APPENDIX A

Characteristic Impedance of a Wide Slot Line on Low Permittivity Substrates*

R. Janaswamy, Student Member
D.H. Schaubert, Senior Member

Department of Electrical and Computer Engineering
University of Massachusetts
Amherst, MA 01003

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Abstract: Computed results on the characteristic impedance of wide slots etched on an electrically thin substrate of low dielectric constant ε_r are presented. These results combined with those in [1] provide design data for these slotlines. Curves are presented for $\varepsilon_r = 2.22$, 3.0, 3.8 and 9.8. Comparison is shown for the characteristic impedance between the present calculations and those available in the literature for high-ε_r substrates. Empirical formulas, based on least square curve fitting, are presented for the normalized slot wavelength λ'/λ_0 and the characteristic impedance Z_0 over the range: $0.0015 \leq W/\lambda_0 \leq 1.0$, $0.006 \leq d/\lambda_0 \leq 0.06$, $2.22 \leq \varepsilon_r \leq 9.8$.

I. Introduction

Impedance properties of a slot line shown in Fig. 1 have been thoroughly treated in the literature by a number of authors [2,3]. All the previous work has been confined to slots on high-ε_r substrates ($\varepsilon_r \geq 9.6$), which are typically used for circuit applications. No data have been reported for slots on low-ε_r substrates, where slot lines have interesting applications as antennas [4-6]. Knowledge of the characteristic impedance of slot lines on these low-ε_r substrates is highly desirable in designing a proper feed and accompanying circuits for these antennas.

In this paper, computed data are presented for the characteristic impedance Z_0 for slots on low-ε_r substrates. The problem is formulated in the spectral domain and the eigenvalue equation for the eigen pair (λ', e^s), where λ' is the slot wavelength and e^s the slot field, is solved by using the spectral Galerkin's method [7]. The slot characteristic impedance Z_0 is calculated in the spectral domain from the slot field.

II. Formulation of the Problem and Numerical Results

The characteristic impedance Z_o of the slot line shown in Fig. 1 is defined as [2]

$$Z_o = \frac{|V_o|^2}{P_f} \tag{1}$$

where V_o is the voltage across the slot in the plane of the slot and is given in terms of the transverse electric field component E_x as

$$V_o = \int_{-W/2}^{W/2} E_x dx = \tilde{E}_x(\alpha)\Big|_{\alpha=0} = \tilde{E}_x(0) \tag{2}$$

'~' denotes quantities Fourier transformed with respect to the x-axis and α is the transform variable. P_f is the real part of the complex power flow (actually real in this case for a propagating mode) along the slot and is given by

$$\begin{aligned} P_f &= \int_x \int_{y \text{ plane}} (E_x H_y^* - E_y H_x^*)\, dxdy \\ &= \frac{1}{2\pi} \int_\alpha \int_{y \text{ plane}} (\tilde{E}_x \tilde{H}_y^* - \tilde{E}_y \tilde{H}_x^*)\, d\alpha dy \end{aligned} \tag{3}$$

where E_x, E_y, H_x, H_y are fields tangential to the z=constant plane. The second equality in (3) follows from Parseval's theorem and * denotes the complex conjugate.

The fields $\tilde{E}$ and $\tilde{H}$ in the spectral domain pertaining to the air and dielectric regions of the slot line can be related to the aperture field (i.e., field in the slot), which is modeled by the method of moments. As was done in [1], the field in the slot region is expanded as

$$\begin{aligned} E_x^s &= \sum_{n=0}^{M_x} a_n e_n^x \\ e_n^x &= \left(\frac{2}{\pi W}\right) \frac{T_{2n}\left(\frac{2x}{W}\right)}{\sqrt{1-\left(\frac{2x}{W}\right)^2}} \quad ; \; n = 0,1,\ldots \end{aligned} \tag{4}$$

$$E_z^s = \sum_{m=1}^{M_z} b_m e_m^z$$

$$e_m^z = \left(\frac{2}{\pi W}\right) \sqrt{1-\left(\frac{2x}{W}\right)^2}\; U_{2m-1}\left(\frac{2x}{W}\right) \quad ; \; m = 1,2,\ldots \tag{5}$$

where T_n and U_n are Tchebycheff polynomials of the I and II kind respectively.

The Fourier transforms of the above basis functions can be found readily in closed form as [8]

$$\tilde{e}_n^x = (-1)^n J_{2n}\left(\frac{\alpha W}{2}\right) \quad , \quad n = 0,1,\ldots \tag{6}$$

$$\tilde{e}_m^z = j(-1)^{m+1}\; 2m \frac{J_{2m}\left(\frac{\alpha W}{2}\right)}{\left(\frac{\alpha W}{2}\right)} \; , \; m = 1,2,\ldots \tag{7}$$

The integration with respect to y in (3) can be done in closed form. However, the integration on the α variable must be done numerically. The slot wavelength λ' is stationary with respect to the slot field and it was found that λ' converges with only one basis function for the longitudinal field as reported in [1]. However, more than one basis function for E_z^s is needed for the convergence of the characteristic impedance Z_o for a wide slot. The maximum number of basis functions needed for E_x^s and E_z^s during the computation of Z_o was 5 and 3, respectively, when the slot width approached one free space wavelength λ_o.

Computer programs have been developed to compute λ' and Z_o for a specified ε_r, λ_o and d. As a check of these programs, Table 1 shows a comparison for Z_o between the present computations and those in [3]. Characteristic impedances for slot lines have been computed for $\varepsilon_r = 2.22$, 3.0, 3.8, 6.5 and 9.8 and for widths varying over $0.0015 \leq W/\lambda_o \leq 1.0$. Computed values of Z_o vs. W/λ_o with d/λ_o as a parameter are plotted in Figure 2.

Empirical formulas have been developed for the normalized slot wavelength λ'/λ_0 and the slot characteristic impedance Z_0 and are given in (8)-(15). These formulas have been obtained by least square curvefitting the computed data. In each case, the average of the absolute percentage error 'Av' and the maximum percentage error 'Max', observed in a systematic sample of 120 data points is given. Also, where possible, the region around which the maximum error has been observed is indicated.

The following formulas are all valid within $0.006 \le d/\lambda_0 \le 0.060$.

$$2.22 \le \varepsilon_r \le 3.8$$

$0.0015 \le W/\lambda_0 \le 0.075$

$$\lambda'/\lambda_0 = 1.045 - 0.365\ln\varepsilon_r + \frac{6.3(W/d)\varepsilon_r^{0.945}}{(238.64 + 100W/d)} - \left[0.148 - \frac{8.81(\varepsilon_r+0.95)}{100\varepsilon_r}\right] \cdot \ln(d/\lambda_0) \quad (8)$$

Av = 0.37%, Max = 2.2% (at one point)

$$Z_0 = 60. + 3.69\sin\left[\frac{(\varepsilon_r-2.22)\pi}{2.36}\right] + 133.5\ln(10\varepsilon_r)\sqrt{W/\lambda_0}$$
$$+ 2.81\left[1-0.011\varepsilon_r(4.48+\ln\varepsilon_r)\right](W/d)\ln(100d/\lambda_0) + 131.1(1.028-\ln\varepsilon_r)\sqrt{d/\lambda_0}$$
$$+ 12.48(1+0.18\ln\varepsilon_r)\frac{W/d}{\sqrt{\varepsilon_r - 2.06 + 0.85(W/d)^2}} \quad (9)$$

Av = 0.67%, Max = 2.7% (at one point)

$0.075 \le W/\lambda_0 \le 1.0$

$$\lambda'/\lambda_o = 1.194 - 0.24\ln\varepsilon_r - \frac{0.621\varepsilon_r^{0.835}(W/\lambda_o)^{0.48}}{(1.344 + W/d)} - 0.0617\left[1.91 - \frac{(\varepsilon_r+2)}{\varepsilon_r}\right]\ln(d/\lambda_o)$$

Av = 0.69%, Max = -2.6% (at two points, for $W/\lambda_o > 0.8$) (10)

$$Z_o = 133. + 10.34(\varepsilon_r-1.8)^2 + 2.87[2.96+(\varepsilon_r-1.582)^2][\{W/d + 2.32\varepsilon_r - 0.56\}.$$
$$.\{(32.5 - 6.67\varepsilon_r)(100d/\lambda_o)^2 - 1\}]^{\frac{1}{2}} - (684.45d/\lambda_o)(\varepsilon_r+1.35)^2$$
$$+ 13.23[(\varepsilon_r-1.722)W/\lambda_o]^2 \quad (11)$$

Av = 1.9%, |Max| = 5.4% (at three points, for $W/\lambda_o > 0.8$)

$$3.8 \leq \varepsilon_r \leq 9.8$$

$0.0015 \leq W/\lambda_o \leq 0.075$

$$\lambda'/\lambda_o = 0.9217 - 0.277\ln\varepsilon_r + 0.0322(W/d)\left[\frac{\varepsilon_r}{(W/d+0.435)}\right]^{\frac{1}{2}} -$$
$$- 0.01\ln(d/\lambda_o)\left[4.6 - \frac{3.65}{\varepsilon_r^2\sqrt{W/\lambda_o}(9.06-100W/\lambda_o)}\right] \quad (12)$$

Av = 0.6%, |Max| = 3% (at three points, occurs for $W/d > 1$ and $\varepsilon_r > 6.0$)

$$Z_o = 73.6 - 2.15\varepsilon_r + (638.9-31.37\varepsilon_r)(W/\lambda_o)^{0.6} + (36.23\sqrt{\varepsilon_r^2 + 41} - 225).$$
$$\frac{W/d}{(W/d+0.876\varepsilon_r-2)} + 0.51(\varepsilon_r+2.12)(W/d)\ln(100d/\lambda_o)-0.753\varepsilon_r(d/\lambda_o)/\sqrt{W/\lambda_o} \quad (13)$$

Av = 1.58%, Max = 5.4% (at three points, occurs for $W/d > 1.67$)

$0.075 \leq W/\lambda_o \leq 1.0$

$$\lambda'/\lambda_o = 1.05 - 0.04\varepsilon_r + 1.411\times10^{-2}(\varepsilon_r-1.421)\ln\{W/d - 2.012(1-0.146\varepsilon_r)\}$$
$$+ 0.111(1-0.366\varepsilon_r)\sqrt{W/\lambda_o} + 0.139(1+0.52\varepsilon_r\ln(14.7-\varepsilon_r))(d/\lambda_o)\ln(d/\lambda_o) \quad (14)$$

Av = 0.75%, |Max| = 3.2% (at two points, occurs for $W/\lambda_o = 0.075$, d/λ_o >0.03)

$$Z_o = 120.75 - 3.74\varepsilon_r + 50[\tan^{-1}(2\varepsilon_r) - 0.8](W/d)^{\left[1.11 + \frac{0.132(\varepsilon_r-27.7)}{(100d/\lambda_o+5)}\right]} \cdot$$

$$\ln\left[100d/\lambda_o + \sqrt{(100d/\lambda_o)^2+1}\,\right] + 14.21(1-0.458\varepsilon_r)(100d/\lambda_o+5.1\ln\varepsilon_r-13.1)$$

$$\cdot(W/\lambda_o+0.33)^2 \tag{15}$$

Av = 2.0%, |Max| = 5.8% (at two points, occurs for $W/\lambda_o < 0.1$)

In the above formula, $\tan^{-1}(.)$ assumes its principal value.

III. Conclusion

A spectral domain Galerkin method is used to compute the characteristic impedance of wide slot lines on low ε_r substrates. Empirical formulas have been presented for the slot wavelength and the characteristic impedance over a wide range of slot widths. The data presented here supplement data already available on high ε_r substrates.

References

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Figures

Table 1

Comparison of calculated slot characteristic impedance Z_o.

ϵ_r	d/λ_o	W/d	$Z_o(\Omega)$ From curves in [3]	Present
9.6	0.06	1.0	140	142
11.0	0.04	1.5	160	160
13.0	0.03	0.4	80	82
16.0	0.025	2.0	150	151
20.0	0.03	1.0	100	101

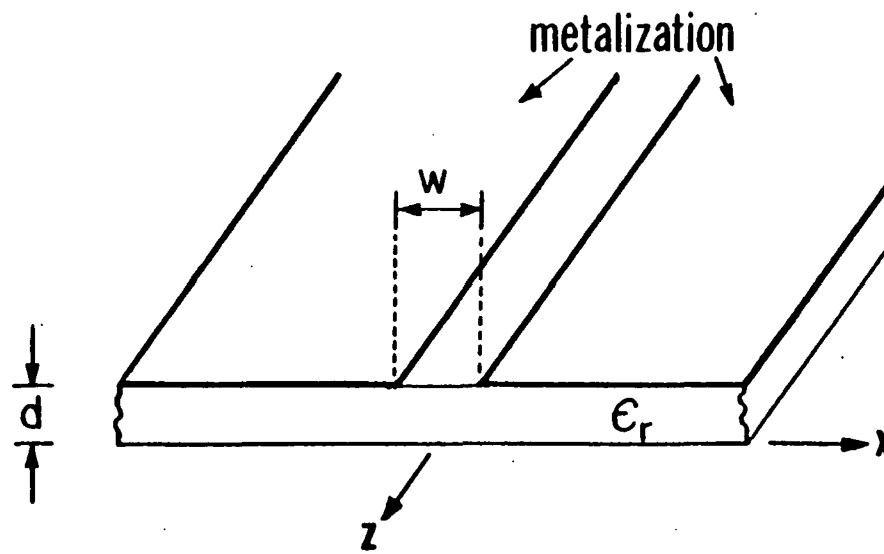
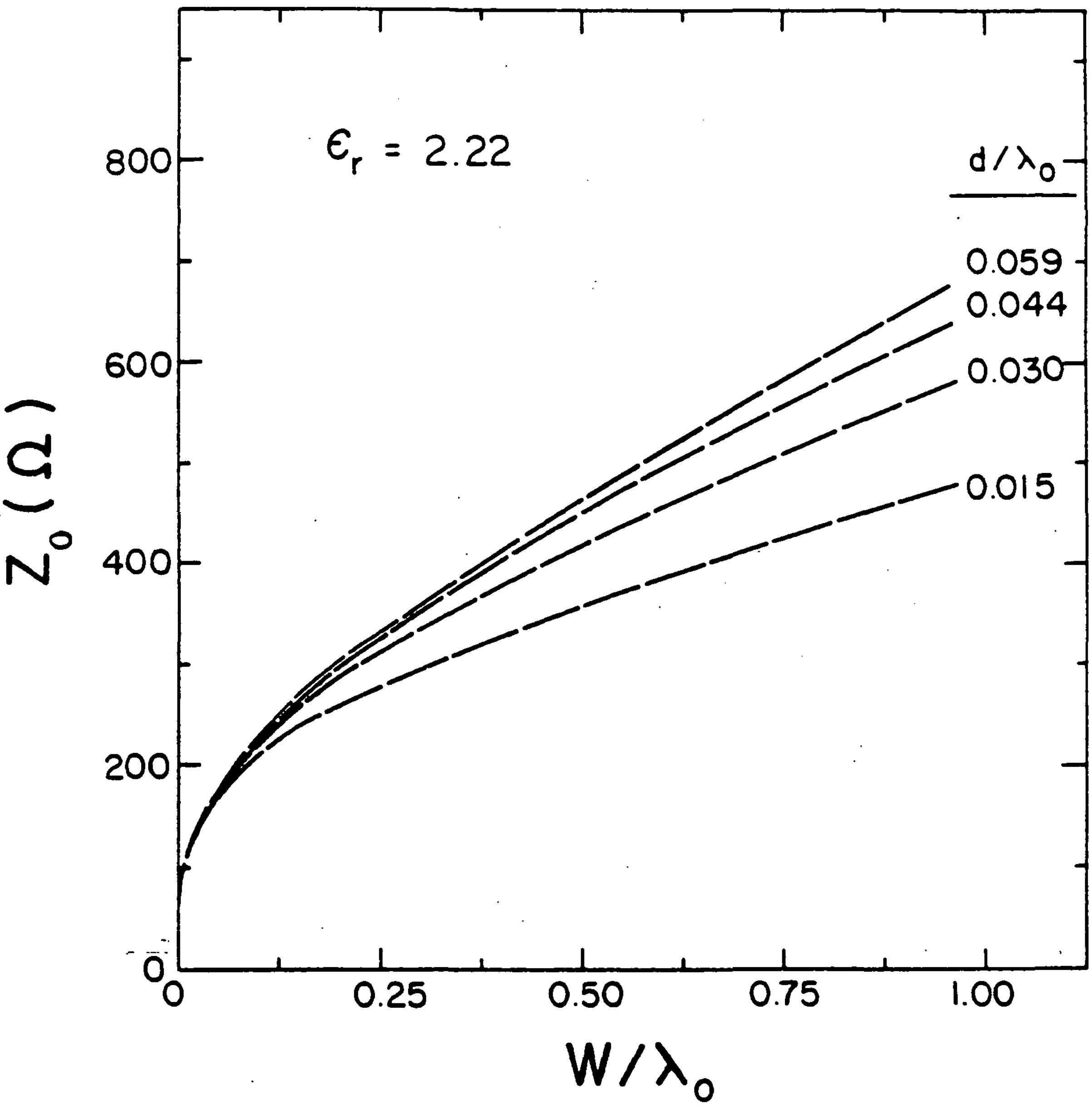
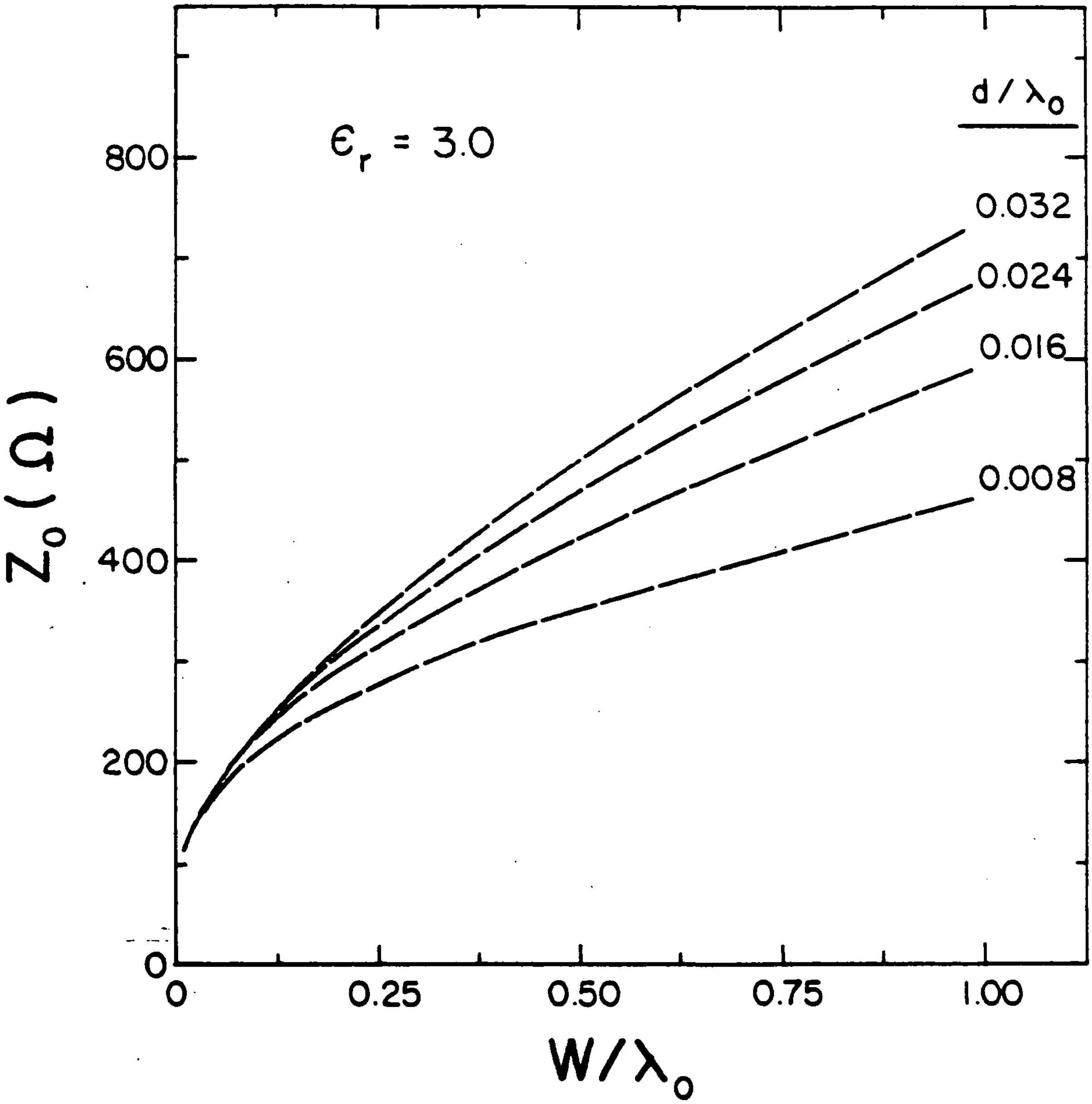


Fig. 1



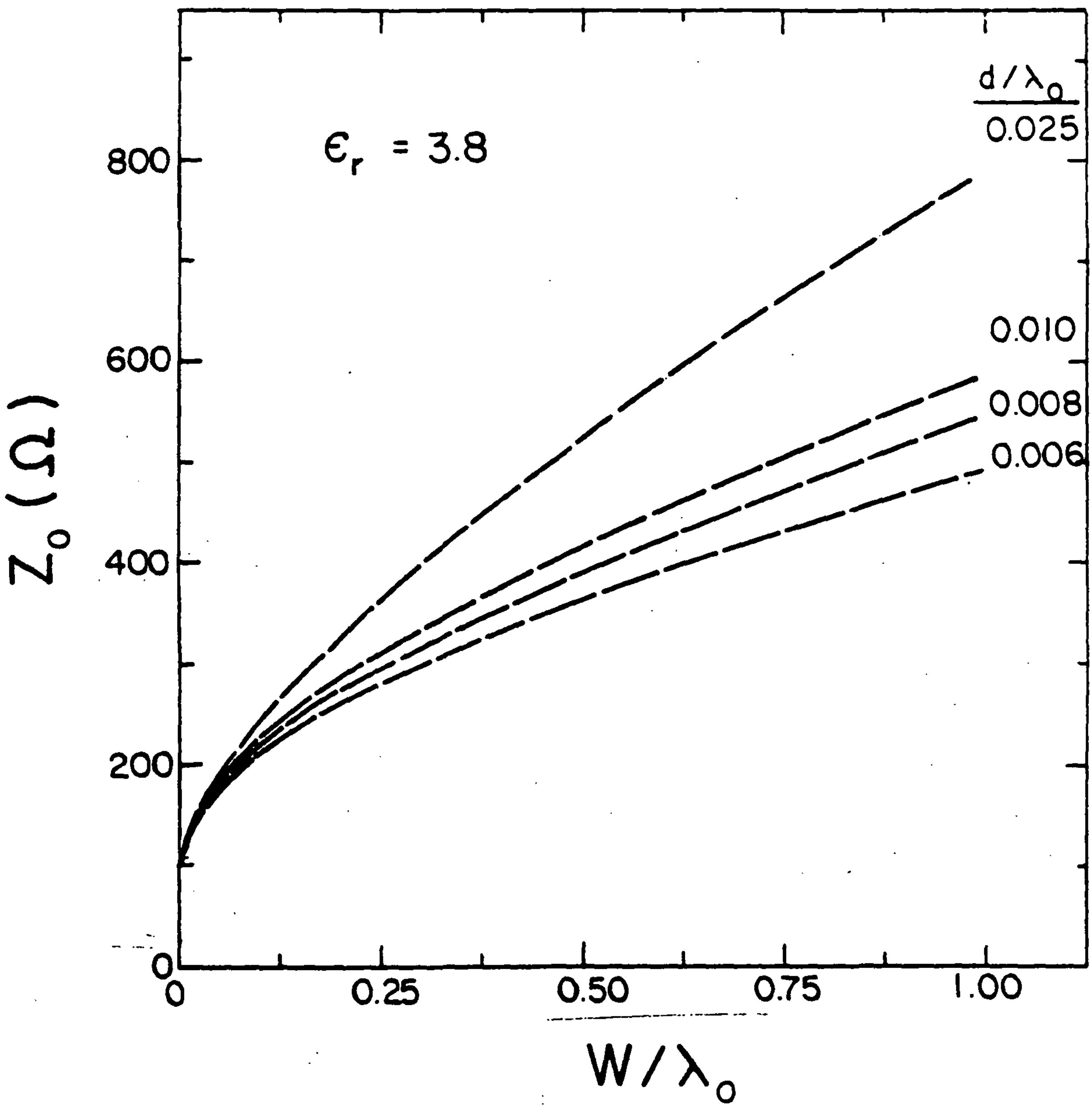
$\epsilon_r = 2.22$
d/λ_0
0.059
0.044
0.030
0.015
800
600
400
200
0
$Z_0 (\Omega)$
0
0.25
0.50
0.75
1.00
W/λ_0

Fig. 2a



$\epsilon_r = 3.0$
d/λ_0
0.032
0.024
0.016
0.008
$Z_0\,(\Omega)$
800
600
400
200
0
0
0.25
0.50
0.75
1.00
W/λ_0

Fig. 2b



d / λo
0.025
0.010
0.008
0.006
εr = 3.8
800
600
400
200
0
Zo (Ω)
0
0.25
0.50
0.75
1.00
W / λo

Fig. 2c

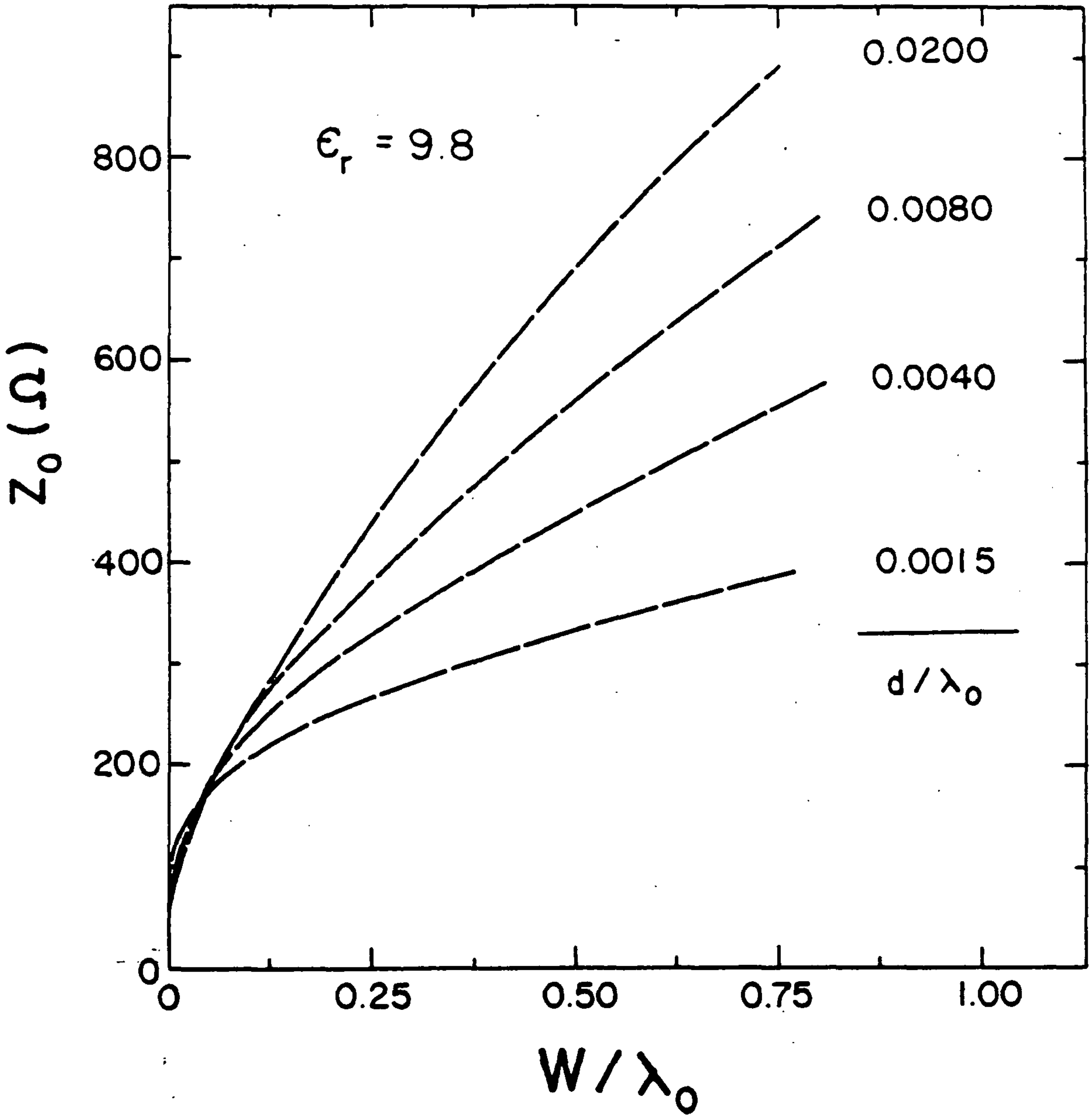

$\epsilon_r = 9.8$
0.0200
0.0080
0.0040
0.0015
d / λ_0
$Z_0\ (\Omega)$
0
200
400
600
800
0
0.25
0.50
0.75
1.00
W / λ_0

Fig. 2d