NASA Technical Memorandum 87749

/U-19606

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(NASA-TM-87749) A SIMPLE NONLINEAR JOINT MODEL (NASA) 7 p CSCL 20K N86-30234

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G3/39 43542

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AUGUST 1986

NASA

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A Simple Nonlinear Joint Model

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**Scientific and Technical
Information Branch**

1986

Summary

Hertzian contact theory is applied to a butt joint with specially mismatched bearing surfaces to devise a simple mathematical model of nonlinear axial force-displacement behavior in jointed members. Normalized tangent stiffness-force plots, for several values of a joint imperfection parameter, are presented for the sample case of solid structural members of circular cross section. The results illustrate the potential problem of high joint compliance at low axial-force levels, as well as the generally desirable stiffening and "linearizing" effects of preload. A nonlinear oscillator problem based on the static model is also formulated and solved to illustrate the effect of amplitude on natural frequency. As expected, natural frequency is low when amplitude is small. The results call attention to the important roles that tight tolerances and preload are expected to play in the design and fabrication of deployable and erectable truss-type space structures.

Introduction

Controlling the shape and attitude of truss-type space structures requires knowledge of their force-displacement behavior, which is likely to be influenced by the behavior of mechanical joints. Most jointed members exhibit some nonlinear force-displacement behavior, especially when lightly loaded. Because of the complex geometries involved, as well as the "looseness" dictated by joint-articulation requirements, a general analysis of force-displacement behavior in jointed members is probably infeasible. However, behavioral trends can often be identified through the study of simplified models that incorporate some of the salient features of actual joints.

In this paper, a simple nonlinear model of a slender member containing an imperfect butt joint is devised and analyzed to obtain expressions for member compressive axial force and tangent stiffness as functions of overall axial strain for arbitrary values of a joint imperfection parameter. The model is also used as a basis for a simple nonlinear spring-mass system, the analysis of which may have implications for the dynamic response of truss-type space structures.

Analysis

Static Model

Conceptual aspects of the nonlinear model are illustrated in figure 1. Two slender members of equal length meet at the butt joint, which, ideally, would involve two perfectly aligned flat surfaces. However, as in many practical situations, there is some surface mismatch. Hence, the amount of joint surface actually in contact varies with compressive axial-force level and generally increases with the force, as does, therefore, the effective joint stiffness. In this paper, the nonlinear joint behavior is modeled according to a Hertzian contact law for the joint faces (e.g., see ref. 1), which should be reasonable as long as the contact area is relatively small. Not illustrated in the figure is a hypothetical device which is assumed to keep the two members in axial alignment, thus assuring stability under axial force, without affecting the force-displacement behavior of the overall configuration.

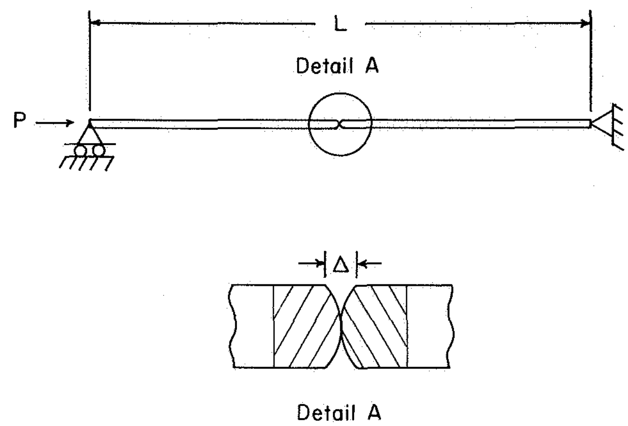


Figure 1. Sketch of static model.

For simplicity, and consistent with the Hertzian contact assumptions, the mismatched joint faces are taken to be shallow paraboloids in contact over a circular interface, the extent of which depends on the axial force and the Young's modulus of the joint material. The slender members, exclusive of the joint body, are assumed to shorten linearly with compressive axial force. The total dimensionless shortening of the jointed column of length L due to the applied axial force P is

$$\varepsilon = \frac{U}{L} = \frac{P}{EA} + \left[\frac{9}{2}\left(1-\nu_j^2\right)^2 \frac{\Delta}{L}\right]^{1/3} \times \left(\frac{E}{E_j}\right)^{2/3} \left(\frac{A}{aL}\right)^{2/3} \left(\frac{P}{EA}\right)^{2/3} \tag{1}$$

where the second term on the right-hand side of the equation is the result of the local nonlinear behavior of the joint and where

U total shortening

E Young's modulus

A member cross-sectional area

a member radius

Δ total joint mismatch

ε overall axial strain

ν Poisson's ratio

j joint properties

The joint-body length is assumed to be small in comparison with the column length.

The Euler buckling force for a simply supported column of length L and cross-sectional moment of inertia I is $P_e = \frac{\pi^2 EI}{L^2}$, and the corresponding critical strain is $\varepsilon_e = \frac{P_e}{EA}$. With the normalizations $\bar{\varepsilon} = \frac{\varepsilon}{\varepsilon_e}$ and $\overline{P} = \frac{P}{P_e}$, equation (1) can be written as

$$\bar{\varepsilon} = \overline{P} + K\overline{P}^{2/3} \tag{2}$$

where

$$K = \left[\frac{9}{2}\left(1-\nu_j^2\right)^2 \frac{\Delta}{L}\right]^{1/3} \left(\frac{E}{E_j}\right)^{2/3} \left(\frac{A^3}{\pi^2 a^2 I}\right)^{1/3} \tag{3}$$

Equation (2) describes the normalized nonlinear force-strain curve. Differentiation of equation (2) yields

$$\frac{d\overline{P}}{d\bar{\varepsilon}} = \frac{1}{1 + \frac{2}{3}\frac{K}{\overline{P}^{1/3}}} \tag{4}$$

When equation (2) is solved inversely for $\overline{P}(\bar{\varepsilon})$ and the results are combined with equation (4), the normalized tangent stiffness as a function of the normalized axial strain is obtained. Of primary interest here, however, is the solution to equation (4), which gives the tangent stiffness as a function of the axial force (or inversely, the amount of prestress needed to induce a particular effective stiffness).

Two cylindrical member types of interest are rods and thin-walled tubes. For rods, $A = \pi a^2$ and $I = \frac{\pi a^4}{4}$. For tubes with wall thickness t, $A = 2\pi a t$ and $I = \pi a^3 t$. Thus, equation (3) becomes

$$K = \begin{cases} \left[18\left(1-\nu_j^2\right)^2 \frac{\Delta}{L}\right]^{1/3} \left(\frac{E}{E_j}\right)^{2/3} & \text{(for rods)} \\ \left[36\left(1-\nu_j^2\right)^2 \frac{\Delta}{L}\right]^{1/3} \left(\frac{E}{E_j}\right)^{2/3} \left(\frac{t}{a}\right)^{2/3} & \text{(for tubes)} \end{cases} \tag{5}$$

For an illustrative example in the case of rods, we choose $\nu_j = \frac{1}{3}$ and $E_j = E$, in which case

$$K = \left(\frac{128\Delta}{9L}\right)^{1/3} \tag{6}$$

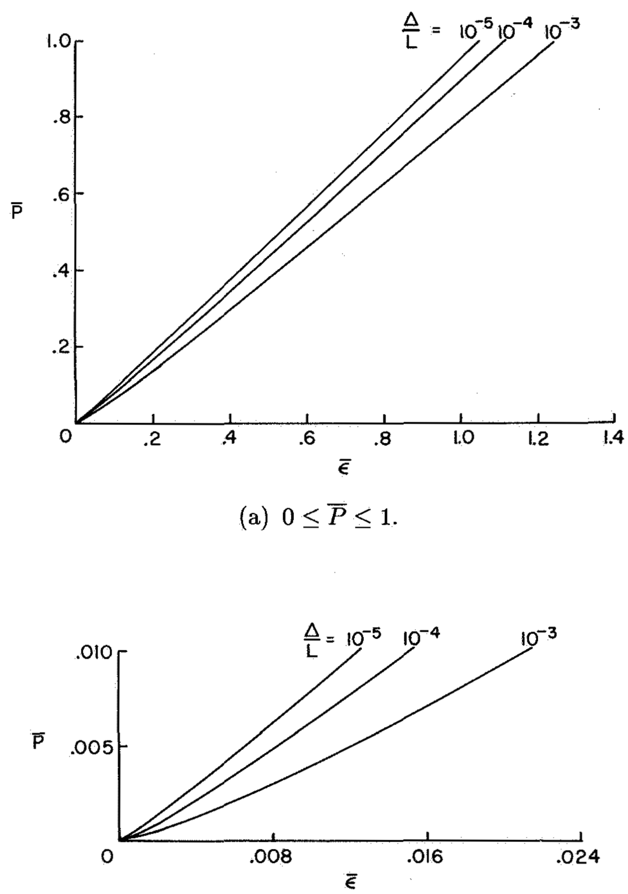
Therefore, equations (2) and (4) become

$$\bar{\varepsilon} = \overline{P} + \left(\frac{128\Delta}{9L}\right)^{1/3} \overline{P}^{2/3} \tag{7}$$

and

$$\frac{d\overline{P}}{d\bar{\varepsilon}} = \frac{1}{1 + \frac{2}{3}\left(\frac{128\Delta}{9L}\right)^{1/3} \frac{1}{\overline{P}^{1/3}}} \tag{8}$$

The solutions of equations (7) and (8) for $\Delta/L = 10^{-5}$, 10^{-4}, and 10^{-3} are plotted in figures 2 and 3. In figure 2(b), the small-strain region of figure 2(a) is magnified to illustrate the nonlinear behavior at small force levels.



(a) $0 \leq \overline{P} \leq 1$.

(b) $0 \leq \overline{P} \leq 0.01$.

Figure 2. Normalized axial force-strain plots.

Vibration Model

To simplify the construction of a numerical example, the force-strain relation of equation (2) is taken to be symmetric about the origin in the $\overline{P} - \bar{\varepsilon}$ plane. This should be a reasonable assumption

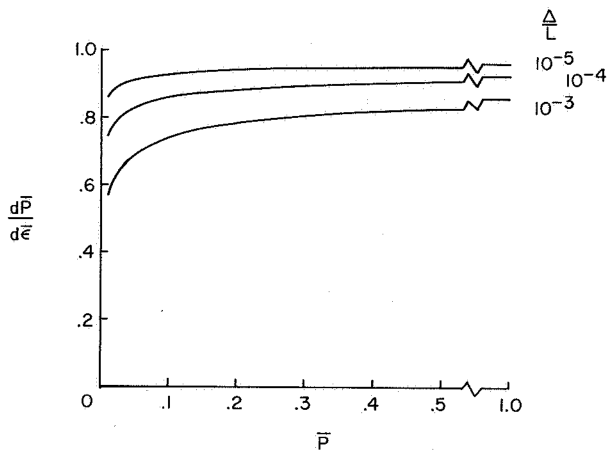


Figure 3. Normalized stiffness-force plots for solid cylindrical members.

for many lightly loaded pin joints with negligible free play. For sufficiently small axial displacements (i.e., $0 \leq \frac{U}{L} \leq \frac{4}{27}K^3$), equation (2) (before normalization) can be solved to yield

$$\frac{P}{EA} = \frac{K^3}{27}\left(\cos\frac{\alpha}{3} + \sqrt{3}\sin\frac{\alpha}{3} - 1\right)^3 \tag{9}$$

where

$$\alpha = \cos^{-1}\left(1 - \frac{27U}{2K^3L}\right)$$

If the entire jointed member is envisioned as a massless spring supporting a mass m, and the axial force P is replaced by its inertial equivalent $\left(\text{i.e., } P = -m\frac{d^2U}{dt^2}\right)$, then the equation of motion for the nonlinear spring-mass system is

$$-\frac{m}{EA}\frac{d^2U}{dt^2} = \frac{K^3}{27}\left(\cos\frac{\alpha}{3} + \sqrt{3}\sin\frac{\alpha}{3} - 1\right)^3 \tag{10}$$

The initial conditions are taken to be

$$U(0) = 0 \qquad \frac{dU}{dt}(0) = V_0 \tag{11}$$

where V_0 is an arbitrary positive quantity consistent with the restriction $0 \leq \frac{U_{\text{max}}}{K^3L} \leq \frac{4}{27}$. The quantity of interest here is the frequency of the nonlinear oscillator. Defining $\mu = \frac{U}{K^3L}$, equations (10) and (11) become

$$\frac{d^2\mu}{dt^2} = -\frac{\omega_l^2}{27}\left(\cos\frac{\alpha}{3} + \sqrt{3}\sin\frac{\alpha}{3} - 1\right)^3 \qquad \alpha = \cos^{-1}\left(1 - \frac{27}{2}\mu\right) \tag{12}$$

and

$$\mu(0) = 0 \qquad \lambda(0) \equiv \frac{d\mu}{dt}(0) = \lambda_0 \tag{13}$$

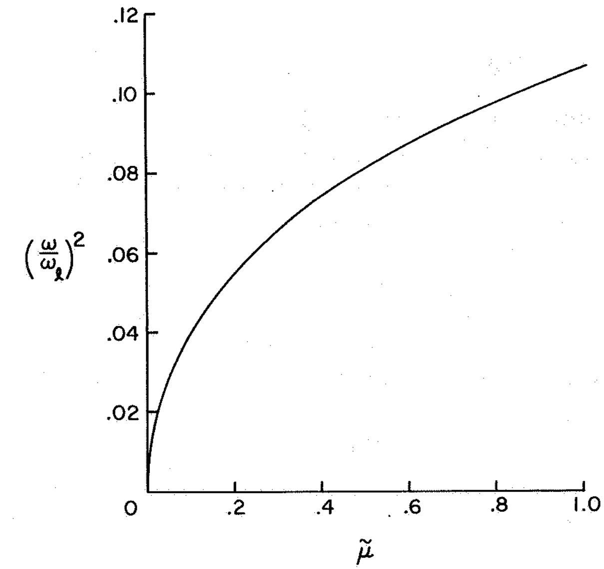


Figure 4. Frequency-amplitude plot for small oscillations.

where $\omega_l = \sqrt{\frac{EA}{mL}}$ is the frequency of the corresponding linear oscillator. One integration of equation (12) and use of the initial conditions in equation (13) yield

$$\left(\frac{d\mu}{dt}\right)^2 = \lambda_0^2 - \frac{2}{27}\omega_l^2\int_0^{\mu}\left(\cos\frac{\alpha}{3} + \sqrt{3}\sin\frac{\alpha}{3} - 1\right)^3 dt \tag{14}$$

where

$$\lambda_0^2 = \frac{2}{27}\omega_l^2\int_0^{\tilde{\mu}}\left(\cos\frac{\alpha}{3} + \sqrt{3}\sin\frac{\alpha}{3} - 1\right)^3 dt$$

and $\tilde{\mu}$ is the maximum value of μ. This maximum excursion occurs at the quarter period (i.e., $\mu\left(\frac{T}{4}\right) = \tilde{\mu}$); hence, integration of equation (14) gives

$$\frac{T}{4} = \sqrt{\frac{27}{2\omega_l^2}}G(\tilde{\mu})$$

where

$$G(\tilde{\mu}) = \int_0^{\tilde{\mu}} \frac{d\mu}{\sqrt{\int_{\mu}^{\tilde{\mu}}\left(\cos\frac{\alpha}{3} + \sqrt{3}\sin\frac{\alpha}{3} - 1\right)^3 dt}} \tag{15}$$

Finally,

$$\left(\frac{\omega}{\omega_l}\right)^2 = \frac{\pi^2}{54\,[G(\tilde{\mu})]^2} \qquad \left(0 \leq \tilde{\mu} \leq \frac{4}{27}\right) \tag{16}$$

where ω is the frequency of the nonlinear oscillator. Numerical integration of equation (15) for numerous values of $\tilde{\mu}$ in the range $0 \leq \tilde{\mu} \leq \frac{4}{27}$ and substitution into equation (16) yields the plot shown in figure 4.

Results and Discussion

Static Model

Figure 3 contains plots of $\frac{d\overline{P}}{d\bar{\varepsilon}}$ as a function of $\overline{P}$ for three values of $\frac{\Delta}{L}$, the joint imperfection parameter. In general, the plots may be thought of as yielding the axial preload needed to induce a desired stiffness level. Clearly, the greater the preload, the greater the axial stiffness and, concurrently, the more nearly linear the force-displacement behavior of the jointed member. Also, the tangent stiffness reaches a particular level sooner (i.e., at lower axial force), when $\frac{\Delta}{L}$ is smaller. In fact, in the limiting case $\frac{\Delta}{L} = 0$, the normalized tangent stiffness graph becomes simply the line $\frac{d\overline{P}}{d\bar{\varepsilon}} = 1$ for all values of $\overline{P}$. Hence, attaining a particular stiffness level (or "degree of linearity") requires lower preload when joint imperfections are smaller.

The values chosen for $\frac{\Delta}{L}$ in the example are somewhat arbitrary. However, though it may be comforting that even modest preload greatly enhances structural stiffness, the model dictates very low stiffness at near-zero axial-force levels for any nonzero value of $\frac{\Delta}{L}$. Therefore, in truss structures containing joints whose force-displacement behavior is reasonably approximated by a Hertzian contact law, high compliance can be expected whenever the forces transmitted by such joints are small. This often undesirable tendency can be mitigated by the addition of pretensioned diagonals or local joint-loading devices, but there will be attendant penalties in terms of mass and complexity.

Vibration Model

As noted in the static case, the Hertzian contact model dictates very low stiffness when axial force (hence, displacement) is small. In the case of the nonlinear oscillator, this characteristic leads to very low natural frequencies when vibration amplitudes are small. This result is also consistent with the nonlinear force-strain plot in figure 2(b). For very small amplitudes, the square of the frequency varies approximately as the square root of the amplitude. (See fig. 4.) Such behavior is likely to impact adversely on the ability to control shape and attitude of a large, multijointed truss structure. As in the static case, however, this undesirable tendency can be mitigated by the introduction of preload through pretensioned diagonals or local joint-loading devices.

Although this highly simplified vibration model clearly does not incorporate all the important dynamic features of a complex truss structure, it does call attention to the potentially troublesome characteristic of high compliance in the absence of preload, which must be dealt with in the design and fabrication stages.

Concluding Remarks

Hertzian contact theory has been used to devise a simple nonlinear model of the axial force-displacement behavior of a truss member containing an imperfect joint. The nonlinear joint model employed here, although too idealized to yield quantitative design information, highlights a behavioral trend which is likely to be exhibited by a variety of truss-type space structures, that is, high compliance at relatively low applied load levels. Similarly, very low natural frequencies can be expected when vibration amplitudes are small. Clearly, one means of moderating this tendency is to build joints with tighter tolerances, an approach which can have undesirable implications for fabrication costs and ease of erection or deployment. Another approach is to design to less demanding tolerances and "linearize" the truss behavior with pretensioned diagonals or local joint-loading devices.

NASA Langley Research Center
Hampton, VA 23665-5225
June 17, 1986

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Standard Bibliographic Page

1. Report No. NASA TM-87749	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle A Simple Nonlinear Joint Model		5. Report Date August 1986
		6. Performing Organization Code 506-43-41-02
7. Author(s) W. B. Fichter		8. Performing Organization Report No. L-16169
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665-5225		10. Work Unit No.
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546-0001		13. Type of Report and Period Covered Technical Memorandum
		14. Sponsoring Agency Code

15. Supplementary Notes

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17. Key Words (Suggested by Authors(s)) Truss structures Elasticity Contact Joints	18. Distribution Statement Unclassified—Unlimited Subject Category 39

19. Security Classif.(of this report) Unclassified	20. Security Classif.(of this page) Unclassified	21. No. of Pages 5	22. Price A02

For sale by the National Technical Information Service, Springfield, Virginia 22161

NASA-Langley, 1986