EFFECT OF COULOMB SPLINE ON ROTOR DYNAMIC RESPONSE

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A rigid rotor system coupled by a coulomb spline is modelled and analyzed by approximate analytical and numerical-analytical methods. Expressions are derived for the variables of the resulting limit cycle and are shown to be quite accurate for a small departure from isotropy.

SYMBOLS

a,b semi-major, minor axis of the elliptical response

a_m $(a+b)/2$

a_d $(a-b)/2$

c external viscous damping coefficient

e amplitude of the circular limit cycle

$\overline{F}$ coulomb spline force vector

I_p equivalent polar moment of inertia

$k_{y,z}$ equivalent stiffness in y,z direction

$\ell_{1,2}$ length of shaft 1,2

m equivalent mass of the system

t time, nondimensionalized using ω_y

T torque transmitted by the coupling

v,w response in the y,z direction (nondimensionalized relative to ℓ_1)

α gyroscopic parameter $= I_p/m\ell_1^2$

α_0 attitude angle of the elliptical response

γ orthotropy parameter $= \omega_z/\omega_y$

ζ viscous damping ratio

λ nondimensional nonlinearity parameter (16 splines)
$= 0.64\mu T \ (1+\ell_1/\ell_2)/m\omega_y^2\ell_1^2$

μ coefficient of friction

$\overline{\Omega}$ nondimensional spin speed of the rotor $= \Omega/\omega_y$

$\overline{\omega}$ nondimensional whirl speed of the rotor $= \omega/\omega_y$

ω_y $\sqrt{k_y/m}$

ω_z $\sqrt{k_z/m}$

INTRODUCTION

A spline coupling connecting two rotors is often used in the turbine engine industry. When it is not sufficiently lubricated, the resulting coulomb friction can produce a driving force that sustains an asynchronous whirling motion.

A simple rigid rotor system coupled by such a spline was analyzed by Williams and Trent (ref. 1) using simulations on an analog computer. Morton (ref. 2) considered the effect of coulomb friction in drive couplings on the response of a flexible shaft and concluded that misalignments had the effect of stabilizing such a system. Marmol, Smalley and Tecza (ref. 3) modelled a coulomb spline by equivalent viscous friction and comparisons of their analytical results to those from a test rig were satisfactory.

The model to be analyzed in this paper is very similar to the one in reference 1, and contains only two degrees of freedom. The supports have, in general, orthotropic stiffness and the model includes the gyroscopic effect.

The emphasis here is on obtaining an improved understanding of the underlying phenomena rather than a detailed quantitative analysis of a large order system. It is for this reason that analytical and numerical-analytical methods have been applied to this problem (in addition to numerical simulation) so that a better insight could be gained.

MATHEMATICAL MODEL

A system consisting of two rigid rotors coupled by a coulomb spline is considered (fig. 1). They are supported on a mount with linear isotropic viscous damping and (in general) anisotropic stiffness. This is essentially a two degree-of-freedom model and the equations of motion are found to be:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{v} \\ \ddot{w} \end{Bmatrix} + \left(\begin{bmatrix} 2\varsigma & 0 \\ 0 & 2\varsigma \end{bmatrix} - \bar{\Omega} \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix} \right) \begin{Bmatrix} \dot{v} \\ \dot{w} \end{Bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & \gamma^2 \end{bmatrix} \begin{Bmatrix} v \\ w \end{Bmatrix} = \bar{F} \qquad (1)$$

The above equations are in nondimensional form and the derivatives are with respect to a nondimensional time. The spin speed of the rotor is denoted by Ω, α denotes a gyroscopic parameter, ς the damping ratio and γ represents an orthotropy parameter ($\gamma = 1$: isotropic support). An equivalent force vector, $\bar{F}$ arises due to the friction excitation in the coulomb spline and is given by:

$$\bar{F} = - \frac{\lambda}{[(\dot{w} - \bar{\Omega}v)^2 + (\dot{v} + \bar{\Omega}w)^2]^{1/2}} \begin{Bmatrix} \dot{v} + \bar{\Omega}w \\ \dot{w} - \bar{\Omega}v \end{Bmatrix} \qquad (2)$$

where, λ is a nondimensional parameter denoting the strength of the nonlinearity.

The system is thus modelled by a set of coupled, second order, nonlinear, autonomous equations (eqn. 1) and a closed form analytical solution is difficult, if not impossible.

SOLUTION: ISOTROPIC CASE

When the support is isotropic ($\gamma=1$), an exact solution can be obtained by assuming a circular response of amplitude e and a frequency $\bar{\omega}$.

i.e.,
$$v = e \cos \bar{\omega}t$$

$$w = e \sin \bar{\omega}t$$
(3)

Substitution of this assumed solution form into equations (1) and (2) yields the following results:

$$\bar{\Omega} < \bar{\omega} \qquad \text{No Limit Cycle Possible}$$

$$\bar{\Omega} > \bar{\omega} \qquad \text{Limit Cycle Exists}$$

with
$$\bar{\omega} = \bar{\Omega}\alpha/2 + \sqrt{1 + (\bar{\Omega}\alpha/2)^2}$$
(4)

$$e = \lambda/2\zeta\bar{\omega}$$
(5)

It is interesting to note that the frequency of the limit cycle is the same as the undamped natural frequency of the linearized system and is independent of the strength of the nonlinearity (λ) and the linear viscous damping ratio (ζ). The amplitude of the resulting circular limit cycle is directly proportional to λ and reduces with increasing spin speed Ω. Figure 2 shows the frequency spectrum of the numerically simulated response at a nondimensional spin speed of 3. Clearly, a single peak occurs at a frequency of 1.56 that agrees with the analytical prediction (eqn. 4).

The onset of instability speed, at which a bifurcation into a limit cycle takes place, can be found to be

$$\bar{\Omega}_{0I} = \frac{1}{\sqrt{1-\alpha}}$$
(6)

Similar results were obtained in reference 1, but without the inclusion of the gyroscopic effect.

Figure 3 shows the force balance diagram for the isotropic system at a spin speed above the onset of instability. From energy considerations, it can also be shown that when the spin speed Ω is less then the whirl speed ω, the coulomb spline dissipates energy and hence cannot sustain a limit cycle. When $\Omega > \omega$ however, the spline gives rise to equivalent negative damping. Then, the amplitude of the limit cycle can be determined by equating the energy dissipated by the linear

viscous damping to the energy added (to the system) by the coulomb spline forces.

The stability of the limit cycle can be investigated by perturbing the limit cycle and writing linear variational equations for the perturbed motion. A detailed stability analysis indicates that the circular limit cycle is stable for all possible values of the parameters.

SOLUTION: ORTHOTROPIC CASE

When the support is not isotropic ($\gamma \neq 1$), the solutions are no longer amenable to an exact analysis. Hence, two approximate methods have been used and they are outlined below.

Trigonometric Collocation Method (TCM)

This is a numerical-analytic method that has been used quite widely for the solution of nonlinear boundary value problems. Ronto (ref. 4) has recently developed and formalized the method for application to periodic solutions and the basic philosophy is outlined below, omitting all the details and proofs. Given a set of nonlinear ODE's

$$\bar{x} = \bar{f}(\bar{x}) , \tag{7}$$

a periodic solution $\bar{x}_0(t)$ is sought in the form:

$$\bar{x}_0(t) = \bar{a}_0 + \sum_{n=1}^{N} (\bar{a}_n \cos n\bar{\omega}t + \bar{b}_n \sin n\bar{\omega}t) \tag{8}$$

where the ith typical coefficients $\bar{a}_i$ and $\bar{b}_i$ and the frequency $\bar{\omega}$ are unknown. This assumed solution is substituted into the differential equations and the resultant errors are forced to be zero at a prescribed number of time points (at least $2N+1$). This leads to a sufficient number of nonlinear algebraic equations in the unknown coefficients and the frequency and can be solved using various numerical methods.

Though this method was developed mainly for the solution of nonautonomous equations in reference 4, it has been successfully developed and applied for autonomous systems as well.

Application of the TCM to the orthotropic system reveals some interesting results. The response is nearly elliptical (as can be expected) but also has several higher frequency components. These higher harmonics have very low amplitudes and hence are not easily observed by numerical simulation.

The effect of the higher harmonics is best seen in a semi-log plot of the frequency spectrum (fig. 4) that has been generated by numerical simulation. The nondimensional spin speed $\bar{\Omega}$ is equal to 3 and peaks are seen to occur at $\bar{\omega}$, $3\bar{\omega}$, $5\bar{\omega}$, etc., where the fundamental frequency $\bar{\omega}$ is ~ 2.92. This may be contrasted with the frequency spectrum for the

isotropic case (fig. 2). The fundamental frequency of the limit cycle is seen to be independent of the strength of the nonlinearity (λ) and the damping (ς). The higher harmonic amplitudes are seen to increase with an increase in orthotropy.

Numerical simulation has been carried out for a range of parameters and the results compare very well with those obtained by the TCM.

Analytical Approximation (AM)

In order to obtain some idea about how the response variables vary with the system parameters, analytical expressions that approximate the response are quite useful. To derive these, since the higher frequency components have low amplitudes, the solution is assumed in the form of a single frequency elliptical component. As the system is autonomous, the time origin of the response is arbitrary. Hence,

$$v = a_m \cos(\bar{\omega}t + \alpha_0) + a_d \cos(\bar{\omega}t - \alpha_0)$$

$$w = a_m \sin(\bar{\omega}t + \alpha_0) - a_d \sin(\bar{\omega}t - \alpha_0)$$

(9)

Substituting this assumed form of the solution into the equations of motion, equation 1, and after substantial algebraic manipulations and retention of only first order approximations, an approximate expression for the fundamental frequency of the limit cycle is seen to be:

$$\bar{\omega}^2 \approx A \{1 + \sqrt{1 - (\gamma/A)^2}\,\}$$

(10)

where:
$$A = (1 + \gamma^2 + \alpha^2\bar{\Omega}^2)/2$$

(11)

and,
$$b/a \approx (\bar{\omega}^2 - 1)/\alpha\bar{\Omega}\bar{\omega}$$

(12)

$$b \approx \lambda/2\varsigma\bar{\omega}$$

(13)

The approximate expression for the fundamental frequency is the same as the undamped natural frequency of the linearized orthotropic system. The general variation of the frequency and amplitude is very similar to the isotropic case. However, in contrast to the isotropic case, the limit cycle appears to exist at all spin speeds. This is consistent with the simulation results of Williams and Trent (ref. 1).

Table 1 shows a comparison between the analytical predictions and the values obtained by the TCM for several values of γ (orthotropy). The frequency prediction is correct to within 10% for γ up to 3.0 and the amplitude prediction has less than 10% error for γ up to 1.6.

The stability of the limit cycle in the orthotropic case has not yet been analyzed and further investigations will be reported elsewhere.

CONCLUSION

A two degree-of-freedom model of rigid motors coupled by a coulomb spline is analyzed. Two different cases are considered.

When the support is isotropic, a bifurcation into a stable, circular limit cycle takes place at a spin speed equal to the forward undamped critical speed. The frequency of the limit cycle is the same as the undamped natural frequency of the system and is independent of the strength of the nonlinearity.

When the support is orthotropic, the limit cycle has many frequency components but is still nearly elliptical. The undamped natural frequency of the system, however, approximates the fundamental whirl frequency of the response to a very good accuracy. The higher harmonic amplitudes increase with an increase in orthotropy. The stability of the response in the orthotropic case is of considerable interest and remains to be investigated.

Similar qualitative phenomena can be expected to occur in large flexible rotor-bearing systems with a spline coupling. Work is continuing along these lines.

REFERENCES

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TABLE 1. - COMPARISON BETWEEN ANALYTICAL AND TCM RESULTS

$$(\Omega = 3.0, \quad \lambda = 0.01867, \quad \zeta = 0.1, \quad \alpha = 0.3)$$

γ	ω_{AM}	ω_{TCM}	$(b/a)_{AM}$	$(b/a)_{TCM}$	$b_{AM} \times 10^2$	$b_{TCM} \times 10^2$
1.0	1.5466	1.5466	1.0000	1.0000	6.0348	6.0348
1.1	1.5470	1.570	1.0007	1.0007	5.8511	6.0169
1.2	1.6495	1.6490	1.1591	1.1513	5.6584	5.8875
1.3	1.7093	1.7081	1.2491	1.2348	5.4605	5.6251
1.5	1.8433	1.8386	1.4453	1.3929	5.0633	4.6291
3.0	3.1466	2.8387	3.1431	1.1148	2.9662	0.4871
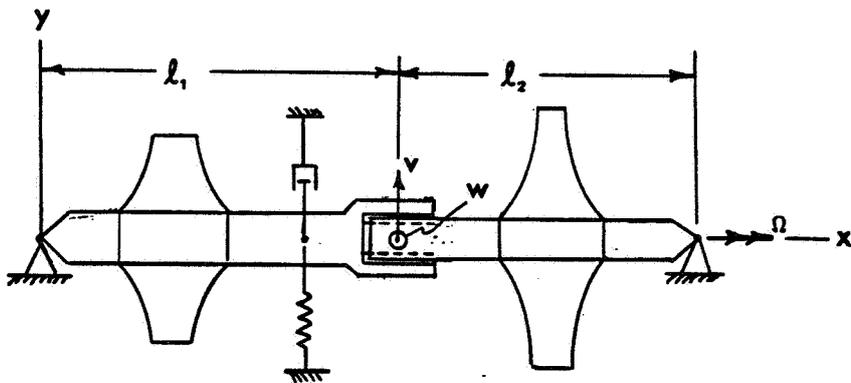
Figure 1. - Schematic of typical rotor.

OMEGA = 3.0 LAMBDA = 0.01867 ZETA = 0.100

GAMMA = 1.00 ALPHA = 0.300
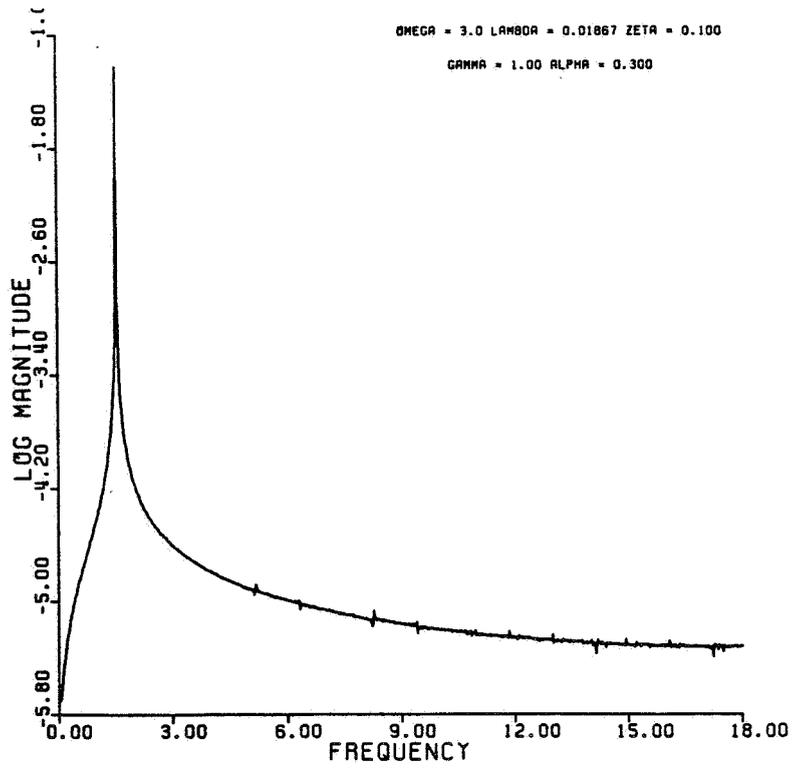
Figure 2. - Response frequency spectrum: isotropic case.
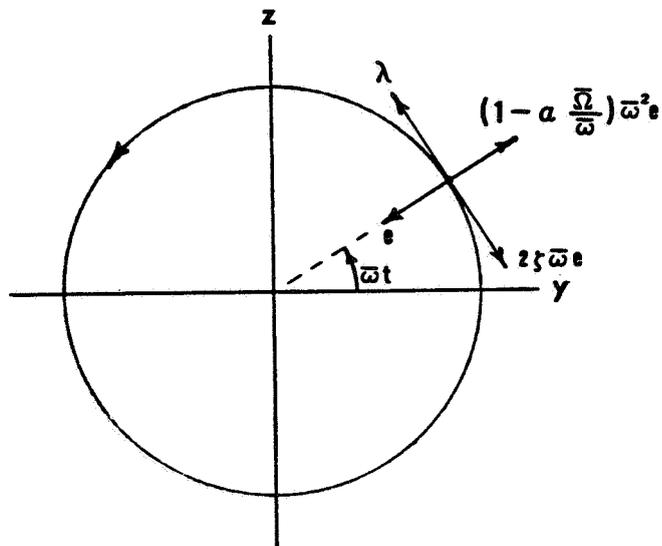
Figure 3. - Force balance diagram: isotropic case.

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OMEGA = 3.0 LAMBDA = 0.01867 ZETA = 0.100

GAMMA = 3.00 ALPHA = 0.300
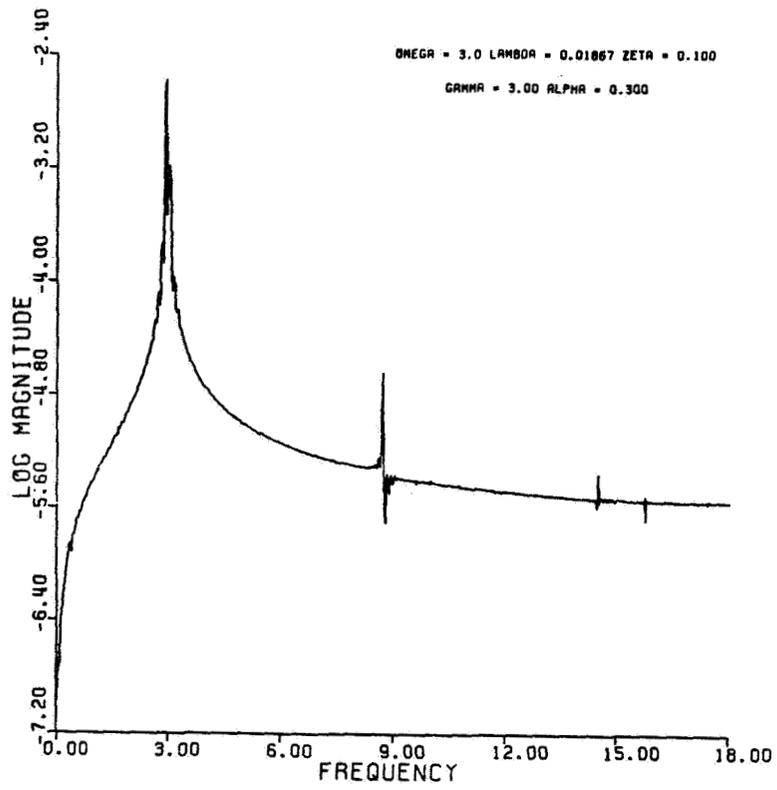
Figure 4. — Response frequency spectrum: orthotropic case.