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DYNAMICS AND CONTROL OF FLEXIBLE SPACECRAFT DURING AND AFTER SLEWING MANEUVERS

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INTRODUCTION

Many future NASA missions would utilize significantly large and flexible spacecrafts and would require very stringent pointing and vibration suppression requirements. The active controller that can achieve these objectives will have to be designed with very accurate knowledge of the dynamic behavior of the spacecraft to ensure performance robustness to a variety of disturbances and uncertainties.

In the past few years, several design approaches were proposed for vibration control during and after slewing maneuvers. NASA Langley Research Center initiated the Spacecraft Control Laboratory Experiment (SCOLE) program [1] to promote direct comparision and realistic test of various control design techniques against a common laboratory article. The article was intended to resemble a large space antenna attached to the space shuttle orbiter by a long flexible mast.

The primary control objective of SCOLE is to direct the RF line-of-sight (LOS) of the antenna-like configuration towards a fixed target under the conditions of minimum time and limited control authority.

This problem of directing the LOS of antenna-like configuration is studied as being composed of two control phases during this research period. In the first phase, the LOS of antenna-like structure is to be changed according to a prespecified target (slewing) together with minimization of vibration amplitudes to ensure stability. The second phase is to achieve the total vibration suppression at the end of slew maneuver with an augmented control law.

BRIEF SUMMARY OF MAJOR ACCOMPLISHMENTS

(a) Nonlinear Model Development:

The focus of the earlier part of the research was in upgrading the dynamics of the SCOLE model to reflect all the kinematic nonlinearities in the previous dynamical model. The dynamical equations of slewing maneuvers of this large flexible spacecraft were developed by formulating Lagrange's equations using an inertial co-ordinate system and a body-fixed coordinate system at the point of attachment of the flexible beam to the shuttle. The generic model used for this analysis consisted of distributed parameter beam with two end masses. The three dimensional linear vibration analysis of this free-free beam model with end masses [2] was incorporated together with rigid-body slewing maneuver dynamics [3] to yield the final set of highly nonlinear and coupled equations.

(b) Slew Maneuvers

The slew maneuvers were analyzed using this enhanced dynamical model in terms of both pure rigid-body slew maneuvers and rigid-body slew maneuvers together with the suppression of the first two flexible modes. The dynamics of the motion during this phase was derived in terms of four Euler parameters, and using the method of nonlinear decoupling, this nonlinear slew maneuver control problem was reduced to calculating a pair of constants for each of the first three Euler parameters and the flexible modes in implementing output feedback.

(c) Vibration Suppression at the End of Slew Maneuver

Since the oscillations of the beam at the end of the slew maneuver were considered to be small, the problem of vibration suppression was considered to be a linear control problem. Thus it was formulated as an infinite-time regulator problem with control spillover and observation spillover terms incorporated into the performance index term. The resultant optimal state feedback control law achieved total vibration suppression. However, the coupling among the modes was found to be a significant problem.

A paper describing some of the details is included in the appendix. This paper is to be presented at the AIAA Guidence, Navigation and Control Conference in Williamsburg, Va. August 18-20, 1986 and is to be published in the volume.

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APPENDIX

Dynamics and Control of Slew Maneuver of Large Flexible Spacecraft

by

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Abstract:

In this paper, the dynamics and control of slewing maneuvers of a large flexible spacecraft namely, NASA-Spacecraft Control Laboratory Experiment (SCOLE) test article are studied. The dynamical equations obtained for slewing maneuvers are highly nonlinear and coupled. The maneuver is expressed in terms of four Euler parameters and is specified as the angular displacement about an arbitrary axis. The slew maneuver control problem is developed in terms of rigid-body slewing and suppression of two elastic modes is analyzed using the method of nonlinear decoupling.

Nomenclature:

$\underline{a}(z)$ -	Position vector of mass element on the beam from the point of attachment
$\underline{c}$ -	Position vector from the point of attachment to the mass center
$\underline{d}(z,t)$ -	Displacement vector of mass element in the body- fixed frame
$\underline{F}_0(t)$ -	Force applied at the orbiter mass center
$\underline{F}(t)$ -	Force applied at the reflector mass center
$\underline{G}_0(t)$ -	Moment applied about the orbiter mass center
I_1 -	Mass moment of inertia matrix of the shuttle
I_2 -	Mass moment of inertia matrix of the reflector
L -	The length of the beam
m -	Total mass of the flexible beam
m_1 -	Mass of the orbiter
m_2 -	Mass of the reflector
$\underline{R}$ -	Position vector of the mass center of the orbiter in the inertial frame
$\underline{r}$ -	Position vector from the orbiter mass center to the point of attachment
$u_x(z,t)$-	The beam deflection in x direction referred to the body-fixed frame
$u_y(z,t)$-	The beam deflection in y direction referred to the body-fixed frame
$u_\theta(z,t)$-	The torsional deflection about z axis in the body-fixed frame
$\underline{V}$ -	Velocity vector of the mass center of the orbiter in the body-fixed frame
$\underline{V}_0$ -	Velocity vector of the point of attachment in the body-fixed frame
ρ -	Mass per unit length
$\underline{\lambda}$ -	Vector representing the axis of rotation

$\underline{\omega}$ - The angular velocity of the orbiter in the body-fixed frame

$\underline{\psi}$ - Angle of rotation

Introduction:

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In the past few years, several design approaches were proposed for vibration control during and after slewing maneuvers. NASA Langley Research Center initiated the Spacecraft Control Laboratory Experiment (SCOLE) program [1] to promote direct comparison and realistic test of various control design techniques against a common laboratory article. The article was intended to resemble a large space antenna attached to the space shuttle orbiter by a long flexible mast.

The primary control objective of SCOLE is to direct the RF Line-of-Sight (LOS) of the antenna-like configuration towards a fixed target under the conditions of minimum time and limited control authority.

The dynamical equations of slewing maneuvers of this large flexible spacecraft are developed by formulating Lagrange's equations using an inertial co-ordinate system and a body-fixed co-ordinate system at the point of attachment of the flexible beam to the shuttle. The generic model used for this analysis consists of a distributed parameter beam with two end masses. The three dimensional linear vibration analysis of this free-free beam model with end masses [2] is incorporated together with rigid-slewing maneuver dynamics [3] to yield the final set of highly nonlinear and coupled equations.

The problem of directing the LOS of antenna-like configuration is being viewed as two phase control problem. In the first phase, the LOS of antenna-like structure is considered to be changed according to a prespecified target (slewing) together with minimization of vibration amplitudes to ensure stability. The second phase is to achieve the total vibration suppression at the end of slew maneuver with an augmented control law.

The first phase of the control law is studied both in terms of pure rigid-body slewing and in terms of rigid-body slewing together with the suppression of the first two flexible modes. The dynamics of the motion during this phase are derived in terms of four Euler parameters, and using the method of nonlinear decoupling, this nonlinear slew maneuver control problem is reduced to calculating a pair of constants for each of the first three Euler parameters and the elastic modes in implementing output feedback.

The second phase of the control is implemented at the end of the maneuver to achieve total vibration suppression, and this is performed by incorporating a state feedback control law derived from an infinite-time regulator problem formulation.

Analytics:

Kinetic Energy:

The dynamics of slew maneuver are developed using two sets of co-ordinate systems and Lagrange's method. The body-fixed frame origin is located at the point of attachment of the flexible beam or mast (Fig.1). The second co-ordinate system is an inertial co-ordinate system. The transformation from the inertial frame to the body-fixed frame is given by the matrix,C as

$$C=\begin{bmatrix} \cos\theta_2\cos\theta_3 & -\cos\theta_2\sin\theta_3 & \sin\theta_2 \\ (\sin\theta_1\sin\theta_2\cos\theta_3 & (-\sin\theta_1\sin\theta_2\sin\theta_3 & -\sin\theta_1\cos\theta_2 \\ +\sin\theta_3\cos\theta_1) & +\cos\theta_3\cos\theta_1) & \\ (-\cos\theta_1\sin\theta_2\cos\theta_3 & (\cos\theta_1\sin\theta_2\sin\theta_3 & \cos\theta_1\cos\theta_2 \\ +\sin\theta_3\sin\theta_1) & +\cos\theta_3\sin\theta_1) & \end{bmatrix} \quad (1)$$

where if $\vec{i},\vec{j},\vec{k}$ represent the dextral set of orthogonal unit vectors fixed in the body-fixed frame, then θ_1 is the rotation of $\vec{i}$, θ_2 is the rotation of $\vec{j}$ and θ_3 is the rotation of $\vec{k}$.

The angular velocity of the orbiter can be transformed from the inertial frame to the body-fixed frame for the body-three angles as

$$\underline{\omega} = M^T\underline{\dot{\theta}} \quad (2)$$

The total kinetic energy expression of the system can be given as [4]

$$T = T_0 + T_1 + T_2 \quad (3)$$

where T_0 is the kinetic energy of the shuttle and is given as

$$T_0 = 1/2\ m_1 \underline{V}^T \underline{V} + 1/2\ \underline{\omega}^T I_1 \underline{\omega} \quad (4)$$

The kinetic energy of the flexible beam is T_1 and it can be shown to be equal to

$$T_1 = 1/2 m \underline{V}_0^T \underline{V}_0 + 1/2\, \underline{\omega}^T J \underline{\omega} - m \underline{V}_0^T \tilde{c} \underline{\omega} + 1/2 \int \dot{\underline{d}}^T \dot{\underline{d}}\, dm$$
$$+ \underline{V}_0^T \int \dot{\underline{d}}\, dm + \underline{\omega}^T \int \tilde{\underline{a}} \dot{\underline{d}}\, dm + 1/2 \int [\dot{u}_x{}', \dot{u}_y{}', \dot{u}_\psi{}'] \, dI \begin{bmatrix} \dot{u}_x{}' \\ \dot{u}_y{}' \\ \dot{u}_\psi{}' \end{bmatrix} \qquad (5)$$

where $\tilde{c} = c\, x$ and J is the mass moment of inertia matrix for the beam. The foregoing kinetic energy expression can be further simplified by using the three-dimensional modal analysis as

$$T_1 = 1/2 m \underline{V}_0^T \underline{V} + 1/2 \underline{\omega}^T J \underline{\omega} - m \underline{V}_0^T \tilde{\underline{c}} \underline{\omega} + m \sum_{i=1}^{n} \dot{q}_i^2 + \underline{V}_0^T \dot{\underline{\alpha}}$$
$$+ \underline{\omega}^T \dot{\underline{\beta}} + 1/4 \rho \left[\sum_{i=1}^{n} p_{5i} \dot{q}_i^2 + \sum_{i=1}^{n} p_{6i} \dot{q}_i^2 \right] \qquad (6)$$

where

$$u_x = \sum_{i=1}^{n} \phi_{xi}(s) q_i(t)$$
$$u_y = \sum_{i=1}^{n} \phi_{yi}(s) q_i(t) \qquad (7)$$
$$u_\psi = \sum_{i=1}^{n} \phi_{\psi i}(s) q_i(t)$$

and

$$p_{1i} = \int_0^L \phi_{xi}(s)\, ds$$
$$p_{2i} = \int_0^L \phi_{yi}(s)\, ds$$
$$p_{3i} = \int_0^L s\phi_{xi}(s)\, ds$$
$$p_{4i} = \int_0^L s\phi_{yi}(s)\, ds \qquad (8)$$
$$p_{5i} = \int_0^L (s\phi_{xi}')^2 ds$$
$$p_{6i} = \int_0^L (s\phi_{yi}')^2 ds$$

and

$$\underline{\dot{\alpha}}(t) = \begin{bmatrix} \sum_{i=1}^{n} p_{1i}\ \dot{q}_i \\ \sum_{i=1}^{n} p_{2i}\ \dot{q}_i \\ 0 \end{bmatrix} \quad (9)$$

$$\underline{\dot{\beta}}(t) = \rho \begin{bmatrix} \sum_{i=1}^{n} p_{4i}\ \dot{q}_i \\ \sum_{i=1}^{n} p_{3i}\ \dot{q}_i \\ 0 \end{bmatrix} \quad (10)$$

The kinetic energy T_2, of the tip mass (the reflector) is

$$T_2 = 1/2\ m_2 \underline{V}_0^T \underline{V}_0 - m_2 \underline{V}_0^T \tilde{\underline{a}}(L)\underline{\omega} + m_2 \underline{V}_0^T \underline{\dot{d}}(L) - 1/2\ m_2 \underline{\omega}^T \tilde{\underline{a}}(L)\tilde{\underline{a}}(L)\underline{\omega} + m_2\ \underline{\omega}^T \tilde{\underline{a}}(L)\underline{\dot{d}}(L) + 1/2\ m_2\ \underline{\dot{d}}^T(L)\underline{\dot{d}}(L) + 1/2\ \underline{\Omega}^T I_2 \underline{\Omega} \quad (11)$$

where

$$\underline{\Omega} = \underline{\omega} + \begin{bmatrix} \dot{u}_x{}'(L) \\ \dot{u}_y{}'(L) \\ \dot{u}_\psi\ (L) \end{bmatrix} \quad (12)$$

Equation (11) can be simplified as

$$T = 1/2\ m_2\ \underline{V}_0^T \underline{V}_0 - m_2 \underline{V}_0^T \tilde{\underline{a}}(L)\underline{\omega} + m_2 \underline{V}_0^T \underline{\dot{d}}(L) - 1/2 m_2 \underline{\omega}^T \tilde{\underline{a}}(L)\tilde{\underline{a}}(L)\underline{\omega} + m_2 \underline{\omega}^T \tilde{\underline{a}}(L)\underline{\dot{d}}(L) + 1/2\ m_2 \left[\sum_{i=1}^{n}\sum_{j=1}^{n}\phi_{xi}(L)\phi_{xj}(L)\dot{q}_i\dot{q}_j + \sum_{i=1}^{n}\sum_{j=1}^{n}\phi_{yi}(L)\phi_{yj}(L)\dot{q}_i\dot{q}_j\right] + 1/2\ \underline{\dot{P}}^T I_2 \underline{\dot{P}} + 1/2 \underline{\omega}^T I_2 \underline{\omega} \quad (13)$$

where

$$\underline{\dot{P}} = \left[\sum_{i=1}^{n}\phi_{xi}'(L)\dot{q}_i(t)\ \ \sum_{i=1}^{n}\phi_{yi}'(L)\dot{q}_i(t)\ \ \sum_{i=1}^{n}\phi_{\psi i}(L)\dot{q}_i(t)\right] \quad (14)$$

Substituting T_0, T_1 and T_2 from the foregoing equations into equation (3), the total kinetic energy expression can be written as

$$T = 1/2\ m_0 \underline{V}^T \underline{V} + \underline{\omega}^T H \underline{V} + 1/2 \underline{\omega}^T I_0 \underline{\omega} + \underline{V}^T A_1 \underline{\dot{q}} + \underline{\omega}^T A_2 \underline{\dot{q}} + 1/2\ \underline{\dot{q}}\ A_3 \underline{\dot{q}} \quad (15)$$

where

$$m_0 = m_1 + \rho L + m_2 \quad (16)$$

$$H = (\rho L + m_0)\underline{\tilde{r}} + m_2\underline{\tilde{a}}(L) + \rho L\underline{\tilde{c}} \quad (17)$$

$$I_0 = I_1 + J + I_2 \quad (18)$$

and also

$$A_1\,\underline{\dot{q}} = \underline{\dot{\alpha}} + m_2\,\underline{\dot{d}}(L) \quad (19)$$

$$A_2\,\underline{\dot{q}} = \underline{\tilde{r}}\,\underline{\dot{\alpha}} + \underline{\dot{\beta}} + m_2\underline{\tilde{r}}\underline{\dot{d}}(L) + m_2\underline{\tilde{a}}(L)\underline{\dot{d}}(L) \quad (20)$$

$$A_3 = \begin{bmatrix} \ddots & & 0 \\ & \rho L + m_2 + p_{5i} + p_{6i} & \\ 0 & & \ddots \end{bmatrix} + \phi^T(L)\, I_2\, \phi(L) \quad (21)$$

The matrix $\phi'^T(L)$ is given as

$$\phi'^T(L) = \begin{bmatrix} \phi'_{1x}(L) & 0 & 0 \\ 0 & \phi'_{1y}(L) & 0 \\ 0 & 0 & \phi'_{1\psi}(L) \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \phi'_{ix}(L) & 0 & 0 \\ 0 & \phi'_{iy}(L) & 0 \\ 0 & 0 & \phi'_{i\psi}(L) \end{bmatrix} \quad (22)$$

Equations of motion:

Lagrange's equations for the case of independent generalized co-ordinates q_k are

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_k} - \frac{\partial T}{\partial q_k} = Q_k - \frac{\partial V}{\partial q_k} \quad (k = 1, 2, \ldots.) \quad (23)$$

where $T(q,\dot{q})$ is the kinetic energy, $V(q)$ is the potential energy and Q_k are the generalized forces arising from nonconservative sources.

However, the kinetic energy developed in equation (15) is given in terms of nonholonomic velocities $\underline{V}, \underline{\omega}$ and generalized velocities $\underline{\dot{q}}$. Using the chain-rule $\partial T/\partial V$ be expressed in terms of generalized velocities. Also, if $\underline{F}(t)$ represents the total applied forces where

$$\underline{F}(t) = \underline{F}_0(t) + \underline{F}_2(t) \quad (24)$$

then the generalized forces are given as $C\underline{F}(t)$. Thus, using the Lagrange's equations the translational equations can be obtained as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \underline{V}}\right) + C^T \dot{C} \left(\frac{\partial T}{\partial \underline{V}}\right) = \underline{F}(t) \quad (25)$$

which can be simplified as

$$m_0 \underline{\dot{V}} - H \underline{\dot{\omega}} + A_1 \underline{\ddot{q}} = \underline{N}_1 + \underline{F}(t) \quad (26)$$

where the nonlinear term N_1 is given as

$$\underline{N}_1 = -C^T \dot{C} \left(m_0 \underline{V} - H \underline{\omega} + A_1 \underline{\dot{q}} \right) \quad (27)$$

$$= \underline{\tilde{\omega}} \left(m_0 \underline{V} - H \underline{\omega} + A_1 \underline{\dot{q}} \right)$$

Similarly, using equation (2) and the chain rule in the Lagrange's equations, the rotational equations are obtained as

$$H \underline{\dot{V}} + I_0 \underline{\dot{\omega}} + A_2 \underline{\ddot{q}} = \underline{G}(t) + \underline{N}_2 \quad (28)$$

where $\underline{G}(t)$ is the net moment about the mass center of the orbiter and is given as

$$\underline{G} = \underline{G}_0 + (\underline{r} + \underline{a}) \times \underline{F}_2 \quad (29)$$

and the nonlinear term $\underline{N}_2$ is given in terms of transformations M and C, and $\underline{\omega}$, $\underline{V}$ and $\underline{\theta}$ .The vibration equations of the beam can be obtained by again using Lagrange's equations and the potential energy function

$$U = 1/2\, \underline{q}^T K \underline{q} \quad (30)$$

where the stiffness matrix K is given as

$$K = \begin{bmatrix} \ddots \frac{EI(\beta_i)^4}{L^3} & \\ & \ddots \end{bmatrix} \quad (31)$$

The vibration equations are

$$A_1^T \underline{\dot{V}} + A_2^T \underline{\dot{\omega}} + A_3 \underline{\ddot{q}} = -K\underline{q} \quad (32)$$

Slew Maneuver Dynamics: Considering the translational velocity and acceleration to be negligible during the slew maneuver, the rotational equations and vibration equations are

$$I_0 \underline{\dot{\omega}} + A_2 \underline{\ddot{q}} = \underline{G}(t) + \underline{N}_2(\underline{\omega}) \quad (33)$$

$$A_2^T \underline{\dot{\omega}} + A_3 \underline{\ddot{q}} = -K\underline{q} \quad (34)$$

Equation (33) can be rewritten as

$$\dot{\underline{\omega}} = I_0^{-1}[\ \underline{G} + \underline{N}_2(\underline{\omega}) - A_2\ddot{\underline{q}}\ ] \tag{35}$$

The first three Euler parameters are defined as

$$\underline{\varepsilon} \triangleq \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \underline{\lambda}\sin\psi/2 \tag{36}$$

$$\varepsilon_4 \triangleq \cos\psi/2 \tag{37}$$

$$\frac{d\underline{\varepsilon}}{dt} \triangleq 1/2\ (\ \varepsilon_4\,\underline{\omega} + \underline{\varepsilon} \times \underline{\omega}) \tag{38}$$

$$\frac{d\varepsilon_4}{dt} = -1/2\ \underline{\omega}\cdot\underline{\varepsilon} \tag{39}$$

$$\underline{\omega} = 2\ (\ \varepsilon_4\frac{d\underline{\varepsilon}}{dt} - \dot{\varepsilon}_4\underline{\varepsilon} - \underline{\varepsilon} \times \frac{d\underline{\varepsilon}}{dt}\ ) \tag{40}$$

$$\dot{\underline{\varepsilon}} = \frac{d\underline{\varepsilon}}{dt} = \underline{h}\ (\ \underline{\varepsilon}, \underline{\omega}) \tag{41}$$

Defining the output vector as

$$\underline{y} = \underline{\varepsilon} \tag{42}$$

$$\dot{\underline{y}} = \dot{\underline{\varepsilon}} = \underline{h}(\ \underline{\varepsilon}\ ,\ \underline{\omega}\ ) \tag{43}$$

$$\ddot{\underline{y}} = \frac{\partial \underline{h}}{\partial \underline{\varepsilon}}\dot{\underline{\varepsilon}} + \frac{\partial \underline{h}}{\partial \underline{\omega}}\dot{\underline{\omega}} \tag{44}$$

$$\ddot{\underline{y}} = \underline{\ell}\ (\ \underline{\varepsilon}\ ,\ \underline{\omega}\ ) + P\ (\ \underline{\varepsilon}\ ,\ \underline{\omega}\ ,\underline{q}\ )\underline{G} \tag{45}$$

Choose $\underline{G}$ as

$$\underline{G} = \bar{P}^{-1}\{-\ \underline{\ell} + \underline{\tau}\ \} \tag{46}$$

It can be shown that $\bar{P}^{-1}$ always exists.
Then

$$\ddot{\underline{y}} = \ddot{\underline{\varepsilon}} = \underline{\tau} \tag{47}$$

form a system of uncoupled equations and they can be expressed in terms of output feedback as

$$\underline{y} = \left[K_p\right]\underline{y} + \left[K_v\right]\dot{\underline{y}} \tag{48}$$

The elements of these gain matrices can be chosen for desired system response.

Results

In figures 2 and 3 , the slewing maneuver of 20° is shown in roll plane. The maneuver shown in figure 2 is for a pure rigid-body slew maneuver whereas figure 3 is the same maneuver with suppression of two first flexible modes. The corresponding case in the pitch plane is shown in figures 4 and 5. A rigid-body slew maneuver is shown about an arbitrary axis in the remaining figures. The effect of first four flexible modes are incorporated in this slew maneuver and figures 6, 7 and 8 in this case represent the moment components to perform this maneuver.

Conclusions

The equations of motion of SCOLE model are highly nonlinear and coupled and this results into the excitation of higher flexible modes when the lower modes are being controlled during the slewing maneuver. Although for this study the beam vibrations at the end of slew maneuver are controlled using Linear Infinite-time Regulator Problem formulation, analysis of higher uncontrolled modes indicates serious control spillover problem due to coupling among the modes.

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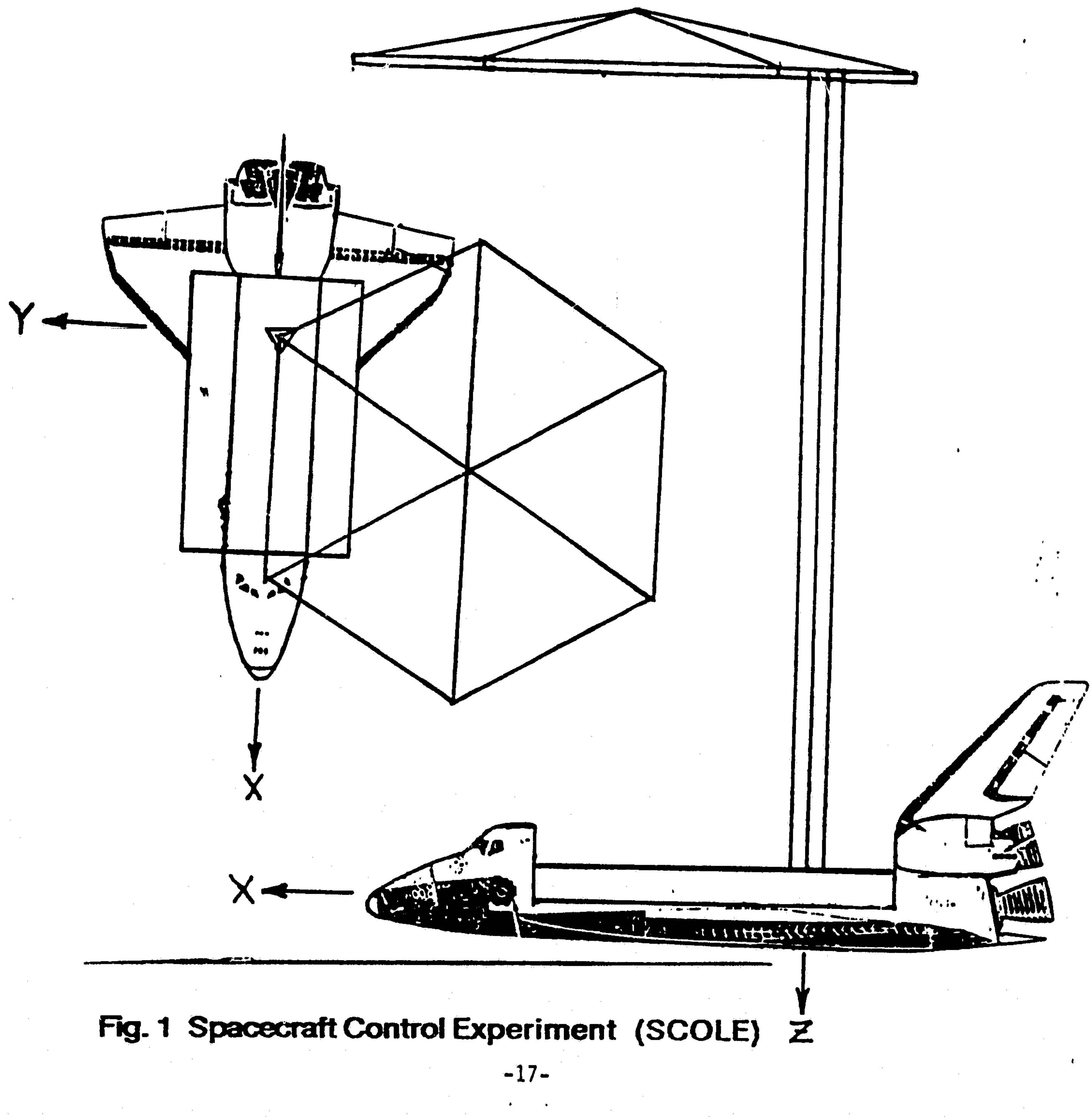

Fig. 1 Spacecraft Control Experiment (SCOLE)

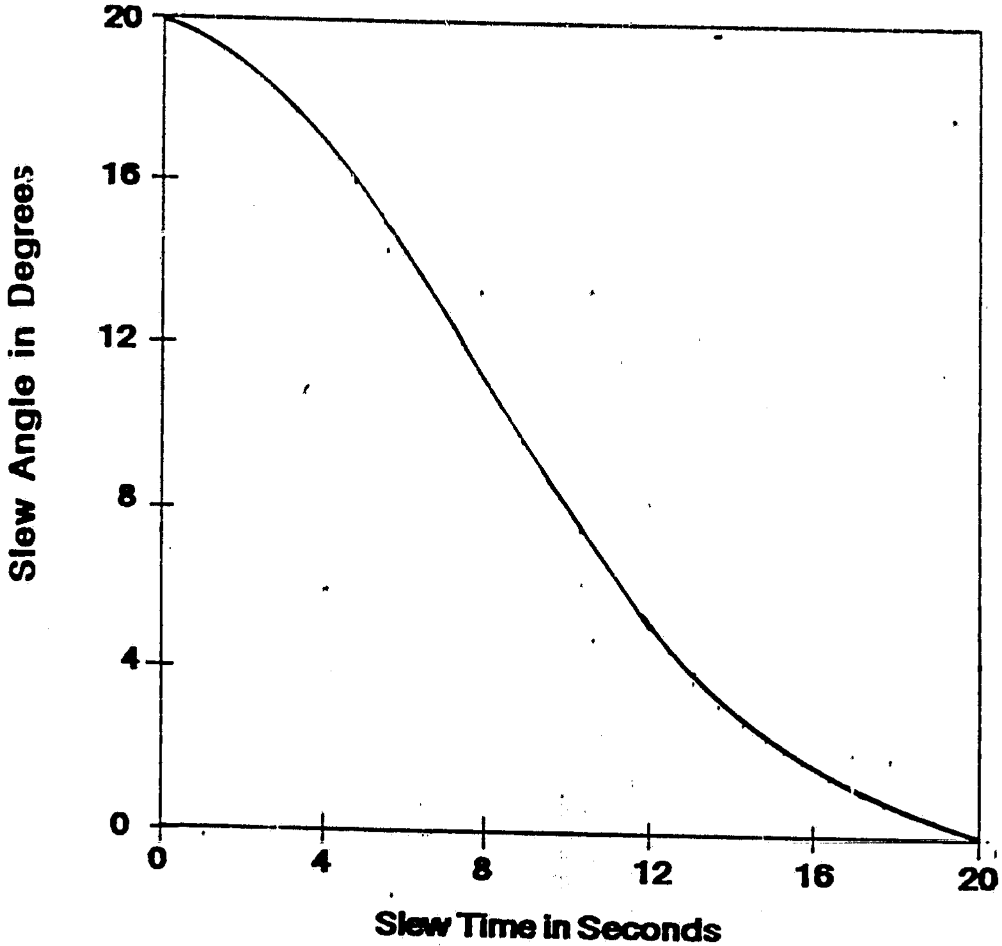

Fig. 2 Slew Angle vs. Time in Roll Plane (Rigid Body Model)

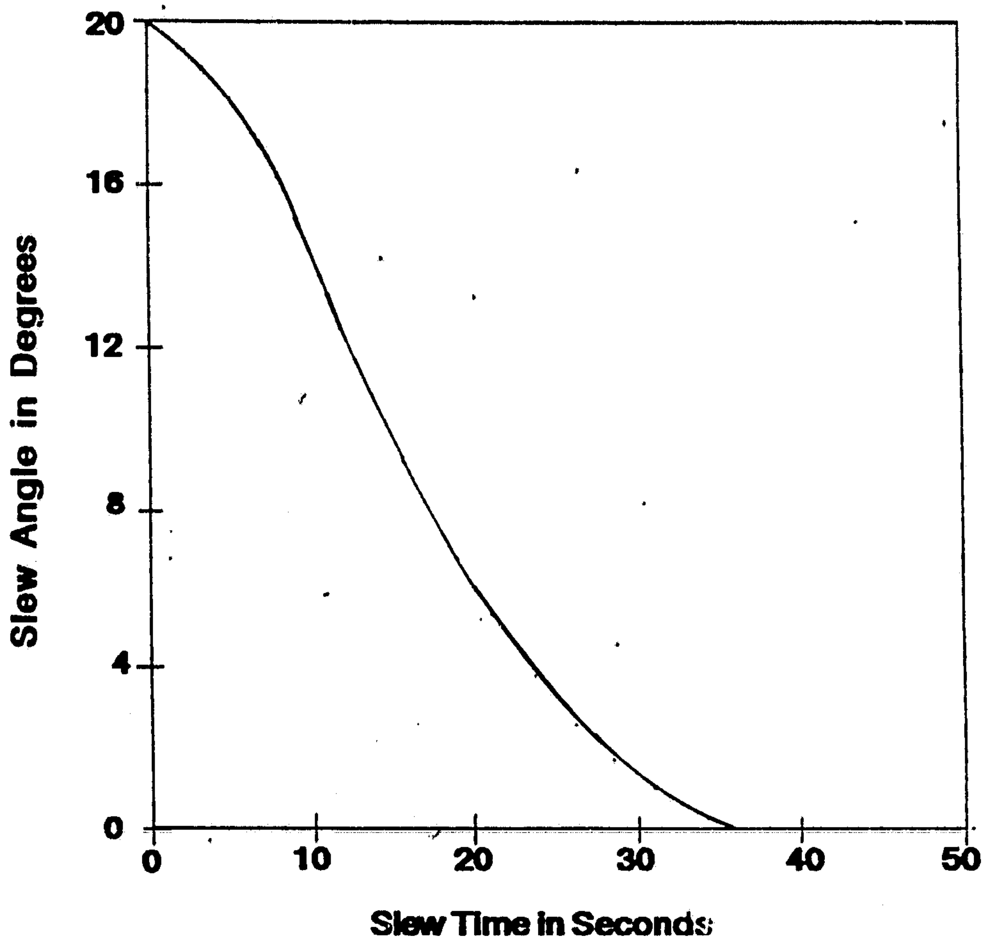

Fig. 3 Slew Angle vs. Time in Roll Plane
(Model with Two Flexible Modes)

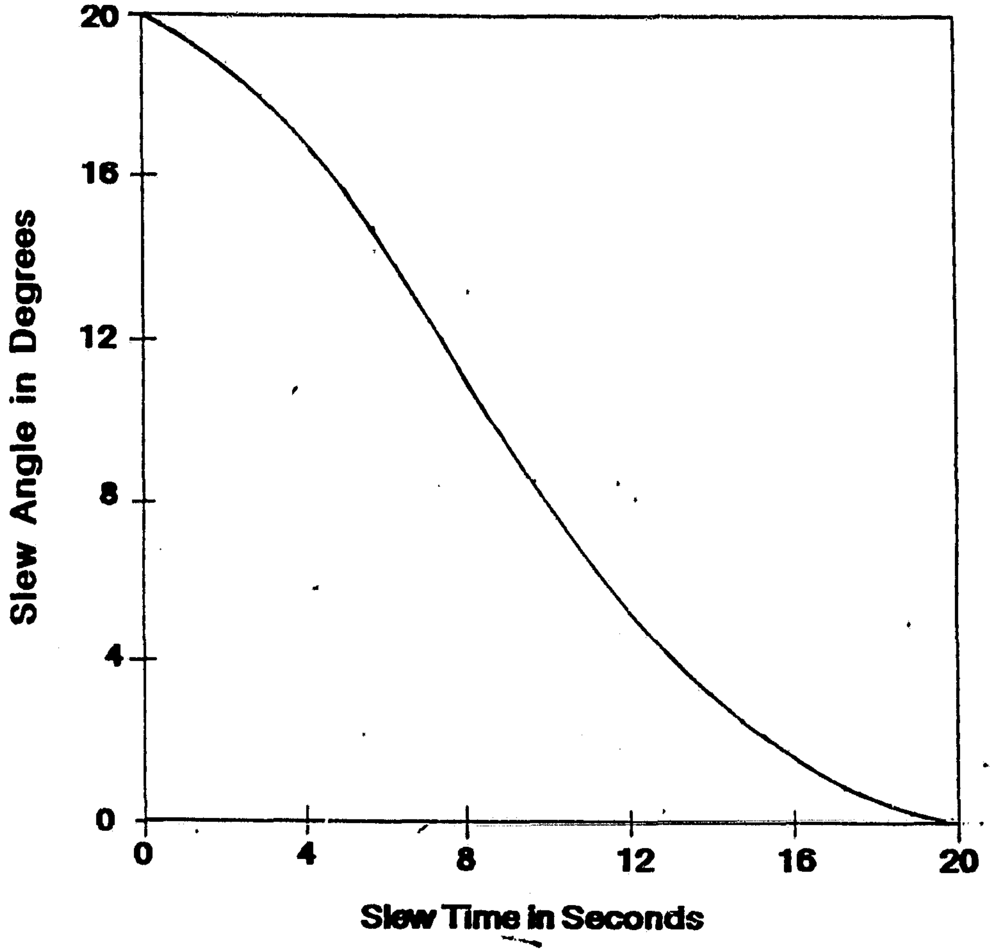

Fig. 4 Slew Angle vs. Time in Pitch Plane (Rigid – Body Model)

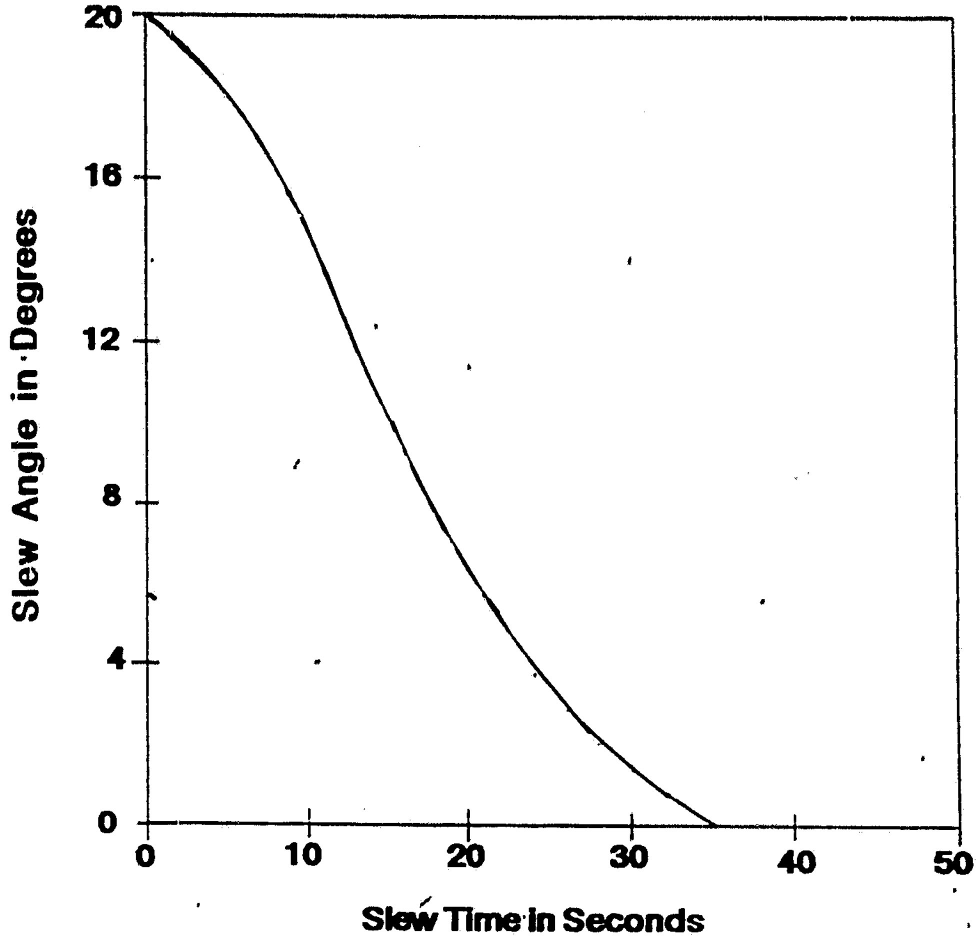

Fig. 5 Slew Angle vs. Time in Pitch Plane (Model with Two Flexible Modes)

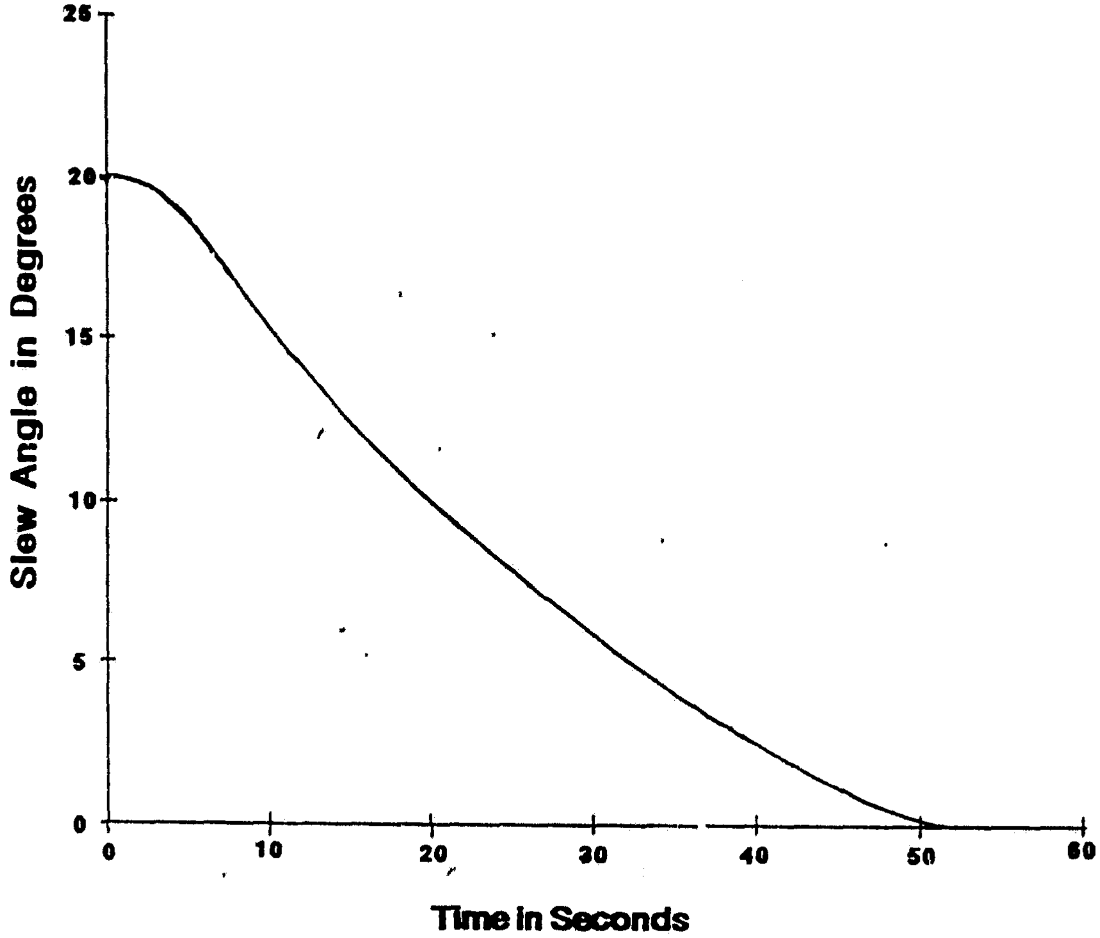

Fig. 6 Slew Angle vs. Time
(Axis of Rotation)
3i +j + 5k

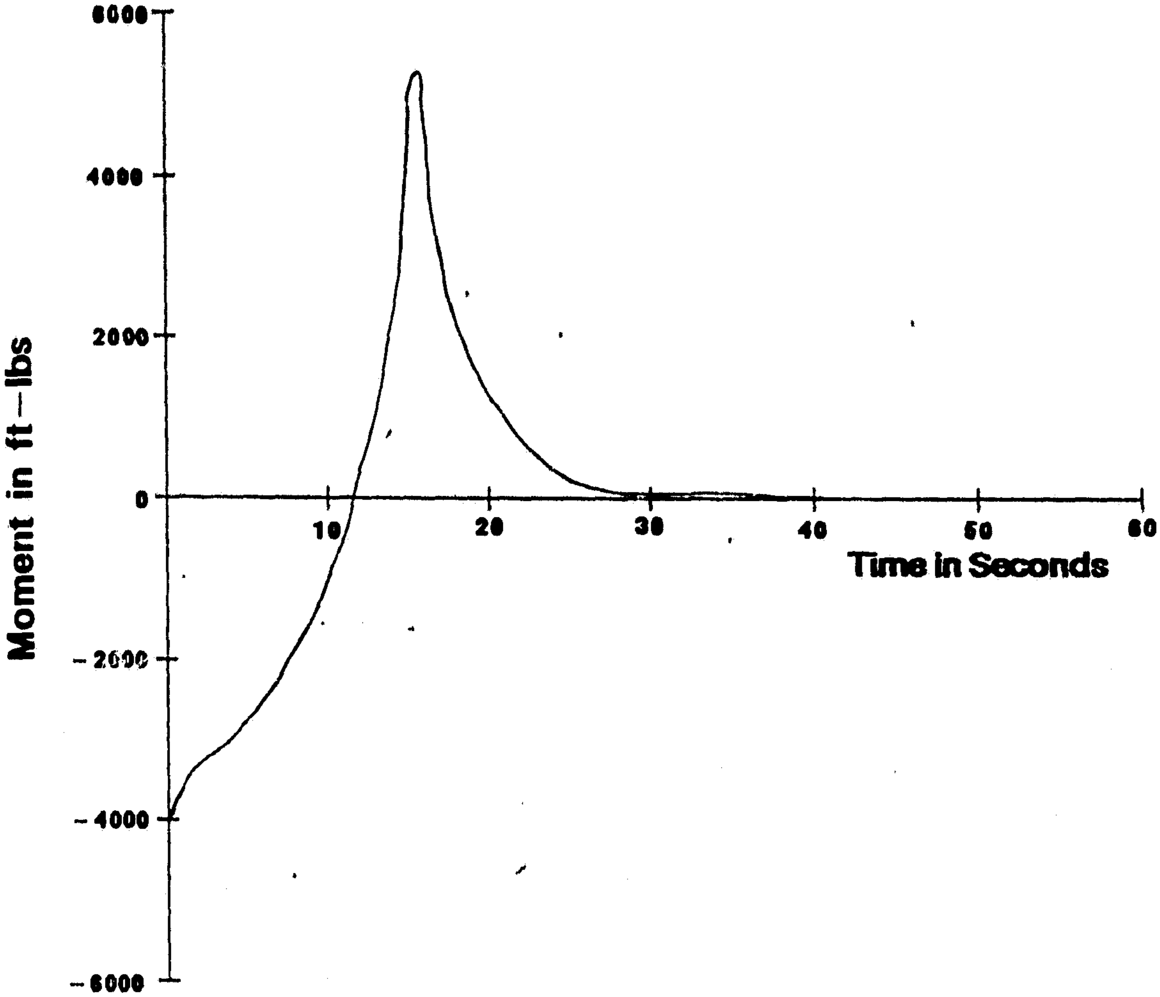

Fig. 7 Moment Component G_1 vs. Time
(Axis of Rotation)
$3i + j + 5k$

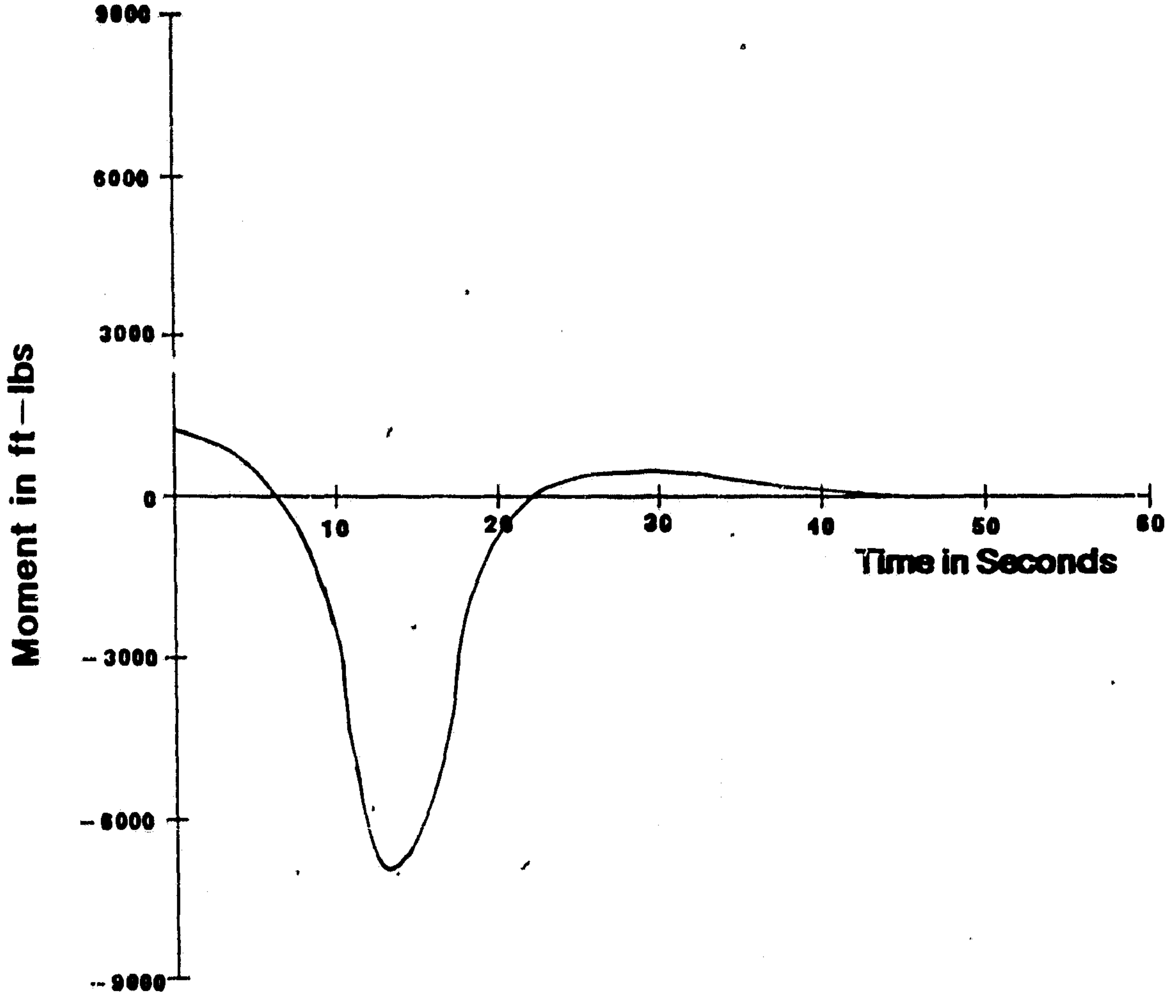

Fig. 8 Moment Component G_2 vs. Time
(Axis of Rotation)
$3i + j + 5k$

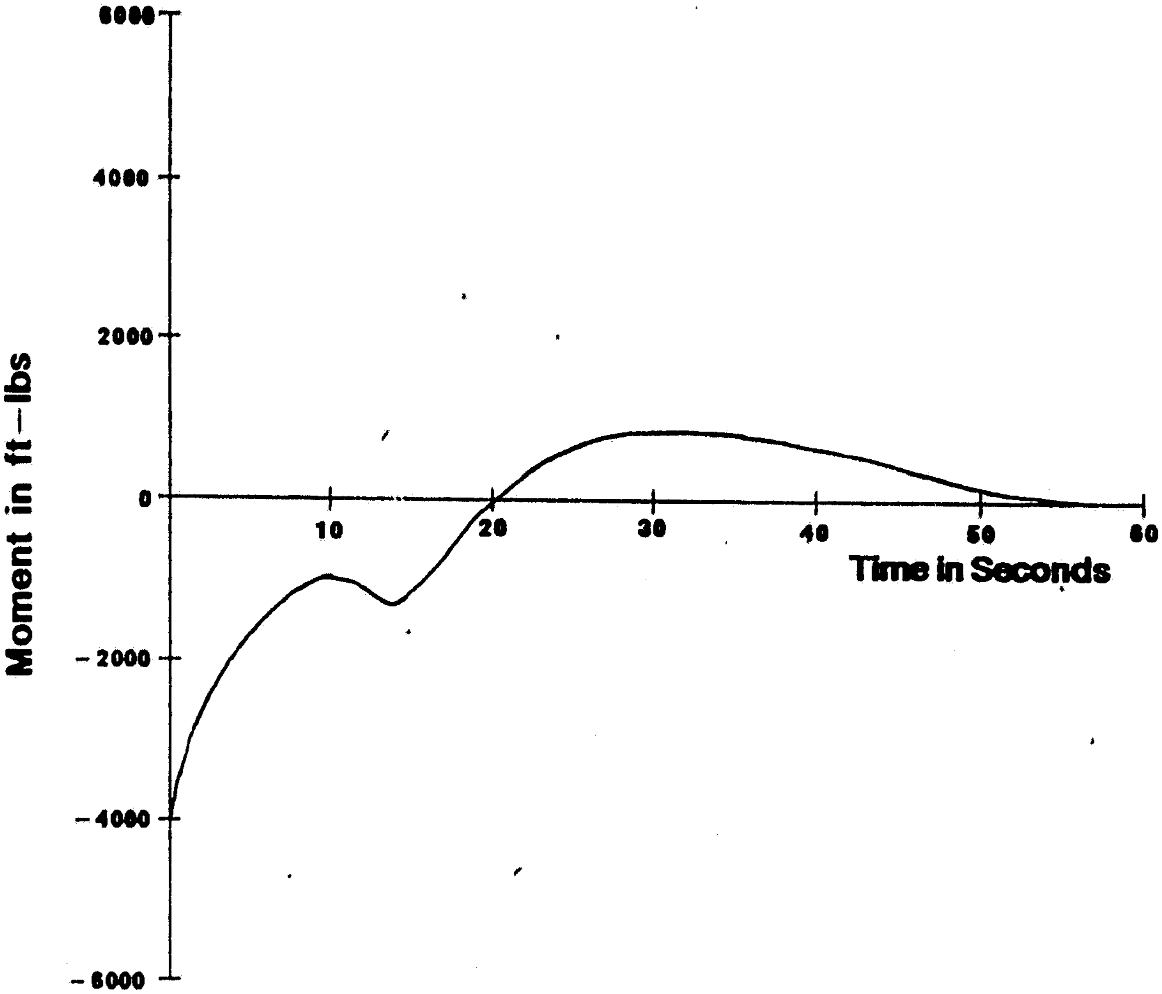

Fig. 9 Moment Component G_3 vs. Time
(Axis of Rotation)
$3i + j + 5k$