IMPROVED ISOPARAMETRIC SOLID AND MEMBRANE ELEMENTS

by

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Summary

Improvements that have been made to the COSMIC NASTRAN elements CIHEX1 and QDMEM1 are described. These elements are isoparametric representations of solid and membrane elastic behavior. Recent papers by the authors have shown the official COSMIC versions of these elements to be inferior to those available in the MacNeal-Schwendler Corporation (MSC) version of NASTRAN in that they are overly stiff for some loadings. Modifications have been made to these elements which reduce the order of integration for shear terms and, for the eight-mode solid element, add additional strain functions. The resulting element formulations give behavior similar to that of the MSC elements. The paper discusses the changes made in the element formulations and compares results of test problems with results from the official COSMIC elements and with the MSC elements.

Introduction

The isoparametric membrane quadrilateral element, QDMEM1, in COSMIC NASTRAN is a stand-alone element for use in modeling problems which exhibit plane stress behavior. It is a stand-alone element because there is no general plate element which currently uses the QDMEM1 for the membrane stiffness. In contrast, the MSC uses the QDMEM1 element for the membrane part of their QUAD4 general plate element.

The results of a finite element idealization study using all of the available membrane elements in NASTRAN was reported in [1]. Although the written version of the paper reported results only for elements available in MSC NASTRAN version 38, the version presented orally at the NASTRAN Colloquium showed results using both MSC-38 and COSMIC-15.5. As presented at the colloquium, there was a marked difference in results for the QDMEM1 elements from these two versions of NASTRAN. At the time, it was surmised that the discrepancy was due to a different manner in which the numerical integration was carried out in the two versions. In particular, it was shown that the COSMIC element exhibited overly stiff behavior for the problems investigated and that the MSC element was vastly superior.

A similar study was conducted by the authors for solid elements and reported in [2]. This study was aimed at finding the best of the available solid elements to model thermal and gravity deformation effects on optical mirrors. Of particular importance was investigation of problems that might be encountered with elements that have aspect ratios in the range of 5 to 10. Use of elements with this range of aspect ratio is necessary in modeling mirrors to avoid the need of extremely large models which would be required if element aspect ratios near unit were necessary. The results of this study again clearly indicated the superiority of the MSC solid elements for modeling relatively thick plates for bending, as would occur for mirrors subjected to thermal gradients. For the eight-node solid isoparametric elements, it was again shown that there was a significant difference between the MSC element (HEXA-8) and the COSMIC element (CIHEX1). In addition, the COSMIC element showed extreme sensitivity to aspect ratio. Elements that had a thickness (in the plate thickness direction) smaller than its in-plane dimensions exhibited large errors. This was to be expected based on the above discussion of the membrane element deficiencies and the similarity of the formulation for the membrane and solid elements.

The purpose of the effort reported herein, then, was to investigate modifications that could be made to the COSMIC QDMEM1 and CIHEX1 elements which would improve their accuracy for modeling bending type behavior.

The next section discusses the cause of the overly stiff behavior of these elements. This has been investigated by others, [3] - [6], and is shown to be due to a parasitic shear that is introduced when these lower order isoparametric elements have bending modes of deformation.

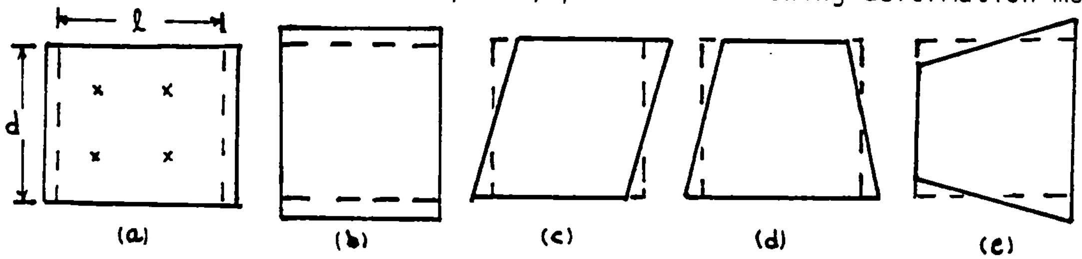
The following section describes the modifications that are required for the CIHEX1 element in order for it to behave as the MSC HEXA-8 element. The authors gratefully acknowledge the MSC for providing the mathematical description [7] of their modifications to the eight-node solid element and the QUAD4 general shell element.

Following this is a brief description of the changes that are required for the QDMEM1 element, which consisted only of reducing the Gaussian integration order for the shear strain terms.

Finally, the results of several problems are presented showing the improvement of the modified COSMIC elements in comparison to the officially installed element.

Parasitic Shear in Bending in Lower Order Isoparametric Elements

The quadrilateral membrane element, QDMEM1, has four grid points with stiffness for the 2 in-plane degrees of freedom at each grid point yielding a total of 8 degrees of freedom for the element. As discussed in [3], the 8 degrees of freedom can be considered to be linear combinations of eight nodes of deformation, consisting of the three rigid body modes (two translation and one rotation in its plane) plus the following deformation modes:



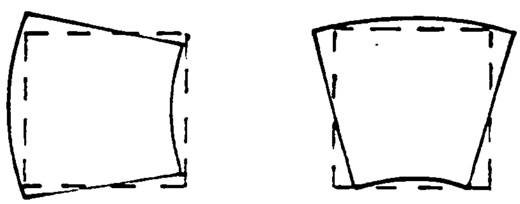
x: location of Gauss Points for 2x2 integration

Modes (a) — (c) are the three constant strain modes while (d) and (e) are similar to bending modes. Generally, Gaussian quadriture is used to evaluate the stiffness matrix for isoparametric elements in which case the 2x2 stiffness matrix for a pair of grid points is

$$\underset{\sim}{K}_{ij} = \sum_{g} w_g J_g \underset{\sim}{C}_{gi}^{T} \underset{\sim}{G}_g \underset{\sim}{C}_{gj}$$

The following section shows the details of this (for the 3-D element) with explanations of the terms.

For now, it is sufficient to point out that the summation in the above equation is over the Gauss points (four in this case) and that $\underset{\sim}{C}_{gi}$ is the matrix relating displacements at grid point i to strains at Gauss point g. At these Gauss points, the terms in the $\underset{\sim}{C}_{gi}$ matrix relating to shear strains are nonzero for the bending modes (d) and (e). Thus (d) and (e) modes will contribute shear strain energy in a situation where the element is used to model pure bending situations. In fact, as the element aspect ratio (ℓ/d) increases, this parasitic shear becomes a dominant part of the strain energy and the element becomes excessively stiff for modeling bending. If, instead of evaluating the terms in $\underset{\sim}{C}_{gi}$ which relate to shear strain at the Gauss points, terms were evaluated at the element center, it would be found that no shear strain energy would result in modes (d) and (e) since the shear strain is zero at the center. Then, since the shear is zero for modes (d) and (e) under this evaluation, modes (d) and (e) would indeed be pure bending; and the "effective" deformation in these modes would be:



This is the motivation behind "reduced integration." One way to enforce the shear terms in $\underset{\sim}{C}_{gi}$ to be evaluated at the center is to use a 1x1 Gaussian integration (g=1) in evaluating the above stiffness matrix equation. However, another way (which preserves the element volume) is to use the required Gaussian integration order needed to exactly evaluate the volume integrals (2x2 in this case) and evaluate the appropriate $\underset{\sim}{C}_{gi}$ shear terms at the element center instead of at the Gauss point. It is this latter approach which is taken in modifying the COSMIC elements.

The solid isoparametric element CIHEX1 has the same difficulty but in three dimensions and is modified with a similar reduced order integration for shear.

Modified CIHEX1 Element

For a 3-D solid isoparametric element, the displacements at any point in the interior of the element are expressed as (see Figure 9)):

$$\underset{\sim}{\bar{\Delta}} = \sum_i N_i \underset{\sim}{\Delta}_i \tag{1}$$

where

$$\underset{\sim}{\bar{\Delta}} = \begin{Bmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{Bmatrix} , \qquad \underset{\sim}{\Delta}_i = \begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} \tag{2}$$

The summation on i is taken over all grid points of the element with $\underset{\sim}{\Delta}_i$ being the vector of grid point i displacements. The N_i are isoparametric interpolating functions in terms of the ξ, η, ζ coordinates which map the general hexahedron, in x,y,z coordinates into a rectangular parallelepiped in ξ, η, ζ coordinates. For an eight-node hex element:

$$N_i = \frac{1}{8}(1+\xi\xi_i)(1+\eta\eta_i)(1+\zeta\zeta_i) \tag{3}$$

with ξ_i, η_i, ζ_i the coordinates of grid point i

The element strains are related to displacements by

$$\underset{\sim}{\epsilon}_g = \sum_i \underset{\sim}{C}_{gi} \underset{\sim}{\Delta}_i \tag{4}$$

where

$$\underset{\sim}{\epsilon}_g^T = \lfloor \epsilon_x \;\; \epsilon_y \;\; \epsilon_z \;\; \gamma_{xy} \;\; \gamma_{xz} \;\; \gamma_{yz} \rfloor_g^T$$

is a vector of the element strains evaluated at the Gauss points

and

$$\underset{\sim}{C}_{gi} = \begin{bmatrix} N_{i,x} & 0 & 0 \\ 0 & N_{i,y} & 0 \\ 0 & 0 & N_{i,z} \\ N_{i,y} & N_{i,x} & 0 \\ 0 & N_{i,z} & N_{i,y} \\ N_{i,z} & 0 & N_{i,x} \end{bmatrix}_g \tag{5}$$

Note: $\sim$ under a quantity indicates it is a matrix.

where the comma denotes partial differentiation with respect to x,y, or z and the subscript g on the matrix indicates that the terms in the matrix are to be evaluated for ξ, η, ζ at a Gauss point. This is the form of the strain-displacement relationship for the classical eight-node hex element and is what is employed in the COSMIC CIHEX1 element. Element stresses at Gauss points are related to strains through

$$\underset{\sim}{\sigma}_g = \underset{\sim}{G}_g(\underset{\sim}{\epsilon}_g - \underset{\sim}{\alpha} T) \tag{6}$$

where $\underset{\sim}{G}_g$ is the 6x6 constitutive matrix of material constants at a Gauss point.

The element stiffness matrix is obtained using Gaussian quadriture from

$$\underset{\sim}{K}_{ij} = \sum_g w_g J_g \underset{\sim}{C}^T_{gi} \underset{\sim}{G}_g \underset{\sim}{C}_{gj} \tag{7}$$

where $\underset{\sim}{K}_{ij}$ is a 3x3 partition of the element stiffness matrix relating forces at grid point i to displacements at grid point j. For exact integration of the eight-node hex element, a 2x2x2 (g=8) Gaussian quadriture must be employed. Wg and Jg are Gauss integration weights and Jacobian.

It has been observed that the stiffness matrix thus derived can exhibit overly stiff behavior when modeling bending situations due to the presence of parasitic shear in the element. One technique of overcoming this difficulty is to employ a reduced integration (i.e., g less than 2x2x2). However, a general reduction of the integration order is not needed and can indeed lead to singular stiffness even when the element is restrained in a rigid body fashion. Instead, selective reduced integration is employed wherein only the terms in equation (7) which relate to shear strains have their integration order reduced.

The HEXA eight-mode element in MSC NASTRAN employs reduced integration for shear. As explained by R. Harder [7], the MSC element uses for $\underset{\sim}{C}_{gi}$ the modified form:

$$\underset{\sim}{C}^r_{gi} = \begin{bmatrix} N_{i,x} & 0 & 0 \\ 0 & N_{i,y} & 0 \\ 0 & 0 & N_{i,z} \\ \bar{N}_{i,y} & \bar{N}_{i,x} & 0 \\ 0 & \bar{N}_{i,z} & \bar{N}_{i,y} \\ \bar{N}_{i,z} & 0 & \bar{N}_{i,x} \end{bmatrix}_g \tag{8}$$

The terms in the upper half of equation (8) are identical to the form used in the classical isoparametric element. The terms in the lower portion of $\underset{\sim}{C}^{r}_{qi}$ are different from their classical counterparts due to employing a reduced integration scheme. Harder proposes to use a Gaussian weighted average of the $N_{i,x}$, etc., to obtain $\bar{N}_{i,x}$, etc.

$$\left(\bar{N}_{i,x}\right)_q = \frac{\sum_{q}^{group} J_q \left(N_{i,x}\right)_q}{\sum_{q}^{group} J_q} \tag{9}$$

with a similar definition for $\bar{N}_{i,y}$ and $\bar{N}_{i,z}$. The summations in equation (9) are taken over some of the Gauss points. In particular, when evaluating terms in the fourth row of equation (8), the group summation is over all Gauss points in the plane in which ζ is a constant. For terms in the fifth row, the group is all points in a plane of constant ξ and, for the sixth row, a plane of constant η. Harder shows that this Gaussian weighted averaging (thus reduced integration) scheme is necessary for the element to maintain its capability to pass a constant strain patch test.

In addition to employing this reduced integration for the shear terms, by averaging the related $\underset{\sim}{C}_{qi}$ coefficients, Harder also employs additional "strain functions" in the MSC element to allow higher order polynomial variation of the direct strain terms. Strain functions are somewhat like bubble modes (see, for example, [8]), which have been used in some elements to also overcome their relatively stiff behavior in bending problems. Conceptually, the additional strain terms are included by modifying the basic strain-displacement relation of equation (4). Following the development in [7]:

$$\underset{\sim}{\epsilon}_q = \sum_i \underset{\sim}{C}^{r}_{qi} \underset{\sim}{\Delta}_i + \underset{\sim}{C}_{qo} \underset{\sim}{\Delta}_o \tag{10}$$

where $\underset{\sim}{C}^{r}_{qi}$ is the classical $\underset{\sim}{C}_{qi}$ modified to represent the reduced order integration for shear.

$\underset{\sim}{C}_{qo}$ is a 6xn matrix of strain coefficients and $\underset{\sim}{\Delta}_o$ a vector of the amplitudes of the n strain functions added. Recognizing that the strain energy for a linear material element is

$$U = \frac{1}{2} \sum_q w_q J_q \underset{\sim}{\epsilon}^{T}_q \underset{\sim}{\sigma}_q$$

and minimizing U with respect to the $\underset{\sim}{\Delta}_o$ amplitudes (keeping mind that $\underset{\sim}{\sigma}_q$ is a function of $\underset{\sim}{\Delta}_o$ through equations (6) and (10), it is found that

$$\frac{\partial U}{\partial \underset{\sim}{\Delta}_o} = 0 = \sum_q w_q J_q \underset{\sim}{C}^{T}_{qo} \underset{\sim}{\sigma}_q \tag{11}$$

Using (6) and (10) in (11), the $\underset{\sim}{\Delta}_o$ amplitudes are found as

$$\underset{\sim}{\Delta}_o = \underset{\sim}{K}^{-1}_{oo} \sum_i \underset{\sim}{K}^{r}_{oi} \underset{\sim}{\Delta}_i \tag{12}$$

where

$$\underset{\sim}{K}_{oo} = \sum_{q} w_q J_q \underset{\sim}{C}^T_{qo} \underset{\sim}{G}_q \underset{\sim}{C}_{qo} \tag{13}$$

$$\underset{\sim}{K}^r_{oi} = \sum_{q} w_q J_q \underset{\sim}{C}^T_{qo} \underset{\sim}{G}_q \underset{\sim}{C}^r_{qi} \tag{14}$$

Finally, in combining (10) and (12), it is found that the strain displacement law in terms of only the physical degrees of freedom, $\underset{\sim}{\Delta}_i$, is

$$\underset{\sim}{\epsilon}_q = \sum_{i} \underset{\sim}{\bar{C}}_{qi} \underset{\sim}{\Delta}_i \tag{15}$$

where

$$\underset{\sim}{\bar{C}}_{qi} = \underset{\sim}{C}^r_{qi} - \underset{\sim}{C}_{qo} \underset{\sim}{K}^{-1}_{oo} \underset{\sim}{K}^r_{oi} \tag{16}$$

With equation (15), the element stiffness matrix for the MSC eight-node hex element is generated as in equation (7) but with $\underset{\sim}{\bar{C}}_{qi}$ being used instead of $\underset{\sim}{C}_{qi}$. It remains to select the strain functions to be added--i.e., the terms in the $\underset{\sim}{C}_{qo}$ matrix. In general, the $\underset{\sim}{C}_{qo}$ terms are added to fill a need in terms of improvement in element accuracy for some particular application. Addition of these terms will almost certainly invalidate the interelement displacement continuity that exists with the classical element. However, this is not as significant as insuring that the modified element will still be capable of passing a constant strain patch test (see [9]) for a discussion of the patch test). Given the fact that the classical element does pass the constant strain patch test, Harder shows that the reduced integration technique will also pass the patch test and in order for the completely modified element to pass the patch test, it is required that:

$$\sum_{q} w_q J_q \underset{\sim}{C}^T_{qo} \underset{\sim}{\sigma}_q = 0 \tag{17}$$

For a constant patch stress, and keeping in mind that $w_q = 1.0$ for the eight-node hex, equation (17) requires

$$\sum_{q} J_q \underset{\sim}{C}^T_{qo} = 0 \tag{18}$$

As pointed out in [7], this can be accomplished if the terms in $\underset{\sim}{C}_{qo}$ are of the form

$$\xi_q/J_q \;,\; \eta_q/J_q \;,\; \zeta_q/J_q \;,\; \xi_q\eta_q/J_q \;, \ldots$$

Thus, the MSC element uses for $\underset{\sim}{C}_{qo}$:

$$\underset{\sim}{C}_{qo} = \frac{1}{J_q}\begin{bmatrix} \xi & 0 & 0 & \xi\eta & 0 & \xi\zeta \\ 0 & \eta & 0 & \xi\eta & \eta\zeta & 0 \\ 0 & 0 & \zeta & 0 & \eta\zeta & \xi\zeta \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_q \tag{19}$$

Only terms are included in the first three rows as only the direct stresses are sought to be modified by the additional strain functions. It is not clear exactly why the particular form for $\underset{\sim}{C}_{q_0}$ was chosen, aside from the considerations in equation (18); however, it is the form identified by Harder in [7].

Equation (16) is the form of the strain-displacement law that was used in the modified CIHEX1 element reported herein, with equations (8), (13), (14), and (19) defining the various terms in (16). This $\underset{\sim}{\bar{C}}_{qi}$ matrix must be used in the development of the stiffness matrix as well as the thermal load vector and in stress data recovery.

Modified QDMEM1 Element

For the 2-D isoparametric element, the stiffness matrix has the same general form as shown in the previous section:

$$\underset{\sim}{K}_{ij} = \sum_q w_q J_q \underset{\sim}{C}^T_{qi} \underset{\sim}{E}_q \underset{\sim}{C}_{qj} \quad , \quad \underset{\sim}{\Delta}_i = \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} \tag{20}$$

However, for this element

$$\underset{\sim}{\epsilon}_q = \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix}_q \tag{21}$$

and

$$\underset{\sim}{C}_{qi} = \begin{bmatrix} N_{i,x} & 0 \\ 0 & N_{i,y} \\ N_{i,y} & N_{i,x} \end{bmatrix}_q \tag{22}$$

$$N_i = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i) \tag{23}$$

Following the procedure of the previous section, reduced integration for shear terms is employed by modifying (22) to

$$\underset{\sim}{C}^r_{qi} = \begin{bmatrix} N_{i,x} & 0 \\ 0 & N_{i,y} \\ \bar{N}_{i,y} & \bar{N}_{i,x} \end{bmatrix}_q \tag{24}$$

As in the previous section, the $\bar{N}$ terms could be defined as Jacobian weighted averages of the terms at the Gauss points. However, for this element, it is found that the same result is obtained if the terms are evaluated at the center of the element ($\xi=\eta=0$). Thus

$$(\bar{N}_{i,x})_q = N_{i,x}(\xi=\eta=0) \quad , \quad (\bar{N}_{i,y})_q = N_{i,y}(\xi=\eta=0)$$

MSC also uses strain functions for this element. From [7], for the membrane part of the QUAD4 general shell element

$$\underset{\sim}{C}_{\xi 0} = \begin{bmatrix} \xi & 0 \\ 0 & \eta \\ 0 & 0 \end{bmatrix}_{\gamma}$$

The complete formulation of $\bar{C}_{\xi i}$ then follows as in the previous section and $\bar{\underset{\sim}{C}}_{\xi i}$ would then be used in equation (20) instead of $\underset{\sim}{C}_{\xi i}$. For the modifications made to the COSMIC QDMEM1 element, however, the additional strain functions in $\underset{\sim}{C}_{\xi 0}$ were <u>not</u> implemented. Only the reduced integration for shear was implemented; <u>and</u>, therefore, $\underset{\sim}{C}^{r}_{\xi i}$ is used in equation (20) instead of $\underset{\sim}{C}_{\xi i}$.

<u>Code Changes to Implement Modified CIHEX1 Element</u>

The purpose of the authors' work was to prove the worth and feasibility of the element formulations. Optimizing the implementation of the code changes was not examined. Therefore, a complete discussion of the modifications will not be provided; however, a mention of the extent of the changes and the subroutines where they appear is appropriate.

The alterations to the QDMEM1 were accomplished with a simple modification to the QDMM1D subroutine. At the point where partitions of the stiffness are numerically integrated, terms which include the shear modulus were separated. This allowed the order of Gaussian integration to be selected separately for in-plane stress and shear stress terms. As discussed earlier, the need was for a linear shear stress formulation. The current implementation does not implement aniosotropic materials.

The XIHEX subroutine, which calculates the mass and stiffness matrices for the CIHEX1, as well as the CIHEX2 and CIHEX3, received extensive modifications. Since the IHEX subroutine recalculates part of the element's stiffness in order to form temperature loads, it required some similar changes. The CIHEX2 and CIHEX3 formulations were not considered, and alterations to the CIHEX1 stress recovery subroutine have not been developed at this time.

The initial formulation of the CIHEX1 element matrices in COSMIC NASTRAN was made in the basic coordinate system. Due to the manner in which the reduced integration for shear was implemented in the modified element, it was necessary to develop the CIHEX1 matrices in a local element coordinate system nearly aligned with the ξ, η, ζ axes and then transform the matrices to the basic coordinate system. Additional subroutines needed to accomplish these tasks were written by the authors. Several options for selection of the "best" initial local system were looked at and are still being evaluated. For rectangular parallelepiped elements, the choice of a local system is trivial. For skewed elements, the ξ, η, ζ directions do not form an orthogonal set of vectors in x,y,z; so the choice of the initial local coordinate system is not obvious.

Test Problems

QDMEM1 Element

The test problems are the same as those utilized in the prior mesh study of the membrane elements reported in [1] and involved a deep cantilevered beam type structure with unit depth and beam aspect ratio (length/depth) of two. Figure 1 shows the geometry, coordinate system, boundary conditions, and beam physical properties used in the study. Membrane elements formed the beam model. The mesh subdivision technique as well as the method used to indicate mesh size and element aspect ratio are demonstrated in Figure 2.

The finite element model used work equivalent grid point forces for separate end moment and end shear loadings. This simulated the applied loads as well as the reactions at the cantilevered end (Figure 3). One should note, from Figure 1, that only kinematic constraints were imposed. Discussion of the theoretical solutions to these loading conditions can be found in [1].

In order to assess the effect of the reduced integration modification to the QDMEM1 element, the figures from [1] for the older membrane element mesh study were utilized. These curves, figure 4 through figure 8 herein, show the error in displacements and stresses at specific points on the beam as a function of mesh refinement or aspect ratio. The MSC element referred to in these figures is a CQUAD4 with only membrane properties specified on its PSHELL card; also, the 2,1 or 4,1 after COSMIC '84 refers to the number of Gaussian integration points for in-plane stress and shear stress terms, respectively. As indicated on the figures, the altered COSMIC element '84 w/4,1 reduced integration produces the same answers as the MSC element; and these answers are an improvement over the old COSMIC element.

CIHEX1 Element

The premise for selecting a test problem in [2] was the fact that solid elements are used to model large optical mirrors of spaceborne telescopes, and these mirrors often have thicknesses of as much as 10 percent of their diameter. The test problem involved a cubic slab of equal dimension in the x-y plane and whose thickness varies between one-twentieth and one half of the x-y plane dimensions. Figure 10 shows the geometry, coordinate system, boundary conditions, and basic material information used in the study. The constraints are kinematic and the problem is symmetric about the x=0 plane. That is, the x displacement is zero along the x=0 plane. Using this constraint, only half the slab needed to be included in the finite element model. The mesh subdivision technique and method used to indicate element aspect ratio is shown in Figure 11.

Originally, the test cases were chosen to measure the accuracy of various solid elements under temperature gradient and gravity loadings. Their value to this paper lies in that they provide separate bending and shear load cases. The linear temperature gradient produced a symmetric bending condition with a known theoretical answer. The gravity loading, however, was found to be non-converging and was used in this study only to show the effect of the changes to CIHEX1 under a shear loading case. Further discussion of these load cases is provided in [2].

In order to assess effects of the changes to CIHEX1, three graphs were extracted from [2]. Data for the altered CIHEX1 was added to these graphs, which already included curves representing the original CIHEX1. Figures 13 and 14 present error in displacement, at a particular point, as a function of mesh size and aspect ratio, respectively. Actual displacements versus mesh size are shown in Figure 15 for the (non-convergent) gravity loading. Once again, the important factor to note in these graphs is that for each case the "improved" CIHEX1 provided the same answers as the comparable eight-node MSC element. For an aspect ratio of 10, not at all unreasonable when modeling large mirrors, the changes to the element totally eliminated a 48 percent error (in the temperature gradient case) when the old CIHEX1 element was used.

Conclusions

Modifications to the isoparametric membrane and solid elements, QDMEM1 and CIHEX1, have been implemented in the COSMIC NASTRAN code.

The modified CIHEX1 element performs identically to the MSC HEXA eight-mode element. With the modifications, especially the reduced shear integration, it is anticipated that the new element will perform better when modeling thick plates when only few elements are used through the thickness. In addition, for pure bending, the element gives exact answers when only one element is used through the thickness.

The modified QDMEM1 element has been shown to be superior to the original element when modeling bending situations. Neither element exhibits aspect ratio sensitivity in the modified form as it did in its original form.

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NOTATION

AR = PROBLEM ASPECT RATIO

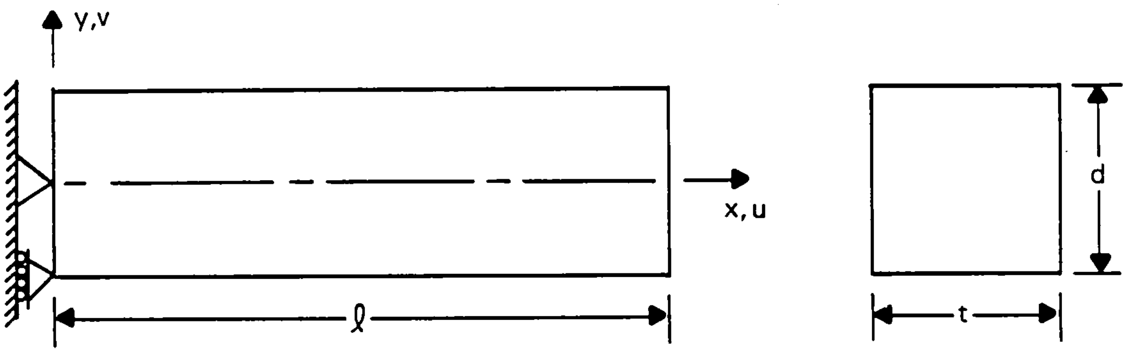
AR_e = ELEMENT ASPECT RATIO

ND = NUMBER OF ELEMENTS THROUGH DEPTH

NL = NUMBER OF ELEMENTS ALONG LENGTH

FIG. 1

BEAM GEOMETRY AND PROPERTIES



$\ell = \begin{cases} .0508 \text{ M (2.0 IN) BASIC DEEP BEAM FOR MESH STUDY} \\ \text{VARIABLE FOR ASPECT RATION STUDY} \end{cases}$

d = .0254 M (1.0 IN)

t = .0254 M (1.0 IN)

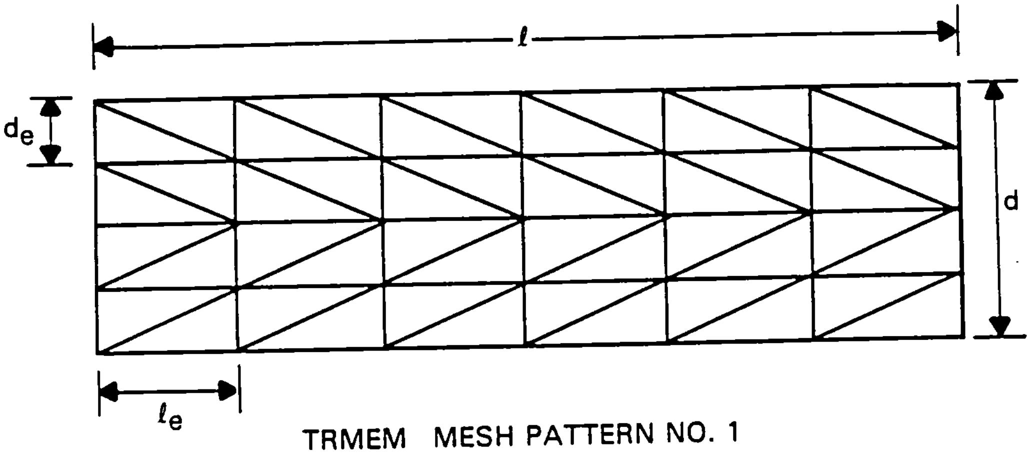
$E = 1.9305 \times 10^{10}$ N/m^2 (28×10^6 LB/IN^2)

$\nu = 0.3$

u = v = o AT x = y = o AND u = o AT x = o, y = −d/2

FIG. 2

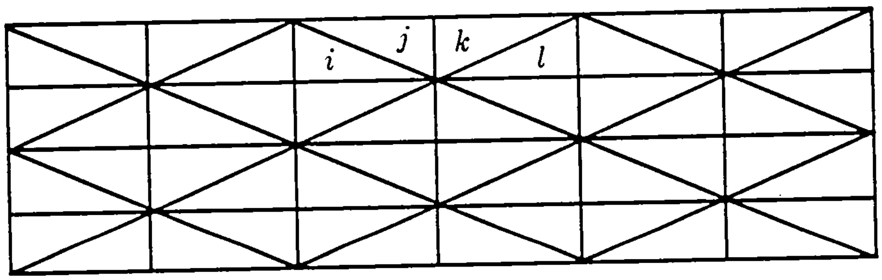
FINITE ELEMENT MODEL MESH PATTERNS MEMBRANE ELEMENTS



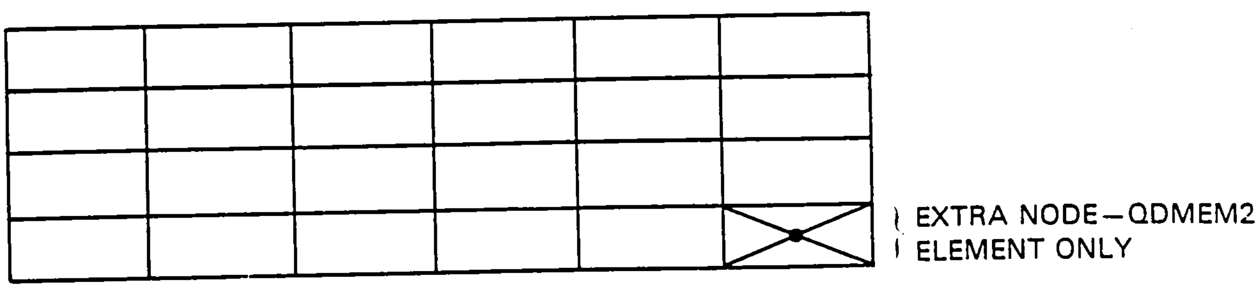
$AR_e = \ell_e / d_e$

$AR = \ell / d$

TRMEM MESH PATTERN NO. 1



TRMEM MESH PATTERN NO. 2



QDMEM, QDMEM1, QDMEM2 MESH PATTERNS

FIG. 3

BEAM LOADS

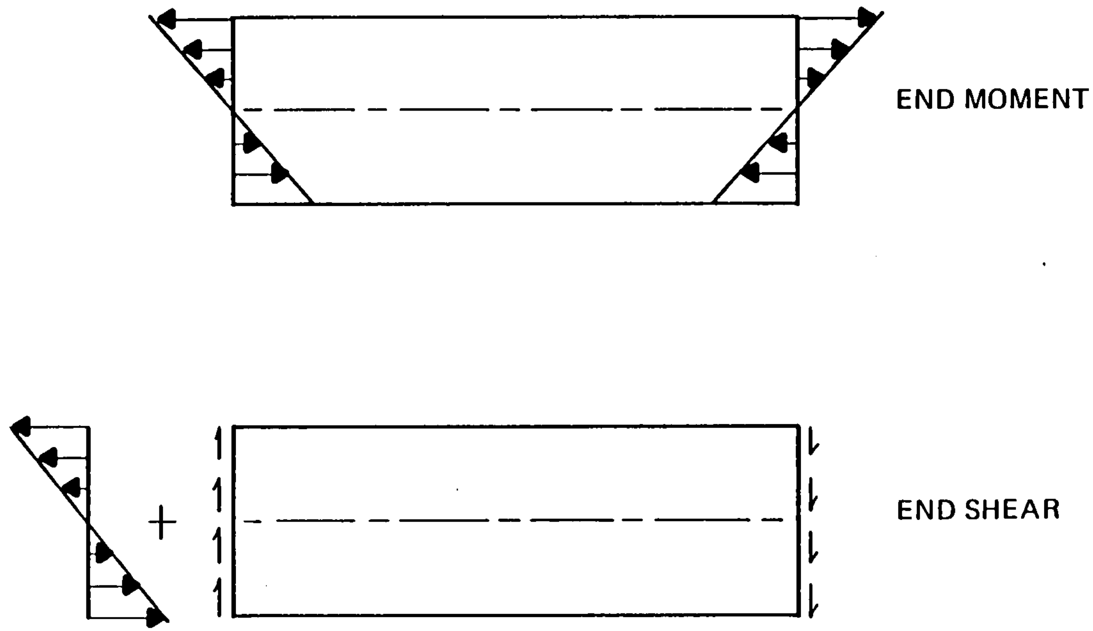


FIG. 4
TIP DEFLECTION ERROR
DEEP BEAM-END MOMENT LOADING
(MESH SIZE STUDY)

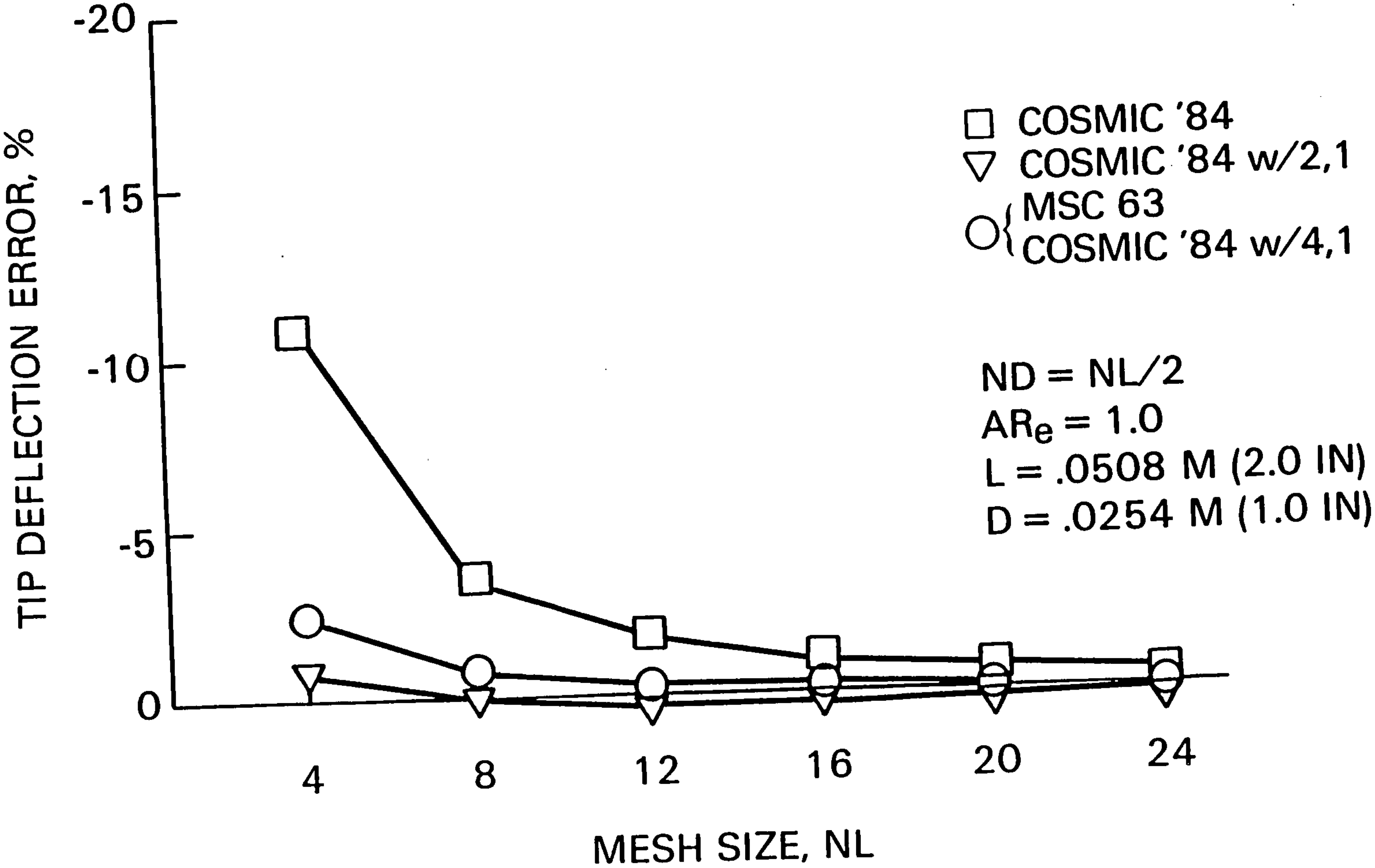


FIG. 5
TIP DEFLECTION ERROR
DEEP BEAM-END SHEAR LOADING
(MESH SIZE STUDY)

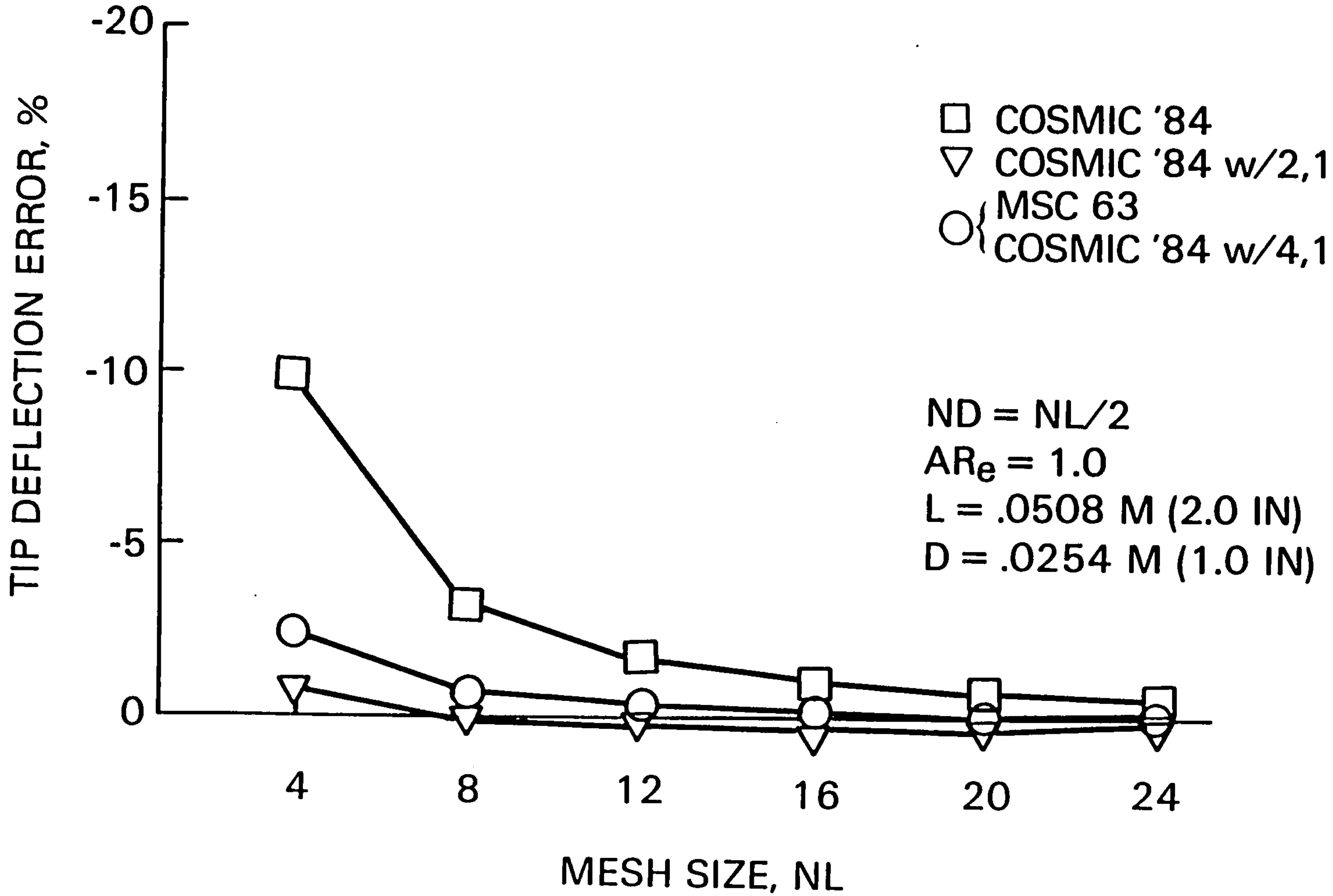


FIG. 6
DIRECT STRESS ERROR
DEEP BEAM-END MOMENT AND END SHEAR LOADING
(MESH SIZE STUDY)

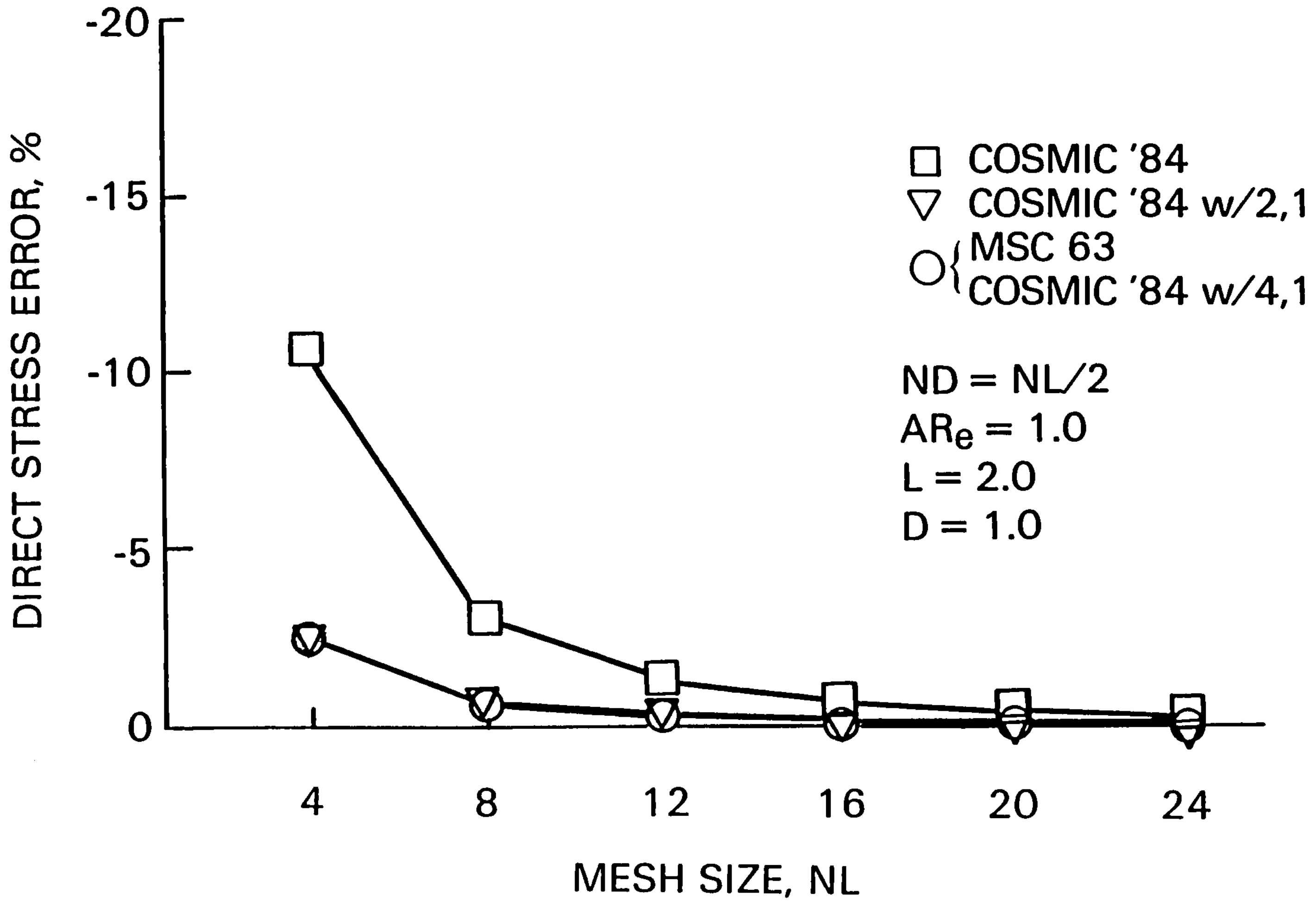


FIG. 7
SHEAR STRESS ERROR
DEEP BEAM-END SHEAR LOADING
(MESH SIZE STUDY)

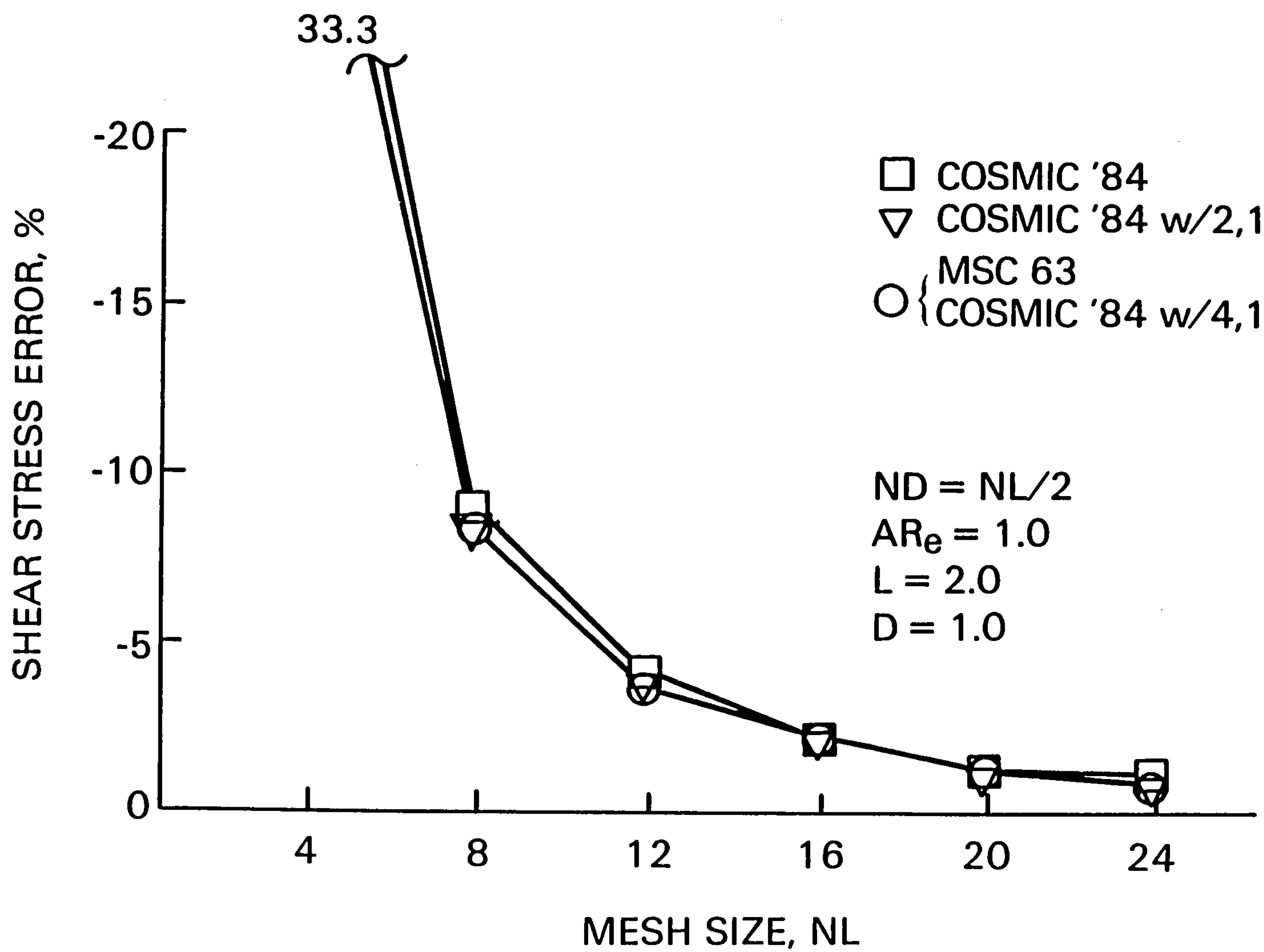


FIG. 8
TIP DEFLECTION ERROR
VARIABLE BEAM-END MOMENT LOADING
(ASPECT RATIO STUDY)

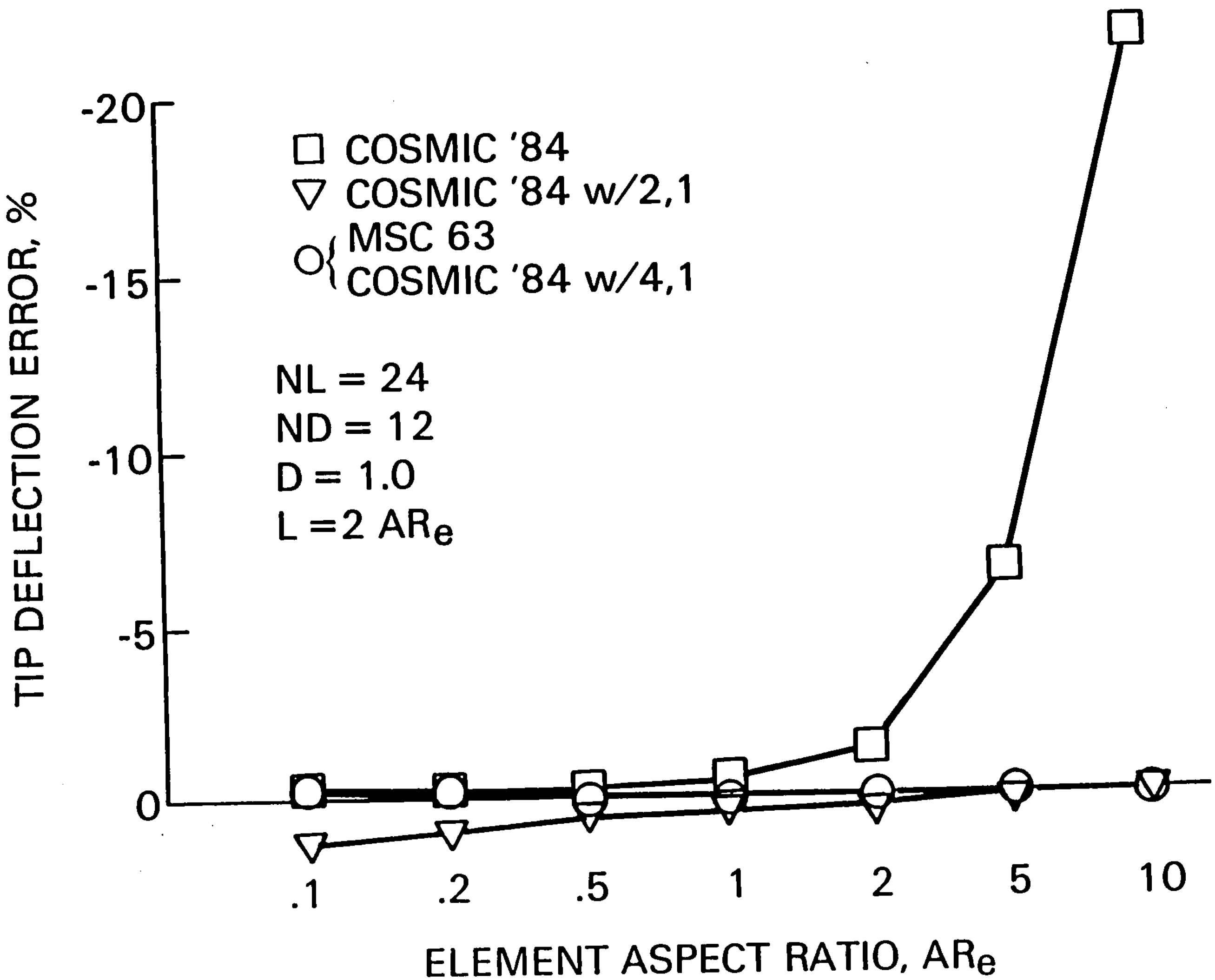
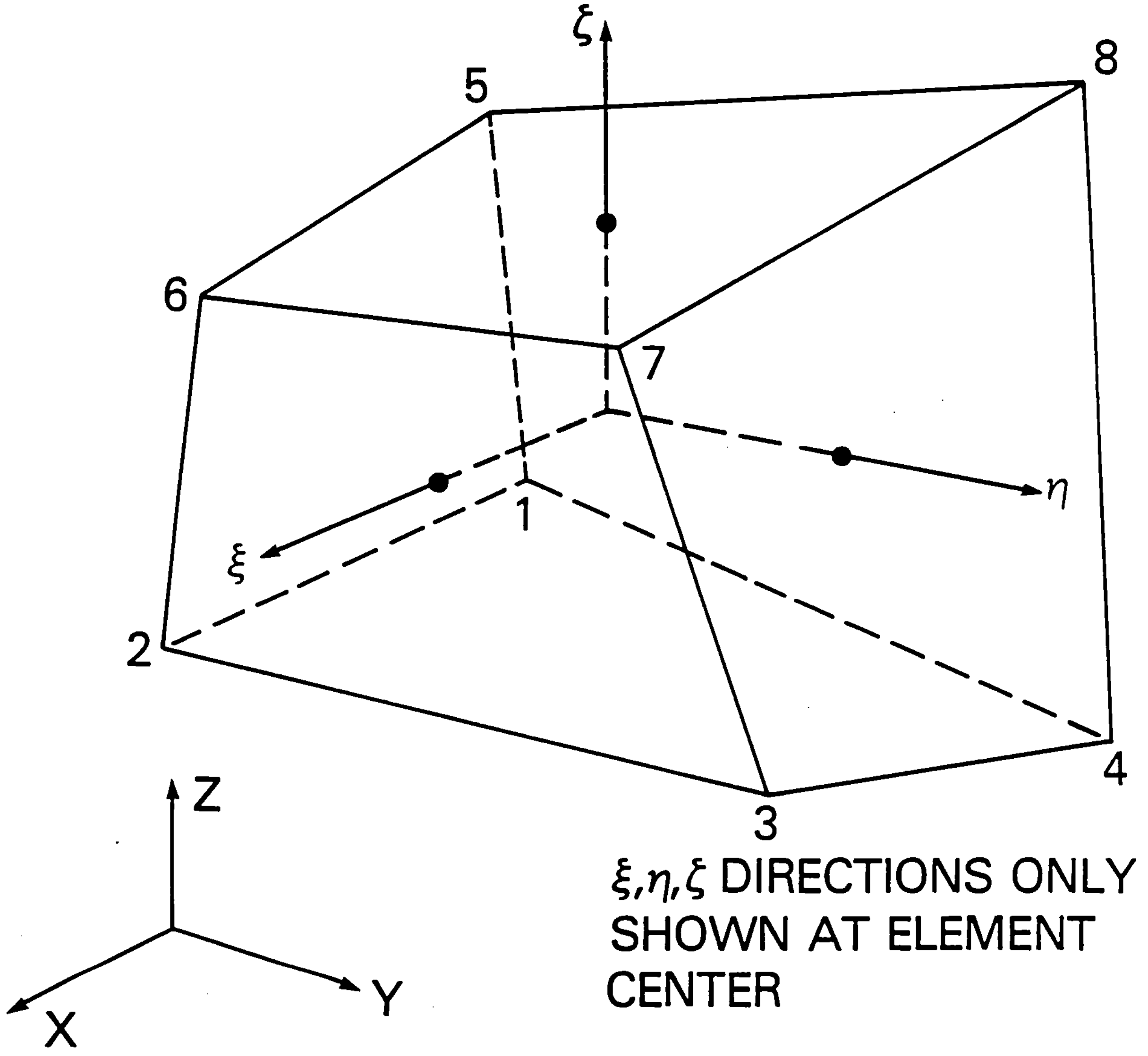


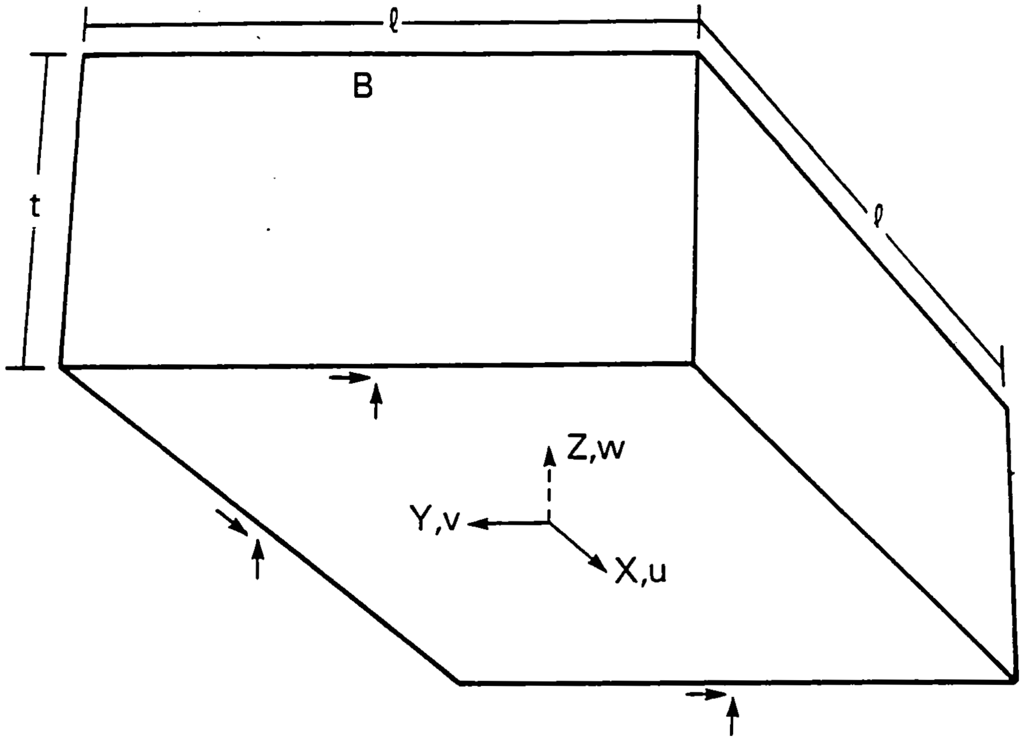
FIG. 9

8 NODE HEX ELEMENT GEOMETRY



ξ, η, ζ DIRECTIONS ONLY SHOWN AT ELEMENT CENTER

FIG. 10
TEST PROBLEM-SOLID ELEMENTS



↑ INDICATES DEGREE OF FREEDOM CONSTRAINED FOR KINEMATIC MOUNTS

$v = w = 0$ AT $Y = Z = 0$, $X = \pm \ell/2$

$u = w = 0$ AT $X = Z = 0$, $Y = \ell/2$

$A = (0.,0.,t)$

$B = (\ell/2,0.,t)$

MATERIAL INFORMATION (ALUMINUM)

$E = 6.89 \times 10^{10}\ N/M^2$ $(10 \times 10^6\ LB/IN^2)$

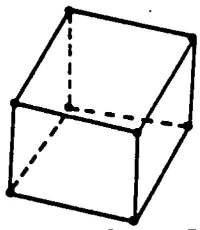
$\rho = 2.71 \times 10^3\ KG/M^3$ $(.098\ LB/IN^3)$

$a = 22.7 \times 10^{-6}\ /K$

$\upsilon = .33$

FIG. 11

ELEMENT TYPES

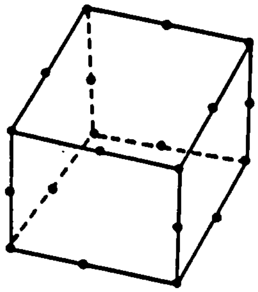


8 NODES

CHEXA2 10 TETRAHEDRA (COSMIC)

CIHEX1 LINEAR ISOPARAMETRIC (COSMIC)

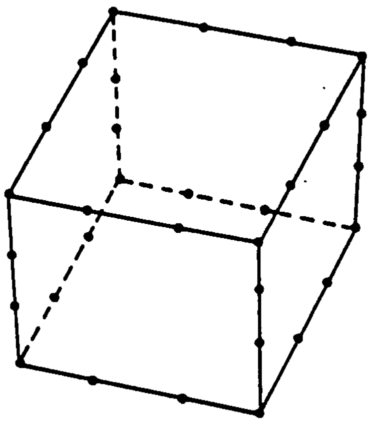
CHEXA LINEAR ISOPARAMETRIC (MSC)



20 NODES

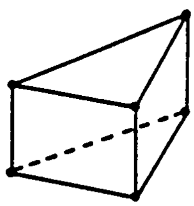
CIHEX2 QUADRATIC ISOPARAMETRIC (COSMIC)

CHEXA QUADRATIC ISOPARAMETRIC (MSC)



32 NODES

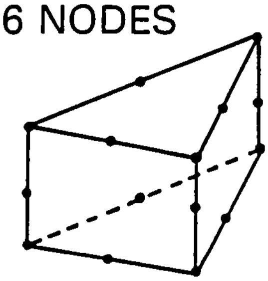
CIHEX3 CUBIC ISOPARAMETRIC (COSMIC)



6 NODES

CWEDGE 3 TETRAHEDRA (COSMIC)

CPENTA LINEAR ISOPARAMETRIC (MSC)

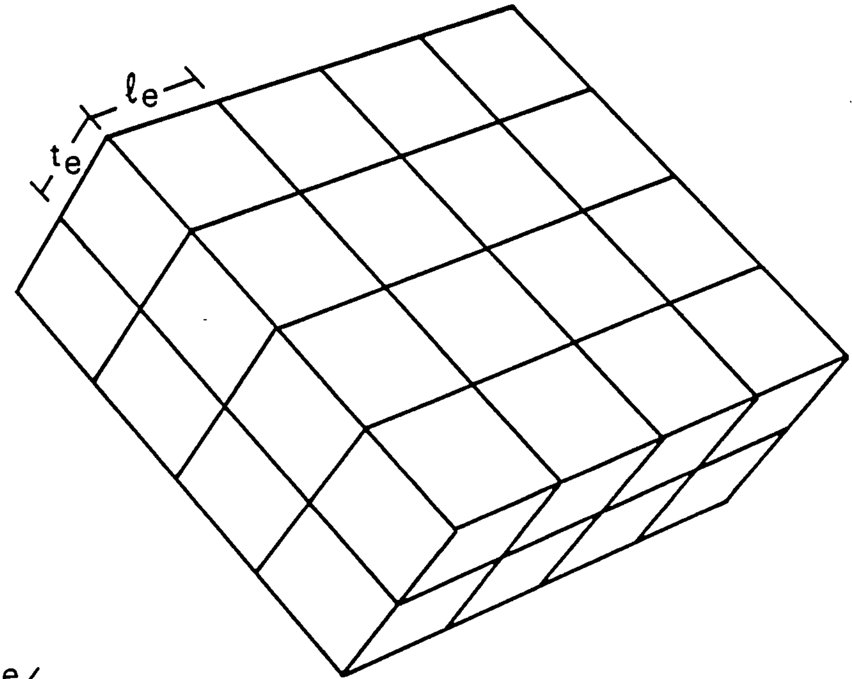


15 NODES

CPENTA QUADRATIC ISOPARAMETRIC (MSC)

FIG. 12

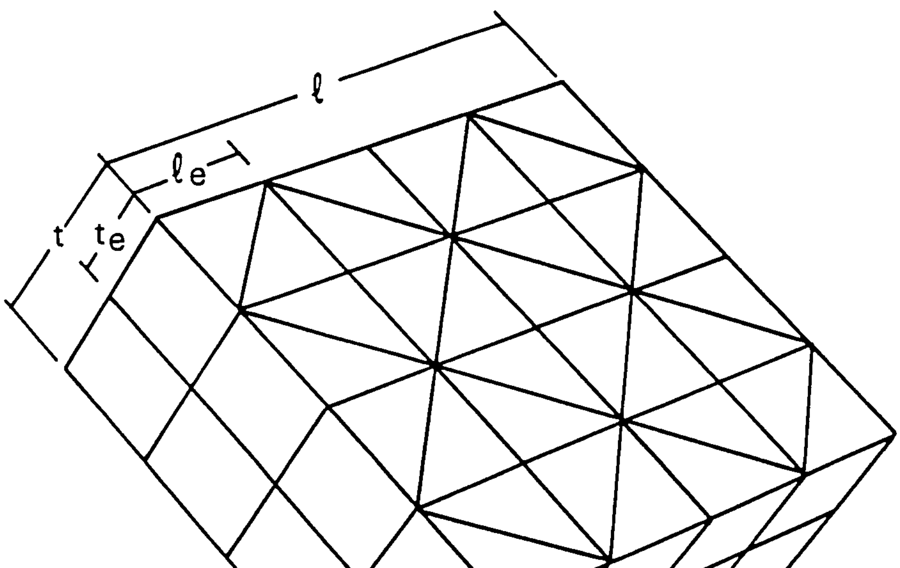
MESH GEOMETRY



$AR_e = \ell_e / t_e$

$AR = \ell / t$

HEXAHEDRAL PATTERN



WEDGE PATTERN

FIG. 13

Z DISPLACEMENT ERROR AT (A)

THICK SLAB—LINEAR TEMPERATURE GRADIENT
(MESH SIZE STUDY)

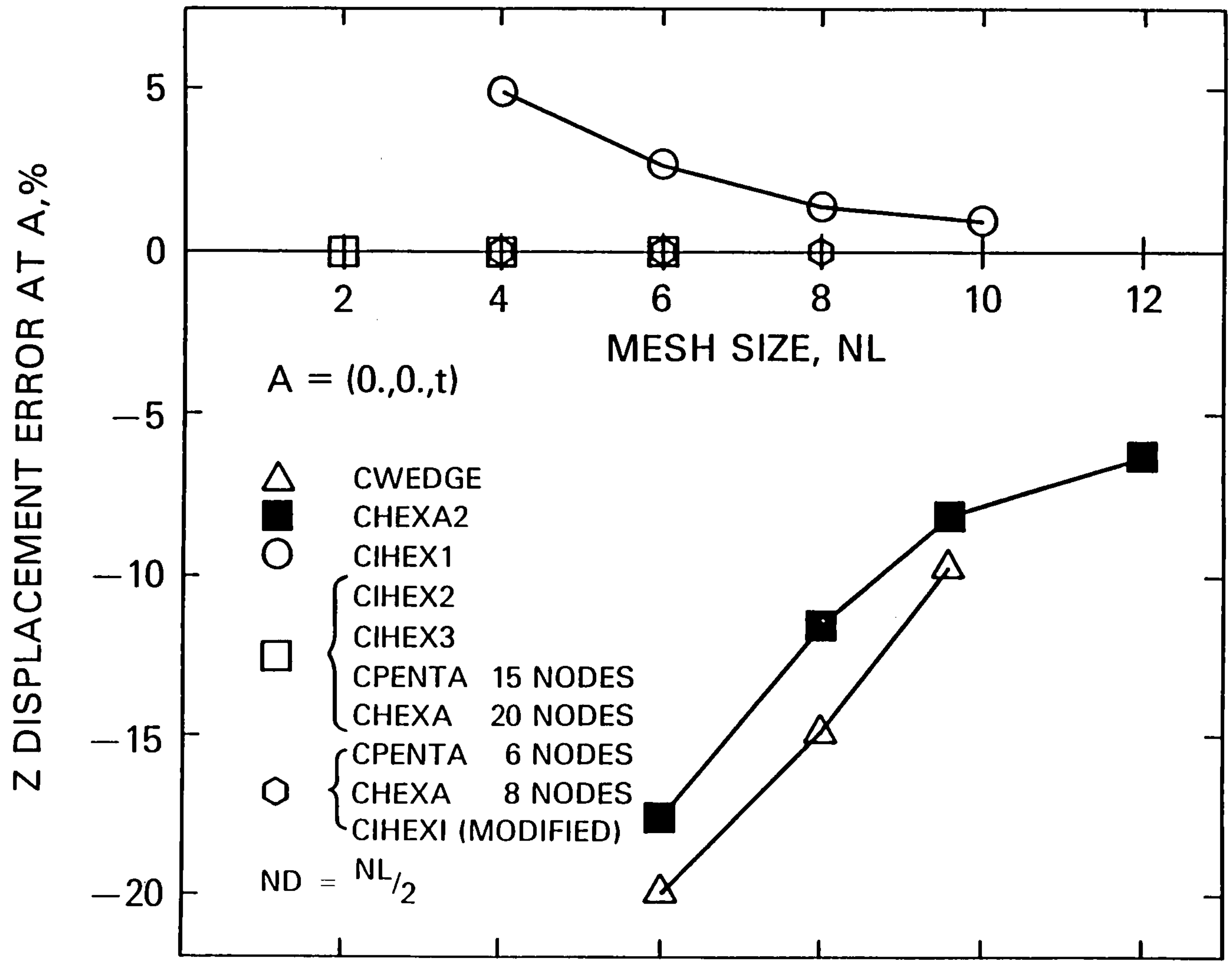


FIG. 14

Z DISPLACEMENT ERROR AT (A)

LINEAR TEMPERATURE GRADIENT
(ASPECT RATIO STUDY)

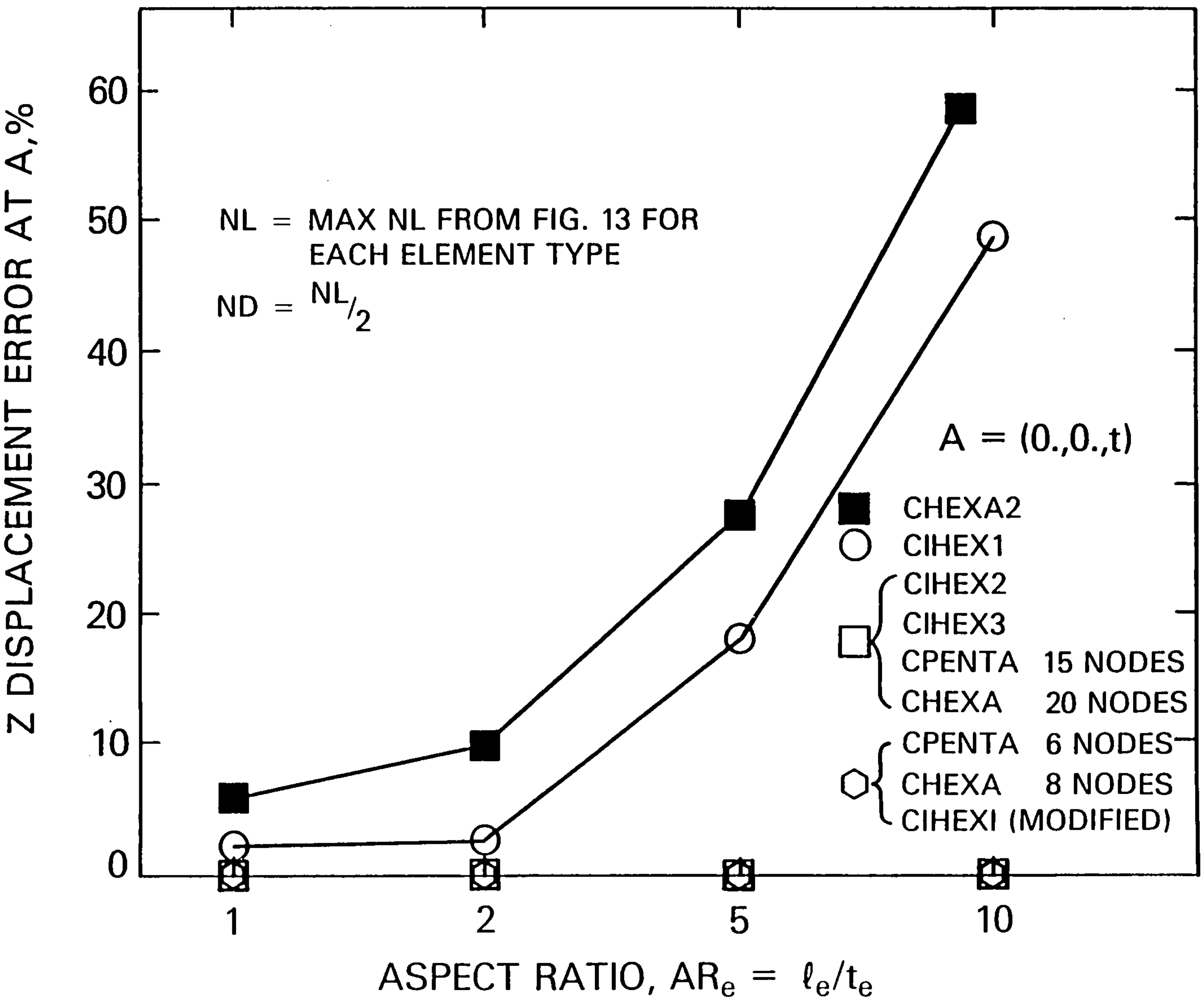


FIG. 15

Z DISPLACEMENT AT (A)

THICK SLAB — GRAVITY