Thermoviscoplastic Nonlinear Constitutive Relationships for Structural Analysis of High Temperature Metal Matrix Composites

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THERMOVISCOPLASTIC NONLINEAR CONSTITUTIVE RELATIONSHIPS
FOR STRUCTURAL ANALYSIS OF HIGH TEMPERATURE METAL MATRIX COMPOSITES

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SUMMARY

A set of thermoviscoplastic nonlinear constitutive relationships (TVP-NCR) is presented. This set is unique and has been developed mainly for application to high-temperature metal-matrix composites (HT-MMC) and is applicable to thermal and mechanical properties. Formulation of the TVP-NCR is based at the micromechanics level. The TVP-NCR are of simple form and readily integrated into nonlinear composite structural analysis. Results show that this unique set of TVP-NCR is computationally effective. It provides a direct means for predicting complex materials behavior at all levels of the composite simulation; that is, from the constituent materials, through the several levels of composite mechanics, and up to the global response of complex HT-MMC structural components.

INTRODUCTION

High temperature metal matrix composites (HT-MMC) are emerging as materials with potentially high payoffs in structural applications. Realization of these payoffs depends on the parallel and synergistic development of (1) a technology base for fabricating HT-MMC structures and components, (2) experimental techniques for measuring their thermal and mechanical characteristics, and (3) computational methodologies for predicting their nonlinear thermovisco-plastic (TVP) behavior in complex service environments. In fact, development of computational methodologies should precede the other two because the structural integrity and durability of HT-MMC can be numerically assessed and the potential payoff for the specific application can be closely estimated. In this way, it is possible to minimize the costly and time consuming experimental effort that would otherwise be required in the absence of a predictive capability.

Recent research at NASA Lewis is directed towards the development of a computational capability to predict the nonlinear TVP behavior of HT-MMC. This capability is schematically depicted in figure 1. As can be seen in this figure the capability consists of several computational modules encompassing the material TVP behavior (bottom), composite mechanics (sides), and the finite element analysis of structural components (top). The TVP computational module consists of mathematical models which formally and explicitly relate the dependence of the constituent material properties on

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(1) temperature, (2) stress, and (3) time. These mathematical models are collectively called thermovisco-plastic nonlinear constitutive relationships (TVP-NCR). The objective of this report is to present the TVP-NCR developed as a part of the computational capability for HT-MMC structures.

The TVP-NCR to be described were developed with an emphasis on computational effectiveness and are unique in their following features: (1) generic form - they are applicable to all constituent material properties (thermal, mechanical, including strength), (2) evolutionary - they are easily extended/modified to accommodate additional effects such as strain rate and material degradation, (3) isomorphic - they have similar structure for all the properties, (4) universal - they are fully coupled at both the increment and cumulative levels, (5) universal - they are applicable to any of the three constituent materials (fibers, matrix, and the interphase), and (6) nondimensional - they are normalized with respect to initial, reference, and ultimate states.

This unique set of TVP-NCR consists of products of terms with unknown exponents. The exponents are determined for the specific material and type of nonlinear dependence. For example, three product terms are required in one TVP-NCR equation to account for: (1) temperature dependence, (2) stress level, and (3) stress rate. Three exponents need to be determined to completely describe this equation. These exponents are determined from available experimental data or estimated from anticipated behavior of the particular product term.

The computational effectiveness of this unique set of TVP-NCR is evaluated using the computational capability depicted schematically in figure 1. The structural response of a turbine blade, made from fiber reinforced superalloy HT-MMC and subject to representative mission loading conditions, is determined. The effectiveness of the TVP-NCR to represent the physical behavior of the material is computationally assessed by perturbing the exponents and comparing the structural response of the blade to the unperturbed case.

**EQUATION FORM/FEATURES**

The generic form selected for the TVP-NCR for the constituents of HT-MMC is as follows:

1. **Mechanical property (moduli, strength) \( P_M \)**
   
   \[
   \frac{P_M}{P_{M_{0}}} = \left[ \frac{1}{M} - 1 \right]^{n} \left[ \frac{S_F - \sigma}{S_F - \sigma_0} \right]^{m} \left[ \frac{S_F - \sigma_0}{S_F - \sigma} \right]^{k} \tag{1}
   \]

2. **Thermal property (expansion coefficients, thermal conductivity, heat capacity) \( P_T \)**
   
   \[
   \frac{P_T}{P_{T_{0}}} = \left[ \frac{1}{T} - 1 \right]^{n} \left[ \frac{S_F - \sigma_0}{S_F - \sigma} \right]^{m} \left[ \frac{S_F - \sigma}{S_F - \sigma_0} \right]^{k} \tag{2}
   \]
The notation used in equations (1) and (2) is as follows:

- \( P_{M,T} \) denotes the current property of interest.
- \( P_0 \) is the corresponding property at reference conditions.
- \( T_M \) is the melting temperature.
- \( T \) is the current temperature.
- \( T_0 \) is the reference temperature at which \( P_0 \) is determined.
- \( S_F \) is the fracture stress determined at \( T_0 \) conditions.
- \( \sigma_0 \) is the reference stress at which \( P_0 \) is determined.
- \( \dot{S}_F \) is an appropriately selected stress rate, for example, the stress rate at which penetration occurs during impact.
- \( \dot{\sigma}_0 \) is the stress rate at which \( P_0 \) is determined, and
- \( \dot{\sigma} \) is the current stress rate.

The exponents \( n, m, \) and \( \varepsilon \) are empirical parameters which are determined from available experimental data or estimated from the anticipated behavior of the particular product term.

The first term on the right side of equation (1) represents the temperature dependence, the second represents the stress dependence, and the third represents the rate dependence or the time dependence, in part. The other part of the time dependence is through the direct time integration as will be described later. Equations (1) and (2) describe, then, material behavior in the temperature-stress-time space.

Each term on the right side of equations (1) and (2) describes a monotonic functional dependence of \( P/P_0 \) from some initial property value to a terminal or ultimate material state. The specific shape of the function depends on the exponent as is shown in figure 2 for a fixed exponent and in figure 3 for a fixed \( T/T_0 \) ratio. By judicious selection of the exponent and the initial and terminal values, a variety of functional dependences can be simulated using equations (1) and (2).

The form of equations (1) and (2) makes it convenient (provides direct feedback) to select the various parameters so that the functional dependence described is consistent with the physical considerations. For example, it is well known that the melting temperature (\( T_M \)) is a fundamental parameter in metals and that the mechanical properties are "zero" at \( T_M \). Also, the stress at fracture for some reference condition is readily determined by simple experiments. In addition, the ultimate value of the stress rate may be determined from high velocity impact penetration tests. It can be seen from figure 2 that the \( P/P_0 \) increases/decreases very rapidly as the melting temperature (or any terminal value) is approached. Furthermore, the form of the equations make it convenient to evaluate the exponents from available data since each term is "isolated" from the others, that is, the other terms...
can be taken at reference conditions and will be unity. The equations are computationally effective since each term requires simple substitution of values and one exponentiation.

The form of the TVP-NCR selected has all the desirable features mentioned in the introduction. These features can be conveniently summarized into three groups: (1) physical, (2) fundamental, and (3) computational, as follows:

(1) Physical - The constitutive relationships describe dependence on: temperature, time, stress, stress rate, and complete property degradation as the ultimate value is approached.

(2) Fundamental - the constitutive relationships are:

- generic - they are applicable to all constituent material properties (fig. 4)
- evolutionary - they are easily extended to include additional dependence, for example cyclic (mechanical, thermal)
- isomorphic - they have the same form for all the properties
- unified - they are fully coupled from the initial to the terminal material state
- universal - they are equally applicable to any three constituents (fibers, matrix, interphase)
- nondimensional - they are normalizable with respect to reference and ultimate values.

(3) Computational - the constitutive relationships are:

- computationally effective - they only require simple substitution and exponentiation
- easily integrated into nonlinear composite mechanics and structural analysis codes - they can be fully integrated using only a few programming statements

APPLICATION

The TVP-NCR were integrated into a special-purpose computer code for structural analysis of turbine blades made from HT-MMC (ref. 1). This code is depicted schematically in figure 1. The TVP-NCR describe the constituent material properties in the material space as shown at the bottom of the figure. Note that the cumulative time, temperature, and stress at the current state are tracked through the integrated computational process as shown in the figure. Once the current properties for the constituent materials have been determined, they are used in the various levels of composite mechanics to generate the quantities required for the global structural analysis (ref. 1).
The finite element model of the component (HT-MMC hollow turbine blade airfoil), selected to demonstrate the computational effectiveness of the IVP-NCR, is shown in figure 5. For the analysis, typical engine mission loads were simulated including temperature, pressure and rotor speeds as shown in figures 6(a) and (b) expanded to show start-up and shut-down details. The airfoil was assumed to be made from tungsten-fiber/superalloy HT-MMC. The volume ratio of the fiber was 0.5. The laminate configuration in the airfoil was a four-ply (±45)_s angleplied laminate. The airfoil structure is a thin-wall hollow shell. The hollow structure is required for internal cooling and minimum weight. Some of the constituent material properties are available in the literature. Others (especially temperature, stress and strain-rate-dependent properties) are estimated or deduced from other relevant data.

The structural analyses were performed to determine global and local structural response. The global structural response variables are tip displacement, untwist, and frequencies. The local response variables are ply stresses and constituent stresses at the various nodes. The examples presented below of local response correspond to the outermost ply (ply no. 4) at the nodal point identified by the arrow in figure 5 (node no. 6). Two sets of structural analyses were performed. The first set was performed to determine the global and local structural response at the baseline properties condition. The second set was performed to determine the sensitivity of the structural response to arbitrary perturbations of the IVP-NCR exponents. The second set of analyses demonstrates the importance of computational simulation since it provides an assessment of the accuracy of the data required to experimentally determine the parameters in the IVP-NCR.

The results obtained from both structural analyses are summarized in table I for the global variables (cruise and residual state) and in table II for the local variables (cruise state only). The results are grouped into cases 1-7. Case 1 is the baseline case and has the "best" values for the parameters in the IVP-NCR. Cases 2-7 constitute the sensitivity analyses. The perturbed values of the exponents and their contribution (increase (I)/decrease (D)) to the particular constituent material property (PM or Pj) are listed next to the case. The percent change with respect to the baseline case is listed in the last column of the tables.

It can be seen from the results in tables I and II that the changes in the global and local structural response variables, due to the perturbations in the exponents, are about 15 percent or less. This is a very significant result as it implies that very careful experiments would be necessary to obtain data for these exponents which is accurate to within 15 percent especially in these high temperature ranges.

The greatest change (16 percent) in the frequency occurs in case 3 (table I) where the perturbations in the exponents decrease the mechanical properties and increase the thermal properties of the matrix. The greatest change (15 percent) in the ply stress occurs in case 6 (table II) where the perturbations in the exponents (1) increase the fiber mechanical properties but decrease those of the matrix, and conversely (2) decrease the thermal properties of the fiber but increase those of the matrix. The residual state ply stresses for all cases are negligible for all practical purposes.
The integrated analysis generates properties at all levels of the composite behavior simulation. Behavior of the longitudinal (fiber direction) modulus \( (E_{11}) \) of the constituents and ply are shown graphically in figures 7(a) and (b) for cases 1 and 6 throughout the mission duration. The modulus decreases initially during the start-up and climb part of the mission. The modulus levels off during the steady state (cruise) part of the mission. Finally, the modulus increases during the landing and engine cut-off part of the mission. The significant point is that the TVP-NCR appear to represent the material behavior as would be intuitively anticipated for this type of flight mission.

The corresponding behavior for the transverse (perpendicular to fiber) modulus \( (E_{22}) \) is shown in figures 8(a) and (b) and for the in-plane shear modulus \( (G_{12}) \) in figures 9(a) and (b). The behavior of these moduli is about the same as that for the longitudinal modulus \( (E_{11}) \).

The behavior of the longitudinal thermal expansion coefficient \( (\alpha_{11}) \) throughout the mission is shown in figures 10(a) and (b) for cases 1 and 6. This coefficient initially increases rapidly (during climb), levels off during cruise, and slowly decreases to about its initial value during landing. This type of behavior is to be expected since the thermal expansion coefficients increase with increasing temperature. Note that even though \( \alpha_{11} \) for the matrix for case 6 is about 30 percent smaller than for case 1, the coefficients for the ply and the interphase are about the same. This illustrates, in part, the restraining influence of the fibers in the HT-MMC behavior and in addition the importance of having TVP-NCR defined at the micromechanics level. The corresponding behavior for the transverse thermal expansion coefficient \( (\alpha_{22}) \) is shown in figures 11(a) and (b). The behavior of \( \alpha_{22} \) is similar to that of \( \alpha_{11} \).

The behavior of the three lowest natural frequencies of the airfoil throughout the flight mission is shown in figures 12(a) and (b) for cases 1 and 6. Note that each frequency behaves somewhat differently. This is expected since each frequency is influenced differently by the centrifugal force. The second and third frequencies decrease during climb, increase during the early part of cruise, remain constant during the major portion of the cruise, and increase gradually to about their initial value during landing and engine cut-off. On the other hand, the first frequency increases sharply during climb (due to centrifugal force stiffening), levels off during cruise, increases slightly during landing (cooling but speed retained) and gradually decreases to approximately its initial value (zero-speed). The coupled behavior of these three frequencies throughout the flight mission further demonstrates the computational effectiveness of the TVP-NCR to represent the physics of the HT-MMC from the constituent materials level to the component global structural response.

The longitudinal stress \( (\sigma_{11}) \) behavior throughout the mission in the constituents and in the ply is shown in figures 13(a) and (b) for cases 1 and 6. The stress in the fiber increases very rapidly during climb, decreases gradually during cruise, decreases rapidly during landing, and decreases gradually to a small residual compressive stress at engine cut-off. The ply stress exhibits the same behavior as the fiber stress but is much lower in magnitude. The stress in the matrix increases (compressively) very rapidly.
during climb, remains compressive during cruise and decreases gradually to a residual tensile value during landing and engine cut-off.

The corresponding behavior for the transverse stress \( \sigma_{22} \) is shown in figures 14(a) and (b) and that for the intralaminar shear stress \( \sigma_{12} \) in figures 15(a) and (b). Note there are three different regions (A, B, and C) for the matrix and two different regions (B and C) for the interphase in which \( \sigma_{22} \) and \( \sigma_{12} \) are computed. These regions correspond to the intralaminar regions shown in figure 4. The interesting points to note are: (1) the matrix in the different regions is subjected to both tensile and compressive transverse stresses which can be of substantial magnitude, (2) the interphase can be subjected to relatively high transverse tensile stresses which may cause interfacial damage, (3) the fiber is subjected to very high transverse tensile stresses which could cause fiber splitting, and (4) the transverse ply stress is relatively small compared to the stress distribution in the constituents.

Collectively, the local stress behavior demonstrates the computational effectiveness of the IVP-NCR to predict the instantaneous behavior of the constituents at the micromechanics level as well as at the ply (macromechanics) level. Determination of the stress behavior in the constituents is possible because the IVP-NCR are referred to the constituent material space and the formulation is based at the composite micromechanics level. The gradual decrease indicated for all the stresses during cruise is caused by the material thermoviscoplastic behavior and may be thought of as a form of creep.

POSSIBLE EXTENSIONS/LIMITATIONS

The IVP-NCR can be extended to include thermal cycle and mechanical cycle effects, diffusion, other material degradation effects, as well as time directly. Though terms for these factors are easily added since they will be of similar form, it is not necessarily clear which of these will contribute to independent material behavior.

In the direct time integration analysis, the effects of temperature are directly accounted for. Any ratchetting, for example, will be part of the residual displacements. Also, residual thermal stresses and other stresses are known and constitute a part of the cumulative stress history. On the other hand, vibratory stress effects are not accounted for in the direct time integration of the structural analysis. Though these effects can be accounted for through the stress rate, vibratory stress effects may indeed contribute to independent behavior. Diffusion or any other material degradation can be incorporated once the type of degradation has been defined. Other extensions will become self evident as HT-MMC start being extensively applied in environments where limited or no property data are available.

Some limitations of the IVP-NCR described herein are that they: (1) must be used at the current instant of time, (2) must be used with a direct time integration nonlinear composite structural analysis, (3) cannot be verified experimentally at all levels of the composite mechanics analysis and at the very high temperatures, and (4) do not incorporate initial tangent unloading or possible shakedown in the classical plasticity sense. Whether (3) and (4) are serious limitations is yet to be determined. At this stage
of the development it is prudent to say that these TVP-NCR must be used judiciously in design studies relying mainly on sensitivity analyses and other judgment factors that are appropriate for the specific case.

CONCLUSIONS

A unique set of thermoviscoplastic nonlinear constitutive relationships (TVP-NCR) for high-temperature metal-matrix composites (HT-MMC) has been developed and is presented. This set of TVP-NCR is of simple form, is applicable to all thermomechanical properties, is fully coupled, is readily integrated into nonlinear composite structural analyses, and is computationally effective. Applicability and computational efficiency were demonstrated through an application to a HT-MMC turbine blade structural analysis. Sensitivity analyses indicated that substantial perturbations in the TVP-NCR exponents have rather minimal effect on the global and local response of the structure. These TVP-NCR make it possible to trace the history of HT-MMC structural components from fabrication through service and from the composite micromechanics to global composite structural response. The TVP-NCR are suitable for preliminary designs and parametric studies. They should be judiciously used in design applications since they have not been experimentally verified as yet.

REFERENCE

TABLE I. - EXPONENT PERTURBATION EFFECTS ON STRUCTURAL RESPONSE

<table>
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<tr>
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P₁ - Thermal Prop.
R - Reference
I - Increase
D - Decrease

Units: Displ. - In: Unit's - Deg: Freq., Hz
# TABLE II. - EXponent Perturbation Effects ON Ply Stress

<table>
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<th>Case</th>
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<th>Exponent</th>
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- $P_T$ - Thermal Prop.
- R - Reference
- I - Increase
- D - Decrease
Fig. 1. - Integrated nonlinear composite structural analysis.

Fig. 2. - Thermoviscoelastic nonlinear constitutive relationships - typical behavior for a given exponent.
Fig. 3. - Thermoviscoelastic nonlinear constitutive relationships typical behavior for given $T/T_F$ ratio.

Fig. 4. - Typical interconstituent regions for composite micromechanics.
Fig. 5. - Finite-element model for hollow turbine blade airfoil of high temperature metal matrix composite.

Fig. 6. - Flight mission profile for turbine blade structural analysis. Node 6; ply 4.
Fig. 7. - Longitudinal modulus behavior during flight mission. Node 6, ply 4.
Fig. 8. - Transverse modulus behavior during flight mission. Node 6; ply 4.
Fig. 9. Intralaminar shear modulus behavior during flight mission. Node 6; ply 4.
Fig. 10. - Longitudinal thermal expansion coefficient behavior during flight mission. Node 6; ply 4.
Fig. 11. - Transverse thermal expansion coefficient behavior during flight mission.
Fig. 12. - Turbine airfoil frequencies behavior during flight mission.
Fig. 13. - Longitudinal stress variation during flight mission. Node 6; ply 4.
Fig. 14. - Transverse stress variation during flight mission. Node 6; ply 4.
Fig. 1.5. Intralaminar shear stress variation during flight mission. Node 6; ply 4.
Thermoviscoplastic Nonlinear Constitutive Relationships for Structural Analysis of High Temperature Metal Matrix Composites

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A set of thermoviscoplastic nonlinear constitutive relationships (TVP-NCR) is presented. This set is unique and has been developed mainly for application to high-temperature metal-matrix composites (HT-MMC) and is applicable to thermal and mechanical properties. Formulation of the TVP-NCR is based at the micro-mechanics level. The TVP-NCR are of simple form and readily integrated into nonlinear composite structural analysis. Results show that this unique set of TVP-NCR is computationally effective. It provides a direct means for predicting complex materials behavior at all levels of the composite simulation; that is, from the constituent materials, through the several levels of composite mechanics, and up to the global response of complex HT-MMC structural components.