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MULTIGRID METHOD FOR A VORTEX BREAKDOWN SIMULATION

Shlomo Ta'asan

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NASA Langley Research Center, Hampton, Virginia 23665

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ABA: Author

ABS: An inviscid model for a steady axisymmetric flow with swirl was studied. The governing equation is a nonlinear el

MULTIGRID METHOD FOR A VORTEX BREAKDOWN SIMULATION

Shlomo Ta'asan
Institute for Computer Applications in Science and Engineering

Dedicated to Milton E. Rose
on Occasion of his 60th Birthday

ABSTRACT

In this paper we study an inviscid model for a steady axisymmetric flow with swirl. The governing equation is a nonlinear elliptic equation which has more than one solution for a certain range of the swirl parameter. The physically interesting solutions have closed streamlines that look like vortex breakdown ("bubble"-like solutions). A multigrid method is used to find these solutions. Using an FMG algorithm (nested iteration), the problem is solved in just a few multigrid cycles.

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1. INTRODUCTION

In this paper we study an inviscid model for steady axisymmetric flow with swirl, which has solutions with closed streamlines. These solutions have a structure similar to that observed experimentally as "bubble"-like solutions when vortex breakdown occurs [4].

Using a streamfunction-vorticity formulation to the axisymmetric incompressible Navier-Stokes equations, it was found [3] that one can reduce the problem to a single nonlinear elliptic equation for the streamfunction, in case of a special inflow flow and some regularity assumption on the vorticity. This nonlinear elliptic equation for the streamfunction has more than one solution. The trivial, represents a uniform flow and is of no physical interest. The other shows a "bubble"-like structure, the target of our numerical study.

In solving the problem numerically, the problem is reformulated in terms of a perturbed streamfunction, i.e., the deviation from the trivial solution. In terms of this perturbed streamfunction, the trivial solution is represented as an identically zero solution. Our goal then is to find non-zero solutions which have "bubble"-like form.

The approach we have taken in finding these solutions is to seek first for a bifurcation point from the trivial branch of solutions. By introducing a continuation parameter, we can then start marching on a branch of non-trivial solutions that bifurcate from that point. One choice of a continuation parameter is arc length [1]. Another choice, which is simpler but may not be good in general, is the norm of the perturbed streamfunction. The natural parameter in the problem, a swirl velocity parameter, is not good enough since it cannot "choose" the non-zero branch as can the former parameter. We therefore choose the norm as a continuation parameter, making the swirl velocity parameter an unknown to be determined by the solution.

The multigrid approach used for solving the problem is similar to the one used in [5] for solving the Bratu problem. The relaxation in this method consists of three steps: (i) a local relaxation to smooth the error; (ii) a step to update the norm of the solution; and (iii) a step to update the swirl velocity parameter. An FMG algorithm (nested iteration) is used. That is, a solution for the prescribed norm is found first on the coarsest level, and then interpolated to finer levels, where on each level a few basic multigrid V-cycles are performed before proceeding to yet finer level.

The coarsest level, when solved to get an initial approximation for finer levels, uses a continuation method. Here the problem was solved first for a small norm, and then the norm is gradually increased until the prescribed norm is reached. Each time the norm is increased, the solution of the previous step was used as initial approximation. By solving for a bifurcation point from the trivial solution, a first approximation for the smallest norm problem was obtained.

Once a solution on the coarsest level is obtained for a prescribed norm, it is possible to solve finer grid problems without continuation.

The same problem we are discussing here was treated by a completely different method and is reported in [3]. There, a single grid method was used with a least squares formulation of the problem. The amount of work needed for that approach is considerably larger than the one reported here. Computed solutions by the two different formulations are in good agreement.

2. ON DERIVATION OF THE GOVERNING EQUATION

We summarize here the derivation of the equations used in the numerical process as given in [3]. In cylindrical coordinates (x,r,θ) the incompressible Navier-Stokes equations can be written in terms of a streamfunction ψ, vorticity ω, and circulation k as

$$r\left(\frac{\psi_r}{r}\right)_r + \psi_{xx} = r\omega \qquad (2.1a)$$

$$(u\omega)_r + (w\omega)_x + \left(\frac{k^2}{r^3}\right)_x = \frac{1}{Re}\left[\omega_{rr} + \frac{1}{r}\omega_r - \frac{\omega}{r^2} + \omega_{xx}\right] \qquad (2.1b)$$

$$uk_r + wk_x = \frac{1}{Re}\left[k_{rr} - \frac{1}{r}k_r + k_{xx}\right] \qquad (2.1c)$$

where $k = rv$, $\omega = w_r - u_x$ and Re is the Reynolds number. The velocity components in the x,r,θ directions are w, u, v, respectively, of which w and u are given in terms of the streamfunction by

$$w = \frac{\psi_r}{r} \qquad (2.2a)$$

$$u = -\frac{\psi_x}{r}\,. \qquad (2.2b)$$

It is shown in [3] that in the inviscid case $(Re = \infty)$, one finds that the circulation k and the vorticity ω are functions of the streamfunction ψ only. Therefore, k and ω can be determined outside the "bubble" from the inflow boundary condition. In the model discussed it is assumed that the same functional dependence of k, ω on ψ is true also inside the bubble (negative ψ). This imposes some regularity on the solution.

For the inflow conditions

$$v(0,r) = \begin{cases} V_0\ r(2 - r^2) & r < 1 \\ V_0/r & r > 1, \end{cases} \tag{2.3a}$$

$$w(0,r) = 1, \tag{2.3b}$$

it is possible to write k and ω in terms of the streamfunction as

$$k^2(0,r) = \begin{cases} 16\ V_0^2\ \psi^2(1 - \psi)^2 & \psi < 1/2 \\ V_0^2 & \psi > 1/2 \end{cases} \tag{2.4a}$$

$$\omega(0,r) = \begin{cases} 16\ V_0^2(1 + 2\psi^2 - 3\psi)(r^2/2 - \psi) & \psi < 1/2 \\ 0 & \psi > 1/2 \end{cases} \tag{2.4b}$$

and therefore, the equation obtained for ψ is

$$r(\psi_r/r)_r + \psi_{xx} = -\ 4V_0^2\ \tilde{\alpha}^2(\psi)(\psi - r^2/2) \tag{2.5a}$$

where

$$\tilde{\alpha}^2(\psi) = \begin{cases} 4(1 + 2\psi^2 - 3\psi) & \psi < 1/2 \\ 0 & \psi > 1/2\,. \end{cases} \tag{25b}$$

The reduction of the governing equations to a single nonlinear elliptic equation is possible if the relation $\psi = f(r)$ in the inflow boundary can be inverted to get $r = g(\psi)$. When $g(\psi)$ is introduced in the expression for

v at the inflow boundary, one has v as a function of ψ in that boundary and therefore $k(\psi)$, $\omega(\psi)$. Note that, in general, one cannot expect to analytically invert the relation $\psi = f(r)$, and so the reduction of the governing equations is possible only for very special inflows.

Numerical experiments were done in terms of $\phi = \psi - \frac{r^2}{2}$, which is a perturbation from the trivial solution $\psi = \frac{r^2}{2}$ that represents a uniform flow.

3. NUMERICAL ALGORITHM

3.1. Discretization

The equation for $\phi = \psi - r^2/2$ is given by

$$r\left(\frac{1}{r}\,\phi_r\right)_r + \phi_{xx} + 4\,V_0^2\,\alpha^2(\phi)\phi = 0, \qquad \Omega = (0,a) \times (0,b) \tag{3.1a}$$

$$\phi = 0, \qquad \text{on} \quad \partial\Omega \tag{3.1b}$$

where

$$\alpha^2(\phi) = \begin{cases} 4\,\phi - 1 + \frac{r^2}{2}\,(2\phi - 1 + r^2) & \phi + \frac{r^2}{2} < \frac{1}{2} \\ 0 & \text{otherwise} \end{cases} \;. \tag{3.1c}$$

Equations (3.1) are discretized as

$$\frac{\phi^h_{i+1,j} - 2\phi^h_{ij} + \phi^h_{i-1,j}}{h^2} + \frac{r_j}{h^2}\left[\frac{2}{r_{j+1} + r_j}\,(\phi^h_{i,j+1} - \phi^h_{ij}) - \frac{2}{r_j + r_{j-1}}\,(\phi^h_{ij} - \phi^h_{i,j-1})\right] + V_0^2\,\alpha^2(\phi^h_{ij})\phi^h_{ij} = 0, \quad \text{in} \quad \Omega^h \tag{3.2a}$$

$$\phi^h_{ij} = 0, \quad \text{on} \quad \partial\Omega^h \tag{3.2b}$$

where $\Omega^h = \{nh,mh), 0 < nh < a, 0 < mh < b\}$.

3.2. General Strategy for Solving the Discretized Equations

Equation (3.2) has the trivial solution $\phi^h = 0$ for any V_0. This solution corresponds to a uniform flow and is not interesting physically. We seek solutions which represent vortex breakdown so that $\|\phi^h\|^2 \neq 0$, where

$$\|\phi^h\|^2 = h^2 \Sigma \phi_{ij}^2. \qquad (3.3)$$

Iterating on equation (3.2) by any iterative method may lead us to the trivial solution. In order to rule out this possibility, we specify the norm of the discrete solution we want to find, while making free the swirl velocity parameter V_0.

To summarize, we solve equation (3.2) for (ϕ^h, V_0) under the constraint

$$\|\phi^h\|^2 = g_0, \qquad (3.4)$$

where g_0 is given.

A relaxation scheme for (ϕ^h,V_0) in equation (3.2) together with the constraint (3.4) is described next.

3.3. Relaxation

Equations (3.2), (3.4) form a nonlinear system of equations for (ϕ^h,V_0). The relaxation used for this system has three steps: (i) a local process for smoothing ϕ^h in equation (3.2); (ii) a global change to satisfy (3.4); and (iii) updating the swirl parameter V_0. That is, one relaxation consists of doing (i), (ii), and (iii) successively.

(i) local relaxation

Scan the point $(i,j) \in \Omega^h$ in lexicographic ordering; at each point (i,j) solve (3.2) approximately for ϕ^h_{ij} by applying one Newton iteration.

(ii) global step

Compute $\beta = \sqrt{g_0/\|\phi^h\|^2}$.

Then make the change

$$\phi^h_{ij} \leftarrow \beta\, \phi^h_{ij}\,, \qquad\qquad (i,j) \in \Omega^h.$$

(iii) updating V_0

Change V_0 such that the following equation holds

$$\langle L^h \phi^h + 4V_0^2\, \alpha^2(\phi^h)\phi^h,\ \phi^h\rangle = \langle f^h,\phi^h\rangle \qquad (3.5)$$

where $L^h \phi^h$ is the discretization of $L\phi = r(\frac{1}{r}\phi_r)_r + \phi_{xx}$, $\langle\cdot,\cdot\rangle$ denotes the inner product, $\langle u,v\rangle = h^2 \sum_{ij} u_{ij} v_{ij}$, and f^h is the right-hand side of equation (3.2). (In a multigrid process f^h is nonzero on coarse grids.)

We now come to the description of the multigrid algorithm used to solve (3.2), (3.4) for (ϕ^h, V_0).

3.4.1. <u>Basic Cycle:</u>

Given a sequence of discretizations with mesh sizes $h_r > h_2 > \cdots > h_m$, where $h_k = 2h_{k+1}$. The h_k-grid equation is generally written as

$$L^k \phi^k = f^k \tag{3.6}$$

where L^k approximates L^{k+1} $(k < m)$ (e.g., they all are finite-difference approximations to the same differential operator). The algorithm for improving a given approximate solution $\tilde{\phi}^k$ to (3.6) is denoted by

$$\tilde{\phi}^k \leftarrow MG(k, \tilde{\phi}^k, f^k) \tag{3.7}$$

and is defined recursively as follows:

If $k = 1$, solve (3.6) by several relaxation sweeps; otherwise do steps (A) - (D):

(A) Perform ν_1 relaxation sweeps on (3.6), resulting in a new approximation $\bar{\phi}^k$.

(B) Starting with $\tilde{\phi}^{k-1} = I_k^{k-1}\, \bar{\phi}^k$, perform one cycle
$\tilde{\phi}^{k-1} \leftarrow MG\left(k-1,\ \tilde{\phi}^{k-1},\ L^{k-1}\, \tilde{\phi}^{k-1} + \bar{I}_k^{k-1}(f^k - L^k\, \bar{\phi}^k)\right)$.

(C) Calculate
$\bar{\bar{\phi}}^k = \bar{\phi}^k + I_{k-1}^k\left(\tilde{\phi}^{k-1} - I_k^{k-1}\, \bar{\phi}^k\right)$.

(D) Perform ν_2 additional relaxation sweeps on (3.6) starting with $\bar{\bar{\phi}}^k$ and yielding the final $\tilde{\phi}^k$ of (3.7).

In this algorithm I_k^{k-1}, $\bar{I}_k^{k-1}$ are fine-to-coarse grid transfer operators; I_{k-1}^k is an interpolation operator. We refer to the above cycle as $MG(\nu_1,\nu_2)$. In the notation of this section (3.6) includes both equations (3.2) and (3.4).

The basic cycle described above is for improving a given approximation on level k. The full multigrid (FMG) process involves solving the problem on the coarsest grid, interpolating it to finer grids, and making the cycle $MG(\nu_1,\nu_2)$ a few times after each refinement.

3.4.2. Full Multigrid Algorithm (FMG)

1. Solve (3.6) for $k = 1$, using a continuation method (see remark below).
2. Set $k = k + 1$ and
   $\tilde{\phi}^k = \Pi_{k-1}^k \tilde{\phi}^{k-1}$, where Π_{k-1}^k is a bicubic interpolation.
3. Perform $\gamma(k)$ times the cycle
   $\tilde{\phi}^k \leftarrow MG(k, \tilde{\phi}^k, f^k)$.
4. If $k < m$, go to step 2; otherwise stop.

A Remark on Step 1 of the FMG Algorithm (Continuation Method)

Since the problem involved is a nonlinear one, and we are using a Newton iteration, a good initial approximation may be needed to get fast convergence for $k = 1$ (the coarsest grid). This has been achieved by using a continuation process where we solve first for a small norm $\|\phi^h\|^2$, then gradually increasing it until the prescribed norm is obtained. Each time the norm is increased, the solution of the previous step is used as an initial approximation. In order to get a good initial approximation for the smallest-norm problem, we have solved for the bifurcation point from the trivial branch of solutions.

3.5 Solving for the Bifurcation Point

At a bifurcation point (ϕ^*, V_0^*), the linearized problem of (3.1) must have a zero eigenvalue, and the corresponding eigenfunction gives rise to a second branch of solutions. Since $\phi = 0$ is a solution for any V_0, we may try to find a bifurcating branch from the trivial one $(0,V_0)$. The linearized equations around $(0,V_0)$ are given by

$$W_{xx} + r\left(\frac{1}{r} W_r\right)_r + 4V_0^2 \tilde{\alpha}^2(0)W = 0, \qquad \text{in } \Omega \tag{3.8a}$$

$$W = 0, \qquad \text{on } \partial\Omega. \tag{3.8b}$$

If there exists a bifurcating branch from the trivial one $(0,V_0)$, equation (3.8) has a solution (W^*,V_0^*) with $\|W^*\|_2 = 1$ where $\| \, \|_2$ denotes the L_2 norm.

We discretize (3.8) in a way similar to the discretization of (3.1). The constraint

$$\|W^h\|^2 = 1,$$

is added to ensure a non-zero solution to the problem. The process of solving the eigenvalue problem is identical to the process of solving (3.2), (3.4).

Once this linear eigenvalue problem is solved, we can use $\phi_0 = \pm\varepsilon W$ as an initial approximation for our original problem with a prescribed norm of ε. The sign is chosen such that ϕ_0 has negative values, to ensure that the total streamfunction $\psi = \frac{r^2}{2} + \phi$ will have closed streamlines with negative values (the bubble).

4. NUMERICAL RESULTS

Experiments were performed with equations (3.2), (3.3) using FMG algorithm of Section 3.4.2. In these experiments the domain was

$$\Omega^h = \{(nh, \ell h),\ 0 < nh < 5,\ 0 < \ell h < 2\}.$$

Three levels were used in the multigrid algorithm where the finest grid problem has mesh size 1/16. On the coarsest level 20 relaxations were performed while on finer grids $\nu_1 = \nu_2 = 3$, $\gamma(k) = 4$. In all numerical experiments $I_k^{k-1} = \bar{I}_k^{k-1}$ is injection, I_k^{k-1} is bilinear interpolation, and Π_{k-1}^k is bicubic interpolation.

Tables I-IX contain the L_2-norm of the residuals and the values of V_0^2 at the end of each cycle on the finest grid. Cycle #0 refers to the approximation obtained from the previous level as an initial guess. Figures 1-9 show the streamlines (contours of ψ) for the different cases. The value of V_0^*, the swirl parameter value for which bifurcation occurs is $V_0^* = 1.0069$ (computed on coarsest level).

The experiments clearly show that the multigrid method suggested is very efficient. In fact, as seen by the convergence history for V_0^2, it is enough to take $\gamma(k) = 2$, instead of $\gamma(k) = 4$, i.e., by 2 FMG cycles the problem is already solved.

The results show that bigger bubbles are obtained for smaller swirl parameters, contradicting to what one would expect. This may be the result of the assumption made in the model, that the same functional dependence of k, ω on ψ holds inside as well as outside the bubble. A future study will investigate this point by solving the full systems (2.1), making no extra assumptions.

REFERENCES

[1] J. H. Bolstad, H. B. Keller, "A Multigrid Continuation Method for Elliptic Problems with Turning Points," to appear in SIAM J. Sci. Stat. Comput.

[2] A. Brandt, Multigrid Techniques: 1984 Guide with Applications to Fluid Dynamics. Monograph available as GMD-Studie No. 85, GMD-FIT, Postfach 1240, D-5205, St. Augustin 1, West Germany.

[3] M. M. Hafez and M. D. Salas: "Vortex Breakdown Simulation Based on a Nonlinear Inviscid Model," Proceedings of ICASE/NASA Workshop on Vortex Dominated Flows, (M. Y. Hussaini and M. D. Salas, eds.), Springer-Verlag, 1986.

[4] S. Leibovich: "Vortex Stability and Breakdown: Survey and Extension," AIAA J., Vol. 22, No. 9, 1984, pp. 1192-1206.

[5] K. Stüben and U. Trottenberg: "Multigrid Methods: Fundamental Algorithms, Model Problem Analysis and Applications," in Multigrid Methods, Lecture Notes in Mathematics, No. 960, (W. Hackbusch and U. Trottenberg, eds.), Springer-Verlag, 1982.

Table I. $\|\phi^h\|^2 = .005$

cycle #	$\|\text{Residuals}\|_2$	V_0^2
0	.362 (-1)	.95088
1	.986 (-3)	.96069
2	.843 (-4)	.96039
3	.148 (-4)	.96041
4	.745 (-5)	.96042

Table II. $\|\phi^h\|^2 = .05$

cycle #	$\|\text{Residuals}\|_2$	V_0^2
0	.948 (-1)	.68322
1	.232 (-2)	.68962
2	.251 (-3)	.68939
3	.113 (-3)	.68941
4	.918 (-4)	.68941

Table III. $\|\phi^h\|^2 = .11$

cycle #	$\|\text{Residuals}\|_2$	V_0^2
0	.122	.59214
1	.233 (-2)	.54739
2	.215 (-3)	.54732
3	.615 (-4)	.54733
4	.542 (-4)	.54733

Table IV. $\|\phi^h\|^2 = .15$

cycle #	$\|\text{Residuals}\|_2$	V_0^2
0	.135	.48347
1	.243 (-2)	.48803
2	.168 (-3)	.48798
3	.474 (-4)	.48798
4	.425 (-4)	.48798

Table V. $\|\phi^h\|^2 = .2$

cycle #	$\|\text{Residuals}\|_2$	V_0^2
0	.150	.42902
1	.242 (-2)	.43301
2	.193 (-3)	.43294
3	.366 (-4)	.43294
4	.266 (-4)	.43294

Table VI. $\|\phi^h\|^2 = .4$

cycle #	$\|\text{Residuals}\|_2$	V_0^2
0	.192	.30435
1	.271 (-2)	.30725
2	.239 (-3)	.30719
3	.177 (-3)	.30719
4	.176 (-3)	.30719

Table VII. $\|\phi^h\|^2 = .6$

cycle #	$\|Residuals\|_2$	V_0^2
0	.230	.24006
1	.303 (-2)	.24335
2	.218 (-3)	.24231
3	.188 (-3)	.24231
4	.175 (-3)	.24231

Table VIII. $\|\phi^h\|^2 = 1.0$

cycle #	$\|Residuals\|_2$	V_0^2
0	.295	.17139
1	.385 (-2)	.17303
2	.363 (-3)	.17302
3	.294 (-3)	.17302
4	.278 (-3)	.17302

Table IX. $\|\phi^h\|^2 = 2.0$

cycle #	$\|Residuals\|_2$	V_0^2
0	.428	.10176
1	.701 (-2)	.10276
2	.777 (-3)	.10275
3	.584 (-3)	.10275
4	.574 (-3)	.10275

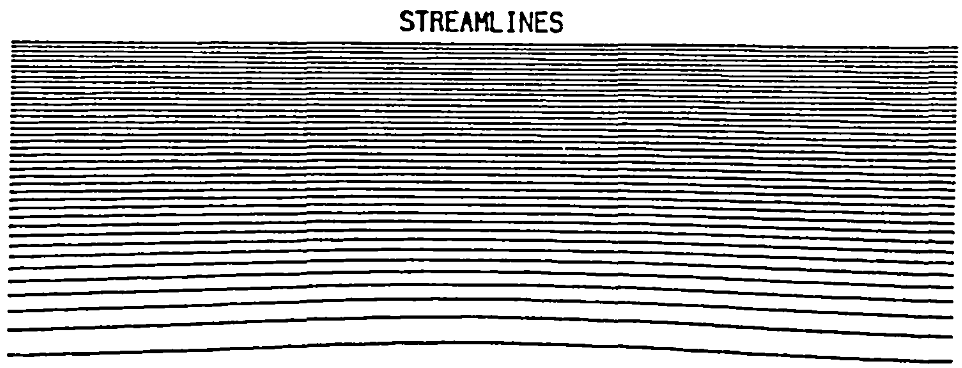

Figure 1. $\|\phi^h\|^2 = .005$, $V_0^2 = .96042$.

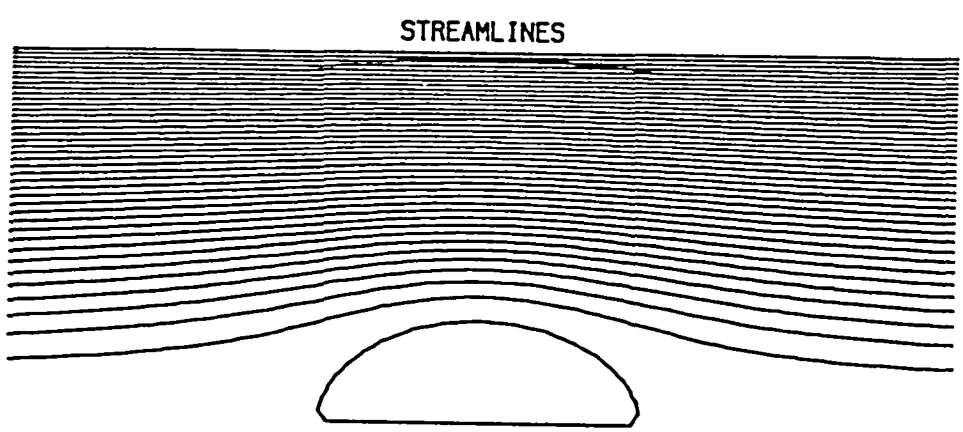

Figure 2. $\|\phi^h\|^2 = .05$, $V_0^2 = .68941$.

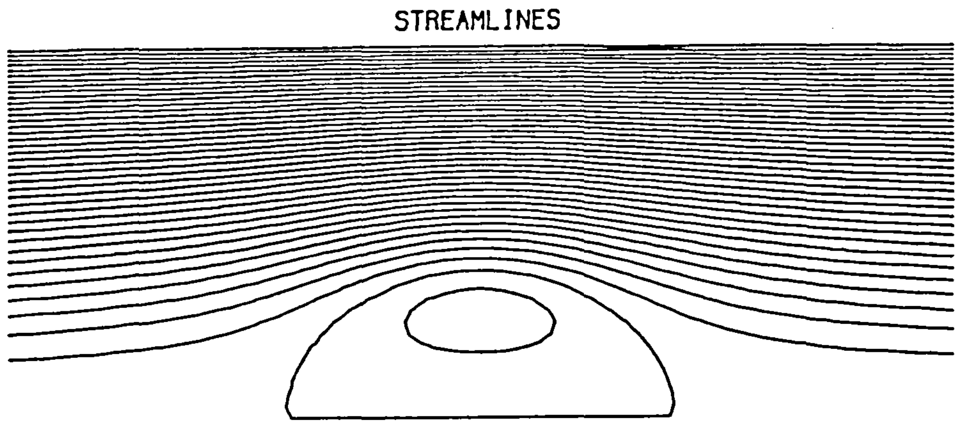

Figure 3. $\|\phi^h\|^2 = .11$, $V_0^2 = .54733$.

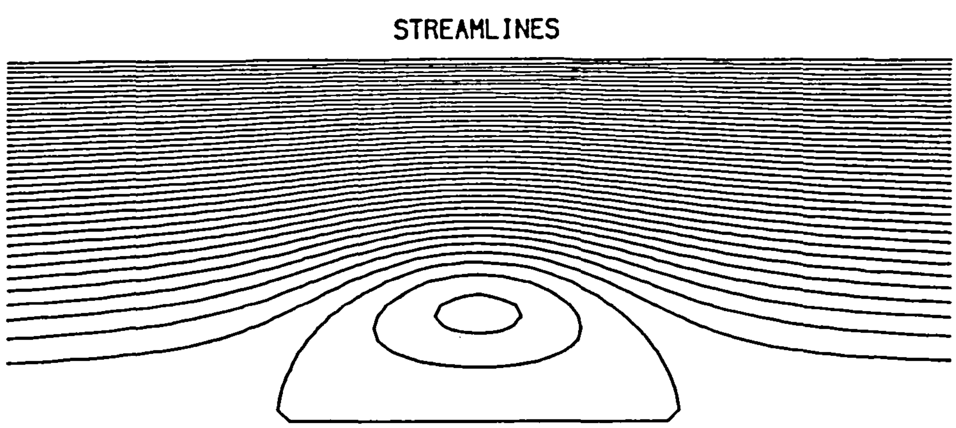

Figure 4. $\|\phi^h\|^2 = .15$, $V_0^2 = .48798$.

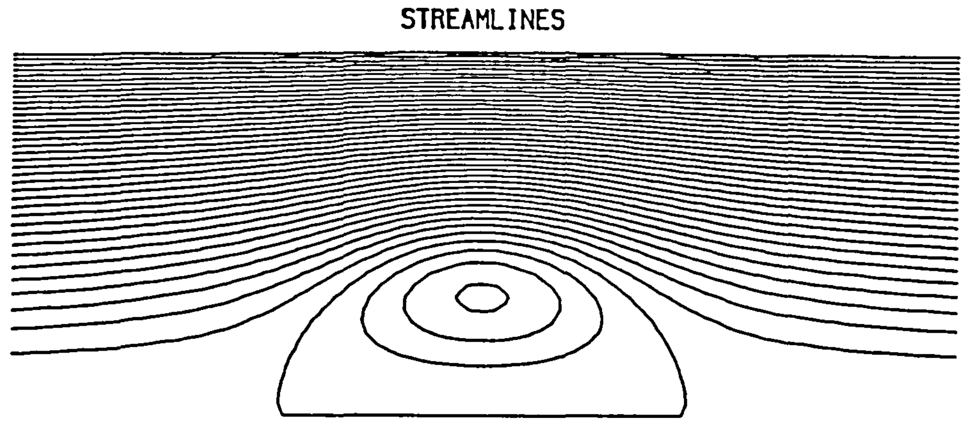

Figure 5. $\|\phi^h\|^2 = .2$, $V_0^2 = .43294$.

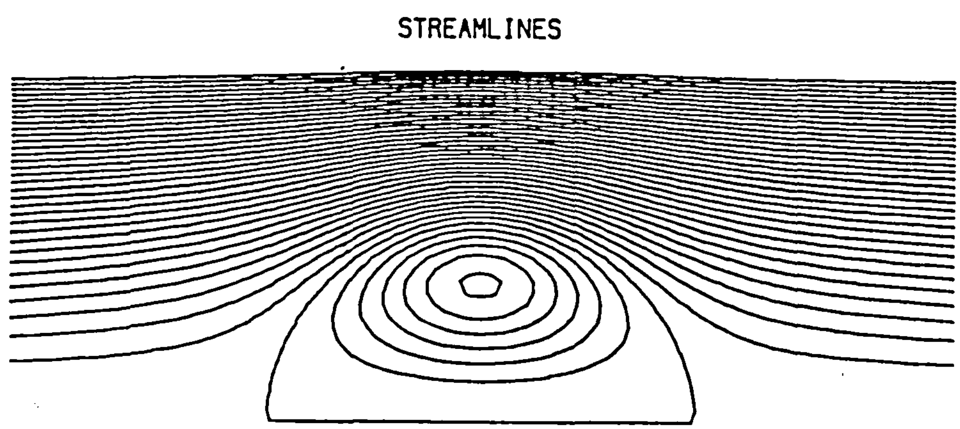

Figure 6. $\|\phi^h\|^2 = .4$, $V_0^2 = .30719$.

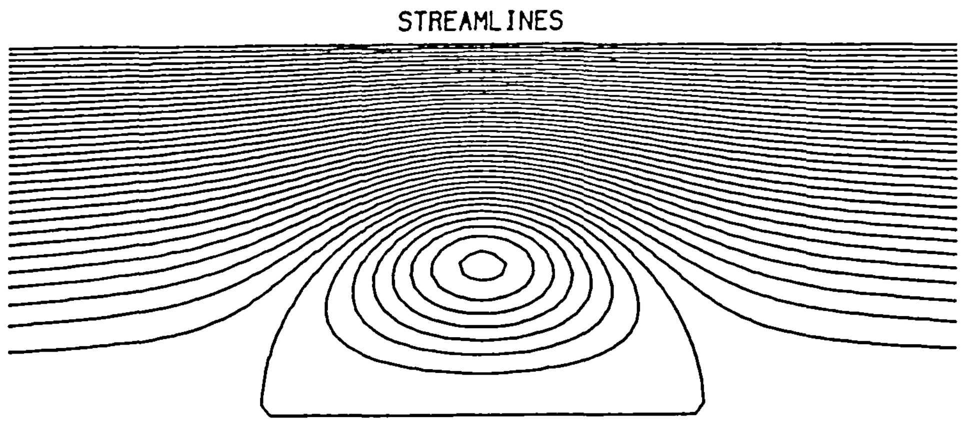

Figure 7. $\|\phi^h\|^2 = .6$, $V_0^2 = .24231$.

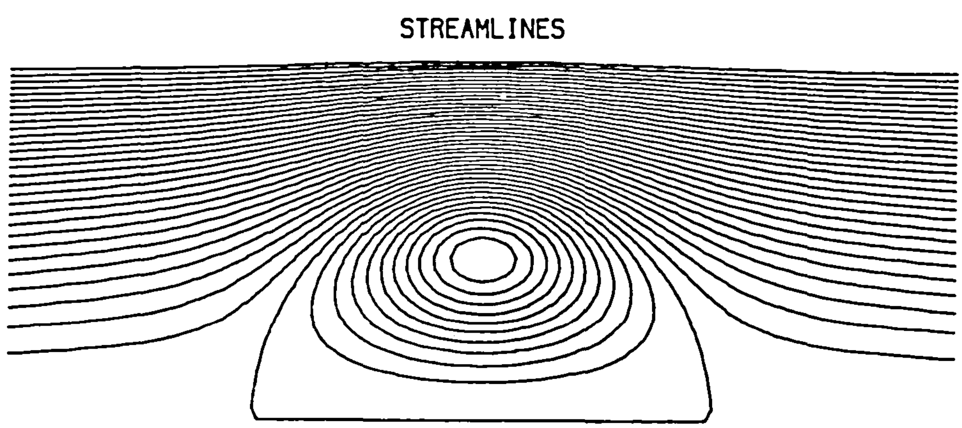

Figure 8. $\|\phi^h\|^2 = 1.0$, $V_0^2 = .17302$.

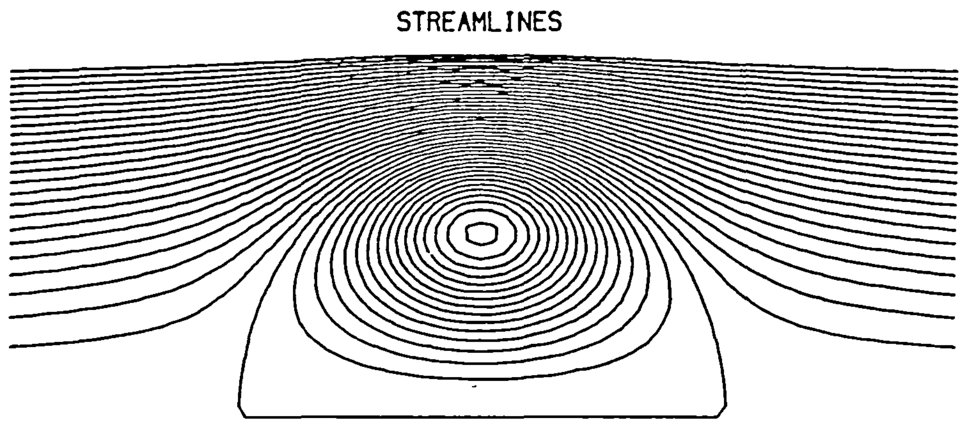

Figure 9. $\|\phi^h\|^2 = 2.0$, $V_0^2 = .10275$.

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16. Abstract

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