IN-13062 9P.

NASA TECHNICAL MEMORANDUM NASA TM-88465

EXPLICIT EXPRESSION TO PREDICT THE EROSIVE BURNING RATE OF SOLID PROPELLANTS

Wang Shoufan

Journal of Engineering Thermophysics, Vol. 6, No. 1, February 1985, Beijing, China, pp 84-86 (A85-37935)

(NASA-TM-88465) EXPLICIT EXPRESSION TO PREDICT THE EROSIVE BURNING RATE OF SOLID PROPELLANTS (National Aeronautics and Space Administration) 9 p HC A02/MF A01 CSCL 21H N86-28259 Unclas G3/28 43342

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON D.C. 20546 JULY 1986

ORIGINAL PAGE IS
OF POOR QUALITY

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-88465		
4. Title and Subtitle EXPLICIT EXPRESSION TO PREDICT THE EROSIVE BURNING RATE OF SOLID PROPELLANTS		5. Report Date JULY 1986
		6. Performing Organization Code
7. Author(s) Wang Shoufan (Wang, S.)		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address SCITRAN Box 5456 Santa Barbara, CA 93108		11. Contract or Grant No. NASw- 4004
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546 NL45748l		14. Sponsoring Agency Code
15. Supplementary Notes Journal of Engineering Thermophysics, Vol. 6, No. 1, February 1985, Beijing China, pp 84-86 (A85-37935)		

16. Abstract

Using the theory of gasdynamics and heat transfer from a turbulent gas flow to the burning surface of propellant along a permeable wall, an explicit expression is derived to predict the burning rate of the solid propellant with crossflow. Results of the calculation have been compared with experimental data and proved to be correct.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	Unclassified and Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	7	

EXPLICIT EXPRESSION TO PREDICT THE EROSIVE BURNING RATE OF SOLID PROPELLANTS

Wang Shoufan*
(Beijing Institute of Technology)

I. Introduction

Erosive burning is an important topic closely related to /84** the design standard of a solid rocket engine. After 30 years of hard work throughout the world, its features are still not fully understood. Representative theories include the L-R equation and K.K. Kuo's chemically reactive turbulent boundary layer theory. In order to clearly understand the law governing erosive burning, the gas flow near the boundary layer of a burning surface is quantitatively analyzed based on existing theory. In addition, an explicit theoretical formula to calculate the erosive burning rate is derived based on macroscopic heat transfer theory.

II. Gasdynamic Analysis of Propellant Surfaces

In order to understand the state of flow of the combustion gas along a surface, the more complicated procedure is to numerically solve a series of equations: the equations of mass, momentum, composition, energy and state of a steady, two-dimensional, chemically reacting turbulent boundary layer. After being simplified, they are as follows:

*This paper presented at the Third Annual Meeting of the Engineering Thermal Physics Society of China held in Wuxi, October 1982

**Numbers in margin indicate foreign pagination

ORIGINAL PAGE IS
OF POOR QUALITY

$$\partial(\rho u)/\partial x + \partial(\rho v^{\circ})/\partial y = 0 \tag{2-1}$$

$$\rho u \partial u/\partial x + \rho v^{\circ} \partial u/\partial y = \partial/\partial y[\mu_{t} \partial u/\partial y - (\nu\mu_{t}/\rho c_{p})\partial p/\partial x] \tag{2-2}$$

$$\rho u \partial Y/\partial x + \rho v^{\circ} \partial Y/\partial y = \partial/\partial y[(\mu/S_c)_{ef} - \partial Y/\partial y] + \dot{\omega} \tag{2-3}$$

$$\rho u \partial H/\partial x + \rho v^{\circ} \partial H/\partial y = \partial/\partial y\{(\mu/Pr)_{t} \partial H/\partial y + [\mu_{ef} - (\mu/Pr)_{ef}]\partial/\partial y(u^2/2)\} \tag{2-4}$$

$$p = \rho RT/W \tag{2-5}$$

where

$$\rho v^{\circ} = \rho v + \overline{\rho' v'}, \qquad \tau_t = -\rho\, \overline{u' v'} = \mu_t \partial u/\partial y.$$

Based on the double equation turbulence model, equations for turbulent kinetic energy and turbulent dissipation factor ε are used to close the loop for equations (2-1) - (2-5), i.e.,

$$\begin{aligned} \rho u \partial K/\partial x + \rho v^{\circ} \partial K/\partial y &= \partial/\partial y[(\mu + \mu_t/\sigma_1)\partial K/\partial y] \\ &+ \mu_t[(\partial u/\partial y)^2 - (\nu/\rho c_p)\partial p/\partial x \partial u/\partial y] - \rho\varepsilon \end{aligned} \tag{2-6}$$

$$\begin{aligned} \rho u \partial \varepsilon/\partial x + \rho v^{\circ} \partial \varepsilon/\partial y &= \partial/\partial y[(\mu + \mu_t/\sigma_2)\partial \varepsilon/\partial y] + c_1\mu_t[(\partial u/\partial y)^2 \\ &- (\nu/\rho c_p)\partial p/\partial x \partial u/\partial y]\varepsilon/K - c_2\rho\varepsilon^2/K \end{aligned} \tag{2-7}$$

By choosing the appropriate boundary conditions to solve the above equations, it is possible to find a numerical solution. Using the AP 25% PBAA/EPON 75% propellant as an example, the values of the functions $K = K(y/\delta)$; $\tau_t = \tau(y/\delta)$ and $u/u_\infty = u(y/\delta)$ are calculated [1]. The calculated turbulent stress follows the following formula: $\tau_1 = a(y/\delta) + b(y/\delta)^2$. Here $\tau_1 = -1000\, \overline{u' v'}/u_\infty^2$, under the flow condition $(u_\infty = 450\text{m/s};\ p = 7.24\text{mPa};\ T_\infty = 2250\text{K})$ the constants are $a = 19.9$ and $b = -55.15$. Based on the definition of aerodynamic friction distance: $y^+ = v_* y/\nu = \sqrt{\tau_1}\, Re_\delta/\sqrt{1000}$, let us choose $y^+ = 30$ as the lower limit of the turbulent region. By substituting it into the equation for y^+ and using the τ_1 expression in iterations, it was calculated that $y/\delta = 0.005$. If $\delta = 1\text{cm}$ at $y > 50\mu\text{m}$, it is in the turbulent region. Under practical working conditions, the exothermic reaction layer of the propellant is usually not greater than 200 μm. Therefore, we can assume

ORIGINAL PAGE IS
OF POOR QUALITY

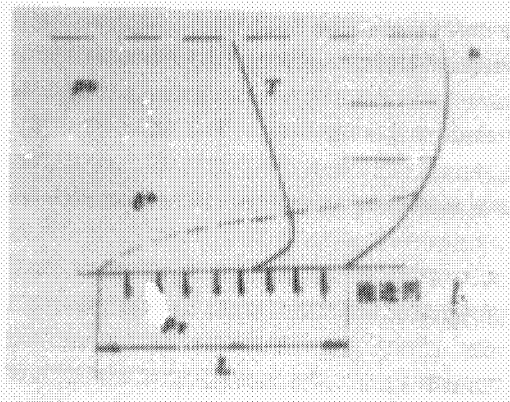

Figure 1. The Flow Model.
1. Propellant.

that gas phase reactions are taking place in the turbulent region.

III. Theoretical Analysis

If the composition of the combustion gas of the propellant sample is different from that of the incident flow, it is possible to assume that there is an interface between these two gases, such as δ^* in Figure 1. Based on the principle of conservation of mass, the following formula can be obtained: /85

$$\delta^* = \sqrt{\rho_p r d \overline{RT} \cdot l / 3.1 g u_\infty p_\infty}$$

ORIGINAL PAGE IS
OF POOR QUALITY

Let us choose $l = 1.3\text{cm}, p = 70\ \text{kg/cm}^2, u_\infty = 500\text{m/s}$.
By calculation $\delta^* = 0.04$. δ^* increases as the value of u_∞ decreases. Under such conditions, the combustion products occupy approximately 400 μm above the specimen itself. Based on Planck's principle, the mixing length $l_m = 0.4\delta^* = 160\mu\text{m}$. This means that at below $400 - 160 = 240\mu\text{m}$, there should be no disturbance due to the constituents of the incident flow. The heat flow density on the surface is increased only due to the turbulence of the combustion gas of the specimen alone.

We can assume that the burning rate of the propellant is proportional to the heat flow density of gas phase feedback. Under transversal flow (turbulent flow) conditions, burning rate can be expressed by the following formula: $r = Cq = C(-\lambda \partial T/\partial y + \rho c_p \overline{v'T'})$. If the two terms are separated and expressed by the burning rate, then $r = r_s + r_t$. The first term r_s represents the effect of laminar heat conduction, and the second term represents the effect of turbulence on heat transfer. Based on conventional heat transfer theory, it is also possible to express r_s as follows: $r_s = Ch(T_f - T_s)$. Based on the principle of conservation of energy, we get $\rho_p c r_s (T_s - T_i) = h(T_f - T_s)$. Here, the proportionality constant C is equivalent to $1/\rho_p c$. By using the heat transfer coefficient formula $h = \rho c_p u_\infty St$, we get

$$r_s = (\rho c_p u_\infty St/\rho_p c)(T_f - T_s)/(T_s - T_i) \tag{3-1}$$

Where there is gas coming in along the wall, the expression for the Stanton number is[2]

$$St = St_0\Psi = C_f Pr^{-2/3}/2 = C_{f_0} Pr^{-2/3}\Psi/2 = C_{f_0} Pr^{-2/3}[2/(\psi^{1/2} + 1)]^2[1 - (b/b_{cr})]^2 \tag{3-2}$$

ORIGINAL PAGE IS
OF POOR QUALITY

Where $\varphi = T_s/T_f,\ b = 2\rho_p r/\rho_\infty u_\infty C_{f_0}$, b_{cr} represents the value of b corresponding to zero stress on the surface. However, $C_{f_0} = C_d(Re_d)^{-0.2},\ \rho_\infty u_\infty = C_p(\overline{\rho u})$. Thus, we get $b = 2\rho_p r / C_d(Re_d)^{-0.2} \cdot C_p(\overline{\rho u})$ where $\overline{\rho u} = (8/d^2)\int_0^{d/2} \rho u y\,dy$. It is also empirically known that $St_0 = 0.023 Pr^{-2/3}(Re_d)^{-0.2}$. By substituting St_0, b, and St into equation (3-1) and then into the formula for burning rate r, we get

$$r = r_o + A(\overline{\rho u})^{0.8} d^{-0.2}[1 - B\rho_p r d^{0.2}/(\overline{\rho u})^{0.8}]^2 \qquad (3\text{-}3)$$

where

$$A = 0.023 C_p \mu^{0.2} Pr^{-2/3}(T_f - T_s) \cdot 4/\rho_p c(T_s - T_i)(\varphi^{1/2} + 1)^2$$
$$B = 2/b_{cr} C_d C_p \mu^{0.2}.$$

When $r = r_o$ (i.e., $r_e = 0$), based on equation (3-3), the following condition should be satisfied: /86
$1 - B\rho_p r_o d^{0.2}/(\overline{\rho u})^{0.8} = 0$. Thus, the erosion boundary condition can be obtained: $(\overline{\rho u})_{th}^{0.8} = B\rho_p r_o d^{0.2}$. In order to facilitate design calculation, implicit function in equation (3-3) is made explicit, i.e.,

$$r/r_o = (\overline{\rho u})^{0.8}/(\overline{\rho u})_{th}^{0.8}\left[1 + (1/2AB\rho_p)\left\{1 - \sqrt{1 + 4AB\rho_p(1 - (\overline{\rho u})_{th}^{0.8}/(\overline{\rho u})^{0.8})}\right\}\right] \qquad (3\text{-}4)$$

IV. Comparison of Theory to Experiment

Based on equation (3-4) and the specifications for the American propellant J.P.N., we calculated r/r_o and $(\overline{pu})$

ORIGINAL PAGE IS
OF POOR QUALITY

($A = 11.48$ kg • cm • s unit; $B = 56.6$). The results are shown in Table 1. As compared to the experimental data obtained by Winblast, the error of calculation is not greater than 1 percent when $u < 600$ m/s.

Table 1. Results Calculated Based on Equation (3-4).

u(m/s)	200	250	300	350	400	450	500	600	700
r/r_0	1.06	1.139	1.22	1.31	1.4	1.49	1.58	1.75	1.93

Symbols

C_{1-4}, C_d, C_p - constants

C_p, C - specific heat for burning gas and propellant

Re_d - ($= \overline{\rho u} d/\mu$)

T_f, T_s, T_j - flame temperature, surface temperature, and initial propellant temperature

Y - mass fraction of burning gas constituent

ρ, ρ_p - density of burning gas and propellant

Subscripts

o - absence of gas blowing onto the surface

′ - fluctuation

References

[1] Razdon, M. K. and K. K. Kuo: "Turbulent Boundary Layer Analysis and Experimental Investigation of Erosive Burning Problem of Composite Solid Propellants", AD/A 067 351, (1978).

[2] S. S. Kutateldze: "Turbulent Boundary Layers in Compressible Gases", London Edward Arnold, Ltd, London, (1964).