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1. INTRODUCTION

The primary purpose of this research effort was to develop personal computer based software for vibrational analysis. The software was developed to analytically determine the natural frequencies and mode shapes for the uncoupled lateral vibrations of the blade and counterweight assemblies used in a single bladed wind turbine. The uncoupled vibration analysis was performed in the flapwise and chordwise directions for static or non-rotating conditions of the counterweight assembly and in the flapwise direction for static conditions of the blade assembly. The effects of rotation on the uncoupled flapwise vibration of the blade and counterweight assemblies was evaluated for various rotor speeds up to 90 rpm. A secondary purpose of this research effort was to attempt to experimentally determine the natural frequencies for the blade and counterweight assemblies for static rotor conditions and to correlate these results with the analytical values.

Section 2 of this report describes the theory used in calculating the natural frequencies and mode shapes. The theory is based upon a lumped mass formulation for the blade and counterweight assemblies. It is this theory which forms the basis for the development of the computer codes which ultimately generate the analytical results. Section 3 describes the computer codes developed and documents how the theory is implemented in the codes. The discussion of each code includes the input and output data structures used. The codes are designed to be as general as possible so that other designs can be readily analyzed. The input for the codes are generally interactive to facilitate usage. The actual code listings are included in the Appendix. Section 4 describes the operation and use of the computer codes. Section 4 also includes an example which demonstrates the application of the the computer software. Section 5 describes the actual results of the analysis of the MOD-O blade and counterweight assemblies at various operational speeds and includes a discussion of the analytical and experimental results. Section 6 contains general conclusions and recommendations.

2. THEORY

The following formulation concerns the uncoupled flapwise bending vibration of the blade and counterweight assemblies including the effects of rotation. Flapwise vibration, by definition, deals with bending in a plane perpendicular to the plane of rotation. Additional reading material on the determination of the vibratory characteristics of nonuniform beams and the effects of rotation on nonuniform rotating beams can be obtained in References (1) through (6).

Figure 1 shows a discrete mass model of a nonuniform rotating cantilevered beam. The nonuniform beam has been modeled by a series of lumped masses connected by massless beam segments of various lengths. The flexural rigidity of each beam segment is allowed to vary along the length of the beam segment. Free body diagrams are illustrated in Figure 2 for mass n and the beam segments on either side of this mass. In Figure 2,

ℓ = the segment length

m = the lumped mass

M = the bending moment

V = the shear force

F = the centrifugal or inertia force due to rotation

The deflection and slope of the beam at mass n can now be formulated in terms of the deflection and slope of the beam at the n-1 mass by letting

d_{Sn} = the deflection of a cantilevered beam at the free end due to a unit shear force applied at the free end

d_{Mn} = the deflection of a cantilevered beam at the free end due to a unit bending moment applied at the free end

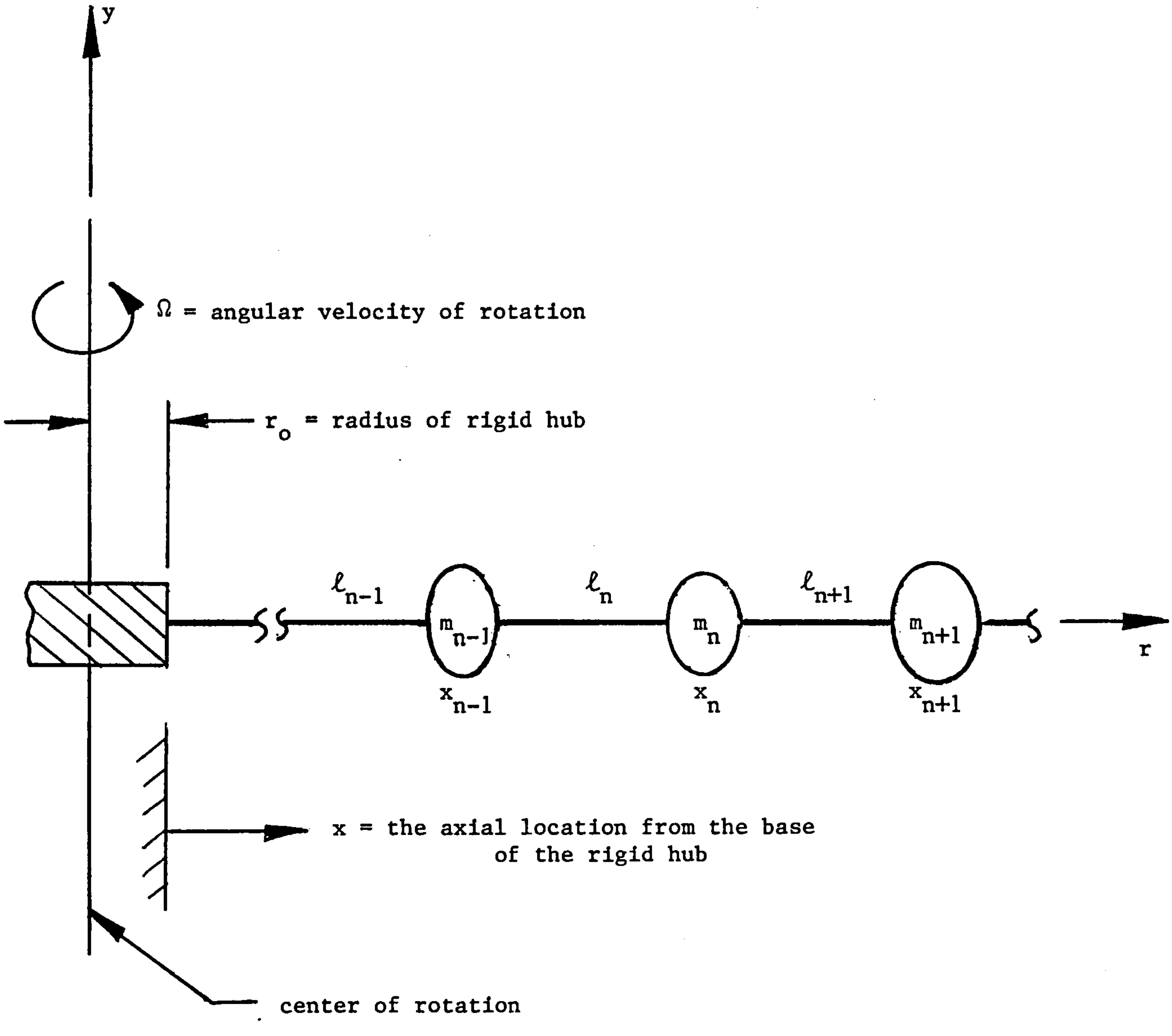


Figure 1 Lumped mass model of a nonuniform rotating beam.

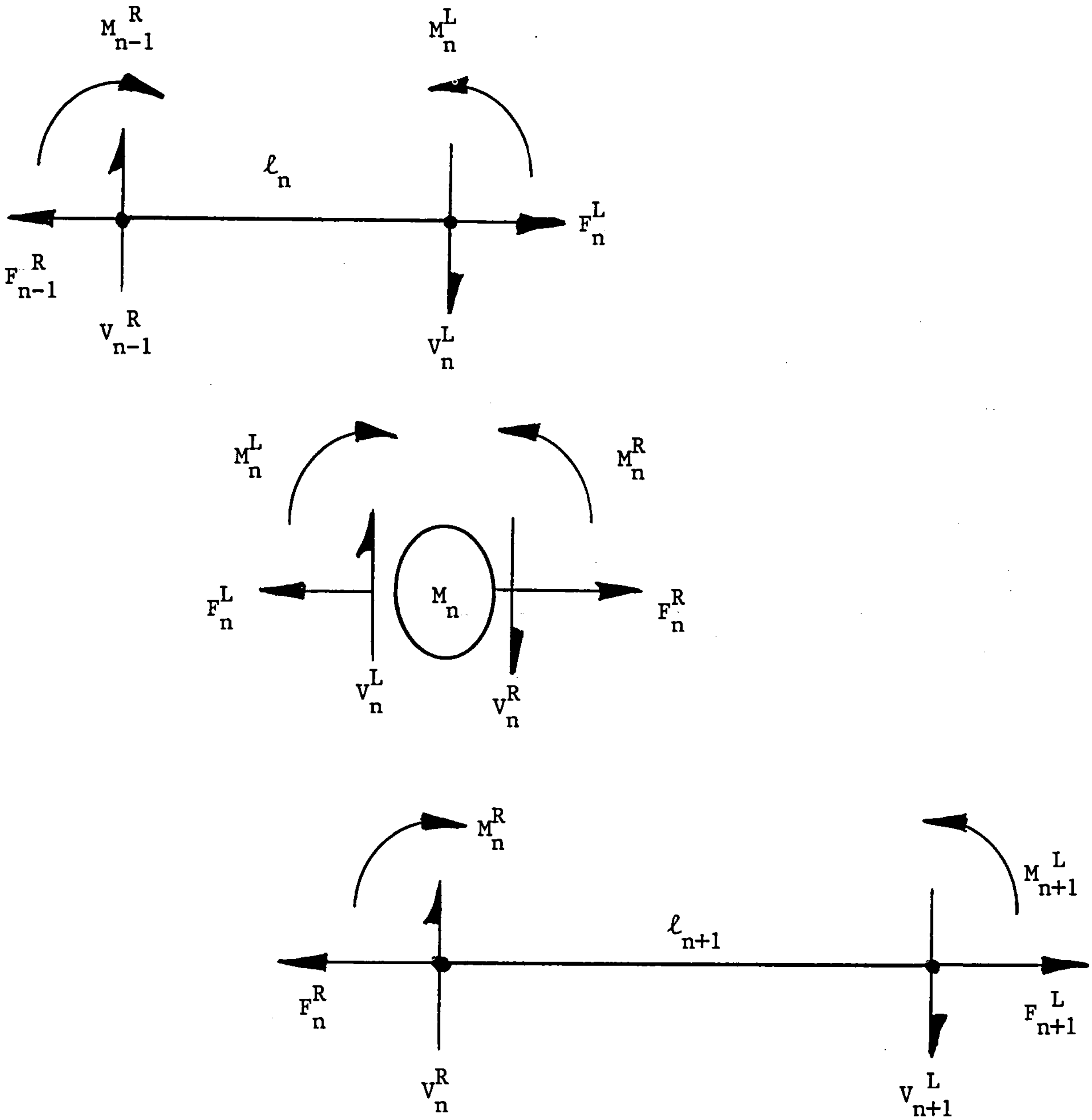


Figure 2 Free body diagrams of the n^{th} mass and the beam segments on either side of the mass.

α_{Sn} = the slope of a cantilevered beam at the free end due to a unit shear force applied at the free end

α_{Mn} = the slope of a cantilevered beam at the free end due to a unit bending moment applied at the free end

Expressions for these coefficients have been developed in detail in Reference (1). Assuming a linear variation in the flexural rigidity for a beam segment of length ℓ_n, these coefficients are given by

$$d_{Sn} = \frac{\ell_n^3}{(EI)_n - (EI)_{n-1}} \left\{ \left[1 + \frac{2(EI)_{n-1}}{(EI)_n - (EI)_{n-1}} + \frac{[(EI)_{n-1}]^2}{[(EI)_n - (EI)_{n-1}]^2} \right] \log \frac{(EI)_n}{(EI)_{n-1}} - \frac{3}{2} - \frac{(EI)_{n-1}}{(EI)_n - (EI)_{n-1}} \right\}$$

$$d_{Mn} = \alpha_{Sn} = \frac{\ell_n^2}{(EI)_n - (EI)_{n-1}} \left[\frac{(EI)_n}{(EI)_n - (EI)_{n-1}} \log \frac{(EI)_n}{(EI)_{n-1}} - 1 \right] \quad (1)$$

$$\alpha_{Mn} = \frac{\ell_n}{(EI)_n - (EI)_{n-1}} \log \frac{(EI)_n}{(EI)_{n-1}}$$

where

$(EI)_n$ = the flexural rigidity of the beam at the location of mass n

$(EI)_{n-1}$ = the flexural rigidity of the beam at the location of the n-1 mass

The centrifugal forces caused by rotation will remain parallel to the original undeformed axis of the beam as the beam vibrates perpendicular to the plane of rotation as illustrated in Figure 3. This effect will cause a stiffening of the beam, thus, increasing the natural frequencies of vibration. As seen in Figure 3, the centrifugal force on the right-hand end of the beam can be resolved into two components. Assuming small displacements and thus small slopes, one component is a tensile force acting on the beam of magnitude F_n^L, and the other component is a downward shear force of magnitude $F_n^L \theta_n^L$. The deflection and slope of the beam segment at the location of mass n can then be determined in terms of the deflection and slope of the beam segment at the n-1 mass location as illustrated in Figure 4. Thus, the deflection at the right-hand end of the beam segment is

$$y_n^L = y_{n-1}^R + \ell_n \theta_{n-1}^R + d_{Mn} M_n^L - d_{Sn} (V_n^L + F_n^L \theta_n^L) \tag{2}$$

and the corresponding slope is

$$\theta_n^L = \theta_{n-1}^R + \alpha_{Mn} M_n^L - \alpha_{Sn} (V_n^L + F_n^L \theta_n^L) \tag{3}$$

Taking moments about the left-hand end of the beam segment yields

$$M_n^L = M_{n-1}^R + V_n^L \ell_n + F_n^L \theta_n^L \ell_n$$

Using the small slope assumption

$$\sin \theta_n^L \cong \tan \theta_n^L \cong \theta_n^L = \frac{y_n^L - y_{n-1}^R}{\ell_n}$$

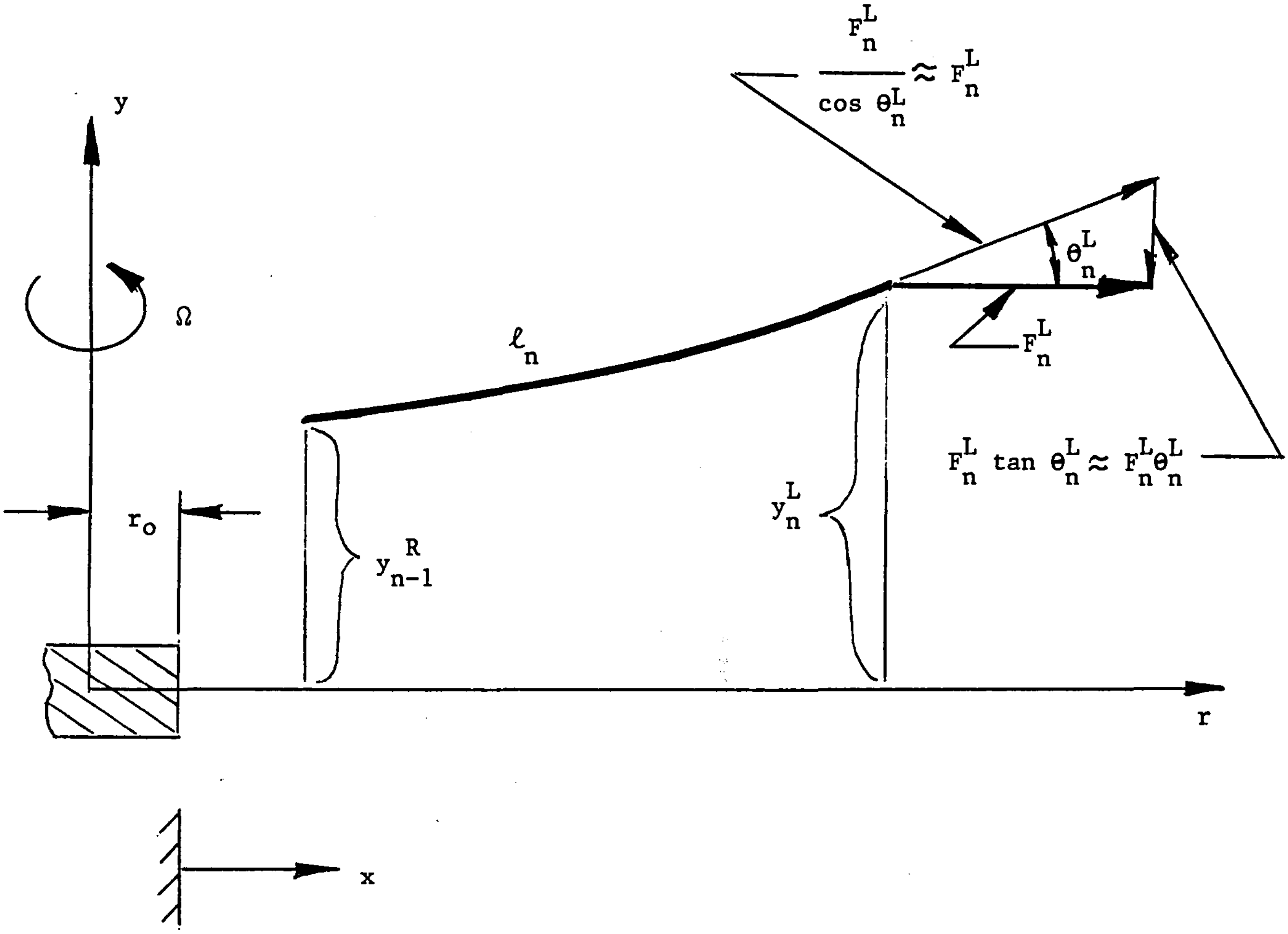


Figure 3 Centrifugal force acting on a rotating beam segment of length ℓ_n.

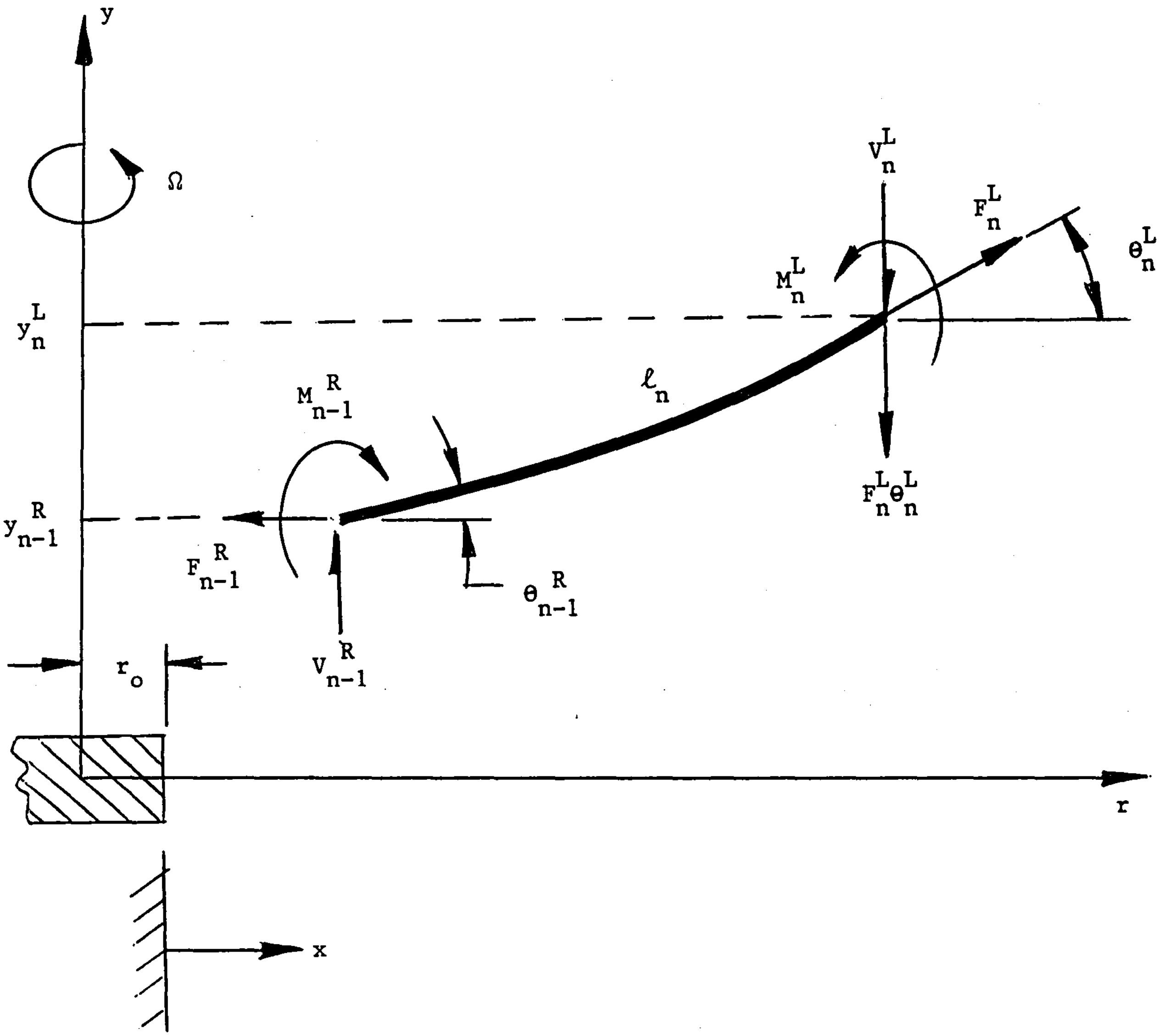


Figure 4   Loading on a deformed beam segment of length ℓ_n.

the bending moment in the beam segment at the location of mass n in the previous equation becomes

$$M_n^L = M_{n-1}^R + V_n^L \ell_n + F_n^L (y_n^L - y_{n-1}^R) \tag{4}$$

Since the beam segment shown in Figure 4 is massless, the summation of the forces in the y-direction yields

$$V_{n-1}^R - V_n^L - F_n^L \theta_n^L + F_n^L \sin\theta_n^L = 0$$

Making use of the small deflection and thus small slope assumption, this equation reduces to

$$V_n^L = V_{n-1}^R \tag{5}$$

Using the free body diagram of mass n shown in Figure 2, Newton's law applied in the radial direction yields

$$F_n^L = F_n^R + m_n r_n \Omega^2 = \sum_{i=n}^{N} m_i (r_o + x_i)\Omega^2 \tag{6}$$

where, as shown in Figure 1,

Ω = the angular velocity of rotation

$r_n = r_o + x$ = the radial location of mass n

r_o = the radius of the rigid hub

N = the total number of lumped masses in the discrete model of the beam

Applying Newton's second law in the radial direction to the free body diagram of the massless beam segment of length ℓ_n shown in Figure 2 yields

$$F_{n-1}^R = F_n^L = \sum_{i=n}^{N} m_i (r_o + x_i)\Omega^2 \tag{7}$$

Using Equations 5 and 7, Equations 2, 3, and 4 become

$$y_n^L = y_{n-1}^R + \ell_n \theta_{n-1}^R + d_{Mn} M_n^L - d_{Sn} (V_{n-1}^R + F_{n-1}^R \theta_n^L) \tag{8}$$

$$\theta_n^L = \theta_{n-1}^R + \alpha_{Mn} M_n^L - \alpha_{Sn} (V_{n-1}^R + F_{n-1}^R \theta_n^L) \tag{9}$$

$$M_n^L = M_{n-1}^R + \ell_n V_{n-1}^R + F_{n-1}^R (y_n^L - y_{n-1}^R) \tag{10}$$

Following the method of formulation as presented in Reference (1), Equations 8 through 10 are solved for the deflection, slope and moment to the left of mass n in terms of the deflection, slope and moment to the right of mass n-1. Writing the resulting equations in matrix form while utilizing equation 5, yields

$$\begin{Bmatrix} y_n^L \\ \theta_n^L \\ M_n^L \\ V_n^L \end{Bmatrix} = \begin{bmatrix} 1 & B_{12} & B_{13} & B_{14} \\ 0 & B_{22} & B_{23} & B_{24} \\ 0 & B_{32} & B_{33} & B_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} y_{n-1}^R \\ \theta_{n-1}^R \\ M_{n-1}^R \\ V_{n-1}^R \end{Bmatrix} \tag{11}$$

where the coefficients in the B matrix are defined by

$$\begin{aligned} B_{12} &= [\ell_n - (d_{Sn} - \ell_n \alpha_{Sn}) F_{n-1}^R]/DEN \\ B_{13} &= [d_{Mn} - (\alpha_{Mn} d_{Sn} - \alpha_{Sn} d_{Mn}) F_{n-1}^R]/DEN \\ B_{14} &= [(\ell_n d_{Mn} - d_{Sn}) - \ell_n (\alpha_{Mn} d_{Sn} - \alpha_{Sn} d_{Mn}) F_{n-1}^R]/DEN \\ B_{22} &= [1 - (d_{Mn} - \ell_n \alpha_{Mn}) F_{n-1}^R]/DEN \end{aligned} \tag{12}$$

$$B_{23} = \alpha_{Mn}/DEN$$

$$B_{24} = [(\ell_n \alpha_{Mn} - \alpha_{Sn}) - (\alpha_{Mn}\, d_{Sn} - \alpha_{Sn}\, d_{Mn}) F^R_{n-1}]/DEN$$

$$B_{32} = B_{12}\, F^R_{n-1} \qquad (12\ \text{cont.})$$

$$B_{33} = [1 + \alpha_{Sn}\, F^R_{n-1}]/DEN$$

$$B_{34} = B_{12}$$

and DEN is given by

$$DEN = 1 + (\alpha_{Sn} - d_{Mn}) F^R_{n-1} + (\alpha_{Mn} d_{Sn} - \alpha_{Sn} d_{Mn})(F^R_{n-1})^2 \tag{13}$$

To include the remaining dynamic effects associated with the vibration of the beam, one considers the free body diagram of the lumped mass illustrated in Figure 2. For continuity of the beam it follows that

$$y^R_n = y^L_n \tag{14}$$

$$\theta^R_n = \theta^L_n \tag{15}$$

and

$$M^R_n = M^L_n \tag{16}$$

Summing the forces in the vertical direction yields

$$\sum F_y = m_n \ddot{y}_n = V^L_n - V^R_n \tag{17}$$

Assuming that each mass is subjected to harmonic motion in the vertical direction at the same frequency,

$$y_n = A_n \sin \omega t$$

and

$$\ddot{y}_n = - \omega^2 A_n \sin \omega t = - \omega^2 y_n \tag{18}$$

Substitution of $\ddot{y}_n$ from Equation 18 into Equation 17, the shear force acting to the right of mass n becomes

$$V_n^R = V_n^L + \omega^2 m_n y_n \tag{19}$$

Rewriting Equations 14, 15, 16 and 19 in matrix form, we have

$$\begin{Bmatrix} y_n^R \\ \theta_n^R \\ M_n^R \\ V_n^R \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \omega^2 m_n & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} y_n^L \\ \theta_n^L \\ M_n^L \\ V_n^L \end{Bmatrix} \tag{20}$$

Substituting for the deflection, slope, moment and shear to the left of mass n as given by Equation 11 into Equation 20 results in

$$\begin{Bmatrix} y_n^R \\ \theta_n^R \\ M_n^R \\ V_n^R \end{Bmatrix} = \begin{bmatrix} 1 & B_{12} & B_{13} & B_{14} \\ 0 & B_{22} & B_{23} & B_{24} \\ 0 & B_{32} & B_{33} & B_{34} \\ (\omega^2 m_n) & (\omega^2 m_n B_{12}) & (\omega^2 m_n B_{13}) & (\omega^2 m_n B_{14} + 1) \end{bmatrix} \begin{Bmatrix} y_{n-1}^R \\ \theta_{n-1}^R \\ M_{n-1}^R \\ V_{n-1}^R \end{Bmatrix} \tag{21}$$

Equation 21 can be written as

$$\begin{Bmatrix} y_n^R \\ \theta_n^R \\ M_n^R \\ V_n^R \end{Bmatrix} = \begin{bmatrix} A_n \end{bmatrix} \begin{Bmatrix} y_{n-1}^R \\ \theta_{n-1}^R \\ M_{n-1}^R \\ V_{n-1}^R \end{Bmatrix} \tag{22}$$

where the coefficients of the A matrix are defined using Equations 21, 12, 13 and 7. It should be noted that when the beam is not rotating, Equation 22 reduces to the equation for the vibration of a static or non-rotating beam developed in Reference (1). Thus, Equation 22 can be used to investigate the chordwise and flapwise vibration of non-rotating beams by setting the angular velocity of the rotor to zero. If the beam is rotating, Equation 22 in its present form can only be used to investigate flapwise vibration.

The determination of the natural frequencies follows the formulation presented in Reference (1). For the discrete model of the beam illustrated in Figure 5, Equation 22 can be applied to each of the lumped masses to yield, for n = 1

$$\begin{Bmatrix} y_1^R \\ \theta_1^R \\ M_1^R \\ V_1^R \end{Bmatrix} = \begin{bmatrix} A_1 \end{bmatrix} \begin{Bmatrix} y_o^R \\ \theta_o^R \\ M_o^R \\ V_o^R \end{Bmatrix} \tag{23}$$

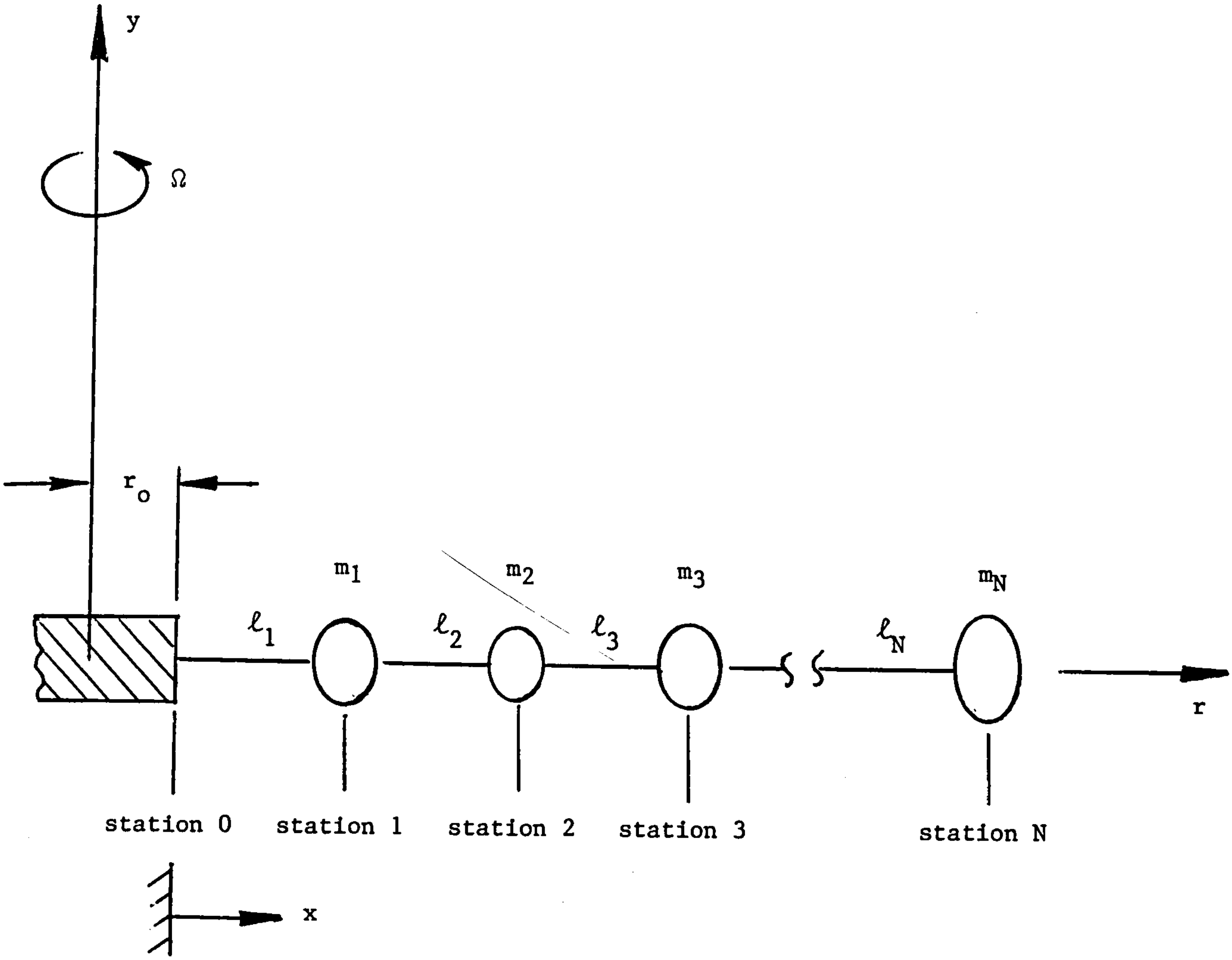


Figure 5 N mass model of a nonuniform rotating beam.

and, for n = 2

$$\begin{Bmatrix} Y_2^R \\ \theta_2^R \\ M_2^R \\ V_2^R \end{Bmatrix} = \begin{bmatrix} A_2 \end{bmatrix} \begin{Bmatrix} Y_1^R \\ \theta_1^R \\ M_1^R \\ V_1^R \end{Bmatrix} \tag{24}$$

and, for n = 3

$$\begin{Bmatrix} Y_3^R \\ \theta_3^R \\ M_3^R \\ V_3^R \end{Bmatrix} = \begin{bmatrix} A_3 \end{bmatrix} \begin{Bmatrix} Y_2^R \\ \theta_2^R \\ M_2^R \\ V_2^R \end{Bmatrix} \tag{25}$$

and finally, for n = N

$$\begin{Bmatrix} Y_N^R \\ \theta_N^R \\ M_N^R \\ V_n^R \end{Bmatrix} = \begin{bmatrix} A_N \end{bmatrix} \begin{Bmatrix} Y_{N-1}^R \\ \theta_{N-1}^R \\ M_{N-1}^R \\ V_{N-1}^R \end{Bmatrix} \tag{26}$$

Concatenating Equations 23 through 26 yields

$$\begin{Bmatrix} y_N^R \\ \theta_N^R \\ M_N^R \\ V_N^R \end{Bmatrix} = \begin{bmatrix} A_N \end{bmatrix} \begin{bmatrix} A_{N-1} \end{bmatrix} \cdots\cdots \begin{bmatrix} A_3 \end{bmatrix} \begin{bmatrix} A_2 \end{bmatrix} \begin{bmatrix} A_1 \end{bmatrix} \begin{Bmatrix} y_o^R \\ \theta_o^R \\ M_o^R \\ V_o^R \end{Bmatrix} \tag{27}$$

Letting the product of all the A matrices equal the matrix U, Equation 27 can be rewritten as

$$\begin{Bmatrix} y_N^R \\ \theta_N^R \\ M_N^R \\ V_N^R \end{Bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{bmatrix} \begin{Bmatrix} y_o^R \\ \theta_o^R \\ M_o^R \\ V_o^R \end{Bmatrix} \tag{28}$$

Since the beam shown in Figure 5 is considered to be fixed at station 0 and free at station N, the boundary conditions can be expressed as

$$\begin{aligned} y_o^R &= 0 \\ \theta_o^R &= 0 \\ M_N^R &= 0 \\ V_N^R &= 0 \end{aligned} \tag{29}$$

Substituting the boundary conditions expressed by Equation 29 into Equation 28 and expanding the resulting matrix yields

$$y_N^R = u_{11}(o) + u_{12}(o) + u_{13} M_o^R + u_{14} V_o^R \tag{30}$$

$$\theta_N^R = u_{21}(o) + u_{22}(o) + u_{23} M_o^R + u_{24} V_o^R \tag{31}$$

$$o = u_{31}(o) + u_{32}(o) + u_{33} M_o^R + u_{34} V_o^R \tag{32}$$

and

$$o = u_{41}(o) + u_{42}(o) + u_{43} M_o^R + u_{44} V_o^R \tag{33}$$

For a non-trivial solution of Equations 32 and 33, that is, for the bending moment and shear force to the right of station 0 to be any value other than zero, the determinant of the coefficients of these terms must be equal to zero. Thus,

$$\begin{vmatrix} u_{33} & u_{34} \\ u_{43} & u_{44} \end{vmatrix} = 0$$

or

$$u_{33}\, u_{44} - u_{34}\, u_{43} = 0 \tag{34}$$

Equation 34 is the characteristic equation or frequency equation for the system. The roots of this equation, that is, the values of the frequencies that satisfy this equation, are the desired natural frequencies of vibration.

Once the natural frequencies have been found using Equation 34 , the corresponding mode shapes can be determined. The mode shapes be can obtained from the deflection of each of the lumped masses. In order to determine the deflection of the masses, Equation 33 can be solved for the bending moment to the right of station 0 yielding

$$M_o^R = - \frac{u_{44} V_o^R}{u_{43}} \tag{35}$$

Substituting the bending moment given by Equation 35 into Equation 30, the deflection at the free end of the beam becomes

$$Y_N^R = - \frac{u_{13} u_{44}}{u_{43}} V_o^R + u_{14} V_o^R \tag{36}$$

If the mode shapes are normalized with respect to the free end of the beam by letting the deflection at the free end be equal to one, the shear force at the fixed end of the beam obtained from Equation 36 becomes

$$V_o^R = \frac{u_{43}}{u_{14} u_{43} - u_{13} u_{44}} \tag{37}$$

Substituting the shear force at the fixed end defined by Equation 37 into Equation 35, the bending moment at the fixed end of the beam can be expressed as

$$M_o^R = \frac{- u_{44}}{u_{14} u_{43} - u_{13} u_{44}} \tag{38}$$

Concatenating Equation 22 starting at mass 1 and going to mass n, the deflection, slope, bending moment and shear force to the right of mass n can be expressed in terms of the same quantities at the fixed end of the beam, station 0, as

$$\begin{Bmatrix} Y_n^R \\ \theta_n^R \\ M_n^R \\ V_n^R \end{Bmatrix} = \begin{bmatrix} A_n \end{bmatrix} \begin{bmatrix} A_{n-1} \end{bmatrix} \cdots \begin{bmatrix} A_2 \end{bmatrix} \begin{bmatrix} A_1 \end{bmatrix} \begin{Bmatrix} Y_o^R \\ \theta_o^R \\ M_o^R \\ V_o^R \end{Bmatrix} \tag{39}$$

Since the deflection and slope at the fixed end of the beam are zero, Equation 39 can be rewritten as

$$\begin{Bmatrix} Y_n^R \\ \theta_n^R \\ M_n^R \\ V_n^R \end{Bmatrix} = \begin{bmatrix} A_n \end{bmatrix}\begin{bmatrix} A_{n-1} \end{bmatrix} \cdots \begin{bmatrix} A_2 \end{bmatrix}\begin{bmatrix} A_1 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ M_o^R \\ V_o^R \end{Bmatrix} \tag{40}$$

The normalized displacement for mass n can then be determined from Equation 40 by concatenating the A matrices as indicated and using the shear force and bending moment at station 0 as defined by Equations 37 and 38. The mode shape for a specific natural frequency is then obtained by determining the normalized deflection of each of the lumped masses.

3. DESCRIPTION OF COMPUTER CODES

There are five computer codes which have been written for the vibrational analysis described in Section 2. Two codes, BLDFLAP and CWTFLAP, generate the discrete or lumped mass models used to analyze the flapwise vibration of the blade and counterweight assemblies respectively. An additional code, CWTCHRD, generates the lumped mass models used to analyze the chordwise vibration of the counterweight assembly. The only differences between the flapwise and chordwise codes are the changes in area moments of inertia due to the change in axes about which bending occurs. The fourth code, BEAM, is a general code for the determination of natural frequencies and mode shapes for cantilevered beams which have been modeled as lumped mass configurations. The fifth code, MODPLT, is a graphics code which displays the mode shapes on a Tektronix storage display terminal. The operation, data requirements, and output for each of the five codes and any associated subprograms are described in the following subsections.

The analysis codes, written in FORTRAN 77, were originally developed on a Digital Equipment Corporation PDP 11/34 minicomputer. The codes were then down loaded to an ITT XTRA personal computer. The ITT XTRA, a fully IBM compatible personal computer, utilized the Microsoft DOS 2.11 operating system and the Microsoft FORTRAN 3.20 compiler. After down loading, minor modifications were made to resolve FORTRAN incompatibilities and the codes were then compiled and linked.

3.1 BLDFLAP

The BLDFLAP computer code permits the user to input the number of lumped masses in the various sections of the blade assembly. This assembly consists of the spool piece (CF 764254), shorty 40 blade, transition section and 6 foot pitchable tip. The program computes the distance between each of the lumped masses, the flexural rigidity at each lumped mass location and the weight of each lumped mass. Flexural rigidity is the product of the modulus of elasticity and area moment of inertia. The area moments of inertia are computed for bending in the flapwise direction. BLDFLAP then writes the title of the problem, the total number of lumped masses, the data for each lumped mass as described above, the flexural rigidity at the base of the spool piece and the radius of the rigid hub to a data file whose name is "VIBIN.DAT". The data which describes the blade geometry is defined in the code and was supplied by NASA (7),(8), (9).

3.2 CWTFLAP

The CWTFLAP computer code permits the user to input the number of lumped masses in the various sections of the counterweight assembly. This assembly consists of the spool piece (CF 764254), the tapered steel spar extension (CF 760549) and the counterweight (CF 764554). The computer code calculates the distance between each of the lumped masses, the weight of each lumped mass and the flexural rigidity at each lumped mass location. The area moments of inertia used in the flexural rigidity calculations are computed for bending in the flapwise direction. CWTFLAP then writes the problem title, the total number of lumped masses, the data for each lumped mass as previously described, the flexural rigidity at the base of the spool piece and the radius of the rigid hub to a data file whose name is "VIBIN.DAT". The data which describes the counterweight geometry is defined in the code and was taken from the drawings as indicated.

3.3 CWTCHRD

The CWTCHRD computer code is identical to the CWTFLAP computer code described in Section 3.2 with the exception being that the area moments of inertia are taken assuming bending in the chordwise direction rather than in the flapwise direction. The only real difference is the moment of inertia of the mass at the free end of the counterweight assembly. The moment of inertia of this mass is not identical in the flapwise and chordwise directions.

3.4 BEAM

The BEAM code is the main vibration analysis code. It reads the lumped mass model data from the "VIBIN.DAT" data file and calculates the natural frequencies and the mode shapes. In addition to reading in the model data from the "VIBIN.DAT" data file, BEAM permits the user to enter the the range of values to be searched for natural frequencies. For flapwise vibration, the user may also specify the rotor speeds to be considered. The mode shape calculations are optional and thus must be requested if desired. The results of the frequency calculations are stored in a file called "VIBOUT.DAT"and the mode shape data are stored in a file entitled "MODES.DAT". The BEAM main program is quite short with the bulk of the input, output and calculations being performed in eight subprograms. The eight subprograms are: DET, FILLA, FORCE, INPUT, MODES, MULT, NFREQ and OUTPUT. The BEAM main program and the eight subprograms are described in the following subsections below.

3.4.1 BEAM

The BEAM main program initially calls the INPUT subroutine which performs all the input functions. BEAM then enters a loop in which all natural frequencies and desired mode shapes are calculated and output for each of the requested rotor speeds. For chordwise vibration, only a static rotor condition is presently permitted. The natural frequencies are calculated by calling the NFREQ subroutine and are output to the "VIBOUT.DAT" data file by calling OUTPUT subroutine. The mode shapes are calculated and output to a file called "MODES.DAT" by calling the MODES subroutine.

3.4.2 DET

The DET function subprogram evaluates the determinant of the characteristic equation for the system. This determinant reflects the boundary conditions of the problem and is a function of the frequency. When the determinant is zero the frequency is a natural frequency. The actual determinant calculation is outlined in the theory section and is performed by first setting the U matrix to the identity matrix. Then the A matrix for each lumped mass is concatenated or multiplied with the existing U matrix This concatenation results in the U matrix for the entire beam. After the A matrices for all masses have been concatenated, the determinant, DET, is computed using the equation

DET=U(3,3)*U(4,4)-U(3,4)*U(4,3).

The values of the frequency at which DET becomes zero are the natural frequencies of a cantilevered beam.

3.4.3 FILLA

The FILLA subroutine fills or defines the A matrix which relates the deflection, slope, moment and shear at mass n-1 to those quantities at mass n. The A matrices are functions of the frequency, centrifugal forces and geometric and material properties. The exact form of the A matrix is given in the theory section. FILLA is called by DET prior to calculating the determinant of the characteristic equation for the system.

3.4.4 FORCE

The FORCE subroutine calculates the centrifugal force acting to the left of each mass for flapwise vibration of the beam. These values are stored in a force array and are then used later by FILLA. This subroutine is called during the initial steps of the natural frequency calculation subroutine, NFREQ. If the non-rotating condition of the blade is being considered, the rotor speed which is passed to FORCE is zero which results in all centrifugal forces being calculated as zero.

3.4.5 INPUT

The INPUT subroutine performs all input for the BEAM program. Initially INPUT reads the problem title, the total number of lumped masses, the lumped mass data, the flexural rigidity at the rigid hub and the radius of the rigid hub. All this data is read from the "VIBIN.DAT" data file which was written by one of the data preparation programs such as CWTFLAP, BLDFLAP or CWTCHRD. The lumped mass data consists of the distance between each lumped mass, the flexural rigidity of the beam at each lumped mass location and the weight of each lumped mass. For flapwise vibration, INPUT then permits the user to interactively enter the number of rotor speeds to be considered and the values of these rotor speeds. The user then enters the lower and upper limits of the frequency range to be searched for natural frequencies and the increment to be used in the initial search. Finally the user enters the number of fractional decimal places of accuracy and indicates if mode shape calculations are desired.

3.4.6 MODES

The MODES subroutine calculates the mode shapes if such calculations were requested in the INPUT subroutine. Mode shapes are calculated for each of the natural frequencies at the various rotor speeds. The relative displacements at each lumped mass, or mode shapes, are output to a file call "MODES.DAT" for subsequent printing or plotting using the MODPLT program. The actual displacements for each natural frequency are calculated by again concatenating the A matrices for all lumped masses into a matrix U and then calculating the shear, VO, and moment, MO, at the fixed end assuming a unit displacement at the free end. The formulas for these calculations are

```
MO=1./(U(1,3)-U(4,3)*U(1,4)/U(4,4))
```

and

```
VO=-U(4,3)*MO/U(4,4).
```

Once the shear and moment are calculated at the fixed end, the displacements at each lumped mass, Y(N), can be calculated by simply concatenating A matrices up to the lumped mass whose displacement is desired and then apply the formula

$$Y(N)=U(1,3)*MO+U(1,4)*VO.$$

3.4.7 MULT

The MULT subroutine is a simple matrix multiplication subroutine. It is used to concatenate the A matrices in the DET and MODES subprograms.

3.4.8 NFREQ

The NFREQ subroutine computes the natural frequencies in the range of frequencies as specified in the INPUT subroutine. Before the subroutine can actually calculate the natural frequencies it first computes the centrifugal force at each lumped mass by calling the FORCE subroutine. Once this is done, the NFREQ subroutine then begins to search for natural frequencies using the method of interval halving. The natural frequencies are those frequencies at which the characteristic determinant of the system is zero. This interval halving method calls on the DET function to evaluate the determinant as described earlier. When a natural frequency is found to the precision requested in the INPUT subroutine, it is stored in an array and the search continues for the next possible frequency in the range of the search. The natural frequencies are ultimately written by the OUTPUT subroutine into the file "VIBOUT.DAT". The NFREQ program, however, also prints the natural frequencies at the terminal so the user can see the progress the program is making.

3.4.9 OUTPUT

The OUTPUT subroutine outputs the input data and natural frequencies to the file "VIBOUT.DAT" for subsequent printing. The output is formatted and labelled to facilitate comprehension. The input data is echoed on the printed output so that the listing of "VIBOUT.DAT" provides one complete summary of the analysis including results and the original discrete model.

3.5 MODPLT

MODPLT is a stand alone program which plots the mode shapes on a Tektronix 4010 or 4014 display terminal. The program reads the "MODES.DAT" data file and permits the user to generate a quick sketch, to scale the results in order to create a pleasing plot, or to generate a finished labelled plot. The program was written with all the graphics subroutine calls in one subroutine called DRAW. This will hopefully minimize the difficulty in moving this program to another machine. The only other subroutine used in MODPLT is a subroutine call MINPUT which actually performs the input from "MODES.DAT".

4. OPERATION AND USE OF COMPUTER CODE

This section illustrates the use of the programs discussed in Section 3 to generate a discrete model, to perform a vibration analysis and to plot the corresponding mode shapes. Section 4.1 illustrates the use of the BLDFLAP program to generate a 107 lumped mass model of the blade assembly for flapwise vibration. Included in this section is a hard-copy of the interaction between the computer and the user taken directly from the computer screen. As in all sections to follow where computer-user interaction is shown, the specific user response has been underlined for the reader's convenience. Section 4.2 contains a listing of the VIBIN data file generated by the BLDFLAP program. Section 4.3 illustrates the use of the BEAM program to perform a flapwise vibration analysis of the blade assembly for rotor speeds of 0, 20 and 40 revolutions per minute. Included in this section is a hard-copy of the computer-user interaction. The results of this analysis are the first three natural frequencies of the blade assembly and the corresponding mode shapes based on the discrete model developed in Section 4.1. Section 4.4 contains a listing of the VIBOUT data file generated by the BEAM program . This data file includes a complete listing of all input data and the values of the natural frequencies as found by the BEAM program. Section 4.5 contains a listing of the MODES data file generated by the BEAM program during the vibration analysis. This data file includes all pertinent information relative to the corresponding mode shapes for the various natural frequencies of vibration. Section 4.6 illustrates the use of the MODPLT program to plot the mode shapes on a Tektronix 4010 or 4014 display terminal. Included in this section is a hard-copy of the computer-user interaction and copies of the three mode shapes for each of the three rotor speeds used in the analysis.

4.1 EXAMPLE OF MODEL CREATION

The following example illustrates the use of the BLDFLAP program which generates the discrete model of the blade assembly. This model can be used for a flapwise vibration analysis performed by the BEAM program. The program is executed by entering BLDFLAP. The program then queries the user for the radius of the rigid hub, the number of divisions for each section of the blade and the title to be assigned to the problem. The values shown in the example are for illustrative purposes only and may be altered to suit the user's requirements. The total weight and the location of the center of gravity of the discrete model are displayed for the user's convenience as well as the individual weight and location of the center gravity for each section of assembly.

This program is typical of the operation of the other model generation programs, CWTFLAP and CWTCHRD. Thus, illustrations for these two programs was deemed unnecessary. The input for all three model generation programs is totally interactive and the output is the VIBIN data file. This output file is denoted as VIBIN because it is used as input to the analysis code, BEAM. The interactive input follows and the output data file is shown in Section 4.2.

B) BLDFLAP

THIS PROGRAM GENERATES THE DATA FOR THE FLAPWISE
VIBRATION OF THE BLADE ASSEMBLY

ENTER THE RADIUS OF THE RIGID HUB (INCHES) : 22.0

SECTION 1-SPOOL PIECE FLANGE L=2.25" W=108.74 LB

SECTION 2-SPOOL PIECE BODY L=13.75" W=207.96 LB

SECTION 3-SPOOL PIECE FLANGE L=1.75" W=82.78 LB

SECTION 4-BLADE SECTION L=441.25" W=2250.00 LB

SECTION 5-TRANSITION & TIP SECTION L=115.56" W=960.00 LB

THE MAXIMUM TOTAL NUMBER OF DIVISIONS YOU
ARE ALLOWED IN ALL 5 SECTIONS IS 699.

ENTER THE NUMBER OF DIVISIONS FOR SECTION 1 : 2

ENTER THE NUMBER OF DIVISIONS FOR SECTION 2 : 3

ENTER THE NUMBER OF DIVISIONS FOR SECTION 3 : 2

ENTER THE NUMBER OF DIVISIONS FOR SECTION 4 : 80

ENTER THE NUMBER OF DIVISIONS FOR SECTION 5 : 20

```
ENTER A TITLE STATEMENT (60 CHAR. MAX.) : FLAPWISE VIBRATION OF THE BLADE

 TOTAL WEIGHT               CG FROM HUB END
    (LBS)                       (INCH)
   3569.472                    266.502

 SPOOL WEIGHT               CG FROM HUB END
    (LBS)                       (INCH)
    399.472                      8.553

 BLADE WEIGHT               CG FROM SPOOL END
    (LBS)                       (INCH)
   2250.000                    200.300

 TRANS. & TIP WEIGHT        CG FROM BLADE END
    (LBS)                       (INCH)
    920.000                     38.004

 TOTAL MOMENT ABOUT CENTERLINE OF HUB
              (LBS-INCH)
               1029800.5
Stop - Program terminated.
```

4.2 EXAMPLE OF VIBIN DATA FILE

The following is a listing of the VIBIN data file generated by the BLDFLAP program for the example problem illustrated in Section 4.1. This data file contains a complete description of the problem including: the problem title, the number of lumped masses, the section lengths, the section rigidities and the lumped weights. The VIBIN data file is an intermediate file and all the information in this file is contained in the analysis code output data file which is formatted with titles suitable for printing.

FLAPWISE VIBRATION OF THE BLADE

107

.562500	229986015274.6	54.367651
1.125000	229986015274.6	54.367651
2.854167	63188041992.2	69.319249
4.583333	63188041992.2	69.319249
4.583333	63188041992.2	69.319249
2.729167	226554261922.3	41.389338
.875000	226554261922.3	41.389338
3.195313	28147549273.9	61.952576
5.527500	27441130208.7	62.264924
5.503750	26737746369.3	64.157750
5.515625	26032844917.0	64.157750
5.515625	25327943464.7	64.157750
5.515625	24623042012.4	64.157750
5.086254	23973014492.5	45.884094
5.944996	23213239107.9	34.154956
5.515625	22508337655.6	34.154956
5.515625	21803436203.3	34.154956
5.515059	21098607134.7	34.147363
5.516191	20504289062.5	34.055675
5.515625	19991335937.5	34.055675
5.515625	19478382812.5	34.055675
5.515625	18965429687.5	34.055675
5.348701	18468000460.9	30.065675
5.682549	17939523437.5	26.719894
5.515625	17426570312.5	26.719894
5.515625	16913617187.5	26.719894
5.515347	16400689925.2	26.717169
5.515903	15887710937.5	26.479413
5.515625	15374757812.5	26.479413
5.515625	14861804687.5	26.479413
5.515625	14348851562.5	26.479413
5.478086	13839389563.3	25.805525
5.553164	13322945312.5	25.071825
5.515625	12809992187.5	25.071825
5.515625	12297039062.5	25.071825
5.515625	11784085937.5	25.071825

5.504605	11377504100.4	23.339200
5.526645	11101171875.0	23.240638
5.515625	10825390625.0	23.240638
5.515625	10549609375.0	23.240638
5.427723	10278223203.5	22.042525
5.603527	9998046875.0	20.343831
5.515625	9722265625.0	20.343831
5.515625	9446484375.0	20.343831
5.515625	9170703125.0	20.343831
5.482032	8896601504.8	18.454506
5.549218	8619140625.0	18.199356
5.515625	8343359375.0	18.199356
5.515625	8067578125.0	18.199356
5.416897	7796733265.7	17.251488
5.614353	7516015625.0	15.479050
5.515625	7240234375.0	15.479050
5.515625	6964453125.0	15.479050
5.515625	6688671875.0	15.479050
5.545001	6475198116.6	16.429450
5.486249	6398390625.0	16.643950
5.515625	6321171875.0	16.643950
5.515625	6243953125.0	16.643950
5.578573	6165853102.5	17.191450
5.452677	6089515625.0	18.576625
5.515625	6012296875.0	18.576625
5.515625	5935078125.0	18.576625
5.515625	5857859375.0	18.576625
5.412902	5782078741.6	16.161813
5.618348	5703421875.0	15.359913
5.515625	5626203125.0	15.359913
5.515625	5548984375.0	15.359913
5.621312	5470286008.8	16.151569
5.409938	5394546875.0	18.989194
5.515625	5317328125.0	18.989194
5.515625	5240109375.0	18.989194
5.515625	5162890625.0	18.989194
5.597277	5078144094.5	20.963006
5.433973	4970673671.2	21.868350

5.515625	4861588380.7	21.868350
5.515625	4752503090.2	21.868350
5.582987	4642085544.3	22.517700
5.448263	4534332509.3	26.113175
5.515625	4425247218.8	26.113175
5.515625	4316161928.3	26.113175
5.515625	4207076637.8	26.113175
5.809453	4092180165.8	36.302150
5.221797	3988906056.9	42.536500
5.515625	3879820766.4	42.536500
5.515625	3770735475.9	42.536500
5.765264	3656712941.5	47.222667
5.265986	3552564894.9	95.898333
5.646812	9133101005.6	99.138314
5.778000	8607303016.7	99.138314
5.778000	8384209507.0	99.138314
5.596266	8253244818.2	81.383133
5.959734	7710909050.0	74.189518
5.778000	7185111061.1	74.189518
5.778000	6659313072.2	74.189518
5.129401	1952000000.0	54.351447
6.426598	1511815488.7	22.649760
5.778000	1471943302.3	22.649760
5.778000	1432522408.7	22.649760
5.766651	1393284319.7	22.555680
5.789349	1353337810.5	21.031920
5.778000	1313651211.2	21.031920
5.778000	1274360812.0	21.031920
5.778000	1234733213.1	21.031920
5.815978	1194602963.2	22.114214
5.740022	1155208014.6	22.511688
5.778000	707573008.8	22.511688
5.778000	235857802.5	22.511688

229986015274.6

22.000

4.3 EXAMPLE OF VIBRATION ANALYSIS

The following illustrates the use of the BEAM program to perform a vibration analysis of the discrete model created by the BLDFLAP program illustrated in Section 4.1. The program is executed by entering BEAM. The program then queries the user for the type of analysis to be performed (chordwise or flapwise vibration), the rotor speeds to be considered (provided a flapwise analysis is requested), the frequency range and search interval, the precision and whether or not the mode shapes are to be determined. For the user's convenience, the program displays the natural frequencies at the various rotor speeds as they are calculated.

B) BEAM

TYPE 1 IF THE ANALYSIS IS FOR CHORDWISE VIBRATION OR
TYPE 2 IF THE ANALYSIS IS FOR FOR FLAPWISE VIBRATION : 2

TYPE IN THE TOTAL NUMBER OF ROTATIONAL
SPEEDS TO BE CONSIDERED (MAX. 10) : 3

TYPE IN ROTOR SPEED (REV/MIN) NO. 1 : 0.0

TYPE IN ROTOR SPEED (REV/MIN) NO. 2 : 20.0

TYPE IN ROTOR SPEED (REV/MIN) NO. 3 : 40.0

TYPE IN THE LOWER LIMIT OF THE FREQUENCY RANGE
YOU WISH SEARCHED FOR NATURAL FREQUENCIES (RAD/SEC) : 0.0

TYPE IN THE UPPER LIMIT OF THE FREQUENCY RANGE
YOU WISH SEARCHED FOR NATURAL FREQUENCIES (RAD/SEC) : 180.

TYPE IN THE FREQUENCY INCREMENT TO BE USED IN
THE INITIAL SEARCH FOR NATURAL FREQUENCIES (RAD/SEC) : 10.

TYPE IN THE NUMBER OF DIGITS OF ACCURACY TO THE
RIGHT OF THE DECIMAL POINT : 3

DO YOU WISH TO CALCULATE MODE SHAPES (Y/N) ? Y

```
** THE PROGRAM IS NOW DETERMINING THE NATURAL
   FREQUENCIES OF VIBRATION **


NATURAL FREQUENCIES AT A ROTOR SPEED OF      .0 RPM
***************************************************

W =            10.341     RAD/SEC

W =            67.686     RAD/SEC

W =           158.607     RAD/SEC


** THE PROGRAM IS CALCULATING THE MODE SHAPES **
```

```
** THE PROGRAM IS NOW DETERMINING THE NATURAL
   FREQUENCIES OF VIBRATION **


NATURAL FREQUENCIES AT A ROTOR SPEED OF    20.0 RPM
*************************************************

W =            10.650    RAD/SEC

W =            68.066    RAD/SEC

W =           158.909    RAD/SEC

** THE PROGRAM IS CALCULATING THE MODE SHAPES **
```

```
** THE PROGRAM IS NOW DETERMINING THE NATURAL
   FREQUENCIES OF VIBRATION **


NATURAL FREQUENCIES AT A ROTOR SPEED OF   40.0 RPM
***************************************************

W =            11.520    RAD/SEC

W =            69.194    RAD/SEC

W =           159.808    RAD/SEC


** THE PROGRAM IS CALCULATING THE MODE SHAPES **


Stop - Program terminated.
```

4.4 EXAMPLE OF VIBOUT DATA FILE

The following is a listing of the VIBOUT data file generated by the BEAM program for the example problem illustrated in the previous sections. This data file contains a complete description of the problem including: the problem title, the number of lumped masses, the section lengths, the section rigidities, the lumped weights, the section rigidity at the base of the model, the radius of the rigid hub, the frequency range and search interval, and the natural frequencies at the various rotor speeds considered. The VIBOUT data file can be spooled to a printer in order to provide the user with a hard-copy of all input and output relative to the vibration analysis performed.

FLAPWISE VIBRATION OF THE BLADE

THE NUMBER OF LUMPED MASSES = 107

SECTION LENGTH (INCH)	SECTION RIGIDITY (LBS-IN*IN)	LUMPED WEIGHT (LBS)
.562500	229986015274.6	54.367651
1.125000	229986015274.6	54.367651
2.854167	63188041992.2	69.319249
4.583333	63188041992.2	69.319249
4.583333	63188041992.2	69.319249
2.729167	226554261922.3	41.389338
.875000	226554261922.3	41.389338
3.195313	28147549273.9	61.952576
5.527500	27441130208.7	62.264924
5.503750	26737746369.3	64.157750

5.515625	26032844917.0	64.157750
5.515625	25327943464.7	64.157750
5.515625	24623042012.4	64.157750
5.086254	23973014492.5	45.884094
5.944996	23213239107.9	34.154956
5.515625	22508337655.6	34.154956
5.515625	21803436203.3	34.154956
5.515059	21098607134.7	34.147363
5.516191	20504289062.5	34.055675
5.515625	19991335937.5	34.055675
5.515625	19478382812.5	34.055675
5.515625	18965429687.5	34.055675
5.348701	18468000460.9	30.065675
5.682549	17939523437.5	26.719894
5.515625	17426570312.5	26.719894
5.515625	16913617187.5	26.719894
5.515347	16400689925.2	26.717169

5.515903	15887710937.5	26.479413
5.515625	15374757812.5	26.479413
5.515625	14861804687.5	26.479413
5.515625	14348851562.5	26.479413
5.478086	13839389563.3	25.805525
5.553164	13322945312.5	25.071825
5.515625	12809992187.5	25.071825
5.515625	12297039062.5	25.071825
5.515625	11784085937.5	25.071825
5.504605	11377504100.4	23.339200
5.526645	11101171875.0	23.240638
5.515625	10825390625.0	23.240638
5.515625	10549609375.0	23.240638
5.427723	10278223203.5	22.042525
5.603527	9998046875.0	20.343831
5.515625	9722265625.0	20.343831
5.515625	9446484375.0	20.343831

5.515625	9170703125.0	20.343831
5.482032	8896601504.8	18.454506
5.549218	8619140625.0	18.199356
5.515625	8343359375.0	18.199356
5.515625	8067578125.0	18.199356
5.416897	7796733265.7	17.251488
5.614353	7516015625.0	15.479050
5.515625	7240234375.0	15.479050
5.515625	6964453125.0	15.479050
5.515625	6688671875.0	15.479050
5.545001	6475198116.6	16.429450
5.486249	6398390625.0	16.643950
5.515625	6321171875.0	16.643950
5.515625	6243953125.0	16.643950
5.578573	6165853102.5	17.191450
5.452677	6089515625.0	18.576625
5.515625	6012296875.0	18.576625
5.515625	5935078125.0	18.576625

5.515625	5857859375.0	18.576625
5.412902	5782078741.6	16.161813
5.618348	5703421875.0	15.359913
5.515625	5626203125.0	15.359913
5.515625	5548984375.0	15.359913
5.621312	5470286008.8	16.151569
5.409938	5394546875.0	18.989194
5.515625	5317328125.0	18.989194
5.515625	5240109375.0	18.989194
5.515625	5162890625.0	18.989194
5.597277	5078144094.5	20.963006
5.433973	4970673671.2	21.868350
5.515625	4861588380.7	21.868350
5.515625	4752503090.2	21.868350
5.582987	4642085544.3	22.517700
5.448263	4534332509.3	26.113175
5.515625	4425247218.8	26.113175

5.515625	4316161928.3	26.113175
5.515625	4207076637.8	26.113175
5.809453	4092180165.8	36.302150
5.221797	3988906056.9	42.536500
5.515625	3879820766.4	42.536500
5.515625	3770735475.9	42.536500
5.765264	3656712941.5	47.222667
5.265986	3552564894.9	95.898333
5.646812	9133101005.6	99.138314
5.778000	8607303016.7	99.138314
5.778000	8384209507.0	99.138314
5.596266	8253244818.2	81.383133
5.959734	7710909050.0	74.189518
5.778000	7185111061.1	74.189518
5.778000	6659313072.2	74.189518
5.129401	1952000000.0	54.351447
6.426598	1511815488.7	22.649760

5.778000	1471943302.3	22.649760
5.778000	1432522408.7	22.649760
5.766651	1393284319.7	22.555680
5.789349	1353337810.5	21.031920
5.778000	1313651211.2	21.031920
5.778000	1274360812.0	21.031920
5.778000	1234733213.1	21.031920
5.815978	1194602963.2	22.114214
5.740022	1155208014.6	22.511688
5.778000	707573008.8	22.511688
5.778000	235857802.5	22.511688

THE SECTION RIGIDITY (AT X=0.0) = 229986015274.6 (LBS-IN*IN)

THE RADIUS OF THE RIGID HUB = 22.000 (IN)

THE FREQUENCY RANGE SEARCHED FOR NATURAL FREQUENCIES STARTING AT: .000
00 AND ENDING AT: 180.00000 (RAD/SEC)

THE INITIAL FREQUENCY INCREMENT USED IN THE SEARCH = 10.00000 (RAD/SEC)

NATURAL FREQUENCIES AT A ROTOR SPEED OF .0 RPM

****

	(RAD/SEC)	(HZ)
1	10.341	1.646
2	67.686	10.773
3	158.607	25.243

NATURAL FREQUENCIES AT A ROTOR SPEED OF 20.0 RPM

****

	(RAD/SEC)	(HZ)
1	10.650	1.695
2	68.066	10.833
3	158.909	25.291

NATURAL FREQUENCIES AT A ROTOR SPEED OF 40.0 RPM
****

	(RAD/SEC)	(HZ)
1	11.520	1.834
2	69.194	11.013
3	159.808	25.434

4.5 EXAMPLE OF MODES DATA FILE

The following is a listing of the MODES data file generated by the BEAM program for the example illustrated in Sections 4.1 and 4.3. This data file contains a complete description of the problem including: the problem title, the total number of lumped masses in the beam model, the axial location of each mass, the number of natural frequencies determined for each of the rotor speeds, the natural frequencies of vibration and the normalized displacements for each of the lumped masses at the natural frequencies. The MODES data file is used by the MODPLT program in order to obtain plots of the mode shapes.

THE PROBLEM TITLE
****

FLAPWISE VIBRATION OF THE BLADE

THE TOTAL NUMBER OF MASSES
****

107

THE AXIAL LOCATION OF MASS N (INCHES)
****

.56250	1.68750	4.54167	9.12500	13.70833	16.43750	17.31250	20.50781
26.03531	31.53906	37.05469	42.57031	48.08594	53.17219	59.11719	64.63281
70.14844	75.66350	81.17969	86.69531	92.21094	97.72656	103.07526	108.75781
114.27344	119.78906	125.30441	130.82031	136.33594	141.85156	147.36719	152.84527
158.39844	163.91406	169.42969	174.94531	180.44992	185.97656	191.49219	197.00781
202.43554	208.03906	213.55469	219.07031	224.58594	230.06797	235.61719	241.13281
246.64844	252.06534	257.67969	263.19531	268.71094	274.22656	279.77156	285.25781
290.77344	296.28906	301.86764	307.32031	312.83594	318.35156	323.86719	329.28009
334.89844	340.41406	345.92969	351.55100	356.96094	362.47656	367.99219	373.50781
379.10509	384.53906	390.05469	395.57031	401.15330	406.60156	412.11719	417.63281
423.14844	428.95789	434.17969	439.69531	445.21094	450.97620	456.24219	461.88900
467.66700	473.44500	479.04127	485.00100	490.77900	496.55700	501.68640	508.11300
513.89100	519.66900	525.43565	531.22500	537.00300	542.78100	548.55900	554.37498
560.11500	565.89300	571.67100					

THE NUMBER OF NATURAL FREQUENCIES AT THE LISTED ROTOR SPEED (RPM)
****

3 .0

THE NATURAL FREQUENCY OF VIBRATION (RAD/SEC)
****

10.34122

THE NORMALIZED DISPLACEMENTS AT THE LUMPED MASSES
****

.00000	.00000	.00001	.00005	.00014	.00022	.00024	.00034
.00061	.00102	.00160	.00233	.00323	.00420	.00551	.00690
.00847	.01021	.01213	.01423	.01651	.01897	.02154	.02446
.02749	.03072	.03414	.03776	.04158	.04560	.04984	.05425
.05894	.06383	.06893	.07427	.07983	.08565	.09170	.09799
.10442	.11130	.11832	.12558	.13310	.14081	.14888	.15714
.16567	.17429	.18349	.19279	.20235	.21219	.22234	.23266
.24331	.25421	.26550	.27679	.28845	.30035	.31249	.32462
.33745	.35026	.36329	.37678	.38996	.40360	.41742	.43142
.44582	.45996	.47447	.48915	.50416	.51895	.53407	.54932
.56469	.58101	.59579	.61151	.62732	.64394	.65921	.67564
.69248	.70936	.72572	.74317	.76010	.77705	.79211	.81103
.82808	.84518	.86227	.87947	.89665	.91385	.93107	.94841
.96553	.98276	1.00000					

THE NATURAL FREQUENCY OF VIBRATION (RAD/SEC)
****

67.68574

THE NORMALIZED DISPLACEMENTS AT THE LUMPED MASSES
****

.00000	-.00001	-.00005	-.00032	-.00090	-.00135	-.00151	-.00211
-.00370	-.00617	-.00952	-.01374	-.01883	-.02426	-.03152	-.03911
-.04751	-.05670	-.06668	-.07742	-.08890	-.10109	-.11358	-.12752
-.14170	-.15649	-.17185	-.18776	-.20419	-.22109	-.23843	-.25605
-.27428	-.29271	-.31142	-.33036	-.34944	-.36873	-.38805	-.40738
-.42636	-.44584	-.46485	-.48362	-.50210	-.52009	-.53786	-.55502
-.57159	-.58722	-.60266	-.61700	-.63043	-.64286	-.65424	-.66433
-.67323	-.68081	-.68708	-.69180	-.69508	-.69683	-.69698	-.69553
-.69232	-.68742	-.68076	-.67212	-.66200	-.64984	-.63579	-.61983
-.60166	-.58210	-.56029	-.53649	-.51036	-.48288	-.45305	-.42120
-.38735	-.34953	-.31365	-.27386	-.23214	-.18651	-.14308	-.09500
-.04498	.00572	.05545	.10903	.16156	.21463	.26224	.32339
.38014	.43842	.49789	.55872	.62035	.68271	.74563	.80936
.87251	.93621	1.00000					

THE NATURAL FREQUENCY OF VIBRATION (RAD/SEC)

****

158.60729

THE NORMALIZED DISPLACEMENTS AT THE LUMPED MASSES

****

.00000	.00001	.00010	.00069	.00189	.00285	.00318	.00443
.00770	.01269	.01937	.02767	.03750	.04788	.06152	.07554
.09080	.10721	.12468	.14310	.16237	.18238	.20239	.22417
.24571	.26752	.28947	.31144	.33329	.35490	.37612	.39668
.41684	.43606	.45431	.47146	.48732	.50183	.51480	.52612
.53555	.54340	.54917	.55289	.55449	.55390	.55103	.54586
.53833	.52860	.51605	.50127	.48405	.46442	.44226	.41802
.39145	.36280	.33187	.29992	.26605	.23082	.19444	.15784
.11916	.08075	.04218	.00298	-.03437	-.07177	-.10820	-.14336
-.17742	-.20861	-.23807	-.26498	-.28927	-.30980	-.32705	-.34040
-.34954	-.35425	-.35388	-.34847	-.33763	-.32025	-.29875	-.27055
-.23877	-.20438	-.16861	-.12795	-.08603	-.04167	.00002	.05928
.12099	.19013	.26563	.34703	.43296	.52265	.61525	.71056
.80596	.90276	1.00000					

THE NUMBER OF NATURAL FREQUENCIES AT THE LISTED ROTOR SPEED (RPM)
****

3 20.0

THE NATURAL FREQUENCY OF VIBRATION (RAD/SEC)
****

10.64968

THE NORMALIZED DISPLACEMENTS AT THE LUMPED MASSES
****

.00000	.00000	.00001	.00005	.00015	.00022	.00025	.00035
.00062	.00104	.00163	.00237	.00328	.00427	.00560	.00702
.00861	.01037	.01232	.01445	.01676	.01926	.02186	.02482
.02789	.03116	.03462	.03828	.04214	.04621	.05049	.05495
.05970	.06463	.06978	.07517	.08077	.08664	.09274	.09908
.10555	.11248	.11955	.12686	.13442	.14218	.15029	.15860
.16716	.17582	.18506	.19439	.20399	.21386	.22404	.23439
.24505	.25598	.26729	.27859	.29026	.30217	.31431	.32645
.33928	.35209	.36511	.37859	.39176	.40538	.41919	.43317
.44754	.46166	.47615	.49079	.50577	.52053	.53561	.55081
.56615	.58243	.59716	.61283	.62859	.64516	.66038	.67675
.69355	.71036	.72667	.74406	.76094	.77783	.79284	.81169
.82868	.84572	.86276	.87989	.89702	.91416	.93131	.94859
.96565	.98282	1.00000					

THE NATURAL FREQUENCY OF VIBRATION (RAD/SEC)
****

68.06614

THE NORMALIZED DISPLACEMENTS AT THE LUMPED MASSES
****

.00000	-.00001	-.00005	-.00032	-.00090	-.00135	-.00151	-.00211
-.00370	-.00617	-.00953	-.01375	-.01884	-.02428	-.03153	-.03912
-.04752	-.05671	-.06669	-.07742	-.08889	-.10108	-.11355	-.12749
-.14165	-.15642	-.17177	-.18766	-.20406	-.22093	-.23824	-.25583
-.27403	-.29242	-.31109	-.32999	-.34903	-.36827	-.38754	-.40682
-.42574	-.44517	-.46412	-.48283	-.50124	-.51917	-.53688	-.55397
-.57048	-.58605	-.60143	-.61571	-.62907	-.64144	-.65277	-.66281
-.67166	-.67920	-.68542	-.69010	-.69335	-.69507	-.69520	-.69374
-.69051	-.68561	-.67894	-.67031	-.66020	-.64806	-.63404	-.61811
-.59997	-.58045	-.55869	-.53494	-.50887	-.48144	-.45167	-.41989
-.38610	-.34836	-.31255	-.27283	-.23118	-.18563	-.14227	-.09425
-.04430	.00634	.05600	.10952	.16198	.21499	.26256	.32365
.38036	.43859	.49804	.55884	.62045	.68279	.74569	.80940
.87253	.93623	1.00000					

THE NATURAL FREQUENCY OF VIBRATION (RAD/SEC)
****

158.90858

THE NORMALIZED DISPLACEMENTS AT THE LUMPED MASSES
****

.00000	.00001	.00010	.00069	.00190	.00286	.00318	.00443
.00770	.01270	.01938	.02768	.03752	.04790	.06153	.07556
.09082	.10722	.12469	.14310	.16237	.18237	.20237	.22414
.24567	.26746	.28940	.31136	.33320	.35478	.37598	.39652
.41666	.43585	.45409	.47121	.48705	.50154	.51448	.52579
.53520	.54303	.54878	.55249	.55408	.55348	.55061	.54544
.53791	.52818	.51564	.50088	.48368	.46406	.44193	.41772
.39118	.36257	.33169	.29978	.26595	.23077	.19443	.15788
.11924	.08088	.04235	.00319	-.03412	-.07149	-.10790	-.14303
-.17707	-.20825	-.23770	-.26461	-.28891	-.30946	-.32672	-.34010
-.34927	-.35402	-.35369	-.34833	-.33755	-.32021	-.29876	-.27061
-.23888	-.20453	-.16881	-.12819	-.08630	-.04199	-.00032	.05892
.12062	.18977	.26528	.34671	.43267	.52241	.61504	.71040
.80585	.90271	1.00000					

THE NUMBER OF NATURAL FREQUENCIES AT THE LISTED ROTOR SPEED (RPM)
****

3 40.0

THE NATURAL FREQUENCY OF VIBRATION (RAD/SEC)
****

11.52035

THE NORMALIZED DISPLACEMENTS AT THE LUMPED MASSES
****

.00000	.00000	.00001	.00006	.00016	.00023	.00026	.00037
.00065	.00110	.00171	.00249	.00345	.00448	.00587	.00735
.00901	.01085	.01288	.01510	.01750	.02010	.02280	.02588
.02906	.03244	.03602	.03981	.04380	.04800	.05241	.05701
.06189	.06696	.07225	.07778	.08353	.08954	.09578	.10226
.10887	.11594	.12314	.13059	.13828	.14617	.15440	.16283
.17151	.18028	.18962	.19906	.20875	.21870	.22897	.23939
.25012	.26110	.27246	.28380	.29551	.30744	.31960	.33175
.34458	.35738	.37038	.38384	.39697	.41055	.42430	.43823
.45253	.46658	.48099	.49554	.51042	.52508	.54005	.55514
.57035	.58650	.60111	.61664	.63226	.64868	.66375	.67997
.69660	.71326	.72941	.74663	.76334	.78007	.79493	.81360
.83042	.84729	.86416	.88112	.89807	.91503	.93201	.94912
.96600	.98300	1.00000					

THE NATURAL FREQUENCY OF VIBRATION (RAD/SEC)
****

69.19415

THE NORMALIZED DISPLACEMENTS AT THE LUMPED MASSES
****

.00000	-.00001	-.00005	-.00033	-.00090	-.00136	-.00151	-.00212
-.00371	-.00619	-.00955	-.01378	-.01887	-.02431	-.03157	-.03916
-.04756	-.05675	-.06671	-.07743	-.08889	-.10105	-.11350	-.12739
-.14152	-.15625	-.17154	-.18737	-.20370	-.22049	-.23772	-.25522
-.27332	-.29161	-.31016	-.32893	-.34784	-.36694	-.38606	-.40519
-.42396	-.44322	-.46200	-.48053	-.49876	-.51651	-.53404	-.55094
-.56726	-.58265	-.59784	-.61194	-.62514	-.63734	-.64851	-.65839
-.66709	-.67450	-.68060	-.68517	-.68833	-.68997	-.69003	-.68852
-.68526	-.68034	-.67368	-.66506	-.65499	-.64291	-.62895	-.61311
-.59508	-.57567	-.55404	-.53044	-.50453	-.47728	-.44770	-.41611
-.38252	-.34499	-.30939	-.26988	-.22844	-.18310	-.13993	-.09212
-.04237	.00808	.05756	.11089	.16318	.21602	.26344	.32437
.38094	.43907	.49843	.55915	.62070	.68298	.74584	.80951
.87260	.93626	1.00000					

THE NATURAL FREQUENCY OF VIBRATION (RAD/SEC)
****

159.80793

THE NORMALIZED DISPLACEMENTS AT THE LUMPED MASSES
****

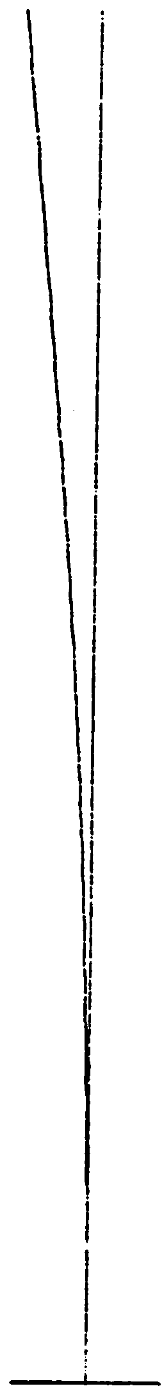
.00000	.00001	.00010	.00069	.00190	.00286	.00319	.00444
.00772	.01272	.01941	.02771	.03756	.04794	.06158	.07561
.09086	.10726	.12472	.14312	.16237	.18236	.20233	.22407
.24556	.26732	.28921	.31112	.33290	.35443	.37558	.39605
.41613	.43526	.45342	.47048	.48625	.50068	.51356	.52480
.53415	.54194	.54764	.55131	.55286	.55225	.54936	.54418
.53666	.52695	.51444	.49971	.48257	.46302	.44097	.41684
.39040	.36190	.33114	.29935	.26566	.23060	.19440	.15799
.11949	.08126	.04285	.00381	-.03339	-.07067	-.10698	-.14204
-.17603	-.20718	-.23661	-.26352	-.28784	-.30843	-.32575	-.33921
-.34848	-.35335	-.35314	-.34792	-.33728	-.32010	-.29879	-.27079
-.23920	-.20499	-.16940	-.12890	-.08713	-.04291	-.00132	.05785
.11953	.18869	.26425	.34576	.43181	.52166	.61444	.70994
.80554	.90255	1.00000					

4.6 EXAMPLE OF MODE SHAPE PLOTTING

The following example illustrates the use of the MODPLT program to plot the mode shapes on a Tektronix 4010 or 4014 display terminal. The program is executed by entering MODPLT. The program then queries the user to the computer interface connected to the terminal and the type of Tektronix terminal being used. The program then allows the user to generate a sketch of the mode shape; to change the scale factor used in sketching the mode shape; to make a complete finished drawing of the mode shape or to go on to the mode shape corresponding to the next natural frequency. For each mode shape, the normal procedure is to generate several sketches in order to determine a suitable scale factor and then generate a finished drawing which includes the title of the problem, the mode of vibration, the corresponding natural frequency, the number of lumped masses in the model, the rotor speed and the current date.

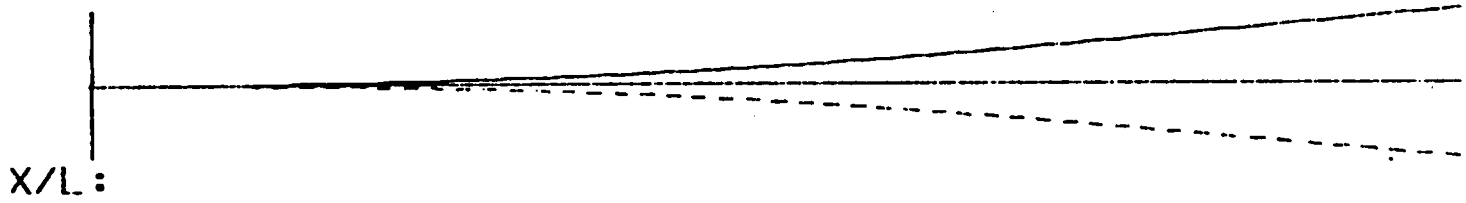
```
A>MODPLT
ENTER THE DATE (DD-MMM-YY)
28-DEC-85
ENTER TYPE OF DISPLAY TERMINAL (4010 OR 4014)
4014
1 = DRAW SKETCH
2 = CHANGE AMPLITUDE SCALE FACTOR, CURRENTLY =    .50
3 = DRAW FINISHED DRAWING(S)
4 = CONSIDER NEXT NATURAL FREQUENCY AND MODE SHAPE
1
```

```
1 = DRAW SKETCH
2 = CHANGE AMPLITUDE SCALE FACTOR, CURRENTLY =    .50
3 = DRAW FINISHED DRAWING(S)
4 = CONSIDER NEXT NATURAL FREQUENCY AND MODE SHAPE
2
ENTER NEW SCALE FACTOR TO BE APPLIED TO AMPLITUDE
.70
1 = DRAW SKETCH
2 = CHANGE AMPLITUDE SCALE FACTOR, CURRENTLY =    .70
3 = DRAW FINISHED DRAWING(S)
4 = CONSIDER NEXT NATURAL FREQUENCY AND MODE SHAPE
1
```



```
1 = DRAW SKETCH
2 = CHANGE AMPLITUDE SCALE FACTOR, CURRENTLY =     .70
3 = DRAW FINISHED DRAWING(S)
4 = CONSIDER NEXT NATURAL FREQUENCY AND MODE SHAPE
3
```

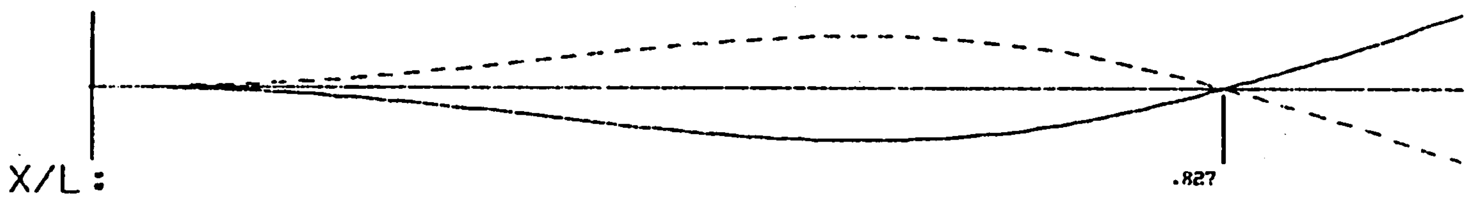
FLAPWISE VIBRATION OF THE BLADE



MODE	NAT. FREQ. (HZ.)	NO. OF MASSES	ROTOR RPM	DATE
1	1.646	107	.0	28-DEC-85

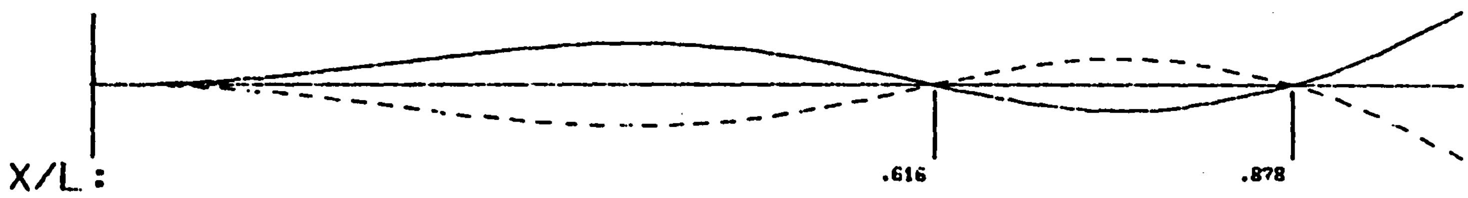
```
1 = DRAW SKETCH
2 = CHANGE AMPLITUDE SCALE FACTOR, CURRENTLY =    .70
3 = DRAW FINISHED DRAWING(S)
4 = CONSIDER NEXT NATURAL FREQUENCY AND MODE SHAPE
4
1 = DRAW SKETCH
2 = CHANGE AMPLITUDE SCALE FACTOR, CURRENTLY =    .70
3 = DRAW FINISHED DRAWING(S)
4 = CONSIDER NEXT NATURAL FREQUENCY AND MODE SHAPE
3
```

FLAPWISE VIBRATION OF THE BLADE



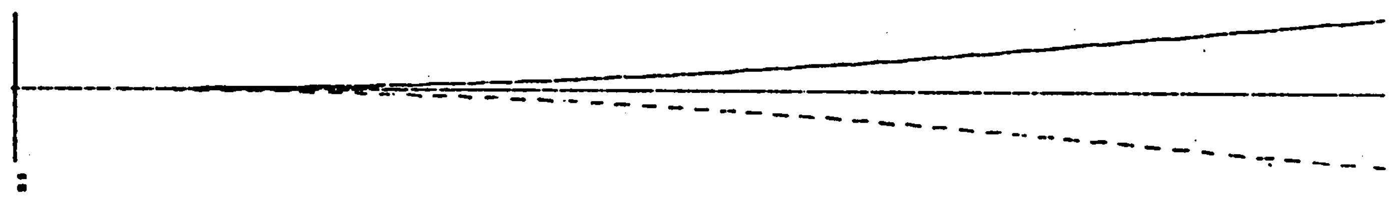
MODE	NAT. FREQ. (HZ.)	NO. OF MASSES	ROTOR RPM	DATE
2	10.773	107	.0	28-DEC-85

FLAPWISE VIBRATION OF THE BLADE



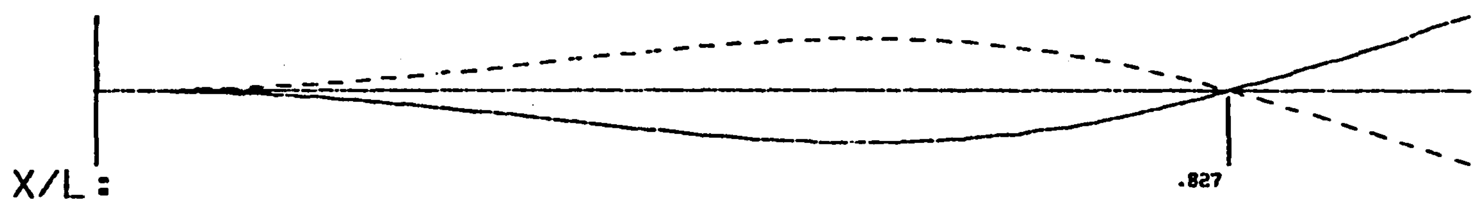
MODE	NAT. FREQ. (HZ.)	NO. OF MASSES	ROTOR RPM	DATE
3	25.243	107	.0	28-DEC-85

FLAPWISE VIBRATION OF THE BLADE



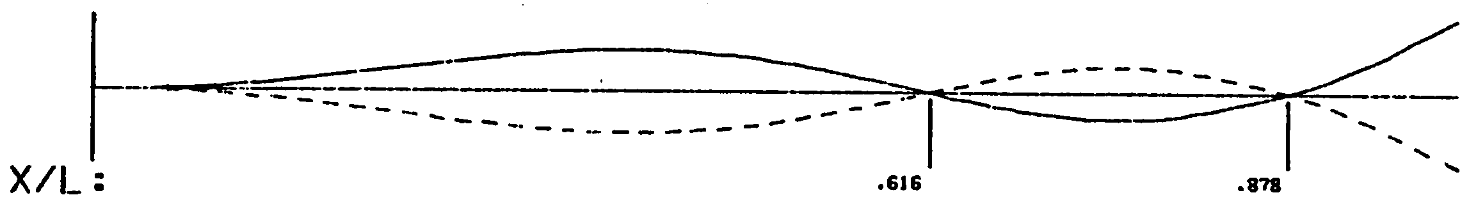
MODE	NAT. FREQ. (HZ.)	NO. OF MASSES	ROTOR RPM	DATE
1	1.695	107	20.0	28-DEC-85

FLAPWISE VIBRATION OF THE BLADE



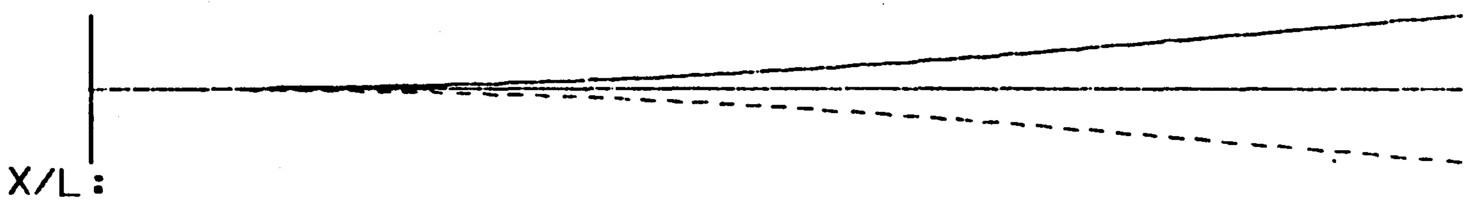
MODE	NAT. FREQ. (HZ.)	NO. OF MASSES	ROTOR RPM	DATE
2	10.833	107	20.0	28-DEC-85

FLAPWISE VIBRATION OF THE BLADE



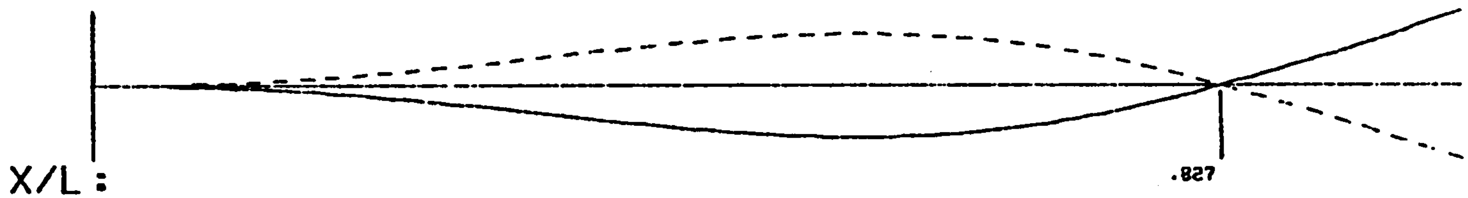
MODE	NAT. FREQ. (HZ.)	NO. OF MASSES	ROTOR RPM	DATE
3	25.291	107	20.0	28-DEC-85

FLAPWISE VIBRATION ØF THE BLADE



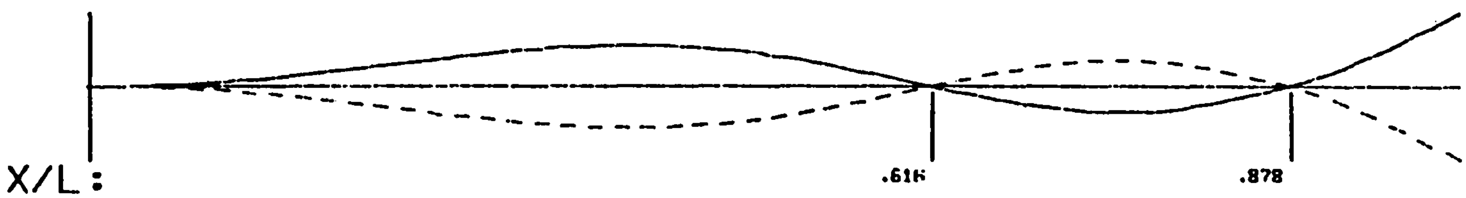
MODE	NAT. FREQ. (HZ.)	NO. OF MASSES	ROTOR RPM	DATE
1	1.834	107	40.0	28-DEC-85

FLAPWISE VIBRATION OF THE BLADE



MODE	NAT. FREQ. (HZ.)	NO. OF MASSES	ROTOR RPM	DATE
2	11.013	107	40.0	28-DEC-85

FLAPWISE VIBRATION OF THE BLADE



MODE	NAT. FREQ. (HZ.)	NO. OF MASSES	ROTOR RPM	DATE
3	25.434	107	40.0	28-DEC-85

5. ANALYTICAL AND EXPERIMENTAL RESULTS

The sections which follow document the analytical and experimental results for the blade and counterweight assemblies. The analytical results were obtained by executing the computer codes which are described in Sections 3 and 4. The expérimental results were obtained using real time spectral analysis techniques.

5.1 ANALYTICAL RESULTS FOR THE MOD-0 BLADE ASSEMBLY

The BLDFLAP computer code was used to generate a 573 lumped mass model of the blade assembly for flapwise vibration. This assembly included a rigid hub having a radius of 22.0 inches, the spool piece (CF 764254), the "Shorty 40" blade, the transition section and a six foot pitchable tip section. The number of lumped masses for each section was :

Spool Piece Inboard Flange	- 2
Spool Piece Body	- 14
Spool Piece Outboard Flange	- 2
Blade Section	- 440
Transition and Tip Sections	- 115

This lumped mass model was used as input to the BEAM code and results were obtained for rotor speeds ranging from 0 rpm (static rotor condition) to 90 rpm at intervals of 10 rpm. Additional input to the BEAM program was:

Lower Limit of Searching Frequency Range	- 0.0 rad/sec
Upper Limit of Searching Frequency Range	- 350.0 rad/sec
Searching Frequency Interval	- 10.0 rad/sec
Number of Digits of Accuracy	- 3

Natural frequencies and mode shapes were obtained for the first four modes of vibration. The natural frequencies for each of the rotor speeds are listed in Table 5.1.

It can be seen that, as predicted, the natural frequencies increase with increasing rotor speed. This effect, although not extremely significant, is less pronounced at the higher modes of vibration.

TABLE 5.1

THE FIRST FOUR NATURAL FREQUENCIES FOR FLAPWISE VIBRATION OF THE BLADE ASSEMBLY (CPS) AT VARIOUS ROTOR SPEEDS

0 RPM	10 RPM	20 RPM	30 RPM	40 RPM
1.64	1.65	1.69	1.75	1.83
10.70	10.72	10.76	10.84	10.94
25.02	25.04	25.07	25.13	25.21
46.13	46.14	46.18	46.24	46.33

50 RPM	60 RPM	70 RPM	80 RPM	90 RPM
1.92	2.03	2.16	2.29	2.43
11.07	11.23	11.42	11.63	11.86
25.32	25.45	25.60	25.77	25.97
46.45	46.59	46.76	46.95	47.16

5.2 ANALYTICAL RESULTS FOR THE COUNTERWEIGHT ASSEMBLY

The CWTFLAP computer code was used to generate a 403 lumped mass model of the counterweight assembly for flapwise vibration. This assembly included a rigid hub having a radius of 22.0 inches, the spool piece (CF 764254), the tapered steel spar extension (CF 760549), and the counterweight (CF 764554). The number of lumped masses for each section was :

Spool Piece Inboard Flange	-	6
Spool Piece Body	-	42
Spool Piece Outboard Flange	-	6
Spar Base Flange	-	6
Spar Inboard Transition Portion	-	15
Spar Tapered Portion	-	309
Spar Outboard Transition Portion	-	12
Spar Tip Flange	-	6
Lumped Rigid Counterweight Mass	-	1

The lumped weight of the rigid counterweight mass located at the free end of the tapered spar extension was 5294 pounds. This lumped mass model was used as input to the BEAM code and results were obtained for rotor speeds ranging from 0 rpm (static rotor condition) to 90 rpm at intervals of 10 rpm. Additional input to the BEAM program was:

Lower Limit of Searching Frequency Range	-	0.0 rad/sec
Upper Limit of Searching Frequency Range	-	7000.0 rad/sec
Searching Frequency Interval	-	100.0 rad/sec
Number of Digits of Accuracy	-	3

Natural frequencies and mode shapes were obtained for the first four modes of vibration. The natural frequencies for each of the rotor speeds are listed in Table 5.2.

Although the natural frequencies of the counterweight assembly increase with increasing rotor speed, the effect is even less pronounced than that of the blade assembly. An identical lumped mass model of the counterweight assembly was used to perform a chordwise vibration analysis for the static rotor condition. The natural frequencies for the first four modes of vibration are listed in Table 5.3. It should be noted that for static rotor conditions, a chordwise and flapwise vibration analysis of the counterweight assembly yielded almost identical results as can be seen in Tables 5.2 and 5.3.

TABLE 5.2

THE FIRST FOUR NATURAL FREQUENCIES FOR FLAPWISE VIBRATION OF THE COUNTERWEIGHT ASSEMBLY (CPS) AT VARIOUS ROTOR SPEEDS

0 RPM	10 RPM	20 RPM	30 RPM	40 RPM
8.21	8.21	8.22	8.23	8.25
171.11	171.12	171.14	171.17	171.21
526.11	526.12	526.14	526.17	526.22
1034.82	1034.83	1034.85	1034.88	1034.93

50 RPM	60 RPM	70 RPM	80 RPM	90 RPM
8.27	8.30	8.33	8.37	8.41
171.27	171.33	171.41	171.50	171.61
526.28	526.35	526.44	526.54	526.65
1034.99	1035.06	1035.15	1035.25	1035.37

TABLE 5.3

THE FIRST FOUR NATURAL FREQUENCIES FOR CHORDWISE VIBRATION OF THE COUNTERWEIGHT ASSEMBLY (CPS) AT A ROTOR SPEED OF ZERO

8.21
171.12
526.16
1035.00

5.3 EXPERIMENTAL RESULTS FOR THE BLADE ASSEMBLY

Frequency domain spectrum were supplied by NASA personnel for a modal analysis conducted at the Plum Brook facility on the blade assembly. The tip sections on the two-bladed rotor assembly were removed and a modal analysis was performed on the resulting assembly in a teetered hub configuration for both chordwise and flapwise vibration. For flapwise vibration, the response spectrum showed peaks at frequencies of 2.92 cps, 9.0 cps, 13.3 cps, 16.8 cps, 25.3 cps, 30.5 cps, and 39.0 cps. The analytical values for the first four modes of vibration of a model including the tip section are given in Table 5.1 as 1.64 cps, 10.70 cps, 25.02 cps and 46.13 cps. The blade model generation program, BLDFLAP, was modified to remove the tip section. As a result of removing the tip section the natural frequencies were found to increase. The analytical values for the first four modes of vibration of the blade assembly without the tip section were found to be 1.94 cps, 13.14 cps, 33.93 cps and 64.83 cps. The additional peaks found experimentally may be due to the rigid body motion of the assembly and the effects of the rubber stops or other nonlinear effects in the system.

5.4 EXPERIMENTAL RESULTS FOR THE COUNTERWEIGHT ASSEMBLY

Two trips were made to the Lewis Research Center to acquire experimental modal analysis data for the counterweight assembly. This data was obtained using both impact excitation techniques which gave the structure an initial velocity without an initial displacement and displacement techniques which resulted in giving the structure an initial displacement without an initial velocity. The response of the structure was obtained using both an accelerometer and a microphone. Output from these sensors was processed using a single channel Spectral Dynamics Real Time Analyzer and the results were recorded using a Hewlett Packard X-Y Plotter. All equipment was provided by the Mechanical Engineering Department of The University of Toledo. The counterweight assembly including the spool piece was mounted on a test fixture supplied by NASA. Data was taken for flapwise vibration of the counterweight assembly for static rotor conditions.

Typical analysis output (frequency domain spectra) are shown in Figure 5.1 for an analysis range of 20 cps and in Figure 5.2 for an analysis range of 2000 cps. The vertical axis represents the amplitude of the signal in terms of a logarithmic scale and the horizontal axis represents the frequency of the spectra in terms of a linear scale. The first peak shown in Figure 5.1 occurs at 1.44 cps and represents internal noise generated within the real time analyzer. The second peak occurs at 6.36 cps and is believed to be the fundamental frequency of the counterweight assembly as mounted on the test fixture. A model of the

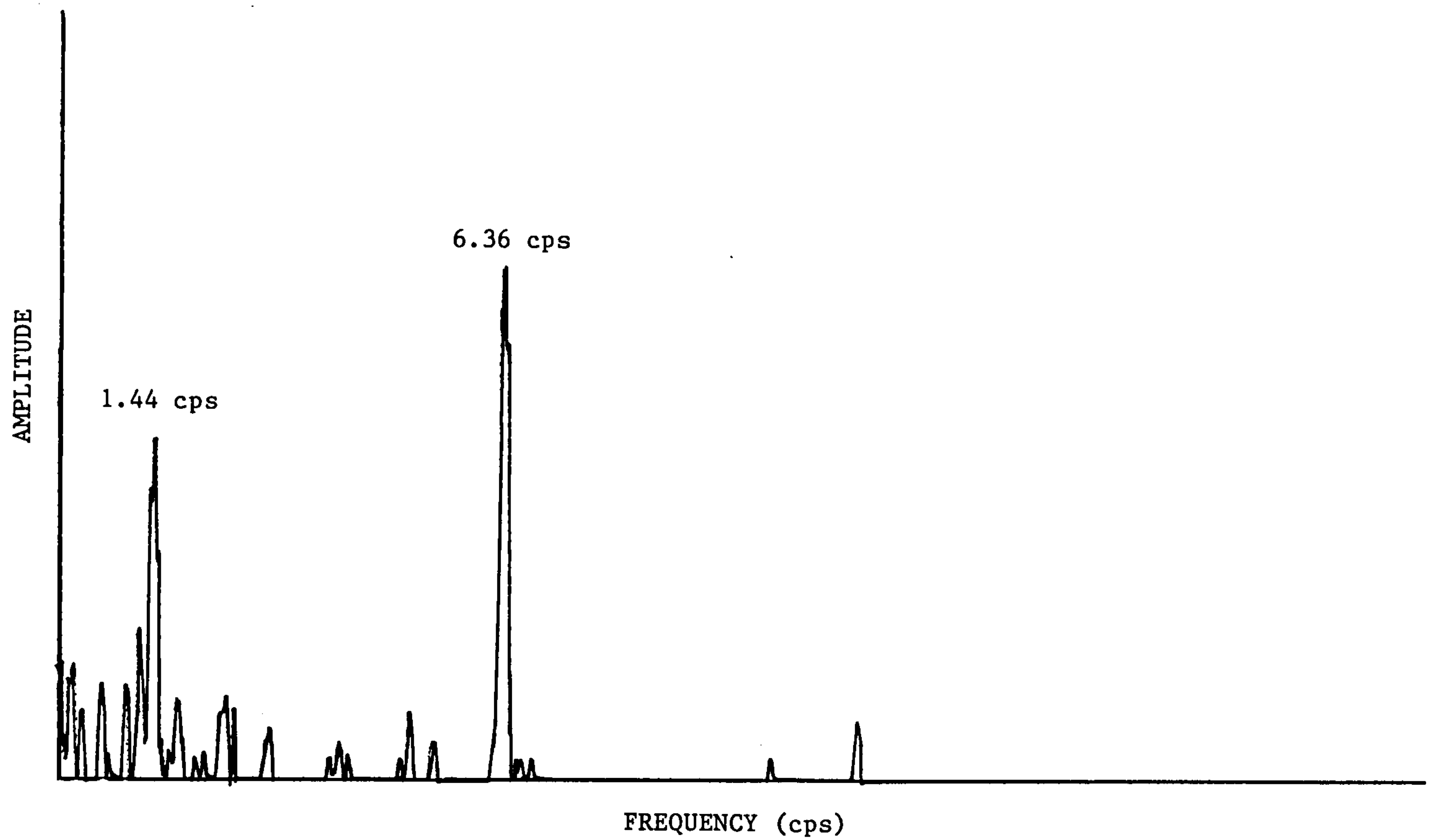


Figure 5.1 Frequency Domain Spectra for Counterweight Assembly.

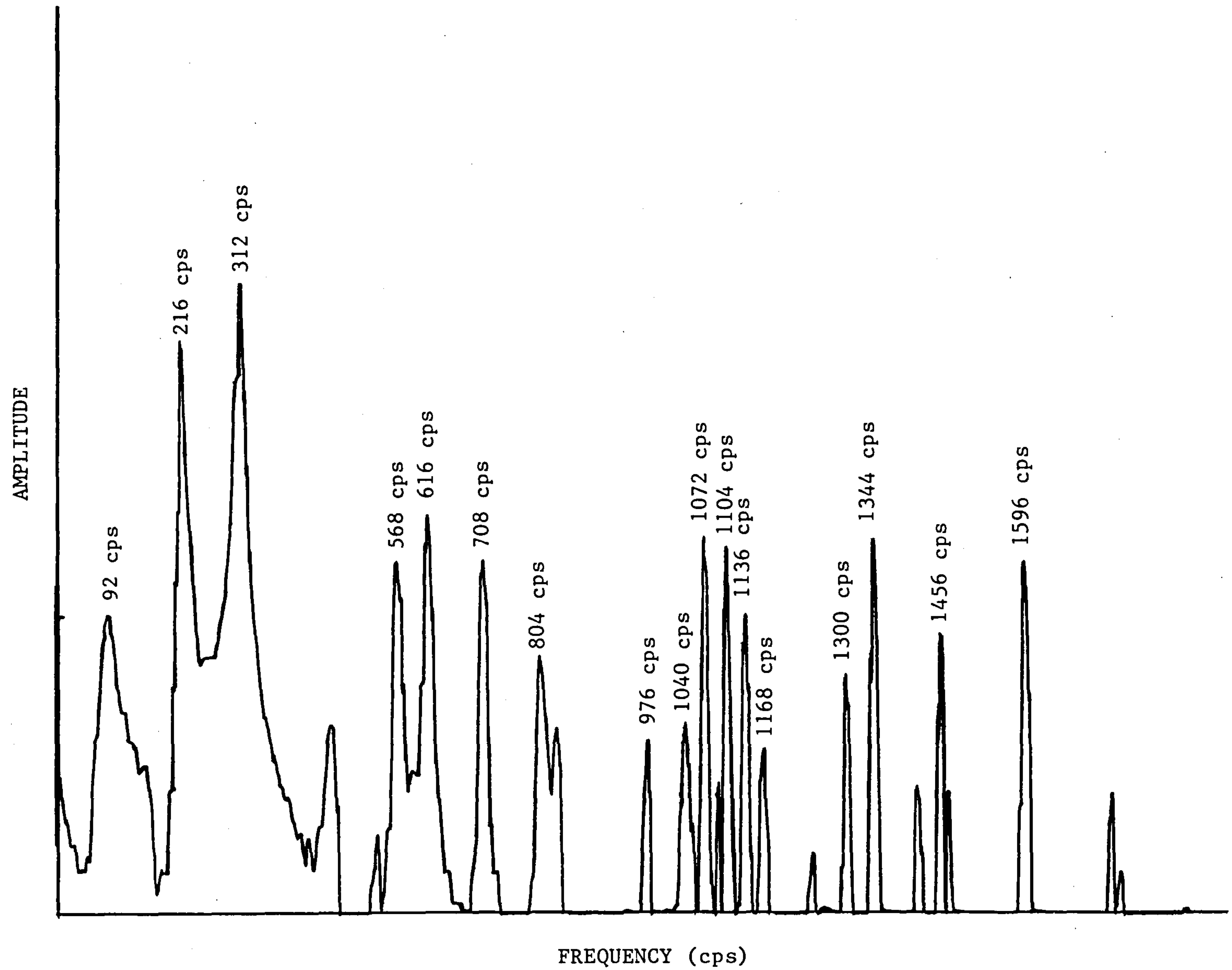


Figure 5.2 Frequency Domain Spectra for Counterweight Assembly

counterweight assembly which did not include the spool piece (the spool piece was originally considered to be rigid with respect to the rest of the assembly) yielded a fundamental frequency of 9.09 cps for flapwise vibration. The counterweight model generation program, CWTFLAP, was then modified to include the effects of the spool piece. The model which included the spool piece yielded an analytical value for the fundamental frequency of 8.21 cps for flapwise vibration of the counterweight assembly. The discrepancy between the experimental value of 6.36 cps and the analytical value of 8.21 cps for the flapwise fundamental frequency of the counterweight assembly may very well be due to the lack of rigidity in the test fixture which supported the assembly. In order to determine if this was indeed the case, one would need to perform a complete modal analysis of the counterweight and test fixture assembly using a two or four channel modal analysis system which is capable of generating animated mode shapes as well as performing a frequency analysis. Such portable equipment was not readily available at the time this study was conducted.

The model of the counterweight which included the spool piece predicted frequencies of 171.11 cps, 526.11 cps and 1034.82 cps for the second, third and fourth modes of vibration. As seen in Figure 5.2 there were many peaks that were experimentally acquired in this range. In fact, there were so many peaks that it was impossible to experimentally determine the remaining natural frequencies of the counterweight assembly. It is believed that these peaks resulted from the ringing effects and the pipe organ effects of the thin shelled steel spar extension. As predicted by the analytical mode shapes and noted in the field, the free end of the assembly where the counterweight is located became a nodal point (a point of zero displacement) for vibration at natural frequencies above the fundamental mode. Thus it was necessary to attach the accelerometer directly to the steel spar extension when trying to determine the natural frequencies for the higher modes of vibration. It is believed that this resulted in picking up not only the vibration of the assembly as a cantilevered beam but the radial vibration of the steel spar extension as a thin shell of revolution as well as the vibration of the column of air trapped inside the assembly. Also, the question of the interaction between the counterweight assembly and the non-rigid test fixture remains unanswered. In order to determine the natural frequencies for flapwise vibration of the counterweight assembly from Figure 5.2, one would need to use a multi-channel modal analysis system. Such a system would not only determine the frequency response functions but would determine the phase lag between the system response and the excitation. This additional information would allow the determination of the animated mode shapes for each of the peaks shown in Figure 5.2 and would thus yield the natural frequencies for flapwise vibration of the assembly.

6. DISCUSSION OF RESULTS AND CONCLUSIONS

The sections which follow discuss the results and conclusions of this study. The results and conclusions of the analysis of the blade assembly and the counterweight assembly are discussed in Sections 6.1 and 6.2 respectively. Section 6.3 compares the application of a personal computer to perform the vibration analysis with the application of a minicomputer. Recommendations for further study are given in Section 6.4.

6.1 DISCUSSION OF THE MOD-0 BLADE ASSEMBLY

It should be emphasized that the discussion which follows is based on the uncoupled vibration of the blade assembly treated as a cantilevered beam. The results presented in Section 5.1 indicate that the natural frequencies for the flapwise vibration of the Mod-0 blade assembly increase with increasing rotor speeds. However, the percentage increase was not as significant at the higher natural frequencies. The natural frequencies for the first four modes of vibration were significantly higher than the rotor speed which was permitted to vary between 0 and 90 rpm. However, considering the rotor is excited twice per revolution, the fundamental natural frequency can be excited at a rotor speed between 50 and 60 rpm. Since the flexural rigidity of the blade assembly is significantly stiffer in the chordwise direction, the natural frequencies are above the operating range and a chordwise analysis was not performed.

6.2 DISCUSSION OF THE COUNTERWEIGHT ASSEMBLY

As in the case of the blade assembly, it should be emphasized that the discussion which follows is based on the uncoupled lateral vibration of the counterweight assembly treated as a cantilevered beam. The results presented in Section 5.2 indicate that the natural frequencies for the flapwise vibration of the counterweight assembly increase with increasing rotor speeds although this effect is even less pronounced than that with the blade assembly. The percentage increase was not as significant at the higher natural frequencies. Natural frequencies for the first four modes of vibration were significantly higher than the rotor speed which was permitted to vary between 0 and 90 rpm. Even, considering the rotor is excited twice per revolution, the fundamental natural frequency remains well above the forcing frequency. Since the flexural rigidity of the counterweight assembly is essentially the same in both the flapwise and chordwise directions, the natural frequencies for vibration in the chordwise direction are almost identical to those for flapwise vibration.

6.3 COMPARISON OF PC AND MINICOMPUTER COMPUTATIONS

One of the primary goals of this research effort was to permit users to employ a personal computer for the vibratory analysis of wind turbines. Traditionally this type of analysis was restricted to mainframe and minicomputer environments. The analysis codes used in this research effort were originally developed on a DEC PDP 11/34 minicomputer. This code was then downloaded and modified to run on an ITT XTRA, IBM compatible, personal computer. Studies were conducted to evaluate the efficiency of running the wind energy programs on a personal computer as compared to the minicomputer. These studies were conducted using a 139 mass model of the counterweight assembly in flapwise vibration at static rotor conditions. In all cases, the first four modes of vibration were determined. The execution times for various cases follow.

System and Options	Time (secs)
PDP 11/34 with floating point hardware	715
ITT PC with default math library but without 8087 math co-processor	6031
ITT PC with 8087 math co-processor and default math library	955
ITT PC with 8087 math co-processor and 8087 math library	815
ITT PC with 8087 math co-processor, 8087 math library, and $NOFLOATCALLS and $STORAGE:2 metacommands	644

One can note that, with the proper combination of hardware and software options, the efficiency of using a PC exceeds that of a minicomputer. It is also noteworthy that, since the quoted studies were conducted, an additional hardware option has been released which further increases the speed of the ITT PC. Thus, the application of personal computers to this type of vibration analysis is very feasible.

6.4 RECOMMENDATIONS FOR FURTHER STUDY

There are at least five additional areas of research which should be investigated in order to guarantee successful operation of a wind turbine rotor assembly. The first research area involves the development of a model generation program for the chordwise vibration of the blade assembly. Currently, models may only be generated for flapwise vibration of the blade assembly. The second research area involves the effect of rotation on the natural frequencies and mode shapes. The BEAM analysis code presently evaluates the effects of rotation for the flapwise vibration of the uncoupled blade and counterweight assemblies. This code should be modified to include the effects of rotation on chordwise vibration. The third research area involves the coupling of the blade and counterweight assemblies into a single rotor configuration. Such a configuration would require that the BEAM program be modified to reflect the altered boundary conditions. The changes are required because the previously assumed cantilevered beam model will have to be supplemented with a centrally supported teetered hub model. The fourth area of research involves the analysis of the tower shadow effects on the forcing frequency applied to the rotor assembly. Applying Fourier Series Analysis techniques to the tower shadow effect may reveal forcing frequencies which differ from the one or two per revolution that were assumed in the current research effort. Lastly, in an effort to obtain more definitive experimental results, a complete modal analysis of the blade and counterweight assemblies should be performed using a multichannel modal analysis system capable of generating animated mode shapes as well as performing a frequency analysis. Because of the various nonlinearities in the entire system, the mode shapes are necessary to properly identify the natural frequencies of vibration.

REFERENCES

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3. "Bending Vibration of a Rotating Blade Vibrating in the Plane of Rotation" by R.L. Sutherland, Journal of Applied Mechanics, December 1949.

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5. "Vibration Analysis" by N.O. Myklested, McGraw-Hill Book Company, 1944.

6. "Lumped Mass Formulation for Rotating Beams having Variable Cross Sections, for Vibration Perpendicular to the Plane of Rotation" by P.R. White and R.R. Little, NASA Progress Report sent to R.D. Corrigan, March 1984.

7. "Shorty 40 and GE Aileron Tip" data sheet, supplied by R.D. Corrigan, NASA, October 1983.

8. "Wind Turbine Analysis Data For 6 FT. Tip RTR", Dean Miller, NASA, September 1983.

9. "Blade Station, I, EI" data sheet, supplied by R.D. Corrigan, NASA, October 1983.

APPENDIX

This appendix contains the listings of all computer codes described in Section 3. The order of the listings and their functions are:

BLDFLAP → blade assembly model generation code

CWTFLAP → counterweight assembly model generation code

CWTCHRD → counterweight assembly model generation code

BEAM → main analysis code

- DET → subroutine to evaluate determinant
- FILLA → subroutine to fill the A matrix
- FORCE → subroutine to calculate centrifugal forces
- INPUT → subroutine to perform all input functions
- MODES → subroutine to calculate mode shapes
- MULT → subroutine to perform matrix multiplication
- NFREQ → subroutine to determine natural frequencies
- OUTPUT → subroutine to perform all output functions

MODPLT → mode shape plotting code

- DRAW → subroutine to draw mode shapes
- MINPUT → subroutine to perform all input functions.

```
$NOFLOATCALLS
$STORAGE:2
C
C          A.1  BLDFLAP - blade assembly model generation code
C
C       THIS PROGRAM GENERATES THE INPUT DATA FOR THE SPOOL
C       PIECE (CF 764254), THE "SHORTY 40" BLADE. TRANSITION
C       SECTION AND THE 6' PITCHABLE TIP FOR FLAPWISE VIBRATION
C       (VIBRATION PERPENDICULAR TO THE PLANE OF ROTATION).  THE
C       PROGRAM CONSIDERS THE ASSEMBLY TO BE BROKEN UP INTO 4
C       SECTIONS AS FOLLOWS:
C
C
C               SECTION 1 - THE FLANGE AT THE BASE OF THE SPOOL PIECE
C                           OF LENGTH 2.25" WEIGHING APPROX. 108.74 LBS.
C
C               SECTION 2 - THE WEBBED MAIN BODY OF THE SPOOL PIECE
C                           OF LENGTH 13.75" WEIGHING APPROX. 207.96 LBS.
C
C               SECTION 3 - THE FLANGE AT THE SPAR END OF THE SPOOL PIECE
C                           OF LENGTH 1.75" WEIGHING APPROX. 82.78 LBS.
C
C               SECTION 4 - THE BLADE SECTION OF LENGTH 441.25"
C                           WEIGHING APPROX. 2250.00 LBS.
C
C               SECTION 5 - THE TRANSITION AND TIP SECTIONS OF LENGTH
C                           115.56" WEIGHING APPROX. 960.00 LBS.
C
C
C
C
C       EI(N)   THE PRODUCT OF MODULUS OF ELASTICITY AND AREA MOMENT
C               OF INERTIA AT MASS N  (LBS*INCH*INCH)
C
C       EII(J)  THE INPUT VALUES OF EI USED TO DEFINE THE RIGIDITY OF
C               THE BLADE, TRANSITION AND TIP SECTIONS.  THESE VALUES
C               ARE ENTERED USING A DATA STATEMENT IN THE PROGRAM.
C               (LBS*INCH*INCH)
C
C       EIO     THE PRODUCT OF MODULUS OF ELASTICITY AND AREA MOMENT
C               OF INERTIA AT ZERO OR THE FIXED END  (LBS*INCH*INCH)
C
C       INEI    THE INPUT NUMBER OF EI VALUES USED TO DEFINE THE
C               RIGIDITY OF THE BLADE, TRANSITION SECTION AND TIP
C               SECTION
C
C       INW     THE INPUT NUMBER OF VALUES OF THE WEIGHT BETWEEN
C               STATIONS USED TO DEFINE THE WEIGHT DISTRIBUTION OF
C               THE BLADE, TRANSITION SECTION AND TIP SECTION
C
C       LEN(N)  THE LENGTH OF THE SECTION TO THE LEFT OF MASS N
C               (INCHES)
C
C       NB      THE MASS NUMBER AT THE BEGINNING OF A SECTION
C
C       NF      THE MASS NUMBER AT THE END OF A SECTION
C
C       ND      THE NUMBER OF DIVISIONS IN A PARTICULAR SECTION
```

```
C
C       NS       THE NUMBER OF THE SECTION UNDER CONSIDERATION
C
C       NTOT     THE TOTAL NUMBER OF MASSES
C
C       RRHUB    THE RADIUS OF THE RIGID HUB (INCHES)
C
C       TITLE    THE PROBLEM TITLE STATEMENT
C
C       WI(L)    THE INPUT VALUES OF THE WEIGHT BETWEEN STATIONS
C                USED TO DEFINE THE WEIGHT DISTRIBUTION OF THE
C                BLADE, TRANSITION AND TIP SECTIONS.  THESE VALUES
C                ARE ENTERED USING A DATA STATEMENT IN THE
C                PROGRAM.  (LBS.)
C
C       WPI(M)   THE WEIGHT OF THE BLADE PER UNIT LENGTH CALCULATED
C                FROM THE INPUT VALUES OF WI(L)  (LBS./INCH)
C
C       WT(N)    THE WEIGHT OF MASS N  (LBS)
C
C       X(N)     THE AXIAL LOCATION OF MASS N  (INCH)
C
C       XEII(J)  THE AXIAL LOCATION WHERE THE INPUT VALUES OF EI
C                ARE SPECIFIED IN ORDER TO DEFINE THE RIGIDITY
C                OF THE BLADE, TRANSITION AND TIP SECTIONS.  THESE
C                VALUES ARE ENTERED USING A DATA STATEMENT IN THE
C                PROGRAM.  (INCH)
C
C       XWI(I)   THE AXIAL LOCATIONS WHERE THE INPUT VALUES OF THE
C                WEIGHT BETWEEN STATIONS ARE SPECIFIED IN ORDER TO
C                DEFINE THE WEIGHT DISTRIBUTION OF THE BLADE,
C                TRANSITION AND TIP SECTIONS.  THESE VALUES ARE ENTERED
C                USING A DATA STATEMENT IN THE PROGRAM.  (INCH)
C
C
C
C
C
        IMPLICIT REAL*8(A-H,O-Z)
        INTEGER R,RM1
        REAL*8 INTR,LEN
        DIMENSION EI(700),LEN(700),TITLE(15),WT(700),X(700),
     1  XEII(21),XWI(25),XWPI(30),EII(21),WI(25),WPI(30)
C
C       INPUT BLADE, TRANSITION SECTION AND TIP SECTION
C       PROPERTIES FOR THE WEIGHT DISTRIBUTION AND
C       FLEXURAL RIGIDITY
C
        DATA INW/25/
        DATA WI/115.13D0,290.80D0,154.81D0,154.36D0,
     1  121.11D0,120.02D0,113.64D0,105.34D0,92.21D0,
     2  82.49D0,70.16D0,75.44D0,84.20D0,69.62D0,86.07D0,
     3  99.12D0,118.36D0,192.80D0,104.32D0,326.0D0,
     4  321.0D0,98.0D0,91.0D0,84.0D0,0.0D0/
        DATA XWI/39.75D0,50.0D0,75.0D0,100.0D0,125.0D0,150.0D0,
     1  175.0D0,200.0D0,225.0D0,250.0D0,275.0D0,300.0D0,325.0D0,
     2  350.0D0,375.0D0,400.0D0,425.0D0,450.0D0,475.0D0,481.0D0,
     3  500.0D0,525.0D0,550.0D0,575.0D0,596.56D0/
```

```
      DATA INEI/21/
      DATA EII/285.D8,208.D8,115.D8,65.D8,51.D8,35.D8,
     1 93.96D8,84.86D8,82.57D8,73.47D8,64.37D8,19.52D8,19.52D8,
     2 15.32D8,14.63D8,13.95D8,13.26D8,12.58D8,11.89D8,
     3 11.53D8,0.0D0/
      DATA XEII/39.75D0,100.D0,200.D0,300.D0,400.D0,480.9D0,
     1 481.D0,491.D0,501.D0,511.D0,521.D0,521.001D0,527.187D0,
     2 527.188D0,537.187D0,547.187D0,557.187D0,567.187D0,577.187D0,
     3 582.437D0,596.56D0/
C
C     INTERPOLATION FUNCTION
C
      INTR(XX,XXL,XXR,FFL,FFR)=(FFR-FFL)*(XX-XXL)/(XXR-XXL)+FFL
C
C     ASSIGN LOGICAL UNIT 1 TO 'VIBIN.DAT'
C
      OPEN(1, FILE ='VIBIN.DAT', STATUS ='NEW')
C
C     READ DATA FROM TERMINAL
C
      WRITE(*,5)
5     FORMAT(//,2X, ' THIS PROGRAM GENERATES THE DATA FOR THE ',
     1 'FLAPWISE ',/,2X, ' VIBRATION OF THE BLADE ASSEMBLY ',
     2 ///,2X,' ENTER THE RADIUS OF THE RIGID HUB (INCHES) : ',$)
      READ(*,*)RRHUB
      WRITE(*,10)
10    FORMAT(//2X,' SECTION 1-SPOOL PIECE FLANGE L=2.25" W=108.74 LB',
     1 //2X,' SECTION 2-SPOOL PIECE BODY L=13.75" W=207.96 LB',
     2 //2X,' SECTION 3-SPOOL PIECE FLANGE L=1.75" W=82.78 LB',
     4 //2X,' SECTION 4-BLADE SECTION L=441.25" W=2250.00 LB'
     5 //2X,' SECTION 5-TRANSITION & TIP SECTION L=115.56" W=960.00 LB')
      WRITE(*,20)
20    FORMAT(/,4X, ' THE MAXIMUM TOTAL NUMBER OF DIVISIONS YOU ' ,
     1 /,4X, ' ARE ALLOWED IN ALL 5 SECTIONS IS 699. ' )
      E=30000000.0
      PI=3.141592654
      NS=1
      NF=0
30    GO TO(40,60,80,100,310,400)NS
C
C     SECTION 1 PARAMETERS
C     ********************
C
40    WRITE(*,50)
50    FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 1 : ',$)
      READ(*,104)ND
      WC=48.3268
      XL=0.
      XR=2.25
      DO=21.63
      DI=15.825
C
C     CALCULATIONS FOR SECTIONS 1 AND 3
C     *********************************
C
55    DN=ND
```

```
      DX=(XR-XL)/DN
      XR=XL+DX
      NB=NF+1
      NF=NF+ND
      SEI=(E*PI*(DO**4-DI**4))/64.0
      DO 57 N=NB,NF
      WT(N)=WC*(XR-XL)
      X(N)=.50*(XR+XL)
      EI(N)=SEI
      XL=XR
57    XR=XL+DX
      NS=NS+1
      GO TO 30
C
C     SECTION 2 PARAMETERS
C     *******************
C
60    WRITE(*,70)
70    FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 2 : ',$)
      READ(*,104)ND
      WC=15.1242
      XR=16.0
C
C     CALCULATIONS FOR SECTION 2
C     *************************
C
      DN=ND
      DX=(XR-XL)/DN
      XR=XL+DX
      NB=NF+1
      NF=NF+ND
      SEI=E*2106.268
      DO 75 N=NB,NF
      WT(N)=WC*DX
      X(N)=.50*(XR+XL)
      EI(N)=SEI
      XL=XR
75    XR=XL+DX
      NS=NS+1
      GO TO 30
C
C     SECTION 3 PARAMETERS
C     *******************
C
80    WRITE(*,90)
90    FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 3 : ',$)
      READ(*,104)ND
      WC=47.3021
      XR=17.75
      DO=21.63
      DI=15.97
      GO TO 55
C
C     SECTION 4 PARAMETERS
C     *******************
```

```
C
100     WRITE(*,102)
102     FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 4 : ',$)
        READ(*,104)ND
104     FORMAT(I5)
        XR=459.00
        XL=17.75
C
C       INITIALIZE LEFT INDEX L.  L WILL POINT TO THE WPI AND XWI VALUE
C       WHICH IS TO THE LEFT OF OR AT XL.
C
        L=1
C
C       CALCULATIONS FOR SECTIONS 4 AND 5
C       ********************************
C
C       CALCULATE THE X COORDINATES AS REFERENCED FROM RIGID HUB
C
        DO 109 I=1,INW
109     XWI(I)=XWI(I)-RRHUB
        DO 110 I=1,INEI
110     XEII(I)=XEII(I)-RRHUB
C
C       CALCULATE THE WT/IN FOR EACH GIVEN SECTION
C
        INWM1=INW-1
        DO 115 I=1,INWM1
        WPI(I)=WI(I)/(XWI(I+1)-XWI(I))
115     CONTINUE
C
C       CALCULATE LUMPED WEIGHTS, AXIAL CG LOCATIONS AND EI VALUES
C
116     DX=(XR-XL)/FLOAT(ND)
C
C       CONSIDER ND MASSES
C
        NB=NF+1
        NF=NF+ND
        DO 300 N=NB,NF
        XR=XL+DX
C
C       FIND RIGHT INDEX R.  R WILL POINT TO THE WPI AND XWI VALUE
C       WHICH CONTIANS XR.
C
        DO 120 R=L,INW
        IF(XR.GE.XWI(R).AND.XR.LE.XWI(R+1)) GO TO 130
120     CONTINUE
        R=INW
C
C       CHECK IF DX IS TOTALLY WITHIN A WPI VALUE (ICASE=1), IF DX
C       SPANS ACROSS TWO WPI VALUES (ICASE=2), OR IF DX SPANS
C       ACROSS 3 OR MORE VALUES (ICASE=3)
C
130     ICASE=R-L+1
        IF(ICASE.GT.3) ICASE=3
        GO TO (140,180,180),ICASE
```

```
C
C       CASE 1 - NEW MASS ENTIRELY BETWEEN WPI VALUES
C
140     WT(N)=WPI(L)*DX
        X(N)=(XL+XR)/2.
        GO TO 240
C
C       CASE 2 AND 3 - NEW MASS SPANS TWO OR MORE WPI VALUES
C
180     RM1=R-1
        LP1=L+1
        WTSUM=WPI(L)*(XWI(LP1)-XL)
        IF(ICASE.EQ.2) GO TO 195
        DO 190 I=LP1,RM1
        WTSUM=WTSUM+WPI(I)*(XWI(I+1)-XWI(I))
190     CONTINUE
C
C       ADD LAST PIECE
C
195     WTSUM=WTSUM+WPI(R)*(XR-XWI(R))
C
C       CALCULATE WT*XCG FOR LEFT PARTIAL PORTION
C
        XCGSUM=WPI(L)*(XWI(LP1)-XL)*(XWI(LP1)+XL)/2.
C
C       ADD WT*XCG FOR RIGHT PARTIAL PORTION TO SUM
C
        XCGSUM=XCGSUM+WPI(R)*(XR-XWI(R))*(XR+XWI(R))/2.
C
C       AVOID ADDING EXTRA TERMS TO SUM FOR CASE 2
C
        IF(ICASE.EQ.2) GO TO 210
        DO 200 I=LP1,RM1
        IP1=I+1
        XCGSUM=XCGSUM+WPI(I)*(XWI(IP1)-XWI(I))*(XWI(IP1)+XWI(I))/2.
200     CONTINUE
210     WT(N)=WTSUM
        X(N)=XCGSUM/WTSUM
C
C       FIND EI AT CALCULATED CG AXIAL LOCATION
C
240     INEIM1=INEI-1
        DO 250 LEI=1,INEIM1
        IF(X(N).GE.XEII(LEI).AND.X(N).LT.XEII(LEI+1)) GO TO 260
250     CONTINUE
        LEI=NEIM1
260     EI(N)=INTR(X(N),XEII(LEI),XEII(LEI+1),EII(LEI),EII(LEI+1))
C
C       ADJUST VALUES FOR NEXT MASS
C
        XL=XR
        DO 280 I=L,INW
        IF(XL.GE.XWI(I).AND.XL.LT.XWI(I+1)) GO TO 290
280     CONTINUE
        L=INW
```

```
      GO TO 300
290   L=I
300   CONTINUE
      NS=NS+1
      GO TO 30
C
C     SECTION 5 PARAMETERS
C     *******************
C
310   WRITE(*,320)
320   FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 5 : ',$)
      READ(*,330)ND
330   FORMAT(I5)
      XR=574.56
      XL=459.00
C
C     INITIALIZE LEFT INDEX L.  L WILL POINT TO THE WPI AND XWI VALUE
C     WHICH IS TO THE LEFT OF OR AT XL.
C
      L=20
      GO TO 116
C
C     ENTER TITLE STATEMENT
C
400   WRITE(*,410)
410   FORMAT(///, ' ENTER A TITLE STATEMENT (60 CHAR. MAX.) : ',$)
      READ(*,420)TITLE
420   FORMAT(15A4)
C
C     CALCULATE EI AT THE BASE OR FIXED END OF THE SPOOL
C     PIECE (X=0.0)
C
      DO=21.63
      DI=15.825
      EIO=(E*PI*(DO**4-DI**4))/64.0
C
C     OUTPUT RESULTS TO VIBIN DATA FILE
C     ********************************
C
700   WRITE(1,710)TITLE
710   FORMAT(6X,15A4)
      NTOT=NF
      WRITE(1,720)NTOT
720   FORMAT(1X,I5)
      LEN(1)=X(1)
      DO 730 N=2,NTOT
730   LEN(N)=X(N)-X(N-1)
      WRITE(1,740)(LEN(N),EI(N),WT(N),N=1,NTOT)
740   FORMAT(1X,F10.6,F20.1,7X,F15.6)
      WRITE(1,750)EIO
750   FORMAT(1X,F20.1)
      WRITE(1,760)RRHUB
760   FORMAT(1X,F20.3)
C
C     CHECK THE TOTAL WEIGHT OF THE DISCRETE MODEL AND THE LOCATION
C     OF THE CG OF THE MODEL WITH RESPECT TO THE BASE FLANGE OF THE
C     SPOOL PIECE (X=0).
```

```
C
      WTOT=0.0
      WX=0.0
      WSP=0.0
      WBD=0.0
      WTT=0.0
      WXSP=0.0
      WXBD=0.0
      WXTT=0.0
      DO 800 N=1,NTOT
      WTOT=WTOT+WT(N)
      IF(X(N).LE.17.75) THEN
C             SPOOL PIECE SECTION
              WXSP=WXSP+WT(N)*X(N)
              WSP=WSP+WT(N)
      ELSE IF(X(N).LE.459.0) THEN
C             BLADE SECTION
              WXBD=WXBD+WT(N)*X(N)
              WBD=WBD+WT(N)
      ELSE
C             TRANSITION AND TIP SECTIONS
              WXTT=WXTT+WT(N)*X(N)
              WTT=WTT+WT(N)
      ENDIF
800   WX=WX+WT(N)*X(N)
      XB=WX/WTOT
      XSP=WXSP/WSP
      XBD=WXBD/WBD-17.75
      XTT=WXTT/WTT-459.0
      WRITE(*,810)
810   FORMAT(//,2X,12HTOTAL WEIGHT,12X,15HCG FROM HUB END)
      WRITE(*,820)
820   FORMAT(5X,5H(LBS),20X,6H(INCH))
      WRITE(*,830)WTOT,XB
830   FORMAT(3X,F9.3,16X,F8.3)
      WRITE(*,840)
840   FORMAT(//,2X,12HSPOOL WEIGHT,12X,15HCG FROM HUB END)
      WRITE(*,820)
      WRITE(*,830)WSP,XSP
      WRITE(*,850)
850   FORMAT(//,2X,12HBLADE WEIGHT,12X,17HCG FROM SPOOL END)
      WRITE(*,820)
      WRITE(*,830)WBD,XBD
      WRITE(*,860)
860   FORMAT(//,2X,19HTRANS. & TIP WEIGHT,5X,17HCG FROM BLADE END)
      WRITE(*,820)
      WRITE(*,830)WTT,XTT
      WRITE(*,870)
870   FORMAT(//,2X,36HTOTAL MOMENT ABOUT CENTERLINE OF HUB)
      WRITE(*,880)
880   FORMAT(15X,10H(LBS-INCH))
      WRITE(*,890) WTOT*(XB+22.0)
890   FORMAT(15X,F10.1)
      STOP
      END
```

```
$nofloatcalls
$storage:2
C
C       A.2  CWTFLAP - counterweight assembly model generation code
C
C        THIS PROGRAM GENERATES THE INPUT DATA FOR THE SPOOL
C        PIECE (CF 764254), THE TAPERED STEEL SPAR EXTENSION
C        (CF 760549) AND THE COUNTERWEIGHT (CF 764554) ASSEMBLY FOR
C        FLAPWISE VIBRATION (VIBRATION PERPENDICULAR TO THE PLANE OF
C        ROTATION).  THE PROGRAM CONSIDERS THE TAPERED SPAR EXTENSION
C        TO BE BROKEN UP INTO 8 SECTIONS AS FOLLOWS :
C
C
C                SECTION 1 - THE FLANGE AT THE BASE OF THE SPOOL PIECE
C                            OF LENGTH 2.25" WEIGHING APPROX. 108.74 LBS.
C
C                SECTION 2 - THE WEBBED MAIN BODY OF THE SPOOL PIECE
C                            OF LENGTH 13.75" WEIGHING APPROX. 207.96 LBS.
C
C                SECTION 3 - THE FLANGE AT THE SPAR END OF THE SPOOL PIECE
C                            OF LENGTH 1.75" WEIGHING APPROX. 82.78 LBS.
C
C                SECTION 4 - THE FLANGE AT THE BASE OF THE SPAR EXTENSION
C                            OF LENGTH 1.80" WEIGHING APPROX. 57.86 LBS.
C
C                SECTION 5 - THE TRANSITION SECTION BETWEEN THE BASE
C                            FLANGE AND THE TAPERED PORTION OF THE SPAR.
C                            THIS SECTION IS 5.20" IN LENGTH AND WEIGHS
C                            56.63 LBS.
C
C                SECTION 6 - THE TAPERED PORTION OF THE SPAR EXTENSION
C                            OF LENGTH 103.80" WEIGHING APPROX.
C                            795.96 LBS.
C
C                SECTION 7 - THE TRANSITION SECTION BETWEEN THE TAPERED
C                            PORTION OF THE SPAR EXTENSION AND THE FLANGE
C                            AT THE TIP OF THE SPAR.  THIS SECTION IS
C                            3.40" IN LENGTH AND WEIGHS 46.13 LBS.
C
C                SECTION 8 - THE FLANGE AT THE TIP OF THE SPAR EXTENSION.
C                            THIS SECTION IS 1.80" IN LENGTH AND WEIGHS
C                            121.56 LBS.
C
C
C
C        CW      THE LUMPED WEIGHT OF THE COUNTERWEIGHT AT THE FREE END OF
C                THE STEEL SPAR EXTENSION  (LBS)
C
C        EI(N)   THE PRODUCT OF MODULUS OF ELASTICITY AND AREA MOMENT
C                OF INERTIA AT MASS N  (LBS*INCH*INCH)
C
C        EIO     THE PRODUCT OF MODULUS OF ELASTICITY AND AREA MOMENT
C                OF INERTIA AT ZERO OR THE FIXED END  (LBS*INCH*INCH)
C
```

```
C       L(N)     THE LENGTH OF SECTION TO LEFT OF MASS N  (INCHES)
C
C       NB       THE MASS NUMBER AT THE BEGINNING OF A SECTION
C
C       NF       THE MASS NUMBER AT THE END OF A SECTION
C
C       ND       THE NUMBER OF DIVISIONS IN A PARTICULAR SECTION
C
C       NS       THE NUMBER OF THE SECTION UNDER CONSIDERATION
C
C       NTOT     THE TOTAL NUMBER OF MASSES
C
C       RRHUB    THE RADIUS OF THE RIGID HUB (INCHES)
C
C       TITLE    THE PROBLEM TITLE STATEMENT
C
C       WT(N)    THE WEIGHT OF MASS N  (LBS)
C
C       X(N)     THE AXIAL LOCATION OF MASS N  (INCH)
C
C
C
        IMPLICIT REAL*8(A-H,O-Z)
        REAL*8 L,M,M1
        DIMENSION EI(700),L(700),TITLE(15),WT(700),X(700)
C
C       ASSIGN LOGICAL UNIT 1 TO 'VIBIN.DAT'
C
        OPEN(1, FILE='VIBIN.DAT',STATUS='NEW')
C
C       READ DATA FROM TERMINAL
C
        WRITE(*,5)
5       FORMAT(//,2X, ' THIS PROGRAM GENERATES THE DATA FOR THE ',
     1  'FLAPWISE ',/,2X, ' VIBRATION OF THE COUNTERWEIGHT ASSEMBLY ',
     2  ///,2X,' ENTER THE RADIUS OF THE RIGID HUB (INCHES) : ',$)
        READ( *,*)RRHUB
        WRITE(*,10)
10      FORMAT(//' SECTION 1 - SPOOL PIECE FLANGE L=2.25" W=108.74 LB',
     1 //' SECTION 2 - SPOOL PIECE BODY L=13.75" W=207.96 LB',
     2 //' SECTION 3 - SPOOL PIECE FLANGE L=1.75" W=82.78 LB',
     3 //' SECTION 4 - SPAR BASE FLANGE L=1.80" W=57.86 LB',
     4 //' SECTION 5 - SPAR TRANSITION PORTION  L=5.20"  W=52.63 LB',
     5 //' SECTION 6 - SPAR TAPERED PORTION  L=103.80"  W=795.96 LB',
     6 //' SECTION 7 - SPAR TRANSITION PORTION  L=3.40"  W=46.13 LB',
     7 //' SECTION 8 - SPAR TIP FLANGE  L=1.80"  W=121.56 LB' )
        WRITE(*,20)
20      FORMAT(/,4X, ' THE MAXIMUM TOTAL NUMBER OF DIVISIONS YOU ' ,
     1 /,4X, ' ARE ALLOWED IN ALL 8 SECTIONS IS 699. ' )
        E=30000000.0
        PI=3.141592654
        NS=1
        NF=0
30      GO TO(40,60,80,100,200,300,400,500,600)NS
```

```
C
C       SECTION 1 PARAMETERS
C
40      WRITE(*,50)
50      FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 1 : ',$)
        READ( *,120)ND
        WC=48.3268
        XL=0.
        XR=2.25
        DO=21.63
        DI=15.825
        GO TO 520
C
C       SECTION 2 PARAMETERS
C
60      WRITE(*,70)
70      FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 2 : ',$)
        READ( *,120)ND
        WC=15.1242
        XR=16.0
C
C       CALCULATIONS FOR SECTION 2
C
        DN=ND
        DX=(XR-XL)/DN
        XR=XL+DX
        NB=NF+1
        NF=NF+ND
        SEI=E*2106.268
        DO 75 N=NB,NF
        WT(N)=WC*DX
        X(N)=.50*(XR+XL)
        EI(N)=SEI
        XL=XR
75      XR=XL+DX
        NS=NS+1
        GO TO 30
C
C       SECTION 3 PARAMETERS
C
80      WRITE(*,90)
90      FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 3 : ',$)
        READ( *,120)ND
        WC=47.3021
        XR=17.75
        DO=21.63
        DI=15.97
        GO TO 520
C
C       SECTION 4 PARAMETERS
C
100     WRITE(*,110)
110     FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 4 : ',$)
        READ( *,120)ND
120     FORMAT(I5)
        WC=32.14160448
        XR=19.55
        DO=20.0
```

```
      DI=15.981
      GO TO 520
C
C     SECTION 5 PARAMETERS
C
200   WRITE(*,210)
210   FORMAT(/, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 5 : ',$)
      READ( *,120)ND
      M=-.139807692
      B=20.46024038
      DI=16.0
      CO=.283*PI/4.
      C1=CO*M**2
      C2=CO*2.*M*B
      C3=CO*(B**2-DI**2)
      XR=24.75
      GO TO 420
C
C     SECTION 6 PARAMETERS
C
300   WRITE(*,310)
310   FORMAT(/, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 6 : ',$)
      READ( *,120)ND
      M=.014450867
      B=16.64234105
      T=.50
      M1=.283*PI*T*M
      B1=.283*PI*T*(B-T)
      XR=128.55
C
C     CALCULATIONS FOR SECTION 6
C
      DN=ND
      DX=(XR-XL)/DN
      XR=XL+DX
      NB=NF+1
      NF=NF+ND
      DO 320 N=NB,NF
      WT(N)=(XR**2-XL**2)*M1/2.0+B1*(XR-XL)
      X(N)=((XR**3-XL**3)*M1/3.0+(XR**2-XL**2)*B1/2.0)/WT(N)
      DO=M*X(N)+B
      EI(N)=(DO**3-3.0*T*DO**2+4.0*DO*T**2-2.0*T**3)*PI*T*E/8.0
      XL=XR
320   XR=XL+DX
      NS=NS+1
      GO TO 30
C
C     SECTION 7 PARAMETERS
C
400   WRITE(*,410)
410   FORMAT(/, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 7 : ',$)
      READ( *,120)ND
      M=.388823530
      B=-31.48326478
      DI=17.5
      C1=CO*M**2
      C2=CO*2.*M*B
      C3=CO*(B**2-DI**2)
```

```
      XR=131.95
C
C     CALCULATIONS FOR SECTIONS 5 OR 7
C
420   DN=ND
      DX=(XR-XL)/DN
      XR=XL+DX
      NB=NF+1
      NF=NF+ND
      DO 430 N=NB,NF
      WT(N)=(XR**3-XL**3)*C1/3.0+(XR**2-XL**2)*C2/2.0+C3*(XR-XL)
      X(N)=((XR**4-XL**4)*C1/4.0+(XR**3-XL**3)*C2/3.0
     1 +(XR**2-XL**2)*C3/2.0)/WT(N)
      DO=M*X(N)+B
      EI(N)=(E*PI*(DO**4-DI**4))/64.0
      XL=XR
430   XR=XL+DX
      NS=NS+1
      GO TO 30
C
C     SECTION 8 PARAMETERS
C
500   WRITE(*,510)
510   FORMAT(/, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 8 : ',$)
      READ( *,120)ND
      WC=67.53381198
      XR=133.75
      DO=24.7
      DI=17.5
C
C     CALCULATIONS FOR SECTIONS 1, 3, 4 OR 8
C
520   DN=ND
      DX=(XR-XL)/DN
      XR=XL+DX
      NB=NF+1
      NF=NF+ND
      SEI=(E*PI*(DO**4-DI**4))/64.0
      DO 530 N=NB,NF
      WT(N)=WC*(XR-XL)
      X(N)=.50*(XR+XL)
      EI(N)=SEI
      XL=XR
530   XR=XL+DX
      NS=NS+1
      GO TO 30
C
C     ACCOUNT FOR THE LUMPED COUNTERWEIGHT AT THE FREE END OF THE
C     SPAR EXTENSION AND CALCULATE EI AT THE BASE OR FIXED END OF
C     THE SPOOL PIECE (X=0.0).
600   DO=21.63
      DI=15.825
      EIO=(E*PI*(DO**4-DI**4))/64.0
```

```
      WRITE(*,610)
610   FORMAT(/, ' ENTER THE LUMPED WEIGHT OF THE COUNTERWEIGHT AT THE ',
     1 /, ' FREE END OF THE TAPERED SPAR EXTENSION (LBS): ',$)
      READ(*,*)CW
      WRITE(*,630)
630   FORMAT(/, ' ENTER A TITLE STATEMENT (60 CHAR. MAX.) : ',$)
      READ(*,640)TITLE
640   FORMAT(15A4)
C     IF NO LUMPED COUNTERWEIGHT EXISTS, OUTPUT THE DATA FOR THE TAPEREI
C     STEEL SPAR MODEL.  IF A LUMPED COUNTERWEIGHT EXISTS, ADD ONE MORE
C     LUMPED WEIGHT TO THE MODEL AT THE CG OF THE COUNTERWEIGHT, THEN
C     ADD AN ADDITIONAL BEAM SEGMENT TO THE MODEL.
C
      IF(CW.EQ.0.0)GO TO 700
C
C     ACCOUNT FOR THE COUNTERWEIGHT AT THE END OF THE SPAR EXTENSION
C
      NF=NF+1
      WT(NF)=CW
      A=19.34
      B=14.50
      EI(NF)=(E*PI*A*B**3)/4.0
      X(NF)=133.75+9.8029
C
C     OUTPUT RESULTS TO VIBIN DATA FILE
C
700   WRITE(1,710)TITLE
710   FORMAT(6X,15A4)
      NTOT=NF
      WRITE(1,720)NTOT
720   FORMAT(1X,I5)
      L(1)=X(1)
      DO 730 N=2,NTOT
730   L(N)=X(N)-X(N-1)
      WRITE(1,740)(L(N),EI(N),WT(N),N=1,NTOT)
740   FORMAT(1X,F10.6,F20.1,7X,F15.6)
      WRITE(1,750)EIO
750   FORMAT(1X,F20.1)
      WRITE(1,760)RRHUB
760   FORMAT(1X,F20.3)
C
C     CHECK THE TOTAL WEIGHT OF THE DISCRETE MODEL AND THE LOCATION
C     OF THE CG OF THE MODEL WITH RESPECT TO THE BASE FLANGE OF THE
C     SPOOL PIECE (X=0).
C
      WTOT=0.0
      WX=0.0
      DO 800 N=1,NTOT
      WTOT=WTOT+WT(N)
800   WX=WX+WT(N)*X(N)
      XB=WX/WTOT
```

```
      WRITE(*,810)
810   FORMAT(/,2X,12HTOTAL WEIGHT,5X,2HCG)
      WRITE(*,820)
820   FORMAT(5X,5H(LBS),7X,6H(INCH))
      WRITE(*,830)WTOT,XB
830   FORMAT(3X,F9.3,3X,F8.3)
      WRITE(*,840)
840   FORMAT(//,2X,36HTOTAL MOMENT ABOUT CENTERLINE OF HUB)
      WRITE(*,850)
850   FORMAT(15X,10H(LBS-INCH))
      WRITE(*,860)WTOT*(XB+22.0)
860   FORMAT(15X,F10.1)
      STOP
      END
```

```
$NOFLOATCALLS
$STORAGE:2
C
C     A.3  CWTCHRD - counterweight assembly model generation code
C
C       THIS PROGRAM GENERATES THE INPUT DATA FOR THE SPOOL
C       PIECE (CF 764254), THE TAPERED STEEL SPAR EXTENSION
C       (CF 760549) AND THE COUNTERWEIGHT (CF 764554) ASSEMBLY FOR
C       CHORDWISE VIBRATION (VIBRATION IN THE PLANE OF
C       ROTATION).  THE PROGRAM CONSIDERS THE TAPERED SPAR EXTENSION
C       TO BE BROKEN UP INTO 8 SECTIONS AS FOLLOWS :
C
C
C               SECTION 1 - THE FLANGE AT THE BASE OF THE SPOOL PIECE
C                           OF LENGTH 2.25" WEIGHING APPROX. 108.74 LBS.
C
C               SECTION 2 - THE WEBBED MAIN BODY OF THE SPOOL PIECE
C                           OF LENGTH 13.75" WEIGHING APPROX. 207.96 LBS.
C
C               SECTION 3 - THE FLANGE AT THE SPAR END OF THE SPOOL PIECE
C                           OF LENGTH 1.75" WEIGHING APPROX. 82.78 LBS.
C
C               SECTION 4 - THE FLANGE AT THE BASE OF THE SPAR EXTENSION
C                           OF LENGTH 1.80" WEIGHING APPROX. 57.86 LBS.
C
C               SECTION 5 - THE TRANSITION SECTION BETWEEN THE BASE
C                           FLANGE AND THE TAPERED PORTION OF THE SPAR.
C                           THIS SECTION IS 5.20" IN LENGTH AND WEIGHS
C                           56.63 LBS.
C
C               SECTION 6 - THE TAPERED PORTION OF THE SPAR EXTENSION
C                           OF LENGTH 103.80" WEIGHING APPROX.
C                           795.96 LBS.
C
C               SECTION 7 - THE TRANSITION SECTION BETWEEN THE TAPERED
C                           PORTION OF THE SPAR EXTENSION AND THE FLANGE
C                           AT THE TIP OF THE SPAR.  THIS SECTION IS
C                           3.40" IN LENGTH AND WEIGHS 46.13 LBS.
C
C               SECTION 8 - THE FLANGE AT THE TIP OF THE SPAR EXTENSION.
C                           THIS SECTION IS 1.80" IN LENGTH AND WEIGHS
C                           121.56 LBS.
C
C
C
C       CW      THE LUMPED WEIGHT OF THE COUNTERWEIGHT AT THE FREE END OF
C               THE STEEL SPAR EXTENSION  (LBS)
C
C       EI(N)   THE PRODUCT OF MODULUS OF ELASTICITY AND AREA MOMENT
C               OF INERTIA AT MASS N  (LBS*INCH*INCH)
C
C       EIO     THE PRODUCT OF MODULUS OF ELASTICITY AND AREA MOMENT
C               OF INERTIA AT ZERO OR THE FIXED END  (LBS*INCH*INCH)
C
C       L(N)    THE LENGTH OF SECTION TO LEFT OF MASS N  (INCHES)
C
```

```
C
C       NB       THE MASS NUMBER AT THE BEGINNING OF A SECTION
C
C       NF       THE MASS NUMBER AT THE END OF A SECTION
C
C       ND       THE NUMBER OF DIVISIONS IN A PARTICULAR SECTION
C
C       NS       THE NUMBER OF THE SECTION UNDER CONSIDERATION
C
C       NTOT     THE TOTAL NUMBER OF MASSES
C
C       RRHUB    THE RADIUS OF THE RIGID HUB (INCHES)
C
C       TITLE    THE PROBLEM TITLE STATEMENT
C
C       WT(N)    THE WEIGHT OF MASS N  (LBS)
C
C       X(N)     THE AXIAL LOCATION OF MASS N  (INCH)
C
C
C
        IMPLICIT REAL*8(A-H,O-Z)
        REAL*8 L,M,M1
        DIMENSION EI(700),L(700),TITLE(15),WT(700),X(700)
C
C       ASSIGN LOGICAL UNIT 1 TO 'VIBIN.DAT'
C
        OPEN(1,FILE ='VIBIN.DAT',STATUS='NEW')
C
C       READ DATA FROM TERMINAL
C
        WRITE(*,5)
5       FORMAT(//,2X, ' THIS PROGRAM GENERATES THE DATA FOR THE ',
     1  'CHORDWISE ',/,2X, ' VIBRATION OF THE COUNTERWEIGHT ASSEMBLY ',
     2  ///,2X,' ENTER THE RADIUS OF THE RIGID HUB (INCHES) : ',$)
        READ(*,*)RRHUB
        WRITE(*,10)
10      FORMAT(//2X,' SECTION 1-SPOOL PIECE FLANGE L=2.25" W=108.74 LB',
     1 //2X,' SECTION 2-SPOOL PIECE BODY L=13.75" W=207.96 LB',
     2 //2X,' SECTION 3-SPOOL PIECE FLANGE L=1.75" W=82.78 LB',
     3 //2X,' SECTION 4-SPAR BASE FLANGE L=1.80" W=57.86 LB',
     4 //2X,' SECTION 5-SPAR TRANSITION PORTION  L=5.20"  W=52.63 LB',
     5 //2X,' SECTION 6-SPAR TAPERED PORTION  L=103.80"  W=795.96 LB',
     6 //2X,' SECTION 7-SPAR TRANSITION PORTION  L=3.40"  W=46.13 LB',
     7  //2X,' SECTION 8-SPAR TIP FLANGE L=1.80" W=121.56 LB')
        WRITE(*,20)
20      FORMAT(/,4X, ' THE MAXIMUM TOTAL NUMBER OF DIVISIONS YOU ' ,
     1 /,4X, ' ARE ALLOWED IN ALL 8 SECTIONS IS 699. ' )
        E=30000000.0
        PI=3.141592654
        NS=1
        NF=0
30      GO TO(40,60,80,100,200,300,400,500,600)NS
```

```
C
C       SECTION 1 PARAMETERS
C
40      WRITE(*,50)
50      FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 1 : ',$)
        READ(*,120)ND
        WC=48.3268
        XL=0.
        XR=2.25
        DO=21.63
        DI=15.825
        GO TO 520
C
C       SECTION 2 PARAMETERS
C
60      WRITE(*,70)
70      FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 2 : ',$)
        READ(*,120)ND
        WC=15.1242
        XR=16.0
C
C       CALCULATIONS FOR SECTION 2
C
        DN=ND
        DX=(XR-XL)/DN
        XR=XL+DX
        NB=NF+1
        NF=NF+ND
        SEI=E*2106.268
        DO 75 N=NB,NF
        WT(N)=WC*DX
        X(N)=.50*(XR+XL)
        EI(N)=SEI
        XL=XR
75      XR=XL+DX
        NS=NS+1
        GO TO 30
C
C       SECTION 3 PARAMETERS
C
80      WRITE(*,90)
90      FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 3 : ',$)
        READ(*,120)ND
        WC=47.3021
        XR=17.75
        DO=21.63
        DI=15.97
        GO TO 520
C
C       SECTION 4 PARAMETERS
C
100     WRITE(*,110)
110     FORMAT(//, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 4 : ',$)
        READ(*,120)ND
```

```
120     FORMAT(I5)
        WC=32.14160448
        XR=19.55
        DO=20.0
        DI=15.981
        GO TO 520
C
C       SECTION 5 PARAMETERS
C
200     WRITE(*,210)
210     FORMAT(/, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 5 : ',$)
        READ(*,120)ND
        M=-.139807692
        B=20.46024038
        DI=16.0
        C0=.283*PI/4.
        C1=C0*M**2
        C2=C0*2.*M*B
        C3=C0*(B**2-DI**2)
        XR=24.75
        GO TO 420
C
C       SECTION 6 PARAMETERS
C
300     WRITE(*,310)
310     FORMAT(/, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 6 : ',$)
        READ(*,120)ND
        M=.014450867
        B=16.64234105
        T=.50
        M1=.283*PI*T*M
        B1=.283*PI*T*(B-T)
        XR=128.55
C
C       CALCULATIONS FOR SECTION 6
C
        DN=ND
        DX=(XR-XL)/DN
        XR=XL+DX
        NB=NF+1
        NF=NF+ND
        DO 320 N=NB,NF
        WT(N)=(XR**2-XL**2)*M1/2.0+B1*(XR-XL)
        X(N)=((XR**3-XL**3)*M1/3.0+(XR**2-XL**2)*B1/2.0)/WT(N)
        DO=M*X(N)+B
        EI(N)=(DO**3-3.0*T*DO**2+4.0*DO*T**2-2.0*T**3)*PI*T*E/8.0
        XL=XR
320     XR=XL+DX
        NS=NS+1
        GO TO 30
C
C       SECTION 7 PARAMETERS
C
400     WRITE(*,410)
410     FORMAT(/, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 7 : ',$)
        READ(*,120)ND
```

```
      M=.388823530
      B=-31.48326478
      DI=17.5
      C1=CO*M**2
      C2=CO*2.*M*B
      C3=CO*(B**2-DI**2)
      XR=131.95
C
C     CALCULATIONS FOR SECTIONS 5 OR 7
C
420   DN=ND
      DX=(XR-XL)/DN
      XR=XL+DX
      NB=NF+1
      NF=NF+ND
      DO 430 N=NB,NF
      WT(N)=(XR**3-XL**3)*C1/3.0+(XR**2-XL**2)*C2/2.0+C3*(XR-XL)
      X(N)=((XR**4-XL**4)*C1/4.0+(XR**3-XL**3)*C2/3.0
     1 +(XR**2-XL**2)*C3/2.0)/WT(N)
      DO=M*X(N)+B
      EI(N)=(E*PI*(DO**4-DI**4))/64.0
      XL=XR
430   XR=XL+DX
      NS=NS+1
      GO TO 30
C
C     SECTION 8 PARAMETERS
C
500   WRITE(*,510)
510   FORMAT(/, ' ENTER THE NUMBER OF DIVISIONS FOR SECTION 8 : ',$)
      READ(*,120)ND
      WC=67.53381198
      XR=133.75
      DO=24.7
      DI=17.5
C
C     CALCULATIONS FOR SECTIONS 1, 3, 4 OR 8
C
520   DN=ND
      DX=(XR-XL)/DN
      XR=XL+DX
      NB=NF+1
      NF=NF+ND
      SEI=(E*PI*(DO**4-DI**4))/64.0
      DO 530 N=NB,NF
      WT(N)=WC*(XR-XL)
      X(N)=.50*(XR+XL)
      EI(N)=SEI
      XL=XR
530   XR=XL+DX
      NS=NS+1
      GO TO 30
C
C     ACCOUNT FOR THE LUMPED COUNTERWEIGHT AT THE FREE END OF THE
C     SPAR EXTENSION AND CALCULATE EI AT THE BASE OR FIXED END OF
C     THE SPOOL PIECE (X=0.0).
```

```
600     DO=21.63
        DI=15.825
        EIO=(E*PI*(DO**4-DI**4))/64.0
        WRITE(*,610)
610     FORMAT(/, ' ENTER THE LUMPED WEIGHT OF THE COUNTERWEIGHT AT THE ',
     1 /, ' FREE END OF THE TAPERED SPAR EXTENSION (LBS): ',$)
        READ(*,*)CW
        WRITE(*,630)
630     FORMAT(/, ' ENTER A TITLE STATEMENT (60 CHAR. MAX.) : ',$)
        READ(*,640)TITLE
640     FORMAT(15A4)
C       IF NO LUMPED COUNTERWEIGHT EXISTS, OUTPUT THE DATA FOR THE TAPEREI
C       STEEL SPAR MODEL.  IF A LUMPED COUNTERWEIGHT EXISTS. ADD ONE MORE
C       LUMPED WEIGHT TO THE MODEL AT THE CG OF THE COUNTERWEIGHT, THEN
C       ADD AN ADDITIONAL BEAM SEGMENT TO THE MODEL.
C
        IF(CW.EQ.0.0)GO TO 700
C
C       ACCOUNT FOR THE COUNTERWEIGHT AT THE END OF THE SPAR EXTENSION
C
        NF=NF+1
        WT(NF)=CW
        A=19.34
        B=14.50
        EI(NF)=(E*PI*B*A**3)/4.0
        X(NF)=133.75+9.8029
C
C       OUTPUT RESULTS TO VIBIN DATA FILE
C
700     WRITE(1,710)TITLE
710     FORMAT(6X,15A4)
        NTOT=NF
        WRITE(1,720)NTOT
720     FORMAT(1X,I5)
        L(1)=X(1)
        DO 730 N=2,NTOT
730     L(N)=X(N)-X(N-1)
        WRITE(1,740)(L(N),EI(N),WT(N),N=1,NTOT)
740     FORMAT(1X,F10.6,F20.1,7X,F15.6)
        WRITE(1,750)EIO
750     FORMAT(1X,F20.1)
        WRITE(1,760)RRHUB
760     FORMAT(1X,F20.3)
C
C       CHECK THE TOTAL WEIGHT OF THE DISCRETE MODEL AND THE LOCATION
C       OF THE CG OF THE MODEL WITH RESPECT TO THE BASE FLANGE OF THE
C       SPOOL PIECE (X=0).
C
```

```
      WTOT=0.0
      WX=0.0
      DO 800 N=1,NTOT
      WTOT=WTOT+WT(N)
800   WX=WX+WT(N)*X(N)
      XB=WX/WTOT
      WRITE(*,810)
810   FORMAT(/,2X,12HTOTAL WEIGHT,5X,2HCG)
      WRITE(*,820)
820   FORMAT(5X,5H(LBS),7X,6H(INCH))
      WRITE(*,830)WTOT,XB
830   FORMAT(3X,F9.3,3X,F8.3)
      WRITE(*,840)
840   FORMAT(//,2X,36HTOTAL MOMENT ABOUT CENTERLINE OF HUB)
      WRITE(*,850)
850   FORMAT(15X,10H(LBS-INSH))
      WRITE(*,860)WTOT*(XB+22.0)
860   FORMAT(15X,F10.1)
      STOP
      END
```

```
$nofloatcalls
$storage:2
C
C                          A.4  BEAM - main analysis code
C
C
C       THIS IS THE MAIN PROGRAM USED FOR THE DETERMINATION OF THE NATURAL
C       FREQUENCIES AND MODE SHAPES FOR THE LATERAL VIBRATION OF A CANTILEVERED
C       BEAM. THE PROGRAM IS BASED ON A LUMPED MASS APPROXIMATION OF A
C       CONTINUOUS SYSTEM.  THE LUMPED MASSES IN THE DISCRETE MODEL OF THE BEAM
C       ARE CONNECTED BY MASSLESS, BUT FLEXIBLE, BEAM SEGMENTS.  THE FLEXURAL
C       RIGIDITY OF THE BEAM, EI, CAN EITHER BE UNIFORM OR IT CAN VARY
C       ALONG THE LENGTH OF THE BEAM.  IF THE FLEXURAL RIGIDITY VARIES ALONG
C       THE LENGTH OF THE BEAM, A LINEAR VARIATION IN EI IS ASSUMED IN
C       EACH BEAM SEGMENT BETWEEN TWO SUCCESSIVE MASSES.   THE EFFECTS OF
C       ROTATION ON THE VIBRATORY CHARACTERISTICS OF THE BEAM CAN ALSO BE
C       DETERMINED FOR FLAPWISE VIBRATION USING THIS PROGRAM.
C
C
C
C
C
C       EI(N)   THE PRODUCT OF MODULUS OF ELASTICITY AND AREA MOMENT
C               OF INERTIA AT MASS N  (LBS*INCH*INCH)
C
C       EIO     THE PRODUCT OF MODULUS OF ELASTICITY AND AREA MOMENT
C               OF INERTIA AT ZERO OR THE FIXED END  (LBS*INCH*INCH)
C
C       ISPEED  AN INDEX CORRESPONDING TO THE PARTICULAR ROTOR
C               SPEED UNDER INVESTIGATION
C
C       L(N)    THE LENGTH OF SECTION TO LEFT OF MASS N  (INCHES)
C
C       NPLACE  THE NUMBER OF DIGITS REQUIRED TO THE RIGHT OF THE
C               DECIMAL POINT
C
C       NRPM    THE TOTAL NUMBER OF ROTATIONAL SPEEDS CONSIDERED
C
C       NTOT    THE TOTAL NUMBER OF MASSES
C
C       NW      THE NUMBER OF NATURAL FREQUENCIES IN THE RANGE FROM
C               WD TO WL
C
C       RPM(N)  THE ROTOR ROTATIONAL SPEED  (REV/MIN)
C
C       RRHUB   THE RADIUS OF THE RIGID HUB  (INCHES)
C
C       TIME    THE ELAPSED TIME DURING EXECUTION (SEC)
C
C       TITLE   THE PROBLEM TITLE STATEMENT
C
C       W(I)    THE NATURAL FREQUENCIES OF VIBRATION  (RAD/SEC)
C
C       WD      THE INITIAL DELTA USED IN SEARCHING FOR NATURAL
C               FREQUENCIES  (RAD/SEC)
```

```
C
C       WF      THE FIRST VALUE TO BE CONSIDERED AS A POSSIBLE NATURAL
C               FREQUENCY  (RAD/SEC)
C
C       WL      THE LAST VALUE TO BE CONSIDERED AS A POSSIBLE NATURAL
C               FREQUENCY  (RAD/SEC)
C
C       WT(N)   THE WEIGHT OF MASS N  (LBS)
C
C
C
C
        IMPLICIT REAL*8(A-H,O-Z)
        REAL*8 L
        CHARACTER*1 KEY
        COMMON EI(700),EIO,FL(700),L(700),NPLACE,NRPM,NTOT,NW,RPM(10)
     1  ,RRHUB,TITLE(15),W(100),WD,WF,WL,WT(700),KEY
C
C       READ INITIAL DATA FROM TERMINAL AND INPUT FILE
C
        CALL INPUT
C
C       CONSIDER EACH OF THE ROTATIONAL SPEED INDIVIDUALLY
C
        DO 10 ISPEED=1,NRPM
C
C               CALCULATE NATURAL FREQUENCIES
C
                CALL NFREQ(ISPEED)
C
C               OUTPUT NATURAL FREQUENCY RESULTS
C
                CALL OUTPUT(ISPEED)
C
C               CALCULATE AND OUTPUT MODE SHAPES IF DESIRED
C
                CALL MODES(ISPEED)
C
10      CONTINUE
C
C       TERMINATE
C
        STOP
        END
```

```
$nofloatcalls       A.5  DET - subroutine  to evaluate determinant
$storage:2
        REAL*8 FUNCTION DET(WW)
C
C       THIS FUNCTION EVALUATES THE DETERMINANT WHICH REFLECTS THE
C       BOUNDARY CONDITIONS OF THE PROBLEM.  WHEN THIS DETERMINANT
C       IS ZERO, THE CORRESPONDING FREQUENCY IS A NATURAL FREQUENCY
C       OF THE SYSTEM.
C
C       A(I,J)    THE MATRIX WHICH RELATES THE DEFLECTION, SLOPE,
C                 MOMENT AND SHEAR AT THE N-1 MASS TO THOSE QUANTITIES
C                 AT MASS N
C
C       NTOT      THE TOTAL NUMBER OF MASSES
C
C       U(I,J)    THE CONCATENATION OF [A] MATRICIES RELATING DEFLECTION,
C                 SLOPE, MOMENT AND SHEAR AT THE FIXED END TO THOSE
C                 QUANTITIES AT THE FREE END
C
C       UNEW(I,J) SAME AS U(I,J)
C
C       WW        THE FREQUENCY AT WHICH THE DETERMINANT IS TO BE COMPUTED
C
        IMPLICIT REAL*8(A-H,O-Z)
        REAL*8 L
        CHARACTER*1 KEY
        COMMON EI(700),EIO,FL(700),L(700),NPLACE,NRPM,NTOT,NW,RPM(10)
     1  ,RRHUB,TITLE(15),W(100),WD,WF,WL,WT(700),KEY
        REAL*8 A(4,4),U(4,4),UNEW(4,4)
C
C       INITIALIZE [U]  (SET IT EQUAL TO THE IDENTITY MATRIX)
C
        DO 10 I=1,4
        DO 10 J=1,4
        U(I,J)=0.
        IF(I.EQ.J) U(I,J)=1.
10      CONTINUE
C
C       COMPUTE [U] = THE CONCATENATION OF THE [A] MATRICES FOR ALL MASSES
C
        DO 20 N=1,NTOT
C       FILL [A] FOR THE MASS N
        CALL FILLA(A,N,WW)
C       CONCATENATE [A] WITH THE EXISTING [U]
        CALL MULT(A,4,4,4,4,U,4,4,4,UNEW,4,4)
C       SET [U] = [UNEW]
        DO 15 I=1,4
        DO 15 J=1,4
15      U(I,J)=UNEW(I,J)
20      CONTINUE
C
C       COMPUTE DETERMINANT WHICH REFLECTS BOUNDARY CONDITIONS
C
        DET=U(3,3)*U(4,4)-U(3,4)*U(4,3)
        RETURN
        END
```

```
$nofloatcalls      A.6  FILLA - subroutine to fill the A matrix
$storage:2

      SUBROUTINE FILLA(A,N,WW)
C
C
C     THIS SUBROUTINE FILLS THE MATRIX A(I,J).  THIS MATRIX IS
C     THE MATRIX WHICH RELATES THE DEFLECTION, SLOPE, MOMENT AND
C     SHEAR AT THE N-1 MASS TO THOSE QUANTITIES AT MASS N.
C
C
C     A(I,J)  THE MATRIX WHICH RELATES THE DEFLECTION, SLOPE, MOMENT
C             AND SHEAR AT THE N-1 MASS TO THOSE QUANTITIES AT MASS N
C
C     AM      THE SLOPE AT THE FREE END OF A CANTILEVERED BEAM
C             DUE TO A UNIT MOMENT APPLIED AT THE FREE END  (RAD/LBS-IN)
C
C     AS      THE SLOPE AT THE FREE END OF A CANTILEVERED BEAM
C             DUE TO A UNIT FORCE APPLIED AT THE FREE END  (RAD/LBS)
C
C     DM      THE DEFLECTION AT THE FREE END OF A CANTILEVERED BEAM
C             DUE TO A UNIT MOMENT APPLIED AT THE FREE END  (1/LBS)
C
C     DS      THE DEFLECTION AT THE FREE END OF A CANTILEVERED BEAM
C             DUE TO A UNIT FORCE APPLIED AT THE FREE END  (INCH/LBS)
C
C     EIAVE   THE AVERAGE VALUE OF EI AT MASS N, EI(N), AND EI AT
C             MASS N-1, EI(N-1)  (LBS*INCH*INCH)
C
C     EIN     THE VALUE OF THE FLEXURAL RIGIDITY AT MASS N, EI(N)
C             (LBS*INCH*INCH)
C
C     EINM1   THE VALUE OF THE FLEXURAL RIGIDITY AT MASS N-1, EI(N-1)
C             (LBS*INCH*INCH)
C
C     EIO     THE VALUE OF THE FLEXURAL RIGIDITY AT ZERO OR THE FIXED
C
C     LN      THE DISTANCE BETWEEN MASS N-1 AND MASS N  (INCHES)
C
C     MASSN   THE MASS OF THE N TH LUMPED MASS  (LBS-SEC*SEC/IN)
C
C     RPM     THE ROTOR ROTATIONAL SPEED  (REV/MIN)
C
C     RRHUB   THE RADIUS OF THE RIGID HUB  (INCHES)
C
C     WW      THE CURRENT FREQUENCY AT WHICH THE DETERMINANT IS TO BE
C             COMPUTED  (RAD/SEC)
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 A(4,4),LN,MASSN,L
      CHARACTER*1 KEY
      COMMON EI(700),EIO,FL(700),L(700),NPLACE,NRPM,NTOT,NW,RPM(10)
     1 ,RRHUB,TITLE(15),W(100),WD,WF,WL,WT(700),KEY
      IF(N.GT.1) GO TO 10
      EINM1=EIO
      A(1,1)=1.0
```

```
      A(2,1)=0.0
      A(3,1)=0.0
      GO TO 20
10    EINM1=EI(N-1)
20    EIN=EI(N)
      MASSN=WT(N)/386.4
      LN=L(N)
      EIAVE=(EIN+EINM1)/2.0
C
C     IF THE VALUE OF EI(N)/EI(N-1) IS BETWEEN .9999 AND 1.00009 THEN
C     THE BEAM IS CONSIDERED TO BE UNIFORM BETWEEN THE TWO MASSES WITH
C     AN AVERAGE VALUE OF EI.  IF NOT, A LINEAR VARIATION IN EI BETWEEN
C     THE TWO MASSES IS ASSUMED.
C
      R=EIN/EINM1
      IF(R.GT..9999.AND.R.LT.1.00009)GO TO 30
C
C     ASSUMING A LINEAR VARIATION IN EI BETWEEN EI(N) AND EI(N-1)
C
      DEI=EIN-EINM1
      REI=EIN/EINM1
      DS=((LN**3)/DEI)*((1.0+2.0*EINM1/DEI+(EINM1/DEI)**2)*
     1 DLOG(REI)-1.5-EINM1/DEI)
      DM=((LN**2)/DEI)*((EIN/DEI)*DLOG(REI)-1.0)
      AM=(LN/DEI)*DLOG(REI)
      GO TO 40
C
C     ASSUMING A UNIFORM VALUE OF EI BETWEEN MASSES
C
30    DS=(LN**3)/(3.0*EIAVE)
      DM=(LN**2)/(2.0*EIAVE)
      AM=LN/EIAVE
C
40    AS=DM
      DEN=1.+(AS-DM)*FL(N)+(AM*DS-AS*DM)*FL(N)**2
      A(4,1)=WW**2*MASSN
      A(1,2)=(LN-(DS-LN*AS)*FL(N))/DEN
      A(2,2)=(1.-(DM-LN*AM)*FL(N))/DEN
      A(3,2)=A(1,2)*FL(N)
      A(4,2)=WW**2*MASSN*A(1,2)
      A(1,3)=(DM-(AM*DS-AS*DM)*FL(N))/DEN
      A(2,3)=AM/DEN
      A(3,3)=(1.+AS*FL(N))/DEN
      A(4,3)=WW**2*MASSN*A(1,3)
      A(1,4)=(LN*DM-DS-LN*(AM*DS-AS*DM)*FL(N))/DEN
      A(2,4)=(LN*AM-AS-(AM*DS-AS*DM)*FL(N))/DEN
      A(3,4)=A(1,2)
      A(4,4)=WW**2*MASSN*A(1,4)+1.
      RETURN
      END
```

```
$nofloatcalls   A.7  FORCE - subroutine to calculate centrifugal forces
$storage:2

      SUBROUTINE FORCE(ISPEED)
C
C
C     THIS SUBROUTINE CALCULATES THE CENTRIFUGAL FORCE ACTING TO THE
C     LEFT OF EACH MASS.
C
C
C     FL(N)            THE CENTRIFUGAL FORCE ACTING TO THE LEFT OF
C                      THE N TH MASS
C
C     ISPEED           AN INDEX CORRESPONDING TO THE PARTICULAR ROTOR
C                      SPEED UNDER INVESTIGATION
C
C     L(N)             THE DISTANCE BETWEEN MASS N-1 AND MASS N  (INCHES)
C
C     NTOT             THE TOTAL NUMBER OF MASSES
C
C     OMEGA            THE ROTOR ROTATIONAL SPEED  (RAD/SEC)
C
C     RPM(ISPEED)      THE CURRENT ROTOR ROTATIONAL SPEED  (REV/MIN)
C
C     RRHUB            THE RADIUS OF THE RIGID HUB  (INCHES)
C
C     WT(N)            THE WEIGHT OF MASS N  (LBS)
C
C     XI               THE DISTANCE FROM THE CENTERLINE OF THE ROTOR
C                      TO THE N TH MASS  (INCHES)
C
C
C
C
C     COMPUTE THE LENGTH TO THE LAST MASS
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 L
      CHARACTER*1 KEY
      COMMON EI(700),EIO,FL(700),L(700),NPLACE,NRPM,NTOT,NW,RPM(10)
     1 ,RRHUB,TITLE(15),W(100),WD,WF,WL,WT(700),KEY
C
      XN=RRHUB
      DO 10 I=1,NTOT
10    XN=XN+L(I)
C
C     COMPUTE THE INERTIAL FORCES BEGINNING AT THE LAST MASS AND
C     PROCEEDING TO MASS 1
C
C
      OMEGA=RPM(ISPEED)*3.14159265/30.
      XI=XN
      FL(NTOT)=WT(NTOT)/386.4*XN*OMEGA**2
      DO 20 I=2,NTOT
      J=NTOT+1-I
      XI=XI-L(J+1)
20    FL(J)=FL(J+1)+WT(J)/386.4*XI*OMEGA**2
      RETURN
      END
```

```
$nofloatcalls     A.8  INPUT - subroutine to perform all input functions
$storage:2

      SUBROUTINE INPUT
C
C     THIS SUBROUTINE PERMITS THE USER TO ENTER INITIAL VALUES FROM
C     THE TERMINAL AND ALSO READS BEAM DATA FROM THE FILE
C     'VIBIN.DAT'.
C
C
C
C     EI(N)   THE PRODUCT OF MODULUS OF ELASTICITY AND AREA MOMENT
C             OF INERTIA AT MASS N  (LBS*INCH*INCH)
C
C     EIO     THE PRODUCT OF MODULUS OF ELASTICITY AND AREA MOMENT
C             OF INERTIA AT ZERO OR THE FIXED END  (LBS*INCH*INCH)
C
C     KEY     A FLAG WHICH IF "Y" WILL CAUSE THE MODE SHAPES TO BE
C             CALCULATED
C
C     L(N)    THE LENGTH OF SECTION TO LEFT OF MASS N  (INCHES)
C
C     NPLACE  THE NUMBER OF DIGITS REQUIRED TO THE RIGHT OF THE
C             DECIMAL POINT
C
C     NRPM    THE TOTAL NUMBER OF ROTATIONAL SPEEDS CONSIDERED
C
C     NTOT    THE TOTAL NUMBER OF MASSES
C
C     RPM(N)  THE ROTOR ROTATIONAL SPEED  (REV/MIN)
C
C     RRHUB   THE RADIUS OF THE RIGID HUB  (INCHES)
C
C     TITLE   THE PROBLEM TITLE STATEMENT
C
C     WD      THE INITIAL DELTA USED IN SEARCHING FOR NATURAL
C             FREQUENCIES  (RAD/SEC)
C
C     WF      THE FIRST VALUE TO BE CONSIDERED AS A POSSIBLE NATURAL
C             FREQUENCY  (RAD/SEC)
C
C     WL      THE LAST VALUE TO BE CONSIDERED AS A POSSIBLE NATURAL
C             FREQUENCY  (RAD/SEC)
C
C     WT(N)   THE WEIGHT OF MASS N  (LBS)
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 L
      CHARACTER*1 KEY,YES
```

```
      COMMON EI(700),EIO,FL(700),L(700),NPLACE,NRPM,NTOT,NW,RPM(10)
     1 ,RRHUB,TITLE(15),W(100),WD,WF,WL,WT(700),KEY
      DATA YES/'Y'/
C
C     ASSIGN LOGICAL UNIT 1 TO 'VIBIN.DAT'
C
      OPEN(1, FILE='VIBIN.DAT', STATUS='OLD')
C
C     ASSIGN LOGICAL UNIT 2 TO 'VIBOUT.DAT'
C
      OPEN(2, FILE='VIBOUT.DAT', STATUS='NEW')
C
C     READ DATA FROM VIBIN.DAT
C
      READ(1,90)TITLE
90    FORMAT(6X,15A4)
      READ(1,100) NTOT
100   FORMAT(1X,I5)
      READ(1,110) (L(N),EI(N),WT(N),N=1,NTOT)
110   FORMAT(1X,F10.6,F20.1,7X,F15.6)
      READ(1,120)EIO
120   FORMAT(1X,F20.1)
      READ(1,130) RRHUB
130   FORMAT(1X,F20.3)
C
C     READ ADDITIONAL DATA FROM TERMINAL KEYBOARD
C
131   WRITE(*,132)
132   FORMAT(/, ' TYPE 1 IF THE ANALYSIS IS FOR CHORDWISE',
     1 ' VIBRATION OR',/,' TYPE 2 IF THE ANALYSIS IS FOR',
     2 ' FOR FLAPWISE VIBRATION : '$)
133   READ( *,*)NA
      IF(NA.LT.1.OR.NA.GT.2)GO TO 131
      IF(NA.EQ.1)GO TO 180
135   WRITE(*,140)
140   FORMAT(/,' TYPE IN THE TOTAL NUMBER OF ROTATIONAL ',
     1 /,' SPEEDS TO BE CONSIDERED (MAX. 10) : '$)
      READ(*,*)NRPM
      IF(NRPM.LT.0.OR.NRPM.GT.10) GO TO 135
      IF(NRPM.EQ.0) GO TO 180
C
C     ENTER ROTATION SPEEDS
C
      DO 160 I=1,NRPM
      WRITE(*,150)I
150   FORMAT(/,' TYPE IN ROTOR SPEED (REV/MIN) NO. ',I2,'  : '$)
      READ(*,*)RPM(I)
160   CONTINUE
      GO TO 190
C
C     NO ROTATIONS REQUESTED
C
180   NRPM=1
      RPM(1)=0.
```

```
190       WRITE(*,200)
200       FORMAT(/,' TYPE IN THE LOWER LIMIT OF THE FREQUENCY RANGE',/,
     1    ' YOU WISH SEARCHED FOR NATURAL FREQUENCIES (RAD/SEC) : '$)
          READ(*,*)WF
          WRITE(*,210)
210       FORMAT(/,' TYPE IN THE UPPER LIMIT OF THE FREQUENCY RANGE',/,
     1    ' YOU WISH SEARCHED FOR NATURAL FREQUENCIES (RAD/SEC) : '$)
          READ(*,*)WL
          WRITE(*,220)
220       FORMAT(/,' TYPE IN THE FREQUENCY INCREMENT TO BE USED IN',/,
     1    ' THE INITIAL SEARCH FOR NATURAL FREQUENCIES (RAD/SEC) : '$)
          READ(*,*)WD
          WRITE(*,230)
230       FORMAT(/,' TYPE IN THE NUMBER OF DIGITS OF ACCURACY TO THE',/,
     1    ' RIGHT OF THE DECIMAL POINT : '$)
          READ(*,*)NPLACE
          WRITE(*,240)
240       FORMAT(//,' DO YOU WISH TO CALCULATE MODE SHAPES (Y/N) ? ',$)
          READ(*,250)KEY
250       FORMAT(A1)
          IF(KEY.NE.YES) GO TO 260
C
C         ASSIGN LOGICAL UNIT 3 TO 'MODES.DAT'
C
          OPEN(3, FILE='MODES.DAT', STATUS='NEW')
260       CLOSE(1)
          RETURN
          END
```

```
$nofloatcalls     A.9  MODES - subroutine to calculate mode shapes
$storage:2
      SUBROUTINE MODES(ISPEED)
C
C     THIS SUBROUTINE CALCULATES THE MODE SHAPES IF DESIRED.
C
C
C
C     A(I,J)   THE MATRIX WHICH RELATES THE DEFLECTION, SLOPE,
C              MOMENT AND SHEAR AT THE N-1 MASS TO THOSE QUANTITIES
C              AT MASS N
C
C     ISPEED   AN INDEX CORRESPONDING TO THE PARTICULAR ROTOR
C              SPEED UNDER INVESTIGATION
C
C     MO       THE BENDING MOMENT AT THE FIXED END ASSUMING A UNIT
C              DISPLACEMENT AT THE FREE END   (INCH-LBS)
C
C     NTOT     THE TOTAL NUMBER OF MASSES
C
C     NW       THE NUMBER OF NATURAL FREQUENCIES IN THE RANGE FROM
C              WF TO WL
C
C     TITLE    THE PROBLEM TITLE STATEMENT
C
C     RPM(N)   THE ROTOR ROTATIONAL SPEED (RPM)
C
C     U(I,J)   THE CONCATENATION OF [A] MATRICES RELATING DEFLECTION,
C              SLOPE, MOMENT AND SHEAR AT THE FIXED END TO THOSE
C              QUANTITIES AT MASS N
C
C     UNEW(I,J)  SAME AS U(I,J)
C
C     VO       THE SHEAR FORCE AT THE FIXED END ASSUMING A UNIT
C              DISPLACEMENT AT THE FREE END   (LBS)
C
C     WF       THE FIRST VALUE TO BE CONSIDERED AS A POSSIBLE NATURAL
C              FREQUENCY   (RAD/SEC)
C
C     WL       THE LAST VALUE TO BE CONSIDERED AS A POSSIBLE NATURAL
C              FREQUENCY   (RAD/SEC)
C
C     W(NF)    THE NATURAL FREQUENCIES OF VIBRATION   (RAD/SEC)
C
C     X(N)     THE AXIAL LOCATION OF MASS N   (INCHES)
C
C     Y(N)     THE NORMALIZED DISPLACEMENT AT MASS N.  THAT IS, THE
C              DISPLACEMENT AT MASS N ASSUMING A UNIT DISPLACEMENT
C              AT THE FREE END OF THE BEAM
C
```

```
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 L
      CHARACTER*1 KEY,YES
      COMMON EI(700),EIO,FL(700),L(700),NPLACE,NRPM,NTOT,NW,RPM(10)
     1 ,RRHUB,TITLE(15),W(100),WD,WF,WL,WT(700),KEY
      REAL*8 A(4,4),U(4,4),UNEW(4,4),MO,Y(700),X(700),LTOT
      DATA YES/'Y'/
      IF(KEY.NE.YES) RETURN
      IF(ISPEED.NE.1) GO TO 35
10    FORMAT(// ' ** THE PROGRAM IS CALCULATING THE MODE SHAPES ** ',//)
C
C     WRITE THE TITLE, THE TOTAL NUMBER OF LUMPED MASSES
C    1 AND THE AXIAL LOCATION OF EACH MASS.
C
      WRITE(3,20)
20    FORMAT(//,1X,'THE PROBLEM TITLE',/,1X,'*****************')
      WRITE(3,25)TITLE
25    FORMAT(/,6X,15A4)
      WRITE(3,27)
27    FORMAT(//,1X,'THE TOTAL NUMBER OF MASSES',/,
     1 1X,'**************************')
      WRITE(3,40)NTOT
      LTOT=0.0
      DO 30 N=1,NTOT
      LTOT=LTOT+L(N)
30    X(N)=LTOT
      WRITE(3,33)
33    FORMAT(//,1X,'THE AXIAL LOCATION OF MASS N (INCHES)',/,
     1 '*************************************',/)
      WRITE(3,120)(X(N),N=1,NTOT)
C
C     WRITE THE TOTAL NUMBER OF MODES OF VIBRATION.
C
35    WRITE(3,37)
37    FORMAT(//,1X,'THE NUMBER OF NATURAL FREQUENCIES AT THE'
     1 ,' LISTED ROTOR SPEED (RPM)',/,
     2 1X,'************************************************'
     3 ,'**************')
      WRITE(3,40)NW,RPM(ISPEED)
40    FORMAT(/,1X,I5,F10.1)
      WRITE(*,10)
C
C     CONSIDER ALL NATURAL FREQUENCIES
C
      DO 130 NF=1,NW
C
C     INITIALIZE [U]  (SET IT EQUAL TO THE IDENTITY MATRIX)
C
      DO 50 I=1,4
      DO 50 J=1,4
```

```
      U(I,J)=0.
      IF(I.EQ.J) U(I,J)=1.
50    CONTINUE
C
C     COMPUTE [U] = THE CONCATENATION OF THE [A] MATRICES FOR ALL MASSES
C
      DO 70 N=1,NTOT
C     FILL [A] FOR THE MASS N
      CALL FILLA(A,N,W(NF))
C     CONCATENATE [A] WITH THE EXISTING [U]
      CALL MULT(A,4,4,4,4,U,4,4,4,UNEW,4,4)
C     SET [U] = [UNEW]
      DO 60 I=1,4
      DO 60 J=1,4
60    U(I,J)=UNEW(I,J)
70    CONTINUE
C
C     CALCULATE THE SHEAR AND MOMENT AT FIXED END ASSUMING A UNIT
C     DISPLACEMENT AT THE FREE END (AUTOMATICALLY NORMALIZES THE MODE
C     SHAPE WITH RESPECT TO THE DISPLACEMENT AT THE FREE END).
C
      MO=1.0/(U(1,3)-U(4,3)*U(1,4)/U(4,4))
      VO=-U(4,3)*MO/U(4,4)
C
C     CALCULATE DISPLACEMENT AT MASS N IN TERMS OF THE MOMENT AND SHEAR AT
C     THE FIXED END (MO AND VO RESPECTIVELY).
C
C     REINITIALIZE [U] (SET IT EQUAL TO THE IDENTITY MATRIX)
C
      DO 80 I=1,4
      DO 80 J=1,4
      U(I,J)=0.
      IF(I.EQ.J) U(I,J)=1.
80    CONTINUE
C
C     COMPUTE [U] = THE CONCATENATION OF THE [A] MATRICES FOR ALL
C     MASSES UP TO AND INCLUDING MASS N
C
      DO 100 N=1,NTOT
C     FILL [A] FOR THE MASS N
      CALL FILLA(A,N,W(NF))
C     CONCATENATE [A] WITH THE EXISTING [U]
      CALL MULT(A,4,4,4,4,U,4,4,4,UNEW,4,4)
C     SET [U] = [UNEW]
      DO 90 I=1,4
      DO 90 J=1,4
90    U(I,J)=UNEW(I,J)
C
```

```
C        CALCULATE THE DISPLACEMENT OF MASS N
C
100      Y(N)=U(1,3)*MO+U(1,4)*VO
         WRITE(3,105)
105      FORMAT(//,1X,'THE NATURAL FREQUENCY OF VIBRATION (RAD/SEC)',/,
     1   ,1X,'********************************************')
         WRITE(3,110) W(NF)
110      FORMAT(/,10X,F15.5)
         WRITE(3,115)
115      FORMAT(//,1X,'THE NORMALIZED DISPLACEMENTS AT THE LUMPED MASSES',/
     1   1X,'*************************************************',/)
         WRITE(3,120) (Y(I),I=1,NTOT)
120      FORMAT(8F10.5)
130      CONTINUE
         RETURN
         END
```

```
$nofloatcalls   A.10  MULT - subroutine to perform matrix multiplication
$storage:2
      SUBROUTINE MULT(A,ROWA,COLA,DROWA,DCOLA,B,COLB,DROWB,DCOLB,C,
     1 DROWC,DCOLC)
C
C     THIS SUBROUTINE MULTIPLIES THE [A] MATRIX TIMES THE [B]
C     MATRIX AND STORES THE RESULTS IN THE [C] MATRIX.  ALL
C     THREE MATRICES MUST BE UNIQUE (I.E. ONE CANNOT HAVE
C     [A] X [B] = [A].
C
C     A(I,J)   THE FIRST MATRIX
C
C     B(I,J)   THE SECOND MATRIX
C
C     C(I,J)   THE RESULT OF [A] X [B] = [C]
C
C     COLA     THE NUMBER OF COLUMNS USED IN [A]
C
C     COLB     THE NUMBER OF COLUMNS USED IN [B]
C
C     DCOLA    THE NUMBER OF COLUMNS DIMENSIONED FOR [A] IN THE CALLING
C              PROGRAM
C
C     DCOLB    THE NUMBER OF COLUMNS DIMENSIONED FOR [B] IN THE CALLING
C              PROGRAM
C
C     DCOLC    THE NUMBER OF COLUMNS DIMENSIONED FOR [C] IN THE CALLING
C              PROGRAM
C
C     DROWA    THE NUMBER OF ROWS DIMENSIONED FOR [A] IN THE CALLING
C              PROGRAM
C
C     DROWB    THE NUMBER OF ROWS DIMENSIONED FOR [B] IN THE CALLING
C              PROGRAM
C
C     DROWC    THE NUMBER OF ROWS DIMENSIONED FOR [C] IN THE CALLING
C              PROGRAM
C
C     ROWA     THE NUMBER OF ROWS USED IN [A]
C
C
      REAL*8 A(DROWA,DCOLA),B(DROWB,DCOLB),C(DROWC,DCOLC),VAL
      INTEGER COLA,COLB,DROWA,DCOLA,DROWB,DCOLB,DROWC,DCOLC,ROWA
      DO 10 I=1,ROWA
      DO 10 J=1,COLB
      VAL=0.
      DO 8 K=1,COLA
8     VAL=VAL+A(I,K)*B(K,J)
10    C(I,J)=VAL
      RETURN
      END
```

```
$nofloatcalls
$storage:2
      SUBROUTINE NFREQ(ISPEED)
C
C     THIS SUBROUTINE COMPUTES THE NATURAL FREQUENCIES IN THE RANGE
C     FROM WF TO WL AND RETURNS THE RESULTS IN W(I)
C
C     THE METHOD OF INTERVAL HALVING IS USED TO SEARCH FOR THE
C     NATURAL FREQUENCIES.
C
C     DET1    THE DETERMINANT OF [U] AT THE STARTING POINT OF
C             AN INTERVAL BOUNDING A NATURAL FREQUENCY
C
C     DET2    THE DETERMINANT OF [U] AT THE ENDING POINT OF
C             AN INTERVAL BOUNDING A NATURAL FREQUENCY
C
C     DETM    THE DETERMINANT OF [U] AT THE MIDPOINT OF
C             THE INTERVAL BOUNDING A NATURAL FREQUENCY
C
C     ISPEED  AN INDEX CORRESPONDING TO THE PARTICULAR ROTOR
C             SPEED UNDER INVESTIGATION
C
C     NPLACE  THE NUMBER OF DIGITS TO THE RIGHT OF THE DECIMAL POINT
C
C     NW      THE NUMBER OF NATURAL FREQUENCIES IN THE RANGE FROM
C             WF TO WL
C
C     W1      THE FREQUENCY AT THE BEGINNING OF AN INTERVAL OF LENGTH
C             WD BOUNDING A NATURAL FREQUENCY (RAD/SEC)
C
C     W1CUR   THE FREQUENCY AT THE BEGINNING OF THE CURRENT INTERVAL
C             OF LENGTH WDCUR BOUNDING A NATURAL FREQUENCY (RAD/SEC)
C
C     W2CUR   THE FREQUENCY AT THE END OF THE CURRENT INTERVAL OF
C             LENGTH WDCUR BOUNDING A NATURAL FREQUENCY (RAD/SEC)
C
C     W(I)    THE NATURAL FREQUENCIES (RAD/SEC)
C
C     WD      THE INITIAL INTERVAL USED IN SEARCHING FOR NATURAL
C             FREQUENCIES (RAD/SEC)
C
C     WDCUR   THE CURRENT FREQUENCY INTERVAL USED IN SEARCHING FOR
C             A NATURAL FREQUENCY (RAD/SEC)
C
C     WF      THE FIRST VALUE TO BE CONSIDERED AS A POSSIBLE NATURAL
C             FREQUENCY (RAD/SEC)
C
C     WL      THE LAST VALUE TO BE CONSIDERED AS A POSSIBLE NATURAL
C             FREQUENCY (RAD/SEC)
C
C     WMCUR   THE FREQUENCY AT THE MIDPOINT OF THE INTERVAL
C             BEING SEARCHED FOR A NATURAL FREQUENCY (RAD/SEC)
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 L
```

A.11 NFREQ - subroutine to determine natural frequencies

```
      CHARACTER*1 KEY
      COMMON EI(700),EIO,FL(700),L(700),NPLACE,NRPM,NTOT,NW,RPM(10)
     1 ,RRHUB,TITLE(15),W(100),WD,WF,WL,WT(700),KEY
      WRITE(*,3)
3     FORMAT(///,2X, ' ** THE PROGRAM IS NOW DETERMINING THE NATURAL ',
     1 /,5X, ' FREQUENCIES OF VIBRATION ** ' )
      WRITE(*,4) RPM(ISPEED)
4     FORMAT(/// ' NATURAL FREQUENCIES AT A ROTOR SPEED OF ',F6.1,,
     1 ' RPM ')
      WRITE(*,5)
5     FORMAT( ' ************************************************ ')
C
C     CALCULATE FORCES DUE TO ROTATION
C
      CALL FORCE(ISPEED)
C
C     INITIALIZE VARIABLES
C
      NW=0
      W1=WF
6     W1CUR=W1
      DET1=DET(W1)
10    W2CUR=W1CUR+WD
C
C     CHECK IF DONE
C
      IF(W1CUR.GE.WL) RETURN
      IF(W2CUR.GT.WL) W2CUR=WL
C
C     NOT DONE, CONTINUE SEARCHING
C
      DET2=DET(W2CUR)
C
C     CHECK IF A NATURAL FREQUENCY OCCURS BETWEEN W1CUR AND W2CUR,
C     IS AT W1CUR OR W2CUR, OR IS NOT IN THE INTERVAL W1CUR TO W2CUR
C
15    IF(DET1*DET2) 21,30,40
C
C     A NATURAL FREQUENCY OCCURS BETWEEN W1CUR AND W2CUR
C
C     COMPUTE THE SIZE OF INTERVAL IN ORDER
C     TO CHECK IF DESIRED ACCURACY HAS BEEN REACHED
C
21    WDCUR=W2CUR-W1CUR
      IF(WDCUR.LT.10.**(-NPLACE-1)) GO TO 25
C
C     WDCUR TOO LARGE, CONSIDER FREQUENCY AT INTERVAL MIDPOINT
C
      WMCUR=(W1CUR+W2CUR)/2.
      DETM=DET(WMCUR)
C
C     DECIDE WHICH ENDPOINT, W1CUR OR W2CUR,
C     TO REDEFINE
```

```
C
        IF(DETM*DET1) 22,23,23
C
C       DETM IS ON OPPOSITE SIDE OF AXIS FROM DET1 SO REDEFINE W2CUR
C
22      DET2=DETM
        W2CUR=WMCUR
        GO TO 15
C
C       DETM IS ON THE SAME SIDE OF AXIS AS DET1 OR IS ZERO
C       SO REDEFINE W1CUR
C
23      DET1=DETM
        W1CUR=WMCUR
        GO TO 15
C
C       FOUND ROOT
C
25      NW=NW+1
        W(NW)=(W1CUR+W2CUR)/2.
        WRITE(*,1000)W(NW)
1000    FORMAT(/ ' W = ' ,F15.3, '     RAD/SEC ')
        W1=W1+WD
        GO TO 6
C
C       ONE OF THE ENDPOINTS WAS A NATURAL FREQUENCY
C
30      IF(DET2.EQ.0.) GO TO 35
C
C       W1CUR IS A NATURAL FREQUENCY, STORE IT
C
        NW=NW+1
        W(NW)=W1CUR
        WRITE(*,1000)W(NW)
C
C       MOVE TO NEXT INTERVAL
C
        W1=W1+WD
        GO TO 6
C
C       W2CUR MUST BE A NATURAL FREQUENCY, STORE IT
C
35      NW=NW+1
        W(NW)=W2CUR
        WRITE(*,1000)W(NW)
C
C       MOVE TO NEXT INTERVAL
C
        W1=W1+2.*WD
        GO TO 6
C
C       NO NATURAL FREQUENCY FOUND, MOVE TO NEXT RANGE
C
40      W1CUR=W2CUR
        W1=W1CUR
        DET1=DET2
        GO TO 10
        END
```

A.12 OUTPUT - subroutine to perform all output functions

```
$nofloatcalls
$storage:2
      SUBROUTINE OUTPUT(ISPEED)
C
C     THIS SUBROUTINE OUTPUTS THE INPUT DATA AND NATURAL FREQUENCIES
C     TO FILE 'VIBOUT.DAT'
C
C
C     EI(N)    THE PRODUCT OF MODULUS OF ELASTICITY AND AREA MOMENT
C              OF INERTIA AT MASS N  (LBS*INCH*INCH)
C
C     EIO      THE PRODUCT OF MODULUS OF ELASTICITY AND AREA MOMENT
C              OF INERTIA AT ZERO OR THE FIXED END  (LBS*INCH*INCH)
C
C     ISPEED   AN INDEX CORRESPONDING TO THE PARTICULAR ROTOR
C              SPEED UNDER INVESTIGATION
C
C     L(N)     THE LENGTH OF SECTION TO LEFT OF MASS N  (INCHES)
C
C     NPLACE   THE NUMBER OF DIGITS REQUIRED TO THE RIGHT OF THE
C              DECIMAL POINT
C
C     NTOT     THE TOTAL NUMBER MASSES
C
C     NW       THE NUMBER OF NATURAL FREQUENCIES IN THE RANGE FROM
C              WD TO WL
C
C     RPM(N)   THE ROTOR ROTATIONAL SPEED  (REV/MIN)
C
C     RRHUB    THE RADIUS OF THE RIGID HUB  (INCHES)
C
C     TITLE    THE PROBLEM TITLE STATEMENT
C
C     W(I)     THE NATURAL FREQUENCIES OF VIBRATION  (RAD/SEC)
C
C     WD       THE INITIAL DELTA USED IN SEARCHING FOR NATURAL
C              FREQUENCIES  (RAD/SEC)
C
C     WF       THE FIRST VALUE TO BE CONSIDERED AS A POSSIBLE NATURAL
C              FREQUENCY (RAD/SEC)
C
C     WL       THE LAST VALUE TO BE CONSIDERED AS A POSSIBLE NATURAL
C              FREQUENCY  (RAD/SEC)
C
C     WT(N)    THE WEIGHT OF MASS N  (LBS)
C
C
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 L
      CHARACTER*1 KEY
```

```
      COMMON EI(700),EIO,FL(700),L(700),NPLACE,NRPM,NTOT,NW,RPM(10)
     1 ,RRHUB,TITLE(15),W(100),WD,WF,WL,WT(700),KEY
      IF(ISPEED.NE.1) GO TO 175
C
C     WRITE OUT DATA READ FROM FILE 'VIBIN.DAT'
C
      WRITE(2,99)TITLE
99    FORMAT(6X,15A4)
      WRITE(2,100) NTOT
100   FORMAT(//' THE NUMBER OF LUMPED MASSES = ',I5)
      WRITE(2,110)
110   FORMAT(//' SECTION LENGTH      SECTION RIGIDITY',
     1 '       LUMPED WEIGHT ')
      WRITE(2,120)
120   FORMAT(/'      (INCH)              (LBS-IN*IN)',
     1 '               (LBS)')
      DO 140 N=1,NTOT
140   WRITE(2,150) L(N),EI(N),WT(N)
150   FORMAT(/,1X,F10.6,3X,F20.1,3X,F15.6)
      WRITE(2,160) EIO
160   FORMAT(//,' THE SECTION RIGIDITY (AT X=0.0) = ',F20.1,
     1 ' (LBS-IN*IN)')
      WRITE(2,165) RRHUB
165   FORMAT(//,' THE RADIUS OF THE RIGID HUB  = ',F10.3, ' (IN) ')
      WRITE(2,170) WF,WL,WD
170   FORMAT(//,' THE FREQUENCY RANGE SEARCHED FOR NATURAL FREQUENCIES ',/,
     1 ' STARTING AT:',F15.5,' AND ENDING AT: ',F15.5, ' (RAD/SEC)',
     2 //,' THE INITIAL FREQUENCY INCREMENT USED IN THE SEARCH = ',F15.5
     3 ,' (RAD/SEC)')
      FACTOR=1./(2.*3.14159265)
175   WRITE(2,180) RPM(ISPEED)
180   FORMAT(////,8X,'NATURAL FREQUENCIES AT A ROTOR SPEED OF ',F6.1,
     1 ' RPM'
     2 ,/,8X,'*************************************************',//,
     3 6X,'(RAD/SEC)',16X,'(HZ)',//)
      II=NPLACE+1
      IF(NPLACE.LT.0)II=1
      IF(NPLACE.GT.4)II=5
      DO 190 I=1,NW
      GO TO(300,310,320,330,340)II
300   WRITE(2,301)I,W(I),W(I)*FACTOR
301   FORMAT(/,1X,I4,F10.0,10X,F10.0)
      GO TO 190
310   WRITE(2,311)I,W(I),W(I)*FACTOR
311   FORMAT(/,1X,I4,F10.1,10X,F10.1)
      GO TO 190
320   WRITE(2,321)I,W(I),W(I)*FACTOR
321   FORMAT(/,1X,I4,F10.2,10X,F10.2)
      GO TO 190
330   WRITE(2,331)I,W(I),W(I)*FACTOR
331   FORMAT(/,1X,I4,F10.3,10X,F10.3)
      GO TO 190
340   WRITE(2,341)I,W(I),W(I)*FACTOR
341   FORMAT(/,1X,I4,F10.4,10X,F10.4)
190   CONTINUE
      RETURN
      END
```

A.13 MODPLT - mode shape plotting code

```
C       THIS PROGRAM PLOTS THE MODE SHAPES FOR THE LUMPED MASS MODEL
C       OF THE BEAM VIBRATION
C
C       AMSF     THE AMPLITUDE SCALE FACTOR
C       ITERM    THE TERMINAL NUMBER (4012 OR 4014) FOR GRAPHICS
C       NC       THE NUMBER OF CHARACTERS IN THE TITLE
C       NTOT     THE TOTAL NUMBER OF LUMPED MASSES
C       NW       THE TOTAL NUMBER OF NATURAL FREQUENCIES
C       TITLE    THE TITLE TO BE PRINTED AT THE TOP OF THE PLOT
C       W        THE CURRENT NATURAL FREQUENCY
C       X(NTOT)  THE X COORDINATES OF THE LUMPED MASSES
C       Y(NTOT)  THE DEFLECTIONS OF THE LUMPED MASSES
C
        BYTE TITLE
        COMMON AMSF,NC,NTOT,NW,TITLE(60),W,X(900),Y(900)
        DATA AMSF/10./
        OPEN (FILE='MODES.DAT',UNIT=1,TYPE='NEW')
        CALL START
        WRITE(*,50)
50      FORMAT(' ENTER TYPE OF DISPLAY TERMINAL (4012 OR 4014) ')
        READ(*,*) ITERM
C
C       READ IN NUMBER OF MASSES, NTOT, AND THE NUMBER OF MODES OF
C       VIBRATION
C
        CALL MINPUT(1)
C
C       READ NATURAL FREQUENCY AND INITIAL MODE SHAPE (IT MAY NOT BE
C       FIRST MODE IF A NON-ZERO INITIAL FREQUENCY WAS GIVEN TO THE
C       BEAM PROGRAM).
C
100     CALL MINPUT(2)
C
C       SCALE DRAWING
C
110     WRITE(*,120) AMSF
120     FORMAT(' 1 = DRAW SKETCH'
     1  ,/,     ' 2 = CHANGE AMPLITUDE SCALE FACTOR, CURRENTLY = ',F6.2
     2  ,/,     ' 3 = DRAW FINISHED DRAWING(S) ')
        READ(*,*) I
        IF(I.LT.1.OR.I.GT.3) GO TO 110
        GO TO (130,140,150),I
C
C       DRAW SKETCH
C
130     CALL DRAW(ITERM,1)
        GO TO 110
C
C       CHANGE SCALE FACTOR
C
140     WRITE(*,145)
145     FORMAT(' ENTER NEW SCALE FACTOR TO BE APPLIED TO AMPLITUDE ')
        READ(*,*) AMSF
```

```
      GO TO 110
C
C     DRAW FINISHED DRAWING(S) FOR ALL FREQUENCIES USING SAME SCALE
C     FACTOR
C
150   IF(NW.LE.0) STOP
      DO 160 I=1,NW
      CALL DRAW(ITERM,2)
C     READ IN NEXT MODE SHAPE IF THERE IS ANOTHER MODE SHAPE IN DATA
C     FILE
      IF(I.LT.NW) CALL MINPUT(2)
160   CONTINUE
C
C     STOP
C
      STOP
      END
```

A.14 DRAW - subroutine to draw mode shapes

```
      SUBROUTINE DRAW(ITERM,ITYPE)
C
C     THIS SUBROUTINE DRAWS A SIMPLE SKETCH OF THE MODE SHAPE
C     (ITYPE = 1) FULL DRAWING COMPLETE WITH TITLE BLOCK (ITYPE = 2).
C     AMSF    THE AMPLITUDE SCALE FACTOR
C     CHARX   THE X SIZE OF THE INDIVIDUAL CHARACTERS TO BE DISPLAYED
C     DAT     THE CHARACTERS FOR THE CURRENT DATE
C     LFTMAR  THE SIZE OF THE LEFT MARGIN ON THE SCREEN
C     MODES   THE CURRENT MODE NUMBER
C     NC      THE NUMBER OF CHARACTERS IN THE TITLE
C     NTOT    THE TOTAL NUMBER OF LUMPED MASSES
C     NW      THE TOTAL NUMBER OF NATURAL FREQUENCIES
C     RGTMAR  THE SIZE OF THE RIGHT MARGIN ON THE SCREEN
C     SIGN    THE SIGN OF THE AMPLITUDE AT THE PREVIOUS LUMPED MASS
C     TITLE   THE TITLE TO BE PRINTED
C     W       THE CURRENT NATURAL FREQUENCY
C     Y(NTOT) THE X COORDINATES OF THE LUMPED MASSES
C     XNODES  THE X COORDINATES OF THE NODES
C     XORG    THE X SCREEN COORDINATE FOR THE AXES ORGIN
C     XSCALE  THE SCALE FACTOR TO BE APPLIED TO ALL X COORDINATES
C     XSCRN   THE WIDTH OF THE SCREEN IN INCHES
C     XSTART  THE X SCREEN COORDINATE WHERE TITLE BLOCK BEGINS
C     Y(NTOT) THE Y DEFELECTIONS OF THE LUMPED MASSES
C     YMAX    THE MAXIMUM DEFLECTION OF THE LUMPED MASSES
C     YMIN    THE MINIMUM DEFLECTION OF THE LUMPED MASSES
C     YORG    THE Y SCREEN COORDINATE FOR THE AXES ORIGIN
C     YP      THE SCALED Y DEFLECTION
C     YPOLD   THE PREVIOUS SCALED Y DEFLECTION
C     YSCRN   THE HEIGHT OT THE SCREEN IN INCHES
C
      BYTE TITLE,DAT(9)
      COMMON AMSF,NC,NTOT,NW,TITLE(60),W,X(900),Y(900)
      REAL LFTMAR,XNODES(100)
      DATA LFTMAR,RGTMAR/.5,.5/
      WRITE(*,100)
100   FORMAT(' PUSH THE RETURN KEY TO DISPLAY NEXT PLOT',/,
     1 ' THEN AFTER PLOT IS DISPLAYED PUSH RETURN KEY FOR NEXT PLOT')
      READ(*,*) I
      CALL PLOTS(ITERM)
      CALL ERASE
C
C     SET TERMINAL VALUES
C
      IF(ITERM.EQ.4014) GO TO 120
C     4012
      XSCRN=7.307
      YSCRN=5.564
      CHARX=.1085
      GO TO 130
C     4014
120   XSCRN=14.62
      YSCRN=11.14
      CHARX=0.196
```

```
C
C       SCALE X
C
130     XSCALE=(XSCRN-LFTMAR-RGTMAR)/X(NTOT)
C
C       LOCATE AXIS
C
        YORG=(YSCRN-.5-1.0)/2.+1.
        XORG=LFTMAR
C
C       PLOT SOLID LINE MODE SHAPE, LOOK FOR AXIS CROSSINGS TO DETERMINE
C       MODE NUMBER AND LOOK FOR MAX AND MIN AMPLITUDE VALUES
C
        MODES=1
        SIGN=0
        CALL PLOT(XORG,YORG,-3)
        YMAX=0
        YMIN=0
        DO 200 N=1,NTOT
        YP=Y(N)*AMSF
        YMAX=AMAX1(YP,YMAX)
        YMIN=AMIN1(YP,YMIN)
        IF(SIGN.NE.0) GO TO 180
        SIGN=YP
        GO TO 190
C       CHECK FOR CROSSING
180     IF(YP*SIGN) 185,185,190
C       CROSSING FOUND
185     MODES=MODES+1

C       COMPUTE NODE X COORDINATE LOCATION
        XNODES(MODES)=X(N-1)+(YPOLD-YP)/(YPOLD*(X(N)-X(N-1)))
        SIGN=YP
C       NO CROSSING FOUND, PLOT POINT
190     CALL PLOT(X(N)*XSCALE,YP,2)
        YPOLD=YP
200     CONTINUE
C
C       PLOT AXIS
C
        CALL PLOT(X(NTOT)*XSCALE,0.,3)
        CALL PLOT(0.,0.,2)
        CALL PLOT(0.,YMAX,2)
        CALL PLOT(0.,YMIN,2)
C       CHECK IF FULL DRAWING IS TO BE DONE
        IF(ITYPE.EQ.2) GO TO 300
C       NO, ONLY SKETCH
        CALL PLOT(XSCRN-XORG,YSCRN-YORG,999)
        READ(*,*) I
        CALL ERASE
        RETURN
C
C       DRAW FULL DRAWING
C
C       BEGIN BY DRAWING DASHED ENVELOPE (SOLID LINE DRAWN ABOVE)
300     IPEN=2
```

```
      CALL PLOT(0.,0.,3)
      DO 320 N=1,NTOT
      CALL PLOT(X(N)*XSCALE,-Y(N)*AMSF,IPEN)
C     SWITCH FROM SOLID TO DRAWN LINE FOR DASHED EFFECT
      IF(IPEN.EQ.2) GO TO 310
      IPEN=2
      GO TO 320
310   IPEN=3
320   CONTINUE
C
C     PLOT X/L LOCATIONS OF NODES
C
C     PLOT HEADER
      CALL PLOT(-4.*CHARX,YMIN-.2,3)
      CALL STRING('X/L:',4,4)
      DO 330 I=1,MODES
      XX=XNODES(I)*XSCALE
      CALL PLOT(XX,-0.05,3)
      CALL PLOT(XX,YMIN,2)
      CALL NUMBER(XX-2.5*CHARX,YMIN-.2,0.11,XNODES(I)/X(NTOT),0.,
     1 '(F6.3)')
330   CONTINUE
C
C     PLOT TITLE AND TITLE BLOCK
C
C     RESET ORIGIN
      CALL PLOT(-XORG,-YORG,-3)
C     PLOT TITLE
      CALL PLOT(XSCRN-FLOAT(NC+1)/2.*CHARX,YSCRN-.5,3)
      CALL STRING(TITLE,NC,4)
C     PLOT TITLE BLOCK
      XSTART=(XSCRN-4.5)/2.
      CALL PLOT(XSTART,0.,3)
      CALL PLOT(XSTART+5.,0.,2)
      CALL PLOT(XSTART+5.,1.,2)
      CALL PLOT(XSTART,1.,2)
      CALL PLOT(XSTART,0.,2)
      CALL PLOT(XSTART,.3,3)
      CALL PLOT(XSTART+5.,.3,2)
      CALL PLOT(XSTART+5.,1.,3)
      CALL PLOT(XSTART+5.,1.,3)
      CALL PLOT(XSTART+5.,0.,2)
      CALL PLOT(XSTART+1.75,1.,3)
      CALL PLOT(XSTART+1.75,0.,2)
      CALL PLOT(XSTART+3.25,1.,3)
      CALL PLOT(XSTART+3.25,0.,2)
      CALL PLOT(XSTART+.1,.6,3)
      CALL STRING('MODE',4,1)
      CALL PLOT(XSTART+.6,.6,3)
      CALL STRING('NAT. FREQ.',10,1)
      CALL PLOT(XSTART+.6,.3,3)
      CALL STRING('(RAD/SEC)',9,1)
      CALL PLOT(XSTART+1.85,.6,3)
      CALL STIRNG('NO. OF MASSES',13,1)
      CALL PLOT(XSTART+3.35,.6,3)
```

```
CALL STRING('    DATE',7,1)
CALL NUMBER(XSTART+.1,.1,0.11,MODES,0.,'(I3)')
CALL NUMBER(XSTART+.6,.1,0.11,W,0.,'(F10.3)')
CALL NUMBER(XSTART+1.85,.1,0.11,NTOT,0.,'(I7)')
CALL DATE(DAT)
CALL PLOT(XSTART+3.35,.1,3)
CALL STRING(DAT,9,1)
CALL PLOT(XSCRN,YSCRN,999)
READ(*,*) I
CALL ERASE
RETURN
END
```

A.15 MINPUT - subroutine to perform all input functions

```
      SUBROUTINE MINPUT(ITYPE)
C
C     THIS SUBROUTINE READS DATA FROM DATA FILE 'MODES.DAT'.
C
C     NC      THE NUMBER OF CHARACTERS IN TITLE
C     NTOT    THE TOTAL NUMBER OF MASSES
C     NW      THE TOTAL NUMBE OF NATURAL FREQUENCIES
C     TITLE   THE TITLE TO BE PRINTED
C     W       THE CURRENT NATURAL FREQUENCY BEING CONSIDERED
C     X(NTOT) THE X COORDINATES OF THE LUMPED MASSES
C     Y(NTOT) THE Y DEFLECTIONS AT THE LUMPED MASSES
C
C     IF ITYPE = 1 THE TITLE, NUMBER OF LUMPED MASSES, THEIR
C     X COORDINATES AND THE NUMBER OF NATURAL FREQUENCIES COMPUTED
C     ARE READ.
C
C     IF ITYPE = 2, THE NATURAL FREQUENCY AND MODE SHAPE AMPLITUDES
C     AT THE LUMPED MASSES ARE READ.
C
      BYTE TITLE
      COMMON AMSF,NC,NTOT,NW,TITLE(60),W,X(900),Y(900)
C
C     BRANCH ON TYPE OF READ
C
      GO TO (100,200),ITYPE
C
C     READ TITLE, NUMBER OF LUMPED MASSES, THEIR X LOCATIONS AND THE
C     NUMBER OF MODES ARE READ.
C
100   READ(1,110) NC,(TITLE(I),I=1,NC)
110   FORMAT(6X,Q,60A1)
      READ(1,120) NTOT
120   FORMAT(1X,I5)
      READ(1,130)  (X(N),N=1,NTOT)
130   FORMAT(8F10.5)

      READ(1,120) NW
      RETURN
C
C     READ NATURAL FREQUENCY AND MODE SHAPE
C
200   READ(1,210) W
210   FORMAT(10X,F15.5)
      READ(1,130)  (Y(N),N=1,NTOT)
      RETURN
      END
```

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16. Abstract

A research effort was undertaken to develop personal computer based software for vibrational analysis. The software was developed to analytically determine the natural frequencies and mode shapes for the uncoupled lateral vibrations of the blade and counterweight assemblies used in a single bladed wind turbine. The uncoupled vibration analysis was performed in both the flapwise and chordwise directions for static rotor conditions. The effects of rotation on the uncoupled flapwise vibration of the blade and counterweight assemblies were evaluated for various rotor speeds up to 90 rpm. The theory, used in the vibrational analysis codes, is based on a lumped mass formulation for the blade and counterweight assemblies. The codes are general so that other designs can be readily analyzed. The input for the codes is generally interactive to facilitate usage. The output of the codes is both tabular and graphical. Listings of the codes are provided. Predicted natural frequencies of the first several modes show reasonable agreement with experimental results. The analysis codes were originally developed on a DEC PDP 11/34 minicomputer and then downloaded and modified to run on an ITT XTRA personal computer. Studies conducted to evaluate the efficiency of running the programs on a personal computer as compared with the minicomputer indicated that, with the proper combination of hardware and software options, the efficiency of using a personal computer exceeds that of a minicomputer.

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