in the structural distance. One idea is to find P such that the distance is equivalent to the covariance of the residuals of the estimated relation $P_e(i)$. Lets suppose that $P = E(P_i P_i^T)$. Then

$$D(i) = [P_e(i) - P_r]^T E(P_i P_i^T) [P_e(i) - P_r]$$

As P_e and P_r are not random

$$D(i) = E[(P_e(i) - P_r)^T (P_i P_i^T) (P_e(i) - P_r)]$$
$$= E[r_e^2(i) - 2 r_e(i) r_r - r_r^2]$$

But as r_r=0 by definition of P_r

$$D(i) = E[r_e^2(i)]$$

The problem is that such a weighing would require the storage of $^1/_2 N^2$ coefficients as in the Kalman filter case. To avoid this only the diagonal terms of $E(P_i P_i^T)$ were taken to create a diagonal matrix.

$$\text{Diagonal (P)} = [\, \underbrace{y_1, \dots, y_1}_{\text{S times}}, \underbrace{y_2, \dots, y_2}_{\text{S times}}, \dots, \underbrace{y_6, \dots, y_6}_{\text{S times}}, \underbrace{u_1, \dots, u_1}_{\text{S times}} \,]$$

The result of the new algorithm is shown in figure 3-7. The residuals are now comparable to those achieved with the Kalman filter. The algorithm is not able to give a better estimate even though no noise is simulated. The usual simulation using the last estimated relation is presented in figure 3-8.

Using the same weighting matrix but λ=0.1, sensor noise at the 10^{-4} level was introduced. The result of the estimation is shown in figure 3-9 compared with the 5% mismatch residual in figure 3-10. The simulation involving the estimated

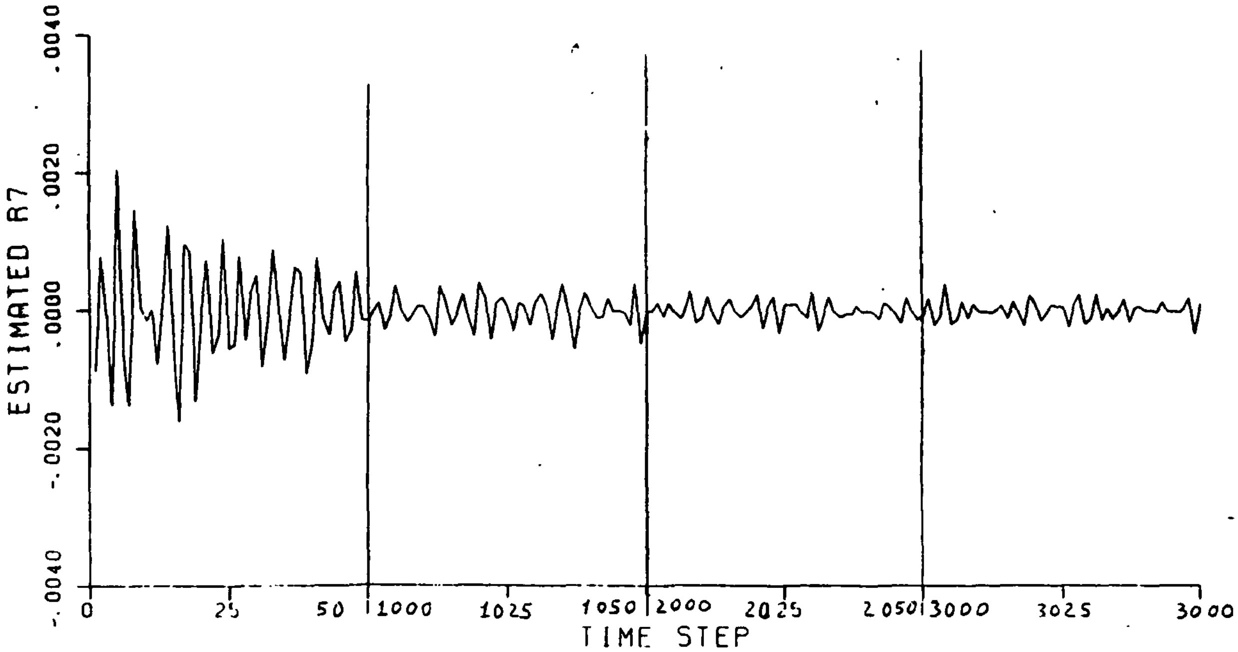


Figure 3-7: Residual of the structural distance with modified P

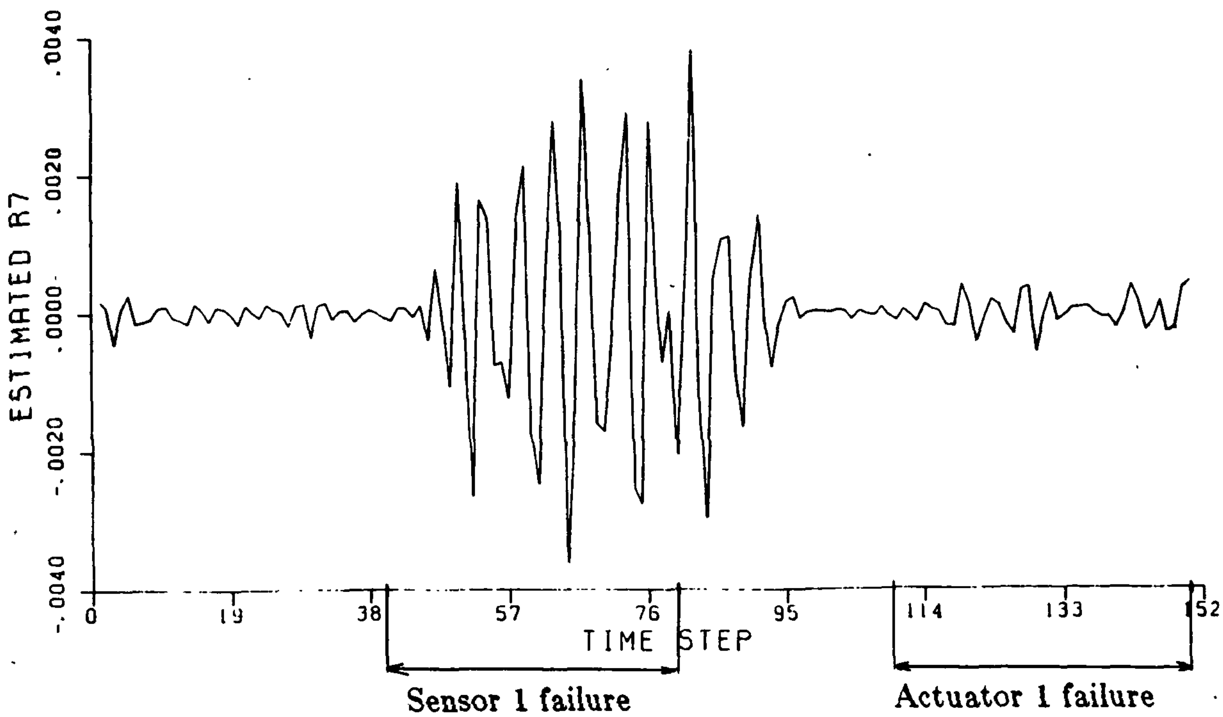


Figure 3-8: Relation estimated without noise using the structural distance

relation is shown in figure 3-11. This shows that the algorithm still performs

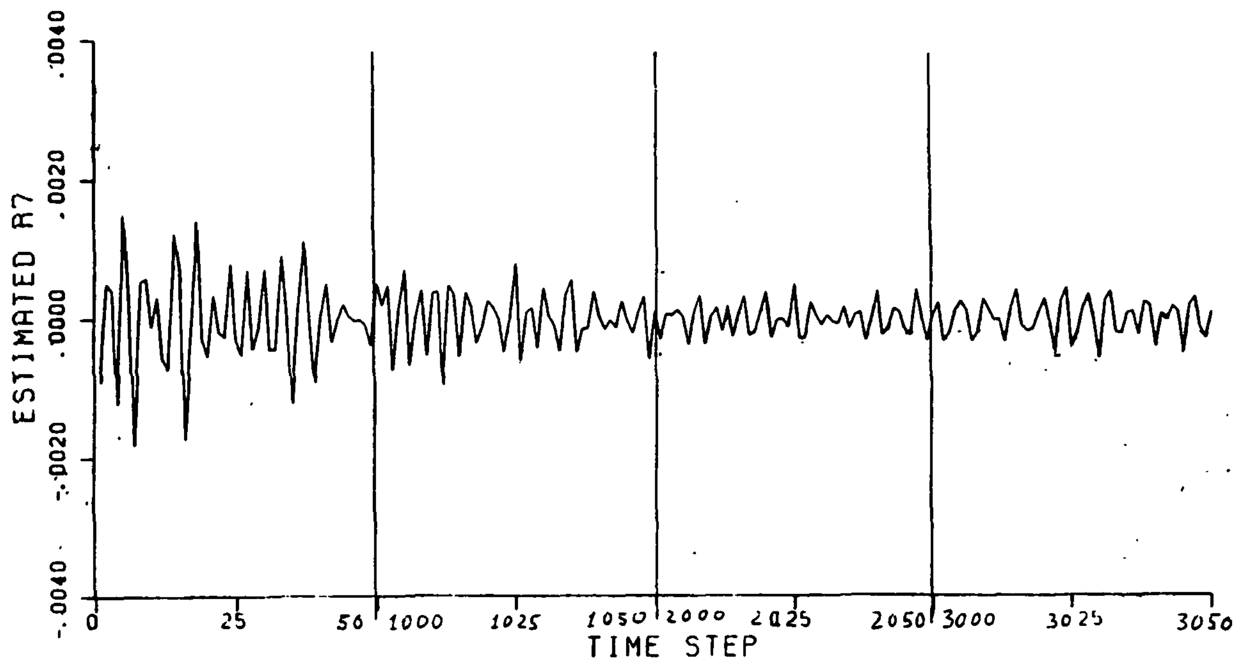


Figure 3-9: Residual of the structural distance with noise

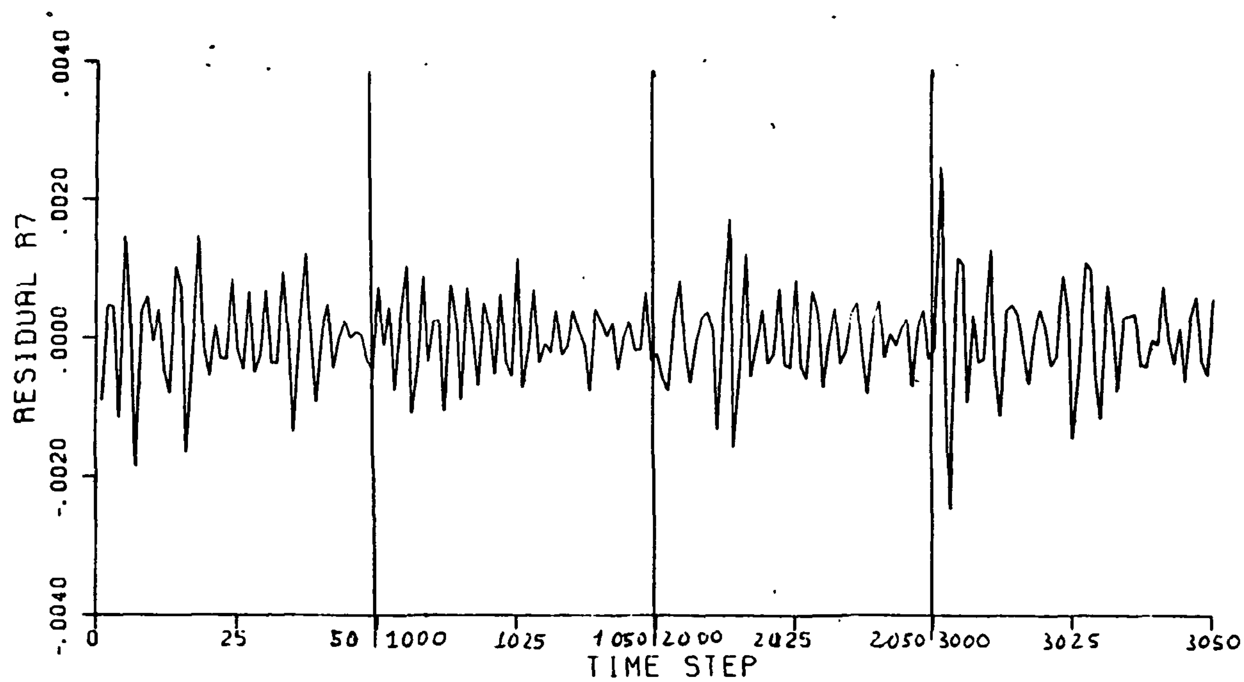


Figure 3-10: Residual of the 5% mismatch case

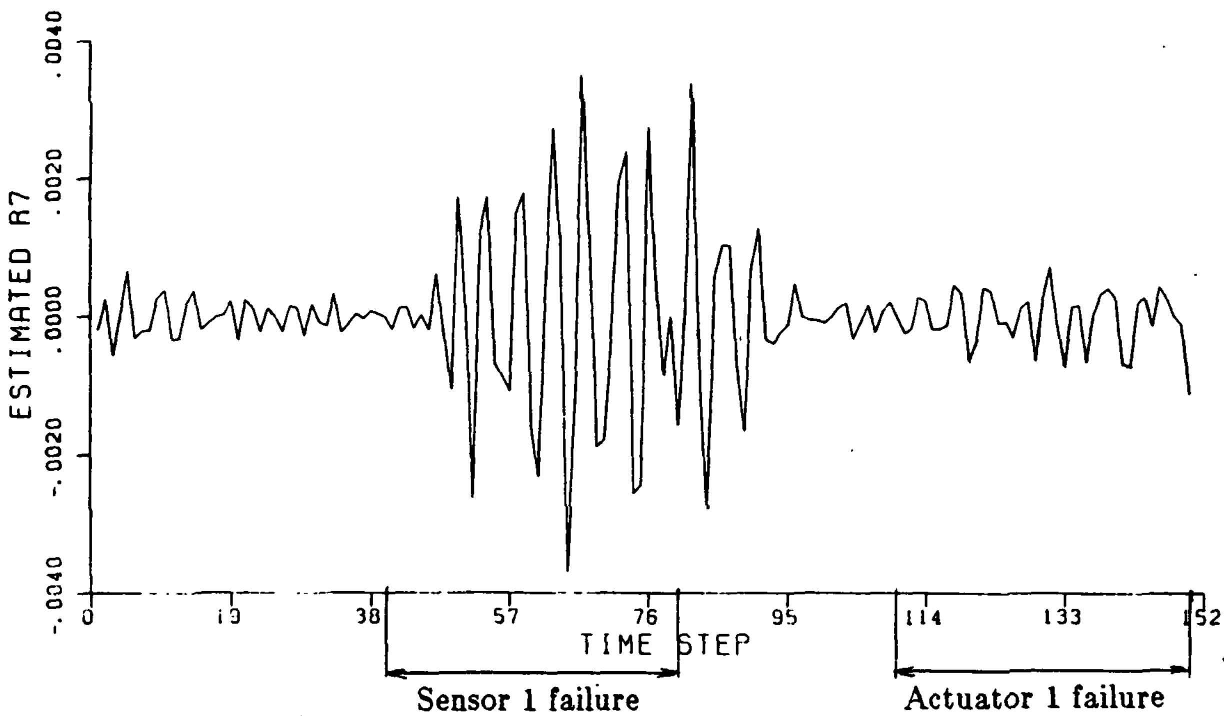


Figure 3-11: Relation estimated with noise using the structural distance

correctly with sensor noise as long as λ is reduced sufficiently.

3.4 Eigenvalue Decomposition of a Parity Information Matrix

The parity vector made of the R coefficients of a parity relation and the corresponding parity information vectors P_i are elements of the same R dimensional vector space. The definition of a parity relation states that when no noise is considered we have

$$r(i) = P_r^T P_i(i) = 0$$

The vector space interpretation of this definition is that the parity vector is the vector orthogonal to all parity information vectors. This property was used in the last section when we estimated the parity relation by performing successive orthogonal projections of our estimate in the direction of the information vector P_i.

Now instead of performing an estimation at each step we build a matrix A made of different information vectors.

$$A = \begin{bmatrix} P_i(1) & P_i(2) & \dots & P_i(n) \end{bmatrix}$$

According to the definition the corresponding parity relation is in the left null space of this matrix. Consequently, without noise, a null space algorithm based on a matrix containing at least R-1 information vectors gives us the corresponding parity relation.

If some noise is introduced the exact null space vector no longer exists. Instead we have to find the vector "as orthogonal as possible" to A as it is called in [1]. This is the vector that minimizes the covariance of the residuals $E(r^2)$.

This vector can be found by performing an eigenvalue decomposition of $A\,A^T$. First we note that

$$\begin{aligned} A\,A^T &= \begin{bmatrix} P_i(1) & \dots & P_i(n) \end{bmatrix} \begin{bmatrix} P_i^T(1) \\ \dots \\ P_i^T(n) \end{bmatrix} \\ &= \sum_{j=1}^{n} [\, P_i(j)\, P_i^T(j) \,] \end{aligned}$$

Then if our estimation is based on n parity information vectors, the covariance of the residual r_r corresponding to the relation P_r is

$$E(r_r^{\,2}) = \frac{1}{n} \sum_{j=1}^{n} [\, (P_r^T P_i(j))\, (P_i^T(j)\, P_r)\,]$$

$$= \frac{1}{n}\; P_r^T\, [\, \sum_{j=1}^{n} (P_i(j)\, P_i^T(j))\,]\, P_r$$

$$= \frac{1}{n}\; P_r^T\, (A\, A^T)\, P_r$$

Now if the decomposition of $A\,A^T$ is

$$A\,A^T = \begin{bmatrix} V_1 & \cdots & V_R \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \cdots & \\ & & \lambda_R \end{bmatrix} \begin{bmatrix} V_1^T \\ \cdots \\ V_R^T \end{bmatrix}$$

where $\lambda_k > 0$ and the V_k are orthonormal
and the λ_k are ordered by magnitude

we have

$$E(r_r^{\,2}) = \frac{1}{n} \sum_{j=1}^{R} \lambda_k\, [\, P_r^T V_k\,]^2$$

If we impose that the norm of P_r be one, then

$$\|P_r\| = \sum_{j=1}^{R} [\, P_r^T V_k\,]^2 = 1$$

And consequently $E(r^2)$ is minimized if P_r is the eigenvector V_1 corresponding to the smallest eigenvalue λ_1. This vector is "as orthogonal as possible" to A.

In our case imposing the norm of P_r to be one is almost the equivalent of having the norm of the sensor coefficients to be one as they are much larger than the actuator coefficients. However if this approximation is not valid, for example if the sensors have different noise levels, then the algorithm can be modified as follows. Let $\| \, \|_N$ represent the N norm that takes into account the different noise levels. Then $E(r_r^2)$ can be written

$$E(r_r^2) = \frac{1}{n} \sum_{j=1}^{R} \frac{\lambda_k}{\|V_k\|_N^2} [\, P_r^T V_k \,]^2 \, \|V_k\|_N^2$$

But as the eigenvectors V_k are an orthogonal basis of our vector space, P_r can be written

$$P_r = \sum_{j=1}^{R} [\, P_r^T V_k \,] \, V_k$$

and we have

$$\|P_r\|_N = \sum_{j=1}^{R} [\, P_r^T V_k \,]^2 \, \|V_k\|_N^2 = 1$$

The best relation is given by the eigenvector V_k corresponding to the smallest ratio $\lambda_k / \|V_k\|_N^2$

In [1] a singular value decomposition of A is used. The parity relation is given by the left singular vector corresponding to the smallest singular value. This technique might be more accurate for small eigenvalues [2]. The eigenvalue technique, however is more interesting when memory capacity is limited. Storing A requires keeping R by n numbers. As $A\,A^T$ is symmetric and can be built by adding up the $P_i(i)\,P_i^T(i)$ matrices it only requires the storage of $^1/_2 R^2$ numbers.

Our single actuator relation is made of 147 coefficients. The estimation would then in theory require 146 time steps. But if the system is slow compared to the sampling rate these vectors might not be representative of the information vector population. To be sure to get an accurate estimation some redundancy was introduced by taking 200 time steps.

In the first run, no sensor noise was introduced. The relation obtained was used in the two failures simulation and the result is shown in figure 3-12. We have here a true parity relation as the residuals are zero in the no fail case. The interesting point is that the failure signatures are quite different from what we had in figure 2-4, even though the inputs and outputs used in both cases are the same. According to section 2.2 the null space vector with S=N+2 is of dimension 1. Consequently the normalized parity relation is unique. In practice we find that normalized relations with very different coefficients can have almost zero residuals in the no fail case.

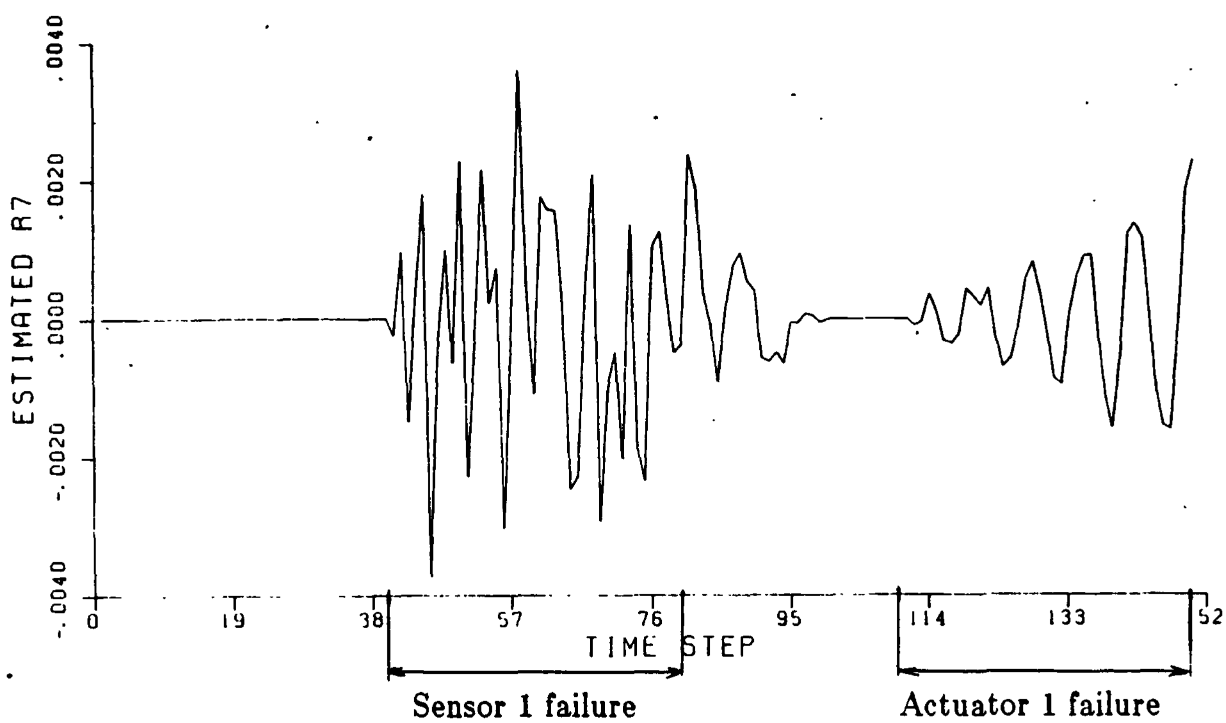


Figure 3-12: Relation estimated without noise using the eigenvalue method

In the second run, sensor noise at the 10^{-4} level was introduced in the estimation and simulation. The result is shown in figure 3-13. The residuals in the no fail case are comparable with what we had with the two other estimators. But here the failure signature is larger so the effectiveness of the relation is improved. But as usual the introduction of noise resulted in a reduction of the failure signature.

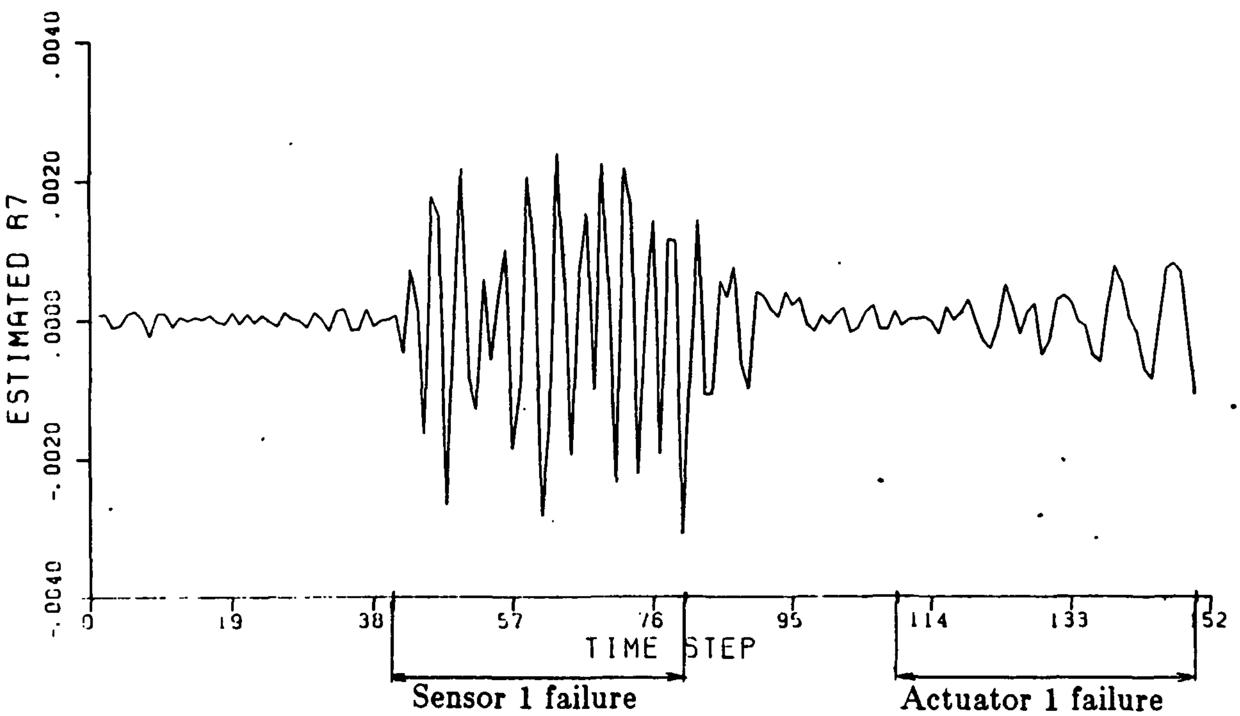


Figure 3-13: Relation estimated with noise using the eigenvalue method

Finally we note that this algorithm through the eigenvectors generates an orthogonal basis of all possible parity relations. For each we have a measurement of the residual in the no-fail case given by the corresponding eigenvalue. For the estimation with sensor noise the 10 smallest eigenvalues are given in table 3-I. They are very close to one another which means that the 10 corresponding relations will have almost the same residuals in the no-fail case. But their respective failure signatures could be very different. Figure 3-14 shows the behavior of relation number 5 in the two failures simulation.

relation number	1	2	3	4	5	6	7	8	9	10
eigenvalue in 10^{-7}	0.97	1.23	1.54	1.76	2.21	2.24	2.42	2.59	2.66	2.70

Table 3-I: 10 smallest eigenvalues of the information matrix

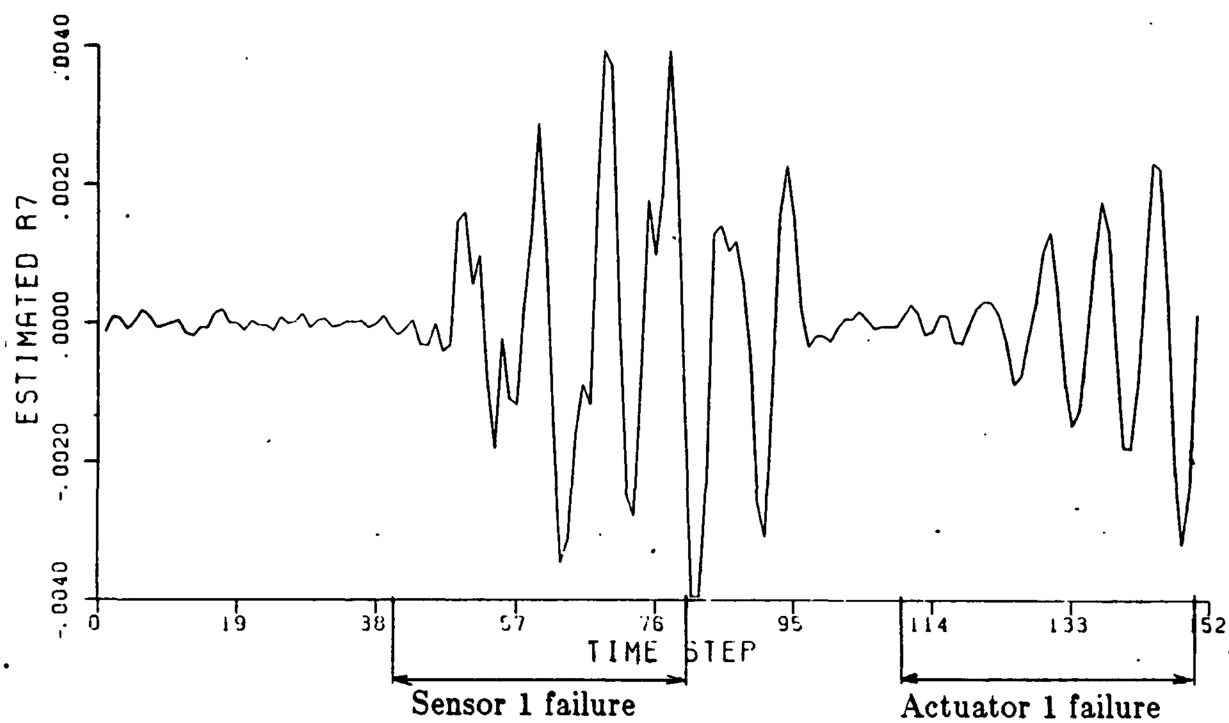


Figure 3-14: One of the Other relations estimated using the eigenvalue method

Chapter 4
Optimization of a Covariance Ratio

4.1 The Covariance Ratio and the Failure Signature

In section 3.4 we saw that as sensor noise is introduced many different relations tend to have equivalent level of residuals in the no fail case. Up to now we selected the one with the lowest level so as to be as close as possible to a parity relation. This is not always the best choice. Comparing figures 3-13 and 3-14 we find that relation 5 corresponding to 3-14 is more suitable for failure detection. Both relations have a comparable level of residuals in the no-fail case but relation 5 has a much larger actuator 1 failure signature. Estimation algorithms could be greatly improved if we could measure the performance of the relations they generate. A good criterion for this would be to compute the ratio of the covariances of the residuals in the fail and the no-fail case. To select the single actuator 1 relation, for example, this would be

$$c_r = \frac{E[r^2(i)] \text{ with failure of actuator 1}}{E[r^2(i)] \text{ without failure}}$$

Any linear relation that can be generated by our estimation algorithm is of the form

$$r(i) = W_s^T Y^*(i) + W_a^T U^*(i)$$

W_s is the vector of sensor coefficients
W_a is the vector of actuator coefficients
r is the residual

The covariance of this residual can be computed quite accurately in the no-fail case as the corresponding inputs and outputs are available. On the other hand, the covariance when a failure is present will have to be estimated. First we must find how a failure affects the residuals. Lets assume that we want to detect an actuator 1 failure. A failure appears when the input computed by the controller U_1^* and the input applied to the system U_{1s}^* are different. The residual becomes

$$r_f(i) = W_s^T Y^*(i) + W_a^T U_1^*(i)$$

This can be written

$$r_f(i) = W_s^T Y^*(i) + W_a^T U_{1s}^*(i) + W_a^T [U_1^*(i) - U_{1s}^*(i)]$$

As U_{1s} and Y^* are the no-fail inputs and outputs of the system, the residual they generate, r_s, is equivalent to the residual in the no-fail case, r_n. Consequently, the signal $U_1^*(i) - U_{1s}^*(i)$ is the one that allows us to detect the failure. It affects the residual through W_a to give the failure signature $W_a[U_1^*(i) - U_{1s}^*(i)]$.

If we assume r_s has the same covariance as r_n, the covariance ratio becomes

$$c_r = 1 + \frac{W_a^T E[(U_1^*(i) - U_{1s}^*(i))\,(U_1^*(i) - U_{1s}^*(i))^T]\, W_a}{E[r_n^2(i)]}$$

If also the failure is such that $U_{1s}^*(i)$ is known, then the covariance ratio can be estimated based only on the inputs and outputs in the no-fail case.

4.2 Model Based Covariance Ratios

To see how this new technique performs we first assume we have available an accurate model of the structure. We want to find the single actuator 1 relation with the best covariance ratio for a zero failure of that actuator.

Going back to relation (2.5), after selecting the lines involving the actuator 1 input we have a relation of the form

$$U_1^*(i) = C^* X(i) + D^* Y^*(i)$$

where C^* and D^* represent the selected rows of $-D_r^{-1} C_r^*$ and D_r^{-1}

This relation can be modified to take into account sensor noise given by relation (2.6). We get

$$U_1^*(i) = C^* X(i) + D^* [Y^*(i) + N(i)] \tag{4.1}$$

Previously, to build a parity relation, we multiplied this relation by the vector W satisfying $W^T C^* = 0$. To have such a vector the number S of coefficients in W had to be at least one greater than the rank N of C^*. Now we will choose this vector, renamed W_a, so as to optimize the covariance ratio and get what will be called an optimized relation. This means that W_a can be of any size. However if the dimension of W_a is greater than N, as we will show later, we are guaranteed that our relation will perform better than a parity relation. Multiplying by W_a, relation (4.1) becomes

$$W_a^T U_1^*(i) = W_a^T C^* X(i) + W_a^T D^* Y^*(i) + W_a^T D^* N(i)$$

and the corresponding residual is

$$r_n(i) = - W_a^T D^* Y^*(i) + W_a^T U_1^*(i) = W_a^T C^* X(i) + W_a^T D^* N(i)$$

Now the unknown state will be an added source of noise. What we hope is that this will be largely compensated by an increase in the failure signature.

The covariance of this residual is given by

$$E[r_n^2(i)] = W_a^T C^* E[X(i) X^T(i)] C^{*T} W_a + W_a^T D^* E[N(i) N^T(i)] D^{*T} W_a \tag{4.2}$$

It depends on two covariance matrices. The noise covariance matrix is known. It is a diagonal matrix with diagonal elements equal to q. The state covariance matrix on the other hand has to be computed. This was done through simulations of the system. Equation (4.2) can be regrouped into one matrix equation.

$$E[r_n^2(i)] = W_a^T M_n W_a$$

As we want to optimize our relation for a zero failure of actuator 1 we have $U_{1s}^*(i) = 0$ and our failure covariance matrix will be

$$M_f = E[U_1^*(i) U_1^{*T}(i)]$$

The covariance ratio becomes

$$c_r = 1 + \frac{W_a^T M_f W_a}{W_a^T M_n W_a} \tag{4.3}$$

Without sensor noise M_n is simply

$$M_n = C^* E[X(i) X^T(i)] C^*$$

With $W^T C^* = 0$ the denominator is zero and c_r is infinite. Consequently in that case parity relations are the optimized relations.

Generally, this is not true and we have to find the vector W_a that maximizes the ratio

$$\frac{W_a^T\ M_f\ W_a}{W_a^T\ M_n\ W_a} \tag{4.4}$$

Seen this way this doesn't seem very easy. The idea is to first find a transformation for which the image of M_n is the identity matrix. Let the eigenvalue decomposition of M_n be

$$M_n = P\ D\ P' \quad \text{with D a diagonal matrix:}\ D = \begin{bmatrix} \lambda_1 & & \\ & \cdots & \\ & & \lambda_S \end{bmatrix}$$

As the eigenvalues of D are positive, it can be written as

$$D = D^{1/2}\, D^{1/2} \quad \text{with} \quad D^{1/2} = \begin{bmatrix} (\lambda_1)^{1/2} & & \\ & \cdots & \\ & & (\lambda_S)^{1/2} \end{bmatrix}$$

$P\ D^{1/2}$ is the transformation changing M_n into I.

$$M_n = P\ D^{1/2}\ I\ [P\ D^{1/2}]^T$$

Using this transformation the ratio can be rewritten as

$$\frac{W_a^T P\ D^{1/2}\ D^{-1/2}\ P^T\ M_f\ P\ D^{-1/2}\ D^{1/2}\ P^T W_a}{W_a^T P\ D^{1/2}\ I\ D^{1/2}\ P^T W_a}$$

If we pose $W_{at} = D^{1/2}\ P^T W_a$ and $M_{ft} = D^{-1/2}\ P^T M_f\ P\ D^{-1/2}$, the ratio becomes

$$\frac{W_{at}^{\top}\ M_{ft}\ W_{at}}{W_{at}^{\top}\ I\ W_{at}}$$

Finding the W_{at} maximizing.this ratio is quite easy. If we impose that $||W_{at}||=1$, then the denominator is one. The W_{at} maximizing the numerator will be given by the eigenvector of M_{ft} correspoding to the largest eigenvalue. This technique is similar to what was done in 3.4. Once the best W_{at} is found, we get the corresponding W_a with

$$W_a = P \cdot D^{-1/2}\, W_{at}$$

Optimized relations that will be generated here have the same number of coefficients, 147, as previous parity relations. They also have the norm of the sensor coefficients equal to one. This will enable us to compare the performances of the two types of relations.

One single actuator 1 relation was generated for each of the two levels of noise. They were then tried in the two failures simulation. For the 10^{-4} level the relation was first tried without simulating any noise to see how the state affects the residual. The result is presented in figure 4-1. This particular case emphasizes the fact that letting through a little amount of the state can dramatically improve the failure signature. Figure 4-2 shows the simulation taking into account noise at the 10^{-4} level. The equivalent simulations for the 10^{-3} level are shown without noise in figure 4-3 and with noise in figure 4-4. In this case much more of the state is visible in the residual. In return we should have a larger failure signature, as this relation must have a better covariance ratio than the preceding one for a level of noise of 10^{-3}. This cannot be seen in figure 4-3 which means that the increase must be quite small.

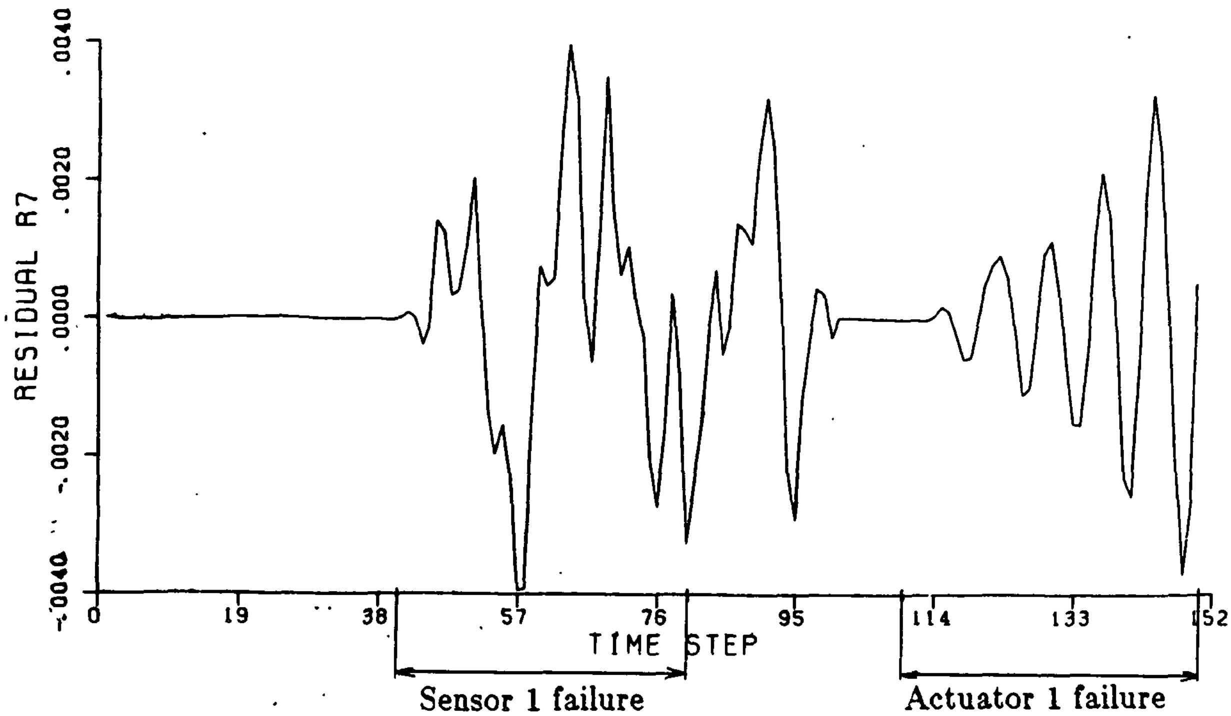


Figure 4-1: Noiseless residual of the 1st model based optimized relation

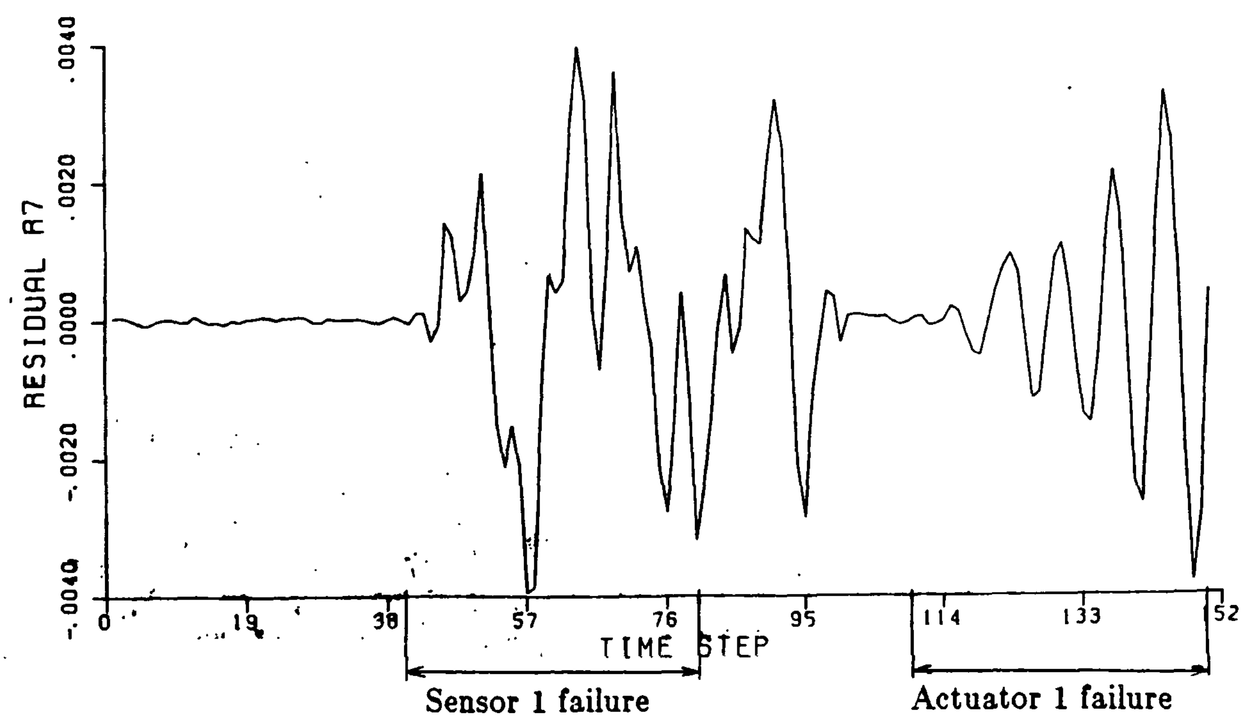
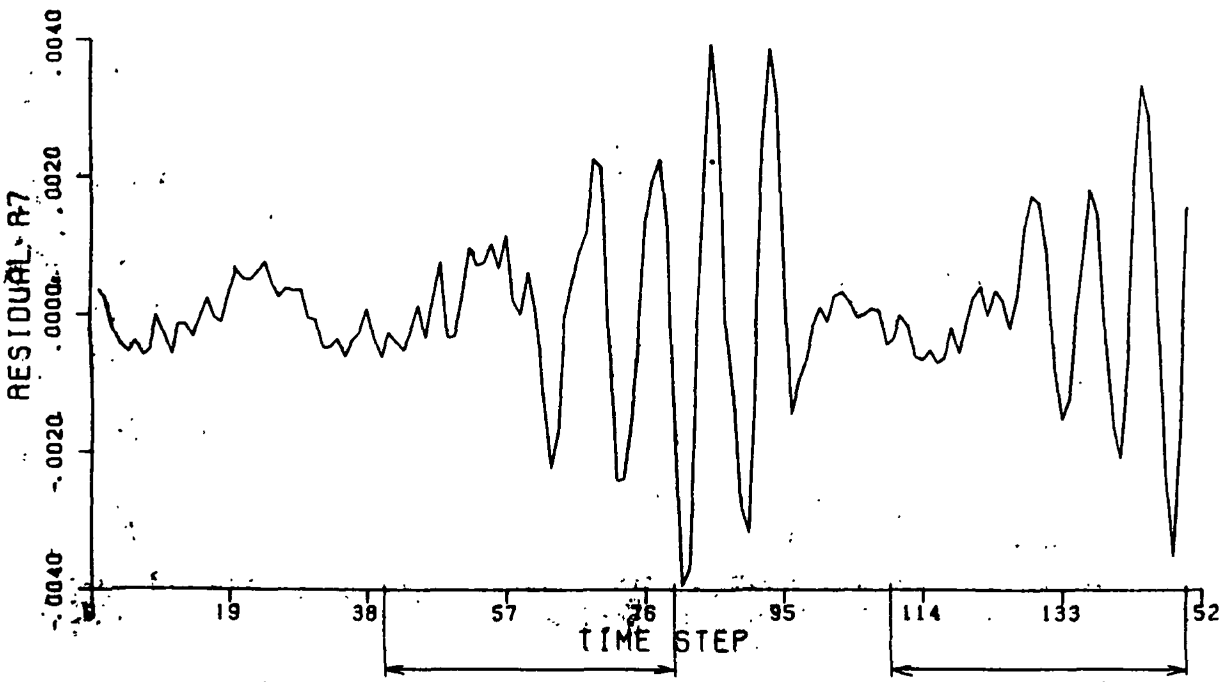


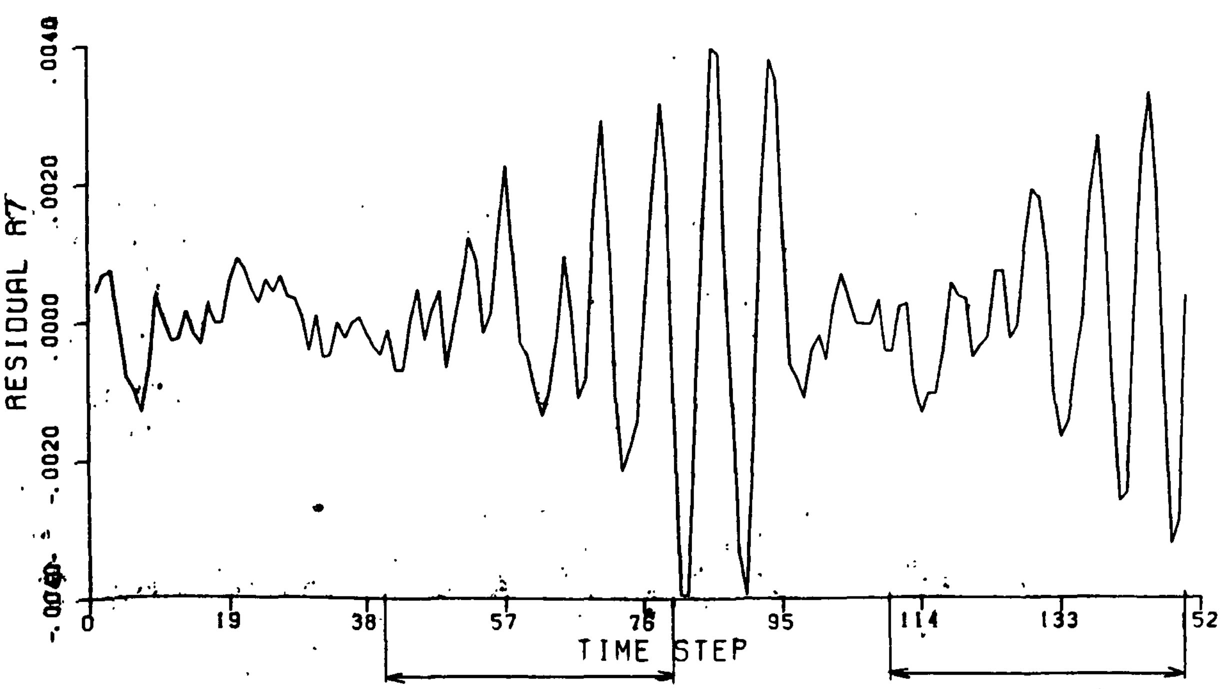
Figure 4-2: Noisy residual of the 1st model based optimized relation



Sensor 1 failure

Actuator 1 failure

Figure 4-3: Noiseless residual of the 2^{nd} model based optimized relation



Sensor 1 failure

Actuator 1 failure

Figure 4-4: Noisy residual of the 2^{nd} model based optimized relation

4.3 Estimation Using the Optimization

We now want to use the covariance ratio with the eigenvalue decomposition method. From section 3.4 we already know how to find the covariance in the no-fail case. The relation is of the form

$$E[r^2(i)] = P_r^T M_n P_r$$

We also know how to compute the covariance in the fail case. If the failure signature is $W_a^T U_1^*(i)$

$$E[r_f^2(i)] = E[r_n^2(i)] + W_a^T E[U_1^*(i)\, U_1^{*T}(i)]\, W_a$$

And if our estimation uses n information vectors, we have

$$E[U_1^*(i)\, U_1^{*T}(i)] = \frac{1}{n} \sum_{j=1}^{n} [U_1^*(j)\, U_1^{*T}(j)]$$

The covariance ratio is

$$c_r = 1 + \frac{W_a^T E[U_1^*(i)\, U_1^{*T}(i)]\, W_a}{P_r^T M_n P_r}$$

To be able to use the transformation technique of the previous section we want to express c_r only as a function of P_r. As $P_r^T = [\, W_s^T, W_a^T\,]$, if we define M_f as

$$M_f = \begin{bmatrix} 0 & 0 \\ 0 & E[U_1^*(i)\, U_1^{*T}(i)] \end{bmatrix}$$

c_r can be rewritten as

$$c_r = 1 + \frac{P_r^T M_f P_r}{P_r^T M_n P_r}$$

This equation is equivalent to equation (4.3) except that now we are estimating all the coefficients of the relation instead of only the actuator ones.

The optimized estimation was performed using 200 parity information vectors. First noise of 10^{-4} was introduced. The resulting relation is shown in figure 4-5 for a simulation taking into account sensor noise. We somehow expected to get the same residual as in figure 4-2. Instead we have a totally different relation with a larger failure signature and a much higher level of residuals in the no fail case. The covariance ratio given by the estimation is 0.79 10^3. Obviously, this number does not correspond to what we really have in the simulation. For the 10^{-3} level of noise, the estimated relation is shown in figure 4-6. Here we have an even greater difference between the estimated ratio, 0.57 10^3, and the simulation. This bad estimation of the covariance ratio indicates that the covariance matrices needed for the estimation cannot be reliably computed with 200 information vectors. Sensor noise is a likely source for the bias in the estimation. In section 3.4 the same amount of information gave good results even when sensor noise was introduced. But if we compare figures 3-13 and 4-5, it seems that a small amount of information and sensor noise affects the estimation of optimized relations more than that of parity relations.

In a second attempt the estimation was carried over 1000 time steps. Figure 4-7 shows the result for the 10^{-4} level of noise and figure 4-8 for the 10^{-3} level. The corresponding estimated covariance ratios are 0.18 10^3 and 0.13 10^3. These numbers correspond more to what we actually see in the simulations. The most interesting point is that these relations have better performances than the ones of

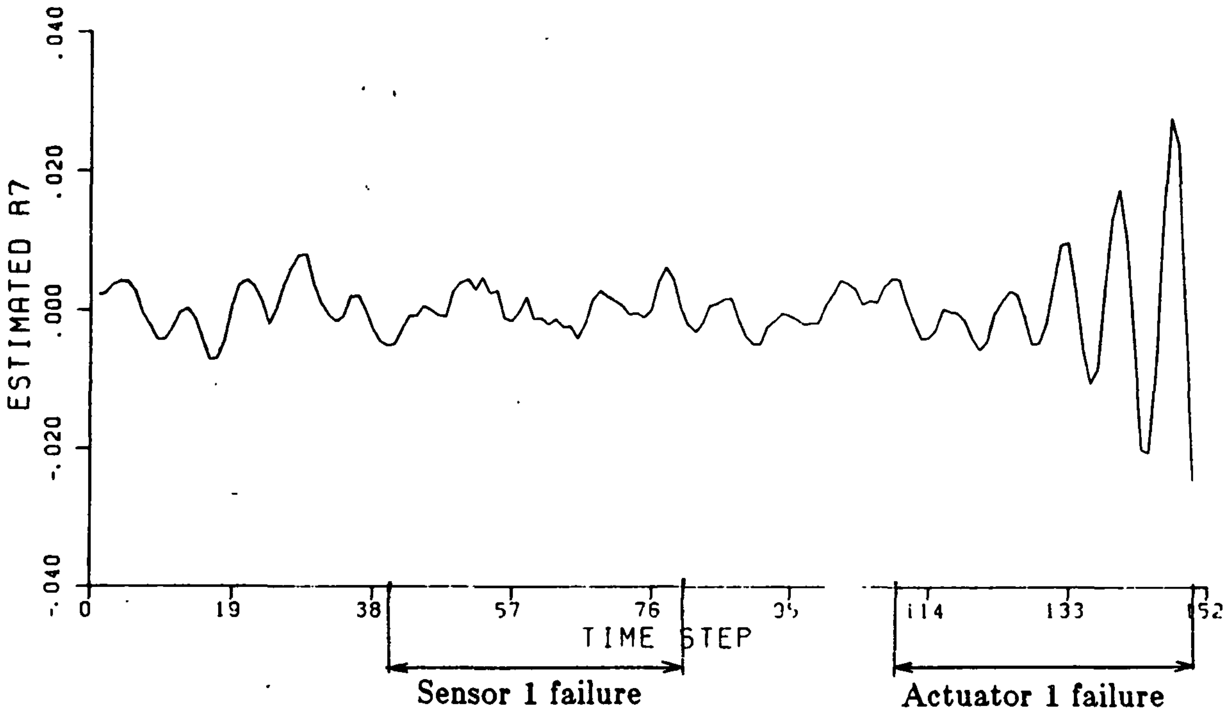


Figure 4-5: 1^{st} optimized relation estimated using 200 steps

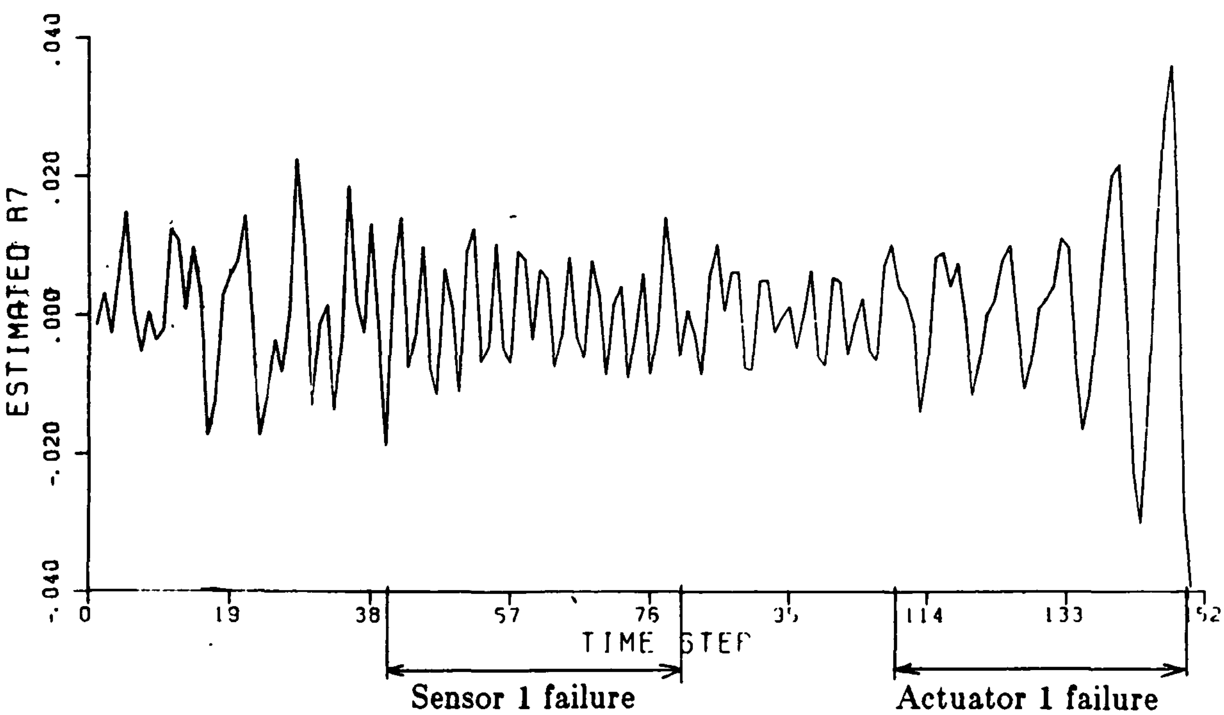


Figure 4-6: 2^{nd} optimized relation estimated using 200 steps

section 4.2. Previously the optimization was done only with respect to the actuator coefficients. Once these were chosen the rest of the relation was given by the matrix D^*. This means that only 21 independent relations were considered. Now the optimization is done with respect to all the coefficients and we are comparing 147 independent relations of which 21 are the previous ones. It turns out that the latter are not the best ones.

Optimized relations are completely different from parity relations as they have a high level of residual in the no-fail case. As a consequence the effect of the sensor 1 failure is no longer visible in the residuals. This might also be the case for any actuator 1 failure other than the "zero" failure.

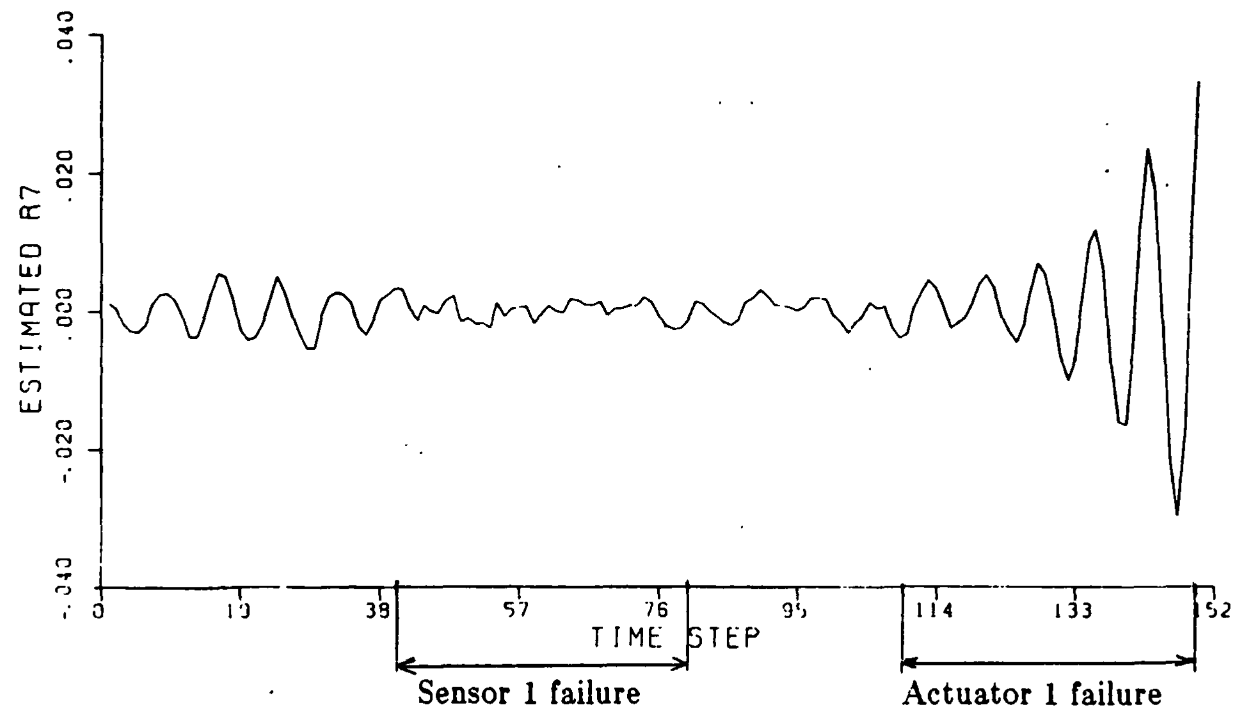


Figure 4-7: 1^{st} optimized relation estimated using 1000 steps

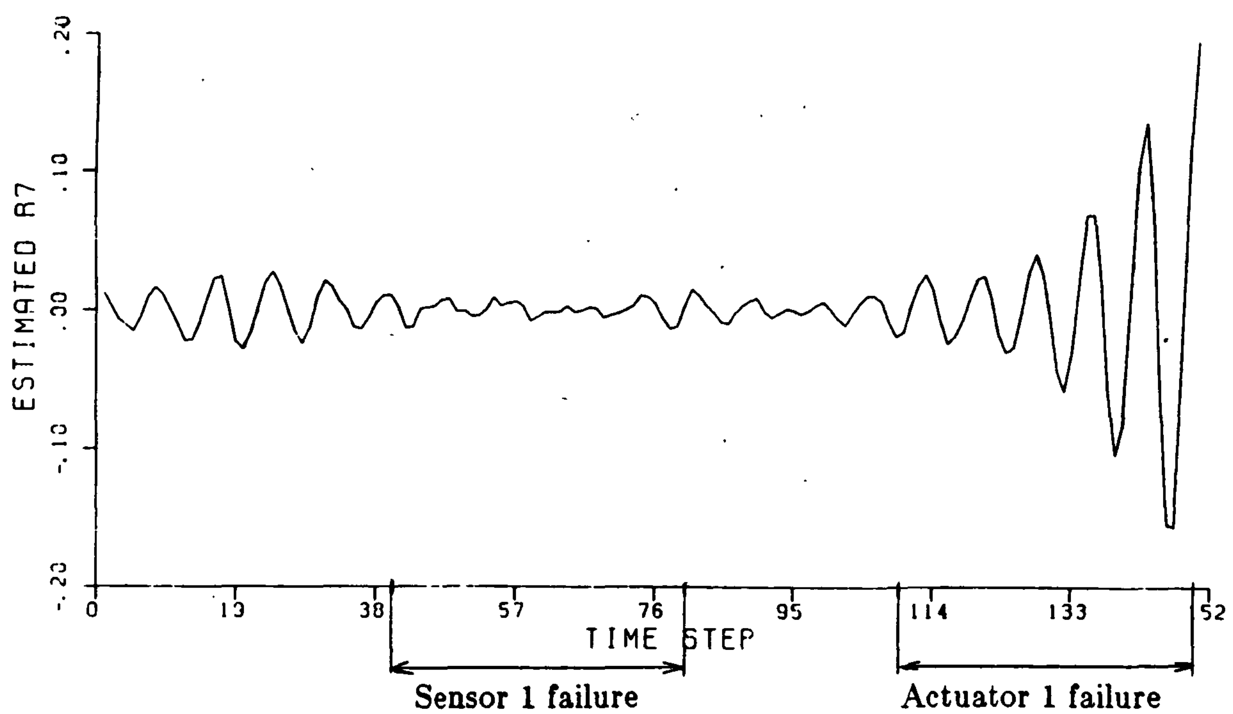


Figure 4-8: 2^{nd} optimized relation estimated using 1000 steps

Chapter 5
Conclusion

5.1 Summary

In chapter 2, parity relations were generated using a model of a grid structure. The goal was to investigate the sensitivity of these relations to modeling errors. 5% error in the model parameters was enough to prevent us from detecting any actuator failure, and with 10%, no failure could be detected. As no accurate model can be found before the structure is built, chapter 3 focused on the estimation of parity relations using the actuator inputs and sensor outputs.

Three on-line estimators were studied. First a Kalman filter was tried. The resulting estimated relation had a reduced detection efficiency compared to the relation generated using the model, due mainly to a reduced failure signature. This trend was observed in all estimators. Also this technique requires an amount of memory proportional to the square of the number of coefficients in the relation. As parity relations for large space structures will have many coefficients and memory on a space based computer is limited this can be a major problem. Consequently, a second estimator, for which the storage requirement is only proportional to the number of coefficients, was developed. It was based on the minimization of a structural distance. The results were comparable to what we had with the Kalman filter. However, the required estimation time is much longer. But this is not a problem in our case. This technique is the most interesting for single sensor parity relations. For a single actuator relation, however, a more accurate estimator is required. The third technique used an eigenvalue decomposition of a covariance matrix. It proved to be slightly more accurate than the others with the same memory requirement as the Kalman filter.

With a few modifications, the last technique was used to generate linear relations other than parity relations. Unlike the latter, these relations take into account the effect of the failure on the input-output relations. As a result they are optimized for the detection of a particular failure. The main interest of such relations is their ability to detect failures in very noisy conditions. It is likely that, in these conditions, they might only detect the failure they are designed for. However, components can usually fail in only one or a few different ways, so we can generally run a different relation for each different failure mode of the same component. If this is computationally too heavy, another solution is to modify the covariance ratio so as to optimize a relation for two or more failures. For example, to detect the failure signatures F1 and F2, the covariance ratio would be

$$c_r = \frac{1/2 \text{ covariance of F1} + 1/2 \text{ covariance of F2}}{\text{covariance of the residual without failure}}$$

5.2 Recommendations

The last chapter showed the importance of the failure signature in the design of failure detection relations. Consequently, further research using this concept could be performed. In particular, the following points could be investigated.

As a remedy to the complexity of the double eigenvalue decomposition, it would be interesting to use the covariance ratio concept with a simplified estimator. For example, we could define a structural distance as the inverse of the covariance ratio and try to decrease it. But as this gives a nonlinear expression, the general framework of section 3.3 is not applicable and a new approach must be found.

We have seen that estimated optimized relations have better performance than model based ones. But, whenever possible, it is more convienient to build relations with a model of the system rather than estimate them. Thus, further work could be done on the improvement of model based optimized relations.

Finally, we can use the failure signature idea with parity relations. If we take the number S of time steps to be greater than the minimum, N+1, we can choose between different independent relations. We can then choose the one with the best covariance ratio. For a parity relation, as the state is no longer present, this ratio is

$$c_r = \frac{\text{covariance of the failure signature}}{\text{covariance of the residual due to noise}}$$

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Appendix

PHI,GAMA,C AND FEEDBACK MATRICES OF THE DISCRETIZED SYSTEM

PHI MATRIX

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0.97304D+00 0.98868D-01 0.00000D+00 0.00000D+00 0.00000D+00
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   0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
-.53636D+00 0.97309D+00 0.00000D+00 0.00000D+00 0.00000D+00
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0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
  0.00000D+00 0.00000D+00 -.86791D+00 -.11925D-01 0.00000D+00
   0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
  0.00000D+00 0.00000D+00 0.15485D+02 -.78724D+00 0.00000D+00
   0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
  0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 -.81023D+00
   -.14219D-01 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
  0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.19715D+02
   -.72517D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
```

```
0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
  0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
    0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
      0.00000D+00 -.50700D-01 -.20421D-01 0.00000D+00 0.00000D+00
0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
  0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
    0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
      0.00000D+00 0.44525D+02 0.61761D-01 0.00000D+00 0.00000D+00
0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
  0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
    0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
      0.00000D+00 0.00000D+00 0.00000D+00 0.91579D+00 0.31906D-02
0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
  0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
    0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
      0.00000D+00 0.00000D+00 0.00000D+00 -.13509D+02 0.10418D+01
```

GAMA MATRIX

```
-.74690D-06 0.10471D-03 -.10559D-03 -.14759D-03 0.14743D-03 -.61615D-06
-.14859D-04 0.20831D-02 -.21006D-02 -.29361D-02 0.29330D-02 -.12258D-04
0.29270D-03 0.17438D-03 0.17958D-03 0.36252D-04 0.29221D-04 0.23339D-03
0.57658D-02 0.34351D-02 0.35375D-02 0.71412D-03 0.57562D-03 0.45976D-02
-.46849D-05 -.23567D-03 0.22932D-03 0.29955D-03 -.30134D-03 -.25229D-05
-.86962D-04 -.43746D-02 0.42567D-02 0.55604D-02 -.55935D-02 -.46832D-04
0.20925D-05 -.39623D-03 0.39790D-03 0.36579D-04 -.34719D-04 -.70175D-06
0.34670D-04 -.65649D-02 0.65925D-02 0.60606D-03 -.57523D-03 -.11627D-04
-.27218D-03 0.16225D-03 0.16801D-03 -.22238D-03 -.21857D-03 0.19189D-03
-.35734D-02 0.21301D-02 0.22057D-02 -.29195D-02 -.28695D-02 0.25192D-02
-.16903D-04 0.20369D-03 -.18677D-03 0.13753D-03 -.14224D-03 -.21490D-04
-.36354D-04 0.43807D-03 -.40168D-03 0.29578D-03 -.30590D-03 -.46218D-04
0.49485D-05 -.75041D-04 0.71816D-04 0.63141D-04 -.62195D-04 0.24307D-05
-.41023D-04 0.62209D-03 -.59536D-03 -.52344D-03 0.51560D-03 -.20150D-04
-.16931D-03 0.62137D-04 0.88369D-04 -.18938D-04 -.21604D-05 -.21026D-03
0.18440D-02 -.67672D-03 -.96242D-03 0.20625D-03 0.23529D-04 0.22899D-02
-.45806D-04 0.73897D-04 0.72432D-04 0.25156D-04 0.23476D-04 -.13264D-04
0.19411D-02 -.31315D-02 -.30694D-02 -.10660D-02 -.99483D-03 0.56210D-03
0.41732D-05 -.23847D-05 -.22677D-05 0.24889D-05 0.25446D-05 -.32142D-05
0.66948D-03 -.38255D-03 -.36379D-03 0.39927D-03 0.40820D-03 -.51563D-03
```

C MATRIX

```
-.29744D+01 0.00000D+00 0.36352D+01 0.00000D+00 0.26448D+01
  0.00000D+00 -.30631D+01 0.00000D+00 -.43970D+01 0.00000D+00
    -.25155D+01 0.00000D+00 -.53252D+01 0.00000D+00 0.44874D+01
      0.00000D+00 -.35185D+01 0.00000D+00 -.55109D+01 0.00000D+00
-.15329D+01 0.00000D+00 0.31972D+01 0.00000D+00 -.10910D+01
  0.00000D+00 -.37494D+01 0.00000D+00 0.14606D+01 0.00000D+00
    0.19290D+01 0.00000D+00 0.97648D+00 0.00000D+00 0.24219D+01
      0.00000D+00 0.38126D+01 0.00000D+00 0.57296D+00 0.00000D+00
-.45793D+00 0.00000D+00 -.19070D-01 0.00000D+00 -.34954D+01
  0.00000D+00 0.16891D+01 0.00000D+00 0.71773D-01 0.00000D+00
    -.41499D+01 0.00000D+00 -.33471D+01 0.00000D+00 0.18233D+00
      0.00000D+00 -.15536D-01 0.00000D+00 -.11965D+00 0.00000D+00
0.00000D+00 0.21070D-01 0.00000D+00 0.35452D-01 0.00000D+00
  -.50777D-01 0.00000D+00 -.94270D-01 0.00000D+00 0.45560D-01
    0.00000D+00 0.93361D-01 0.00000D+00 -.52166D-01 0.00000D+00
      0.47592D-01 0.00000D+00 0.15335D+00 0.00000D+00 -.11990D+00
0.00000D+00 0.29666D-01 0.00000D+00 0.59407D-02 0.00000D+00
  -.64926D-01 0.00000D+00 -.82602D-02 0.00000D+00 -.61374D-01
    0.00000D+00 -.65193D-01 0.00000D+00 -.43236D-01 0.00000D+00
      -.16547D-02 0.00000D+00 0.48717D-01 0.00000D+00 0.12794D+00
0.00000D+00 -.12398D-03 0.00000D+00 0.47450D-01 0.00000D+00
  -.54359D-03 0.00000D+00 -.16696D-03 0.00000D+00 0.53881D-01
    0.00000D+00 -.98498D-02 0.00000D+00 0.16897D-02 0.00000D+00
      -.16104D+00 0.00000D+00 -.27526D-01 0.00000D+00 -.16161D+00
```

FEEDBACK MATRIX

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FEEDBACK MATRIX
---------------
0.14950D+03 0.13786D+02 -.72620D+02 -.22225D+02 -.20548D+03
  -.22394D+02 -.17827D+03 0.37739D+01 0.19074D+03 0.28213D+02
    -.25806D+03 0.15741D+02 -.36949D+03 -.99498D+01 0.12244D+04
      -.25387D+02 -.13704D+04 -.23778D+02 -.54601D+03 0.11580D+03
0.50290D+02 0.43523D+01 -.84909D+02 -.11459D+02 -.14638D+02
  0.57319D+00 0.24451D+02 0.78455D+01 0.13988D+02 -.19359D+00
    0.34932D+02 -.28341D+01 0.19625D+02 -.61247D+00 0.45736D+03
      -.95877D+01 0.77038D+02 0.29286D+01 -.63617D+02 0.75571D+01
0.11623D+03 0.15096D+02 -.10481D+03 -.13465D+02 0.59882D+02
  -.26782D+01 -.62854D+01 0.23082D+01 0.13151D+03 0.11916D+02
    -.21246D+03 0.96474D+01 -.35754D+03 -.60771D+00 0.51946D+03
      -.47152D+01 0.35546D+03 -.61223D+01 -.28655D+03 0.54749D+02
0.14620D+03 0.23374D+02 -.31285D+03 -.61244D+01 0.33897D+03
  0.16542D+02 0.23268D+03 0.16369D+02 -.20604D+03 -.14725D+02
    0.26666D+02 0.42290D+01 -.23755D+03 0.10910D+02 -.19795D+04
      0.91990D+02 0.16074D+04 0.20036D+02 0.12976D+04 -.11655D+03
0.14380D+03 0.21557D+02 -.29216D+03 -.47220D+01 0.42863D+03
  0.34292D+02 0.22286D+03 0.10510D+02 -.24284D+03 -.18468D+02
    -.12686D+03 0.53227D+00 -.27647D+03 0.83669D+01 -.19063D+04
      0.87011D+02 0.15243D+04 0.22254D+02 0.13626D+04 -.12981D+03
0.26148D+03 0.41327D+02 -.42751D+03 -.13664D+02 0.59954D+03
  0.32157D+02 0.10517D+03 0.15766D+02 0.52898D+02 -.67932D+01
    -.21183D+03 0.34604D+02 -.66617D+03 0.11061D+02 -.28997D+04
      0.13820D+03 0.86976D+03 0.18481D+02 0.72096D+03 -.55849D+02
```