NASA-TM- 86708

NASA Technical Memorandum 86708

NASA-TM-86708 19860020159

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Space Administration

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N86-29631#

SYMBOLS

c = speed of sound

c_j = complex harmonic amplitude of the jth duct mode

$\mathrm{d}\vec{\mathrm{p}}_d$ = vector of normal derivative of harmonic pressure at downstream boundary

$\mathrm{d}\vec{\mathrm{p}}_u$ = vector of normal derivative of harmonic pressure at upstream boundary

e = 2.718282

f = source term in Helmholtz equation

i = $\sqrt{-1}$

j = index for eigenvalues in x direction

k = wave number, $\frac{\omega}{c}$

k_j^+ = downstream x wave number (eigenvalue) for the jth duct mode

k_j^- = upstream x wave number (eigenvalue) for the jth duct mode

k_d = diagonal matrix of duct downstream eigenvalues

k_u = diagonal matrix of duct upstream eigenvalues

l = index for control points

M = freestream wind tunnel Mach number, $\mathrm{U}_\infty/\mathrm{c}$

n	=	normal to the duct surface, positive toward duct center
n, m	=	indicies for eigenvalues in y and z direction
N_n, N_m	=	normalizing constants for eigenmodes
P	=	acoustic pressure
p	=	complex pressure coefficient for single frequency, $\mathrm{P} = \Re(e^{-i\omega t}\mathrm{p})$
$\vec{\mathrm{p}}_d$	=	vector of harmonic pressures on downstream outflow boundary
$\hat{\mathrm{p}}_{d_j}$	=	harmonic pressure amplitude of jth duct eigenmode for downstream propagation
$\vec{\mathrm{p}}_u$	=	vector of harmonic pressures on upstream outflow boundary
$\hat{\mathrm{p}}_{u_j}$	=	harmonic pressure amplitude of jth eigenmode for upstream propagating wave
S	=	surface of control volume
S_d	=	downstream outflow control surface
S_u	=	upstream outflow control surface
S_w	=	duct wall control surface
u_n	=	componant of acoustic velocity normal to surface
U_∞	=	axial componant of uniform flow in duct
V	=	control volume in duct
x, y, z	=	coordinates

x_s, y_s, z_s	=	acoustic source position
z	=	acoustic impedance of a surface
α	=	boundary condition coefficient
π	=	3.141593
$\psi_j^{\pm}$	=	duct eigenmodes
$\mathbf{\Psi}$	=	matrix of duct eigenvectors
ω	=	acoustic frequency

AN OUTFLOW ACOUSTIC BOUNDARY CONDITION FOR INTERNAL DUCT FLOWS

Marianne Mosher

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SUMMARY

A boundary condition for the linear acoustic equation has been developed that allows the acoustic pressure waves to propagate out of the computational domain boundary, just as they would propagate in an infinitely long duct. The problem is divided into two domains: numerical and analytical. The boundary condition provides a matching of the two domains. Examples show this method works well in an acoustic panel program for a model problem (simple source in a rectangular duct with several propagating modes present). This report describes the boundary condition so that it can be used with various duct geometries and numerical methods.

INTRODUCTION

When computing acoustic properties in large or infinite regions, it is often desirable to restrict the computation to a subregion. This requires a condition to be imposed at an artificial boundary that will represent the acoustic field in the region outside the computational domain. The boundary condition must be very accurate for acoustic problems, otherwise the acoustic waves will reflect off the artificial boundary back into the computational domain. Acoustic fields in ducts are particularly sensitive; a poor boundary condition can change the result by 100% or more.

Unlike a wall boundary, no simple relation exists between the acoustic pressure and its derivative at the artificial outflow boundary. The relationship between the two depends on the solution; both on the acoustic source, and how the acoustic field propagates in the duct away from the control volume.

The simplest boundary condition that might be used is a linear relation between p and its derivative: $\partial p/\partial n = \alpha p$. This would allow a plane wave with wave number in the x-direction of $k_x = -i\alpha$ to pass through the boundary without spurious reflections. All other wave numbers would be partially reflected.

In reference 1, Bayliss and Turkel present a series of boundary conditions that allow several modes to pass through the boundary. Their boundary condition is

$$\prod_{j=1}^{j=jmax} (\frac{\partial}{\partial x} - ik_j)p = 0 \tag{1}$$

When all of the infinite terms are included, this exactly models the acoustic waves propagating in the duct away from the source. At least one term of the product must be included for each propagating wave that exists at the boundary. When many terms are used it becomes difficult to apply this boundary condition as it involves many high order derivatives. This paper develops a boundary condition that involves only one normal derivative.

The following method may be used for linear acoustic duct problems solved in the frequency domain. At the boundary, the acoustic field for one frequency can be expressed as a linear combination of eigenmodes, truncated to a finite number of propagating eigenmodes and decaying eigenmodes. The eigenvalues and eigenmodes depend on the duct shape, wall boundary, and duct terminations; thus they can be determined independently of the source. An equation can be constructed which relates p and $\partial p/\partial n$ at the boundary. The discrete version of this equation can be used as the boundary condition and combined with the discrete equations to be solved. Solving the system then produces the amplitudes of the eigenmodes.

MATHEMATICAL MODEL

For subsonic flow with Mach number M in the +x direction the equation describing the acoustic field is the convected Helmholtz equation:

$$\nabla^2 p - M^2 \frac{\partial^2 p}{\partial x^2} + 2ikM\frac{\partial p}{\partial x} + k^2 p = f(x, y, z, k) \tag{2}$$

where f is the source. The boundary condition on the wall is:

$$\alpha p + \frac{\partial p}{\partial n} = 0, \text{ on S} \tag{3}$$

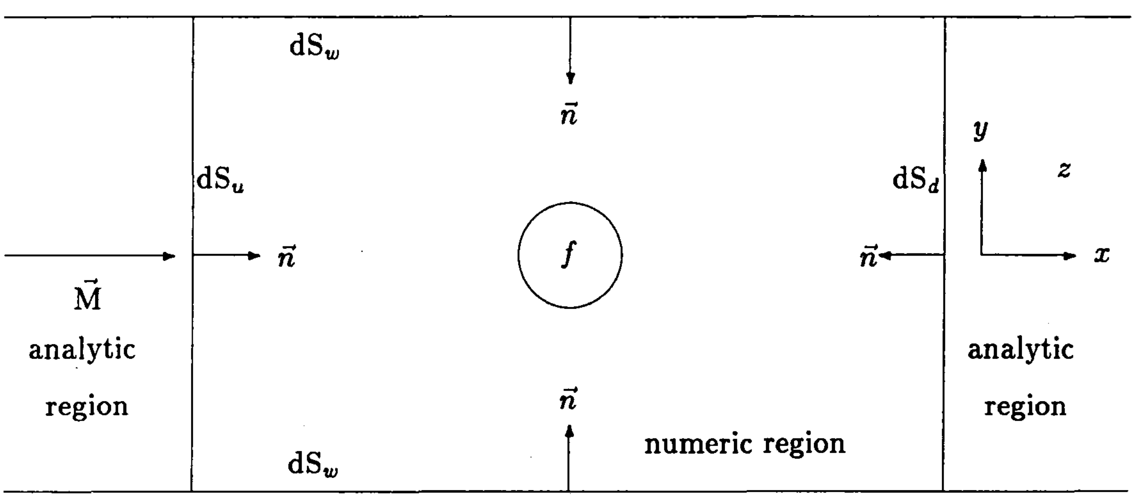


Figure 1. – Control Volume

Figure 1 shows the duct geometry.

The homogeneous solution consists of acoustic modes propagating downstream in the (+x) direction and propagating upstream in the (-x) direction .

$$\mathrm{p}^+ = \sum_{j=0}^{j=\infty} c_j^+ \psi_j^+(y,z) e^{ik_j^+(x-x_s)}, x > x_s \tag{4a}$$

$$\mathrm{p}^- = \sum_{j=0}^{j=\infty} c_j^- \psi_j^-(y,z) e^{-ik_j^-(x-x_s)}, x < x_s \tag{4b}$$

The functions $\psi_j^{\pm}$ are the duct modes for the eigenvalues $k_j^{\pm}$.

Any linear acoustic field in a constant cross-section duct can be represented by a linear combination of duct modes. Far away from all acoustic sources, a finite number of modes represents the acoustic field well. The amplitude of most modes decays exponentially with distance away from the source. Thus, at the outflow boundaries, the sum of a finite number of modes represents the acoustic pressure well. For example, at the downstream boundary, where $x > x_s$ for all the acoustic sources, the harmonic acoustic pressure may be written,

$$\mathrm{p}(x,y,z) = \sum_{j=1}^{j=\infty} c_j \psi_j(y,z) e^{ik_j^+(x-x_s)} \approx \sum_{j=1}^{j_{max}} c_j \psi_j(y,z) e^{ik_j^+(x-x_s)} \tag{5}$$

where k_j is the jth eigenvalue, ψ_j is the jth eigenmode, and c_j is the amplitude of the jth mode. With this restriction to the eigenmodes of the physical duct, both the harmonic acoustic pressure and its normal derivative can be expressed in terms of the same variables, $\hat{\mathrm{p}}_{d_j} = e^{ik_j^+(x_{max}-x_s)} c_j^+$, the complex amplitudes of the j_{max} modes at the boundary $x = x_{max}$.

$$\mathrm{p}(x_{max}, y, z) \approx \sum_{j=1}^{j_{max}} \psi_j(y,z) \hat{\mathrm{p}}_{d_j} \tag{6a}$$

$$\frac{\partial \mathrm{p}(x_{max}, y, z)}{\partial n} = -\frac{\partial \mathrm{p}(x_{max}, y, z)}{\partial x} \approx -\sum_{j=1}^{j_{max}} ik_j^+ \psi_j(y,z)\hat{\mathrm{p}}_{d_j} \tag{6b}$$

Likewise at the upstream boundary with $x < x_s$

$$\mathrm{p}(x_{min}, y, z) \approx \sum_{j=1}^{j_{max}} \psi_j(y,z)\hat{\mathrm{p}}_{u_j} \tag{6c}$$

$$\frac{\partial \mathrm{p}(x_{min}, y, z)}{\partial n} = \frac{\partial \mathrm{p}(x_{min}, y, z)}{\partial x} \approx -\sum_{j=1}^{j_{max}} ik_j^- \psi_j(y,z)\hat{\mathrm{p}}_{u_j} \tag{6d}$$

Next, consider the discretized variables: evaluate the quantities at discrete (y,z) points on the boundary. All the relations in equation (6) can be represented by matrices:

$$\vec{\mathrm{p}}_u = \mathbf{\Psi}\vec{\hat{\mathrm{p}}}_{u_j} \tag{7a}$$

$$\mathrm{d}\vec{\mathrm{p}}_u = i\mathbf{\Psi}\mathrm{k}_u\vec{\hat{\mathrm{p}}}_{u_j} \tag{7b}$$

$$\vec{\mathrm{p}}_d = \mathbf{\Psi}\vec{\hat{\mathrm{p}}}_{d_j} \tag{7c}$$

$$\mathrm{d}\vec{\mathrm{p}}_d = i\mathbf{\Psi}\mathrm{k}_d\vec{\hat{\mathrm{p}}}_{d_j} \tag{7d}$$

where,

$$\vec{\mathrm{p}}_u = (\ldots\mathrm{p}(x_{min}, y_l, z_l)\ldots)^{\mathrm{T}} \tag{8a}$$

$$\mathrm{d}\vec{\mathrm{p}}_u = (\ldots\frac{\partial \mathrm{p}(x_{min}, y_l, z_l)}{\partial n}\ldots)^{\mathrm{T}} \tag{8b}$$

$$\vec{\mathrm{p}}_d = (\ldots\mathrm{p}(x_{max}, y_l, z_l)\ldots)^{\mathrm{T}} \tag{8c}$$

$$\mathrm{d}\vec{\mathrm{p}}_d = (\ldots\frac{\partial \mathrm{p}(x_{max}, y_l, z_l)}{\partial n}\ldots)^{\mathrm{T}} \tag{8d}$$

When the number of control points on the outflow plane equals the number of modes included in the representation, $\hat{\mathrm{p}}$ can be eliminated by direct inversion to obtain a matrix equation relating $\vec{\mathrm{p}}$ and $\mathrm{d}\vec{\mathrm{p}}$.

$$i\mathbf{\Psi}\mathrm{k_u}\mathbf{\Psi}^{-1}\vec{\mathrm{p}}_u - \mathrm{d}\vec{\mathrm{p}} = 0 \tag{9a}$$

$$i\mathbf{\Psi}\mathrm{k_d}\mathbf{\Psi}^{-1}\vec{\mathrm{p}}_d - \mathrm{d}\vec{\mathrm{p}} = 0 \tag{9b}$$

When the number of control points is greater than the number of modes, a least squares solution for $\hat{\mathrm{p}}$ gives:

$$i\mathbf{\Psi}\mathrm{k_u}[\mathbf{\Psi}^{\mathbf{T}}\mathbf{\Psi}]^{-1}\mathbf{\Psi}^{\mathbf{T}}\vec{\mathrm{p}}_u - \mathrm{d}\vec{\mathrm{p}} = 0 \tag{9c}$$

$$i\mathbf{\Psi}\mathrm{k_u}[\mathbf{\Psi}^{\mathbf{T}}\mathbf{\Psi}]^{-1}\mathbf{\Psi}^{\mathbf{T}}\vec{\mathrm{p}}_d - \mathrm{d}\vec{\mathrm{p}} = 0 \tag{9d}$$

Outside the control volume a sum of eigenmodes represents the acoustic field.

$$\mathrm{p} = \sum_{j=1}^{j_{max}} \psi_j(y,z)e^{ik_j^+(x-x_{max})}\hat{\mathrm{p}}_{d_j}, x > x_{max} \tag{10a}$$

$$\mathrm{p} = \sum_{j=1}^{j_{max}} \psi_j(y,z)e^{-ik_j^-(x-x_{min})}\hat{\mathrm{p}}_{u_j}, x < x_{min} \tag{10b}$$

The boundary condition matches the acoustic field in the control volume to an analytic solution outside the control volume.

This form can be used because the acoustic field inside the duct can be expressed as a linear sum of modes. Any type of duct can be used as long as the eigenvalues and eigenmodes can be determined. Often the duct is simple enough (separable coordinates and simple boundary conditions on the wall) that the eigenvalues and eigenmodes have analytic expressions. For more complicated ducts, the eigenvalues and eigenmodes must be determined numerically.

All modes with any significant amplitude at the boundary must be included in the eigenmode representation. Generally, those will be all propagating modes, and possibly a few decaying modes. The boundary should be placed far enough away from the sources so that decaying modes will have a chance to decay.

EXAMPLES

Model Problem

This boundary condition has been incorporated into an acoustic panel method and applied successfully to a model problem of a point acoustic source in an infinitely long rectangular duct. The duct is bound by $y = \pm b/2$ and $z = \pm d/2$. The eigenvalues and eigenmodes may be determined analytically.

Eigenvalues and Eigenmodes

By the method of separation of variables as described in reference 2, the solution to equation (2) in the rectangular duct with boundary condition in equation (3) can be found. Acoustic modes propagating downstream in the +x direction have the form:

$$\mathrm{p}^{+} = \sum_{\substack{n=\infty \\ m=0}}^{\substack{n=\infty \\ m=\infty}} \mathrm{cs}(k_{ym}y)\mathrm{cs}(k_{zn}z)e^{ik_j^{+}(x-x_s)}\mathrm{p}_j^{+}, x > x_s \tag{11a}$$

and propagating in the upstream (-x) direction have the form:

$$\mathrm{p}^{-} = \sum_{\substack{n=\infty \\ m=0}}^{\substack{n=\infty \\ m=\infty}} \mathrm{cs}(k_{ym}y)\mathrm{cs}(k_{zn}z)e^{-ik_j^{-}(x-x_s)}\mathrm{p}_j^{-}, x < x_s \tag{11b}$$

The function $\mathrm{cs}(k_n)$ is defined:

$$\mathrm{cs}(k_n) = \begin{cases} \cos(k_n), n\,\mathrm{even} \\ \sin(k_n)\,, n\,\mathrm{odd} \end{cases} \tag{12}$$

The boundary conditions give the following trancsendental equations for the eigenvalues k_{ym} and k_{zn}:

$$\alpha = k_{ym}\tan(k_{ym}\frac{b}{2}),\ m\,\mathrm{even} \tag{13a}$$

$$\alpha = k_{ym}\tan(\frac{\pi}{2} + k_{ym}\frac{b}{2}),\ m\,\mathrm{odd} \tag{13b}$$

$$\alpha = k_{zn} \tan(k_{zn} \frac{d}{2}),\ n \,\text{even} \tag{13c}$$

$$\alpha = k_{zn} \tan(\frac{\pi}{2} + k_{zn} \frac{d}{2}),\ n \,\text{odd} \tag{13d}$$

A solution for each eigenvalue will exist for each period of π . The solutions will have the form:

$$k_{ym} = \frac{m\pi}{b} + \epsilon_{zm} \tag{14a}$$

$$k_{zn} = \frac{n\pi}{d} + \epsilon_{yn}. \tag{14b}$$

When the wall is close to being a hard wall, α is small, and the solutions give approximate values of:

$$\epsilon_{ym} \sim \begin{cases} \sqrt{\frac{2\alpha}{b}}, & m = 0 \\ \frac{8\alpha}{b\pi^2 m^2}, & m > 0 \end{cases} \tag{15a}$$

$$\epsilon_{zm} \sim \begin{cases} \sqrt{\frac{2\alpha}{d}}, & n = 0 \\ \frac{8\alpha}{d\pi^2 n^2}, & n > 0. \end{cases} \tag{15b}$$

The eigenvalues, k^+_{xmn} and k^-_{xmn} for propagation downstream and upstream, are found.

$$k^+_{xmn} = \frac{-kM \pm \sqrt{k^2 - (1 - M^2)((k_{ym})^2 + (k_{zn})^2)}}{(1 - M^2)} \tag{16a}$$

and

$$k^-_{xmn} = \frac{kM \pm \sqrt{k^2 - (1 - M^2)((k_{ym})^2 + (k_{zn})^2)}}{(1 - M^2)}. \tag{16b}$$

The eigenvalues are chosen such that the imaginary part of $k^\pm_{xmn} > 0$ to ensure that the decaying waves are propagating away from the source.

An ordering of the eigenvalues, which includes at least all of the propagating modes, maps the m, n indices into the j mode indices. Thus, we have $\mathbf{\Psi}(l, j) = \text{cs}(k_{ym} y_l)\text{cs}(k_{zn} z_l)$ is the mode shape of the jth mode at position (y_l, z_l), $\mathrm{k}_u(j, j) = k^-_j$ is the jth upstream eigenvalue, and $\mathrm{k}_d(j, j) = k^+_j$ is the jth downstream eigenvalue.

Analytic Solution

The partial differential equation for a monopole in a duct is:

$$\nabla^2 \mathrm{p} - \mathrm{M}^2 \frac{\partial^2 \mathrm{p}}{\partial x^2} + 2ik\mathrm{M}\frac{\partial \mathrm{p}}{\partial x} + k^2 \mathrm{p} = \delta(x - x_s)\delta(y - y_s)\delta(z - z_s). \tag{17}$$

The solution will have the following form:

$$\mathrm{p}^{\pm} = \sum_{\substack{m=0 \\ n=0}}^{\substack{m=\infty \\ n=\infty}} \mathrm{cs}(k_{ym}y)\mathrm{cs}(k_{zn}z) f^{\pm}_{mn}(x) \tag{18}$$

Substituting equation (18) into (17), and solving by a Fourier Transform method gives:

$$f^{\pm}_{mn} = \frac{\mathrm{cs}(k_{ym}y_s)\mathrm{cs}(k_{zn}z_s)}{(1 - \mathrm{M}^2)N_m N_n} \frac{i}{2(k^{\pm}_{xmn} \pm \frac{k\mathrm{M}}{1-\mathrm{M}^2})} e^{\pm i k^{\pm}_{xmn}(x - x_s)}. \tag{19}$$

The eigenfunctions are orthogonal with respect to the duct area; however, they are not necessarily normal. Let N_m and N_n be the normalizing constants:

$$N_m = \int_{\frac{-b}{2}}^{\frac{b}{2}} \mathrm{cs}^2(k_{my}y)\mathrm{d}y = b + (-1)^m \frac{sin(k_{my}b)}{2k_{my}} \tag{20a}$$

$$N_n = \int_{\frac{-d}{2}}^{\frac{d}{2}} \mathrm{cs}^2(k_{nz}z)\mathrm{d}z = d + (-1)^n \frac{sin(k_{nz}d)}{2k_{nz}}. \tag{20b}$$

Results

Two cases will be shown for a simple monopole source inside an infinitely long rectangular duct without flow. The hard-walled duct has an aspect ratio of 0.7. The source is centered longitudinaly in the control volume at $x = 0$. In the first case, the source has a low frequency, producing 1 propagating mode(plane wave) with outflow boundaries located at $x = \pm 1.5$, and source located off axis at $y = 0.09$ and $z = 0.08$ In the second case, the source has a higher frequency producing 4 propagating modes with outflow boundaries at $x = \pm 1.2$, and source located off axis at $y = 0.20$ and $z = 0.15$

Three solutions are shown for each case: the analytic series solution represented by equation (18) (which converges everywhere except in the plane of the source, $x = 0$), a computation with the new matched boundary condition, and a computation with a simple boundary condition specifying the impedance of the lowest mode at the outflow boundary. This simple boundary condition is equivalent to using one mode in this matching procedure or one term in the boundary condition of Bayliss and Turkel.

For the the low frequency case, six modes are matched at the outflow boundaries, 3 eigenvalues in the y direction and 2 eigenvalues in the z direction. Table 1 shows the 6 eigenvalues and their coefficients at the outflow boundary. Good agreement exists between the analytic coefficients and the coefficients computed with the new procedure. The first coefficient computed with the simple boundary condition also shows good agreement with the analytic coefficient. Figure 2 shows the real part of the pressure solution along 4 longitudinal lines in two horizontal planes. The dotted line indicates the location of the outflow boundary. Both computations agree well with the analytic solution. Figure 3 shows the amplitude of acoustic pressure.

For the higher frequency model problem, 20 modes are matched at the outflow boundaries, 5 eigenvalues in the y direction and 4 eigenvalues in the z direction. Table 2 shows the 20 eigenvalues and their coefficients at the outflow boundary. Good agreement exists between the analytic coefficients and those computed with the new matched boundary condition procedure, particularly the lower order propagating modes. The coefficient for the first propagating mode predicted with the simple boundary condition fails to match the analytically determined coefficient at all. Figures 4 and 5 show the real part and amplitude of the acoustic field respectively. The computation with the new matched boundary condition agrees well with the analytic solution. The computation with the simple boundary condition differs considerably from the analytic solution.

DISCUSSION

This method has some limitations and problems. It can only be applied to linear problems where the acoustic field beyond the boundary can be decomposed into an eigenvalue problem, and the eigenvalues and eigenmodes must be found at this location. In some geometries, they can be found analytically; however, for many geometries they must be found numerically. The boundary condition has been implemented in a panel method. It can also be applied to a finite element or finite difference method; however, this boundary condition will destroy the block tri-/penta- diagonal structure of the problem whenever more than one propagating mode is present. Thus, the solution procedure will have to be altered to allow a small, yet full, submatrix at any outflow boundary.

SUMMARY

Introducing artificial boundaries in acoustic computations can be difficult. The boundary condition must be very accurate in order to avoid reflections which are inconsistent with the actual acoustic field. To do this the boundary condition must represent the acoustic field outside the boundary. Simple boundary conditions specifying acoustic pressure, its normal derivative, or a combination of these will not work in most cases.

A boundary condition that matches the computation to a series representation of the outside acoustic field has been presented. A model problem shows that this boundary condition works well, even when several modes are present.

REFERENCES

1. Bayliss, A. and Turkel, E. : Far Field Boundary Condition for Compressible Flows, Numerical Boundary Condition Procedures, NASA CR 2201, Oct. 1981.

2. Morse, P. and Ingard, K. : *Theoretical Acoustics*, Mcgraw-Hill Book Company Inc., New York, 1968.

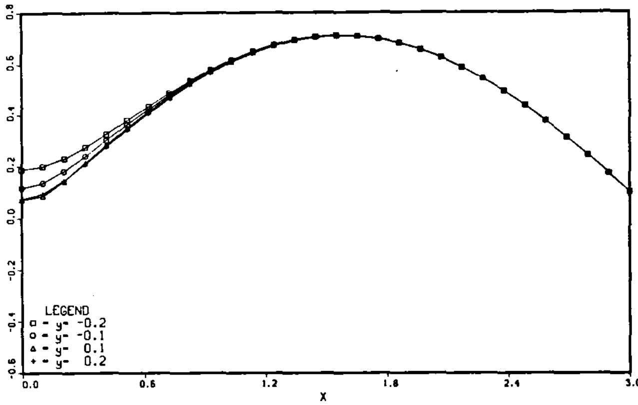
Table 1. - EIGENVALUES FOR TEST CASE WITH ONE PROPAGATING MODE, OUTFLOW BOUNDARY AT $x = \pm 1.5$.

n	m	k_j	$\hat{p}_{d_j}$ analytic	$\hat{p}_{d_j}$ new b.c.	$\hat{p}_{d_j}$ simple b.c.
1	1	(1.,0.)	(0.7090,-0.0503)	(0.7043,-0.0835)	(0.7036,-0.0720)
1	2	(0.,4.36)	(-.00016,0.)	(-.0002,.0000)	(.0000,.0000)
2	1	(0.,2.97)	(-.00154,0.)	(-.0019,.0000)	(.0000,.0000)
2	2	(0.,5.37)	(-.000016,0.)	(.0000,.0000)	(.0000,.0000)
3	1	(0.,6.19)	(-.000018,0.)	(.0015,.0050)	(.0000,.0000)
3	2	(0.,7.64)	(-.0000012,0.)	(.0000,.0000)	(.0000,.0000)

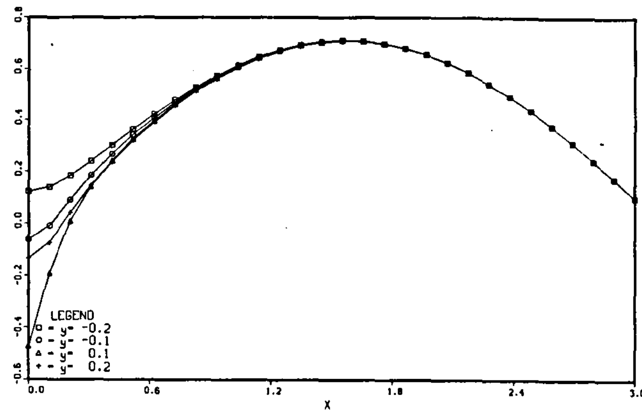
Table 2. – EIGENVALUES FOR TEST CASE WITH THREE PROPAGATING MODES, OUTFLOW BOUNDARIES AT $x = \pm 1.2$.

n	m	k_j	$\hat{p}_{d_j}$ analytic	$\hat{p}_{d_j}$ new b.c.	$\hat{p}_{d_j}$ simple b.c.
1	1	(6.200, 0.00000)	(0.1049, -4.611E-02)	(0.1048 -0.0486)	(0.0431,-0.0927)
1	2	(4.290, .00000)	(-.1051,-.0491)	(-0.1066 -0.0454)	(.0000,.0000)
1	3	(.0000, 6.455)	(7.192E-05, .0000)	(0.0008 0.0010)	(.0000,.0000)
1	4	(.0000, 11.90)	(6.530E-08, .0000)	(-0.0001 -0.0001)	(.0000,.0000)
2	1	(5.348, .0000)	(9.988E-03, -7.332E-02)	(0.0075 -0.0739)	(.0000,.0000)
2	2	(2.929, .00000)	(-3.458E-02, 8.812E-02)	(-0.0233 0.0873)	(.0000,.0000)
2	3	(.0000,7.176)	(-1.514E-05, .0000)	(0.0003 0.0000)	(.0000,.0000)
2	4	(.0000, 12.31)	(-2.160E-08, .0000)	(0.0000 0.0000)	(.0000,.0000)
3	1	(.00000, .9386)	(-.4152, .0000)	(-0.2927 -0.0738)	(.0000,.0000)
3	2	(.00000,4.572)	(-7.620E-04, .0000)	(-0.0008 -0.0004)	(.0000,.0000)
3	3	(.0000, 8.999)	(-4.108E-06, .0000)	(-0.0001 0.0009)	(.0000,.0000)
3	4	(.00000, 13.45)	(-1.519E-08, .0000)	(0.0000 0.0000)	(.0000,.0000)
4	1	(.0000, 7.073)	(-3.098E-05, .0000)	(0.0001 -0.0001)	(.0000,.0000)
4	2	(.0000, 8.370)	(-3.872E-06, .0000)	(0.0000 0.0001)	(.0000,.0000)
4	3	(.0000, 11.40)	(-1.597E-07, .0000)	(0.0000 0.0000)	(.0000,.0000)
4	2	(.0000, 15.17)	(-1.523E-09, .0000)	(0.0000 0.0000)	(.0000,.0000)
5	1	(.0000, 10.90)	(-1.161E-07, .0000)	(-0.0004 0.0002)	(.0000,.0000)
5	2	(.0000, 11.78)	(-2.611E-08, .0000)	(0.0000 0.0001)	(.0000,.0000)
5	3	(.0000, 14.10)	(-2.900E-09, .0000)	(0.0000 -0.0001)	(.0000,.0000)
5	4	(.0000, 17.29)	(-6.000E-11, .0000)	(.0000,.0000)	(.0000,.0000)

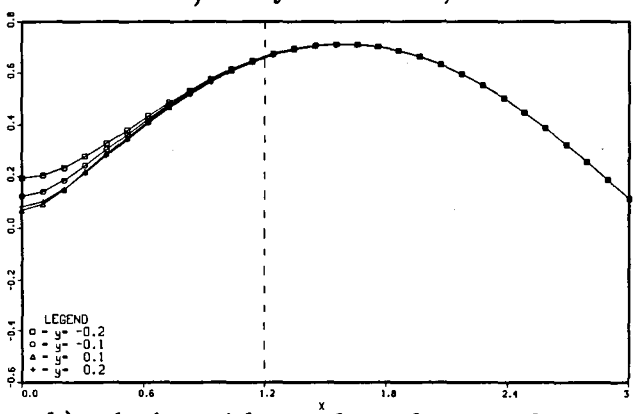
FIGURES



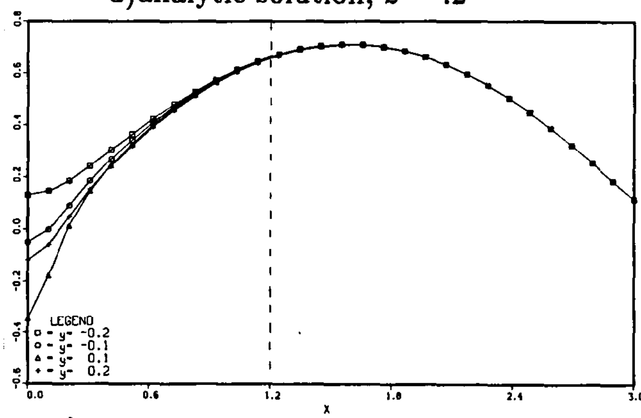
a) analytic solution, $z = -.2$



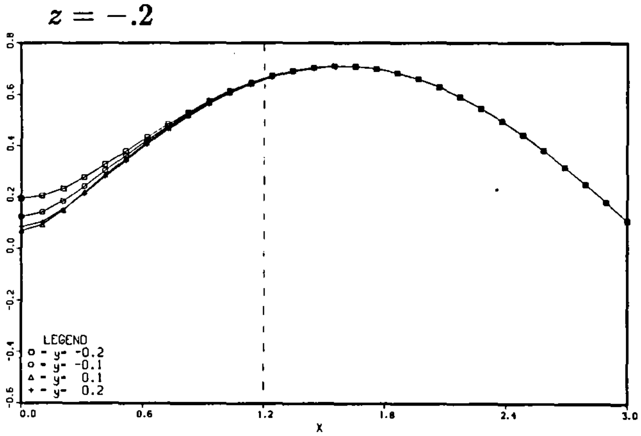
d)analytic solution, $z = .2$



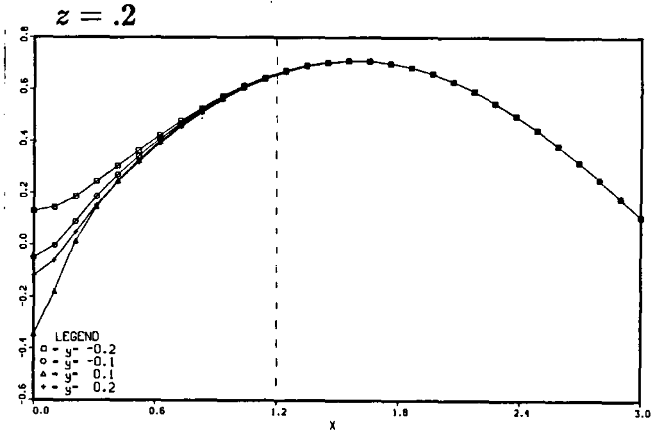
b) solution with new boundary condition, $z = -.2$



e) solution with new boundary condition, $z = .2$

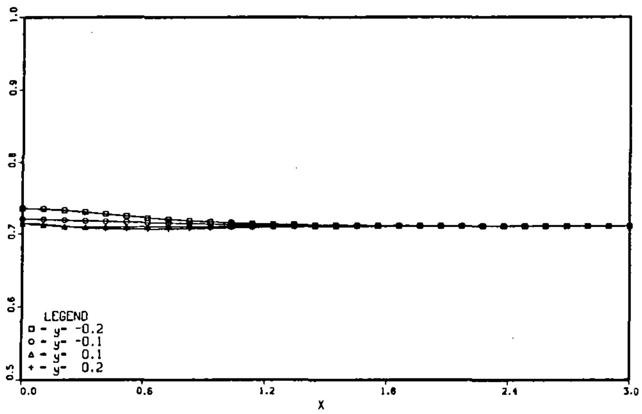


c) solution with simple boundary condition, $z = -.2$

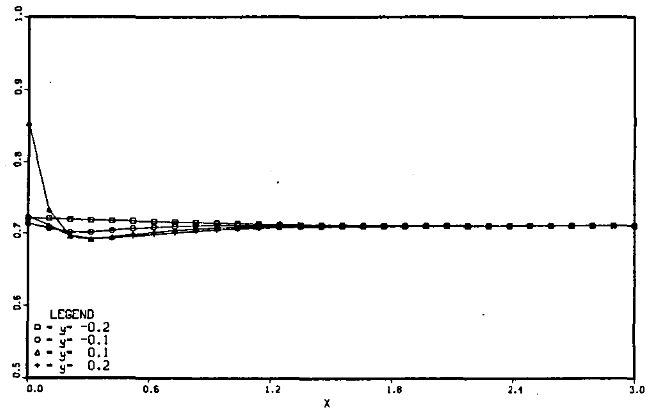


f) solution with simple boundary condition, $z = .2$

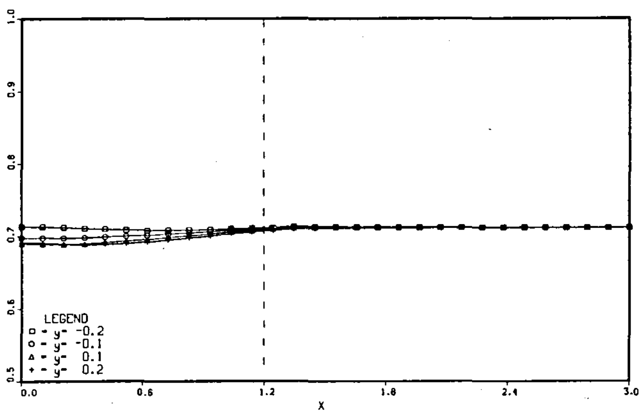
Figure 2 – $\Re(p)$ for one propagating mode



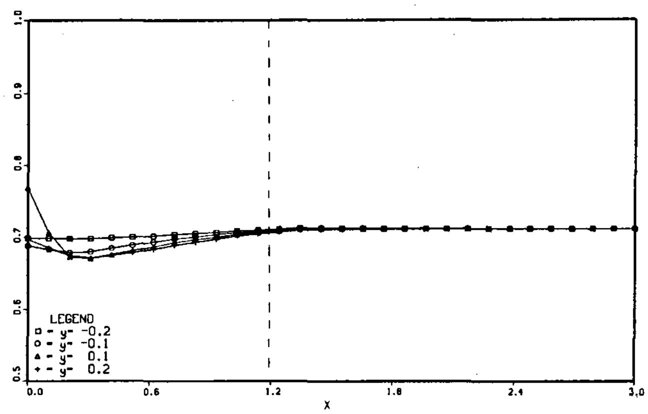
a) analytic solution, $z = -.2$



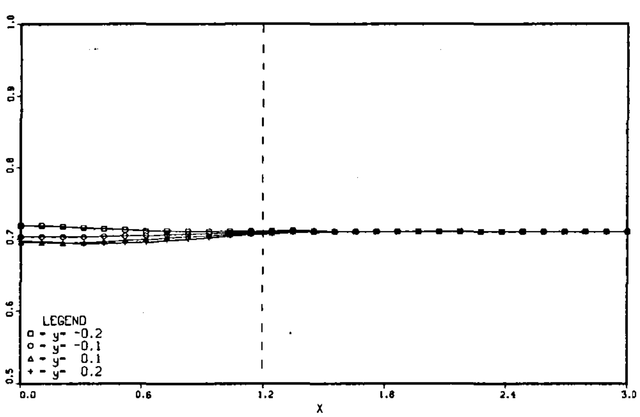
d) analytic solution, $z = .2$



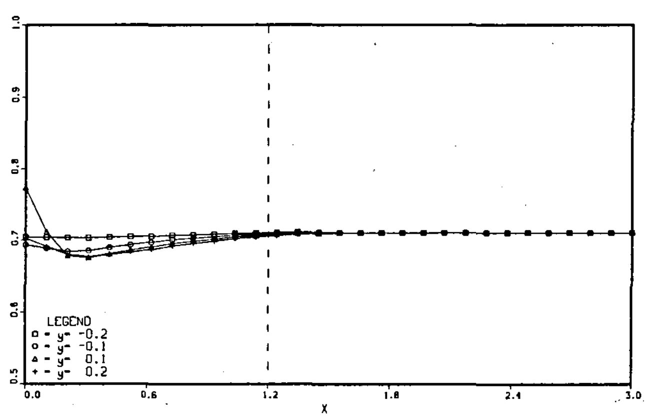
b) solution with new boundary condition, $z = -.2$



e) solution with new boundary condition, $z = .2$

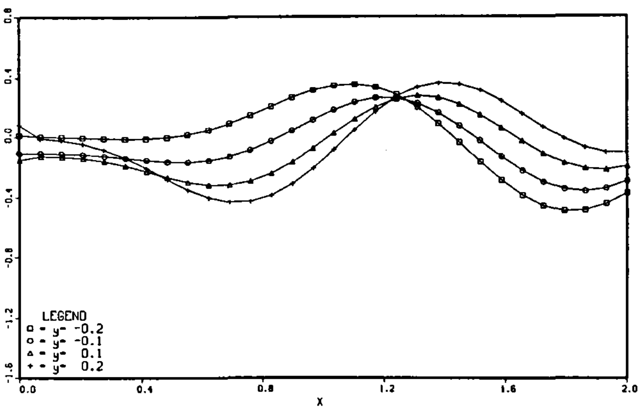


c) solution with simple boundary condition, $z = -.2$

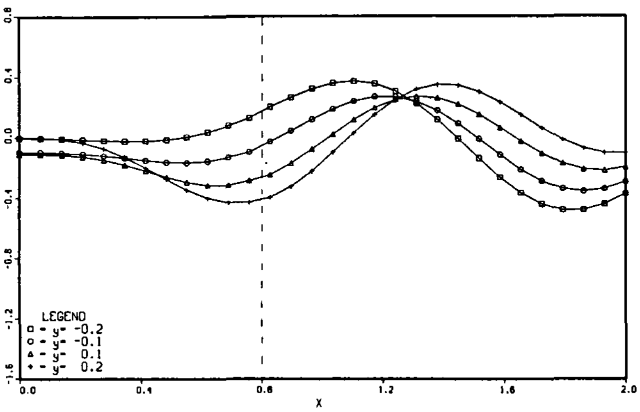


f) solution with simple boundary condition, $z = .2$

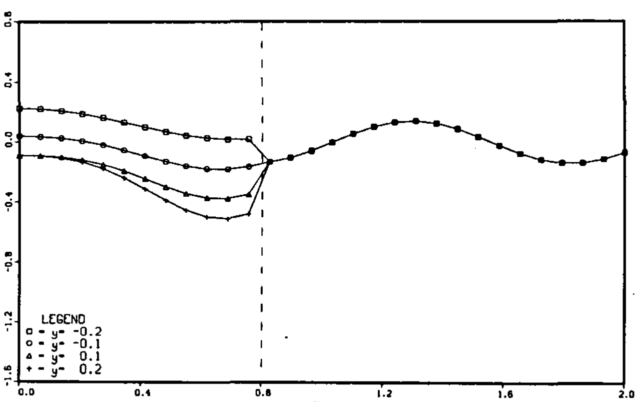
Figure 3 – $|p|$ for one propagating mode



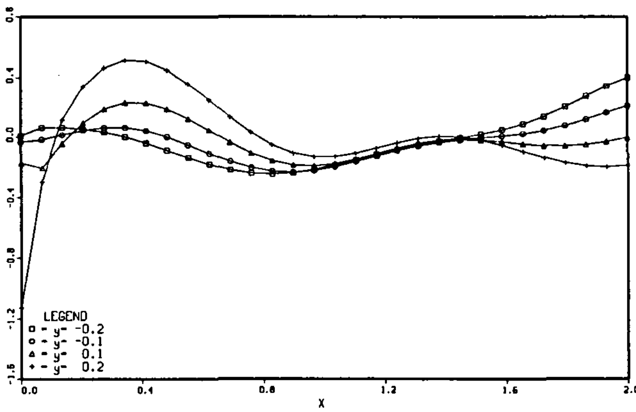
a) analytic solution, $z = -.2$



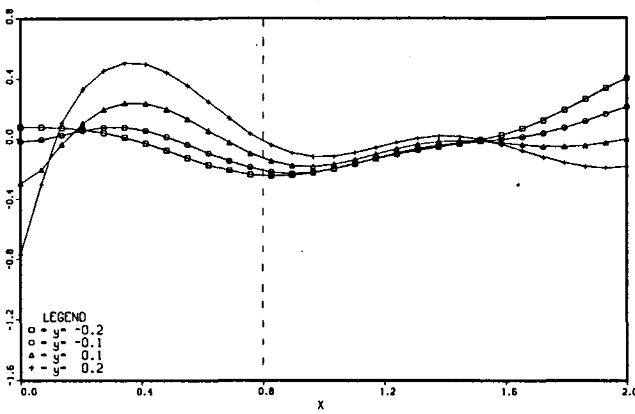
b) solution with new boundary condition, $z = -.2$



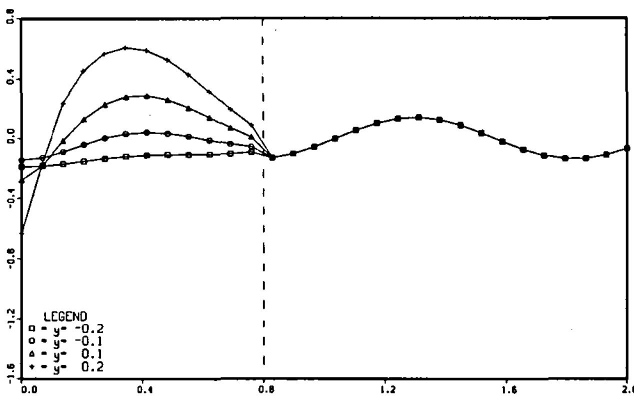
c) solution with simple boundary condition, $z = -.2$



d) analytic solution, $z = .2$

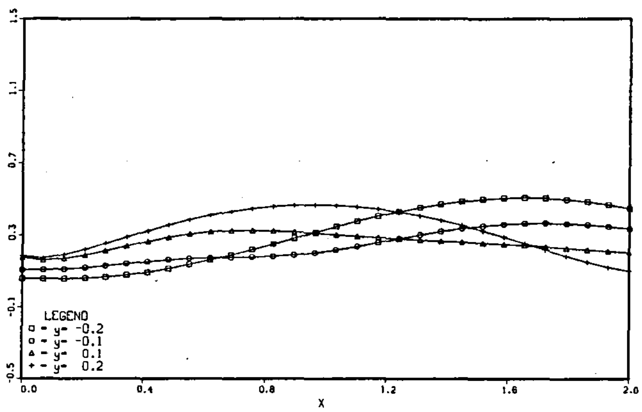


e) solution with new boundary condition, $z = .2$

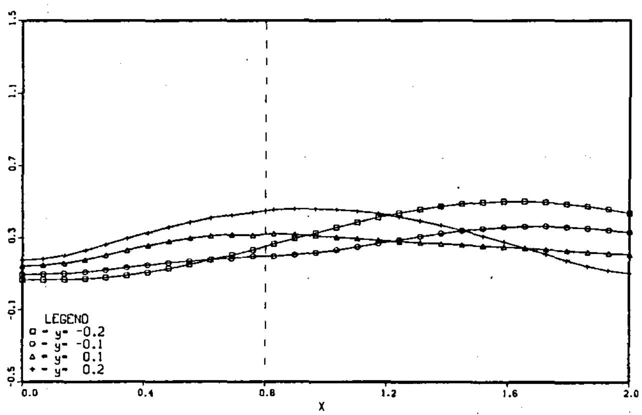


f) solution with simple boundary condition, $z = .2$

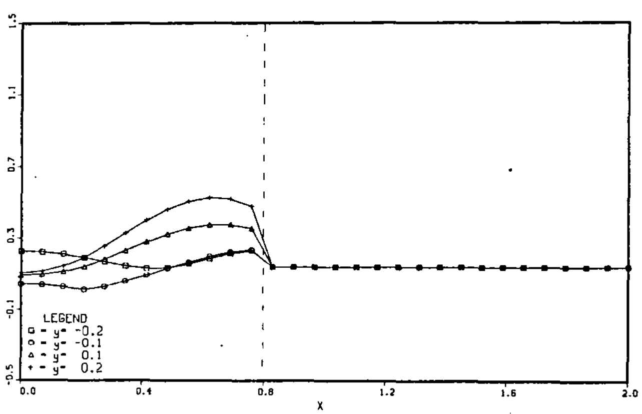
Figure 4 – $\Re(p)$ for five propagating modes



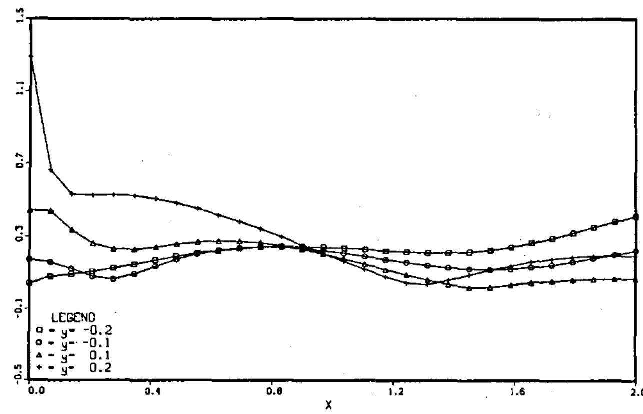
a) analytic solution, $z = -.2$



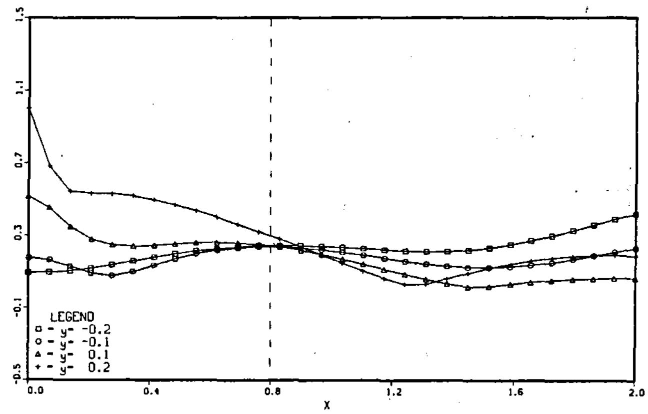
b) solution with new boundary condition, $z = -.2$



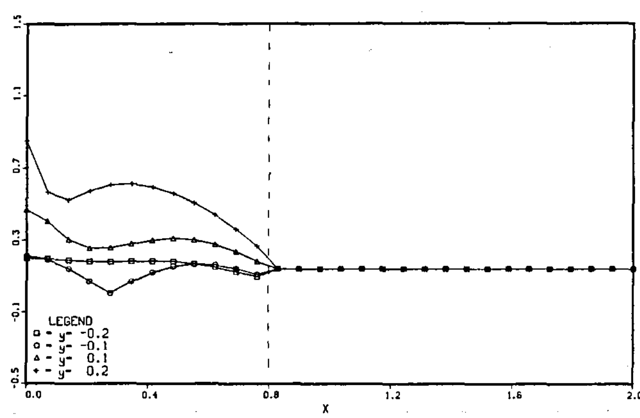
c) solution with simple boundary condition, $z = -.2$



d) analytic solution, $z = .2$



e) solution with new boundary condition, $z = .2$



f) solution with simple boundary condition, $z = .2$

Figure 5 – |p| for five propagating modes

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-86708		

4. Title and Subtitle	5. Report Date
AN OUTFLOW ACOUSTIC BOUNDARY CONDITION FOR INTERNAL DUCT FLOWS	February 1986
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Marianne Mosher	86105
	10. Work Unit No.
9. Performing Organization Name and Address	11. Contract or Grant No.
Ames Research Center Moffett Field, CA 94035	
	13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address	Technical Memorandum
National Aeronautics and Space Administration Washington, DC 20546	14. Sponsoring Agency Code 505-61-51

15. Supplementary Notes

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16. Abstract

A boundary condition for the linear acoustic equation has been developed that allows the acoustic pressure waves to propagate out of the computational domain boundary, just as they would propagate in an infinitely long duct. The problem is divided into two domains: numerical and analytical. The boundary condition provides a matching of the two domains. Examples show this method works well in an acoustic panel program for a model problem (simple source in a rectangular duct with several propagating modes present). This report describes the boundary condition so that it can be used with various duct geometries and numerical methods.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Acoustic Outflow boundary condition Unsteady boundary condition Internal flow Matched solution	Unlimited Subject Category: 71

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	20	A02

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