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ANALOG SIMULATION OF FLUX-SUMMING SERVO MODEL

PHASES I and II

FINAL REPORT

Covering the Period December 1, 1982-September 30, 1984

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1. ABSTRACT

The analog simulation was developed for a closed-loop system having an electrohydraulic flux-summing servo valve and actuator with associated inertial load. One-fourth of the system's total forward gain is carried by each of four channels. The present study successfully applied failure-mode management techniques to the problem of channel failure. Digital logic circuitry was developed to maintain the overall forward gain of the system at a constant value, in the presence of channel failure.

Finally, the stability of the system was verified, and performance characteristics were determined through the use of frequency response methods.

2. SYSTEM DESCRIPTION

The system under study is shown in block diagram form in Figure 1. The major component of the system is a multi-coil flux-summing electrohydraulic servo valve for which second-order underdamped dynamics have been assumed. The valve is assumed to have a dimensionless damping ratio, ζ, equal to 0.5; an undamped natural frequency, ω_n, equal to 200 rad/sec; and a gain constant, K_{QSV} = 0.0351 cis/ma.

Reduction techniques applied to the block diagram of the actuator yielded a standard second-order form. Details are shown in Appendix I. For the parameter values shown in Table 1, the actuator is a lightly damped, second-order system having a dimensionless damping ratio equal to 0.036, and a natural frequency equal to 3607 rad/sec.

It is assumed that the servo amplifier in the forward path and the demodulator in the feedback path have negligible time constants. Since the presence of an inertial load is assumed, the overall system is fifth-order, Type One.

System variables, their units, and maximum values are listed in Table 1. Also shown are values of all system parameters.

The system also contains a system of four independent redundant channels. It is crucial that the net forward gain of the system be maintained in the event of channel failure, or in the event of the loss of the feedback signal on one channel. Digital logic was developed to simulate the compensation of forward gains in the event of such occurrences. Elements used to implement the required logic included amplifiers, potentiometers, comparators, AND gates, and selector switches. The overall system block diagram showing the points of failure considered in this analysis is given in Figure 2. The digital logic implementation is shown in Figure 3.

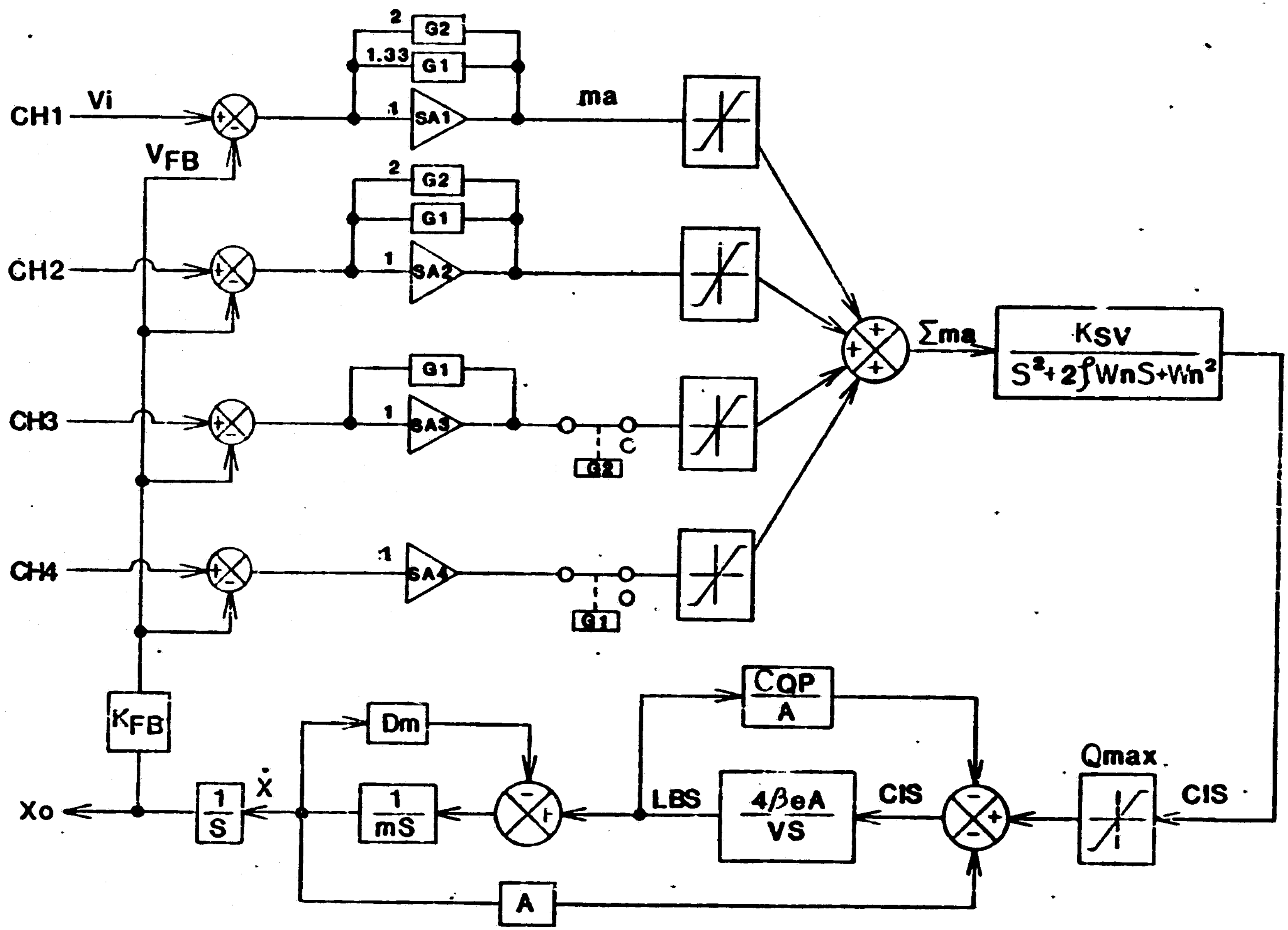


Figure 1. Block Diagram of Overall Closed-Loop Flux-Summing Servo System.

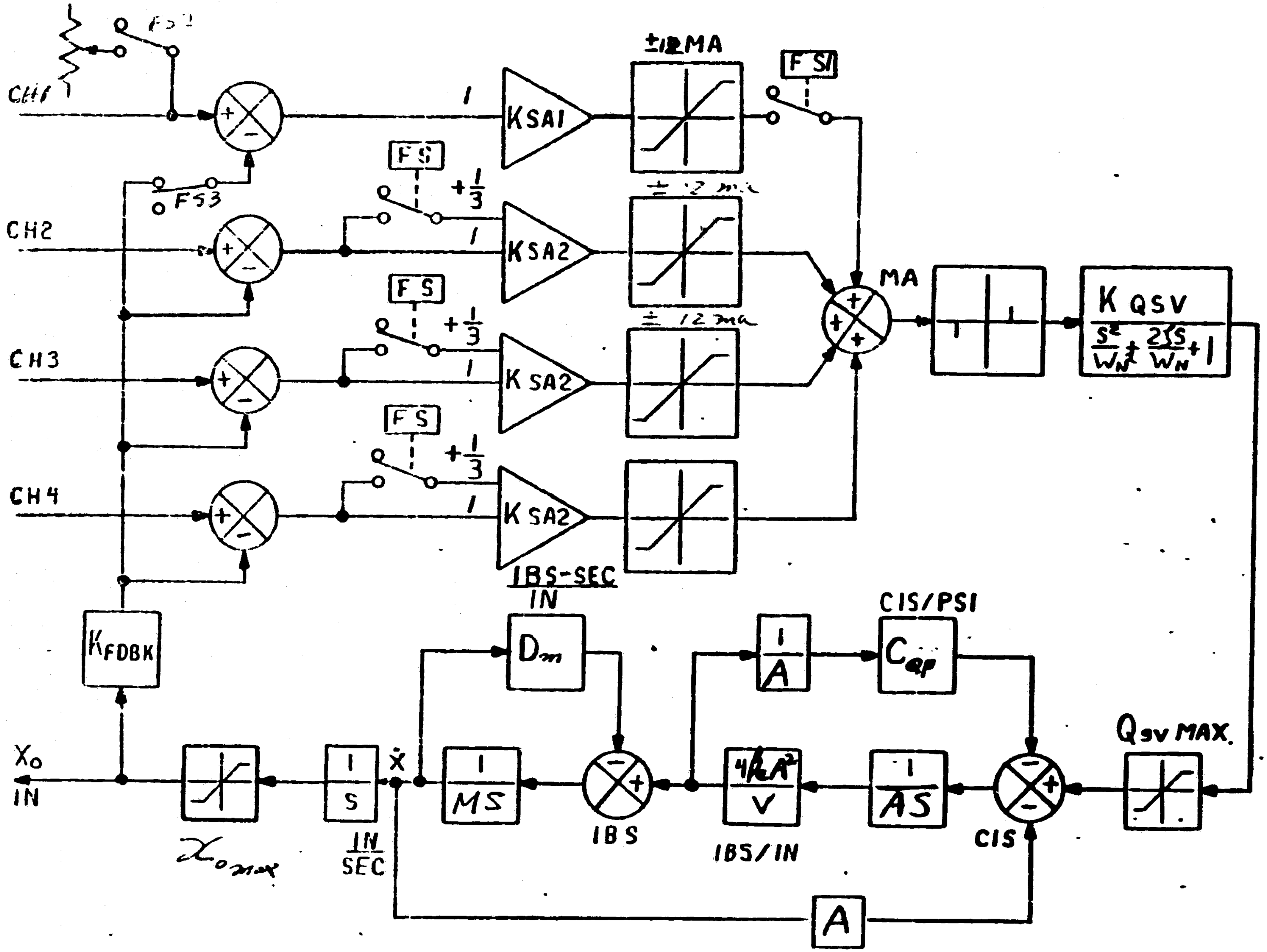


Figure 2. Block Diagram of Overall System Showing Points of Failure Considered.

Table 1.

DESCRIPTION OF CONSTANTS AND VARIABLES IN NASA MODEL OF SERVOVALVE AND ACTUATOR

Symbol	Description	Value/Units
A	Actuator Piston Area	$0.152\ in^2$
C_{QP}	Linearized Flow Coefficient	2.248×10^{-6}
K_{SA}	Servoamplifier Gain	15.6 ma/v
K_{FB}	Feedback Gain (LVDT)	22.22 V/in
M_P	Actuator Piston Mass	$5.2 \times 10^{-3}\ \frac{lb_f\text{-}sec^2}{in}$
V	Actuator Oil Volume	$0.137\ in^3$
β_e	Fluid Bulk Modulus	10^5 psi
D_m	Actuator Piston Damping	$1.32\ \frac{lb_m\text{-}sec}{in}$
K_{QSV}	Servovalve Flow Gain	0.0351 cis/ma

3. ANALOG SIMULATION

Differential equations were recovered from transfer functions of the servo valve and actuator subsvstems. Since the coefficients in the differential equations were widely different in magnitude, scaling was necessary.

For example, let us consider the servo valve initially. The transfer function is:

$$\frac{Y}{X}(s) = \frac{K_{QSV}}{\left(\frac{S}{\omega_{n_1}}\right)^2 + 2\frac{\zeta_1}{\omega_{n_1}} S + 1}$$

$$= \frac{K_{QSV}\,\omega_{n_1}^{\;2}}{S^2 + 2\zeta_1\omega_{n_1} S + \omega_n^{\;2}}$$

$$(S^2 + 2\zeta_1\,\omega_{n_1} S + \omega_{n_1}^{\;2})\; Y\,(S) = K_{QSV}\,\omega_{n_1}^{\;2}\, X(S)$$

INVERTING, WE HAVE:

$$\frac{d^2y}{dt^2} + 2\zeta_1\omega_{n_1}\frac{dy}{dt} + \omega_{n_1}^{\;2}\, y = K_{QSV}\,\omega_{n_1}^{\;2}\, x$$

PARAMETER VALUES ARE:

$K_{QSV} = 0.0351$ cis/ma $\qquad \zeta_1 = 0.5 \qquad \omega_{n_1} = 200$ rad/sec

SUBSTITUTING THESE VALUES INTO THE DIFFERENTIAL EQUATION, WE HAVE:

$$\frac{d^2y}{dt^2} + 200\frac{dy}{dt} + 40{,}000y = 1404x$$

Note that the coefficients differ by more than two orders of magnitude. Scaling is therefore required to reduce the magnitudes of the coefficients and to bring the values of the coefficients closer together. The first method used was unit magnitude scaling. A discussion of this method is given in Appendix 2.

The maximum value of the valve output is estimated at 0.56 CIS. Maximum values of the derivatives of y then form a geometric progression with the undamped natural frequency as the ratio between successive terms, as shown below. It is convenient to round off the values obtained.

$$W_n = 200 \text{ rad/sec}$$

$$Y_{max} = 0.56 \longrightarrow 1$$

$$\dot{Y}_{max} = 0.56(200) = 112 \longrightarrow 100$$

$$\ddot{Y}_{max} = 0.56(200)^2 = 22,4000 \longrightarrow 25,000$$

These scale factors can then be used to obtain the final scaled equations as follows:

$$25,000 \left[\frac{\ddot{y}}{25,000}\right] = -200(100)\left[\frac{\dot{y}}{100}\right] - 40,000 \left[\frac{y}{1}\right] + 1404(20) \left[\frac{x}{20}\right]$$

$$\left[\frac{\ddot{y}}{25,000}\right] = -\frac{200(100)}{25,000} \left[\frac{\dot{y}}{100}\right] - \frac{40,000}{25,000} \left[\frac{y}{1}\right] + \frac{1404(20)}{25,000} \left[\frac{x}{20}\right]$$

THEN, THE SCALED EQUATION IS:

$$\left[\frac{\ddot{y}}{25,000}\right] = -0.8 \left[\frac{\dot{y}}{100}\right] - 1.6 \left[\frac{y}{1}\right] + 1.123 \left[\frac{x}{20}\right]$$

We can see from the coefficient values, that this method of scaling has been effective in this case. A summary of these results is shown below, together with the details of analog programming.

3a. SERVO VALVE SIMULATION

$$X(S) \longrightarrow \boxed{\dfrac{K_{QSV}}{\left(\dfrac{S}{\omega_{n_1}}\right)^2 + 2\dfrac{\zeta_1}{\omega_{n_1}} S + 1}} \longrightarrow Y(S)$$

$$\frac{Y}{X}(S) = \frac{K_{QSV}}{\dfrac{S}{\omega_{n_1}}^2 + 2\dfrac{\zeta_1}{\omega_{n_1}} S + 1}$$

$$= \frac{K_{QSV}\, \omega_{n_1}^{\;2}}{S^2 + 2\zeta_1\, \omega_{n_1} S + \omega_{n_1}^{\;2}}$$

SYSTEM CONSTANTS:

$$K_{QSV} = 0.0351 \frac{\text{cis}}{\text{ma}}\ ;\ \zeta_1 = 0.5;\ \omega_{n_1} = 200 \frac{\text{rad}}{\text{sec}}$$

UNSCALED SYSTEM EQUATION:

$$\frac{d^2y}{dt^2} = -\ 200\ \dot{y} - 40{,}000y + 1404X$$

SCALE FACTORS:

$$y_{max} = 0.56 \longrightarrow 1$$

$$\dot{y}_{max} = 0.56(200) = 112 \longrightarrow 100$$

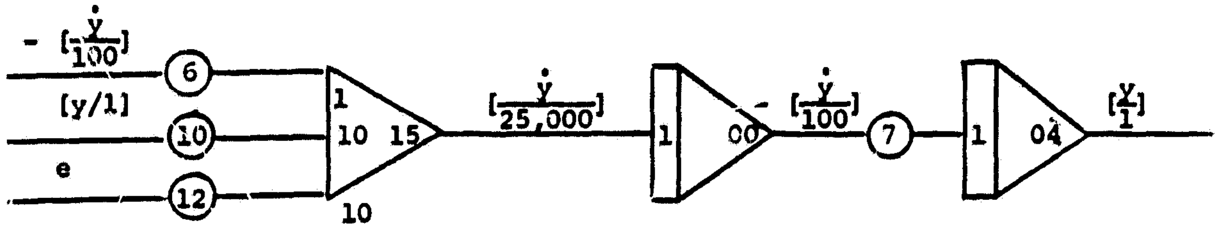
$$\ddot{y}_{max} = 0.56(200)^2 = 22{,}400 \longrightarrow 25{,}000$$

THE SCALED EQUATION:

$$\left[\frac{\ddot{y}}{25{,}000}\right] = -\ 0.8\ \left[\frac{\dot{y}}{100}\right] - 1.6\ \left[\frac{y}{1}\right] + 1.123\ \left[\frac{X}{20}\right]$$

3a. Continued

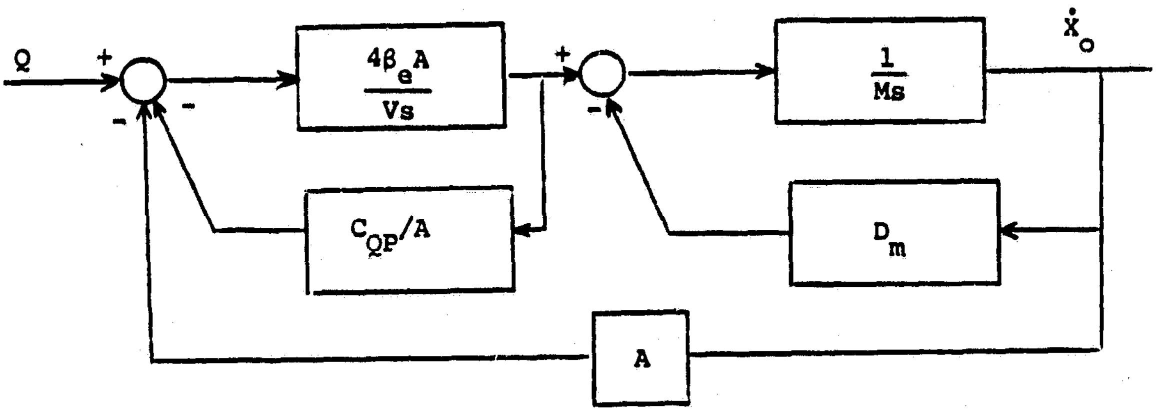
ANALOG SIMULATION



POT NO.	SETTING
6	- 0.8
10	0.16
12	- 0.112

3b. ACTUATOR SIMULATION

THE BLOCK DIAGRAM



$\beta_e = 10^5$ PSI

$D_m = 1.32 \dfrac{lb_f - SEC}{IN}$

$M = 0.0052\ lb_f - SEC^2$

$C_{QP} = 2.248 \times 10^{-6}$ CIS/PSI

$A_p = 0.152\ IN^2$

$V = 0.137\ IN^2$

3b. Continued

STANDARD SECOND-ORDER FORM

$$Q = X \longrightarrow \boxed{\dfrac{1/A}{\left(\dfrac{s}{\omega_{n_2}}\right)^2 + 2\,\dfrac{\zeta_2}{\omega_{n_2}}\,s + 1}} \longrightarrow X_o = Y$$

$\zeta_2 = 0.036$ $\qquad\qquad$ $\omega_{n_2} = 3607$ rad/sec

UNSCALED DIFFERENTIAL EQUATION

$$\frac{d^2y}{dt^2} = -259.9\,\frac{dy}{dt} - 1.303 \times 10^7 y + 8.57 \times 10^7 x$$

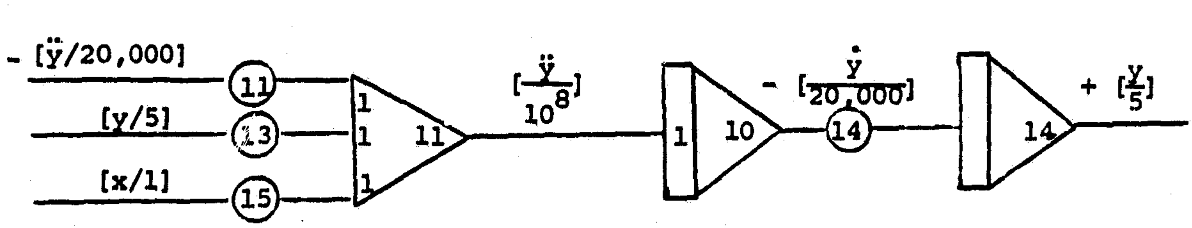
SCALE FACTORS

$$x_{max} = 1.0$$
$$y_{max} = 4.5 \longrightarrow 5$$
$$\dot{y}_{max} = 4.5\ (3607) = 16{,}250 \longrightarrow 20{,}000$$
$$\ddot{y}_{max} = 4.5\ (3607)^2 = 5.86 \times 10^7 \longrightarrow 10^8$$

THE SCALED EQUATION

$$\left[\frac{\ddot{y}}{10^8}\right] = -0.052\left[\frac{\dot{y}}{20{,}000}\right] - 0.651\left[\frac{y}{5}\right] + 0.858\left[\frac{x}{1}\right]$$

THE ANALOG SIMULATION



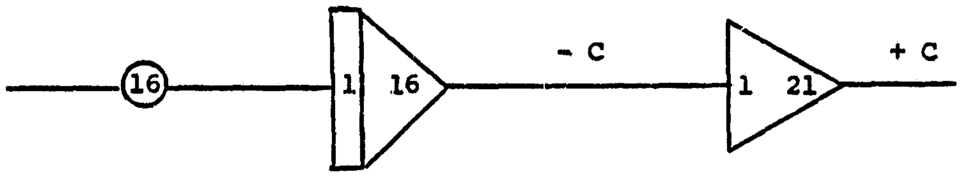
3b. Continued

THE ANALOG SIMULATION (continued)

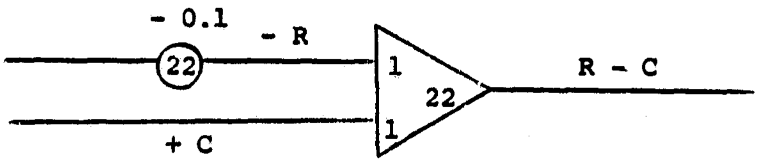
POT. NO.	SETTING
11	- 0.052
13	0.651
15	- 0.858

3c. ANALOG SIMULATION DIAGRAMS

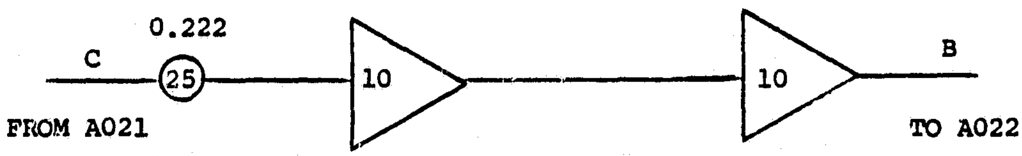
THE VELOCITY ELEMENT



ERROR DETECTOR



KFB



4. FAILURE-MODE MANAGEMENT CIRCUITRY

One of the most important aspects of the present study was the consideration of failure-mode management. Digital logic circuitry was developed which ensured the maintenance of the overall system forward gain at a desired level, in the presence of channel failures.

The complete implementation of the failure-mode management capability is shown in Figure 3. The actual physical system will have four channels. The present implementation, however, was designed for only three channels based on current availability of Ames EAI - 2000 Analog processor logic components. Circuitry for the fourth channel can be added when additional components become available.

The logic is designed with limited function to simulate the input channel or feedback open failure only at the summing junction of summing amplifier. The purpose is to evaluate the failure mode management gain compensation network and the failure mode dynamic performance. It is not designed for cross channel monitoring techniques evaluation.

Points of failure are indicated in Figure 2. FSI can be used to simulate the loss of signal in the feedback path to Channel 1, and FS2 permits the introduction of a disturbance signal (bias voltage) to Channel 1.

The essential requirement is that, in the event of a failure in one of the channels, the forward gain in each of the other two channels is increased by a factor of one-sixth. The net forward gain is thus maintained at the constant desired level. It was observed during tests that the excitation of system dynamics was minimal when such failures occurred. This was to be expected since the logic elements respond so much faster than the analog elements, that, for practical purposes, they may be considered to act instantaneously. For the open-loop tests, the results, which consist of element output values, are shown in Table 2.

4. FAILURE-MODE MANAGEMENT CIRCUITRY (continued)

The basic operation of the logic elements are shown in Appendix 3. The comparator acts as an analog to digital interface; i.e., its inputs are analog signals, and its outputs are logic signals. If the difference in value of its two inputs is positive, the output equals logic 1, if the difference is negative, the output is logic 0. The logic gate is a three input AND (NAND). The selector switch selects one of two analog inputs depending on the value of the logic control input, SELA. If SELA = 0, the switch output is input A. If SELA = 1, the switch output is input B.

Now let us consider several tests in detail. Refer to Figure 2 for component identification. It is assumed throughout the subsequent discussion that proper system operation requires that the net input signal (algebraic sum of all input signals) to each channel lies within the range, ± 1 volt. If a signal lies outside this range, the channel output is set to zero, and the channel is effectively taken off line. The gain in each of the other two channels is then increased by a factor of one-sixth so that the total forward gain is maintained.

In the computations below, appropriate element input and/or output values are computed, starting with the leftmost amplifiers of Figure 2. Computations of element output values continue to the rightmost amplifiers, unless it is determined that a signal lies outside the range, ± 1 volt, and is, therefore, blocked.

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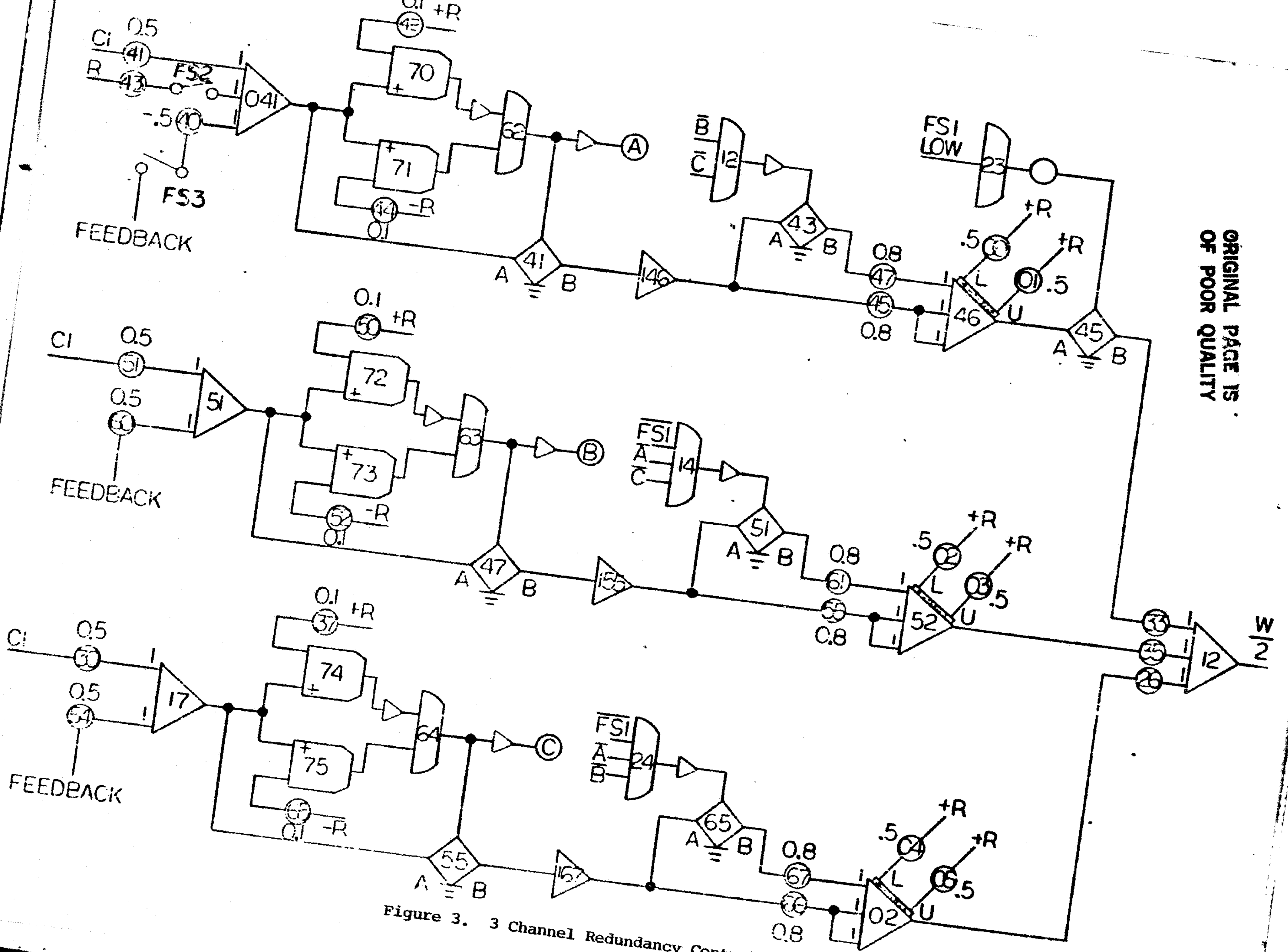


Figure 3. 3 Channel Redundancy Control Logic.

TEST I

CHECK WHETHER SIGNALS LESS THAN -1 VOLT ARE BLOCKED.

FS2 Closed, FS3 Open (Failure in Feedback signal)

POT 41 = 0.5, POT 43 = 0.1

Then, e_i = - 5 v., = r = - 1 v

NOW COMPUTE ELEMENT INPUT AND OUTPUT VALUES, STARTING WITH AMPLIFIER 41, AND WORKING TO THE RIGHT

Output AMP 41 = - 5 - 1 = - 6 v. (Outside acceptable range)

POT 42 = 0.1

Input COMP 70 = - 6 - 1 = - 7v (<0)

Output COMP 70 = LOGIC 0

POT 44 = 0.1

Input COMP 71 = - 6 - (-1) = - 5v (<0)

Output COMP 71 = LOGIC 0

Output Gate 62 = 0·1 = 0

THE OUTPUT OF GATE 62 IS THE CONTROL LOGIC SIGNAL FOR SELECTOR SWITCH 41. SINCE THIS SIGNAL HAS VALUE ZERO, OUTPUT B, WHICH IS EQUAL TO ZERO VOLTS, APPEARS AT THE SWITCH OUTPUT. THEREFOR SWITCH 41 BLOCKS THE - 6 VOLT SIGNAL AND CHANNEL 1 IS EFFECTIVELY TAKEN OFF-LINE.

TEST 2

INDUCE FAILURE ON THE HIGH SIDE OF THE ACCEPTABLE RANGE

FS2 Closed, FS3 Open (as in Test I)

POT 42 = - 0.5; POT 43 = + 0.1

e_i = - 5.0 v, r = 1.0 v

Output AMP 41 = - (- 5 + 1) = + 4v (Out of range on the high side.)

Input COMP 70 = + 4 - 1 = + 3 >0

Output COMP 70 = 1

Input COMP 71 = + 4 + 1 = + 5v

Output COMP 71 = LOGIC 1

Output GATE 62 = 1·0 = 0

LOGIC Input, Switch 41 = 0

Output, Switch 41 = 0

THEREFORE, THE OUTPUT OF AMPLIFIER 41 IS AGAIN EFFECTIVELY BLOCKED.

TEST 3

CHECK FOR PROPER OPERATION - SIGNAL IS WITHIN LIMITS - INDUCE FAILURE IN FS1.

FS2 Closed, FS3 Closed

Set POT 43 = 0.01, POT 41 = 0.5, POT 40 = - 0.5

Output, AMP 41 = - (5.0 + 0.1 - 5.0) = - 0.1 v

Input, COMP 70 = (- 0.1 v) - 1.0 = - 1.1 v

Output, COMP 70 = 0

Input, Gate 62 = $\bar{0}$ = 1

Input, COMP 71 = - 0.1 - (- 1.0) = + 0.9 v

Output, COMP 71 = LOGIC 1

Output, Gate 62 = 1·1 = 1

LOGIC Input, Switch 41 = 1

Output, Switch 41 = - 0.1 v

Output. AMP 45 = + 0.1 v

CONTROL SIGNAL TO SWITCH 43 IS EQUAL TO ZERO, UNLESS THERE IS A FAILURE IN EITHER CHANNEL 2 OR CHANNEL 3. THEREFORE, THE OUTPUT OF SWITCH 43 = 0. (NO COMPENSATION REQUIRED)

Output, AMP 46 = - (2) (0.8) (+ 0.1 v)

= - 0.16 v (Normal System Output)

NOW CONSIDER TWO OPTIONS FOR FS1

I. PROPER OPERATION - (Note that unpatched logic inputs are high)

FS1 LOW MEANS PROPER OPERATION

OUTPUT, GATE 23 = 0

LOGIC INPUT, SWITCH 45 = 1

OUTPUT, SWITCH 45 = - 0.16 v (Signal goes through, correct operation)

II. FS1 HIGH MEANS CHANNEL FAILURE

OUTPUT, GATE 23 = 1

LOGIC INPUT, SWITCH 45 = 0

OUTPUT, SWITCH 45 = 0 (Channel failure, output blocked)

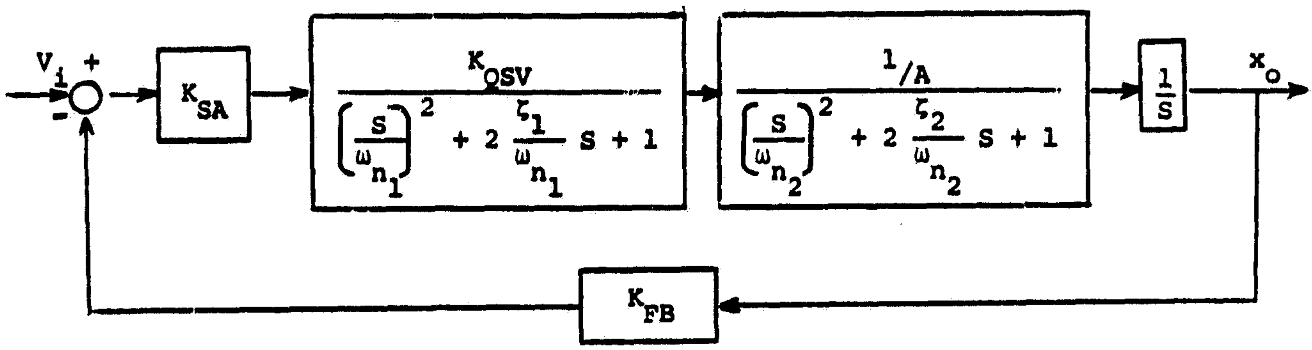
THE ABOVE DISCUSSION GIVES THE ESSENCE OF THE DESIGN. THESE THREE TESTS, AS WELL AS SEVERAL OTHERS ARE TABULATED IN TABLE 2.

Table 2.

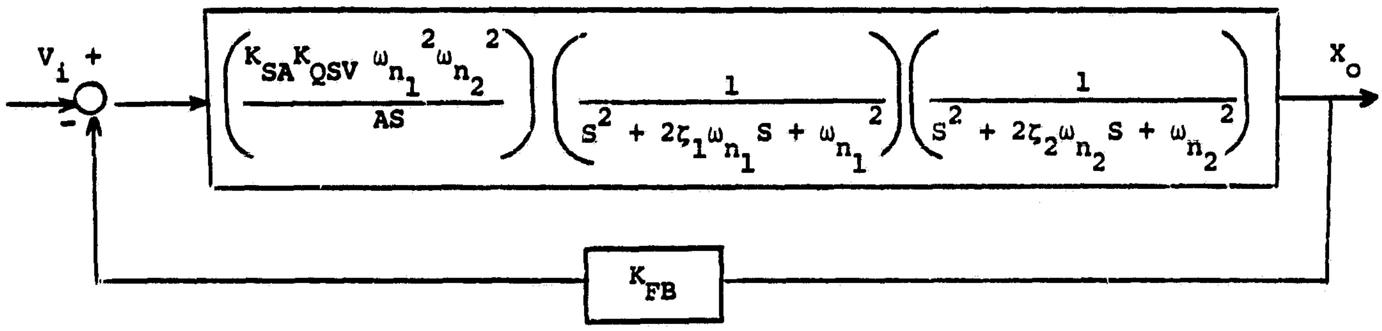
TEST RUNS (OPEN-LOOP), DIGITAL LOGIC CIRCUITRY

TEST NO.	FS1	FS2	FS3	POT 41	POT 43	e_i	r	A41	COMP 70	COMP 71	GATE 62	A B	SS41	A45	SS43	A46	GATE 23	SS45
1.	Closed	Closed	Open	0.5	0.1	5v	1v	-6	0	0	0	1	0	CHANNEL 1 OUTPUT BLOCKED				
2.	"	"	"	-0.5	0.1	-5v	1v	+4	+1	1	0	1	0					
3.	Low	Closed	Closed	0.5	0.01	5.0	0.1v	-0.1v	0	1	1	0	-0.1v	+0.1		-0.16	0 → 1	-0.16
	High	Other conditions as above - Channel Failure																
4.	Closed	Closed	Closed	0.5	0.09	5.0	0.9	-0.9v	0 → 1	1	1	0	-0.9	0.9v		1.44		
	CHANNEL 2 TESTS																	
				POT 51	POT 60	e_i	r	A51	COMP 72	COMP 73	GATE 63		SS47	A55	SS51	A46		
1.				0.4	0.6	4.0v	-6.0v	2.0v	1 → 0	0	0		0		SIGNAL BLOCKED			
2.				0.5	-0.55	5.0v	-5.5v	+0.5v	0 → 1	1	1		0.50		SIGNAL GOES THROUGH			

5. DERIVATION OF CLOSED-LOOP TRANSFER FUNCTION



ALGEBRAIC MANIPULATION YIELDS:



$$\text{DEFINE } K' = \frac{K_{SA}K_{QSV}\omega_{n_1}^{2}\omega_{n_2}^{2}}{A}$$

THEN,

$$\frac{X_o}{V_i} = \frac{K'}{S(S^2 + 2\zeta_1\omega_{n_1} S + \omega_{n_1}^{2})(S^2 + 2\zeta_2\omega_{n_2} S + \omega_{n_2}^{2}) + K'K_{FB}}$$

$$= \frac{K'}{S^5 + 2(\zeta_1\omega_{n_1} + \zeta_2\omega_{n_2})S^4 + (\omega_{n_1}^{2} + \omega_{n_2}^{2} + 4\zeta_1\omega_{n_1}\zeta_2\omega_{n_2})S^3 + \\ + 2\omega_{n_1}\omega_{n_2}(\zeta_2\omega_{n_1} + \zeta_1\omega_{n_2})S^2 + \omega_{n_1}^{2}\omega_{n_2}^{2}S + K'K_{FB}}$$

5. DERIVATION OF CLOSED-LOOP TRANSFER FUNCTION (continued)

$$= \frac{1.875 \times 10^{12}}{s^5 + 4.597 \times 10^2 s^4 + 1.311 \times 10^7 s^3 + 2.746 \times 10^4 s^2 + 5.205 \times 10^{11} s + 4.162 \times 10^{13}}$$

$$2(\zeta_1 \omega_{n_1} + \zeta_2 \omega_{n_2}) = 2[(0.5)(200) + (0.036)(3607)] = 4.597 \times 10^2$$

$$\omega_{n_1}^{\ 2} + \omega_{n_2}^{\ 2} + 4\zeta_1 \omega_{n_1} \zeta_2 \omega_{n_2} = (200)^2 + (3607)^2 + 4(0.5)(0.036)(200)(360) = 1.311 \times 10^7$$

$$2\omega_{n_1} \omega_{n_2} (\zeta_2 \omega_{n_1} + \zeta_1 \omega_{n_1}) = 2(200)(3607)[0.1(200) + 0.5\ (3607)] = 2.746 \times 10^9$$

$$\omega_{n_1}^{\ 2} \omega_{n_2}^{\ 2} = (200)^2 (360)^2 = 5.204 \times 10^{11}$$

$$K' = \frac{K_{SA} K_{QSV} \omega_{n_1}^{\ 2} \omega_{n_2}^{\ 2}}{A} = \frac{(15.6)(0.0351)(200)^2 (3607)^2}{0.156} = 1.875 \times 10^{12}$$

$$K' K_{FB} = 1.875 \times 10^{12} \times 22.2 = 4.162 \times 10^{13}$$

6. ROUTH'S CRITERION - SYSTEM STABILITY DETERMINATION

$$s^5 + 4.597 \times 10^2 s^4 + 1.311 \times 10^7 s^3 + 2.746 \times 10^9 s^2 + 5.204 \times 10^{11} s + 4.162 \times 10^{13}$$

THE ROUTH ARRAY:

s^5	1	1.311×10^7	5.204×10^{11}
s^4	4.597×10^2	2.746×10^9	4.162×10^{13}
s^3	7.137×10^6	4.299×10^{11}	
s^2	2.741×10^9	4.162×10^{13}	
s^1	3.214×10^{11}		
s^0	4.162×10^{13}		

NO SIGN CHANGES IN LEFT COLUMN, SYSTEM IS STABLE.

7. RESPONSE CHARACTERISTICS OF SUBSYSTEMS AND OVERALL SYSTEMS

7a. SUBSYSTEMS

THE TWO BASIC DYNAMIC ELEMENTS IN THE SYSTEM ARE THE SERVO VALVE AND THE ACTUATOR. BOTH SUBSYSTEMS ARE ASSUMED TO HAVE SECOND-ORDER DYNAMICS. LET US CONSIDER THE PERFORMANCE CHARACTERISTICS OF THESE TWO SUBSYSTEMS SEPARATELY, BEFORE WE CONSIDER THE PERFORMANCE OF THE OVERALL SYSTEM.

THE TRANSFER FUNCTION OF THE VALVE IS:

$$\frac{Y}{S}(s) = \frac{K_{QSV}}{\left[\frac{s}{\omega_{n_1}}\right]^2 + 2\,\frac{\zeta_1}{\omega_{n_1}}\,s + 1}$$

$$= \frac{0.0351}{\left(\frac{s}{200}\right)^2 + 2\,\frac{(0.5)}{(200)}\,s + 1}$$

SINCE THE ELEMENT IS ASSUMED TO HAVE SECC ORDER DYNAMICS, WE CAN ESTIMATE ITS IMPORTANT TIME-DOMAIN CHARACTERISTICS FROM THE FOLLOWING RELATIONS:

$$M_p = 1 + \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) = 1 + \exp\left(\frac{(0.5)\pi}{\sqrt{1-(0.5)^2}}\right)$$

$$= 1.16$$

% OVERSHOOT = 16%

t_p = time to peak overshoot

$$= \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{200\sqrt{1-(0.5)^2}}$$

$$= 0.0234 \text{ sec.}$$

$$= 23.4 \text{ msec.}$$

7a. SUBSYSTEMS (continued)

THE FREQUENCY RESPONSE FIGURES OF MERIT ARE:

$$M_m = \frac{1}{2\zeta\sqrt{1 - \zeta^2}} = \frac{1}{2(0.5)\sqrt{1 - (0.5)^2}} = 1.154$$

$$M_{m_{db}} = 20 \log 1.154 = 1.24\text{db}$$

NOTE THE CLOSE AGREEMENT WITH M_p FOR THIS VALUE OF ζ.

$$\omega_n = \omega_n \sqrt{1 - 252} = 200 \sqrt{1 - 2(0.5)^2}$$

$$= 200 \quad 1 - 2(025)$$

$$= 200 \sqrt{0.5}$$

$$= 141.4 \text{ r/s}$$

THE TRANSFER FUNCTION OF THE ACTUATOR IS:

$$\frac{Y}{X}(S) = \frac{1/A}{\left[\frac{S}{\omega_{n_2}}\right]^2 + 2\left[\frac{\zeta_2}{\omega_{n_2}}\right] S + 1}$$

WHERE $\zeta_2 = 0.036$, $\omega_{n_2} = 3607$

TIME-DOMAIN CHARACTERISTICS:

$$M_p = 1 + \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) = 1 + \exp\left(\frac{-(0.036)(\pi)}{\sqrt{1-(0.036)^2}}\right)$$

$$= 1.893$$

% OVERSHOOT = 89.3%

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{(3607)\sqrt{1-(0.036)^2}} = \frac{\pi}{(3607)(0.9994)}$$

$$= 0.0008715 \text{ sec.}$$

= $\underline{0.872}$ ms VERY FAST SYSTEM

$$M_m = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{1}{2(0.036)} = \frac{1}{\sqrt{1-(0.036)^2}}$$

$$= \frac{1}{0.072(0.994)} = 13.98 = 22.9 \text{ db}$$

$$\omega_m = \omega_n\sqrt{1-2\zeta^2}$$

TIME-DOMAIN CHARACTERISTICS: (continued

$$= 3607 \sqrt{1 - 2(0.036)^2}$$

$$= 3607 (0.9987)$$

$$= 3602 \text{ r/s}$$

THE ACTUATOR'S VERY LOW VALUE OF DAMPING RATIO, OF COURSE, ENSURES A VERY RAPID RESPONSE. THE CONSEQUENT HIGH RESONANT PEAK COULD LEAD TO HIGH FREQUENCY NOISE PROBLEMS. A CLOSED-LOOP FREQUENCY DIAGRAM SHOWING THE SYSTEM GAIN AT FREQUENCIES NEAR THE ACTUATOR'S RESONANT PEAK WILL SHOW WHETHER THIS IS A REAL PROBLEM. FROM THE CLOSED-LOOP FREQUENCY RESPONSE DIAGRAM OF THE OVERALL SYSTEM, IT IS CLEAR THAT ANY INPUTS NEAR THE ACTUATOR'S RESONANT PEAK WILL BE ATTENUATED BY A FACTOR OF ONE THOUSAND. See Figure 4 on following page.

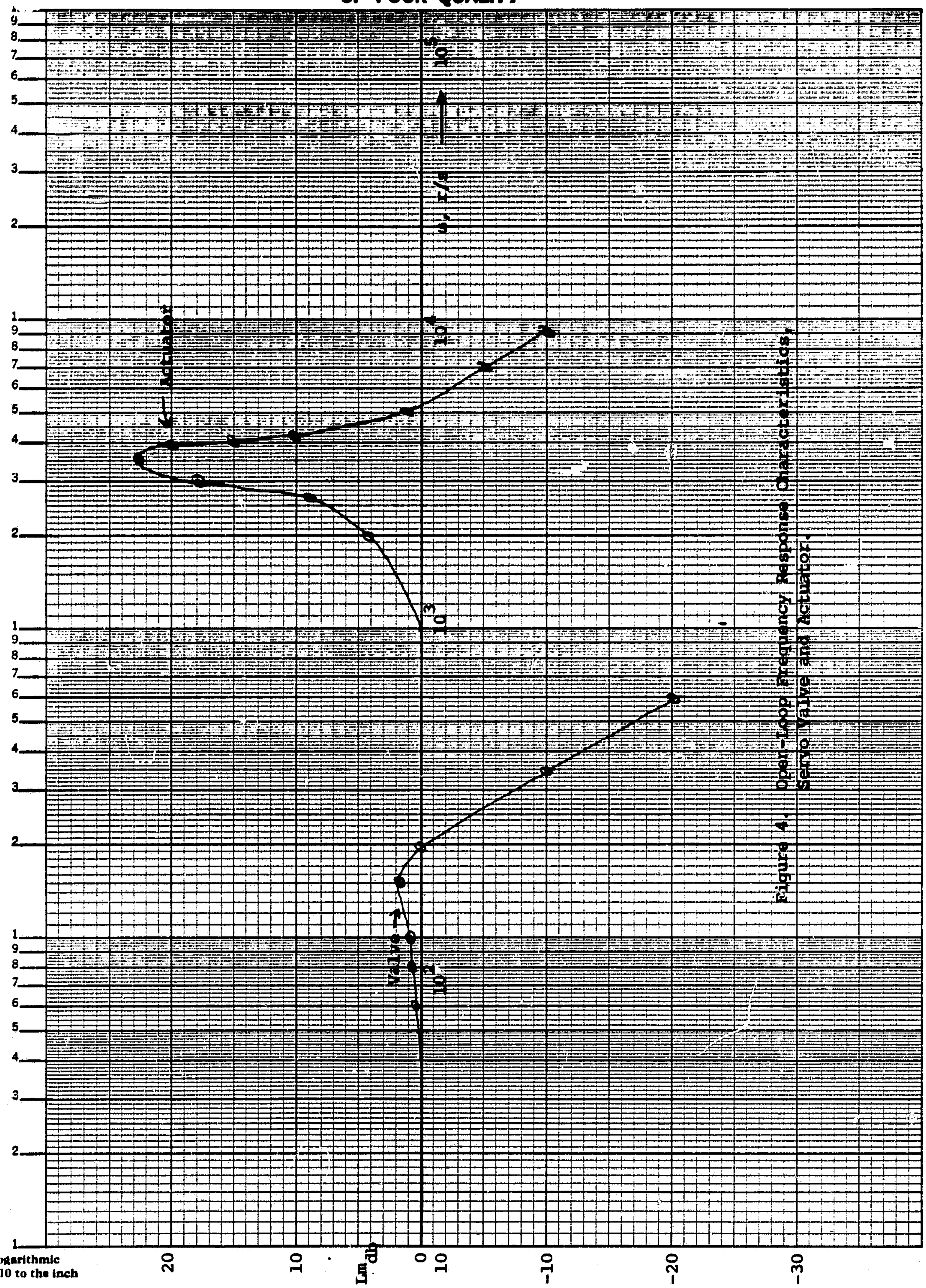
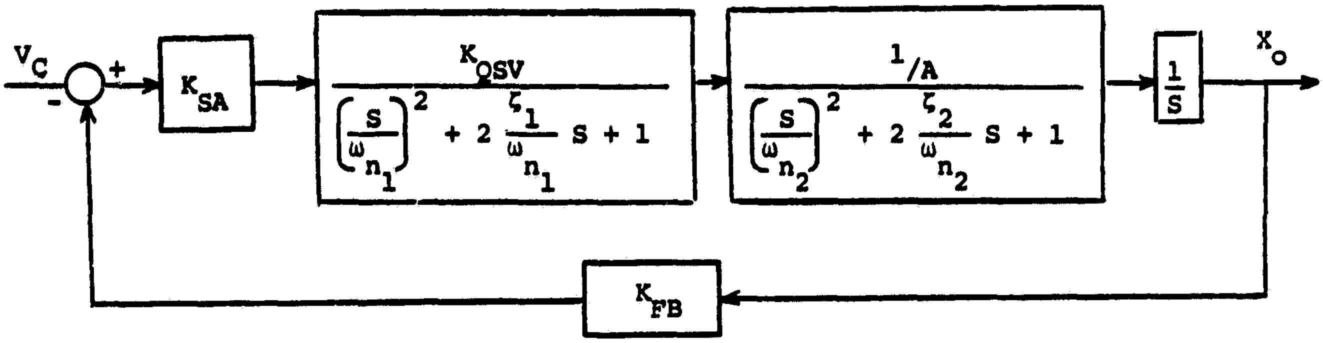


Figure 4. Open-Loop Frequency Response Characteristics, Servo Valve and Actuator.

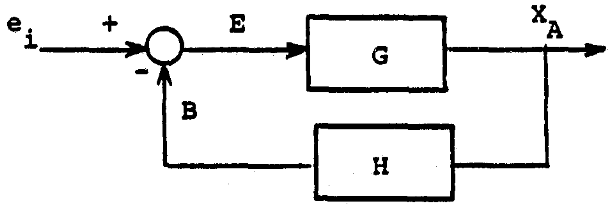
7b. OVERALL SYSTEM

BLOCK DIAGRAM, OVERALL SYSTEM



GAINS:	**VALVE:**	**ACTUATOR:**
$K_{SA} = 15.6$	$\omega_{n_1} = 200$ r/s	$\omega_{n_2} = 3607$ r/s
$K_{QSV} = 0.0351$	$\zeta = 0.5$	$\zeta_2 = 0.036$
$\frac{1}{A} = 6.58$		
$K_{FB} = 22.2$		

RELATE TO FORM



$$G = \frac{K_{SA}K_{QSV}}{A} \quad \frac{1}{S\left(\left(\dfrac{S}{\omega_{n_1}}\right)^2 + 2\dfrac{\zeta_1}{\omega_{n_1}} S + 1\right)\left(\left(\dfrac{S}{\omega_{n_2}}\right)^2 + 2\dfrac{\zeta_1}{\omega_{n_2}} S + 1\right)}$$

$$K' = \frac{K_{SA}K_{QSV}}{A} = 15.6(0.0351)(6.58) = 3.603 = 11.1 \text{ db}$$

$$\frac{1}{H} = \frac{1}{K_{FB}} = \frac{1}{22.2} = 0.045 = -26.9 \text{ db}$$

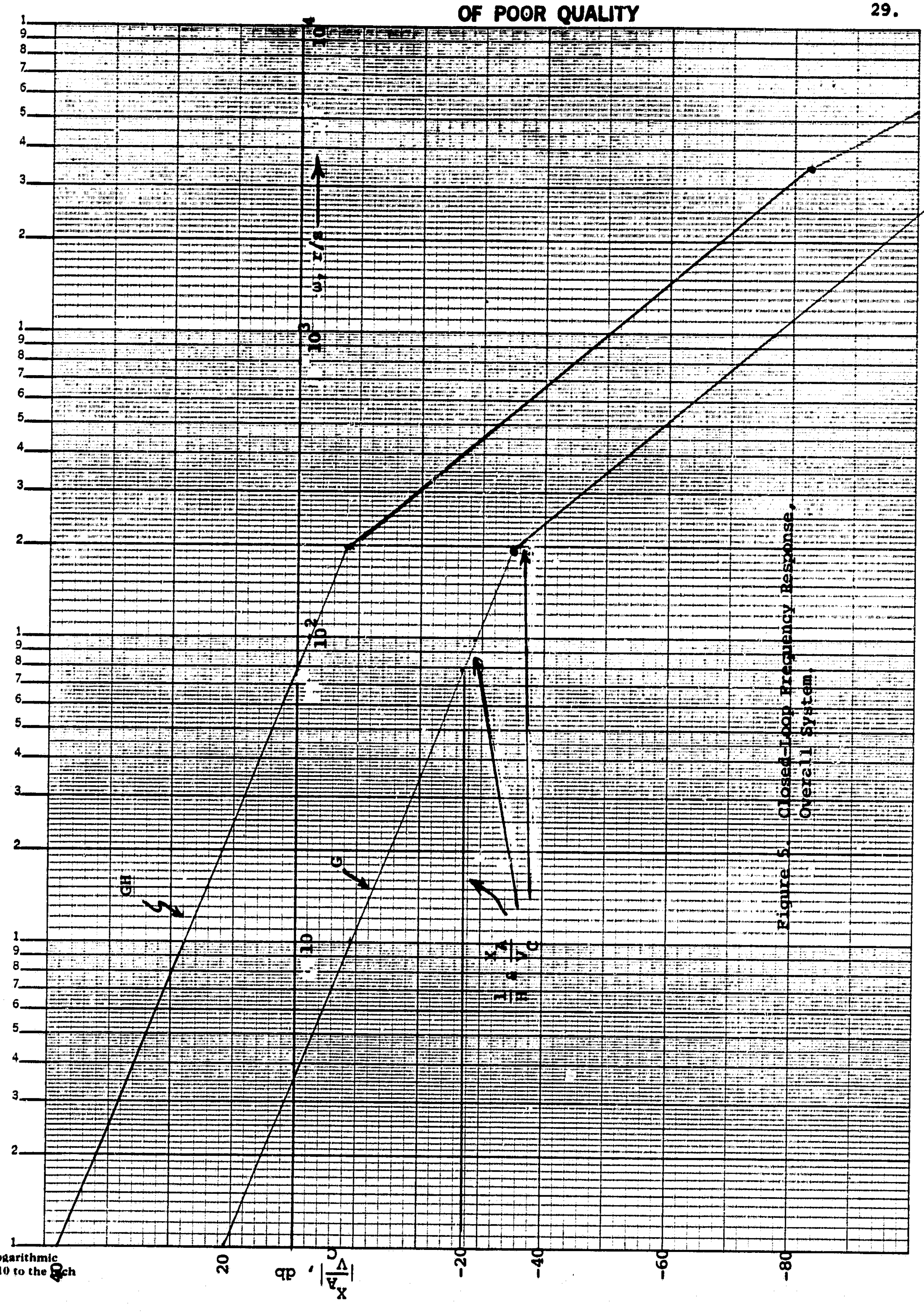


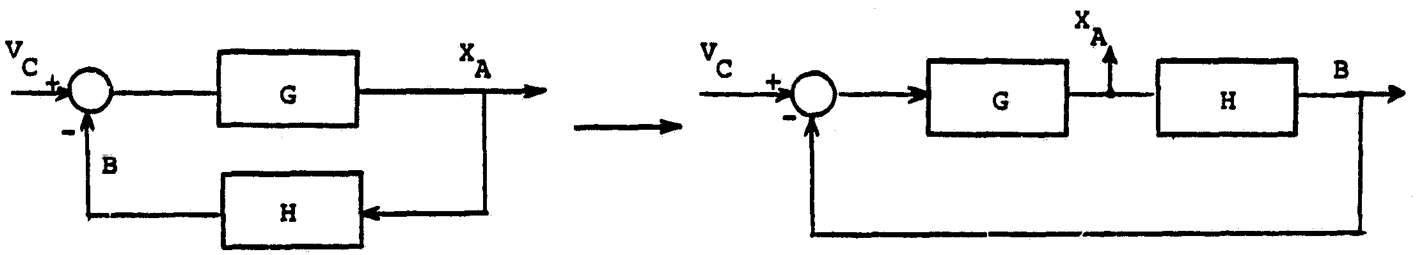
Figure 5. Closed-Loop Frequency Response, Overall System.

BLOCK DIAGRAM, OVERALL SYSTEM (continued)

$$GH = K_{FB}G = 22.2G = K'' \frac{1}{s\left[\left(\frac{s}{\omega_{n_1}}\right)^2 + 2\frac{\zeta_1}{\omega_{n_1}}s + 1\right]\left[\left(\frac{s}{\omega_{n_2}}\right)^2 + 2\frac{\zeta_2}{\omega_{n_2}}s + 1\right]}$$

$$K'' = 22.2(3.603) = 80.06 = 38.07 \text{ db}$$

THE FREQUENCY RESPONSE CHARACTERISTICS CAN BE NORMALIZED BY FINDING THE FEEDBACK TRANSFER FUNCTION, B/V_A



$$\frac{B}{V_C} = \frac{GH}{1 + GH}$$

$$\left|\frac{B}{V_C}\right| \cong 1, \quad GH >> 1.0$$

$$\cong GH, \quad GH << 1.0$$

USE OF THIS TRANSFER FUNCTION ENSURES A GAIN OF 1 (0 db) AT LOW FREQUENCIES. THE CLOSED-LOOP FREQUENCY RESPONSE DIAGRAM FOLLOWS. GH IS AS GIVEN PREVIOUSLY.

$$GH = \frac{80.06}{s\left[\left(\frac{s}{200}\right)^2 + 2\frac{(0.5)}{(200)}s + 1\right]\left[\left(\frac{s}{3607}\right)^2 + 2\frac{(0.036)}{(3607)}s + 1\right]}$$

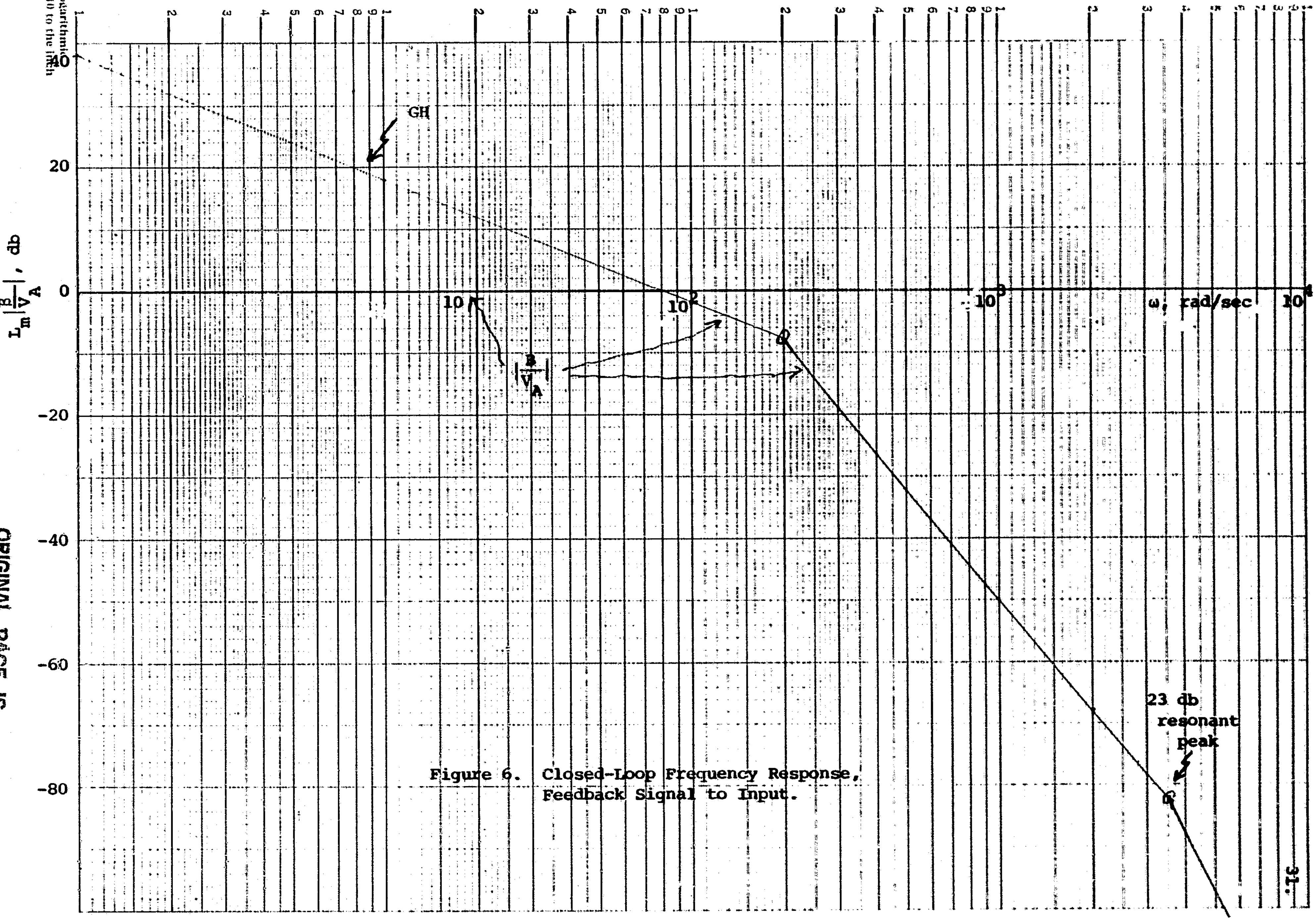


Figure 6. Closed-Loop Frequency Response, Feedback Signal to Input.

8. RESULTS AND RECOMMENDATIONS

The 3-channel redundancy logic system was tested for all possible combinations of channel failure, and the outputs of all amplifiers were verified as correct. Results of Test Runs are shown in Table 2. Difficulties with scaling were encountered with the analog system, and the overall system was not tested closed-loop for actual coefficient values. However, an analog model having the same basic structure as the actual system (i.e., Type One 5th Order) but with simplified coefficients to obviate the need for scaling, was used to verify that the overall system with redundancy logic, could operate properly closed-loop.

The present system does not identify which system has failed, in the event of channel failure. It is recommended that threshold logic circuitry be designed for this purpose. A cross-channel monitoring technique could be used to continuously monitor the differences in signal levels between pairs of channels (i.e., Ch. 1 - Ch. 2, Ch. 2 - Ch. 3, etc.). It is desired that signal levels in all four channels be identical; therefore, ideally, all differences would be zero. If the signals differ by more than a prescribed threshold value, say 10%, this would indicate a channel failure, and an error signal would be sent. The differences indicated above could then be used, through appropriate digital circuitry, to determine which channel has failed. This extension would require a substantial expansion of the present EAI-2000 system.

The nonlinearities were not tested because the remainder of the system used all available circuitry.

A rough check was made of equipment needs for the overall system with 4-channel redundancy logic, the proposed threshold logic cross-channel monitoring system, and the dead-zone nonlinearities. It was determined that even a fully expanded EAI-2000 would be insufficient. In that event the use of a digital computer in conjunction with the EAI-2000 would give hybrid capability and much more flexibility.

8. RESULTS AND RECOMMENDATIONS (continued)

This leads to the question of which digital computer should be used: the PDP-11, the Pacer 100, or the Apple II. The answer to the question depends on whether the digital computer is going to be used extensively for other tasks or will be primarily dedicated to the hybrid configuration. The PDP-11 and Apple II are certainly more versatile in terms of doing other tasks, but the Pacer 100 can be used in the stand-alone mode also.

If the major objective is to have a powerful hybrid system, the Pacer 100 seems to be the best choice. It was designed to interface with an analog machine like the EAI-2000. Further, the hybrid software and library routines are available for the Pacer 100.

9. BIBLIOGRAPHY

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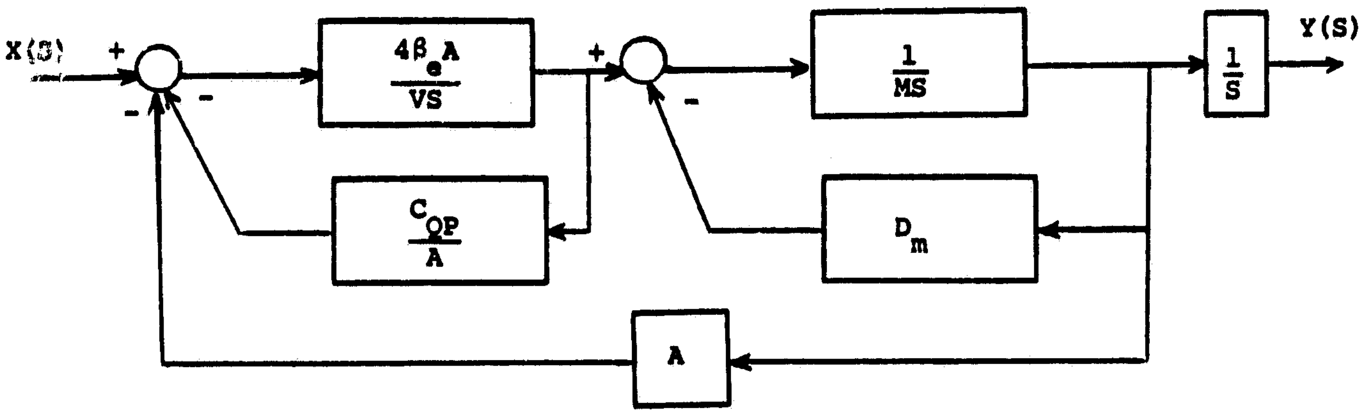
APPENDIX 1 - DERIVATION OF ACTUATOR TRANSFER FUNCTION



$\beta_e = 10^5$ psi

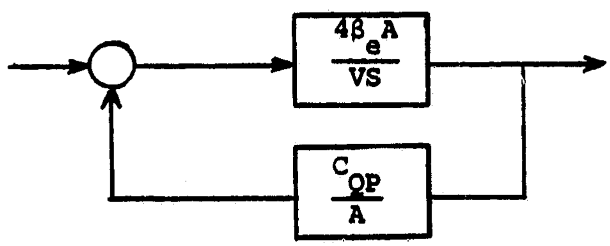
$A = 0.152\ in^2$

$C_{QP} = 2.248 \times 10^{-6}\ \frac{cis}{psi}$

$M = 0.0052\ lb_f - sec^2/in$

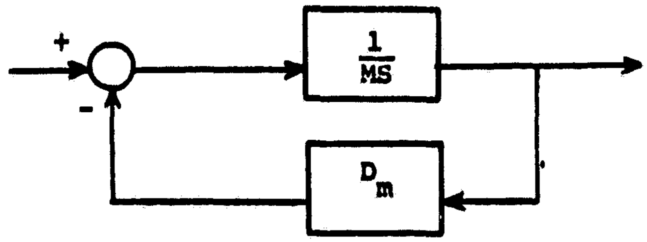
$D_m = 1.32\ lb_f - sec/in$

$V = 0.137\ in^3$



$$\frac{4\beta_e \frac{A}{VS}}{1 + \frac{4\beta_e A}{VS}\left(\frac{C_{QP}}{A}\right)} = \frac{\frac{A}{C_{QP}}}{\frac{V}{4\beta_e C_{QP}} S + 1} = \frac{6.77 \times 10^4}{0.1524\ S + 1}$$

APPENDIX 1 - DERIVATION OF ACTUATOR TRANSFER FUNCTION (continued)



$$\frac{\frac{1}{MS}}{1+\frac{D_m}{MS}} = \frac{1}{MS + D_m} = \frac{\frac{1}{D_m}}{\frac{M}{D_m} S + 1} = \frac{0.7576}{0.003939\ S + 1}$$

FORWARD TRANSFER FUNCTION:

$$G = \frac{6.77 \times 10^4}{0.1524\ S + 1} \quad \frac{0.7576}{0.003939\ S + 1}$$

$$= \frac{5.135 \times 10^4}{0.0006\ S^2 + 0.1563\ S + 1}$$

APPENDIX 1 - DERIVATION OF ACTUATOR TRANSFER FUNCTION (continued)

CLOSED-LOOP TRANSFER FUNCTION

$$\frac{G}{1 + AG} = \frac{5.135 \times 10^4}{0.0006\ S^2 + 0.1563\ S + 1 + (0.152)(5.135 \times 10^4)}$$

$$= \frac{5.135 \times 10^4}{0.0006\ S^2 + 0.1563\ S + 7.805 \times 10^3}$$

$$= \frac{8.558 \times 10^7}{S^2 + 260\ S + 1.301 \times 10^7}$$

2nd. ORDER UNDERDAMPED FORM.

$$\omega_n = \sqrt{1.301 \times 10^7} = 3607\ r/s$$

$$2\zeta\omega_n = 260$$

$$\zeta = \frac{260}{2(3607)} = 0.036$$

THEN

$$\frac{G}{1 + AG} = \frac{6.571}{\left(\frac{S}{3607}\right)^2 + 2\ \frac{(0.036)}{3607}\ S + 1}$$

THIS LATTER FORM IS CONVENIENT FOR FREQUENCY RESPONSE PLOTTING.

APPENDIX 2 - UNIT MAGNITUDE SCALING

Frequently, in system's analysis, the maximum value of a variable is known or specified, but the maximum values of the derivatives of the variables are not known. Then, these maximum values can be estimated from the maximum values of the variables and the natural frequencies of the system. To illustrate the method, consider two functions: the exponential function and the sinusoidal function. Then compute derivatives as follows:

$$x = Ae^{kt}$$
$$\dot{x} = kAe^{kt}$$
$$\ddot{x} = k^2Ae^{kt}$$

Assume that the given system is stable, i.e., $k<0$, then

$$x_{max} = A$$
$$\dot{x}_{max} = kA$$
$$\ddot{x}_{max} = k^2A$$

where k is the root of the characteristic equation.

Note that the maximum values form a geometric progression having k as the ratio between successive terms.

For the sinusoidal function, a somewhat similar result can be obtained:

$$x = Ae^{\sigma t}\sin(\omega t + \phi)$$
$$\dot{x} = A[e^{\sigma t}\omega\cos(\omega t + \phi) + \sigma e^{\sigma t}\sin(\omega t + \phi)]$$
$$= Ae^{\sigma t}\sqrt{\omega^2 + \sigma^2}\sin(\omega t + \phi')$$

where ϕ' depends upon ω and σ. Then,

$$x = A\sqrt{\sigma^2 + \omega^2}\, e^{\sigma t} \sin(\omega t + \phi')$$

and

$$\dot{x}_{max} \doteq \sqrt{\sigma^2 + \omega^2}\, A = \omega_n A$$

Thus it follows that

$$\ddot{x}_{max} \doteq \omega_n^{\,2} A$$

$$\dddot{x}_{max} \doteq \omega_n^{\,3} A$$

etc.

For the general nth-order characteristic equation

$$a_n s^n + a_{n-1} s^{n-1} + a_{n-2} s^{n-2} + \cdots + a_1 k + a_o = 0$$

a good "average" natural frequency to use for scaling purposes is:

$$\bar{\omega} = \sqrt[n]{|a_o|a_n|}$$

The formula works well when the roots of the characteristic equation do not differ too widely in magnitude. It is a basic theorem in the theory of polynomials that the product of all the roots is equal to $\frac{a_o}{a_n}$. Therefore, the average frequency, $\bar{\omega}_n$, defined above is the geometric mean of the absolute values of all the roots. This is frequently a good "average" to use in estimating the maximum magnitudes of the derivatives. Therefore to estimate the maximum values of the derivatives for the differential equation:

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \cdots a_1 \frac{dx}{dt} + a_o x = 0$$

The maximum value of x must be known. Then

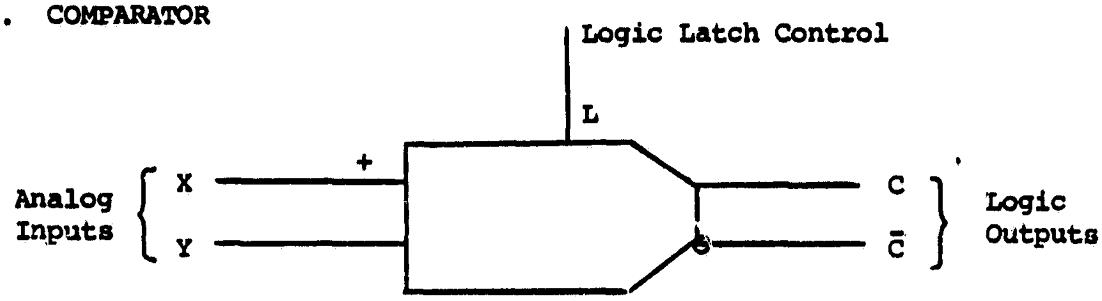
$$\dot{x}_{max} \doteq \bar{\omega}_n x_{max}$$

$$\ddot{x}_{max} \doteq \bar{\omega}^{\,2} x_{max}$$

etc.

APPENDIX 3 - OPERATION OF BASIC LOGIC ELEMENTS

I. COMPARATOR

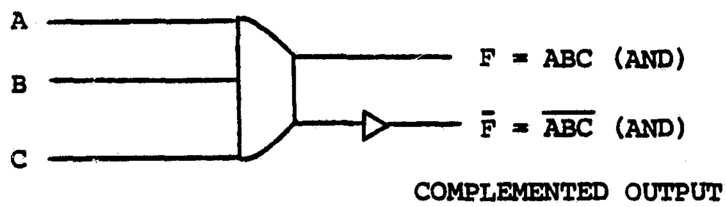


ANALOG TO LOGIC INTERFACE

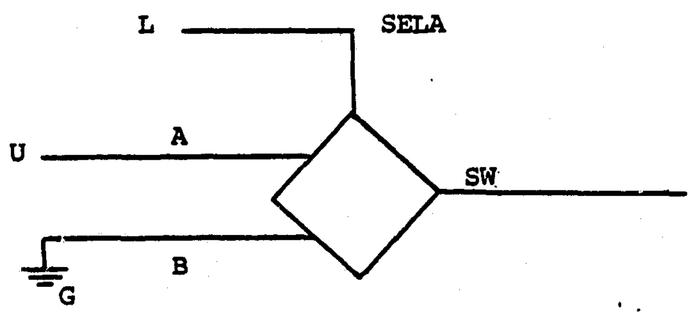
$$C = \begin{cases} 1, & X - Y > 0 \\ 0, & S - Y < 0 \end{cases}$$

(LOGIC LATCH CONTROL FEATURE NOT USED)

II. LOGIC GATE: AND (and NAND)



III. SELECTOR SWITCHES



$$Y = \begin{cases} U, & L = 1 \\ G, & L = 0 \end{cases}$$

APPENDIX 4 - TAFCO MODEL

Since the Tafco Model is so similar to the NASA Model, it may be instructive to compare them in some respects. The block diagram for the Tafco Model is shown in Figure 7 , and the constants are given in Table 3 .

THE CLOSED-LOOP TRANSFER FUNCTION GIVEN IS:

$$\frac{X_A}{V_C}(S) = \frac{0.045\left(\frac{S}{638}+1\right)}{\left(\frac{S}{140 \pm j\ 87.08}+1\right)\left(\frac{S}{898.91 \pm j\,135.36}+1\right)\left(\frac{S}{131.8 \pm j\,3712.8}+1\right)}$$

NOTE THAT THERE ARE THREE CONJUGATE PAIRS OF UNDERDAMPED CLOSED-LOOP POLES. FIND THE CORRESPONDING DAMPING RATIOS AND NATURAL FREQUENCIES.

$$S_{11,2} = -140 \pm j\ 87.08$$

$$(S + 140)^2 + (87.08)^2 = S^2 + 280 + 27182.9$$

$$\omega_n = \sqrt{27182.9} = 164.9 \text{ rad/sec} = 26.2 \text{ hz}$$

$$2\zeta\omega_n = 280$$

$$\zeta = \frac{280}{2\omega} = \underline{\underline{0.849}}$$

$$S_{3,4} = -\ 898.91 \pm j\ 135.36$$

$$(S + 898.91)^2 + (135.36)^2 = S^2 + 17965 + 824726$$

$$\omega_n = \sqrt{824726} = 908.1 \text{ r/s}$$

$$\zeta = \frac{1796}{2\omega_n} = \frac{1796}{2\,(908.1)} = 0.989 \text{ (Near Critical Damping)}$$

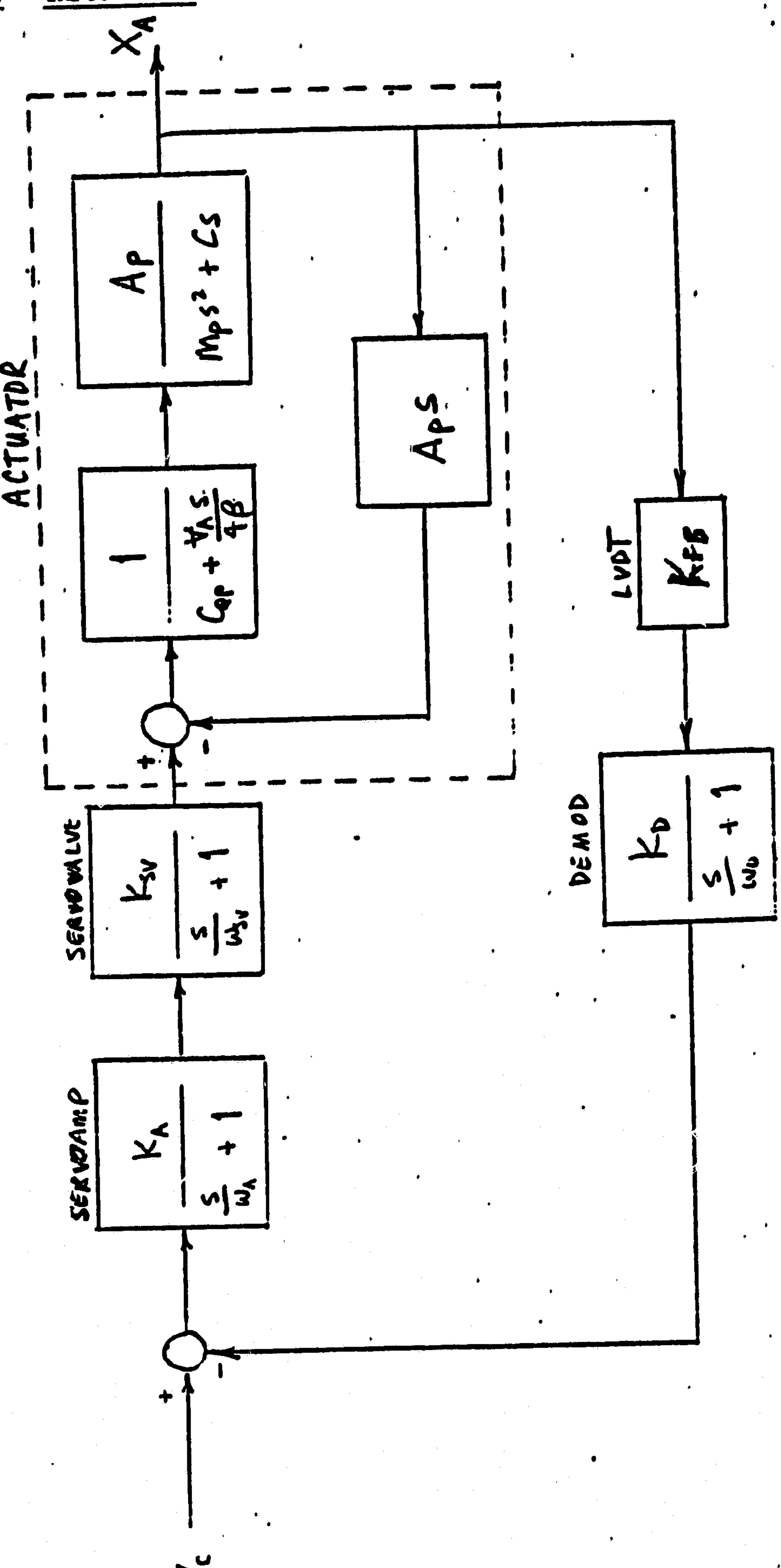


Figure 7. Block Diagram for Linear Model of Unloaded Tafco Actuator

APPENDIX 4 - TAFCO MODEL (continued)

$$S_{5,6} = - 131.8 \pm j\ 3712.8$$

$$(S + 131.8)^2 + (3712.8)^2$$

$$= S^2 + 263.65 + 13,802,255$$

$$\left.\begin{array}{l} \omega_n = \sqrt{13,802,255} = \underline{3715.4}\ r/s \\ \\ \zeta = \dfrac{263.4}{2(3715.1)} = \underline{0.035} \end{array}\right\}\ \text{From Actuator}$$

A BLOCK DIAGRAM FOR THE UNLOADED TAFCO ACTUATOR CONTROL LOOP IS SHOWN IN APPENDIX 4. THE DOMINANT CLOSED-LOOP POLES FOR THIS SYSTEM HAVE A DAMPING RATIO OF 0.849. THIS CORRESPONDS TO A PERCENT OVERSHOOT OF:

$$M_P = 1 + \exp \left(\frac{- \zeta\pi}{1 - \zeta^2}\right)$$

$$= 1 + \exp \left(\frac{- 0.849\pi}{1 - (0.849)^2}\right)$$

$$= 1 + 0.0064 = 1.0064$$

$$\%\ \text{Overshoot} = 0.69\%$$

THUS THE ACTUATOR HAS RELATIVELY LITTLE EFFECT ON THE PERFORMANCE OF THE OVERALL SYSTEM.

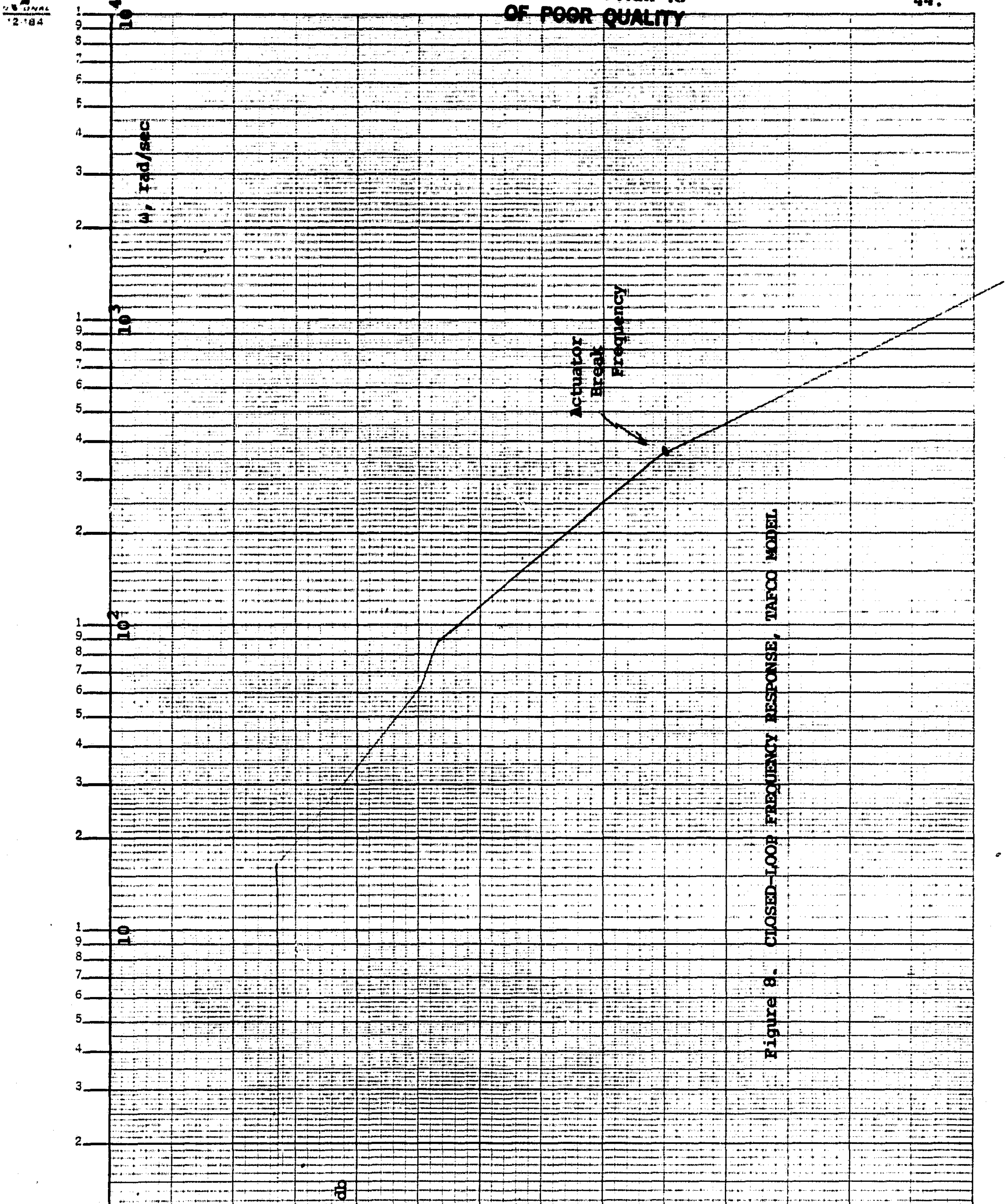


Figure 8. CLOSED-LOOP FREQUENCY RESPONSE, TAFCO MODEL

APPENDIX 4 - TAFCO MODEL (continued)

Table 3

Description of Constants and Variables in Linear Model of Unloaded Tafco Actuator

Symbol		Description		Value/Dimension	
A_P	-	Actuator piston area	-	.152	in^2
C	-	Actuator piston damping coefficient	-	1.32	$\frac{lbf\ sec}{in}$
C_{QP}	-	Linearized flow coefficient	-	2.808×10^{-6}	$\frac{cis}{psi}$
K_A	-	Servoamplifier gain	-	15.6	$\frac{ma}{v}$
K_D	-	Demodulator gain	-	1	$\frac{vdc}{vrms}$
K_{FB}	-	LVDT gain	-	22.22	$\frac{vrms}{in.}$
K_{SV}	-	Servovalve flow gain	-	.0351	$\frac{cis}{ma}$
M_P	-	Actuator piston mass	-	.005182	$\frac{lbf\ sec^2}{in}$
s	-	Laplace variable	-		$\frac{1}{sec}$
V_A	-	Actuator oil volume (both sides)	-	.137	in^3
V_C	-	Command voltage	-	±10	v
β	-	Fluid bulk modulus	-	100,000	psi
ω_A	-	Corner frequency of servoamp response	-	1000	$\frac{rad}{sec}$
ω_D	-	Corner frequency of demod response	-	638	$\frac{rad}{sec}$
ω_{SV}	-	Corner frequency of servovalve response	-	440	$\frac{rad}{sec}$