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Performance of Oil Pumping Rings

An Analytical and Experimental Study

M.W. Euseip, J.A. Walowit,
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NOMENCLATURE

A	Thermal constant, $\frac{6\mu_o U_o L}{\rho c_p C^2}$, (degree)
C	Radial clearance
C_M	Clearance at upstream edge when $\delta \neq 0$
d	Diameter
D	Flexural rigidity, $Et^3/[12(1 - \nu^2)]$
E	Elastic modulus
F	Radial shear force/unit circumferential length
$\tilde{F}$	Dimensionless shear force $FC^2/(6\mu U_o L^2)$
Hz	Hertz
K	Dimensionless flow rate, $\frac{Q}{\pi R U_o C}$
K_{eff}	Effective value of K corrected for starvation
L	Hydrodynamic land width
L_1	Effective distance from fixed end to start of film
$\tilde{L}_1$	Dimensionless length, L_1/L
$P(x)$	Radial loading function; see Equation (2-2)
Q	Volumetric flow rate
Q_o	Flow for $p_f = 0$
R	Radius of ring
T	Temperature
$\tilde{T}$	Dimensionless Temperature, $\frac{\rho c_p C^2 (T - T_o)}{6\mu_o U_o L}$
U_o	Average rod velocity
V	Normal velocity
c_p	Specific heat
e	Length over which p_o acts

f	Frequency
h	Film thickness
h_1	Film thickness at $x = 0$
h_2	Film thickness at $x = L$
h_2'	Film thickness slope dh_2/dx at $x = L$
$\tilde{h}$	(h/C)
k	Integration constant
p	Hydrodynamic pressure
p_f	Sealed Pressure
p_{fm}	Maximum sealed pressure $(Q = 0)$
p_o	Clamping pressure
$\tilde{p}$	Dimensionless pressure $pC^2/(6\mu U_oL)$
$\tilde{p}_f$	Dimensionless sealed pressure $p_fC^2/(6\mu U_oL)$
s	Stroke
t	Ring thickness; time
u	Velocity
w	Elastic deflection
x	Position variable
x_c	Position variable at cavitation point
y	Variable across film
α	Geometric bending parameter, $\dfrac{t^2(R + \frac{t}{2})^2}{12L^4(1 - \nu^2)}$
β	Elastohydrodynamic parameter, $\dfrac{6\mu U_oL}{C^2}\ \dfrac{(R + \frac{t}{2})^2}{CtE}$
δ	Slope of tapered surface, $(C - C_M)/L$
ε	Dimensionless loading length, e/L

NOMENCLATURE (CONT'D)

η	Dimensionless film height, (y/h)
μ	Viscosity
μ_o	Inlet reference viscosity for thermal analysis
μ	Viscous heating function, rate of heat generated/unit area
ϕ	μ/μ_o
ρ	Density
ν	Poisson's Ratio
ξ	Dimensionless position variable x/L
σ	Squeeze film parameter (4L/S)
ψ	Stream function
λ	Dimensionless starvation factor

SUBSCRIPTS

c	Cavitation
F	Forward flow
E	Elastic
f	At x = L
m	Maximum
R	Back flow; backstroke
1	At leading edge
2	At trailing edge

SUMMARY

A steady-state design computer program has been developed to predict the performance of pumping rings as functions of geometry, applied loading, speed, ring modulus, and fluid viscosity. Additional analyses have been developed to predict transient behavior of the ring and the effects of temperature rises occurring in the hydrodynamic film between the ring and shaft. The analysis was initially compared with previous experimental data and then used to design additional rings for further testing.

Tests were performed with Rulon, carbon-graphite, and babbitt rings. Two different shaft diameters were used for the babbitt rings. The design analysis was used to size all of the rings and to select the ranges of clearances, thickness, and loading. Although full quantitative agreement was lacking, relative agreement existed in that rings that were predicted to perform well theoretically, generally performed well experimentally. Some causes for discrepancies between theory and experiment are believed to be due to starvation, leakage past the secondary seal at high pressures, and uncertainties in the small clearances and local inlet temperatures to the pumping ring.

The design criteria that evolved require the applied loading to be of the order of the desired pumping pressure, the flow requirements and tolerances to dictate clearance, and the elastic modulus and ring compliance to be such that the deflection under load statically results in clamping at very small interferences so that back flow is inhibited, but that excessive power loss and wear do not occur.

It was found that the pumping ring could be used to generate its own loading pressure without any priming if an initial taper was applied to the ring. However, for untapered rings, an initial loading had to be applied before self-pumping could be obtained.

A separate preliminary analysis has been performed for a pumping Leningrader seal. This analysis can be used to predict the film thickness and flow rate through the seal as a function of pressure, speed, loading, and geometry.

1.0 INTRODUCTION

An analysis of pumping rings was performed under certain simplifying conditions in previous phases of this work [1]. These conditions consisted of first ignoring the contribution, if any, of the back flow occurring during the return stroke of the rod. Other simplifications related to the use of constant or average parameters, namely constant viscosity and mean rod velocity, throughout the stroke. The latter also presumed the neglect of squeeze film forces due to the variation of film thickness engendered by the harmonic motion of the rod.

The comparison of experimental data with theoretical results based on the simplified analysis showed good agreement with respect to maximum pressures generated by the pumping ring. The flows produced at reservoir pressures below the maximum, however, were consistently lower in the experiments than those indicated by the analysis. The agreement between theory and experiment for the carbon-graphite rings [1] has been found to be incorrect due to an erroneous use of an excessively low viscosity in the theoretical computation.

The present work, an extension of the previous effort, is aimed at advancing the analysis of pumping rings. Thermal effects and variable rod velocity were included and, with it, the effects of squeeze film action in the fluid film. The analysis was extended to include the backstroke and concurrent cavitation and their effect on net flow. The new analysis was then used to run a parametric study in order to obtain optimized configurations of pumping rings of different shapes and materials including the effects of a geometric taper. The results of tests run on these optimized designs were then compared with calculations based on the new analysis.

A separate, preliminary analysis of the pumping Leningrader seal has also been performed. Since this analysis is separate from the pumping ring work, its results are presented in Appendix A.

2.0 ANALYSIS WITH CONSTANT PARAMETERS

2.1 Basic Equations

The equation governing deflection, w, of an axisymmetric shell under bending is

$$\frac{E\,t^3}{12(1-\nu^2)}\frac{d^4w}{dx^4} + \frac{Et}{R^2}w = -P(x) \qquad (2\text{-}1)$$

Referring to Figure 2-1, the radially outward loading, P(x), may be expressed in terms of the clamping load, p_o, and the hydrodynamic pressure, p, as follows:

$$P(x) = \begin{cases} 0, & -L_1 \le x \le 0 \\ p, & 0 \le x \le L - e \\ p-p_o, & L - e \le x \le L \end{cases} \qquad (2\text{-}2)$$

The hydrodynamic pressure, p, is determined from the solution of the Reynolds Equation

$$\frac{dp}{dx} = 6\mu U_o\left(\frac{h-k}{h^3}\right) \qquad (2\text{-}3)$$

with the geometry of the hydrodynamic film as given in Figure 2-2. k is a constant of integration related to the flow, and U_o is the average speed, which is related to the frequency, f, and the stroke, s, by:

$$U_o = 2fs$$

This velocity represents the average of the sinusoidal velocity over each half cycle. If the system contains two opposing pumping rings, it is double acting and the average velocity, U_o is assumed to prevail over the entire cycle (though in two different pumping rings) for the forward stroke, as well as for the back-stroke. In this manner, the transient problem is reduced to and simplified into a quasi-steady-state process. For a single ring, the resulting flow should be divided by 2.

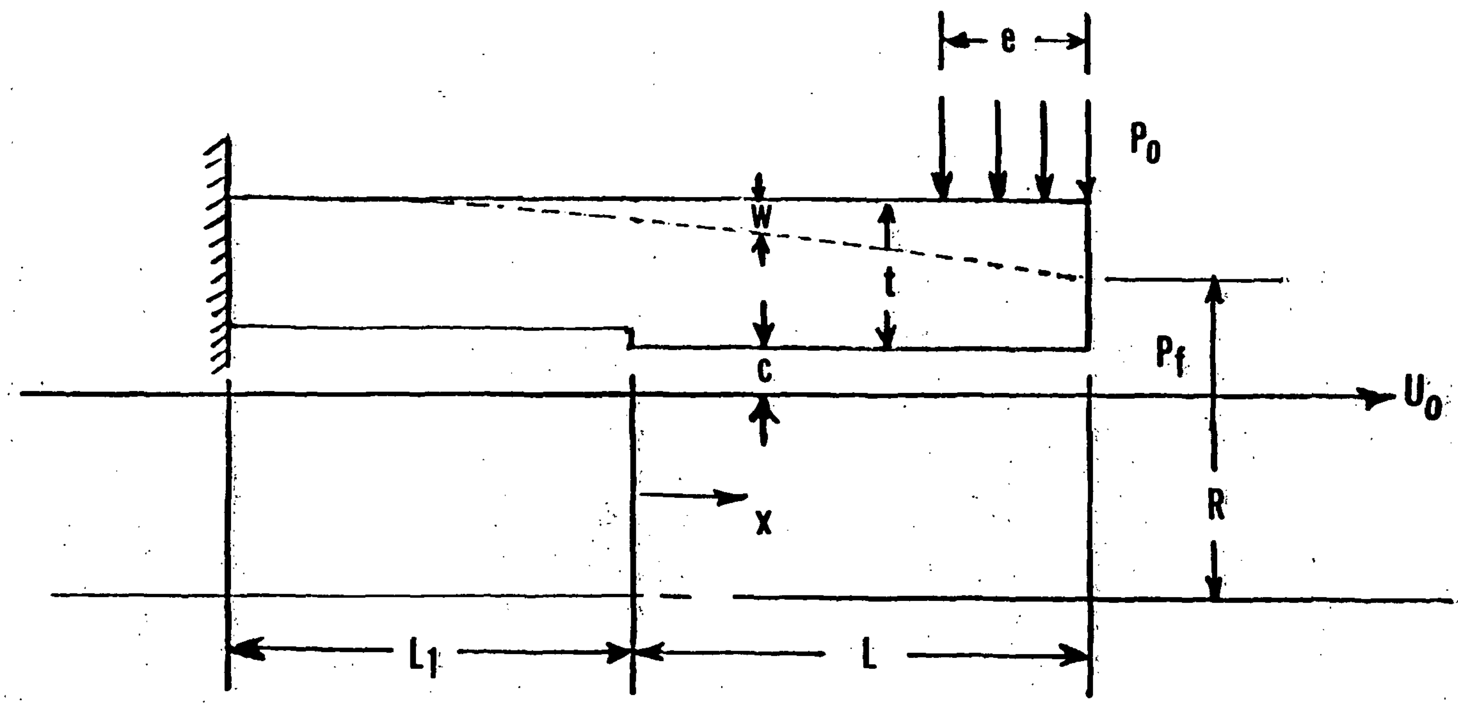

Fig. 2-1 Geometry of Pumping Ring

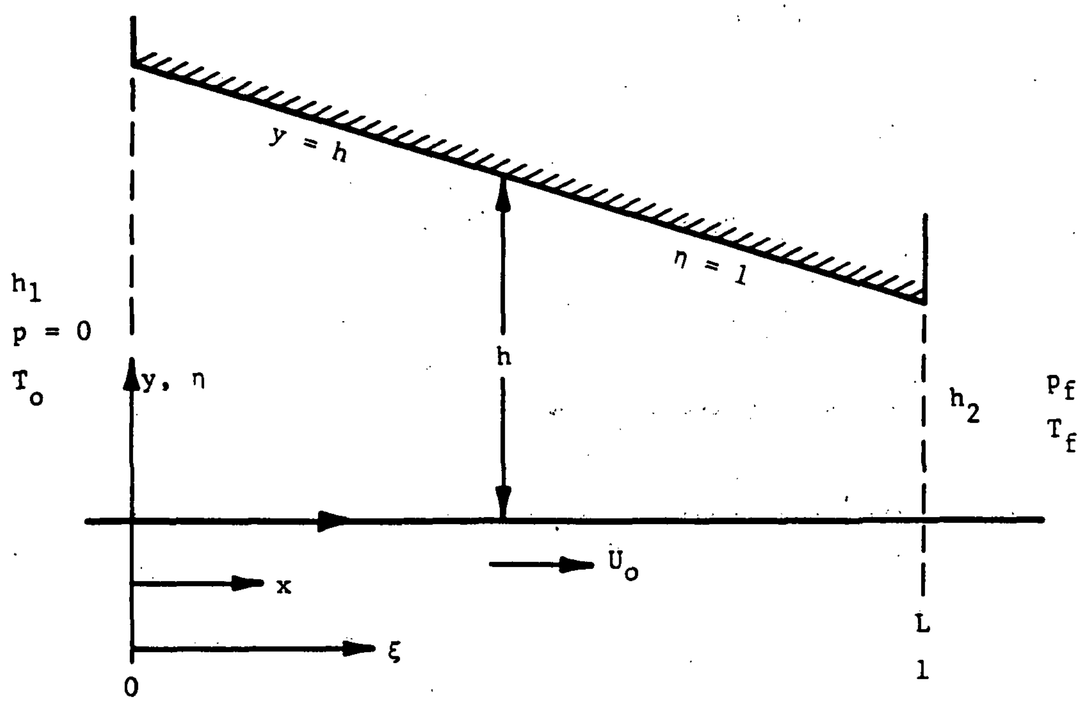

Fig. 2-2 The Hydrodynamic Film

The film thickness, h, appearing in Equation (2-3) is related to the deflection, w, by

$$h = (C - w), \quad 0 \leq x \leq L$$

The system of equations given by Equations (2-1) through (2-3) represents a fifth-order set of differential equations requiring five boundary conditions, in addition to a sixth condition for the evaluation of the constant, k, appearing in Equation (2-3). Two conditions result from the clamped-end requirement

$$w = \frac{dw}{dx} = 0 \text{ at } x = -L_1$$

Two conditions resulting from the prescribed pressures at x = 0 and L

$$p = 0 \text{ at } x = 0 \qquad p = p_f \text{ at } x = L$$

where p_f is the sealed pressure. The remaining two conditions relate to the free-end requiring zero moment and zero shear, or

$$\frac{d^2w}{dx^2} = \frac{d^3w}{dx^3} = 0 \text{ at } x = L$$

The method of solving this set of equations subject to the specified boundary conditions is outlined in Reference [1]. The solution and the results of this analysis, reported in Reference [1], are based on the constancy of both viscosity and speed, namely

$$\mu = \mu_o = \text{constant}$$
$$U = U_o = 2fs = \text{constant}$$

The analysis in Reference [1] did not consider the backstroke and was thus only applicable to sufficiently high loads and low elastic moduli to result in clamping during the backstroke.

2.2 Simplified Approach

A scrutiny of the pumping ring solutions formulated in Section 2.1 shows that the shape of the film over the hydrodynamic portion of the ring is nearly linear for the entire range of ring parameters and operating conditions. A few such examples are shown in Figure 2-3. Consequently, the problem can be considerably simplified if one postulates that the configuration of the film is tapered similar to that of a plane slider. In addition, a constant taper (shown in Figure 2-2) makes it possible to later treat the backstroke and accompanying cavitation. Mathematically a constant taper implies that h' = constant. The pertinent expressions for the hydrodynamic component of pumping ring action are then given by

$$\tilde{h}(\xi) = \tilde{h}_2 + \Delta\tilde{h}\,(1 - \xi) \tag{2-4}$$

$$\tilde{p}(\tilde{h}) = 1/(\tilde{h}_2 - \tilde{h}_1) - 1/\tilde{h} + k'/(2\tilde{h}^2) + C_1 \tag{2-5}$$

where

$$k' = [2\tilde{h}_1\tilde{h}_2/(\tilde{h}_1 + \tilde{h}_2)][1 - (\tilde{h}_1\tilde{h}_2\tilde{p}_f)]$$

$$C_1 = [1/(\tilde{h}_1\tilde{h}_2)][1 + (\tilde{h}_2^2\,\tilde{p}_f)]$$

The elastic deformation equation remains the same and may be written in dimensionless form as

$$\alpha\,(d^4\tilde{h}/d\xi^4) + \tilde{h} = \begin{cases} 1 \text{ for } -\tilde{L}_1 \le \xi \le 0 \\ 1 + \beta\tilde{p} \text{ for } 0 \le \xi \le 1 - \varepsilon \\ 1 + \beta(\tilde{p} - \tilde{p}_o) \text{ for } 1 - \varepsilon \le \xi \le 1 \end{cases} \tag{2-6}$$

The solution algorithm thus consists of determining values for $\tilde{h}_2$ and $\Delta\tilde{h}$ by the use of the secant method. Equation (2-6) is solved subject to the previously stated elasticity boundary conditions. Convergence is achieved

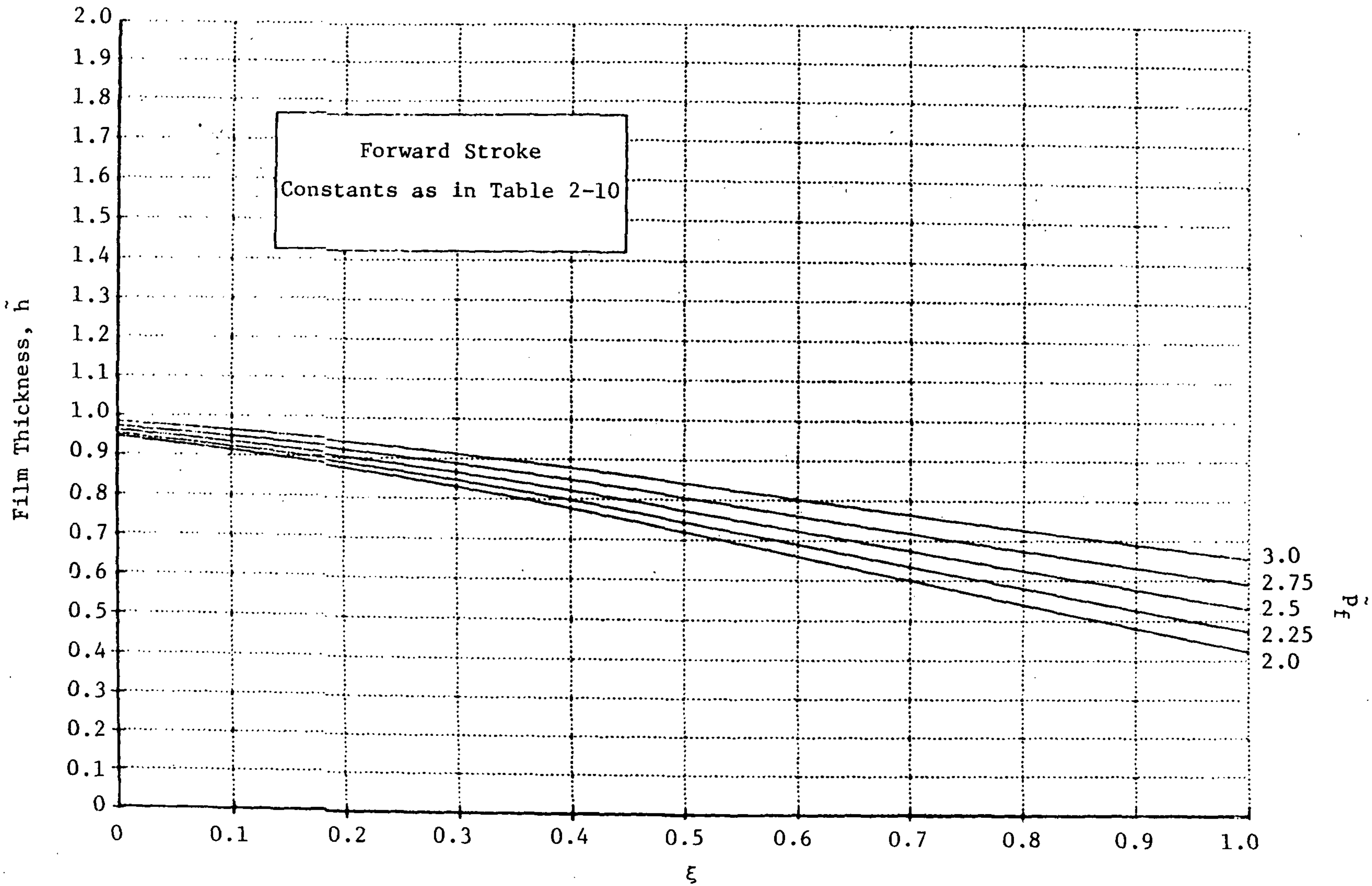

Fig. 2-3 Shape of Film Thickness for Various Values of $\tilde{p}_f$

when the values of $\tilde{h}(1)$ and $\tilde{h}'(1)$ computed from the solution to Equation (2-6) are within the prescribed tolerance limit of the values of $\tilde{h}_2$ and $-\Delta\tilde{h}$ used in calculating $\tilde{p}$ from Equations (2-4) and (2-5).

2.3 The Backstroke

2.3.1 Analytical Approach

The previous analysis assumed clamping during the backstroke. This is valid for very high loadings, or for highly flexible pumping rings. Lower ring loadings which do not cause clamping during the entire backstroke may be desirable for pumping ring design in order to reduce wear. Also, when the upstream pressure is high, the ring may stay open during the reverse stroke, even under high clamping forces. Thus, backstroke effects are here added on to the analytical model.

The basic equations remain unchanged except for two important aspects. One is that the shaft motion will be in the negative x direction; thus Reynolds Equation assumes the form

$$d\tilde{p}/d\xi = -[(\tilde{h} - K)/\tilde{h}^3] \tag{2-7}$$

The other critical modification consists in the appearance of cavitation. As shown in Figure 2-4, due to the divergence of the film in the direction of motion, there may not be enough fluid to fill the gap at sufficiently high values of h. The fluid film will then break up into a pattern of streamers similar to that which occurs in the diverging films of a journal bearing. From continuity requirements, the downstream boundary conditions of ξ_c, where the film ends, must then satisfy the following boundary conditions:

$$\begin{aligned} &(d\tilde{p}/d\xi) \text{ at } \xi = \xi_c = 0 \\ &\tilde{p}(\xi_c) = 0 \end{aligned} \tag{2-8}$$

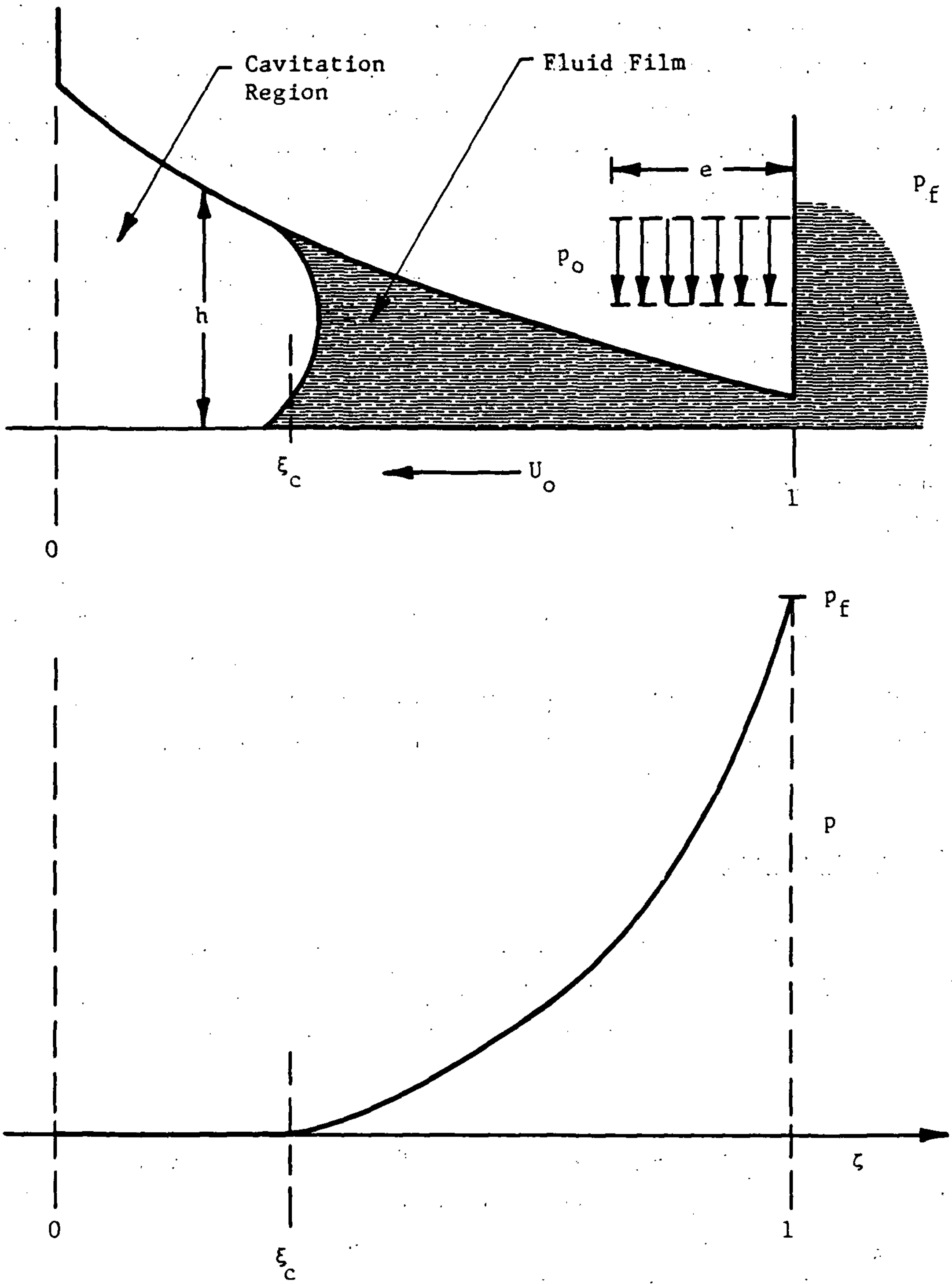

Fig. 2-4 Pumping Ring During Reverse Stroke

To account for cavitation during the backstroke, the equations are integrated backward from $\xi = 1$. As for the forward stroke solution, values of $\tilde{h}_2$ and $\Delta\tilde{h}$ must be determined by using the secant method. Selected values for $\tilde{h}_2$ and $\Delta\tilde{h}$ are used in determining $\tilde{h}(\xi)$ from Equation (2-4) which is in turn substituted in the Reynolds Equation (2-7). The resulting equation is then integrated analytically, subject to the constraints that $\tilde{p}(1) = \tilde{p}_f$, and either the condition given by Equation (2-8) for $0 < \xi_c < 1$ or the condition $\tilde{p}(0) = 0$ if cavitation is not predicted to occur. These conditions are sufficient to determine the constant, K, ξ_c (if applicable), and the pressure distribution, $\tilde{p}(\xi)$. Equation (2-6) may now be integrated backward from $\xi = 1$, with the boundary condition at $\xi = 1$ such that

$$\tilde{h}(1) = \tilde{h}_2$$
$$\tilde{h}'(1) = -\Delta\tilde{h}$$
$$\tilde{h}''(1) = \tilde{h}'''(1) = 0$$

The secant method is then used to find the values of $\tilde{h}_2$ and $\Delta\tilde{h}$ that make the quantities $|\tilde{h}(-\tilde{L}_1) - 1|$ and $|\tilde{h}'(-\tilde{L}_1)|$ be within prescribed tolerance limits. The solutions so obtained provide dimensionless values of the film thickness profile, the pressure profiles and the flow rates for prescribed values of parameters α, β, $\tilde{p}_o$, $\tilde{p}_f$, ε, and $\tilde{L}_1$.

2.3.2 Nature of Solution

The nature of the solution for the backstroke depends very much on the value of p_f relative to the clamping force, p_o. It is thus first necessary to describe the conditions which are apt to generate either high or low levels of p_f.

If a pumping ring delivers oil to a closed reservoir of finite volume, the parameters α, β, ε, and $\tilde{p}_o$ describing the pumping ring will predetermine the maximum p_f that the pumping ring is likely to generate. At that point, i.e., when p_{fm} is reached, there will be no further inflow into the reservoir, and the pressure in the reservoir will be maintained by pumping ring action at p_{fm}. In Figure 2-5 this situation would be represented by both valves A and B being closed. However, should there be an outflow from the reservoir, that

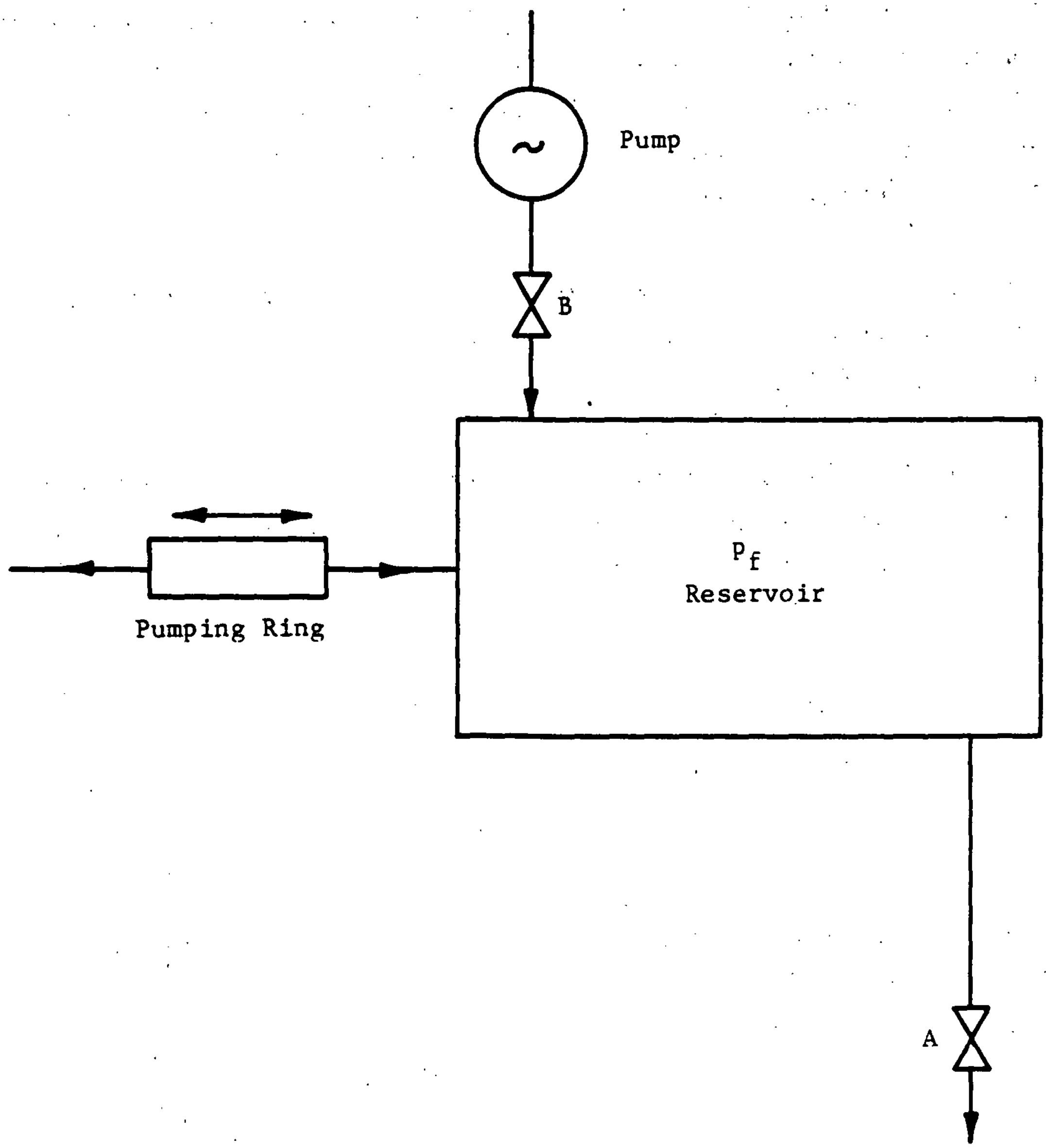

Fig. 2-5 Conditions Determining Level of p_f

is, were valve A open, then the pumping ring would be delivering a net inflow equal to the amount of outflow, with the pressure in the reservoir below the maximum, i.e., $p_f < p_{fm}$. On the other hand, if an external pump were to deliver a certain flow, then it is possible to have $p_f > p_{fm}$. The pumping ring would then be overloaded. The net flow over a cycle would be negative and the outflow through the pumping ring would equal the inflow delivered by the pump. Since there is no external pump in the present system, the overloaded condition will be of academic interest only.

The interaction of pumping ring behavior versus conditions in the reservoir has a direct bearing on the nature of the backstroke solution. The mechanism can be summarized as follows:

- If p_f is low, i.e., if there is an appreciable outflow from the reservoir, the film during the backstroke always cavitates. It is only by having a cavitating backstroke that a net inflow can be maintained over a complete cycle.

- At high values of p_f, there are generally two possible solutions, namely:
  - High h_2 with a noncavitating film
  - Low h_2 with a cavitating film

  Such a double set of possible solutions is shown in Figure 2-6, both the same $\tilde{p}_f = 2.7$ and $\tilde{p}_o = 3.5$. The performance of the pumping ring under these two sets of conditions is as follows:

	Noncavitating	Cavitating
Back $\tilde{h}_2$	0.482	0.120
Forward $\tilde{h}_2$	0.580	0.580
Back K	-1.433	-0.225
Forward K	-0.42	-0.42
Net K	-1.85	-0.642
ξ_{cav}	--	0.87

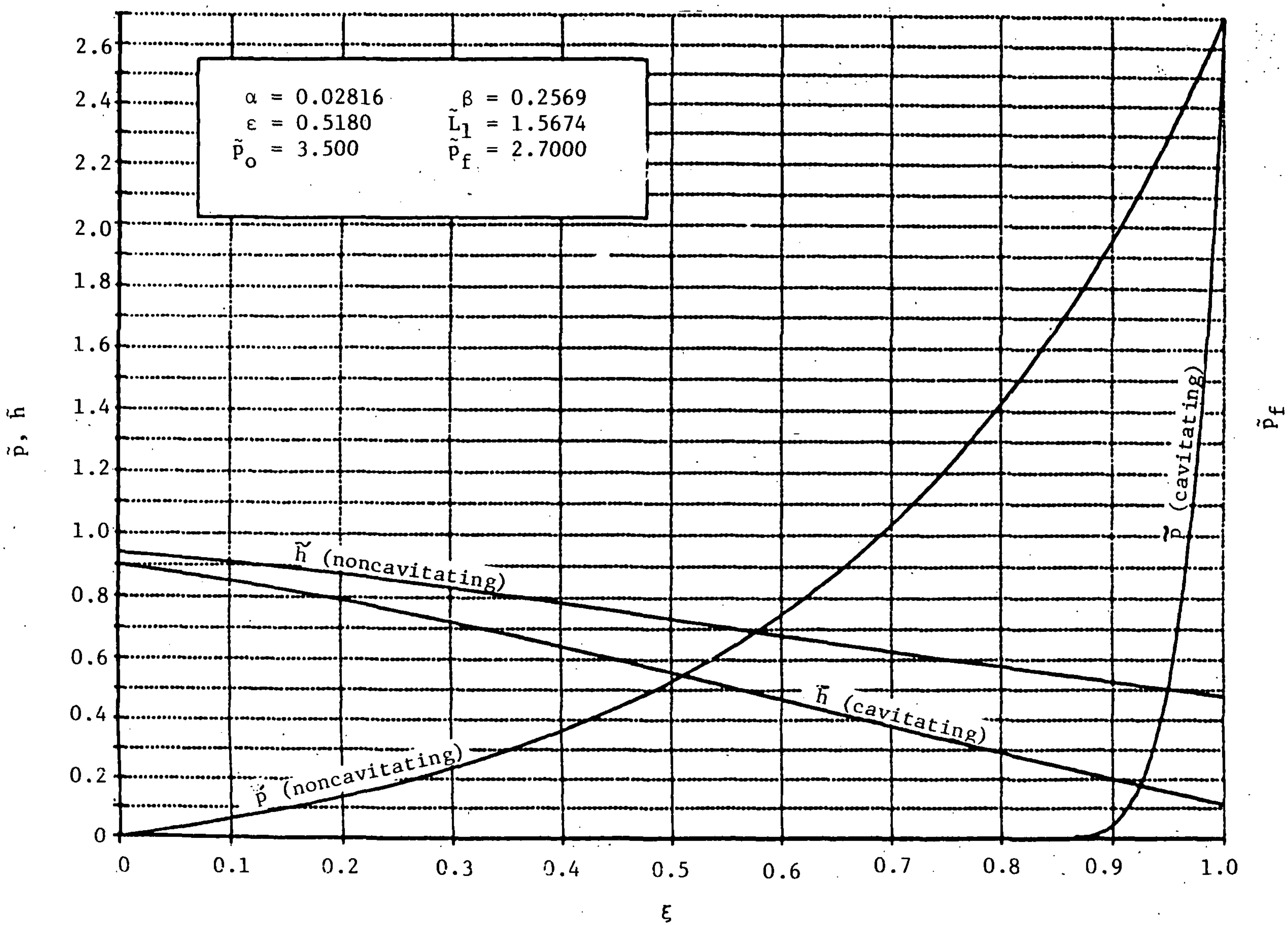

Fig. 2-6 Two Possible Reverse Stroke Solutions for High Values of $\tilde{p}_f$

- If there is no supply of oil to the reservoir by a pump, that is, if the pressure, p_f, is generated by the pumping ring only, then the solution of a high h_2 with a noncavitating film is practically impossible, since a thick noncavitating film would produce a net outflow so that no high p_f producing that outflow could possibly be generated in the first place.

Thus, in conclusion, for practical cases of an active pumping ring, the backstroke is always accompanied by cavitation.

2.3.3 Performance Characteristics

For the practical range of ring operation, the pumping ring will, in most cases, cavitate during the backstroke. When the condition of $(\tilde{d}p/\tilde{d}\xi)|\xi_c = 0$ is introduced during the backstroke, the resulting film thickness, pressures, and flows are significantly altered from the full film case. Table 2-1 gives the detailed characteristics of cavitating pumping rings as a function of sealed pressure, $\tilde{p}_f$. Particularly significant for these purposes is the amount of back flow and its effect on the net pumping accomplished over a complete cycle. Figures 2-7 and 2-8 show the pressure distribution during the forward and reverse strokes for a range of $\tilde{p}_f$ from 0 to 5. As seen, cases of $0 \leq \tilde{p}_f \leq 3$ produce cavitation, and the fluid film and corresponding positive pressures for these cases occupy merely a fraction of the interspace. This reduced pressure profile naturally has a strong effect on the film thickness distribution shown in Figures 2-9 and 2-10. Although $\tilde{h}_2$ for the noncavitating cases, $\tilde{p}_f > 3$, is about the same for the forward and backstrokes, there is a manyfold (5 to 10 times) decrease in $\tilde{h}_2$ during a cavitating backstroke. The shapes of the pressure profiles for a range of values of $\tilde{p}_o$ (all previous plots were for $\tilde{p}_o = 3.0$) are shown in Figure 2-11.

A scrutiny of the data contained in Table 2-1, as well as of the accompanying plots, reveals the following interesting features regarding cavitating versus noncavitating films.

TABLE 2-1

PERFORMANCE OF CAVITATING PUMPING RINGS

$\alpha = 0.0282$ $\beta = 0.257$ $\varepsilon = 0.518$ $\tilde{p} = 3.5$ $\tilde{L}_1 = 1.57$

$\tilde{p}_f$	Flow, K			ξ_c	$\tilde{h}_{min}$		(p_{max}/p_f)
	Forward Stroke	Reverse Stroke	Net		Forward Stroke	Reverse Stroke	
0	0.355	0.0452	0.310	1.0	0.213	0.045	0.74
1.0	0.315	0.0635	0.252	0.983	0.285	0.045	1.011
1.5	0.244	0.0759	0.168	0.975	0.342	0.052	1.0
2.0	0.0877	0.0954	-0.008	0.963	0.422	0.059	1.0
2.5	-0.227	0.903	-1.130	0.0604	0.527	0.342	1.0
3.0	-0.786	2.068	-2.854	None	0.652	0.603	1.0
3.5	-1.6650	3.284	-4.949	None	0.790	0.771	1.0

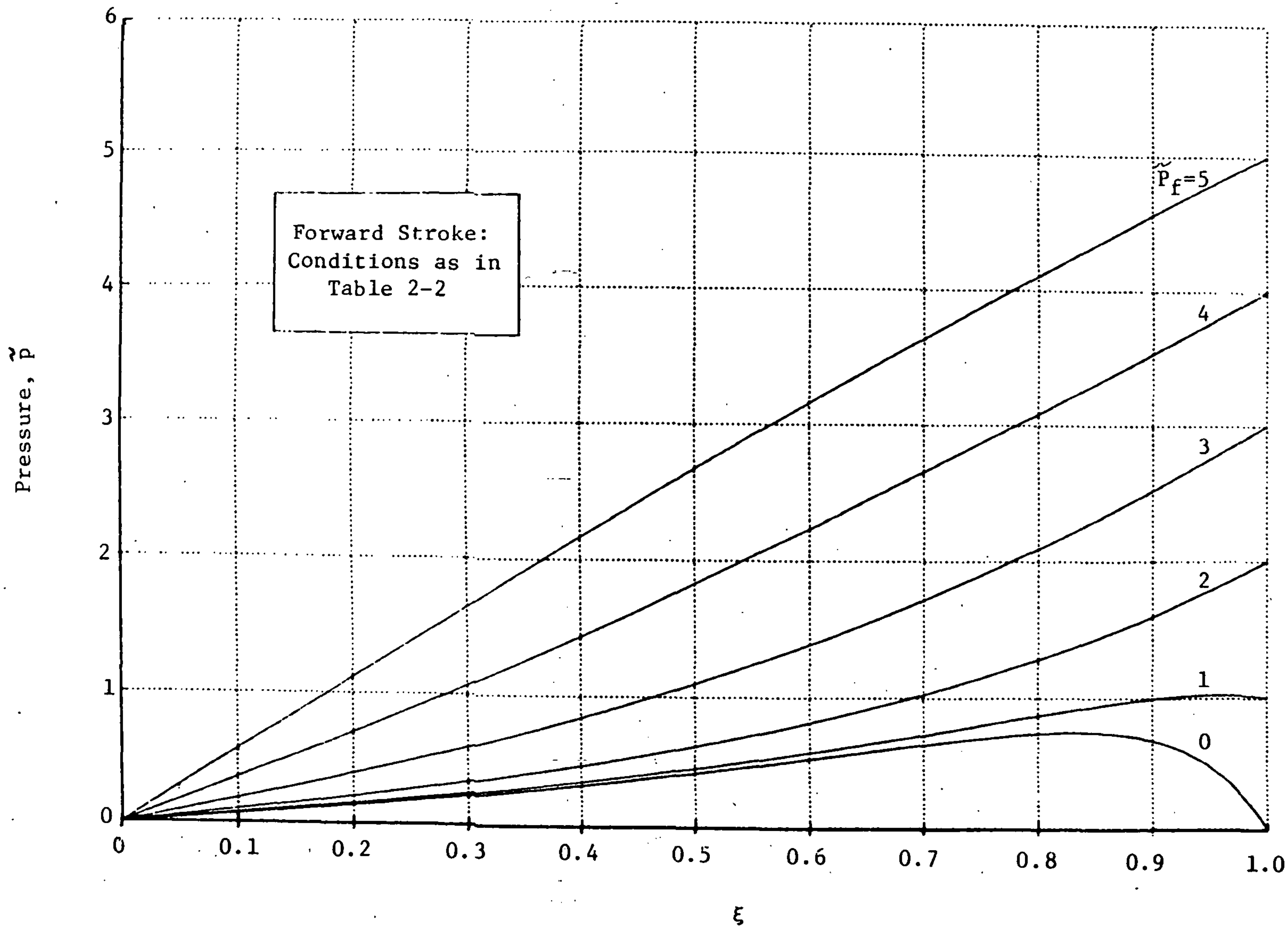

Fig. 2-7 Pressure Distribution During Forward Stroke

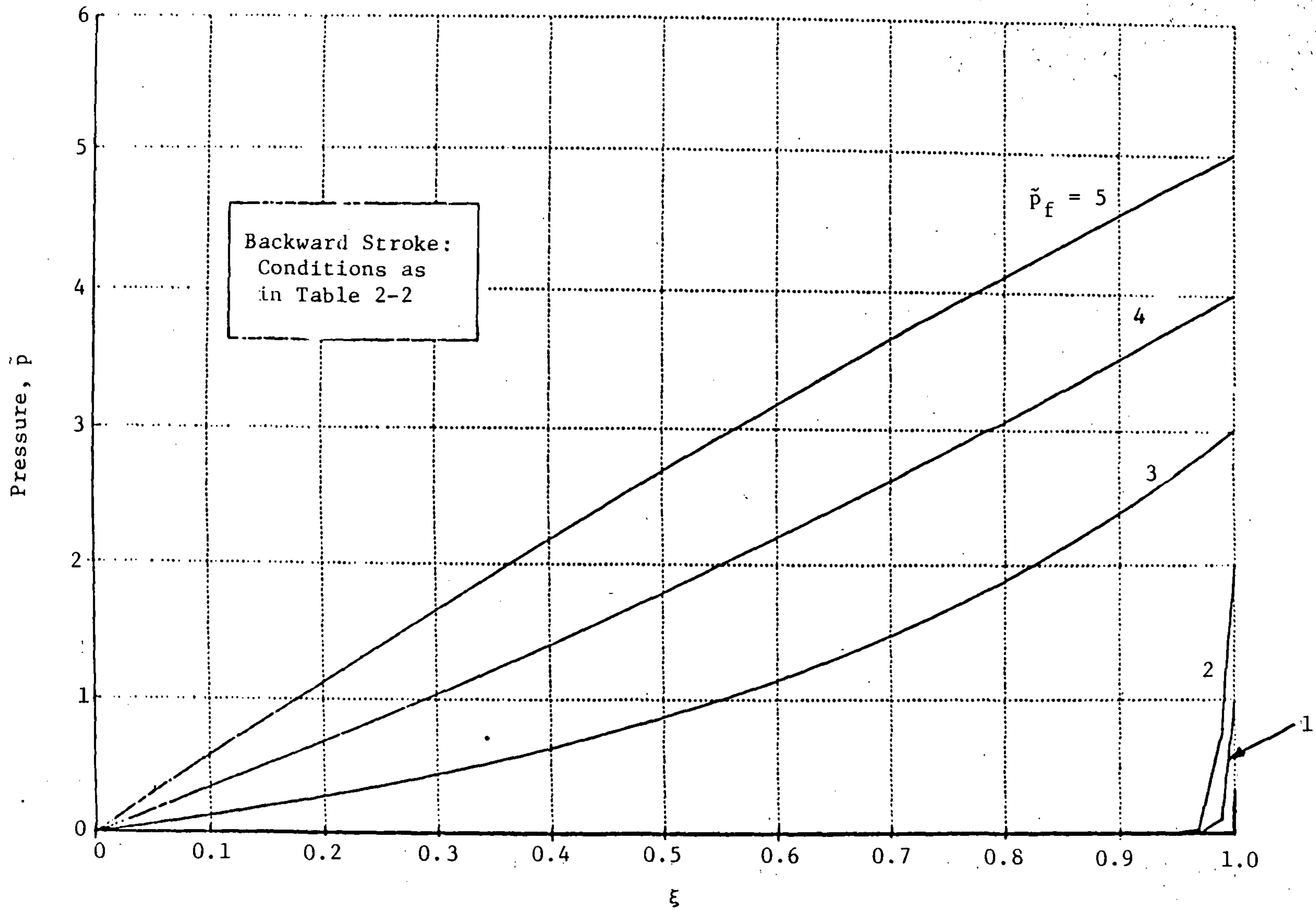

Fig. 2-8 Pressure Distribution During Reverse Stroke

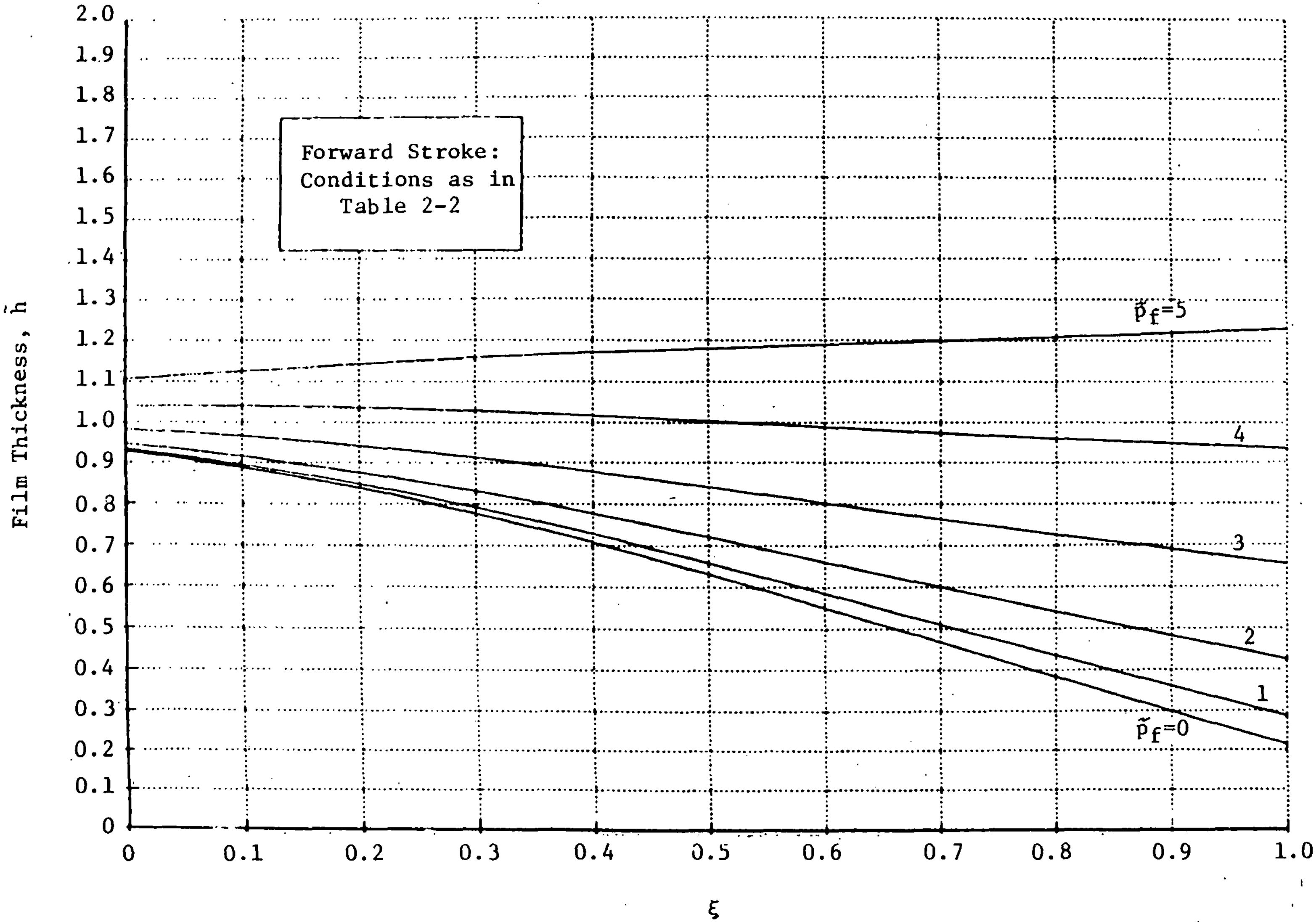

Fig. 2-9 Film Thickness Distribution during Forward Stroke

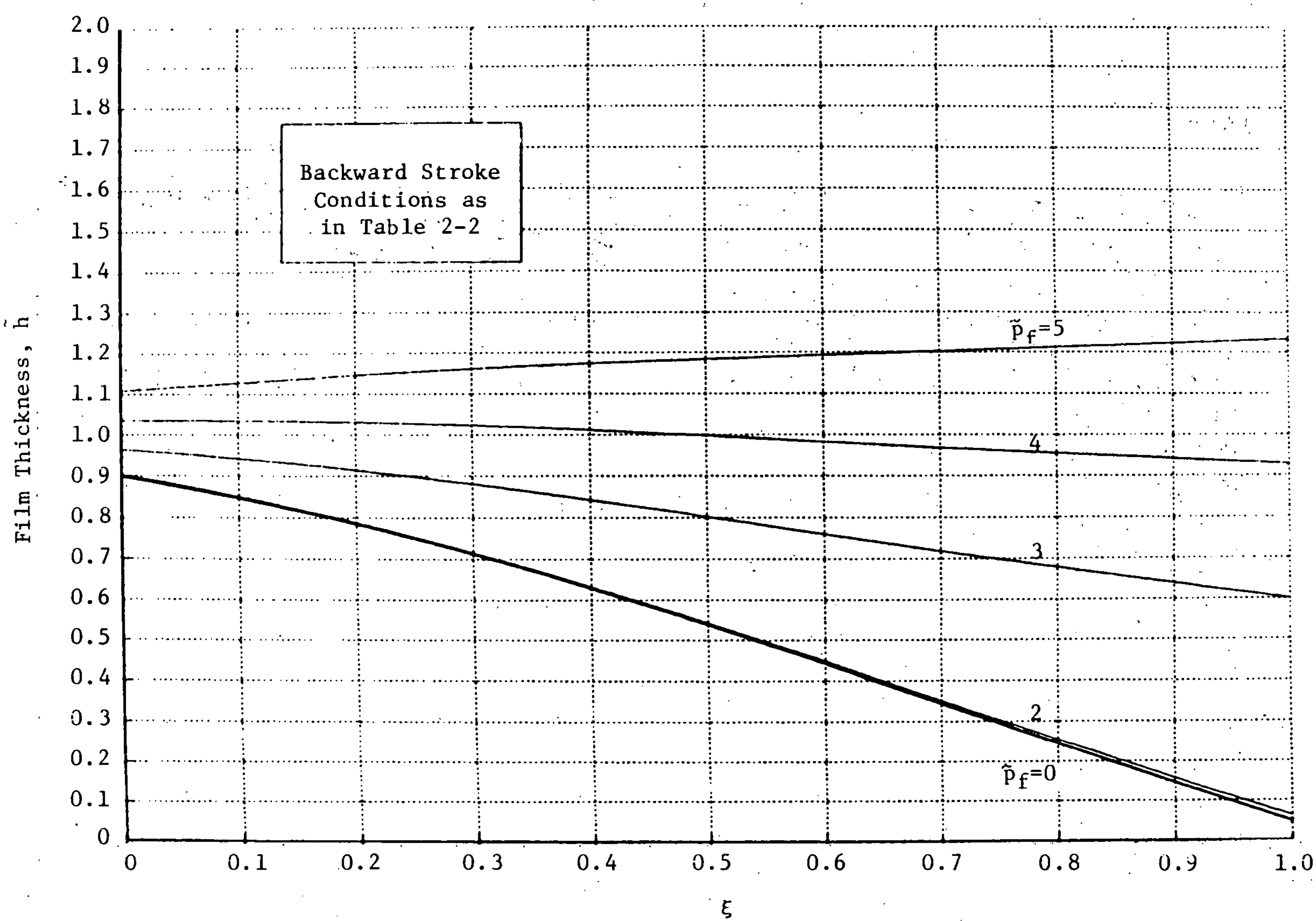

Fig. 2-10 Film Thickness Distribution during Reverse Stroke

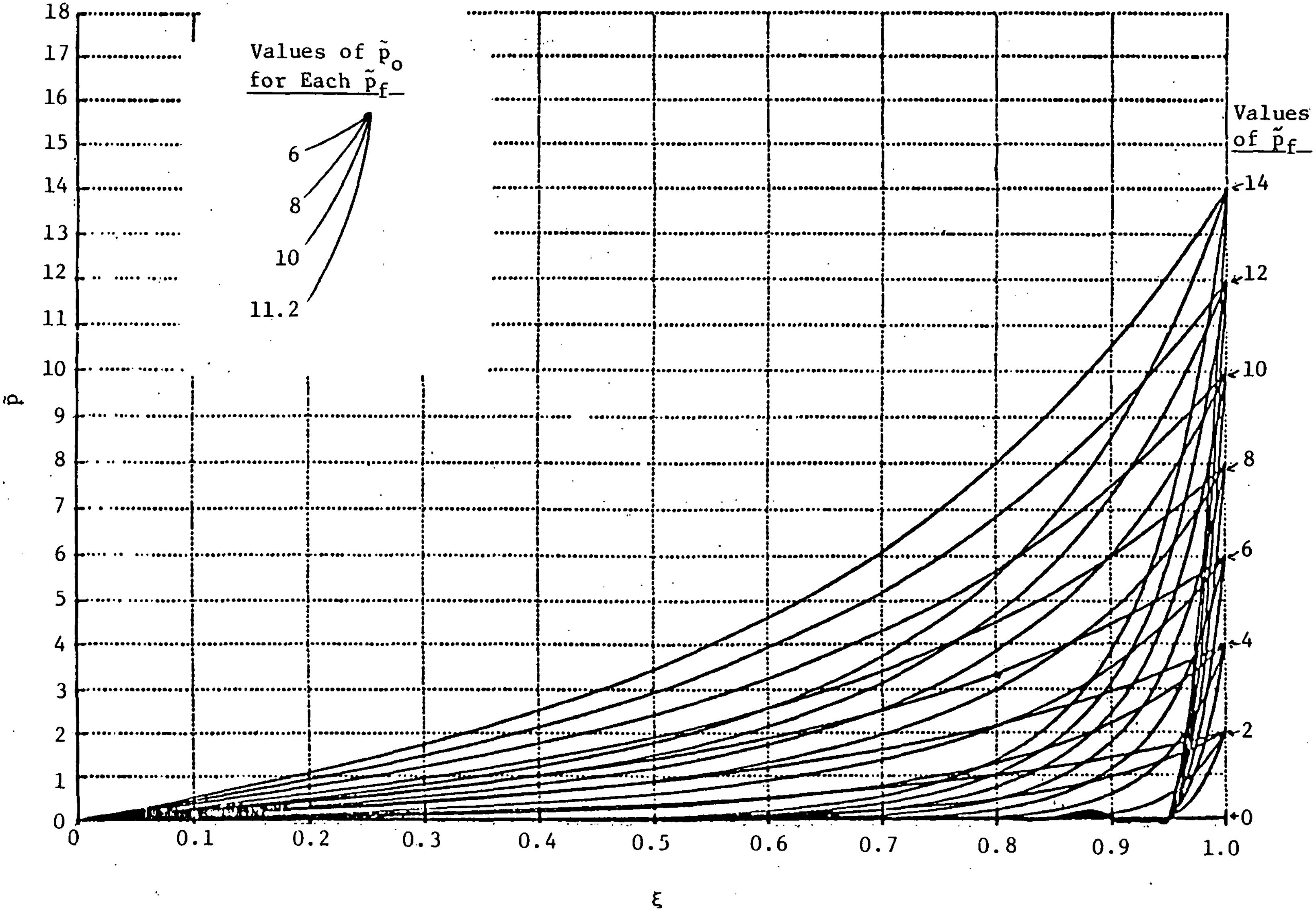

Fig. 2-11 Pressure Profile for Reverse Stroke

- The change from a cavitating to a noncavitating film is rapid. This is shown in Figures 2-12 and 2-13 where the transition region ($2.0 < \tilde{p}_f < 3.0$) is plotted in some detail. The onset of cavitation seems to occur at about $\tilde{p}_f = 2.5$ where $\xi_c \simeq 0$.

- Noncavitating films seem to be accompanied by a net reverse flow even during the forward stroke, emphasizing the previous remark that such cases would be of little practical interest in the application of pumping rings.

- The total net flow, including both the forward and backstrokes, becomes negative, even for cavitating films at about $\tilde{p}_f \simeq 2.0$.

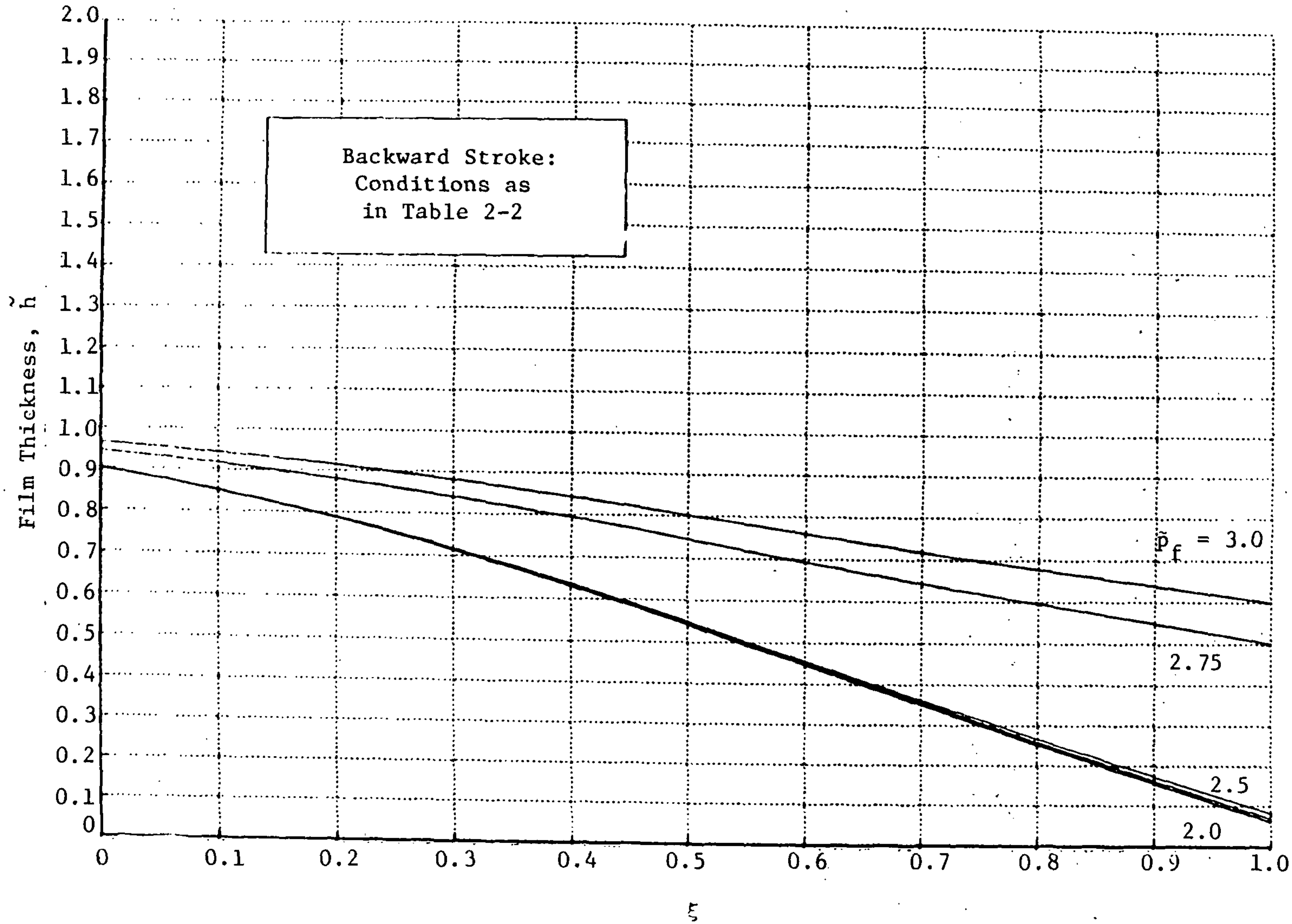

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Fig. 2-12 Film Thickness at Transition from Cavitating to Noncavitating Condition

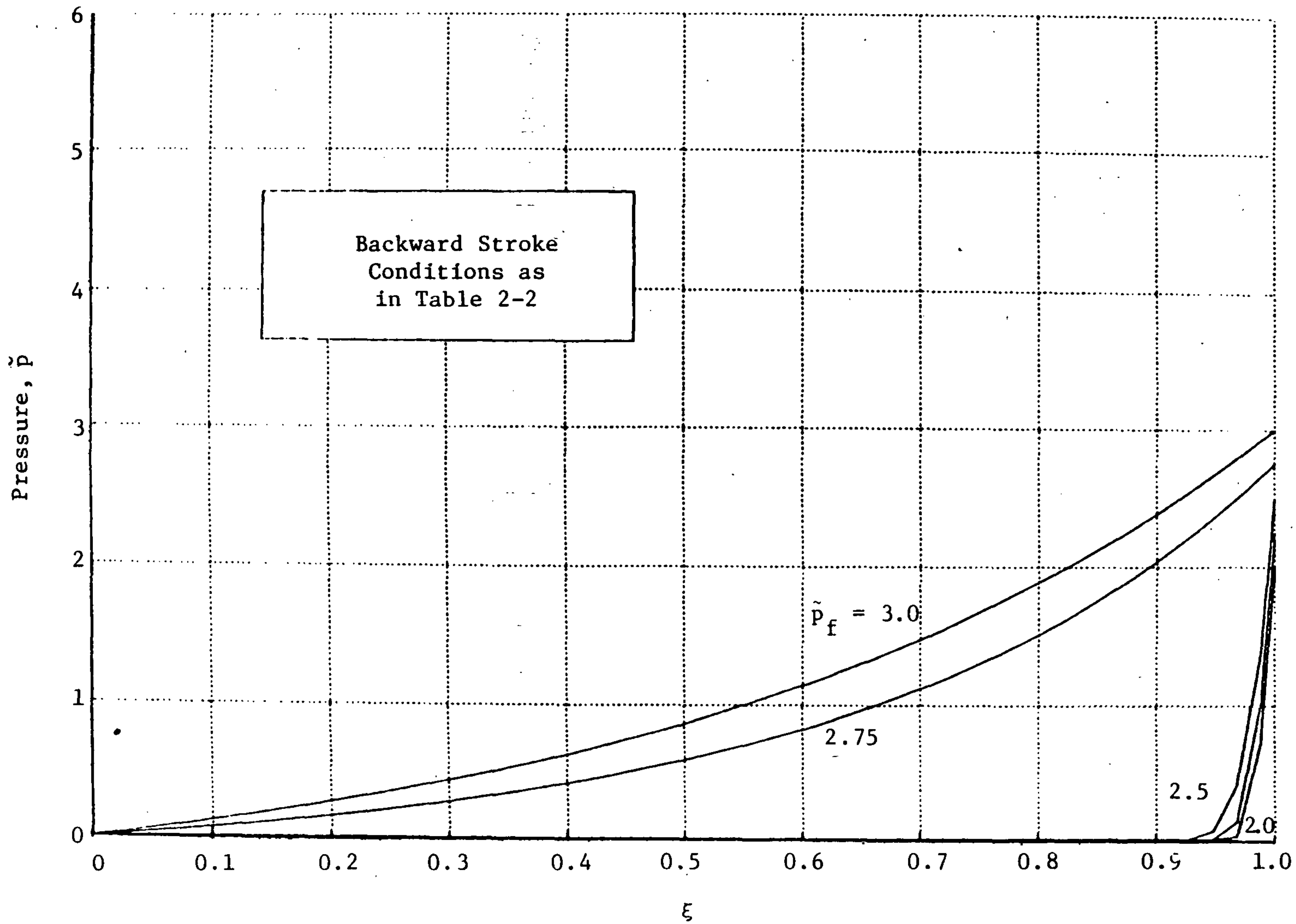


Fig. 2-13 Pressure Profile at Transition from Cavitating to Noncavitating Condition

3.0 ANALYSIS WITH VARIABLE PARAMETERS

This part of the analysis not only abandons the assumptions of constant speed and constant viscosity but also considers some other factors which were previously neglected. Specifically, the analysis included the following elements:

- Thermal effects
- Variable speed
- Squeeze film action
- Starvation
- Nonparallel contours.

Of course, the analysis also retained the backstroke and accompanying cavitation. However, since for the cavitating backstroke, the film thickness is of limited extent, the refinements of variable temperature were not included for that part of the cycle. Thermal effects and transient effects were considered separately to obtain their individual influence. This provides the quantitative corrections associated with these effects without adding the high degree of complexity of a fully coupled analysis.

3.1 Thermal Effects

3.1.1 The Energy Equation

Since no side leakage exists, the one-dimensional energy equation could be used. This, of course, assumes that temperature variation with y can be averaged, a fact which, as will be seen later, is particularly relevant here. It was also assumed that all the heat generated is convected away by the lubricant.

Ignoring conduction, the one-dimensional energy equation can be written as,

$$\rho c u(\partial T/\partial x) = \mu[(\partial u/\partial y)^2] \qquad (3\text{-}1)$$

Since, from one-dimensional bearing theory

$$u = \frac{1}{2\mu}\left(\frac{\partial p}{\partial x}\right) y\,(y - h) + U_o\left(\frac{h - y}{y}\right) \qquad (3\text{-}2)$$

Equation (3-1) can be integrated with respect to y in the interval $0 \le y \le h$ to yield

$$\rho c \left(\frac{U_o h}{2} - \frac{h^3}{12\mu}\frac{\partial p}{\partial x}\right)\left(\frac{\partial T}{\partial x}\right) = \left[\frac{h^3}{12\mu}\left(\frac{\partial p}{\partial x}\right)^2 + \frac{\mu U_o}{h}\right. \tag{3-3}$$

or

$$\left(\frac{\partial T}{\partial x}\right) = \frac{1}{\rho c}\left[\frac{\frac{h^3}{12\mu}\left(\frac{\partial p}{\partial x}\right)^2 + \frac{\mu U_o^2}{h}}{\frac{U_o h}{2} - \frac{h^3}{12\mu}\frac{\partial p}{\partial x}}\right] \tag{3-4}$$

Normalizing and utilizing the relationship

$$(1/\tilde{\mu})(\partial\tilde{p}/\partial\xi) = (\tilde{h} - K)/(\tilde{h}^3)$$

the following can be obtained

$$\partial T/\partial\xi = [\tilde{\mu}(\xi)/K]\ \{[(\tilde{h} - K)^2/(\tilde{h}^3)] + [1/(3\tilde{h})]\} \tag{3-5}$$

where

$$\tilde{T} = \frac{6\mu_o U_o L(T - T_o)}{\rho c(C^2)}$$

3.1.2 Lubricant Flow

The flow pattern at the interface of a pumping ring was somewhat problematic. The nature of this flow can be visualized in Figure 3-1. As shown in Figure 3-1a, at zero or low upstream pressure, the flow is mainly forward, with only a small pocket of fluid circulating near the inlet of the film. This recirculating flow is induced by the adverse hydrodynamic pressure gradient prevailing near $x = 0$. When the level of p_f rises, more and more of the forward flow near

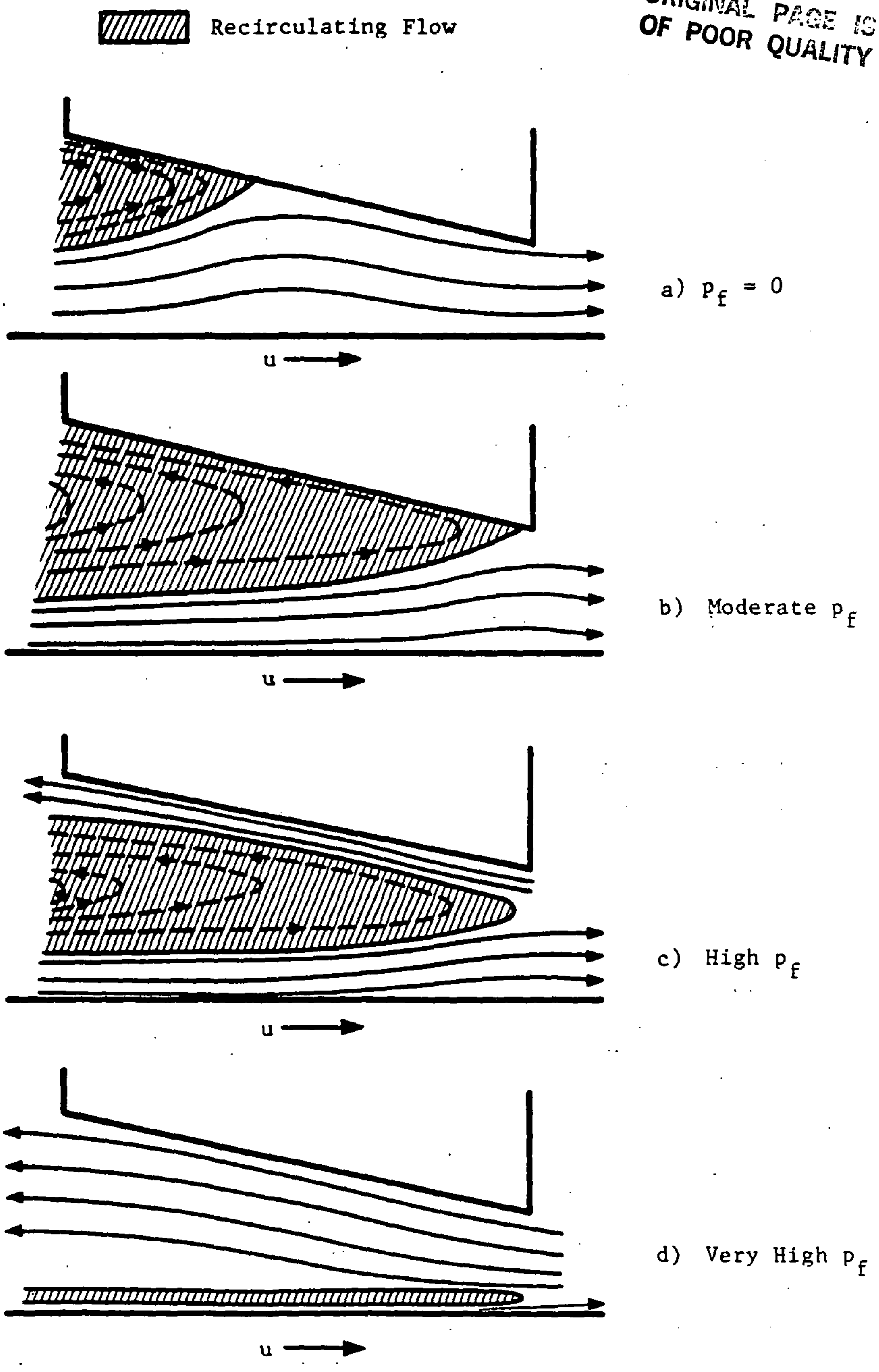


Fig. 3-1 Lubricant Flow in the Fluid Film

the stationary surface is blocked and eventually reversed (Figure 3-1c) by the static pressure gradient induced by p_f. Eventually, when the upstream pressure is sufficiently high, fluid from the sealed chamber begins to leak backwards along the stationary surface. When the magnitude of p_f becomes very high (Figure 3-1d), most of the flow is backward, with only thin, vanishing layers of forward and recirculating flows maintained near the moving surface.

Thus, in general, there are three layers of flow possessing the following characteristics:

- Forward Flow - This flow is along the moving surface. It enters at a temperature, T_o, that prevails at $x = 0$ and is heated on its way to the reservoir to some T_{max} at $x = L$.

- Recirculating Flow - This flow also enters at a temperature, T_o, but various portions of that flow penetrate only part way into the film before they are reversed and returned to their source. It should be noted that the bulk of the flow recirculates near the entrance, where h is large. It undergoes relatively little viscous shearing, resulting in low energy dissipation to the fluid.

- Reverse Flow - This flow originates in the reservoir entering at a temperature, T_f, and is heated while traveling upstream. Its maximum temperature is reached at the entrance to the pumping ring, at $x = 0$.

In view of these characteristics, only fluid which transverses the whole length, L, i.e., only the forward and reverse flows, was considered instrumental in carrying away the dissipated heat. Since the bulk of the intermediate layer recirculates near the entrance where the temperature differential is relatively small, its effect is left out of the heat balance. This treatment represents a conservative approach because inclusion of the recirculating flow would yield lower temperatures and thus safer operating conditions than those predicted by the present method.

The first task was to find expressions for the three flow regimes in terms of ring geometry and its operating conditions. Defining a dimensionless transverse coordinate and a dimensionless velocity by

$$\tilde{u}(y) = (u/U_o); \; \eta(\xi) = [y/h]$$

the velocity from Equation (3-2) may be expressed in dimensionless form as

$$u = 3(1 - K/h)\,\eta(\eta - 1) + (1 - \eta) \tag{3-6}$$

The line of zero velocity, or the line below which all fluid flows forward and above which the flow is backward, is easily obtained from Equation (3-6) by writing u = 0 which yields the locus

$$\eta(\xi)\Big|_{u=0} = \eta^*(\xi) = \frac{1}{3[1-K/\tilde{h}(\xi)]} \tag{3-7}$$

At a given ξ, the total flow contained between the moving surface and any point $(\eta,\ \xi)$ line is given by

$$\psi(\eta,\xi) = \tilde{h}\int_o^{\eta} u(\eta')\,d\eta'$$

and after integration yields

$$\psi(\eta,\xi) = \tilde{h}\eta(\eta - 1)^2 + K\eta^2\,(3/2 - \eta) \tag{3-8}$$

By assigning to $\psi(\eta,\xi)$ various constant values, the flow streamlines are obtained. The net flow, of course, is contained between $\eta = 0$ and $\eta = 1$ or

$$\psi(1,\xi) = \tilde{q}_{NET} = (K/2)$$

which is shear flow at $\tilde{h} = K$ where $(d\tilde{p}/d\xi) = 0$.

The film thickness is given by

$$\tilde{h} = \tilde{h}_2[1 + (a - 1)(1 - \xi)]$$

So that the value of (K/2) for an isoviscous, linear slider is given by

$$\frac{K}{\tilde{h}_2} = \frac{2a}{a+1}(1 - a\tilde{p}_f\tilde{h}_2^{\,2}) \tag{3-9}$$

where

$$a = \left(\frac{\tilde{h}_2 - h_2'}{\tilde{h}_2}\right)$$

For the variable viscosity case where $\mu = \mu(\xi)$, the value of K is

$$K = \frac{-\tilde{p}_f + \int_0^1 \frac{\tilde{\mu}\,d\xi}{\tilde{h}^2}}{\int_0^1 \frac{\tilde{\mu}\,d\xi}{\tilde{h}^2}} \tag{3-10}$$

The individual streamlines are obtained by assigning different constants to $\psi(\eta,\xi)$ in Equation (3-7). A sample plot of such streamlines for a = 3 and different values of $\tilde{p}_f\tilde{h}_2^{\,2}$ is shown in Figure 3-2. Reverse flow will commence when the dividing streamline between the forward and recirculating flow reaches $\xi = 1$. This, from Equation (3-7), occurs at

$$\eta^*(1) = \frac{1}{3[1 - K/\tilde{h}_2]} = 1 \tag{3-11}$$

or at $(K/\tilde{h}_2) = 2/3$. At values of $K/\tilde{h}_2^{\,2} < 2/3$, the tangent point of the recirculating envelope at $\xi = 1$ will lie below $\eta^* = 1$, opening up a passage for reverse flow. Below the recirculating envelope the fluid will, as shown in Figure 3-3, flow forward at a rate of

$$\tilde{q}_F = \psi[\eta^*(1), 1]$$

whereas, above the envelope there will be reverse flow at a rate of

$$\tilde{q}_R = \tilde{q}_F - \tilde{q}_{NET} = \psi[\eta^*(1), 1] - (K/2) \tag{3-12}$$

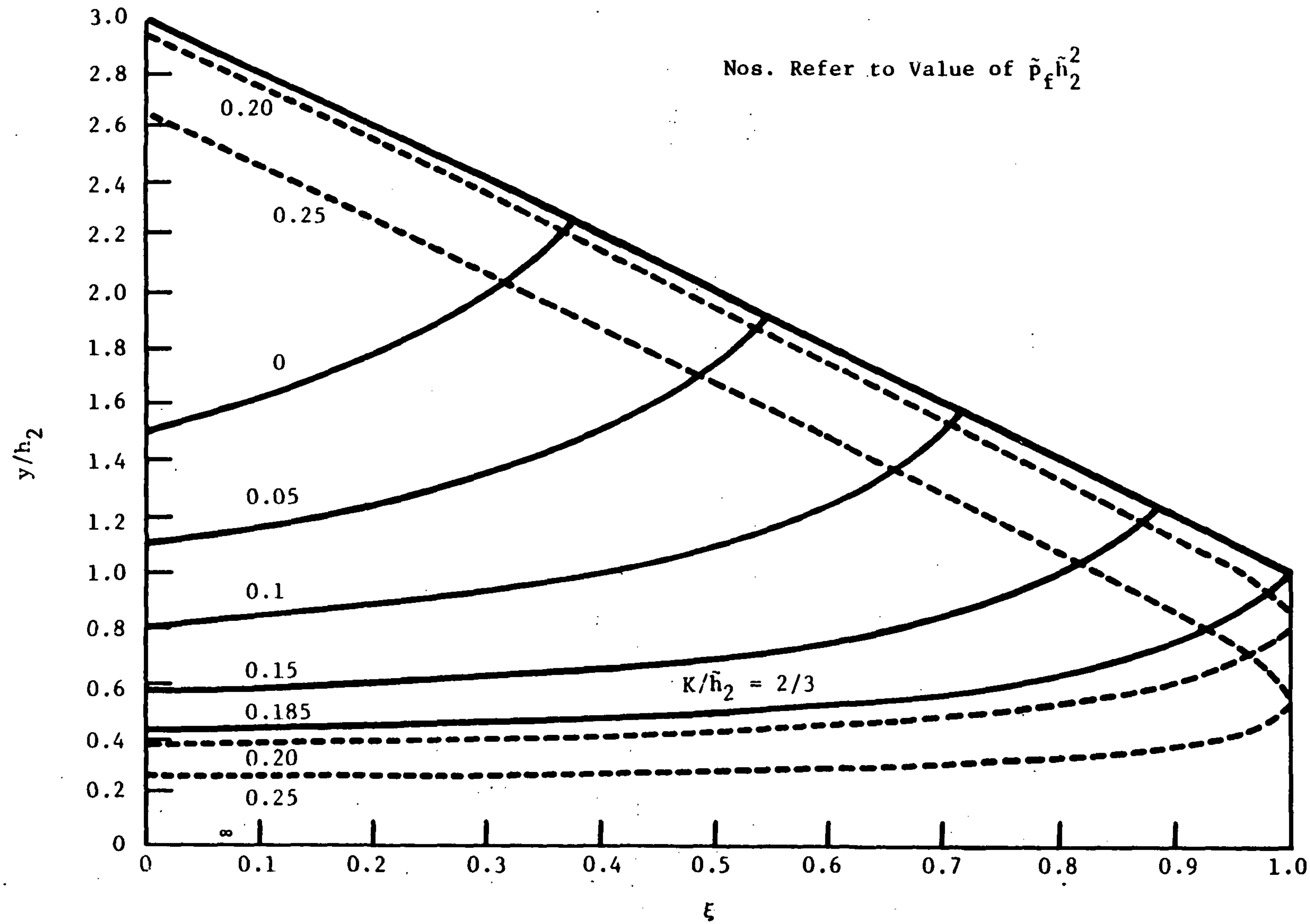


Fig. 3-2 Fluid Film Streamlines for a = 3

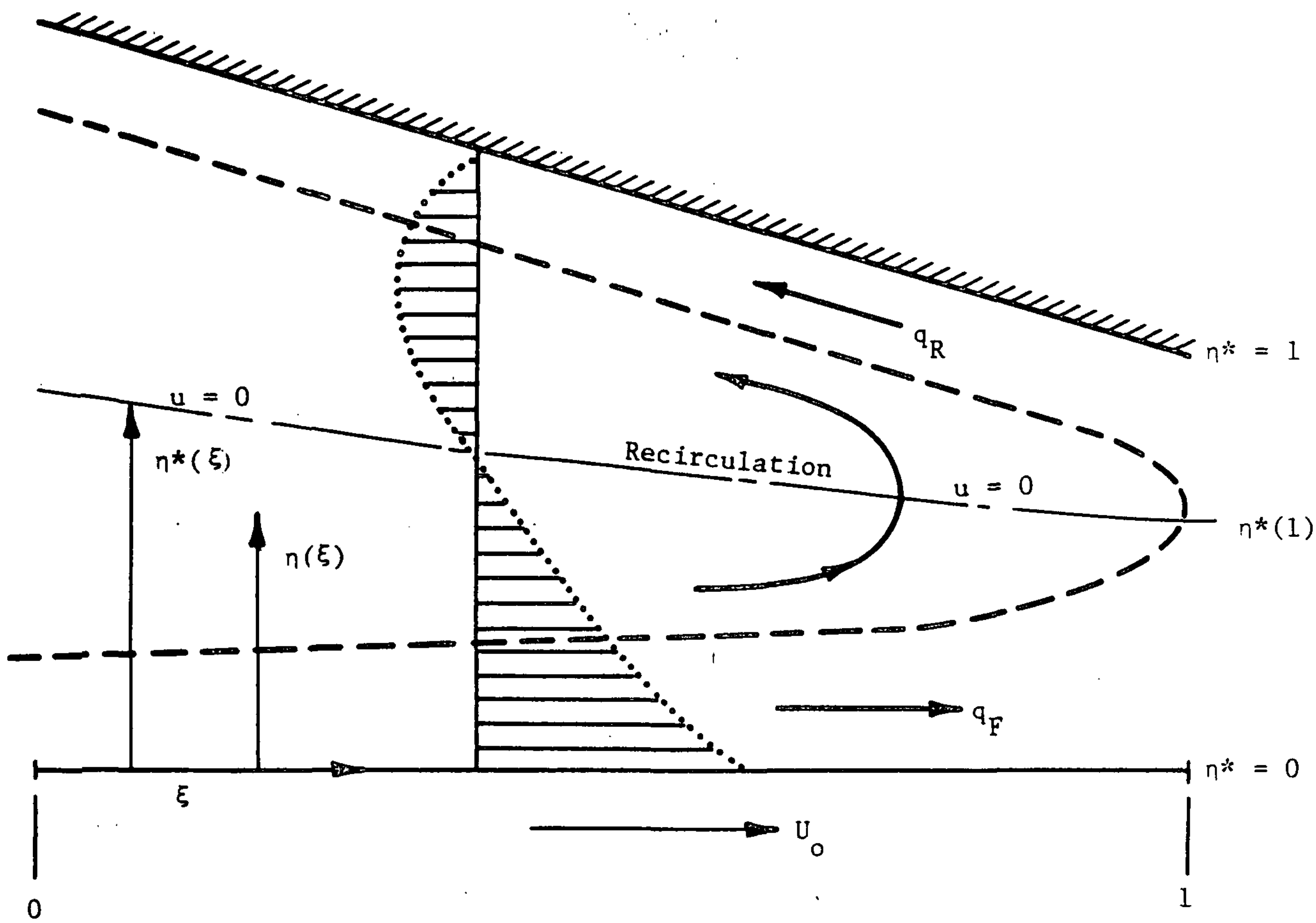


Fig. 3-3 Mapping of the Three Flow Regions

3.1.3 Modeling of Thermal Problem

As stated previously, the viscous heating is assumed to be convected away only by the fluid that passes the interspace, be it to the right or to the left. For this purpose, the interspace is modeled so as to be filled with these two streams, i.e. a forward flow, q_F, and a reverse flow, q_R, as shown in Figure 3-4. By designating

$$f = q_F/(q_F + q_R) \tag{3-13}$$

$f\theta(x)$ will be the heat convected to the right by q_F and $(1-f)\ \theta$ the heat connected to the left by q_R. Likewise, temperatures will be averaged across the film (i.e., in the y but not in the x direction) with $T_F(x)$ and $T_R(x)$ representing the temperature profiles generated in the two flow layers, q_F and q_R. The overall average temperature profile will be obtained from

$$T(x) = f\ T_F(x) + (1 - f)\ T_R(x) \tag{3-14}$$

The viscosity at each x station will be based on this averaged temperature, i.e.

$$\mu(x) = \mu[T(x)].$$

The expression for the viscous heating (Section 3.1.1) is exact in the sense that the losses are calculated over the exact velocity profile, including the region of recirculating flow, namely.

$$\phi(x) = \mu(x) \int_0^h (du/dy)^2 dy \tag{3-15}$$

where u (x,y) is the velocity profile shown in Figure 3-3. Consistent with the flow model, $\mu(x)$ in the calculation of this viscous shear will be that corresponding to the average temperature T(x).

It should be noted that, whereas with no reverse flow ($q_R = 0$), the temperature at the inlet to the film is a constant equal to T_o, this is no longer true when reverse flow sets in. The reverse flow, starting at an initial temperature, T_f,

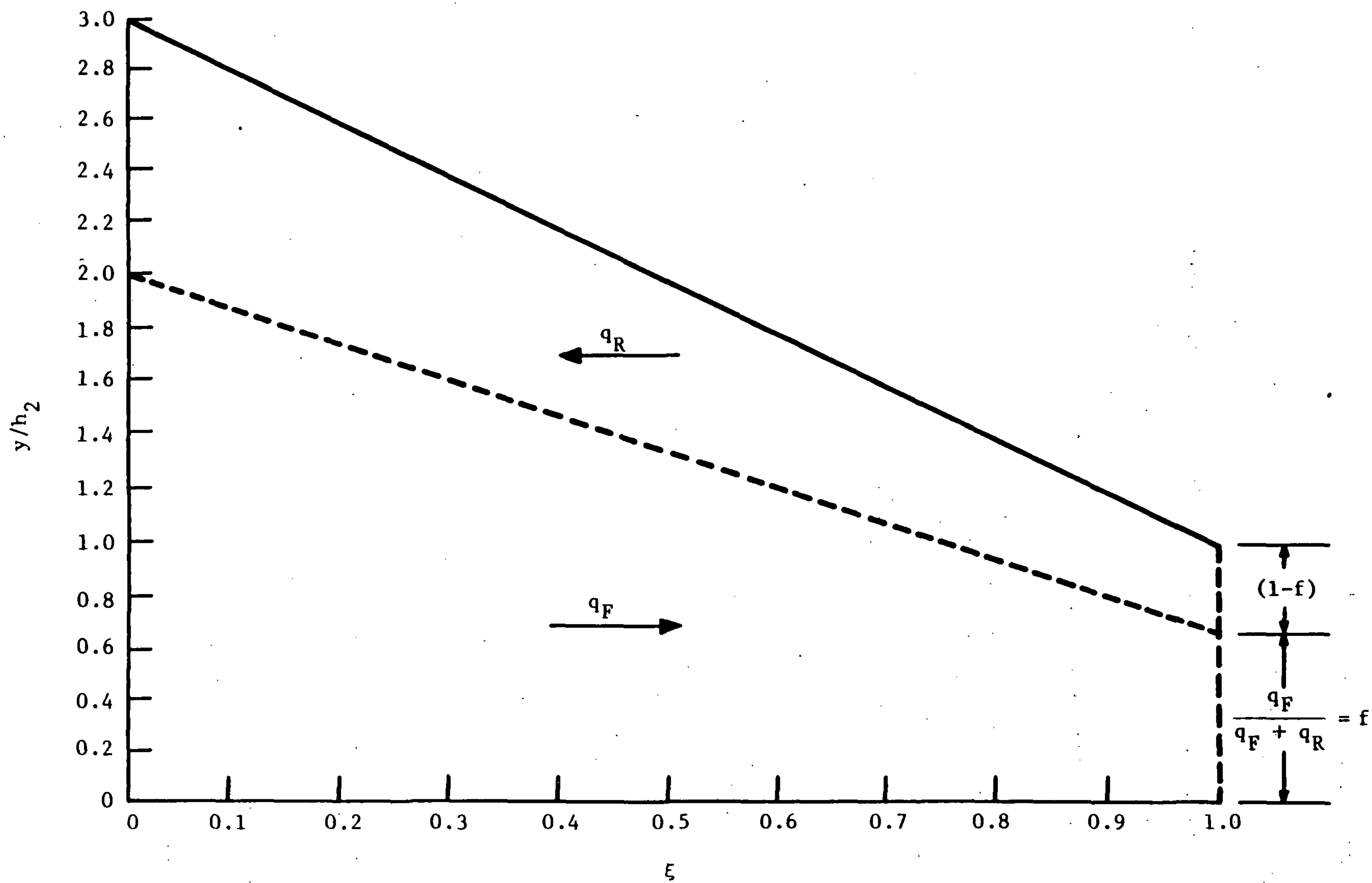


Fig. 3-4 Subdivision of Flow for Convective Heat Transfer

will reach a maximum at x = 0 and, since temperatures are averaged across y, the inlet temperature will be some $T_o' > T_o$.

3.1.4 Calculation Procedure

The procedure to be followed in calculating thermal effects in the fluid film is, in view of the previous discussion, as follows:

a. The differential equation to be solved is

$$\frac{d\tilde{p}}{d\xi} = \tilde{\mu}(\xi)\left[\frac{\tilde{h} - K}{\tilde{h}^3}\right]$$

where due to the variable viscosity $\mu(\xi)$

$$K = \frac{\int_0^{\tilde{h}} \frac{\tilde{\mu}(\xi)}{\tilde{h}^2}\, d\xi - \tilde{p}_f}{\int_0^1 \frac{\tilde{\mu}(\xi)}{h^3}\, d\xi}$$

and

$$\mu(\xi) = F\left[T_o + \frac{6\mu_o U_o L}{\rho c\, C^2}\, \tilde{T}(\xi)\right]$$

b. If $(K/\tilde{h}_2) > 2/3$, there is no reverse flow and

$$\frac{d\tilde{T}}{d\xi} = \frac{\tilde{\mu}(\xi)}{K}\left[\frac{(\tilde{h} - K)^2}{\tilde{h}^3} + \frac{1}{3\tilde{h}}\right] \tag{3-16}$$

with $T = T_o$ at $\xi = 0$.

c. If $(K/\tilde{h}_2) < 2/3$, there is reverse flow, and the following quantities have to be established in order to account for both the forward and backward streams:

$$\eta^* = \eta^*(1) = \frac{1}{3\left[1 - \frac{K}{\tilde{h}_2}\right]}$$

$$K_F = 2\tilde{h}_2\ \eta^*(\eta^* - 1)^2 + 2K(\eta^*)^2\ [3/2 - \eta^*] = 2\tilde{q}_F$$

$$K_R = K_F - K = 2q_R$$

$$K_T = K_F + K_R = 2K_F - K = 2(q_F + q_R) = 2q_T$$

$$\frac{d\tilde{T}}{d\xi} = \frac{\tilde{\mu}(\xi)}{K_T}\left[\frac{(\tilde{h} - K)^2}{\tilde{h}^3} + \frac{1}{3\tilde{h}}\right] \qquad (3\text{-}17)$$

Equation (3-17) needs a solution with two different boundary conditions:

- For $T_F\ (\xi)$, $T = T_o$ at $\xi = 0$
- For $T_R\ (\xi)$, $T = T_f$ at $\xi = 1$

The averaged temperature, any given ξ, is thus

$$\tilde{T}(\xi) + \frac{\tilde{T}_R K_F + \tilde{T}_L K_R}{K_T} \qquad (3\text{-}18)$$

where

$$\tilde{T} = \frac{\rho c\ C^2(T - T_o)}{6\mu_o U_o L}$$

A sample solution for the temperature profile in the fluid film for various values of upstream pressure is given in Figure 3-5. The solutions are for the case of equal upstream and downstream boundary temperatures. Curves which start at T = 86°F are those without reverse flow and thus have what may be called a conventional profile. However, at $\tilde{p}_f > 1.5$, there is reverse flow and, due to the averaging of temperatures across y, the inlet temperature is higher than

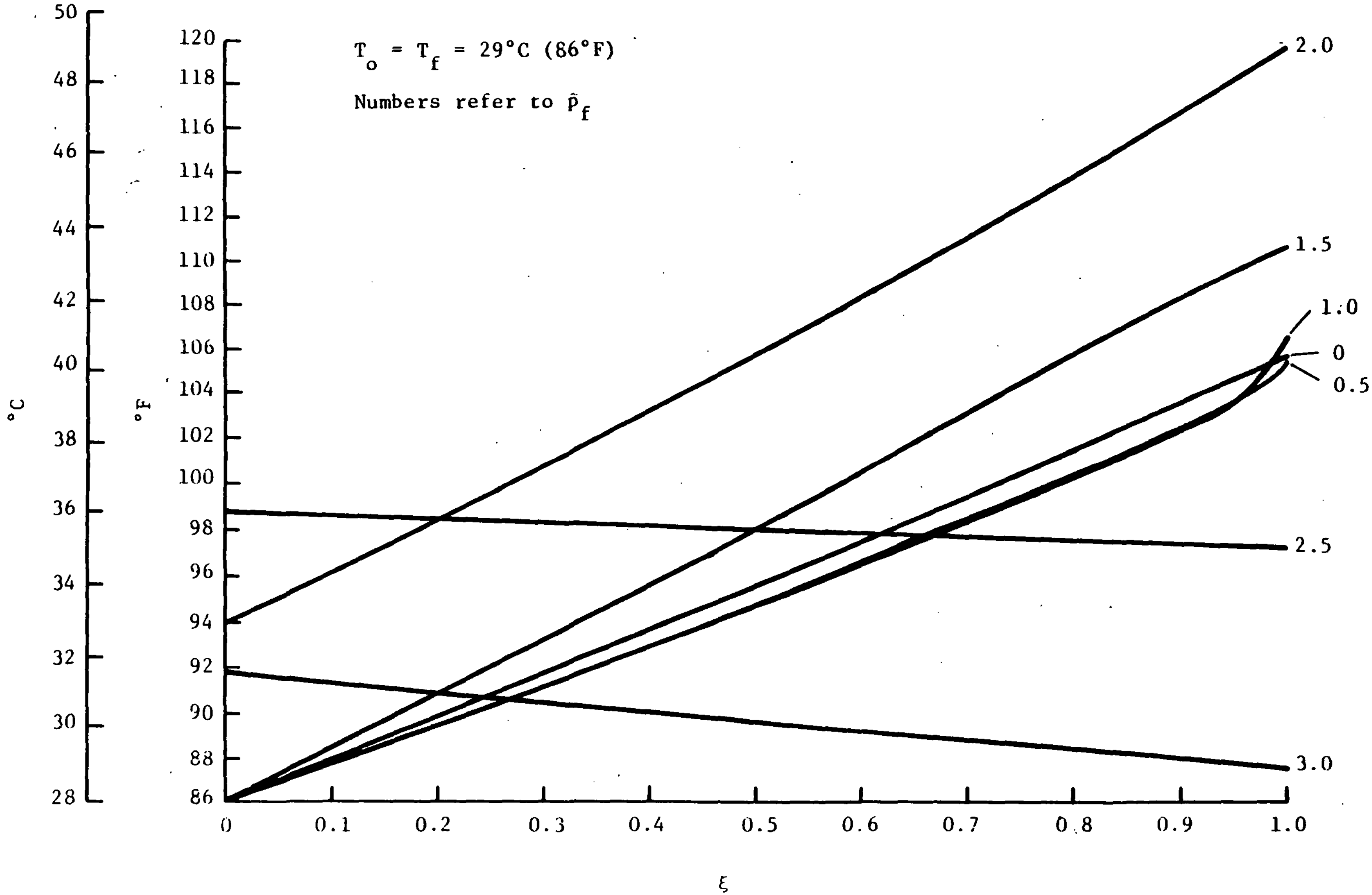


Fig. 3-5 Temperature Profile as Function of $\tilde{p}_f$

86°F. It is interesting to note that, due to the cooling effect of the backward flow, the maximum temperatures in the film actually decrease as the back flow increases above a certain level. Thus, for $p_f = 0$, $T_{max} = 107°F$, and for $p_f = 3.0$, $T_{max} = 92°F$ and occurs not at the trailing but at the leading edge of the film. The highest temperatures occur at some intermediate combination of forward and backward flows; in this particular example it happens at $p_f = 2.0$ with T_{max} reaching 120°F.

3.2 Variable Velocity and Squeeze-Film Effects

Throughout the previous discussions, the rod velocity was considered to be constant, given by $U_o = 2sf$. This, of course, represents the average velocity over each half cycle. In actuality, the rod, driving a crankshaft, moves with a variable velocity given by

$$u = u_{max} \cos\pi 2ft$$

where

$$u_{max} = fs = \pi U_o/2$$

Consequently, a normal velocity component and squeeze film forces are imposed on the ring, as shown in Figure 3-6. These are generated by a variation of the hydrodynamic forces and of film thickness, as a function of the variable velocity. The normal velocity introduces perturbations on nearly all the relevant quantities, such as film thickness, pressures, flows, extent of cavitation, and others.

When variable velocity and squeeze-film effects are included, the Reynolds Equation becomes:

$$(\partial/\partial\xi)\ [\tilde{h}^3(\partial\tilde{p}/\partial\xi)] = (\pi/2)[\cos\tau(\partial\tilde{h}/\partial\xi) + \sigma(\partial\tilde{h}/\partial\tau)] \qquad (3\text{-}19)$$

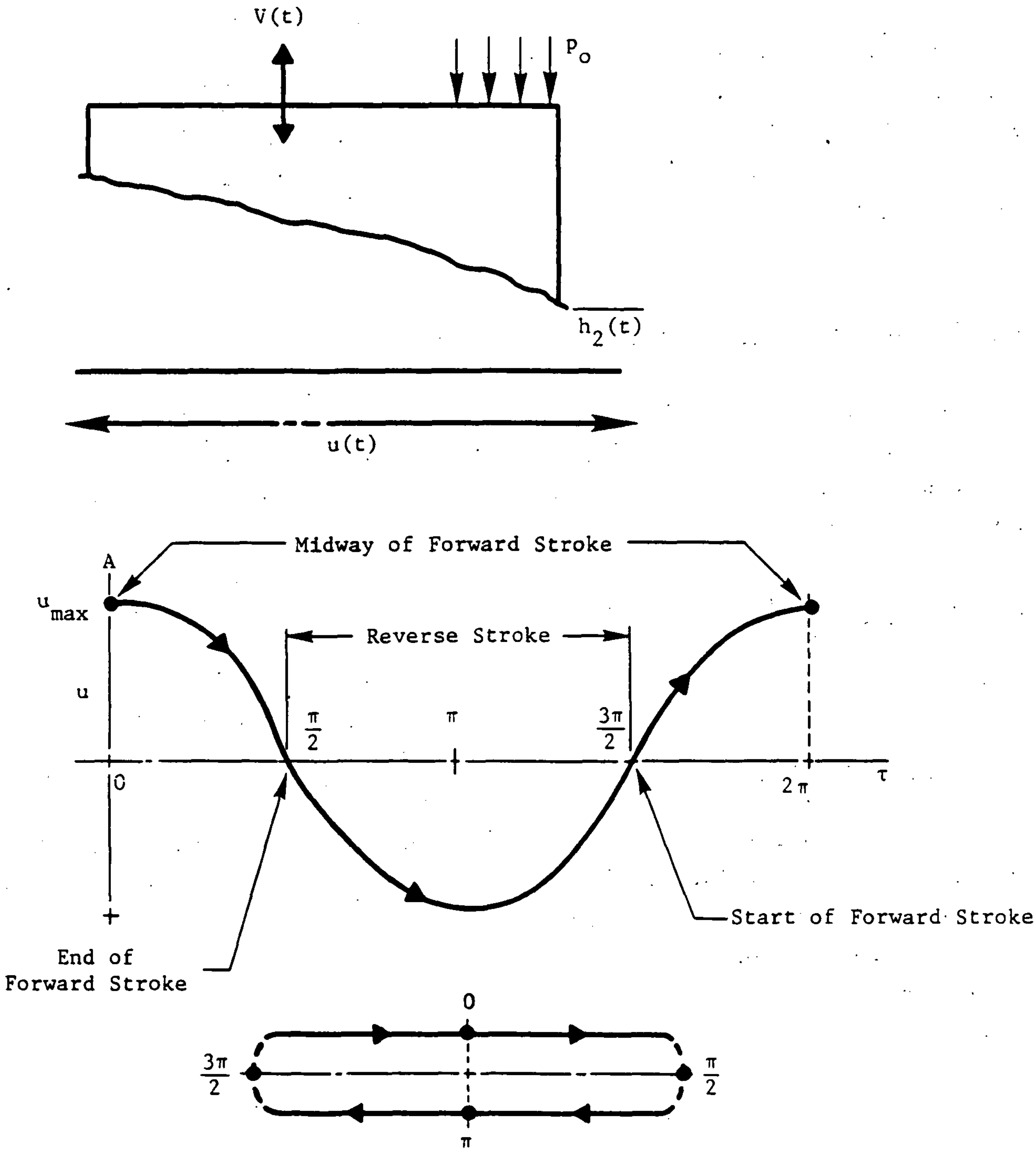


Fig. 3-6 Variable Velocity and Squeeze Film Effects

where σ is the squeeze-film parameter given by $\sigma = 4L/s$, and τ is the dimensionless time $\tau = 2\pi ft$.

The relevant boundary conditions are:

- For no caviation: $\tilde{p}(0) = 0$; $\tilde{p}(1) = \tilde{p}_f$
- For cavitation: $\tilde{p}(1) = \tilde{p}_f$, $\left.\frac{d\tilde{p}}{d\xi}\right|_{\xi = \xi_c} = 0$

The one-dimensional, transient Reynolds Equation, given by Equation (3-19), may be solved with the use of the implicit time-transient method. $\partial h/d\tau$ may be written as:

$$(\partial\tilde{h}/\partial\tau) \approx [\tilde{h}_{(i)} - \tilde{h}_{(i-1)}]/\Delta\tau$$

where the subscript i in parentheses denotes the i^{th} time step, and $\Delta\tau$ is the interval between the i^{th} and $(i-1)^{th}$ time steps. Hence

$$\tilde{h}_{(i)} = \tilde{h}_{2(i)} + \Delta\tilde{h}_{(i)}\,(1 - \xi)$$

and the discretized form of Equation (3-19) is written as

$$\frac{d}{d\xi}\left[\tilde{h}^3_{(i)}\frac{d\tilde{p}_{(i)}}{d\xi}\right] = \frac{\pi}{2}\left\{[-\cos\tau_{(i)}\Delta\tilde{h}_{(i)}] + \sigma\frac{[\tilde{h}_{(i)} - \tilde{h}_{(i-1)}]}{\Delta\tau}\right\} \qquad (3\text{-}20)$$

If it is assumed that $\tilde{h}_{(i-1)}$ is known, then Equation (3-20) can be integrated analytically to obtain $\tilde{p}_{(i)}$ as a function of ξ at the i^{th} time step. The elasticity equation, Equation (2-6), remains unchanged except for the time dependence of $\tilde{p}$, and may be coupled with the solution to Equation (3-20) and solved at each time step as described previously for the steady-state solution. Values of $\tilde{h}_2$ and $\Delta\tilde{h}$ at the previous time step ($\tilde{h}_{2(i-1)}$, $\Delta\tilde{h}_{(i-1)}$) were used in evaluating $\tilde{h}_{(i-1)}$ in Equation (3-20) and for initial guesses for $\tilde{h}_{2(i)}$ and $\Delta\tilde{h}_{(i)}$ for use in the secant method.

Quasi-static solutions ($\sigma = 0$) were obtained at the middle of the forward stroke ($\tau = 0$) to start the marching procedure. Solutions were then computed at successive time steps until periodic solutions were obtained.

Sample solutions were run on a pumping ring having the following dimensionless characteristics:

$$\alpha = 0.0282,\ \beta = 0.2569,\ \varepsilon = 0.518,\ \tilde{L}_1 = 1.567,\ \sigma = 4.0$$

Three cases were considered, at different combinations of $\tilde{p}_f$ and $\tilde{p}_o$; the $\tilde{p}_o = 4.0$ solution also represents a case where the ring is clamped on the backstroke. The three cases were all solved for steady-state ($u = u_o$), quasi-static ($\sigma = 0$), and transient conditions to show the effects of squeeze film on ring performance. The value of $\sigma = 4.0$ is quite large for practical applications as it corresponds to a stroke, s, equal to the bearing land, L. For present applications, σ is generally less than 1. The large value was used to exaggerate the effects of squeeze film for purposes of illustration and interpretation. Graphs for film thickness, flow, and pressure profiles are given in Figures 3-7 through 3-14, whereas the values of the component flows are itemized in Table 3-1. Note should be taken that $\tau = 0$ corresponds in these plots to rod position at $u = u_{max}$, i.e., at the midpoint of its forward stroke.

Considering first the effects of variable velocity alone (quasi-static solution), the plots show the following trends:

- The film thickness variation during the forward stroke follows the sinusoidal shape of the velocity curve. During the backstroke velocity, variation has no effect whenever cavitation commences at the trailing edge and a small effect when cavitation is located at $\xi < 1$ (Figure 3-9).

- The flow curve follows the shape of the velocity curve throughout the cycle, except, of course, when, due to a clamped ring, the flow during the backstroke is zero. In the particular case of Figure 3-14, the flow, after peaking at u_{max}, became negative at the end of the forward stroke ($u \to 0$). However, upon commencing the backstroke, the ring clamped and flow ceased.

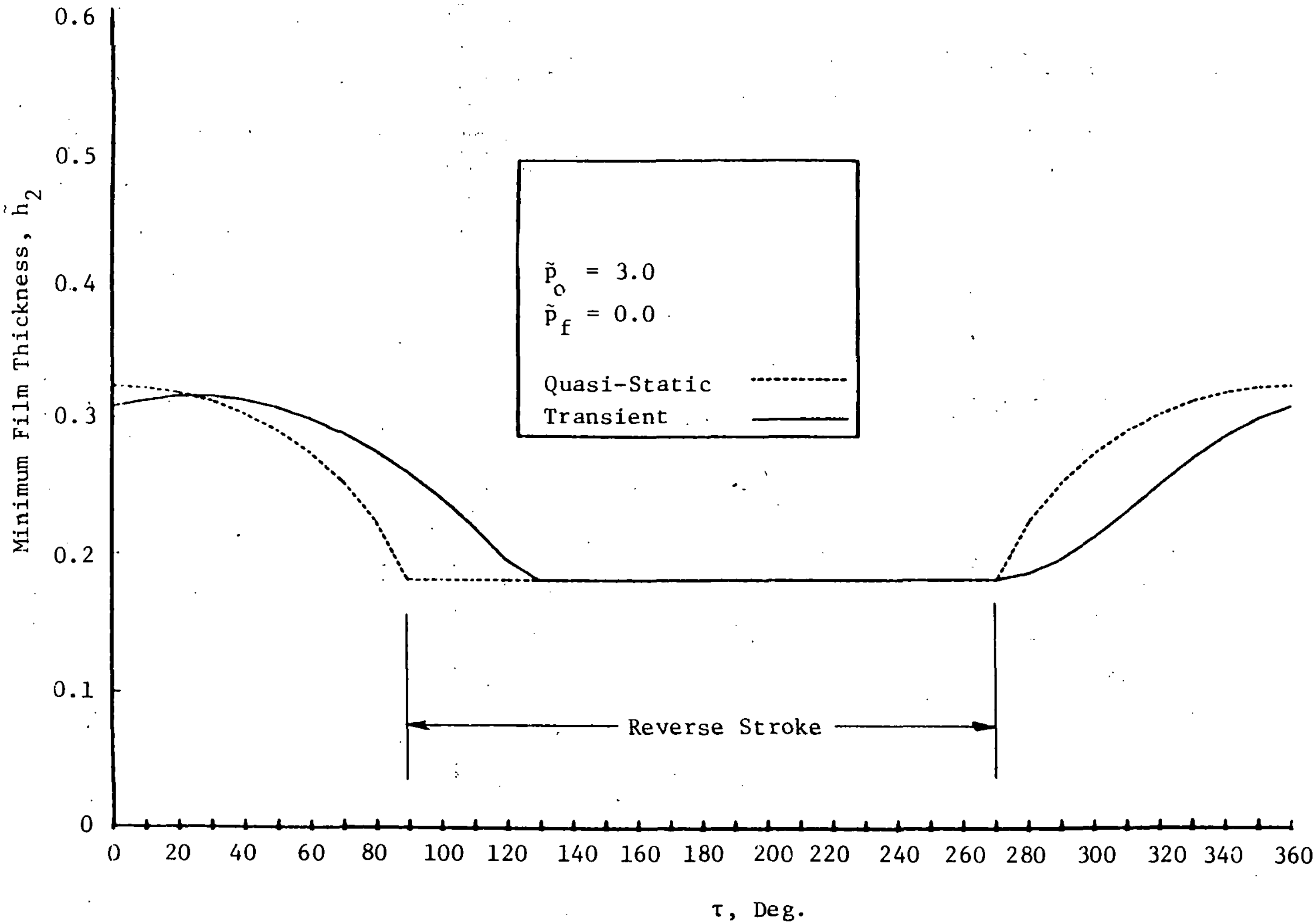


Fig. 3-7 Film Thickness $\tilde{h}_2$ as Function of Time

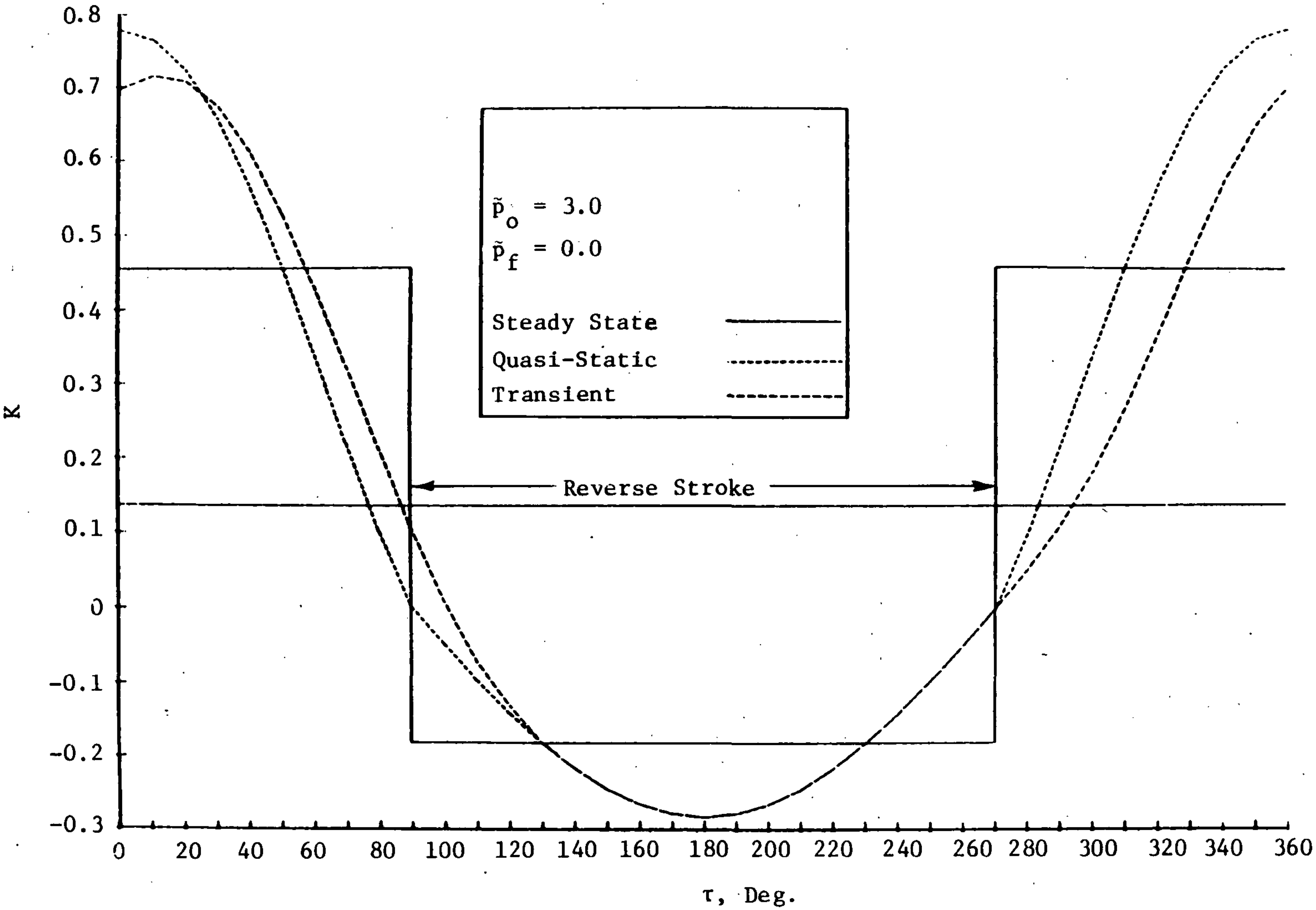


Fig. 3-8 Flow Parameter Versus Time

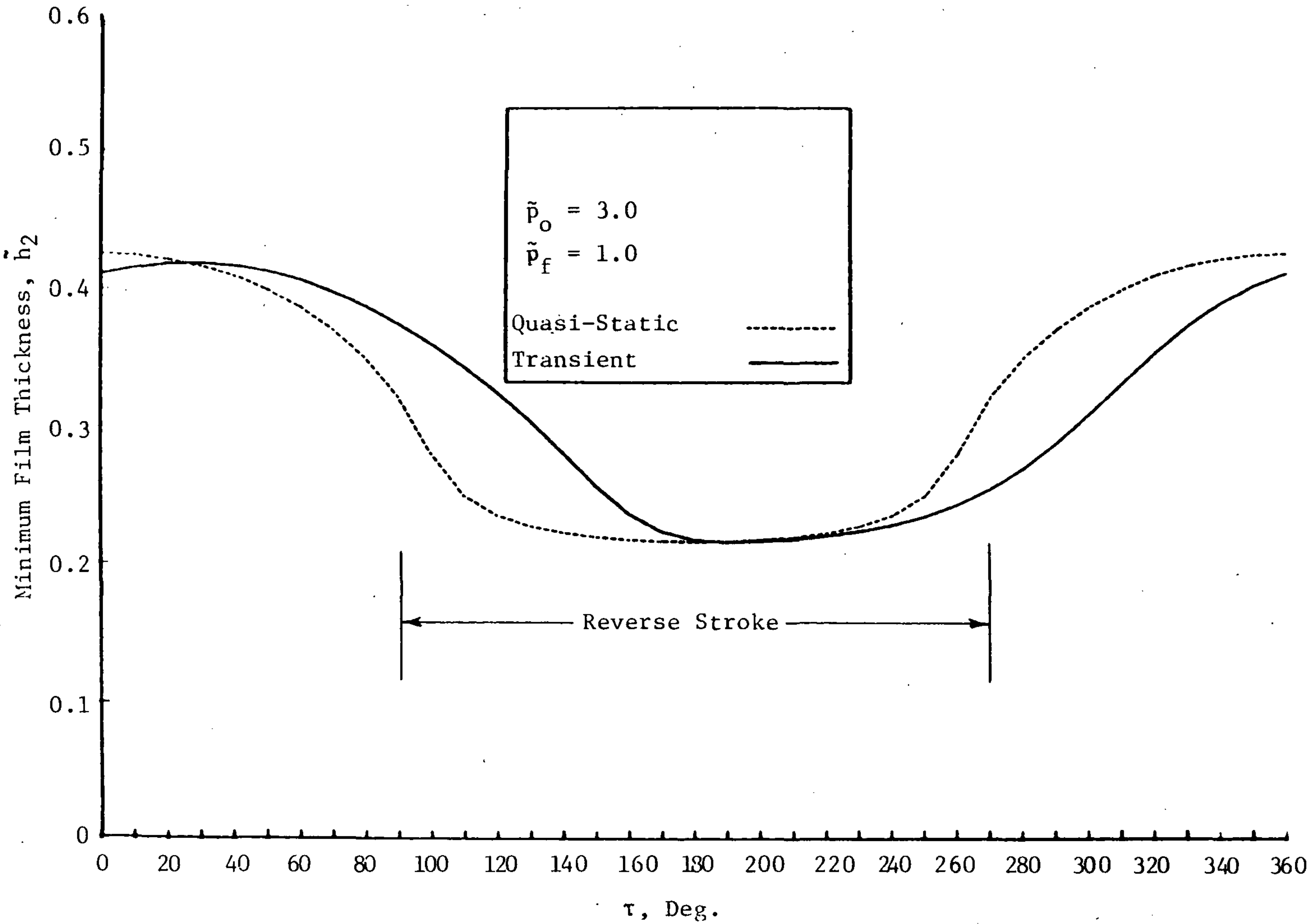


Fig. 3-9 Film Thickness $\tilde{h}_2$ as Function of Time

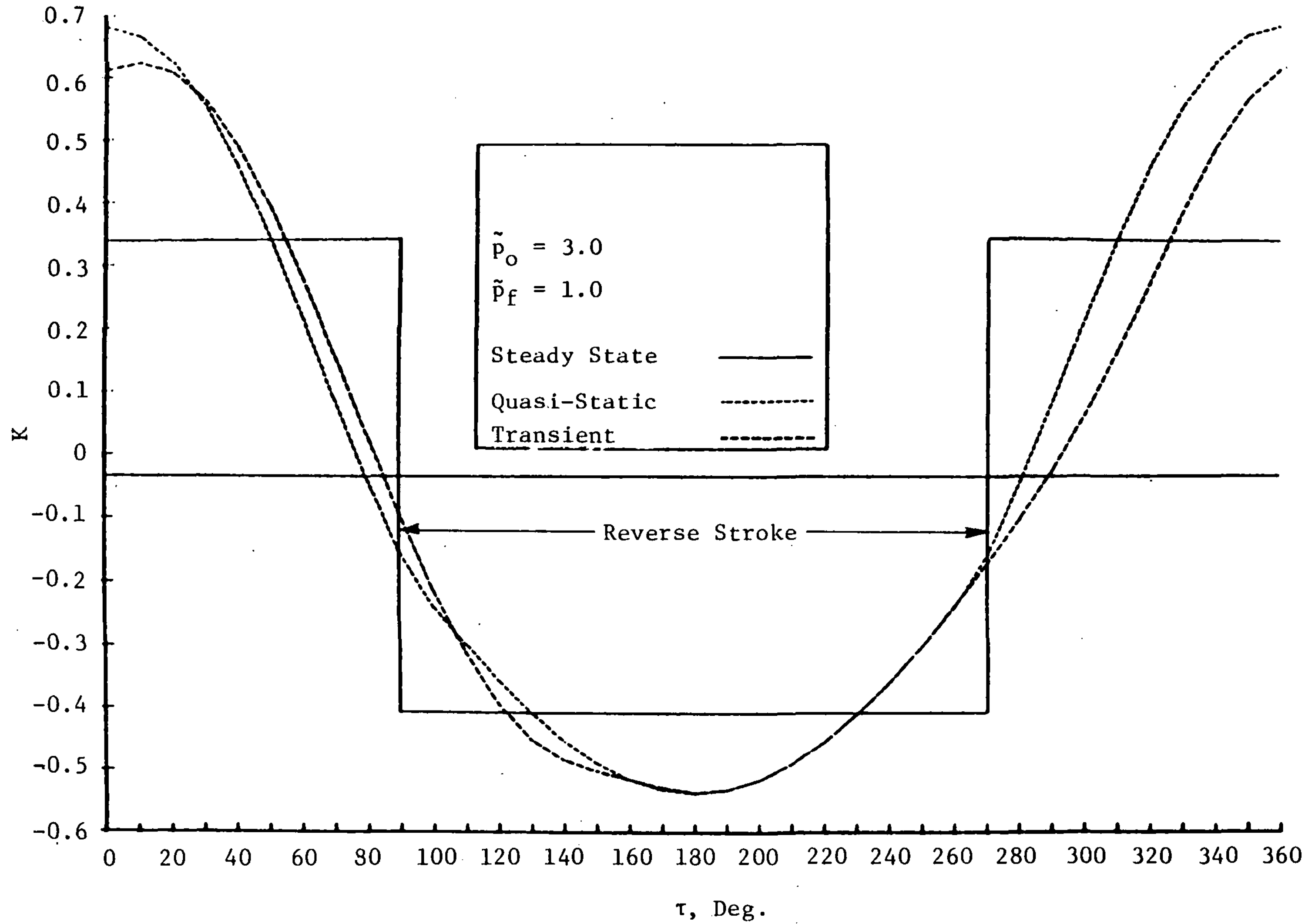


Fig. 3-10 Flow Parameter Versus Time

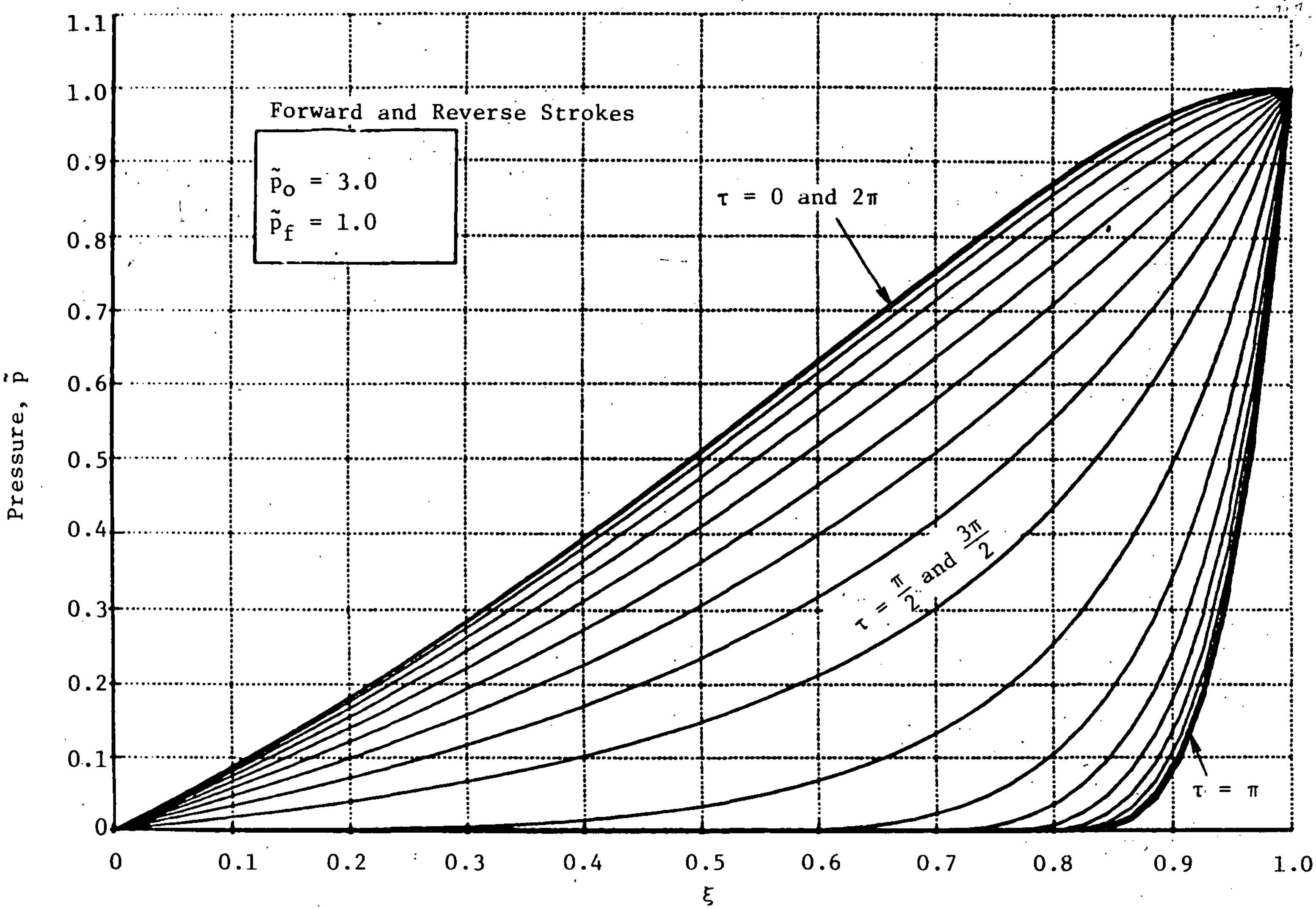


Fig. 3-11 Pressure Profiles as Function of Time for Quasi-Static Analysis

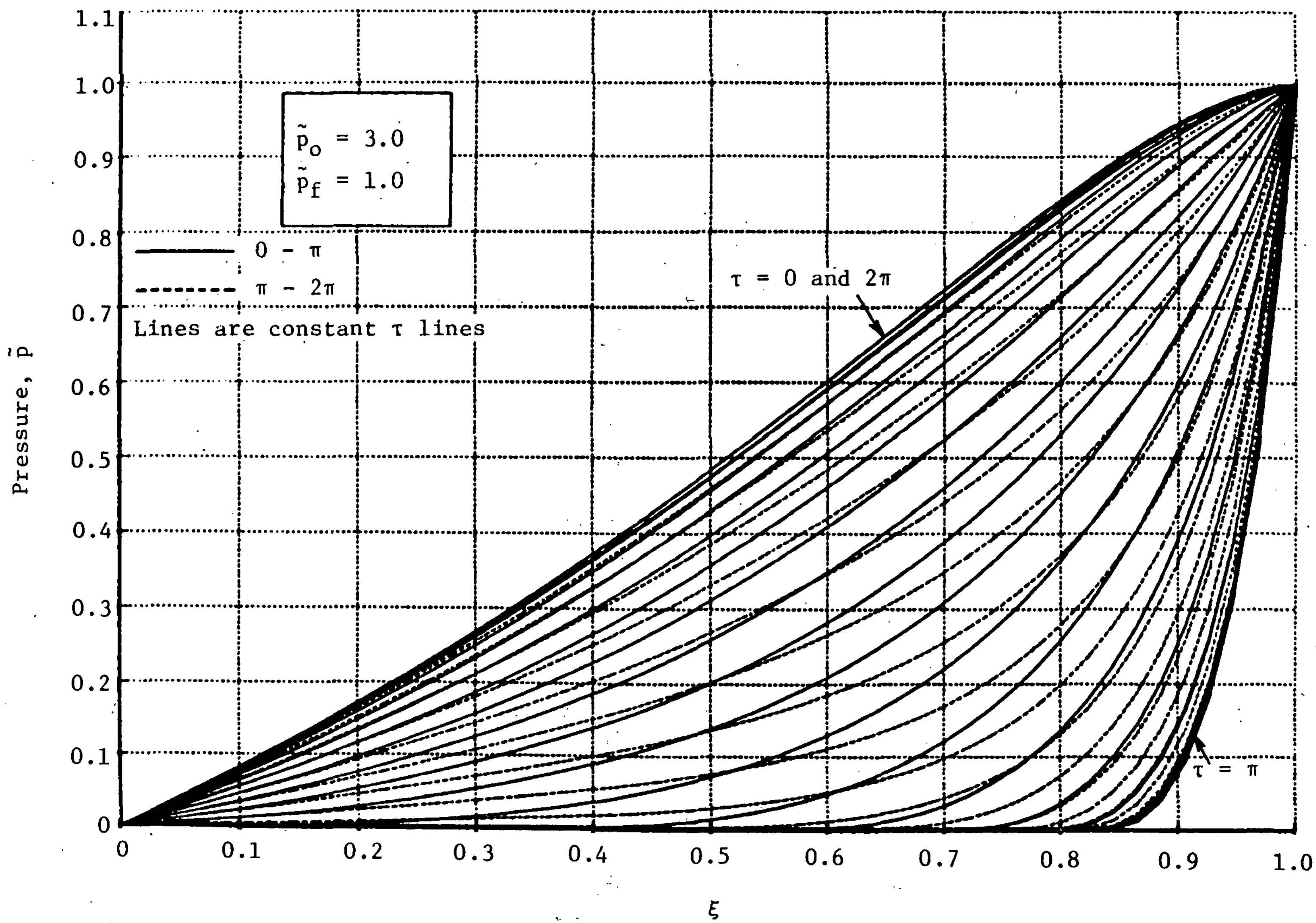


Fig. 3-12 Pressure Profiles as Function of Time for Transient Analysis

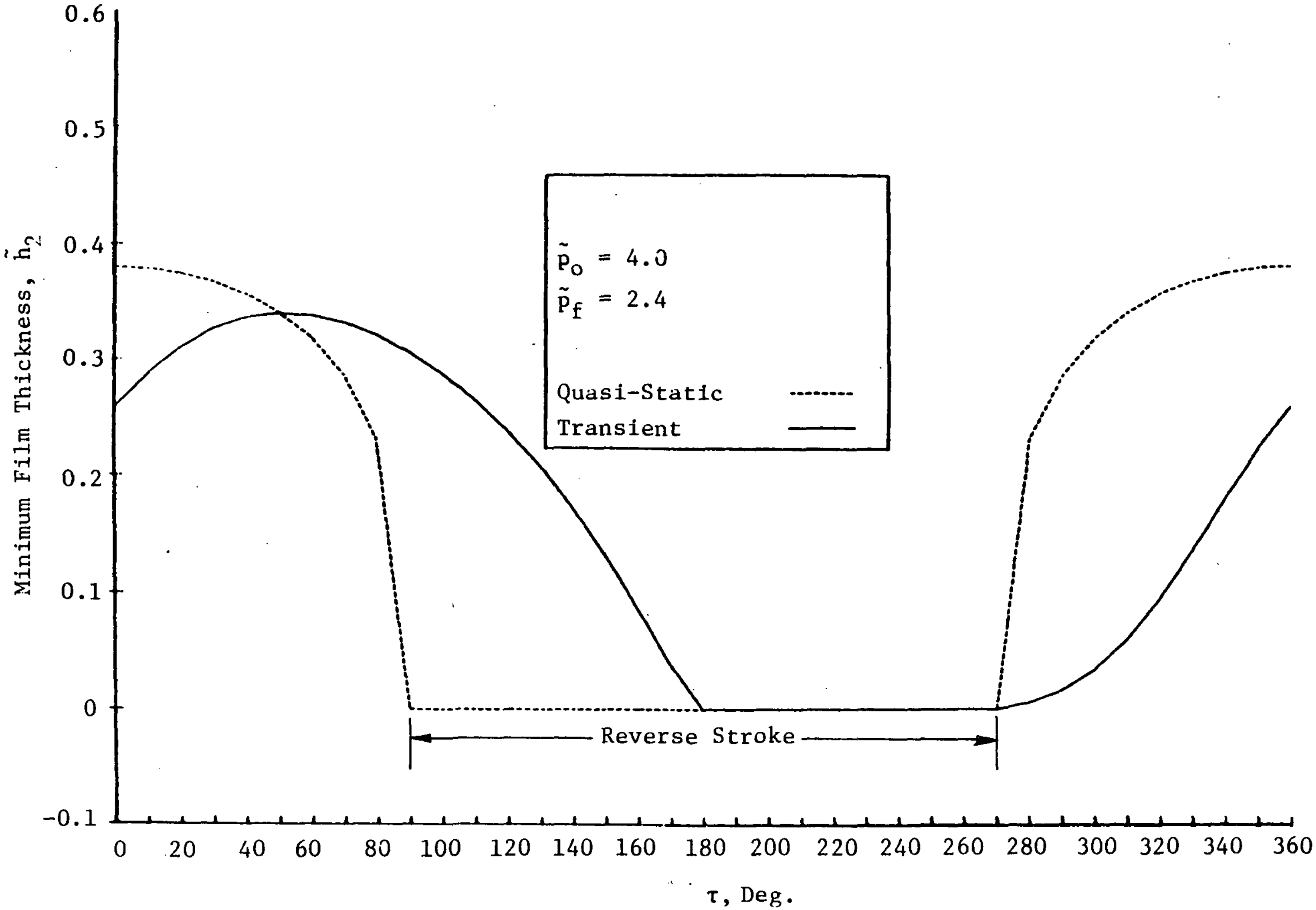


Fig. 3-13 Film Thickness $\tilde{h}_2$ Versus Time

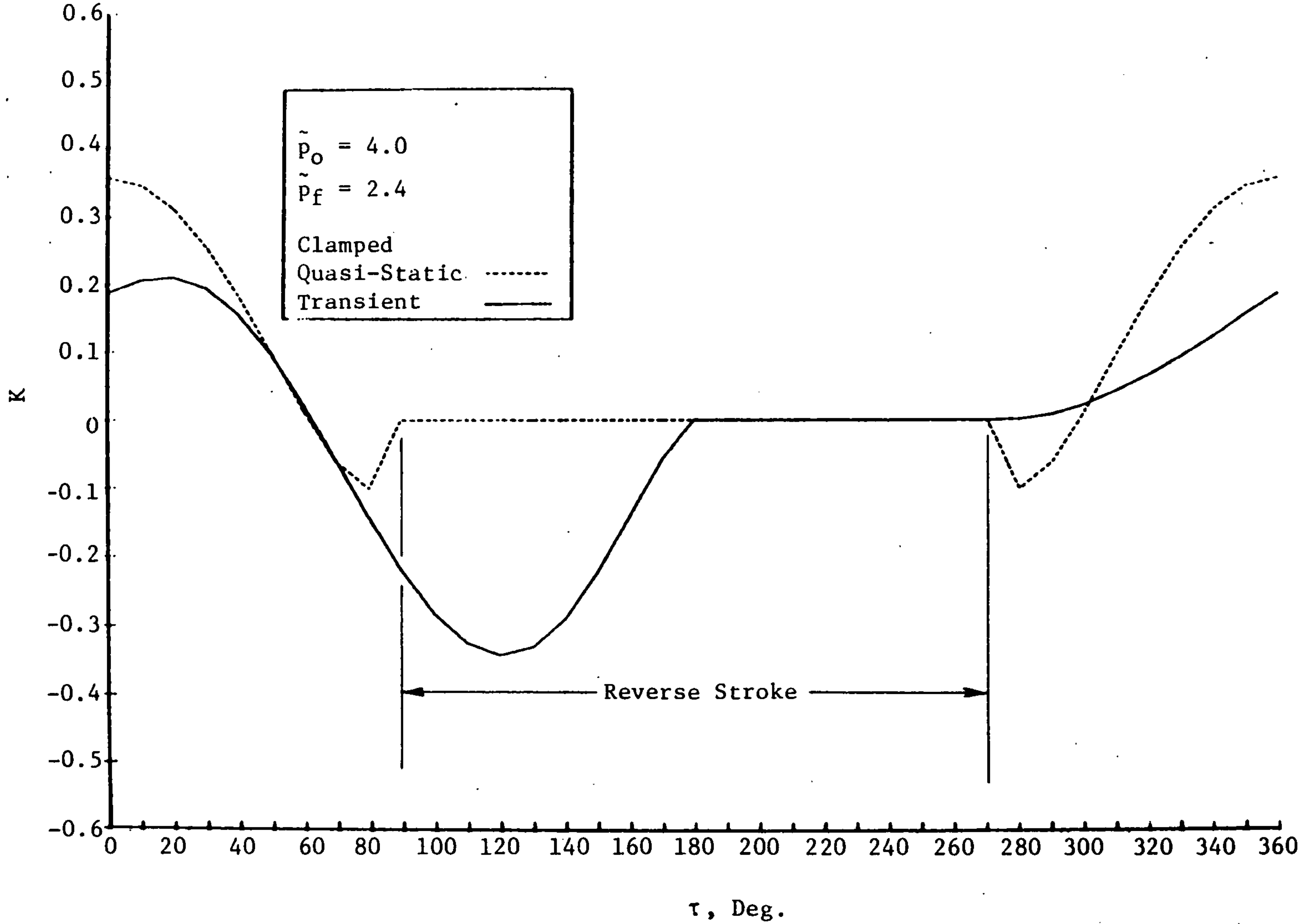


Fig. 3-14 Flow Parameters Versus Time

TABLE 3-1

EFFECTS OF VARIABLE VELOCITY AND SQUEEZE FILM ON FLOW

$\alpha = 0.0282$ $\beta = 0.2569$ $\varepsilon = 0.518$ $\tilde{L}_1 = 1.5614$

$\sigma = 4.0$

$\tilde{p}_o$	$\tilde{p}_f$	Analysis*	FORWARD		REVERSE		NET	
			$\tilde{Q}$	$\Delta\tilde{Q}$, %	$\tilde{Q}$	$\Delta\tilde{Q}$, %	$\tilde{Q}$	$\Delta\tilde{Q}$, %
3.0	0	S	0.4542	Ref.	0.1816	Ref.	0.1363	Ref
		QS	0.4687	+ 3.2	0.1811	0.28	0.1438	+ 5.5
		T	0.4231	- 6.9	0.1731	4.7	0.1250	- 8.3
3.0	1.0	S	0.3392	Ref.	0.4080	Ref.	- 0.0344	Ref.
		QS	0.3524	+ 3.9	0.4059	- 0.8	- 0.0268	|22|
		T	0.3020	- 11.0	0.4110	+ 0.7	- 0.0545	|58|
4.0	2.4	S	0.0948	Ref.	0	--	0.0474	Ref.
		QS	0.1348	+ 42	0	--	0.0677	+ 43
		T	0.0711	+ 25	0.1165	--	0.0227	- 148

* S - Static QS - Quasi-static T - Transient

- Variable velocity has the effect of increasing the absolute values of all flows; that is, even though it boosts both the forward and back flows, the net flow is increased. Table 3-1 shows this increase to be of the order of several percent for the case of $p_f = 0$. The high percentage changes shown for other cases should not be given too much signficance since they center about very low flow levels when a small variation is apt to produce deceptively large effects in terms of percentages.

- The pressures are, of course, positive over the entire ring during the forward stroke, as shown by the $0-\pi$ range of the constant lines on Figure 3-11. Their magnitude, too, follows the velocity curve. During the backstroke, $\pi/2 < \tau < 3\pi/2$, cavitation sets in as high negative velocities are approached ($\tau \rightarrow \pi$); they tend, however, to disappear at the beginning and end of the backstroke, when the negative velocities are low.

The inclusion of squeeze-film effects have the following effect:

- The largest film thickness does not occur at u_{max}. The peak is delayed so that it is reached after u_{max}. Likewise, the onset of constant h during the reverse stroke does not commence at $u = 0$, but is also delayed. Thus, squeeze films have the effect of producing a phase shift in the film thickness curve. The shift is fairly large, varying from 50° to 90° in the three cases considered. However, the absolute values of h seem not to be affected.

- The effect on the flow curves is similar in that there is a phase shift with little change in their magnitude. Both the forward and net flows are reduced in comparison to either the steady-state or quasi-static solutions; the amount of back flow tends to increase under the influence of squeeze film forces. Since the quasi-static solution gives higher flows and the static solution gives lower flow than the constant parameter approach, the inclusion of variable velocity without squeeze film effects would yield errors of at least 10%.

- Unlike the behavior in the quasi-static solution, the pressure curves are not identical over the 0-π and π-2 halves of the cycle. The pressures are lower when squeeze film effects are included, except over the latter half of the backstroke, when a closing gap helps to increase the pressures above those of the quasi-static analysis.

3.3 Starvation

An estimate of the effect of starvation can be obtained by first considering the clamped case where the area under the bearing land, $0 < x < L$, is predicted to be completely dry at the end of the backstroke. The average film thickness in this area is $\Delta h/2$, and the average Couette flow per unit of circumferential length is $U_o \Delta h/4$. The time, t_s, for the starved volume to fill up would thus be the cross-sectional area $\Delta hL/2$ divided by the flow per unit of circumferential length. Thus,

$$t_s = (\Delta hL/2)/(U_o \Delta h/4)$$

The distance traveled prior to flooding is $U_o t_s = 2L$. This would reduce the effective forward stroke by an amount equal to 2L. The value of t_s given above will be somewhat low in that it does not account for the development of a resisting pressure gradient that will occur as the starved volume fills up nor does it account for any inertial effects in the entrance region that could impede the start of the filling process. In order to account for these effects when analyzing experimental data, multiply t_s by a factor λ which will in general be greater than or equal to 1. Thus,

$$s_{eff} = s - 2L\lambda$$

and the dimensionless flow rate K_{eff} becomes

$$K_{eff} = K\,(s_{eff}/s) = K\,[1 - \lambda(2L/s)] \tag{3-21}$$

The starvation process is, of course, much more complex than that treated here, especially in the unclamped case where there is a partial film and the film is exposed to the sealed pressure, p_o, so that the cavity could be trapped in the middle. In order to understand more fully the influence of starvation on pumping ring performance, transparent model experiments should be performed so that the size and behavior of the partial film can be observed and modeled. In the interim, a simple model was adopted to estimate the effect of starvation on pumping for either the clamped or cavitating, nonclamped case.

If the film cavitates during the backstroke at $\xi = \xi_c$, the film thickness at the point, h_c, is

$$h_c = h_2 + \Delta h(1 - \xi_c)$$

and the void volume (volume of gas or vapor in the cavitated region) per unit of circumferential length, V_c, would be

$$V_c = (h_2 + \Delta h - h_c)\, L\, \xi_c/2 = L\, \xi_c^{\,2}\, \Delta h/2$$

If it is assumed that the void volume must fill up before significant pumping can begin and if it fills based on Couette flow at the average film thickness of the cavitated region, the incoming flow per unit of circumferential length would be $U_o\ (h_2 + \Delta h + h_c)/4$, and the starvation time, t_s, would be V_c divided by that flow

$$t_s = (2L\xi_c^{\,2}\Delta h)/[U_o(h_2 + \Delta h + h_c)]$$

As for the clamped case, the effective forward stroke would be reduced by λU_o t_s. If the dimensionless flow in the forward stroke is denoted by K_f and the backstroke by K_R, then

$$K_{f\ eff} = K_f(s - \lambda U_o t_s)/s$$

$$= K_f \left[1 - 2\lambda \left(\frac{L}{s} \right) \frac{\xi^2 \Delta h}{h_2 + \Delta h + h_c} \right]$$

$$= K_f \left[1 - 2\lambda \left(\frac{L}{s} \right) \frac{\xi^2 \Delta h}{2h_2 + \Delta h\ (2 - \xi_c)} \right]$$

and

$$K_{eff} = K_{f\ eff} - K_R \qquad (3\text{-}22)$$

For the clamped condition, $K_R = 0$, $\xi_c = 1$ and $\tilde{h}_2 = 0$; thus Equation (3-22) reduces to Equation (3-21).

In order to show the effects of starvation, the data for a clamped carbon graphite ring is given in Figure 3-15. The dotted curves include the effects of starvation for a value of $\lambda = 1$. It can be seen that, although the inclusion of the effects of starvation reduces the discrepancy between theory and experiment, it does not resolve the lack of agreement. The effect of values of $\lambda > 1$ will be shown later when comparing theory to experiment.

As a result of the uncertainty regarding the mechanisms of the starvation process, the effects of starvation are not included in the parametric studies in Section 4.0. It is recommended, however, that Equation (3-22) be used to estimate potential effects of starvation when designing pumping rings. The dimensionless starved net flow is given as an output to the computer program RING listed in Appendix B of this report.

3.4 Nonparallel Contours

Essentially, the contours considered are tapered surfaces with a constant slope as shown in Figure 3-16. The reference clearance is that at the trailing edge of the ring; the leading edge clearance is designated by C_M. The slope parameter, δ, is then given by

$$\delta = (dh/dx) = (C - C_M)/L \qquad (3\text{-}23)$$

In terms of the nondimensional quantities $\tilde{h}$ and ξ, the slope is given by

$$\tilde{\delta} = (d\tilde{h}/d\xi) = (C - C_M)/C = (1 - \tilde{C}_M) \qquad (3\text{-}24)$$

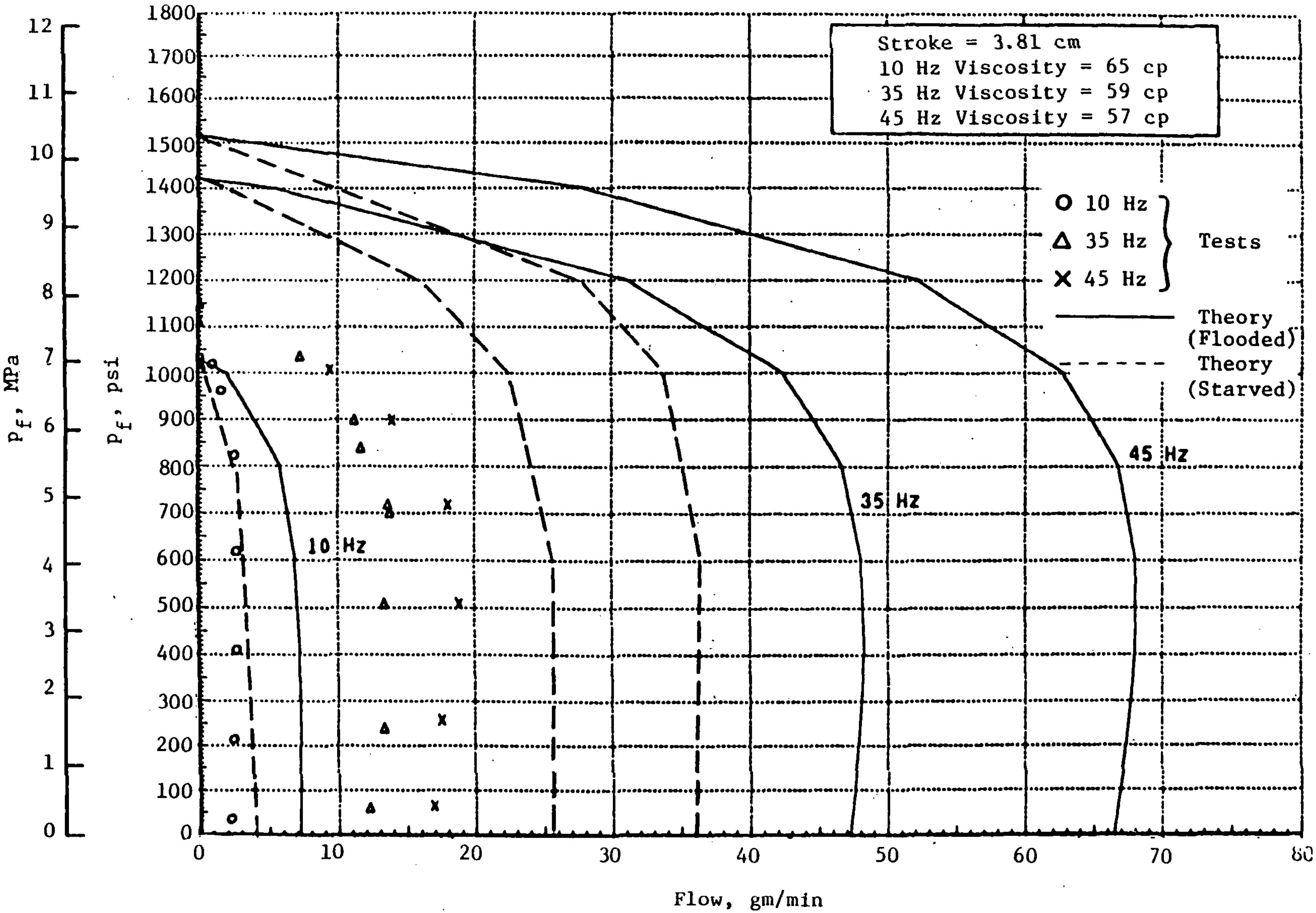
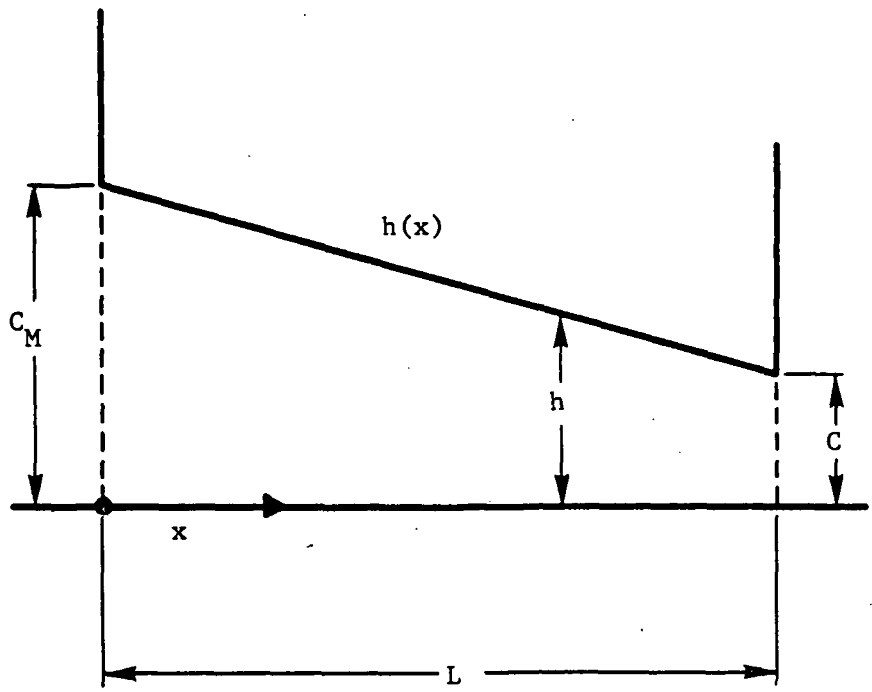


Fig. 3-15 Effect of Starvation on Performance of Carbon Pumping Ring



$$\delta = \frac{C - C_M}{L}$$

$$\tilde{C}_M = (C_M/C)$$

Fig. 3-16 Geometric Taper on Pumping Ring

The clearance ratio $\tilde{C}_M = (C_M/C)$ in terms of this δ is then given by:

$$\tilde{C}_M = 1 - L\delta/C \tag{3-25}$$

The effect of using tapers on the performance of pumping rings is discussed in Section 5.0. Briefly, it can be said that the use of tapers has little effect, except in cases of low clamping load, p_o, or high values of E, i.e., when there is little elastic deflection of the ring. Clearly, when there is no deflection at all, a taper would constitute the sole mechanism of generating hydrodynamic pressures. In practical applications where deflections are present, the effect of tapers is minimal.

4.0 PARAMETRIC STUDY

A study was made regarding the impact of various structural and operational parameters on the performance of the pumping rings. These parameters include:

- Modulus of Elasticity (E)
- Poisson's Ratio (ν)
- Clearance (C)
- Length (L & L_1)
- Ring Thickness (t)
- Taper (δ)
- Loading (p_o and e).

The computer runs that provide the results of the parametric study are based on the constant-parameter approach without starvation but including the effects of the backstroke and attendant cavitation. Thermal and transient effects were omitted because, as shown on Figure 4-1, their effect is not of the sort as to qualitatively change the behavior of the pumping ring. The constant parameter analysis should be sufficient to reveal the essential features of the parametric relationships without the undue complications of the more elaborate analysis.

Since it would ultimately be desirable to optimize the design of pumping rings, the question arises as to what constitutes an optimum. Given the purposes for which these devices are customarily used, an optimum ring was deemed to be one which, within given constraints such as reasonable values of p_o, C, etc., can maintain the highest possible sealing pressure without excessive wear. This implies the highest values of p_f and a low, preferably zero, frictional force F. Thus, a ring design which generates high p_f and just clamps shut upon reversing the stroke would fulfill this criterion. An additional quantity of interest, perhaps, is also the maximum flow Q_o (flow at $p_f = 0$), that such a ring can produce. Thus, in the plots that follow, the items of p_f, F, and Q_o will be the items against which ring performance will be measured.

The parametric study was conducted by first establishing a standard or reference design and then by varying its parameters, one at a time, to values below

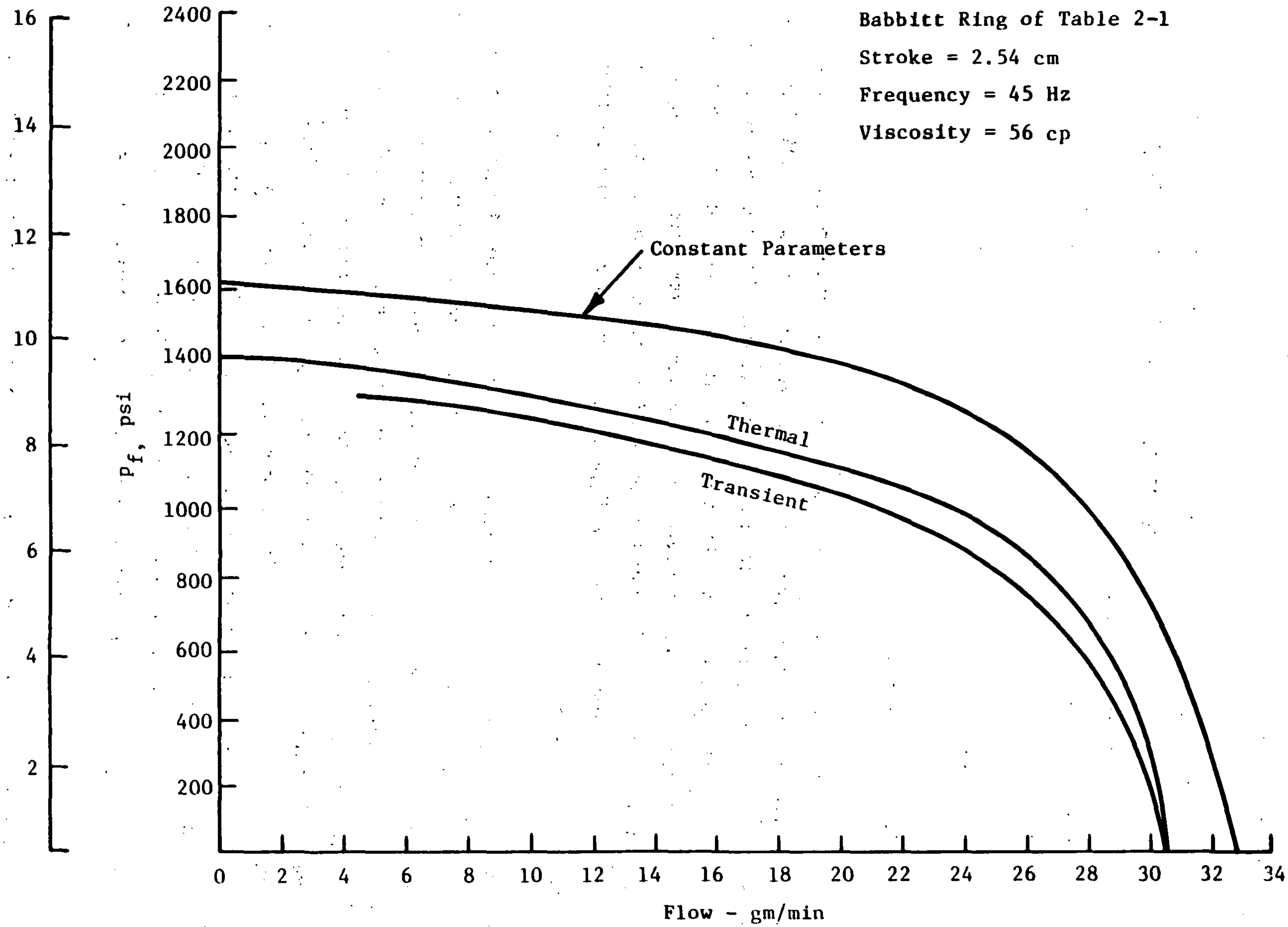


Fig. 4-1 Modifications Due to Thermal and Transient Effects

and above the reference quantities. The parameters for this standard ring are given in Table 4-1 along with selected departures from the standard set. Including the standard, most parameters have four computed points from which optimization plots can be made. The results of the parametric study are summarized in Table 4-2; more relevant plots are portrayed in Figures 4-2 through 4-11. In Figure 4-4, the reason the film thickness h_2 drops with an increase in clearance, is due to the drop in p_{fm}; a high p_{fm} usually helps to contract the clamping pressure and thus maintain a high h_2. In Figure 4-6, the explanation for an increase in flow with a rise in length L is that with a greater L the hydrodynamic effects are increased, and thus, also the flow. As seen, four parameters, the lengths L and L_1, taper δ, and Poisson's ratio have little effect on performance. Of the two important variables, namely, clearance and clamping force, the first has to be raised above its optimum for maximum p_f, and the latter reduced below the optimum, if large frictional forces are to be avoided. Since the clamping force is given by the product of p_o and e, Figure 4-12 shows the effects of using the same clamping force, 31.96 KN/m (182.5 lb per in.) of circumference, but by varying the relative values of p_o and e. As seen, higher values of p_f are achieved when p_o is high, while higher values of Q_o are achieved when e is high. The differences, however, are not striking.

It should be pointed out that the particular optima recorded in Table 4-2, such as t = 2.54 mm (0.1 in.), C = 25.4 microns (1 mil), etc., are valid in the range of parameters characterizing the particular ring specified in Table 4-1. Thus, for example, for a material with a much higher E value, optimum results would be achieved with lower values of clearance and ring thickness. The opposite would be true for a material with a value of E much below the 34.5 GPa (5×10^6 psi) assigned to the standard design.

TABLE 4-1

STANDARD DESIGN FOR PARAMETRIC STUDY

p_o = 10.3 MPa (1500 psi)

e = 3.175 mm (0.125 in.)

T = 49°C (120°F), $\mu = 59 \times 10^{-3}$ Pa-sec, (8.6×10^{-6} Reyns)

E = $3.45 \cdot 10^4$ MPa ($5 \cdot 10^6$ psi)

ν = 0.36

s = 12.7 mm (0.5 in.)

f = 35 Hz

R = 12.7 mm (0.5 in.)

L = 6.35 mm (0.25 in.)

L_1 = 10.2 mm (0.4 in.)

C = 0.019 mm (0.75×10^{-3} in.)

t = 1.9 mm (0.075 in.)

δ = 0

VARIATIONS IN PARAMETERS

C: 0.0063 mm, 0.0127 mm, 0.0381 mm (0.25×10^{-3} in., 0.5×10^{-3} in., 1.5×10^{-3} in.)

E: $6.895 \cdot 10^3$ MPa, $20.7 \cdot 10^3$ MPa, $68.95 \cdot 10^3$ MPa, $207 \cdot 10^3$ MPa (10^6 psi, $3 \cdot 10^6$ psi, $10 \cdot 10^6$ psi, $30 \cdot 10^6$ psi)

L: 5.1 mm, 7.6 mm, 10.2 mm (0.2 in., 0.3 in., 0.4 in.)

p_o: 5.17 MPa, 6.895 MPa, 13.79 MPa (750 psi, 1,000 psi, 2,000 psi)

ν: 0.25, 0.5

t: 1.27 mm, 2.52 mm, 3.81 mm (0.05 in., 0.1 in., 0.15 in.)

L 6.35 mm, 15.2 mm (0.25 in, 0.6 in.)

δ: -10^{-3}, $-2 \cdot 10^{-3}$, $-3 \cdot 10^{-3}$

TABLE 4-2

OPTIMUM PARAMETERS FOR STANDARD RING

(See Table 4-1)

ITEM		RANGE EVALUATED	For p_{fm} SUBJECT TO F=0	FOR Q_o
E	psi 10^{-6}	1 - 30	5.5	6
	MPa 10^{-3}	6.89 - 207	41.3	41.3
ν		0.25 - 0.5	No Effect	No Effect
C	mils	0.25 - 1.5	1.0	1.0
	microns	6.35 - 38.1	25.4	25.4
L	in	0.2 - 0.4	No Effect	Highest
	mm	5.08 - 10.16		
L_1	in	0.25 - 0.6	No Effect	No Effect
	mm	6.35 - 15.2		
t	in	0.05 - 0.15	0.100	0.102
	mm	1.27 - 3.81	2.54	2.64
δ		0 - $(-3 \cdot 10^{-3})$	No Effect	0
p_o*	psi	750 - 2,000	1100	1050
	MPa	5.17 - 13.8	7.5	7.20

*See Also Fig. 4-12.

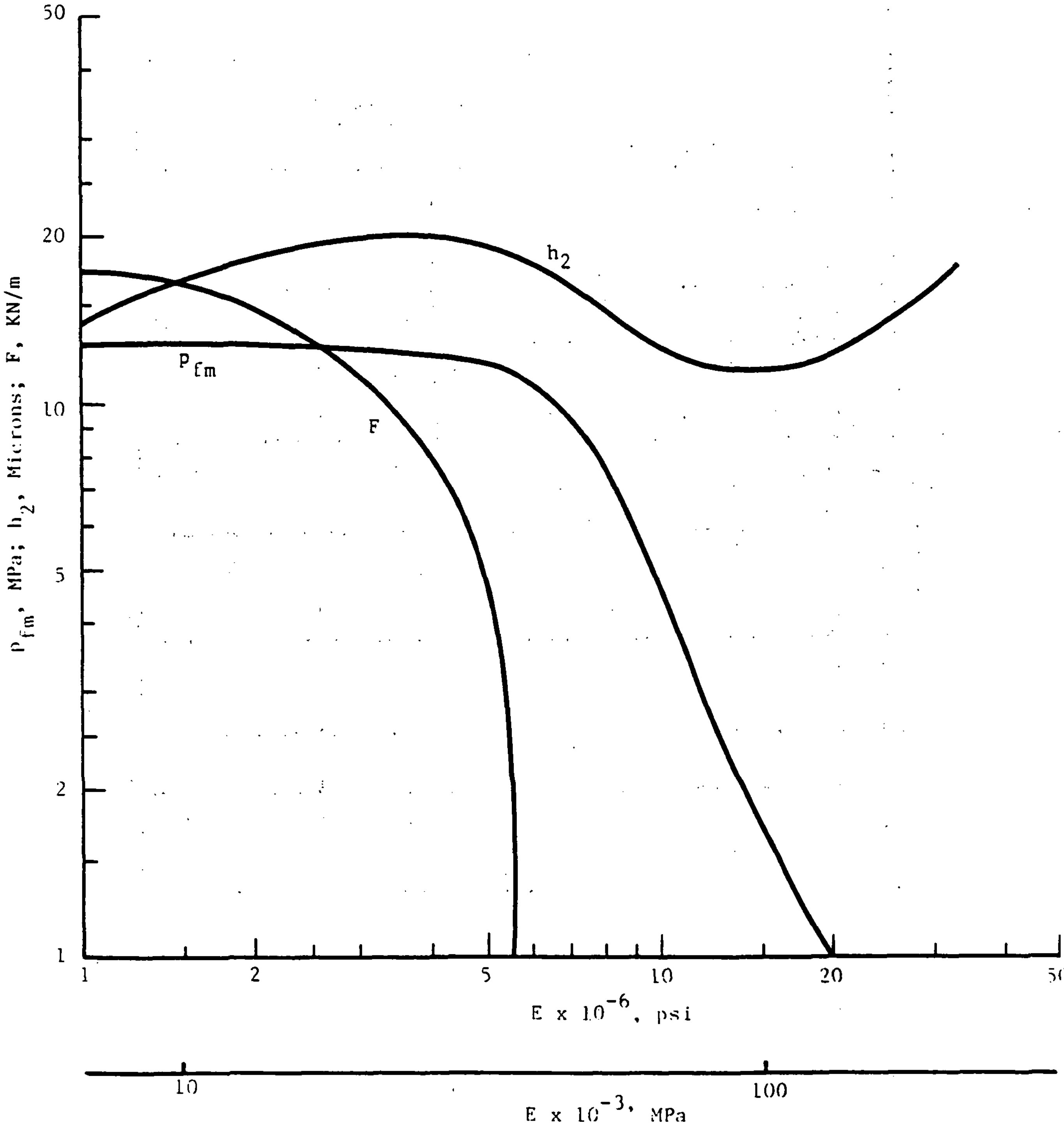


Fig. 4-2 Effect of E on Performance Standard Ring (See Table 3-1)

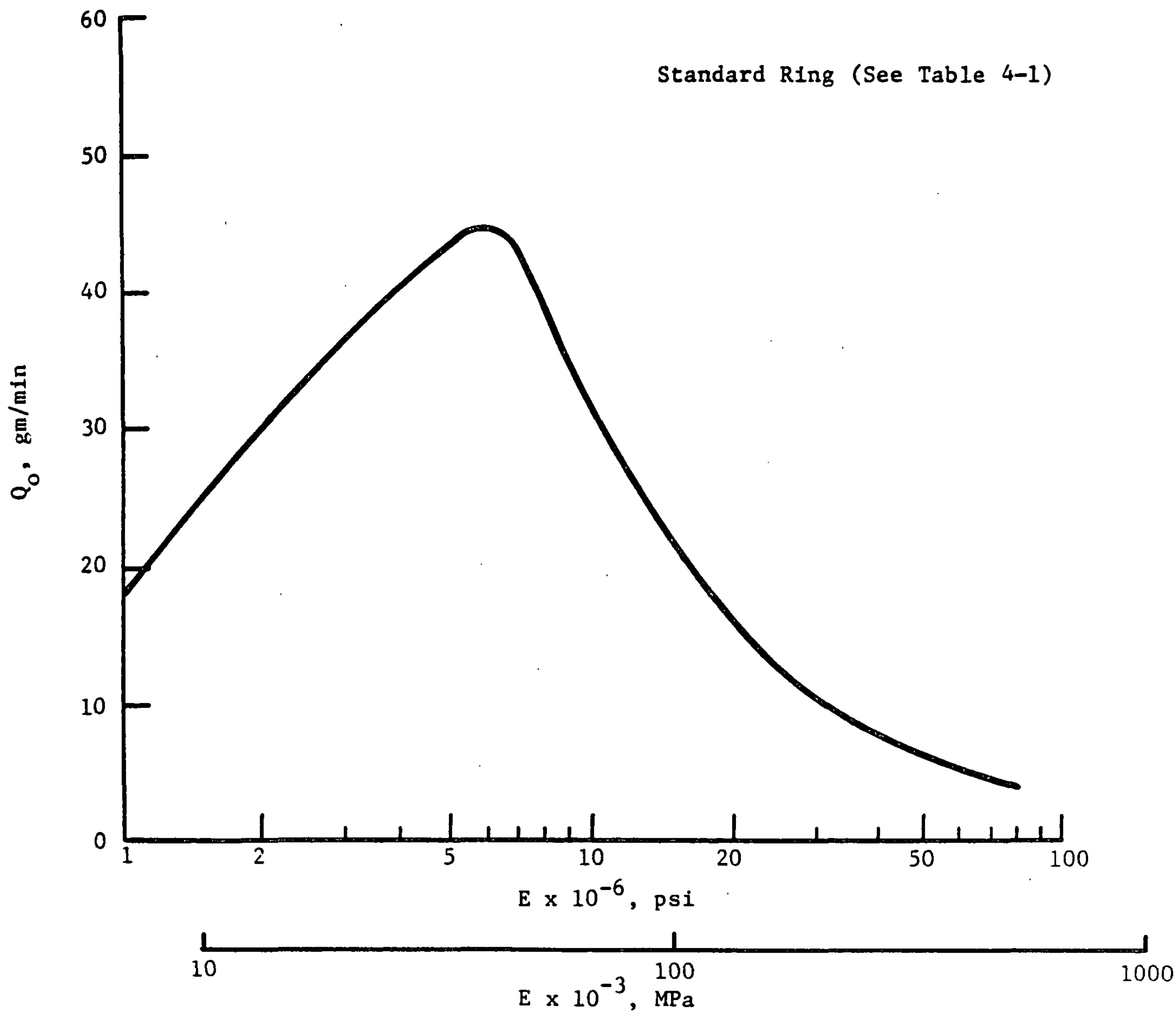


Fig. 4-3 Effect of E on Q_o

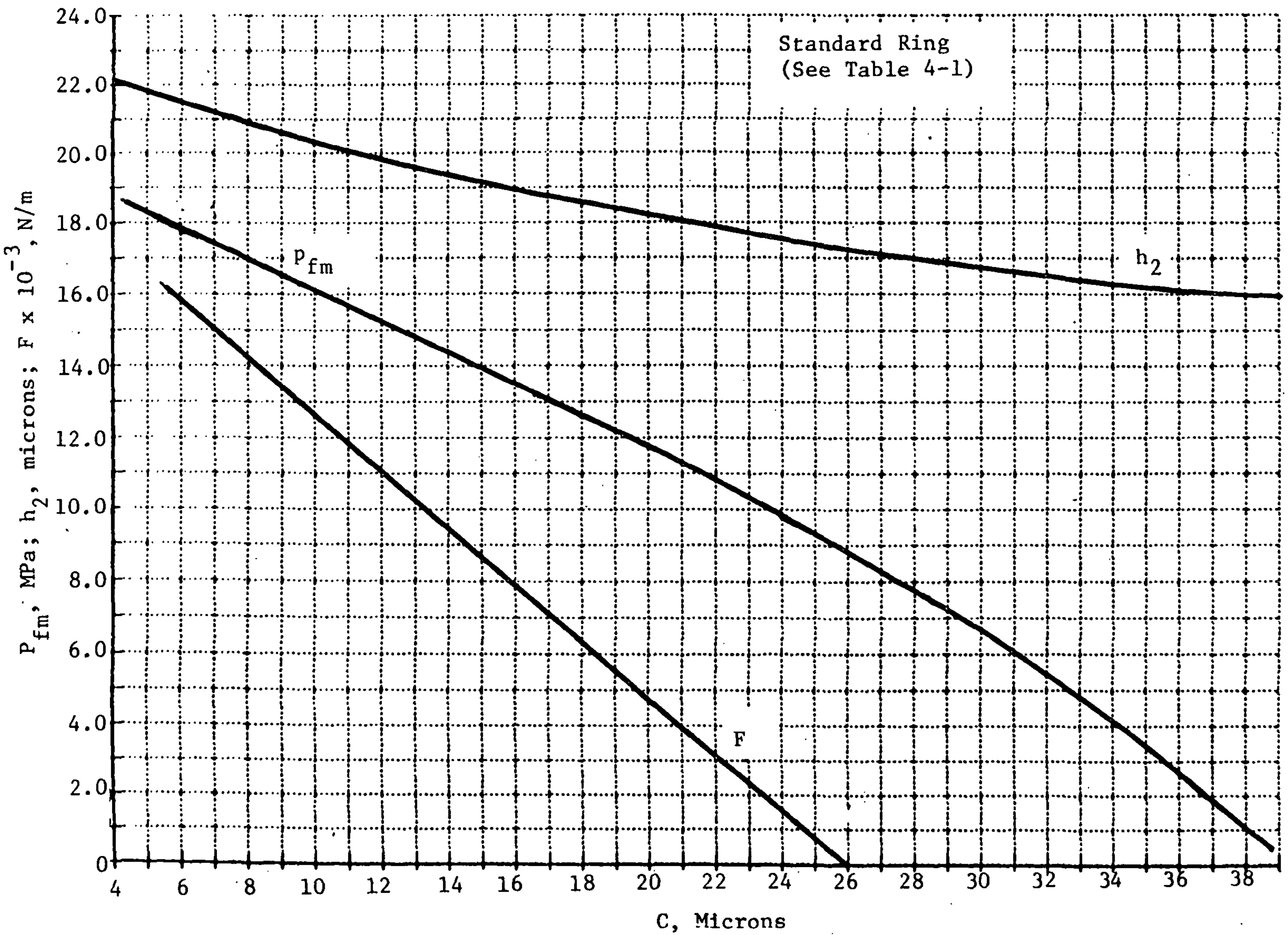


Fig. 4-4 Effect of C on Performance

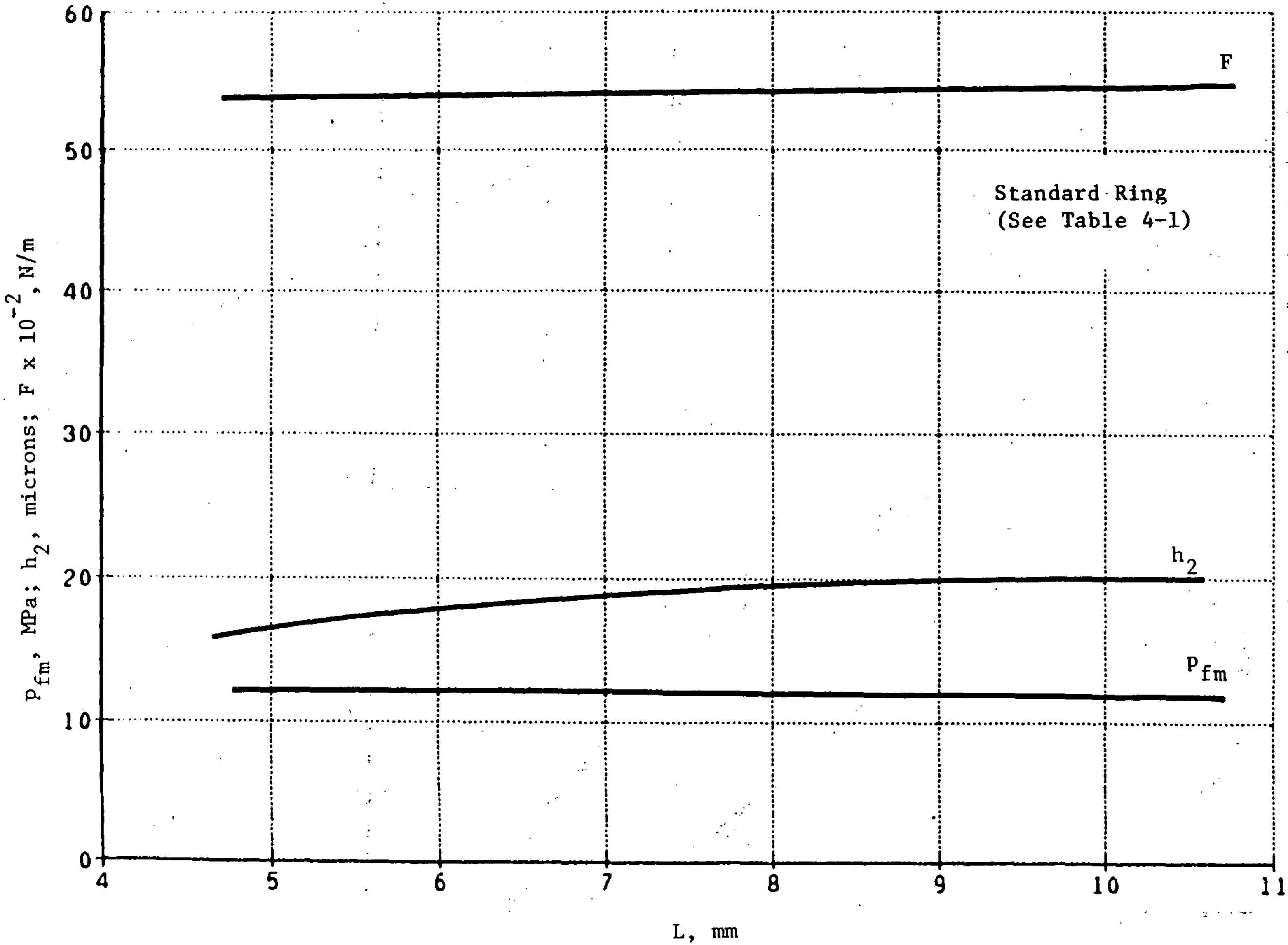


Fig. 4-5 Effect of L on Performance

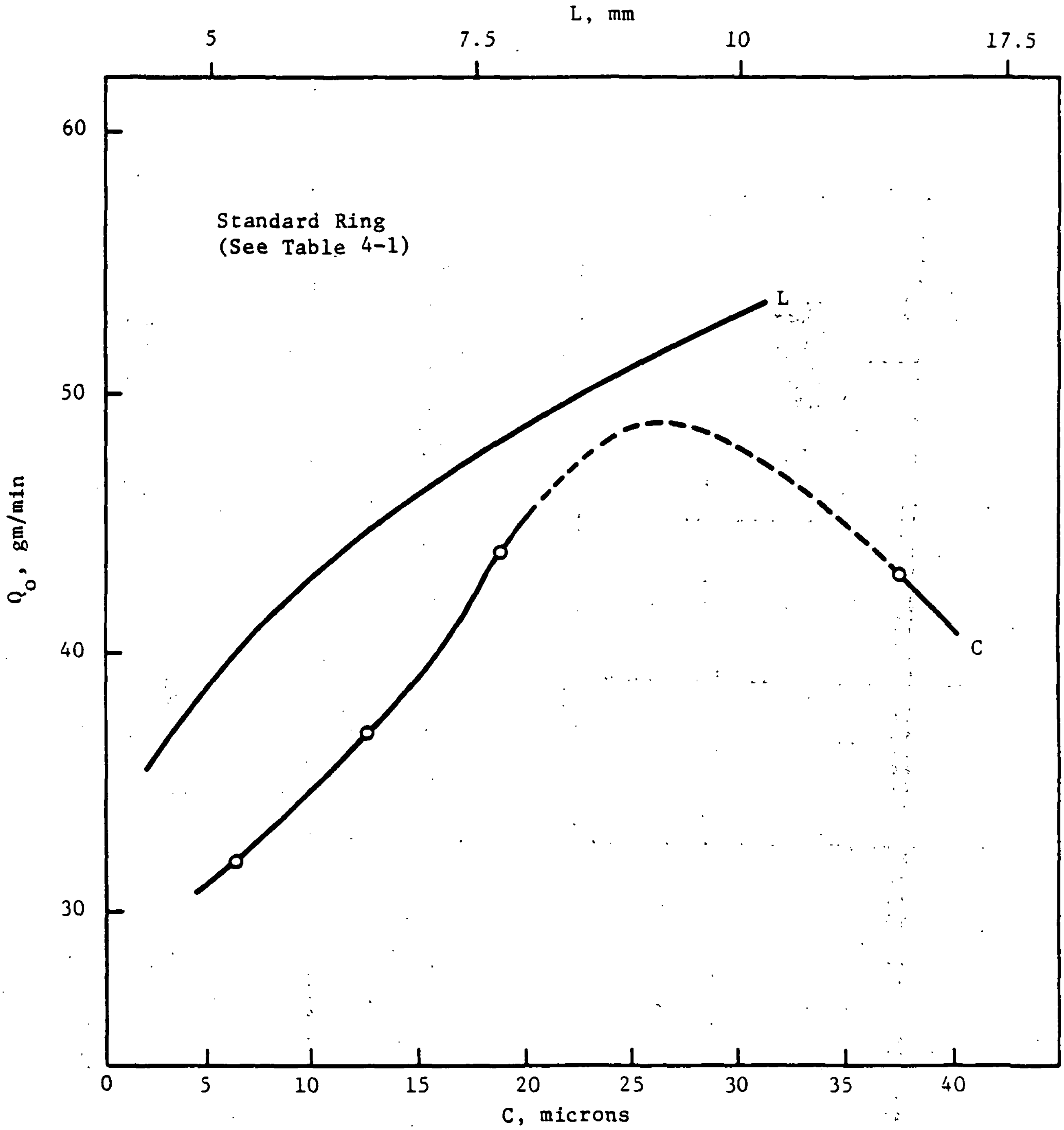


Fig. 4-6 Effects of C and L on Q_o

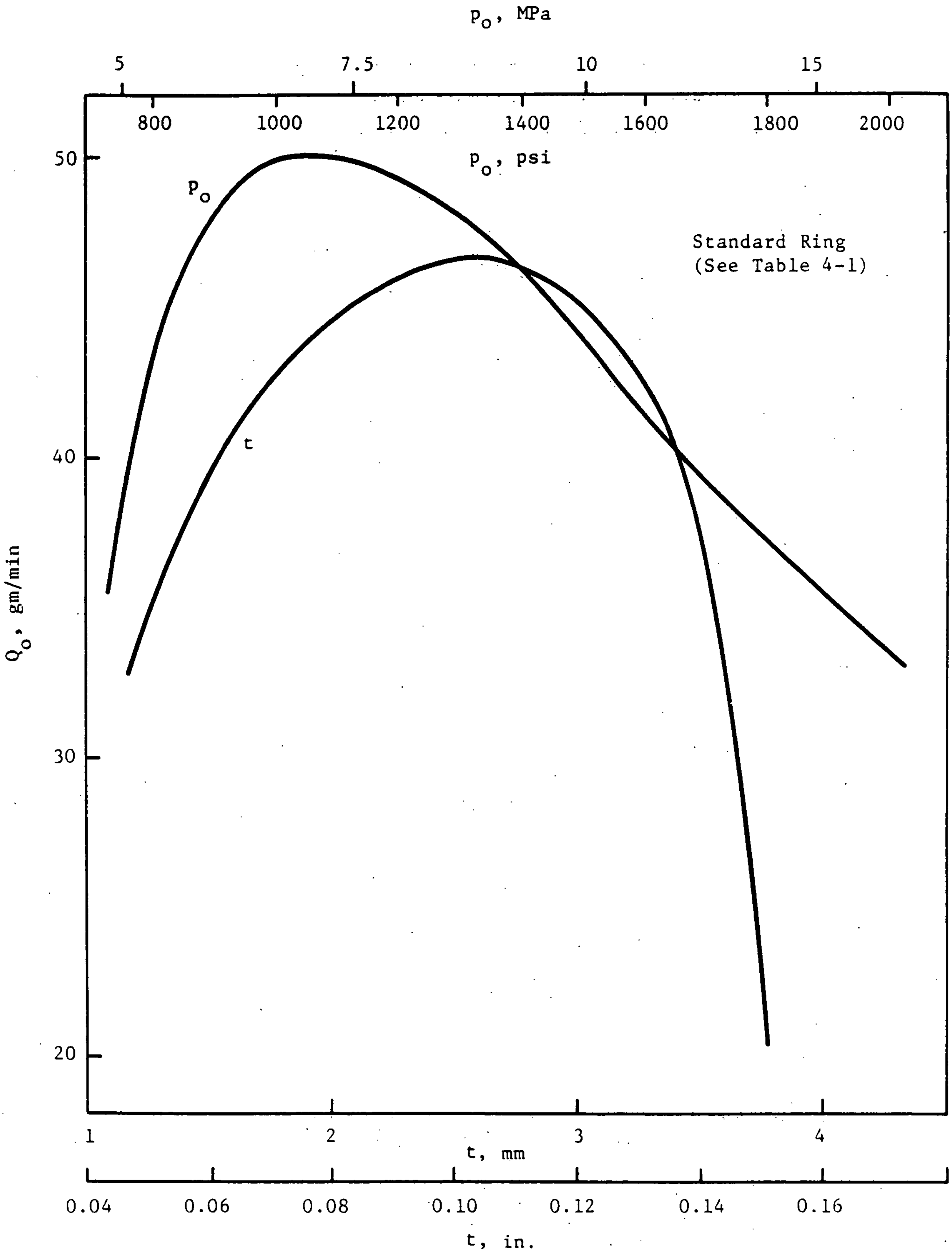


Fig. 4-7 Effects of t and p_o on Q_o

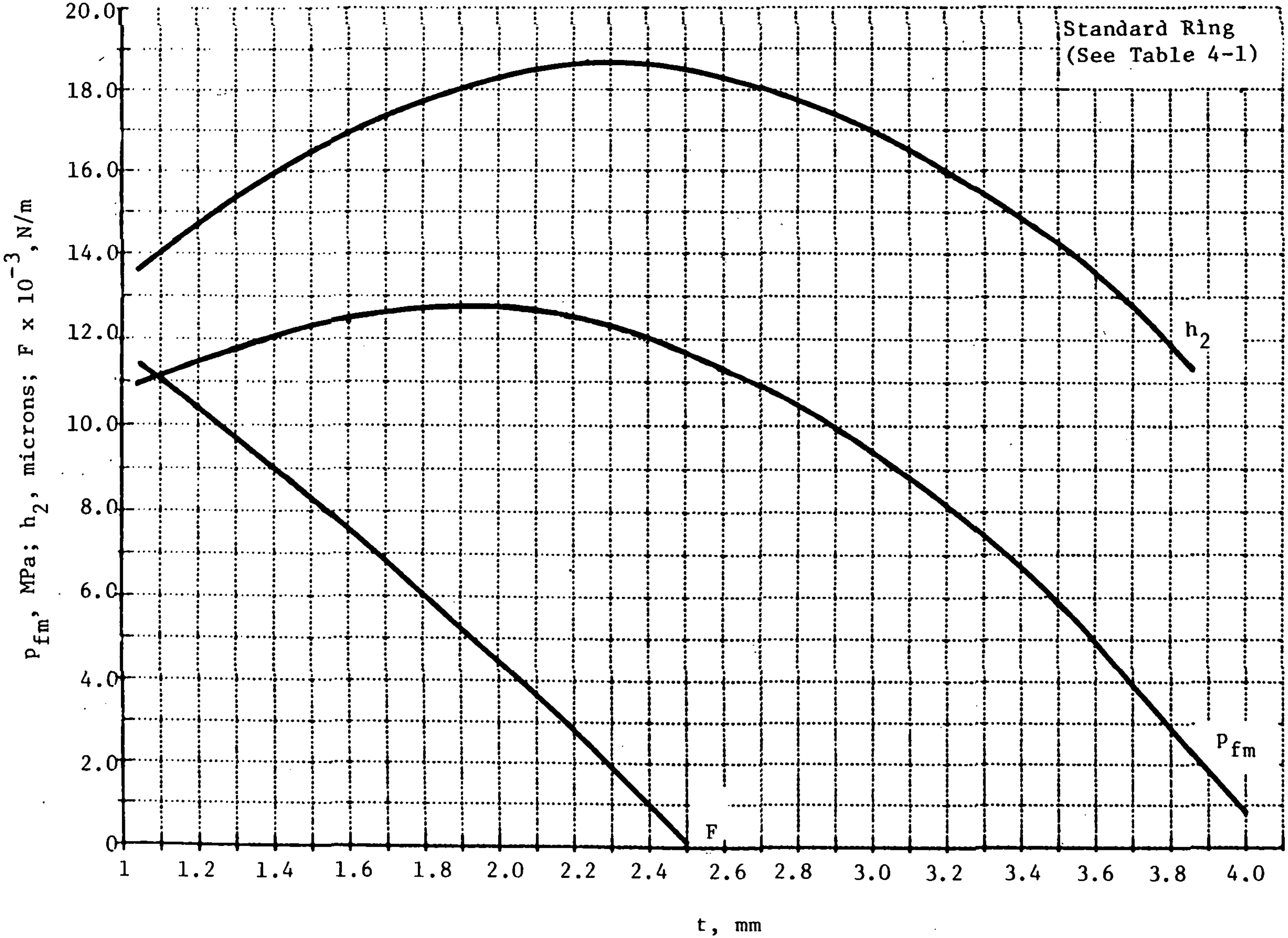


Fig. 4-8 Effect of t on Performance

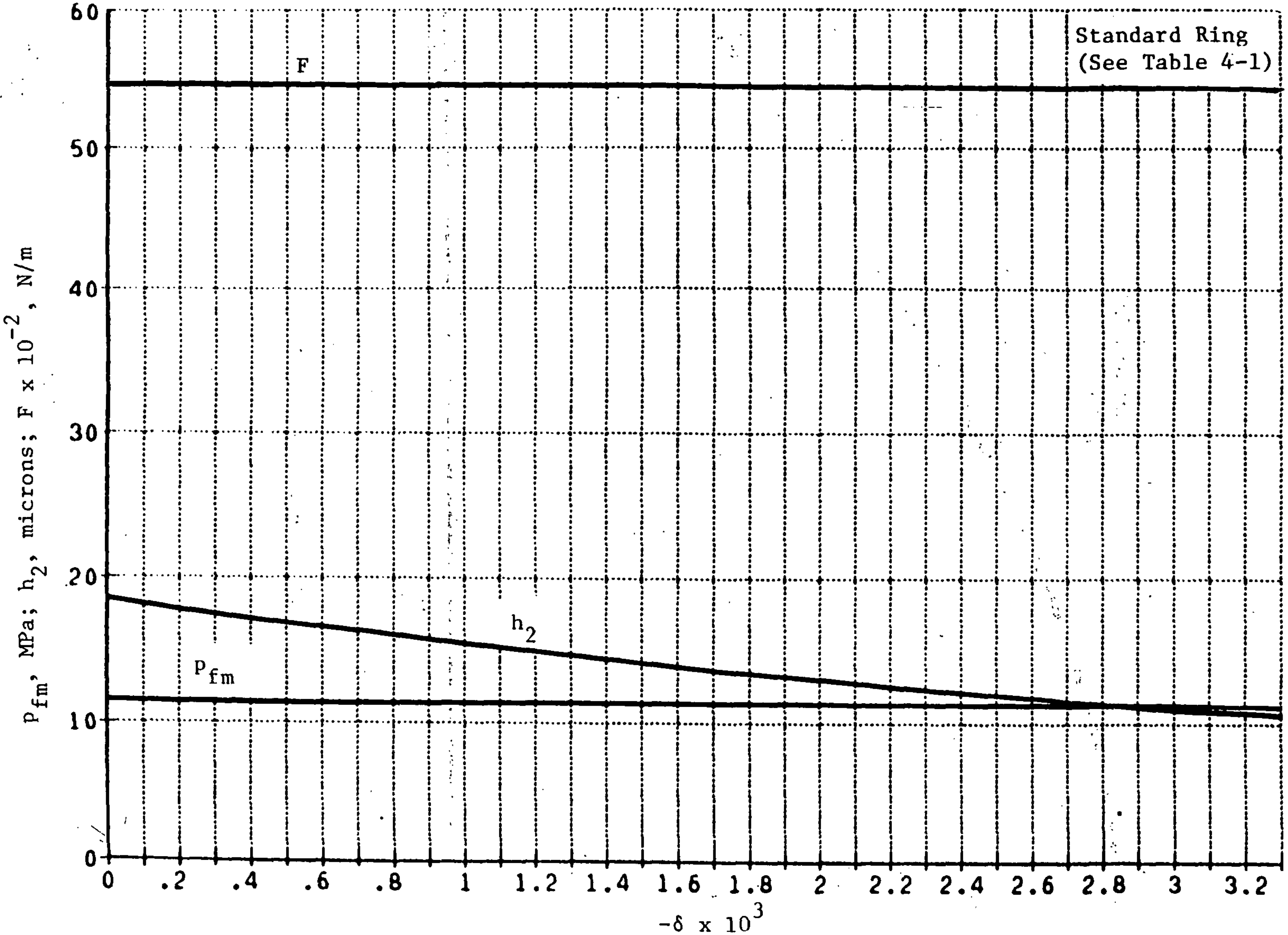


Fig. 4-9 Effects of Taper on Performance

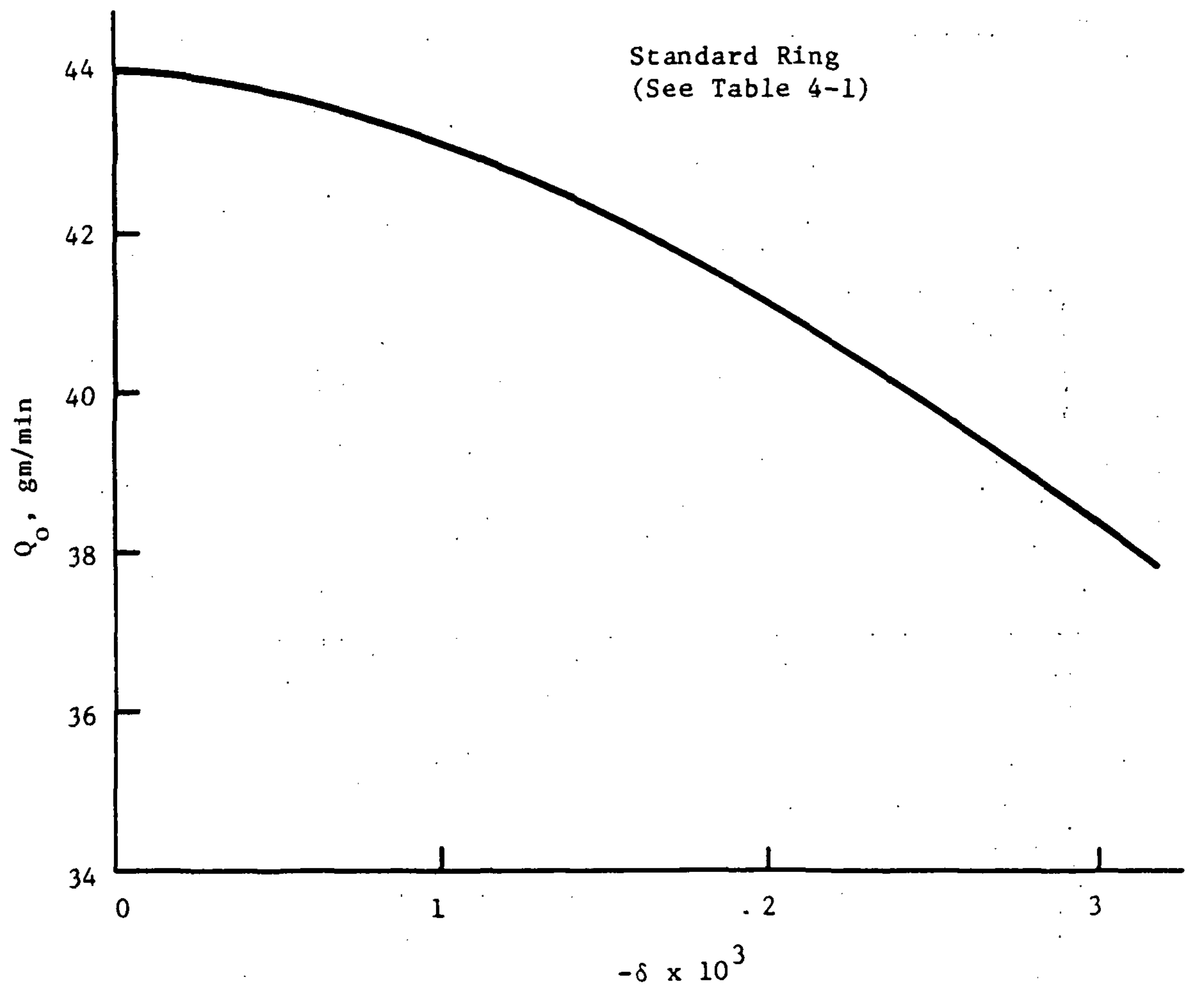


Fig. 4-10 Effect of δ on Q_o

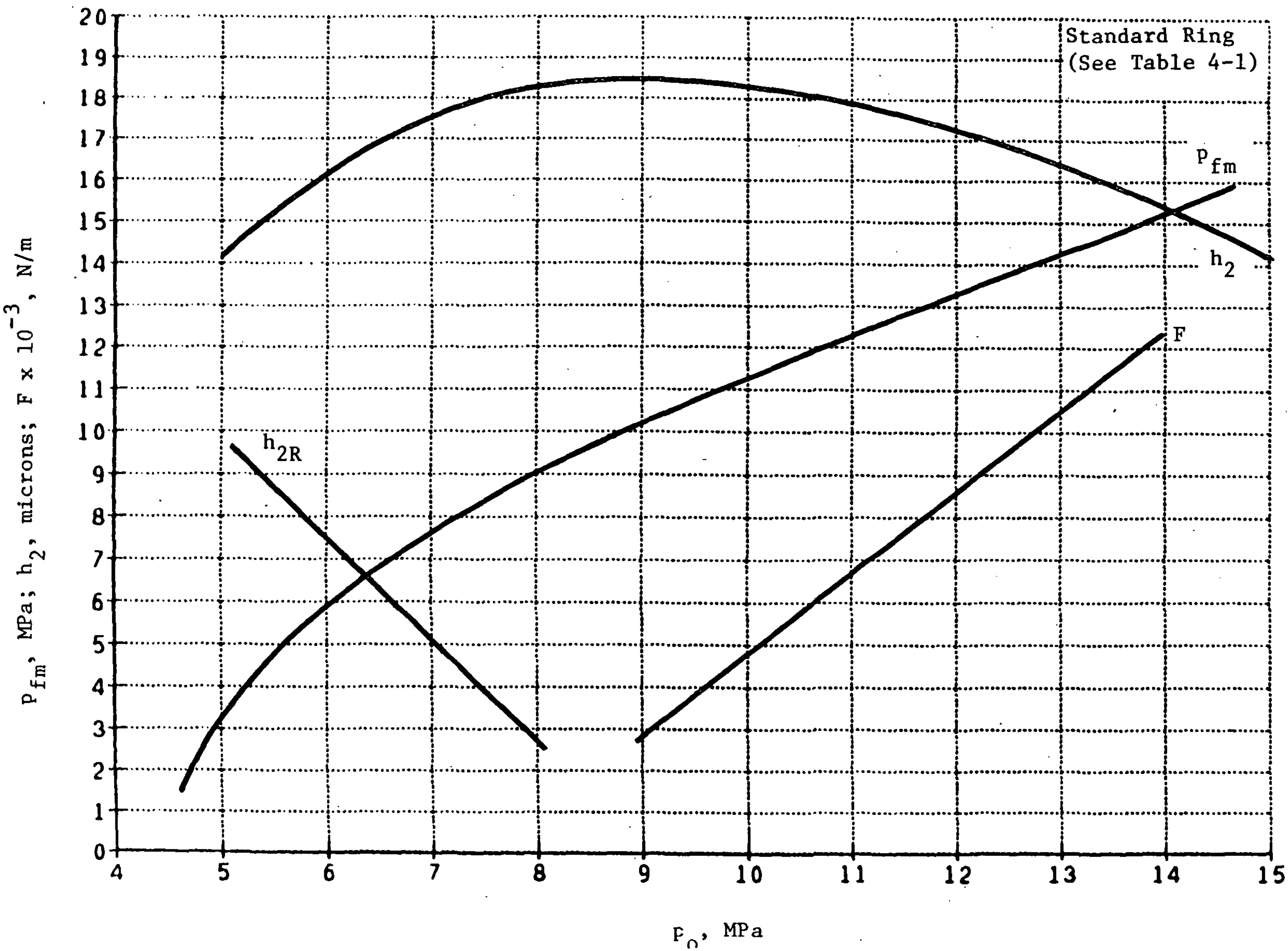


Fig. 4-11 Effect of p_o on Performance

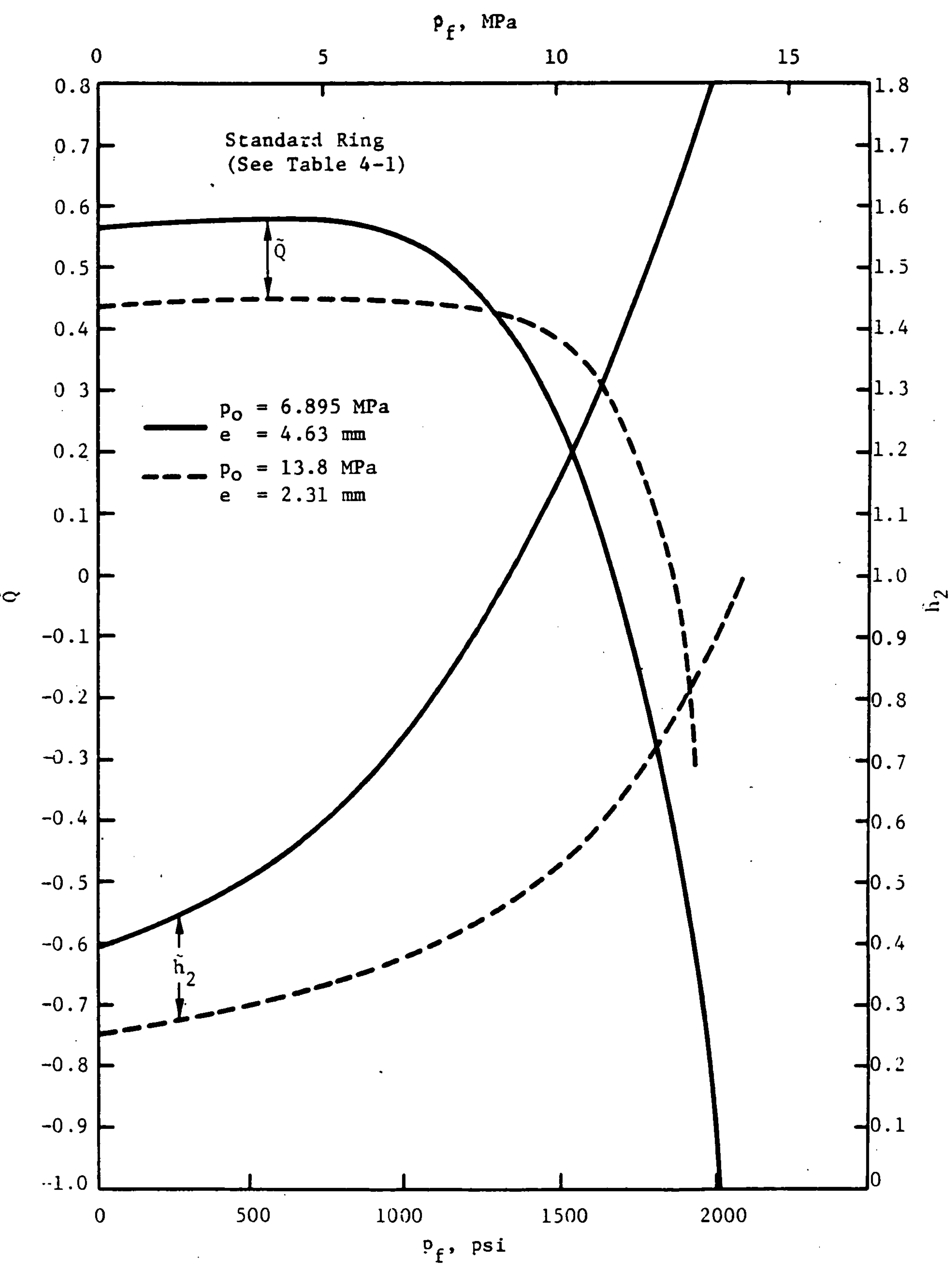


Fig. 4-12 Combined Effect of p_o and e

5.0 EXPERIMENTAL PROGRAM

Following the optimization of the candidate rings, tests were run on three rings to verify their performance and their agreement with theoretical predictions. The test rig and the experimental procedures were the same as those used on the tests described in Ref. [1].

The pumping ring program, given in Appendix B based on the analysis performed here, has been used to select ring designs for experimental testing. Initially, a set of studies was performed with steel, babbitt, and Rulon J rings as examples of a high, medium, and low modulus pumping ring. Overall dimensions were selected consistent with the experimental test rig. The principal design variables to be evaluated within the applied pressure, p_o, the clearance, c, and the average thickness, t. A target value of p_o of 1500 psi was used; however, it was found not feasible to design to this pressure with the low modulus Rulon ring. Sample geometries that have been arrived at are shown in Table 5-1. The general configuration of the pumping rings submitted for test is shown in Figure 5-1.

The principal criterion used involves being able to significantly deflect the ring to obtain pumping action without excessive clamping during the backstroke, which would result in excessive wear.

Based on the criterion above, the design of the steel ring would require relatively small thickness and a small clearance to obtain suitable compliance and deflection characteristics. The tolerances required seem excessive for the present application, and it is thought that steel rings would only be applicable in situations requiring much higher pressures. It was thus decided to confine testing to medium and low modulus materials ranging between babbitt, carbon graphite, and Rulon. The test rig and experimental procedures were the same as those used on the test.

One problem area revealed in the course of testing concerns the clamping load. Difficulties were encountered in testing the carbon and Rulon rings, and often such attempts at testing led to breakage of the specimens. It was also noted that some of the babbitt rings were severely worn, not at the downstream end

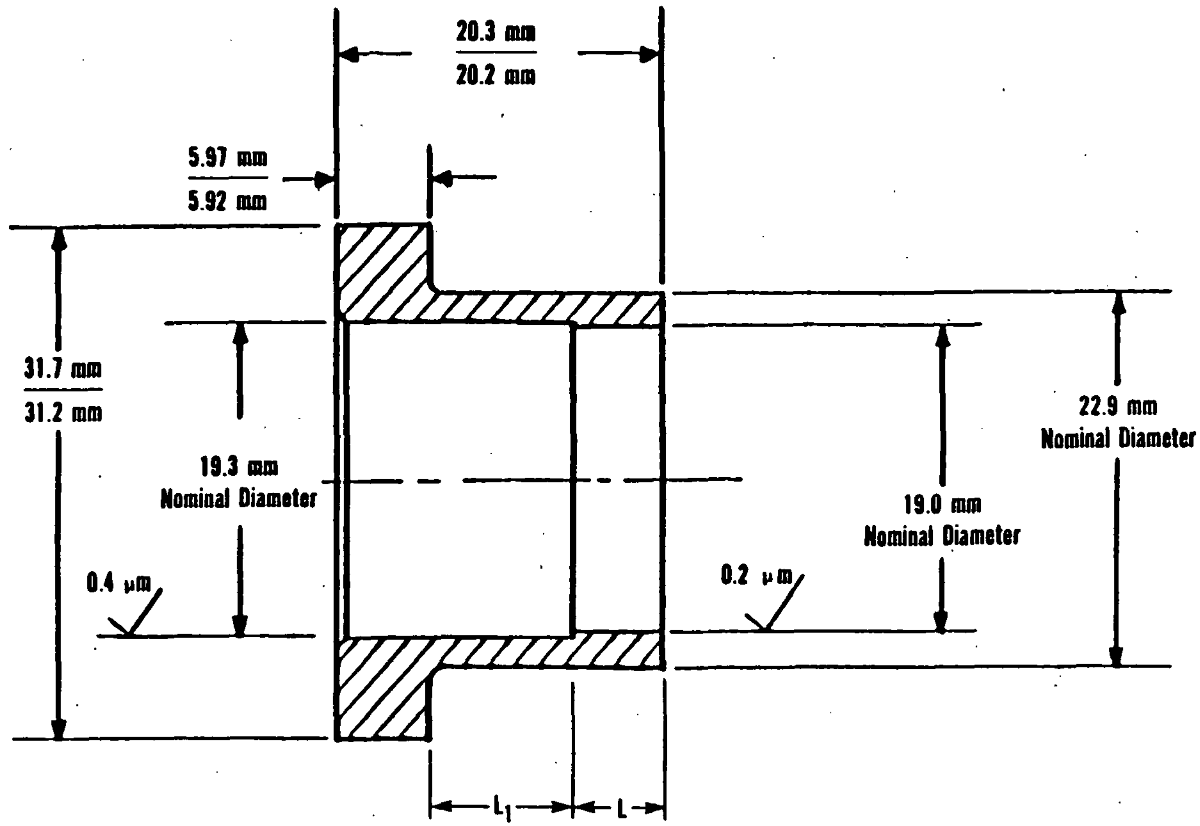


Fig. 5-1 Hydrodynamic Oil Pumping Ring Dimensions

TABLE 5-1

SAMPLE GEOMETRIES AND CONDITIONS FOR PARAMETRIC STUDY OF CANDIDATE TEST RINGS

L = 6.73 mm (0.265 in.), L_1 = 7.62 mm (0.30 in.), δ = 0, s = 38.1 mm (1.5 in.), f = 35 Hz.

ITEM	STEEL	BABBITT	RULON J
R, mm (in.)	9.5 (0.38)	9.5 (0.38)	9.5 (0.38)
E, MPa (psi)	$207\cdot10^3$ (30×10^6)	$51\cdot10^3$ (7.5×10^6)	$1.72\cdot10^3$ (0.25×10^6)
ν	0.3	0.36	0.46
p_o, MPa (psi)	10.34 (1500)	10.34 (1500)	5.17 (750)
C, microns (mils)	8.96 (0.35)	19 (0.75)	50.8 (2)
t, mm (in.)	1.27 (0.05)	1.90 (0.075)	3.81 (0.15)

where wear might be anticipated but far upstream, ahead of the actual pumping area, as shown in Figure 5-2a. This led to the conclusion shown in Figure 5-2b that the high-pressure oil used for clamping leaked past the O-ring and, by pressing against bracket A, effectively produced a sealed chamber over the OD of the ring. The pumping ring was thus loaded with a pressure, p_o, over the entire length (L_1 + L) instead of only the narrow region, e, covered by the O-ring. This unpresidented pressure loading gave the ring the flared shape shown in Figure 5-2b which led to high wear upstream of the active pumping area and to breakage of the weaker rings.

The problem was resolved by opening relief passages behind bracket A so that any fluid leaking past the O-ring would be scavenged outside. Since the leakage past the O-ring occurs only when p_f approaches p_o, such leakage should not affect the build-up of sealed pressure p_f at levels below p_o. With the relief grooves in place, all unsatisfactory tests were repeated; no further difficulties in testing the Rulon and carbon rings were encountered. It should also be noted here that the sealed pressure p_f was not observed to significantly exceed the loading pressure p_o. This is believed to be a result of leakage past the O-ring, which occurs when p_s starts to exceed p_o, as indicated in Figure 5-3.

The rings tested were made of either babbitt, carbon graphite, or Rulon. The geometry and dimensions of the various rings tested are as summarized in Table 5-2 and, except for one item (Item VII), had the following two dimensions in common:

Shaft Diameter - 19.05 mm

Inside Diameter of Back Section (over length L_1) - 19.3 mm

The viscosities of the oil used are those given in Figure C-1 of Appendix C.

5.1 Tests with Large Babbitt Ring

The 19.05-mm diameter babbitt ring was tested at three frequencies, 60, 35, and 10 Hz, and at two strokes, 50.8 mm and 25.4 mm. The parameters investigated were the pumping ring length (L), clearance (C), and the effect of a taper (δ). Tables C-1 through C-4 in Appendix C give the detailed results of

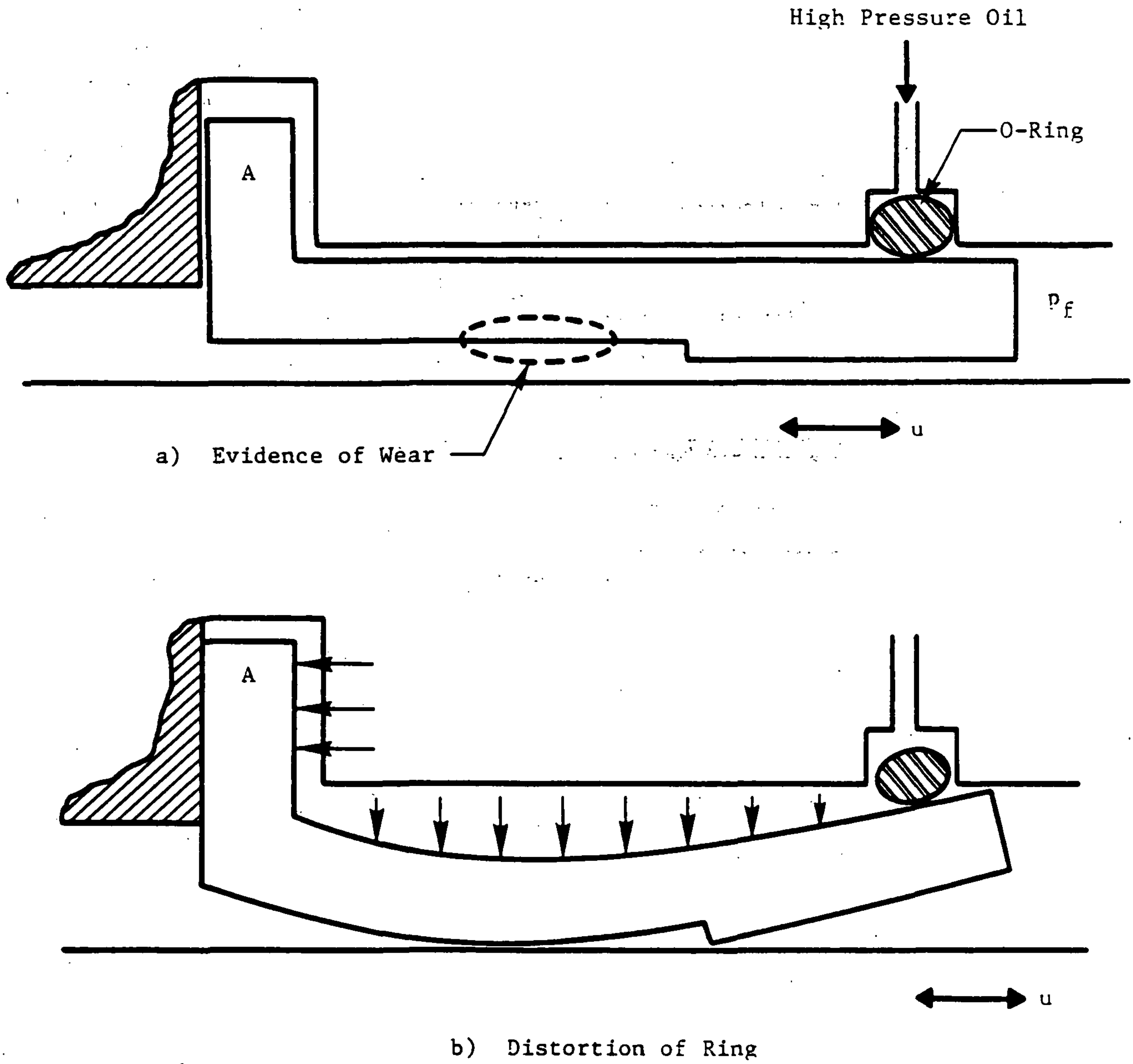


Fig. 5-2 Leakage of High Pressure Oil Past O-Ring

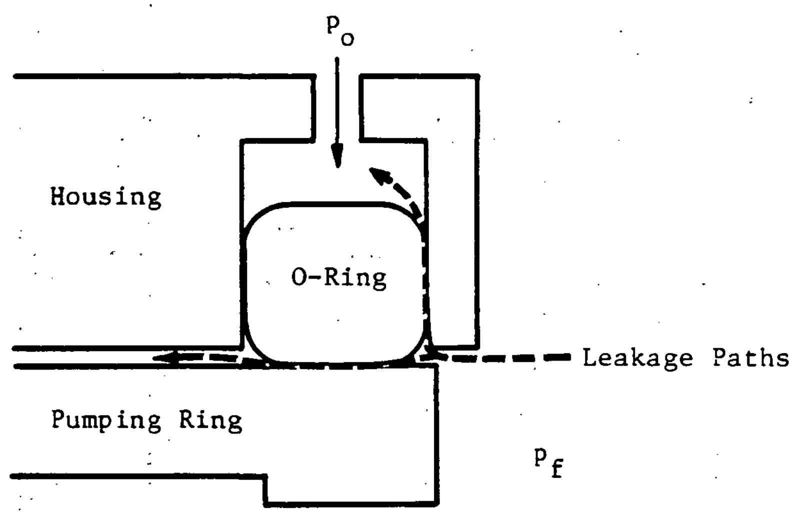


Fig. 5-3 Schematic of Leakage Past O-Ring at High Sealed Pressures

TABLE 5-2

LIST OF PUMPING RINGS TESTED

I.D. NO.	MATERIAL	R mm (in.)	O.D. mm (in.)	L_1 mm (in.)	L mm (in.)	C microns (mils)	$-\delta \times 10^3$	TEST I.D. NO.
I	Babbitt	9.52 (.375)	21.6 (.850)	7.56 (0.298)	6.78 (.267)	11.43 (0.45)	0	A-1-A-1
II	↓	↓	21.5 (.846)	↓	6.78 (.267)	19.05 (0.75)	↓	A-2-A-1
III	↓	↓	21.4 (.843)	↓	4.83 (.190)	12.70 (0.50)*	↓	A-1-B-1
IV	↓	↓	21.4 (.843)	↓	6.78 (.267)	25.4 (1.0)*	3.75	A-1-A-2
V	Carbon	9.52 (.375)	22.07 (.869)	7.56 (0.298)	6.78 (.267)	20.32 (0.8)	0	C-1-A-1
VI	Rulon	9.52 (.375)	24.15 (.951)	7.56 (0.298)	6.78 (.267)	42.55 (1,675)*	↓	B-1-A-1
VII	Babbitt	6.00 (.236)	13.92 (.548)	5.59 (0.22)	5.08 (.200)	8.89 (0.35)	↓	D-1-A-1

* Average of two rings.

the tests. The self-pressurization runs represent an arrangement in which the clamping pressure was provided by the sealed pressures generated by the pumping ring. This represents the case of $p_o = p_f$ at any instance. It turned out that, in order to start the untapered pumping ring, a priming pressure was required (that is, when at the start, $p_f = 0$, no self-clamping was possible). Some initial $p_{fo} > 0$ had to be provided by external means in order to enable the pumping ring to apply self-pressurization and build up a p_f beyond the initially supplied priming pressure, p_{fo}. The tapered rings were found to be self-priming.

With regard to the effect of the several variables tested, the plots suggest the following:

- Effect of Clearance - Table 5-3 shows the changes in maximum flow and pressure induced by changing the clearance from 11.4 microns to 19 microns. These changes are minimal, indicating that the optimum lies within the range of values.

- Effect of Length - As shown in Table 5-4, there is a definite advantage to a higher length, L, with regard to maximum flow rates; but the shorter rings produced higher maximum levels of sealed pressure, p_{fm}.

- Effect of Taper - Table 5-5 confirms what can be intuitively deduced as the desirability of having a geometric taper. At high clamping pressures when ring deflections are large, the geometric taper is counterproductive. However, at $p_o = 3.45$ MPa, a taper produces higher flow rates and higher levels of maximum pressure.

5.2 Tests with the Rulon and Carbon Ring

The detailed results of the tests on the Rulon and carbon rings are given in Tables C-5 and C-6 in Appendix C. In both rings, the higher clamping pressures produced higher reservoir pressures, p_{fm}, but the lower values of p_o yielded higher maximum flow rates. This trend was already noticeable with the babbitt ring, but as the value of E drops, it becomes more pronounced so that,

TABLE 5-3

EFFECT OF CLEARANCE

RUNS I AND II

p_o MPa (psi)	s mm (in.)	f, Hz	Q_0, gm/min C = 0.0114 mm (0.45 mil)	Q_0, gm/min C = 0.019 mm (0.75 mil)	p_{fm}, MPa C = 0.0114 mm (0.45 mil)	p_{fm}, MPa C = 0.019 mm (0.75 mil)
8.62 (1250)	50.8 (2)	60	43	42	9	9
		35	21	22	9	9
		10	4	4	8.8	8.8
6.895 (1000)	50.8 (2)	60	50	51	7.2	7.2
		35	24	27	7.2	7.2
		10	4	4	7.2	7.2
8.62 (1250)	25.4 (1)	60	7	7.5	8.8	9.2

TABLE 5-4

EFFECT OF REDUCED LENGTH L

RUNS I AND III

p_o MPa (psi)	s mm (in.)	f Hz	Q_0, gm/min		p_{fm}, MPa	
			L=6.78 mm (0.267 in.)	L=4.83 mm (0.19 in.)	L=6.78 mm (0.267 in.)	L=4.83 mm (0.19 in.)
6.895 (1000)	50.8 (2)	60	50	34	7.2	7.8
		35	25	17	7.2	7.8
		10	4	2.5	7.2	7.0
3.45 (500)	50.8 (2)	60	59	51	3.8	7.2
		35	28	27	3.8	7.2
		10	3.5	3.5	3.8	7.2
8.62 (1250)	25.4 (1)	60	70	60	8.8	8.0

TABLE 5-5

EFFECT OF TAPER

RUNS I AND IV

p_o MPa (psi)	s mm (in.)	f, Hz	Q_o, gm/min		p_{fm}, MPa	
			$\delta = 0$	$\delta=-3.75\cdot10^{-3}$	$\delta = 0$	$\delta=-3.75\cdot10^{-3}$
8.62 (1250)	50.8 (2)	60	42	32		
		35	22	17	9.2	9.2
		10	4	4	8.9	9.0
6.895 (1000)	50.8 (2)	60	50	42	9.2	7.2
		35	24	18		
		10	4	2		7.5
3.45 (500)	50.8 (2)	60	60	> 80	3.8	4.2
		35	28	38	3.8	4.2
		10	4	4	3.8	3.8
8.62 (1250)	25.4 (1)	60	7	2.5	8.8	8.8

for the Rulon rings, Q_o at p_o = 1.72 MPa (250 psi) is two or three times the value of Q_o at p_o = 5.17 (750 psi).

5.3 Tests with the Small Babbitt Ring

The results for the 6-mm radius babbitt ring are given in Table C-7. The diameter was reduced roughly 1/3 as compared to the large rings. Neither the flows, Q_o, nor the maximum sealed pressure, p_{fm}, were much affected by the change in size.

6.0 DISCUSSION OF RESULTS

Three sets of comparative results are presented in Appendix D.

- The babbitt rings, which comprise the large size with two different clearances and a small size ring
- The carbon graphite ring
- The Rulon ring.

The theoretical curves presented in Appendix D were obtained without the starvation correction and without allowance for the fact that the sealed pressure does not exceed the loading pressure due to leakage past the O-ring.

A number of qualitative conclusions can be drawn from a study of the figures in Appendix D. The major discrepancy observable is the fact that the measured flows are substantially smaller than those predicted by theory at zero sealed pressure. This is believed to be partially due to effects of starvation as will be discussed later. For practical purposes, this discrepancy is not believed to be a major one in that the pumping rings will normally be used to develop a buffer pressure as opposed to being designed to deliver prescribed flow rate. The predicted maximum developable pressure at zero net flow was found to be in better agreement with the experimental data. Again, all of the curves that predict sealed pressure higher than the loading pressures should be truncated at the loading pressure value due to leakage past the secondary seal.

It is important to note that all of the data presented in Appendix D was obtained with seals that were sized and designed with the analysis contained here. In all cases, as predicted, the seals did perform the task of developing and maintaining the designed sealed pressures. Significant pressures could not be developed for steel rings where the theory indicated that sufficient deflection would not be obtainable to provide adequate pumping. Although the analysis presented so far does not accurately predict deliverable flow, it can be used reliably to size and design pumping rings. In order to obtain and improve fit to the flow data, a two-constant empirical relationship has been determined from the data for babbitt pumping rings to attempt to account for starvation. The results of this fit when compared with experimental data are described below.

6.1 Empirical Correction for Starvation

An empirical relationship for the starvation factor, λ, as a function of L/s of the form

$$\lambda = 0.74\ (s/L)^{0.58}$$

has been obtained by fitting flow rates at zero sealed pressure for ring No. I at p_o = 8.62 MPa. Comparisons have been made for rings I - III over a range of frequencies, strokes, loading pressures, sealed pressures, and geometries. Values of λ obtained from the above equations varied between 1.6 and 2.4 for all the cases shown in Figures 6-1 through 6-10.

Comparisons of the data for a 50.8-mm stroke at 35 Hz in Figure 6-1 with those for a 25.4-mm stroke at 60 Hz shown in Figure 6-4 indicate that, even though the speeds (product of stroke and frequency) are fairly close together, the short stroke data in Figure 6-4 shows a factor of 3 less flow. This is very much in keeping with the starvation analysis that predicts that reduced flow will occur when the land length becomes an appreciable fraction of the stroke. If starvation is neglected, the theory would predict the flows to be solely a function of the product of frequency and stroke. Even though only a two-constant fit was used, the agreement between theory and experiment for all of the cases involving the small clearance babbitt ring looks reasonably good.

Figure 6-6 shows the comparison between theory and experiment with the larger clearance babbitt ring. The shaded area shows the difference between predictions for a 0.75-mil clearance and a 0.65-mil clearance. The difference being well within the limit of the measurements. In general, the data tend to indicate that the 0.65-mil clearance is probably closer to the truth. This appears to be particularly true when one looks at the data at 10 Hz. Again, the agreement is reasonably good and no additional constants were used in fitting the data for the large clearance ring.

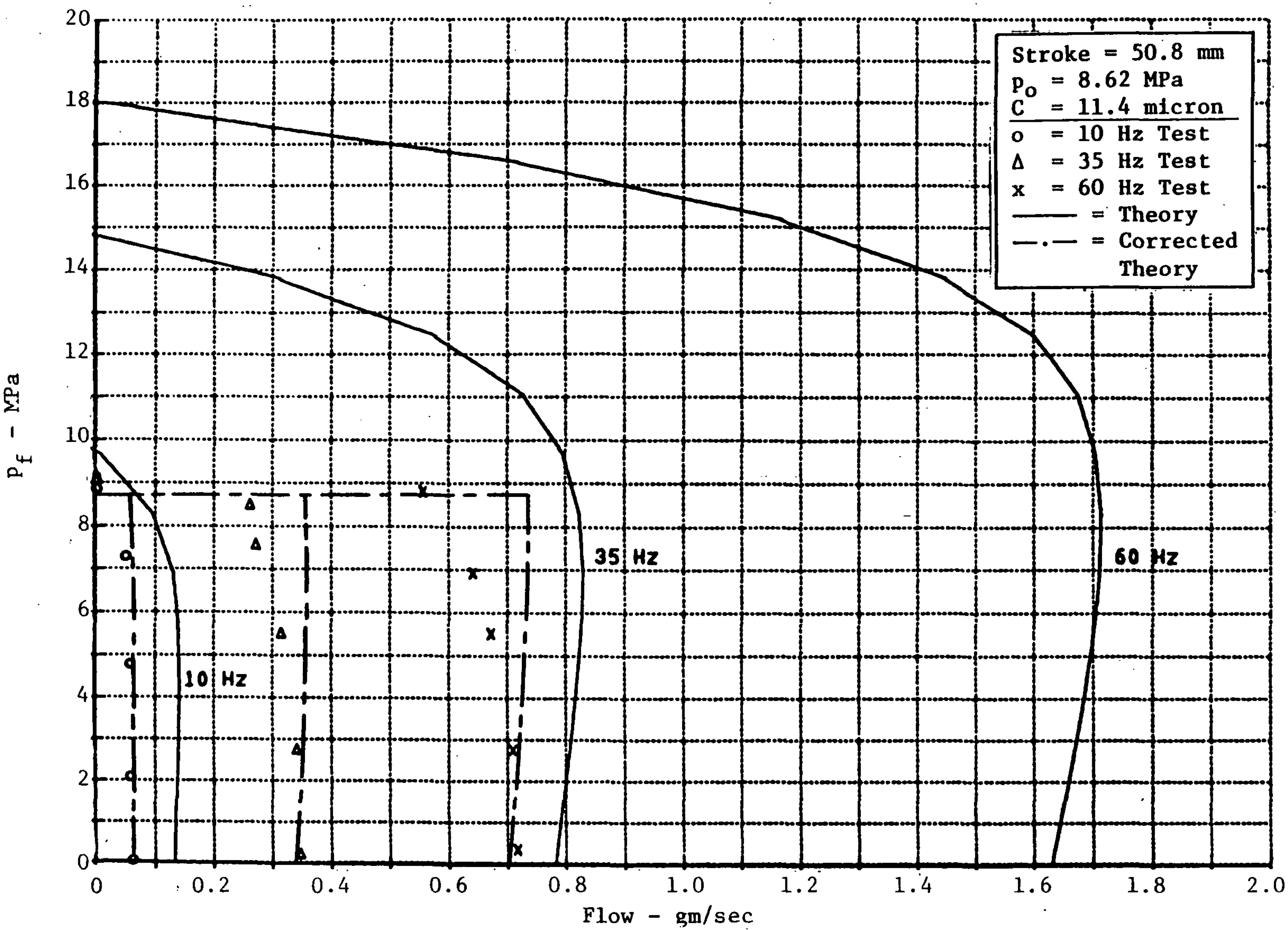


Fig. 6-1 Comparison Between Corrected Theory and Experiment for Babbitt Pumping Ring I (s = 50.8 mm; p_o = 8.62 MPa)

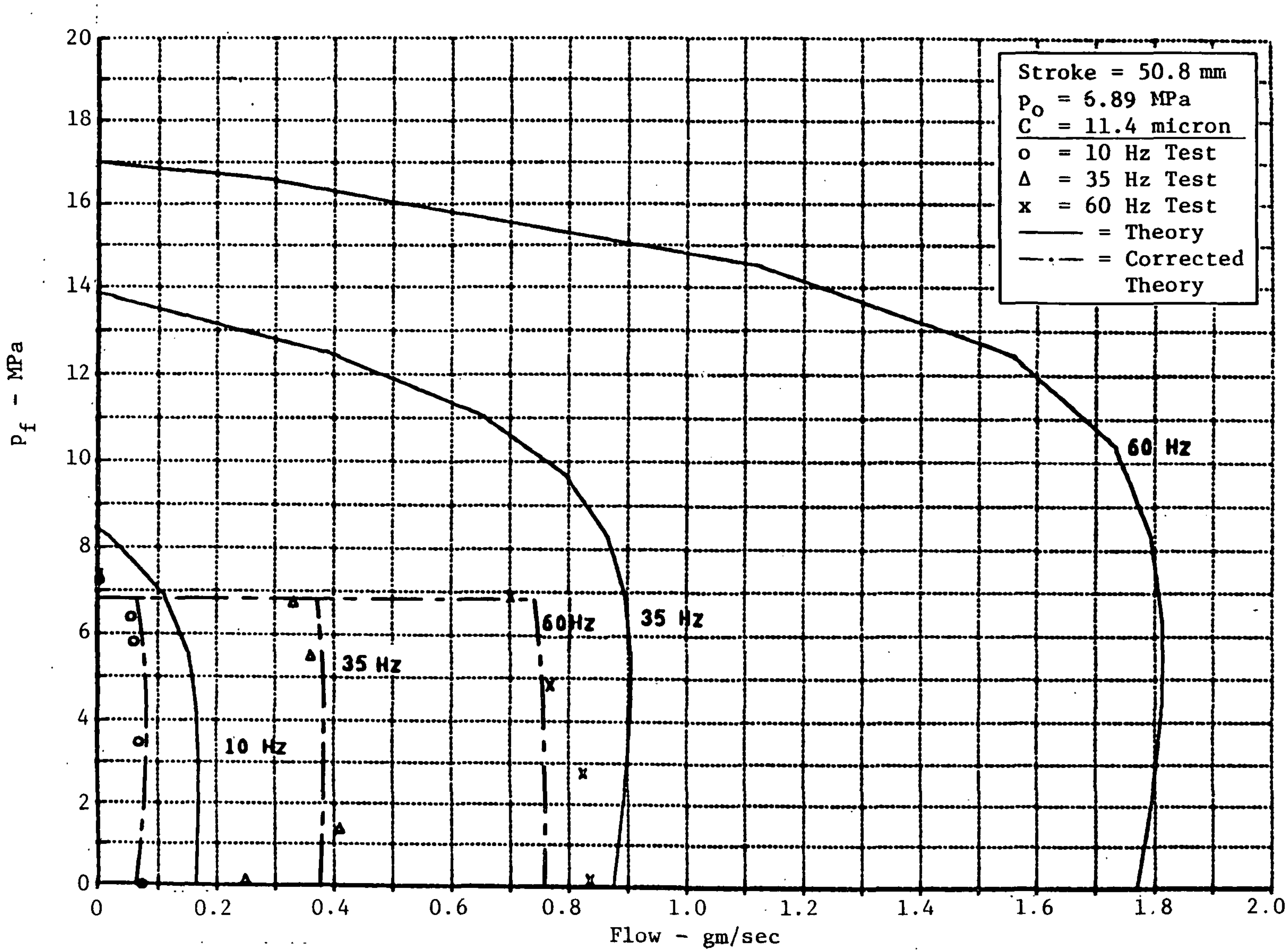


Fig. 6-2 Comparison Between Corrected Theory and Experiment for Babbitt Pumping Ring I (s = 50.8 mm; p_o = 6.89 MPa)

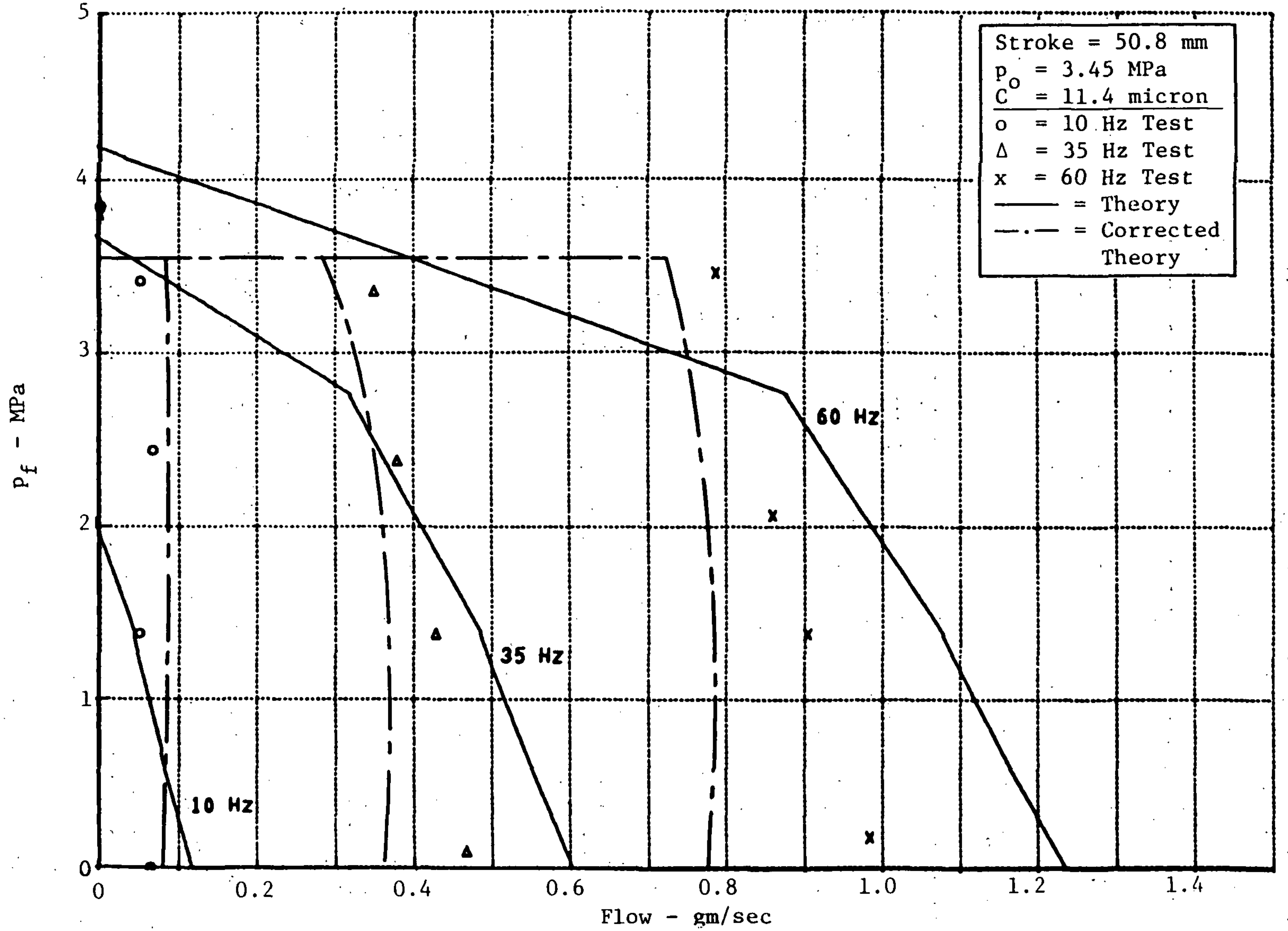


Fig. 6-3 Comparison Between Corrected Theory and Experiment for Babbitt Pumping Ring I (s = 50.8 mm; p_o = 3.45 MPa)

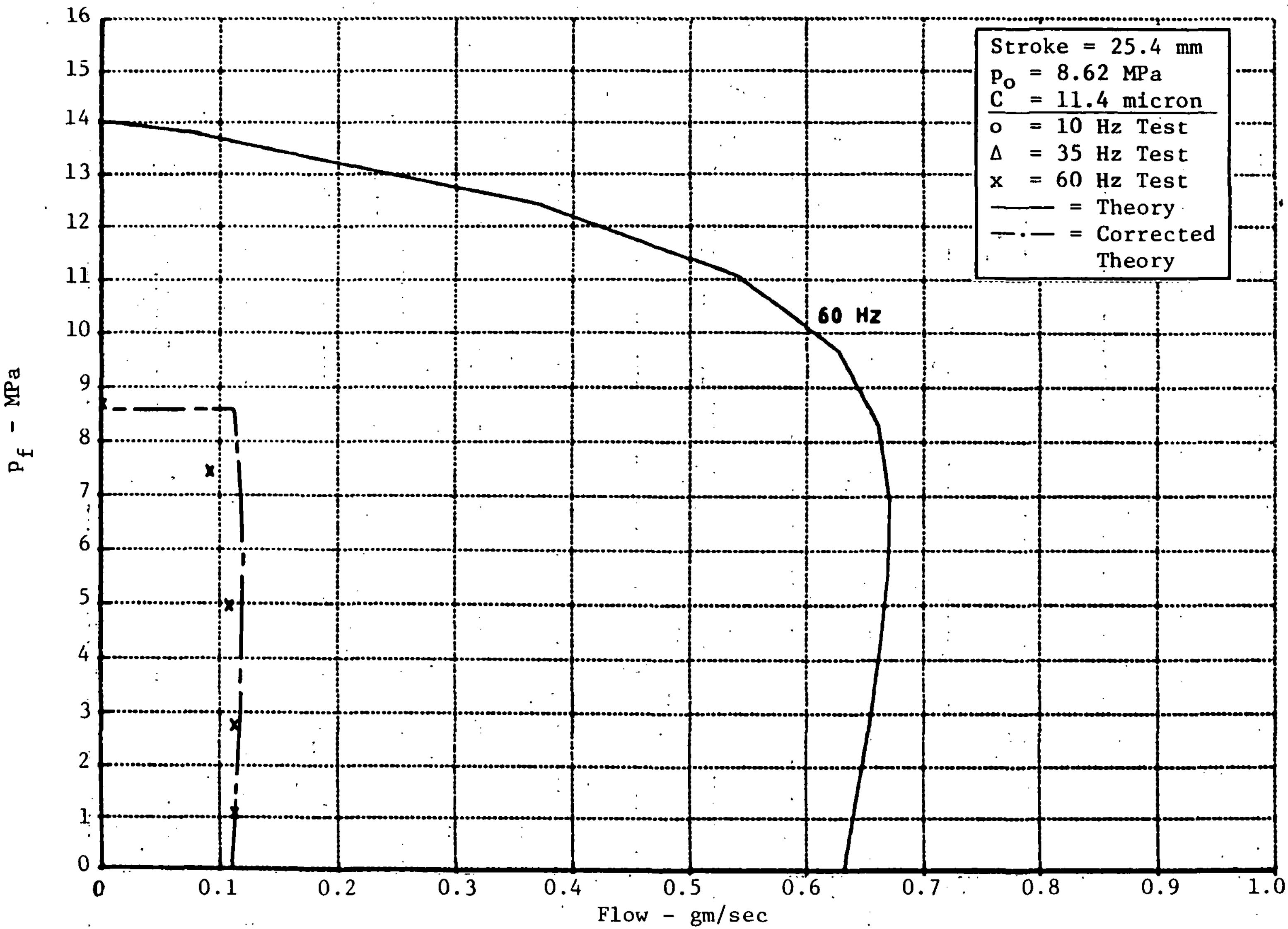


Fig. 6-4 Comparison Between Corrected Theory and Experiment for Babbitt Pumping Ring I (s = 25.4 mm; p_o = 8.62 MPa)

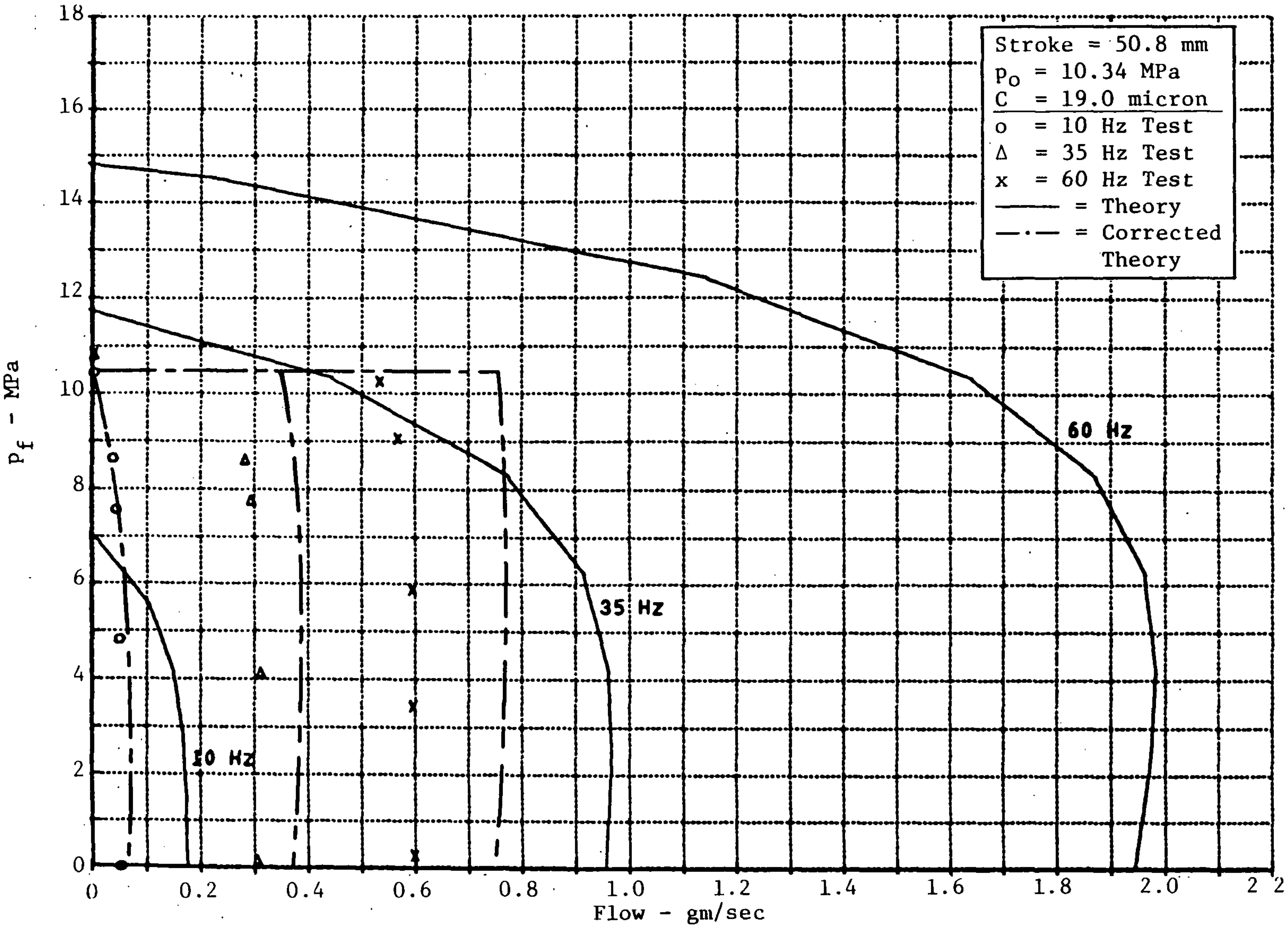


Fig. 6-5 Comparison Between Corrected Theory and Experiment for Babbitt Pumping Ring II (s = 50.8 mm; p_o = 10.34 MPa)

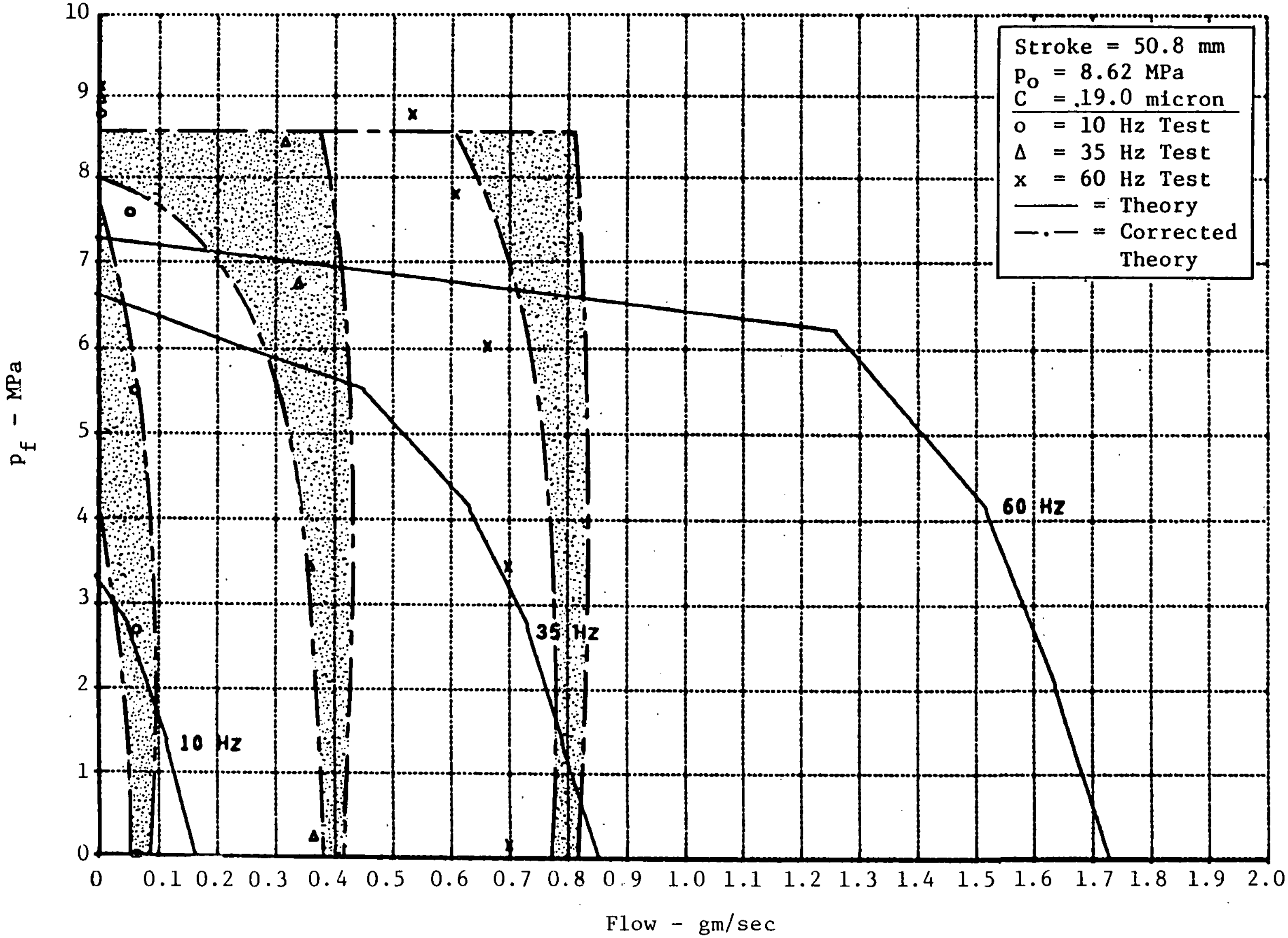


Fig. 6-6 Comparison Between Corrected Theory and Experiment for Babbitt Pumping Ring II (s = 50.8 mm; p_o = 8.62 MPa)

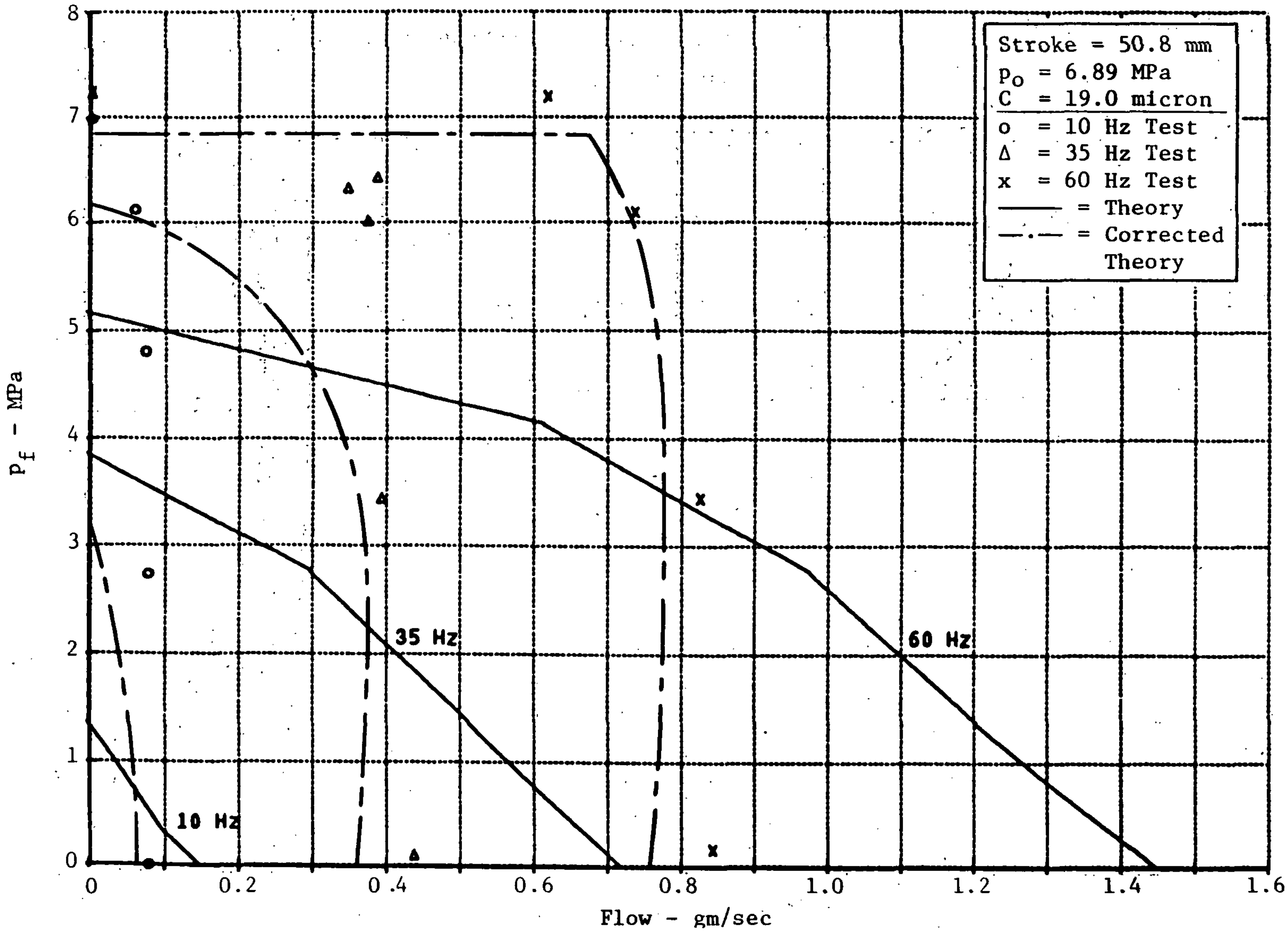


Fig. 6-7 Comparison Between Corrected Theory and Experiment for Babbitt Pumping Ring II (s = 50.8 mm; p_o = 6.89 MPa)

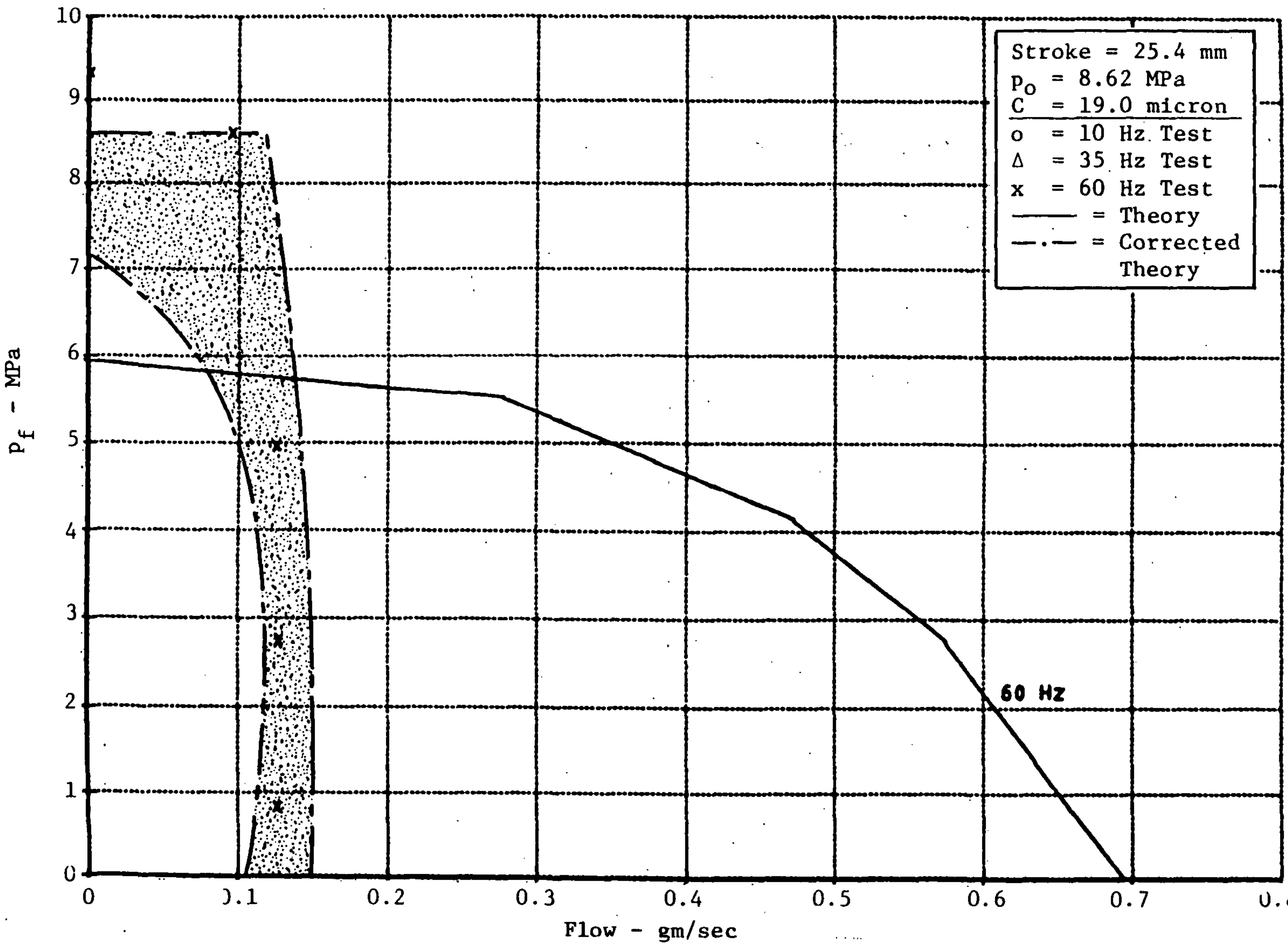


Fig. 6-8 Comparison Between Corrected Theory and Experiment for Babbitt Pumping Ring II
(s = 25.4 mm; p_o = 8.62 MPa)

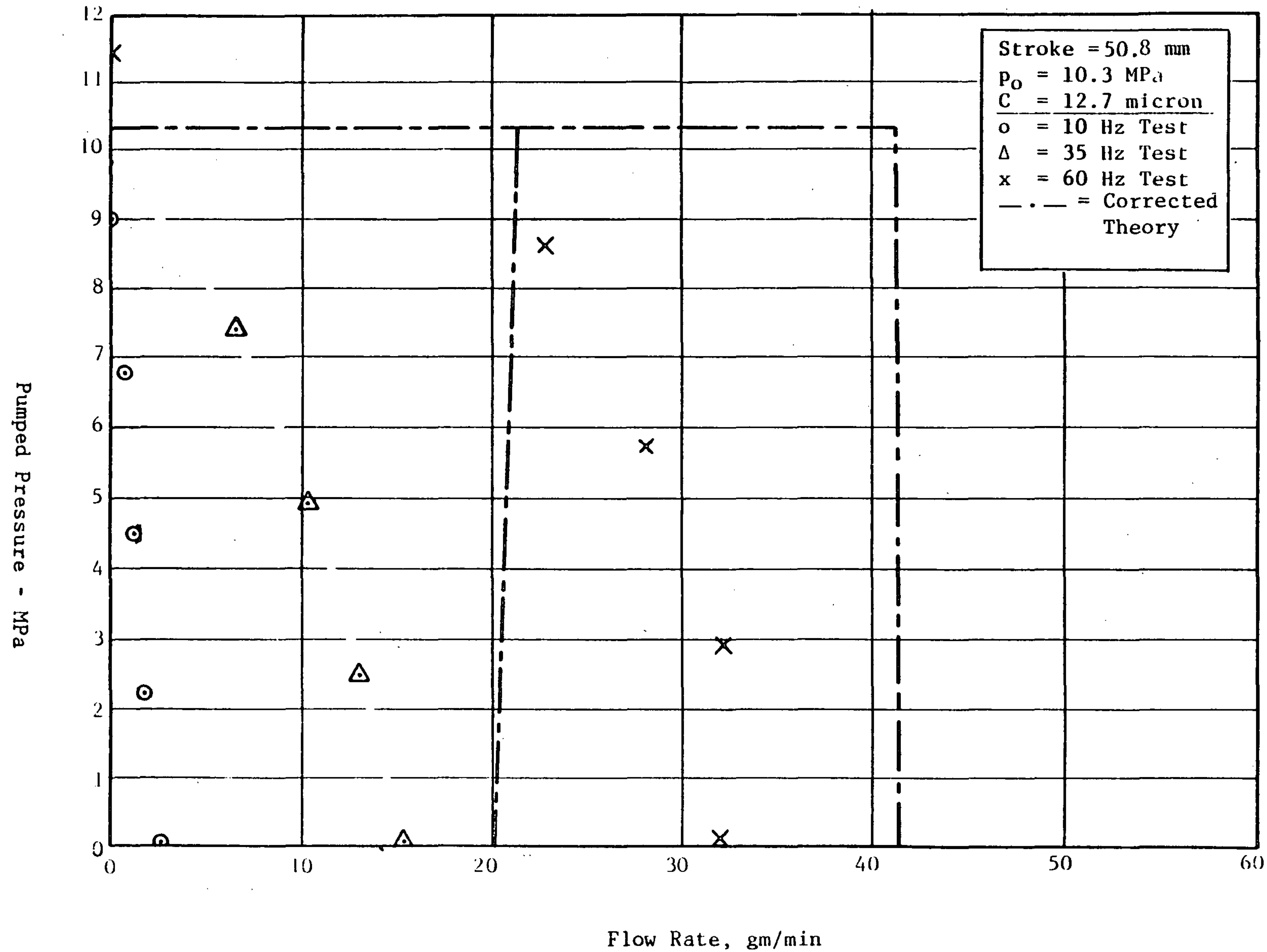


Fig. 6-9 Comparison Between Corrected Theory and Experiment for Babbitt Pumping Ring III (s - 50.8 mm; p_o = 10.3 MPa)

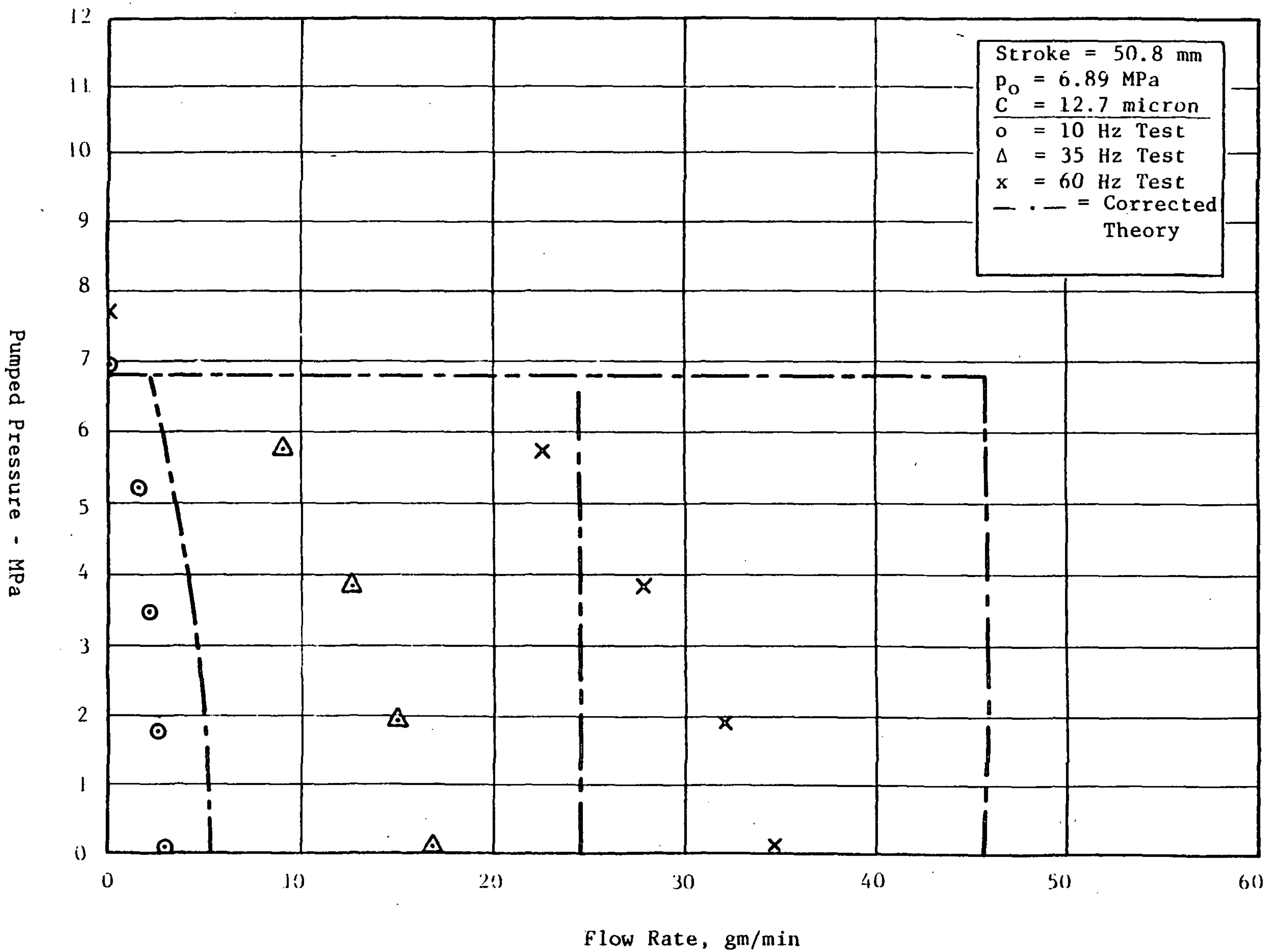


Fig. 6-10 Comparison Between Corrected Theory and Experiment for Babbitt Pumping Ring III (s = 50.8 mm; p_o = 6.89 MPa)

Figures 6-9 and 6-10 show comparisons between theory and experiment for the shorter length ring. The increased discrepancies indicate that the starvation correction depends more on the stroke than on the seal land. This could be an effect of inertia in the inlet region. In general, the starvation correction, along with the constraint that the end pressure doesn't exceed the loading pressure, provides much better agreement between theory and experiments but is still somewhat incomplete.

6.2 Effects of Viscosity

Since the viscosity cannot be measured at the interface (it is, in fact, measured several inches upstream of the inlet) and since the clearances are very small, both parameters could, in actuality, differ from the assumed values. Major reductions in viscosity, such as that which would occur if the actual temperature were 100°F higher than the measured inlet temperature, would provide remarkably improved agreement between theory and experiment. This fact was observed in numerous parametric studies which are not presented here in that no justification has been found for this temperature rise.

6.3 Suggested Design Procedure

The most effective use of a pumping ring is believed to be for generating a buffer pressure to back up another seal. Thus, if one were to seal gas at 10 MPa from oil at ambient pressure, the pumping ring should be used to develop the 10 MPa backup pressure in a buffer reservoir, thus alleviating the pressure gradient on the primary seal. This should provide prolonged life for the sealing system since the pumping ring is essentially a "light contact" device.

Clearances should be selected based on shaft diameters and machining tolerances. Values of C/R should be of the order of 10^{-3}, thus resembling bearing clearances rather than the tight clearances (or interference) that one would generally find in a seal.

The loading pressure should be of the order of the desired buffer pressure. Back leakage past the O-ring could thus provide a potential relief valve to protect against excessive pressure buildup in the buffer chamber.

The computer program RING (given in Appendix B) should then be used to arrive at a combination of material properties (E, ν) and geometries (t, e, L, L_1) so that the computed value of P_{fm} is equal to the desired buffer pressure, and a small amount of clamping is predicted to occur during the back stroke. Allowances should be made for some wear to occur due to friction resulting from the radial shear force F.

If the pumping ring is designed to generate its own loading pressure, $p_o = p_f$, an initial taper should be used. Values of C_M/C of the order of 2-3 have been shown to be adequate for obtaining self-pumping without the need for external priming. Caution should be used here in providing pressure relief to avoid overloading.

Finally, if the pumping ring is used as a flow device, rather than a pressure generator, allowances should be made for the fact that the analysis tends to overpredict flow under many circumstances. In general, the "starved net flow" prediction generated by RING should be within a factor of 2 of the flow that one would expect to obtain in practice, based upon the measurements reported here.

7.0 CONCLUSIONS

A design analysis has been developed for predicting pressure-flow relationships for various input geometries, speeds, fluid viscosities, and elastic moduli of pumping rings. The analysis can be used to size and design pumping rings for various applications. Results of testing with Rulon, babbitt, and carbon-graphite rings at various loads, speeds, and geometries indicate that the analysis can consistently be used to design rings that work well over a fairly broad range of parameters.

The design criteria for selecting the clearance would be dictated by tolerance and flow rate consideration. The applied loading, p_o, should be the order of the desired pumping pressure. The choice of materials and geometry are then dictated by elasticity considerations so that the deflected ring, under static load, clamps the shaft with a very small force to nearly eliminate the back flow without resulting in excessive friction and ensuing wear and power loss.

The greatest discrepancy between theory and experiment lies in the prediction of flow at zero sealed pressure, Q_o. Predicted values of Q_o are substantially higher than those observed experimentally in almost all cases. Predictions of the maximum sealed pressure at zero net flow, p_{fm}, differ from those observed experimentally, in that values of p_{fm} substantially greater than the loading pressure, p_o, are frequently predicted but have never been observed.

The analysis shows that starvation can have a significant influence on Q_o, although not sufficient to completely explain the discrepancies, and that leakage past the O-ring could cause p_{fm} not to exceed p_o. Further causes of discrepancy could be due to uncertainties in the clearance and the local temperature at the inlet to the pumping ring.

It was found experimentally that the use of tapers on pumping rings enabled the rings to self-pump up to high sealed pressures. The untapered rings, in general, needed to be initially loaded until a sufficiently high sealed pressure could be generated to load the ring. No priming was necessary for the tapered rings. Theory indicates that performance of a tapered ring and an untapered ring at the same minimum film thickness are similar under significant loading, but the tapered ring is predicted to pump even when unloaded.

8.0 REFERENCES

1. "Experimental Evaluation of Oil Pumping Rings," M. W. Eusepi, J. A. Walowit, M. Cohen, DOE/NASA/0119-81/1, NASA CR-165271, U.S. Department of Energy, April 1981.

APPENDIX A

PRELIMINARY ANALYSIS OF PUMPING LENINGRADER SEAL

NOMENCLATURE

δ	Interference
w	Radial inward deflection
h	Film thickness
x_1, x_2	Coordinate variables
p	Interface pressure
p_g	Sealed gas pressure
p_i	Interference pressure
p_o	Ambient oil pressure
E	Elastic modulus of seal
ν	Poisson's ratio
t	Thickness of steel
R	Radius
$D = Et^3/[12(1-\nu^2)]$	Flexural rigidity
$f(x)$	Shape of undeformed seal ring relative to shaft radius
μ	Viscosity
θ	Inlet slope
L, L_1	Lengths shown in Figure A-4

Figure A-1 is a schematic of a pumping Leningrader seal cross section mounted on a shaft. The seal, which is preloaded against the shaft with an interference fit, is loaded by a backup spring and by high-pressure gas, which acts behind the seal and is separated from the oil by the static sealing land. The long, chamfered inlet region provides the pumping action in that it forms a gradual inlet which tends to trap or to provide a preferred direction for flow when the shaft moves toward the cooling supply. (This is referred to as the forward stroke.) During the backstroke, the seal (as drawn in Figure A-1) tends to rub and wipe oil away. Thus, any oil that leaks toward the high-pressure gas tends to be pumped back toward the cooling oil reservoir.

Figure A-2 shows the actual dimensions of the seal used in an automotive Stirling engine. The dotted line denotes the geometry used in the initial modeling of the seal discussed below.

A preliminary model has been developed for a seal with a constant thickness as shown in Figure A-3. The pressure profile drawn under the seal is the pressure anticipated to occur between the seal and the shaft during the forward stroke. The pressure on the far left corresponds to the high-pressure gas, and the pressure on the right represents the oil pressure. The constant pressure, p_i, denotes the compressive radial stress associated with the interference fit of the ring on the shaft. The gas pressure is assumed to act along the outside of the seal with an imaginary secondary seal separating the gas from the oil. The deformation of the seal is assumed to be governed by the elasticity equations for a thin axisymmetric cylindrical shell.

$$D(d^4w/dx^4) + (Et/R^2)w = -(p - p_g) \qquad \text{(A-1)}$$

The geometric variables and coordinate system are shown in Figure A-4; w denotes the inward radial deflection, E denotes the elastic modulus of the seal, and D denotes the flexural rigidity defined as:

$$D = Et^3/[12(1 - \nu^2)]$$

where ν is Poissons ratio for the seal.

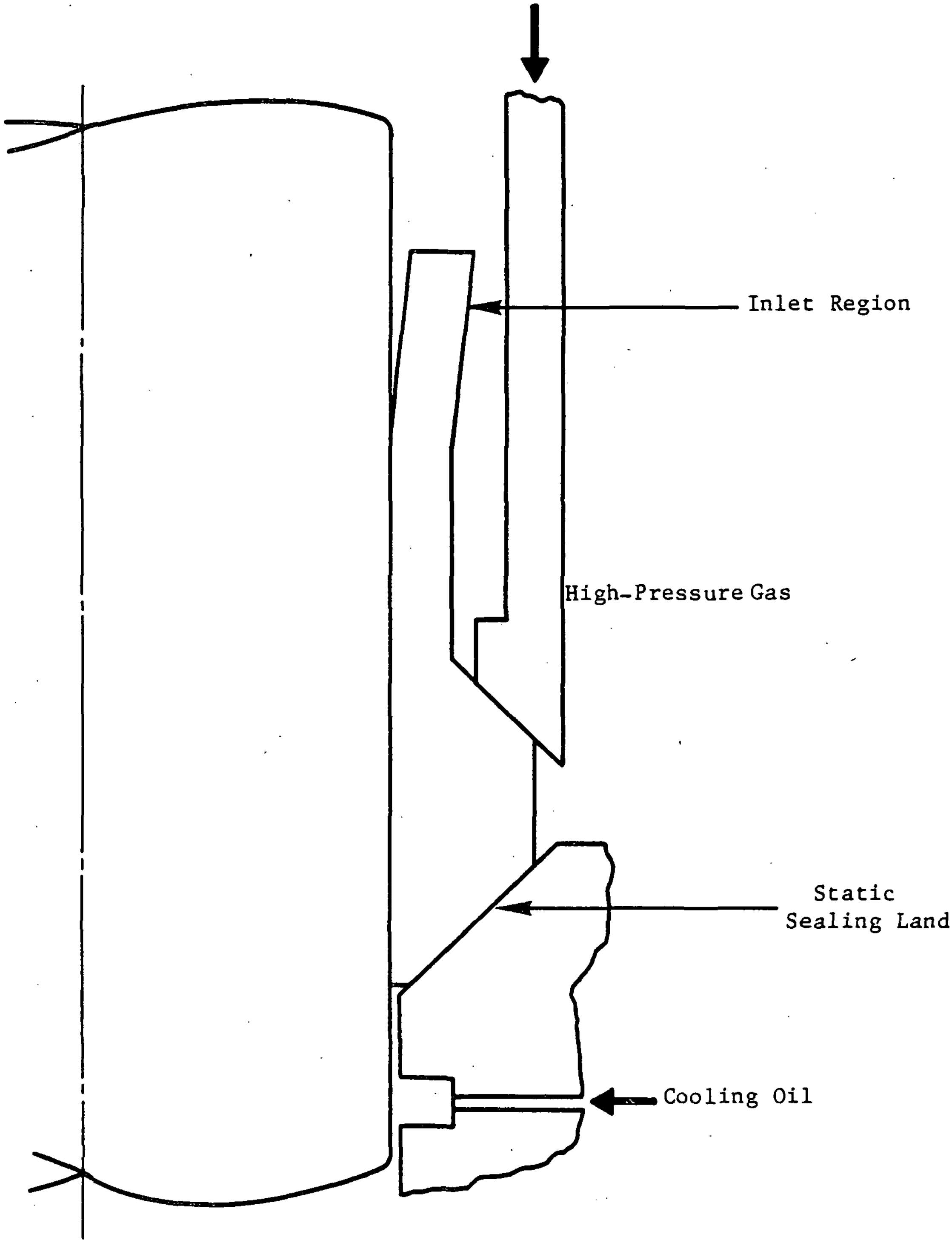


Fig. A-1 Pumping Leningrader Seal

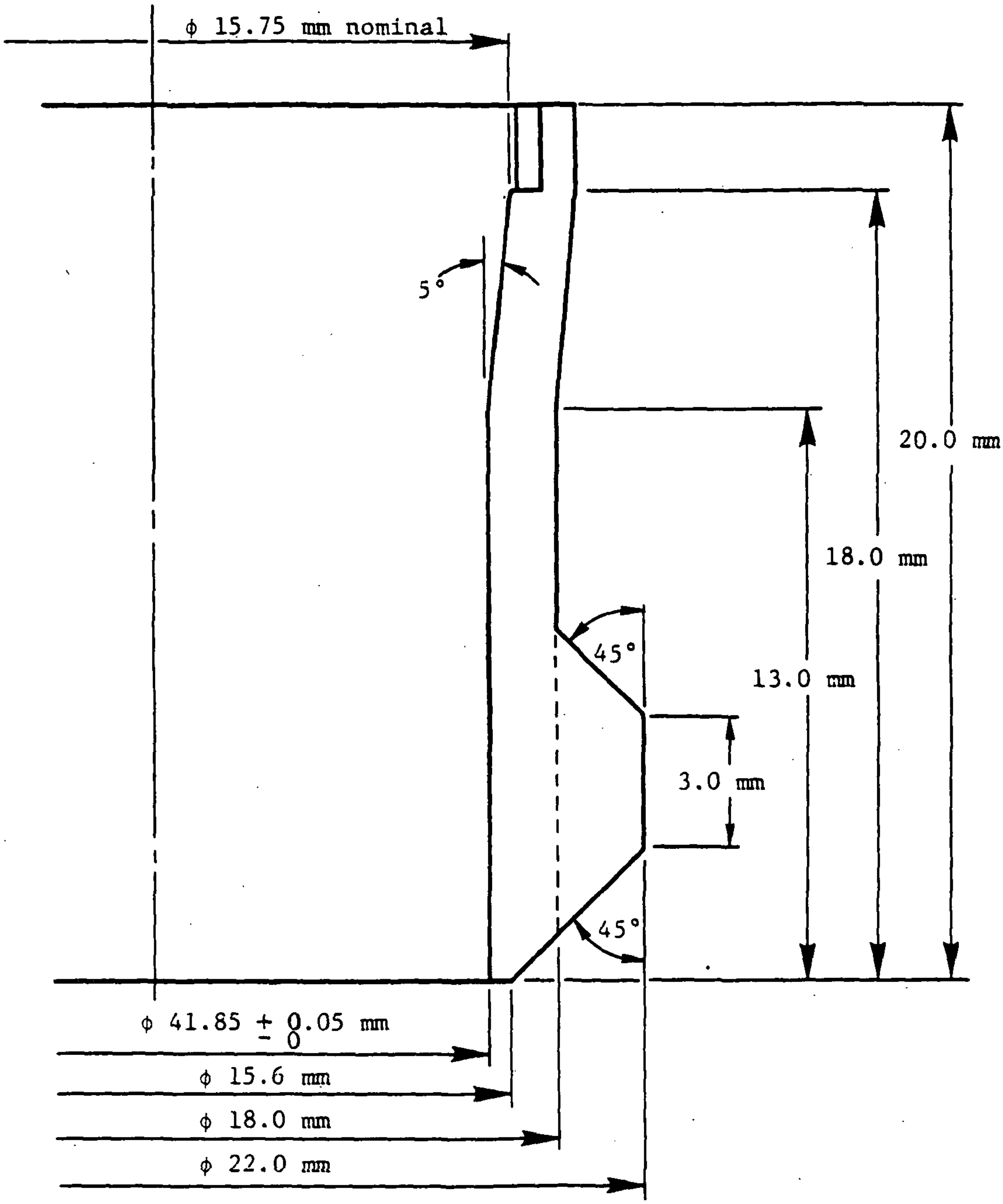


Fig. A-2 Seal Geometry for Automotive Stirling Engine Application

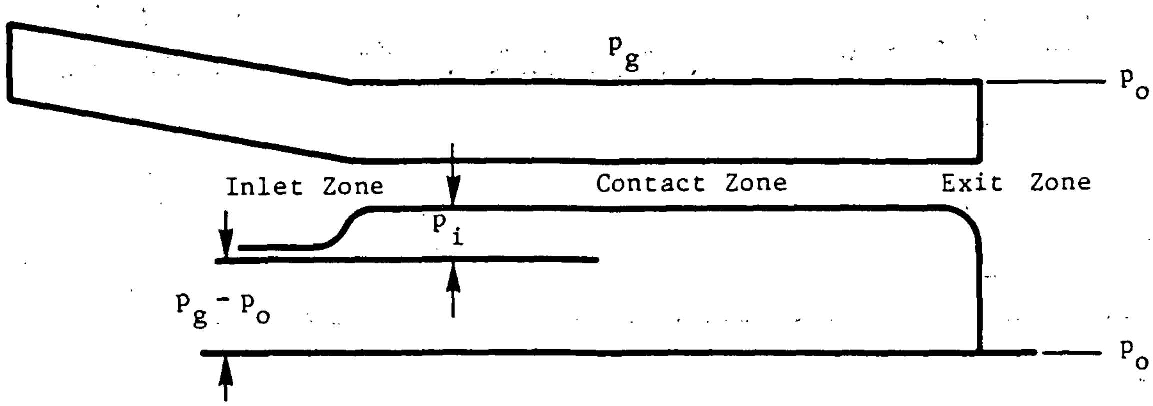


Fig. A-3 Pressures Acting on Pumping Leningrader Seal Used in Preliminary Analysis

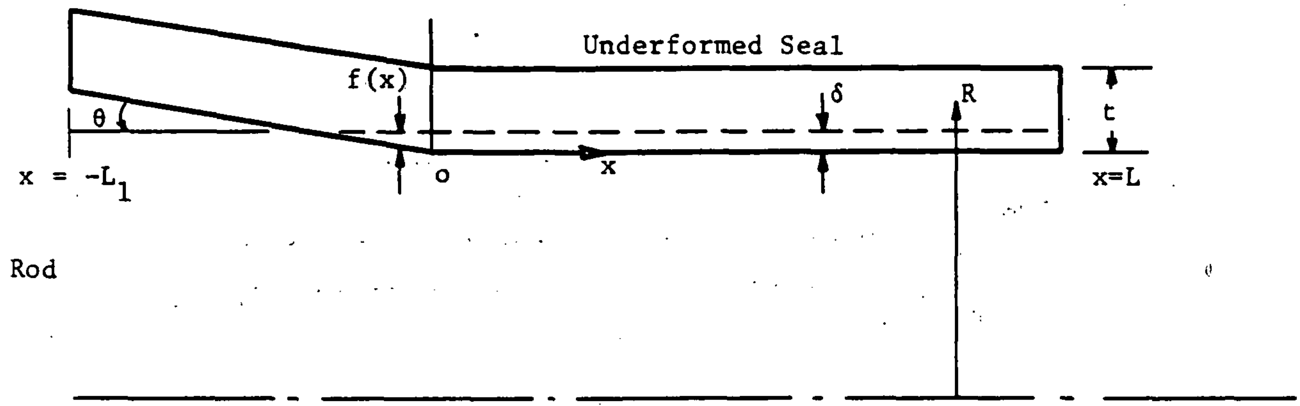


Fig. A-4 Schematic of Geometry Used in the Analysis

The quantity, p, enotes the pressure in the interface between the shaft and the seal which will be determined by the Reynolds Equation for axisymmetric steady-state motion during the forward stroke given by:

$$dp/dx = 6\mu U_o[(h - h^*)/h^3] \qquad \text{(A-2)}$$

where h is the local hydrodynamic film thickness, μ is the viscosity of the oil, U_o is the average velocity during the forward stroke, and h^* is a constant of integration to be determined from the boundary conditions.

The film thickness h is related to the elastic deflection w by:

$$h = f(x) - w \qquad \text{(A-3)}$$

where f(x) is the radius of the undeformed seal relative to the shaft radius. For the model depicted in Figure A-4, f(x) is given by:

$$f(x) = \begin{cases} -\delta \quad , & 0 \le x \le L \\ -\delta - \theta x, & -L_1 \le x \le 0 \end{cases} \qquad \text{(A-4)}$$

The boundary conditions on the elasticity equation come from the requirement that both ends of the seal are free (forces and moments are zero at each end of the seal). The boundary conditions on the Reynolds equation comes from the requirements that the hydrodynamic pressures equal the prescribed gas pressure at the high-pressure side and the prescribed oil pressure on the low-pressure side. These may be written as:

$$d^2w/dx^2 = d^3w/dx^3 = 0 \text{ at } x = -L_1 \text{ and } x = L \qquad \text{(A-5)}$$

$$p = p_g \text{ at } x = -L_1 \qquad \text{(A-6)}$$

$$p = p_o \text{ at } x = L \qquad \text{(A-7)}$$

Equations (A-1, through (A-7) represent a fifth-order nonlinear system corresponding to the mo'el under consideration.

A number of approximations that are characteristic of elastohydrodynamic lubrication may be introduced. These are based largely on the fact that the hydrodynamic film thickness, h, will be small compared with the interference fit, δ. The elastohydrodynamic behavior may be characterized by three zones:

- an inlet zone where the film develops
- a contact zone where the film thickness is nearly constant
- an exit zone where the pressure drops to ambient.

This type of behavior has been seen in many other types of elastohydrodynamic lubrication, e.g., Hertzian contacts in rolling element bearings, elastomeric bearings and foil bearings.

A.1 Contact Zone Solution

The contact zone is the region extending from $x = 0$ to the start of the exit zone which will be localized near $x = L$. In that region, from Equation (A-4), $f(x) = -\delta$ and from Equation (A-3), $w = - \delta - h$, which may be approximated by $w = \delta$ since $|h/\delta| << 1$. With w being constant, the left-hand term in Equation (A-1) will vanish; hence the relationship

$$p - p_g \equiv p_i = (Et\delta)/R^2, \text{ contact zone pressure} \qquad \text{(A-8)}$$

Since p as determined in Equation (A-8) is constant in the contact zone, the left-hand hand term in Equation (A-2) will vanish and the result will be $h = h^*$, constant (contact zone film thickness).

Equation (A-8) is essentially the dry contact pressure resulting from the interference fit. The dry contact profile would also contain a radial shear load at the sharp corner at $x = 0$. With a small amount of wear, however, this corner will be rounded, thus alleviating the radial shear load without radically affecting the inlet film shape. Equation (A-8) will then be assumed to prevail throughout the contact zone with the value of h^* to be determined by matching the pressure obtained from the solution to the equations for the inlet zone.

A.2 Inlet Zone Solution

In order to match the solution in the inlet zone with the contact zone solution, the pressure and flow must be continuous at $x = 0$. This results in the constraint

$$p = p_g + p_i \text{ at } x = 0 \qquad \text{(A-9)}$$

and

$$dp/dx = 0 \text{ at } x = 0$$

which is equivalent to

$$h = h^* \text{ at } x = 0 \qquad \text{(A-10)}$$

For values of θ of the order of a few degrees or more, the pressure will vary from its limiting values of $p_g + p_i$ at $x = 0$ to p_g over a relatively small portion of the inlet zone as sketched in Figure A-3. If the shape of the inlet zone is assumed to be unaffected by the pressure distribution in the region, the film thickness relationship is

$$h = h^* - \theta x \quad , x < 0$$

The stretched variable $\xi = -\theta x/h^*$ may now be introduced into Equation (A-2) to obtain

$$dp/d\xi = -6\mu U_o \xi / [\theta h^*(1 + \xi)^3] \qquad \text{(A-11)}$$

with the constraints

$$p = p_g + p_i \text{ at } \xi = 0 \qquad \text{(A-12)}$$

$$\text{and } p = p_g \text{ at } \xi = L_1\theta/h^* \qquad \text{(A-13)}$$

The quantity $L_1\theta/h^*$ will be a large number for practical cases. Equation (A-13) may be replaced by its limiting form

$$p = p_g \text{ at } \xi \to \infty \tag{A-14}$$

Equation (A-11) may readily be integrated to yield

$$p_i = \frac{6\mu U_o}{\theta h^*} \int_o^\infty \frac{\xi}{(1 + \xi)^3} d\xi = 3\mu U_o /(\theta h^*)$$

Thus the film thickness h^* is given by

$$h^* = 3\mu U_o/(\theta p_i) = 3\mu U_o R^2/(Et\delta\theta) \tag{A-15}$$

which may, in turn, be used to calculate flow.

A.3 Exit Zone Solution

The exit zone solution is the region in the neighborhood of $x = L$ where the pressure must drop from $p_g + p_i$ down to the ambient value of p_o. This will, again, occur over a relatively short region. The film thickness deflection relationship for the exit zone as determined from Equations (A-3) and (A-4) is the same as that for the contact zone and may be written as

$$w = -(h + \delta)$$

The above relationship may be substituted for w in Equation (A-1) to obtain

$$D(d^4h/dx_1{}^4) + Et(\delta + h)/R^2 = p - p_g$$

where $x_1 = L - x$.

Neglecting h compared with δ in the second term of the above equation and employing Equation (A-8), one obtains

$$D(d^4h/dx_1^4) = p - (p_g + p_i) \tag{A-16}$$

One may now differentiate Equation (A-16) with respect to x_1 and substitute Equation (A-2) for $-dp/dx_1$ to obtain the exit zone equation

$$D(d^5h/dx_1^5) = -6\mu U_o\ (h - h^*)/h^3 \tag{A-17}$$

where h^* is now determined from Equation (A-15).

The boundary conditions at $x_1 = 0$ may be obtained from Equations (A-5), (A-7) and (A-16)

$$d^2h/dx_1^2 = d^3h/dx_1^3 = 0 \text{ at } x_1 = 0 \tag{A-18}$$

and

$$D(d^4h/dx_1^4) = p_o - p_g - p_i \text{ at } x = 0 \tag{A-19}$$

Without going through the formality of introducing a stretching transformation as was done for the inlet zone, the exit zone solution must merge with the contact zone solution as $x_1 \to \infty$. Thus, we look for a bounded solution with $h \to h^*$ as $x_1 \to \infty$.

The above system of equations may be solved numerically by Runge-Kutta integration, however; if the variation in h is not excessive ($|h = h^*| << h^*$), then one may replace h^3 appearing in the denominator of the right-hand side of Equation (A-17) with h^{*3} to obtain a linear system of equations with constant coefficients that may readily be solved algebraicly.

The algebraic solution is in the form:

$$h/h^* = 1 + A_1e^{-\zeta} + e^{-\lambda_1\zeta}\ (A_2 \cos \lambda_2\zeta + A_3 \sin \lambda_2\zeta)$$

where

$$\zeta = [6\mu U_o/(h^{*3}D)]^{1/5}\ x_1\ ,\ \lambda_1 = \cos\ (2\pi/5) \text{ and } \lambda_2 = \sin\ (2\pi/5).$$

The constants A_1, A_2 and A_3 are readily determined from Equations (A-18) and (A-19).

A.4 Sample Solution

Film thickness and pressure profiles have been calculated corresponding to the geometry shown in Figure A-2 with the trapezoidal section modeled by the dotted line shown in the figure. A fluid viscosity of 55 cp, an elastic modulus of 1.72 GPa, and a Poisson's ratio of 0.41 have been used in the computation.

As shown on the scale in Figure A-5, the inlet zone film thickness profile is very steep, indicating that only the portion of the inlet zone very near the contact zone is relevant in affecting the film thickness profile, and that the major portion of the seal to the left of this region is probably not necessary. This is again shown in Figure A-6, where the pressure, approximately 1 mm before the start of the contact zone, is very nearly that of the gas (10 MPa). The pressure rises very steeply and joins the pressure in the contact zone smoothly, although it appears to have a sharp corner on the scale in Figure A-6. The pressure profile in the contact zone is constant and equal to the sum of the gas pressure plus the interference pressure (the pressure that would be associated with interference fit statically). Near the exit region, the pressure starts to oscillate, peaks to 18 MPa, and then falls rapidly to 0 (the oil pressure).

Examination of the rapid falloff in pressure at the exit region shows that if the exit oil pressure were raised to be equal to, or even greater than, the gas pressure, there would be little change in the pressure profile except in the immediate vicinity of the exit zone. The film thickness in the contact zone would be virtually unaffected. This indicates that the ambient pressure difference has little effect on the film thickness or the flow rate since the flow is completely dominated by viscous forces. Thus, the seal could be made to provide a film thickness in either direction by adding a chamfer on the reverse side. Such chamfers, which were recommended and tested as part of the Automotive Stirling Engine Program, have been shown to be effective in providing lubrication without excessive leakage in either direction.

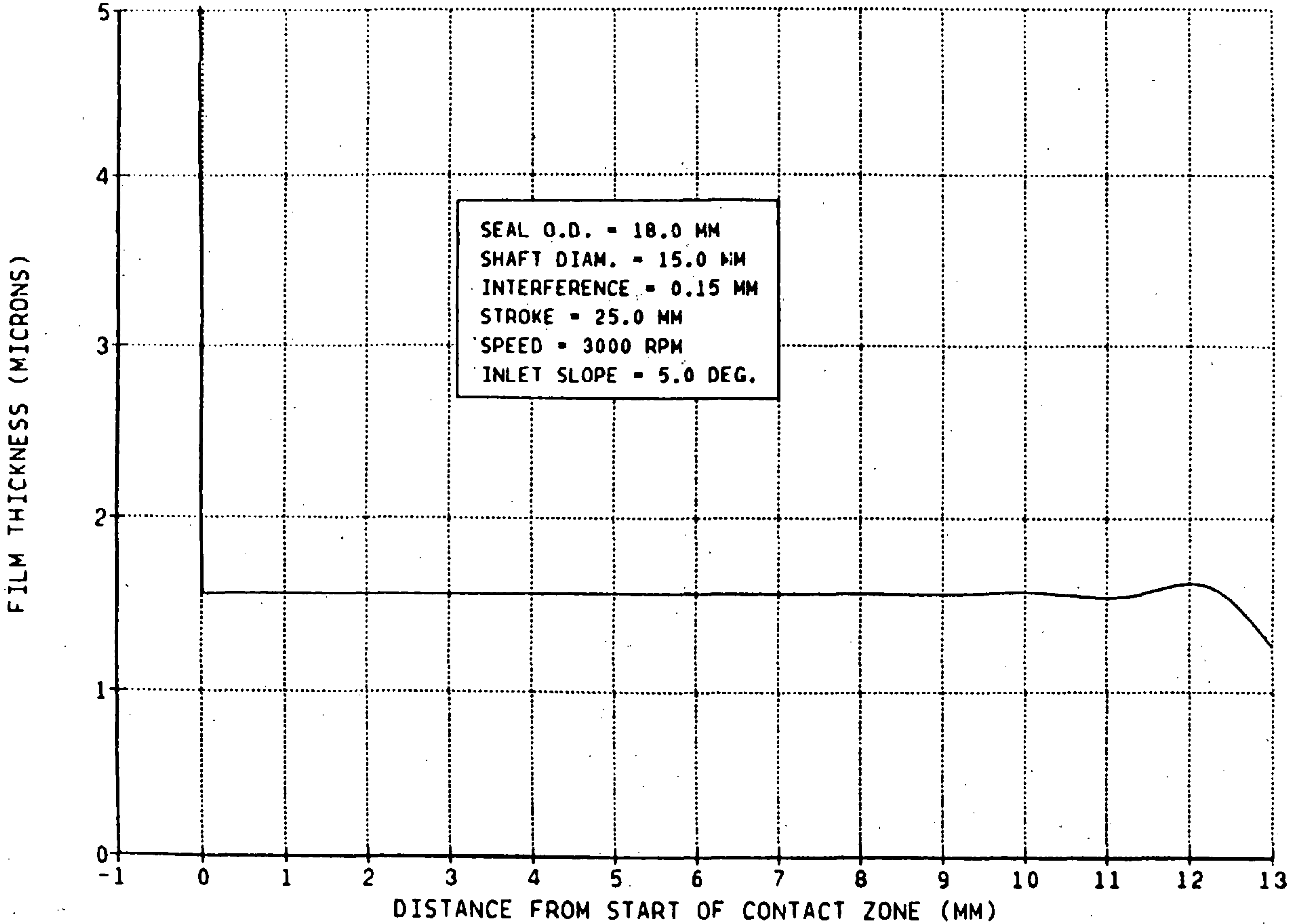


Fig. A-5 Film Thickness Profile

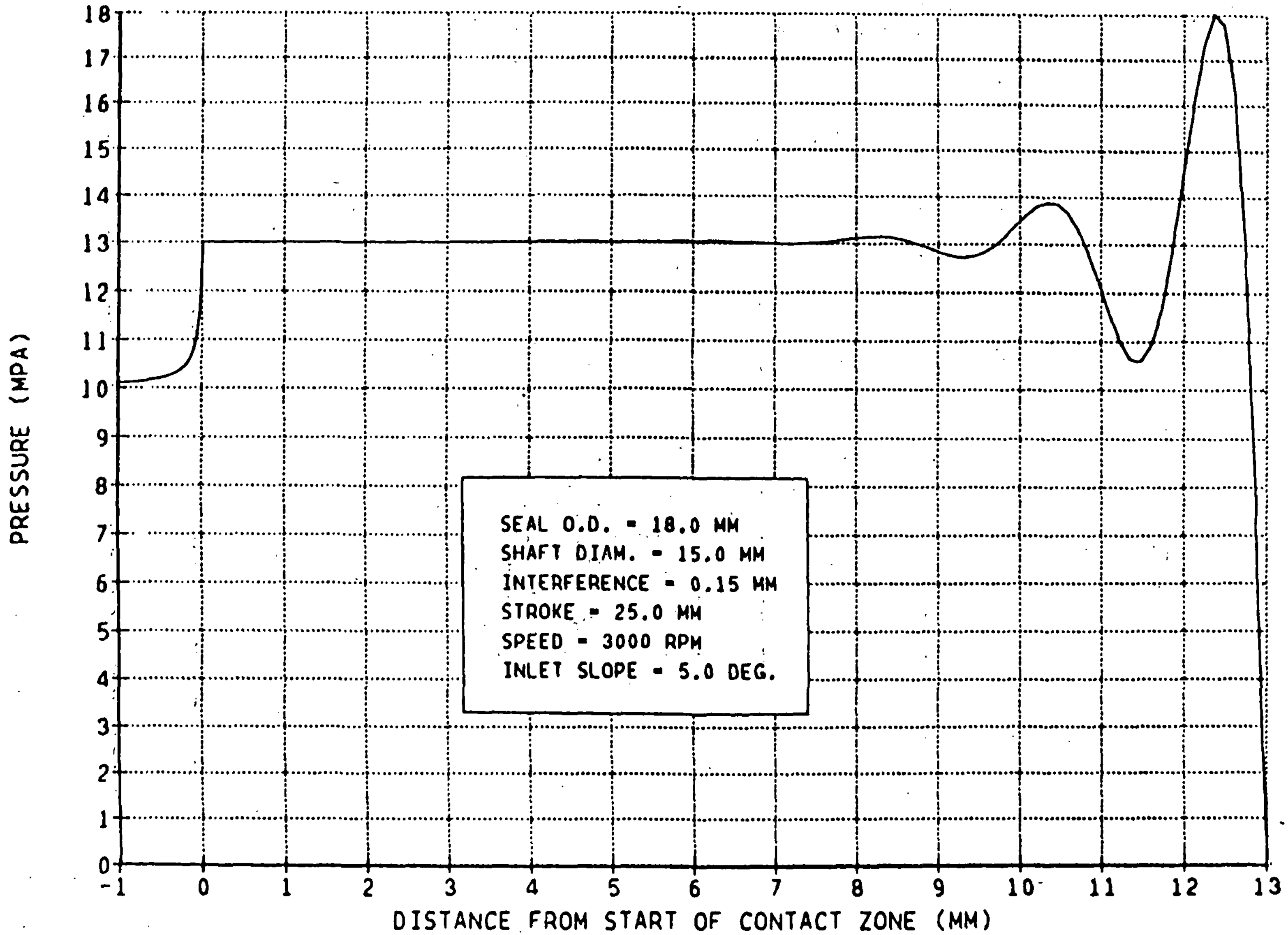


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Fig. A-6 Pressure Profile

APPENDIX B

COMPUTER PROGRAM "RING"

The computer program RING which has been used throughout the report is listed herein. It is written in IBM FORTRAN 77 (VS), and uses an IMSL subroutine ZSCNT. A FORTRAN IV listing of ZSCNT and its subroutines is also included for completeness. The FORTRAN listings are preceded with an input description, an output description, and sample inputs and outputs.

B.1 INPUT DESCRIPTION

All input variables are on a namelist (NAMELIST/INPUTS/). Symbols in the left column denote the FORTRAN names of the namelist variables. Symbols appearing on the right correspond to the nomenclature given at the beginning of the report.

Input Variable Definitions

STRING	A character string of up to 60 characters to identify job
IDIM	Type of input
	0 - Dimensionless (default value)
	1 - Dimensional (any consistent set of units)

Dimensional Inputs (used only if IDIM = 1)

NI	Frequency (Hz), f
SI	Stroke (L), s
CI	Clearance (L), c
ELI	Bearing length (L), L
EL1I	Length of nonbearing portion of ring (L), L_1
EI	Length over which p_o acts (L), e
RI	Shaft radius (L), R
TI	Ring thickness (L), t
C1I	Initial taper, δ (defaults to 0)
POI	Preload pressure (F/L^2), p_o
PFI	Reservoir pressure (F/L^2), p_f
RMUI	Lubricant viscosity (FT/L^2), μ
EMOD	Ring modulus of elasticity (F/L^2), E
POIS	Ring Poisson's Ratio, ν

Nondimensional Inputs

AL	α
BET	β
EPS	ε

EL1	L_1
C1	$\delta L/C$ (defaults to 0)
PO	Po
PF	Pf
ELOS	L/s for starvation calculation (defaults to 0)

NDX — A vector of length 3 containing number of increments for Runge-Kutta integration

NDX(1) = Number of increments in $1 - \varepsilon \leq \xi \leq 1$ (defaults to 40)

NDX(2) = Number of increments in $0 \leq \xi \leq 1 - \varepsilon$ (defaults to 25)

NDX(3) = Number of increments in $-L_1 \leq \xi \leq 0$ (defaults to 25)

NDP — A vector of length (3) containing number of points at which pressures etc. are saved for printout. NDX(i) should be integral multiples of NDP(i). (Defaults are NDP(1) = 10, NDP(2) = 25, NDP(3) = 25.)

ICAV — Flag calculation of cavitation (backstroke)

0 - No cavitation (negative pressures allowed)

2 - Cavitation included, no negative pressures (default value)

IPR — Print Flag

0 - Short output (default value)

1 - Longer output (print pressure profile etc.)

NMAX — Maximum iterations for secant method solution (default is 10)

NSIG — Number of significant digits in accuracy of solution using secant method (default is 4)

XF — A vector of length (2) containing initial guesses for forward stroke secant solution

XF(1) = Initial guess for h at $\xi = 1$

XF(2) = Initial guess for $dh/d\xi$ at $\xi = 1$

If XF(1) and XF(2) are both set equal to 0, the program will attempt to find its own initial guesses. In subsequent runs, values determined from the previous run are used unless explicitly specified.

XB A vector length (2) containing initial guesses for h and $dh/d\xi$ for reverse stroke secant solution. Same rules apply to XB as for XF.

B.2 OUTPUT DESCRIPTION

The output starts with a listing of the inputs which are defined in D.1. Dimensional inputs are listed only if IDIM = 1. All outputs are dimensionless.

The Elastic Solution refers to the deflection of the ring when hydrodynamic forces are not present. If the ring is unclampled, $h(1) > 0$, one line of output is given for the Elastic Solution. If the ring is clamped, $h(1) < 0$, two lines of output are given. The first line refers to the ring deflection relative to an imaginary shaft that would occur if the shaft were not present (the negative value of h noting the interference). The second line of output includes the effect of the interference force exerted by the shaft on the ring at $\xi = 1$ when $h(1) = 0$.

Thw Pumping Flow Solution and Back Flow Solution refer to forward and reverse strokes respectively. They will contain one line of output each at IPR = 0 or multiple outputs if IPR = 1.

The following table relates the output caption with the symbols for variables as defined in the nomenclature for the elastic, pumping flow and back flow solution.

Output Caption	Algebraic Symbol
X	ξ
H	h
H'	$dh/d\xi$
H"	$d^2h/d\xi^2$
H'''	$d^3h/d\xi^3$
PRES	p
FORCE	F

The final output listed under flows relates to dimensionless flow and cavitation output and are given as follows:

Output Caption	Algebraic Symbol
PUMPING	K_F (Forward Stroke)
BACK	K_R (Reverse Stroke)
XCAV	ξ_c
NET FLOW	$K_F - K_R$
STARVED NET FLOW	$K_{F\,eff} - K_R$

B.3 SAMPLE INPUT AND OUTPUT

The sample input and output contained herein correspond to three solutions for a babbitt pumping ring with loading pressures, p_o, of 3.45, 5.17 and 6.89 MPa (500, 750 and 1000 psi). A full printout (IPR = 1) was requested at p_o = 3.45 MPa (500 psi). It should be noted that the ring is predicted to be clamped during the backstroke at p_o = 6.89 MPa (1000 psi).

```
&INPUTS
STRING='BABBIT RING A-1-A-1  P0=500 COMPLETE OUTPUT (IPR=1)'
IDIM=1,              NI =35.,            SI = 2.,            C1I =0.,
CI =.5E-3,           RMUI =.885E-5,      ELI =.267,          POI =500.,
PFI =0.,             EI =.115,           EL1I =.298,         RI =.375,
TI =.047,            EMOD =7.5E6,        POIS =.36,
XF = 0.,0.,          XB = 0.,0.,         IPR=1,
&END
&INPUTS POI=750.,STRING ='BABBIT RING A-1-A-1  P0=750',IPR=0,&END
&INPUTS POI=1000.,STRING ='BABBIT RING A-1-A-1  P0=1000',&END
```

* PUMPING RING ANALYSIS PROGRAM *

INPUTS

```
****************************************************************************
STRING = BABBIT RING A-1-A-1  PO=500 COMPLETE OUTPUT (IPR=1)
****************************************************************************
DIMENSIONAL INPUTS

ELI    = 0.26700000D+00; EL1I  = 0.29800000D+00;
EI     = 0.11500000D+00; CI    = 0.50000000D-03;
C1I    = 0.00000000D+00; TI    = 0.47000000D-01;
SI     = 0.20000000D+01; RI    = 0.37500000D+00;
RMUI   = 0.88500000D-05; NI    = 0.35000000D+02;

POI    = 0.50000000D+03; PFI   = 0.00000000D+00;
EMOD   = 0.75000000D+07; POIS  = 0.36000000D+00;
****************************************************************************
IDIM  = 1;
AL    = 0.66085654D-02; BET   = 0.71535454D+01;
EPS   = 0.43071161D+00; PO    = 0.62976163D-01;
EL1   = 0.11161049D+01; PF    = 0.00000000D+00;
STARV = 0.32041966D+00; C1    = 0.00000000D+00;

NDX   =   40,   25,   25;
NDP   =   10,   25,   25;

NMAX  =   10; NSIG  =    4;
IPR   =    1; ICAV  =    2;
XF    =   0.00000D+00     0.00000D+00
XB    =   0.00000D+00     0.00000D+00
****************************************************************************
```

OUTPUTS

```
****************************************************************************
----------------------------------------------------------------------------
```

* ELASTIC SOLUTION *

X	H	H'	H''	H'''	PRES	FORCE
1.0000	0.48845	-0.67281	0.00000	0.00000	0.00000	0.00000

* PUMPING FLOW SOLUTION *

X	H	H'	H''	H'''	PRES
1.0000	0.81814	-0.69559	0.00000	0.00000	0.00000
0.9914	0.82413	-0.69558	-0.00147	0.33645	0.00375
0.9742	0.83611	-0.69548	-0.01254	0.93187	0.01074
0.9569	0.84809	-0.69510	-0.03304	1.43418	0.01709
0.9397	0.86006	-0.69429	-0.06149	1.85487	0.02285
0.9225	0.87201	-0.69294	-0.09655	2.20463	0.02805
0.9052	0.88393	-0.69093	-0.13710	2.49334	0.03274
0.8880	0.89582	-0.68819	-0.18216	2.73019	0.03695
0.8708	0.90764	-0.68464	-0.23092	2.92369	0.04071
0.8536	0.91940	-0.68022	-0.28270	3.08173	0.04405
0.8363	0.93108	-0.67488	-0.33695	3.21161	0.04701
0.8191	0.94265	-0.66859	-0.39324	3.32008	0.04960
0.8019	0.95411	-0.66132	-0.45126	3.41339	0.05186

0.7846	0.96543	-0.65304	-0.51080	3.49729	0.05380
0.7674	0.97660	-0.64371	-0.57175	3.57709	0.05545
0.7502	0.98761	-0.63333	-0.63406	3.65768	0.05682
0.7330	0.99842	-0.62186	-0.69781	3.74354	0.05793
0.7157	1.00903	-0.60927	-0.76311	3.83880	0.05881
0.6985	1.01941	-0.59555	-0.83016	3.94719	0.05946
0.6813	1.02954	-0.58066	-0.89921	4.07216	0.05990
0.6640	1.03941	-0.56456	-0.97058	4.21679	0.06015
0.6468	1.04899	-0.54720	-1.04464	4.38389	0.06021
0.6296	1.05825	-0.52854	-1.12178	4.57598	0.06011
0.6124	1.06719	-0.50853	-1.20247	4.79530	0.05984
0.5951	1.07577	-0.48709	-1.28718	5.04382	0.05943
0.5779	1.08396	-0.46415	-1.37644	5.32328	0.05887
0.5693	1.08791	-0.45209	-1.42295	5.47508	0.05855
0.5579	1.09297	-0.43555	-1.48207	4.91228	0.05807
0.5351	1.10249	-0.40062	-1.58150	3.83132	0.05695
0.5124	1.11120	-0.36370	-1.65702	2.81167	0.05564
0.4896	1.11904	-0.32532	-1.71003	1.85497	0.05416
0.4668	1.12600	-0.28598	-1.74199	0.96235	0.05252
0.4440	1.13206	-0.24614	-1.75435	0.13448	0.05073
0.4213	1.13721	-0.20622	-1.74861	-0.62831	0.04881
0.3985	1.14146	-0.16663	-1.72623	-1.32603	0.04676
0.3757	1.14481	-0.12772	-1.68871	-1.95894	0.04461
0.3530	1.14728	-0.08982	-1.63751	-2.52748	0.04236
0.3302	1.14891	-0.05323	-1.57408	-3.03230	0.04001
0.3074	1.14972	-0.01821	-1.49988	-3.47412	0.03759
0.2846	1.14975	0.01500	-1.41633	-3.85377	0.03509
0.2619	1.14905	0.04623	-1.32484	-4.17212	0.03252
0.2391	1.14766	0.07529	-1.22678	-4.43005	0.02989
0.2163	1.14564	0.10206	-1.12353	-4.62844	0.02721
0.1936	1.14303	0.12643	-1.01643	-4.76813	0.02448
0.1708	1.13990	0.14833	-0.90681	-4.84992	0.02171
0.1480	1.13630	0.16772	-0.79599	-4.87453	0.01890
0.1252	1.13228	0.18459	-0.68524	-4.84260	0.01606
0.1025	1.12791	0.19894	-0.57586	-4.75471	0.01319
0.0797	1.12324	0.21083	-0.46912	-4.61129	0.01029
0.0569	1.11833	0.22034	-0.36627	-4.41272	0.00737
0.0342	1.11322	0.22755	-0.26857	-4.15924	0.00443
0.0114	1.10798	0.23262	-0.17726	-3.85102	0.00148
-0.0000	1.10532	0.23439	-0.13439	-3.67640	0.00000

---

* BACK FLOW SOLUTION *

X	H	H'	H''	H'''	PRES
1.0000	0.48845	-0.67281	0.00000	0.00000	0.00000
0.9914	0.49424	-0.67281	0.00033	-0.07580	0.00000
0.9742	0.50584	-0.67284	0.00279	-0.20473	0.00000
0.9569	0.51743	-0.67292	0.00721	-0.30344	0.00000
0.9397	0.52902	-0.67309	0.01307	-0.37193	0.00000
0.9225	0.54062	-0.67338	0.01985	-0.41019	0.00000
0.9052	0.55223	-0.67378	0.02703	-0.41820	0.00000
0.8880	0.56384	-0.67431	0.03409	-0.39595	0.00000
0.8708	0.57546	-0.67495	0.04050	-0.34341	0.00000
0.8536	0.58710	-0.67570	0.04575	-0.26055	0.00000
0.8363	0.59875	-0.67652	0.04931	-0.14735	0.00000

0.8191	0.61041	-0.67738	0.05065	-0.00376	0.00000
0.8019	0.62209	-0.67825	0.04926	0.17025	0.00000
0.7846	0.63378	-0.67906	0.04461	0.37472	0.00000
0.7674	0.64548	-0.67976	0.03617	0.60970	0.00000
0.7502	0.65720	-0.68028	0.02343	0.87520	0.00000
0.7330	0.66892	-0.68054	0.00584	1.17126	0.00000
0.7157	0.68065	-0.68045	-0.01711	1.49788	0.00000
0.6985	0.69237	-0.67992	-0.04595	1.85507	0.00000
0.6813	0.70407	-0.67883	-0.08120	2.24279	0.00000
0.6640	0.71575	-0.67708	-0.12340	2.66099	0.00000
0.6468	0.72740	-0.67454	-0.17307	3.10961	0.00000
0.6296	0.73899	-0.67107	-0.23072	3.58851	0.00000
0.6124	0.75051	-0.66654	-0.29689	4.09756	0.00000
0.5951	0.76195	-0.66079	-0.37208	4.63653	0.00000
0.5779	0.77327	-0.65366	-0.45682	5.20518	0.00000
0.5693	0.77889	-0.64953	-0.50292	5.50054	0.00000
0.5579	0.78625	-0.64346	-0.56341	5.12594	0.00000
0.5351	0.80075	-0.62936	-0.67194	4.41448	0.00000
0.5124	0.81490	-0.61297	-0.76483	3.75238	0.00000
0.4896	0.82865	-0.59464	-0.84320	3.13837	0.00000
0.4668	0.84196	-0.57467	-0.90812	2.57100	0.00000
0.4440	0.85481	-0.55337	-0.96063	2.04872	0.00000
0.4213	0.86716	-0.53101	-1.00175	1.56985	0.00000
0.3985	0.87899	-0.50783	-1.03244	1.13265	0.00000
0.3757	0.89028	-0.48406	-1.05364	0.73528	0.00000
0.3530	0.90103	-0.45991	-1.06622	0.37590	0.00000
0.3302	0.91123	-0.43556	-1.07103	0.05260	0.00000
0.3074	0.92087	-0.41118	-1.06887	-0.23653	0.00000
0.2846	0.92995	-0.38693	-1.06050	-0.49338	0.00000
0.2619	0.93849	-0.36293	-1.04663	-0.71988	0.00000
0.2391	0.94649	-0.33930	-1.02793	-0.91790	0.00000
0.2163	0.95395	-0.31614	-1.00503	-1.08929	0.00000
0.1936	0.96089	-0.29355	-0.97851	-1.23587	0.00000
0.1708	0.96732	-0.27160	-0.94892	-1.35941	0.00000
0.1480	0.97326	-0.25036	-0.91676	-1.46164	0.00000
0.1252	0.97873	-0.22987	-0.88250	-1.54421	0.00000
0.1025	0.98374	-0.21018	-0.84657	-1.60875	0.00000
0.0797	0.98831	-0.19132	-0.80936	-1.65679	0.00000
0.0569	0.99246	-0.17332	-0.77123	-1.68981	0.00000
0.0342	0.99621	-0.15620	-0.73251	-1.70922	0.00000
0.0114	0.99958	-0.13996	-0.69348	-1.71637	0.00000
-0.0000	1.00113	-0.13218	-0.67394	-1.71575	0.00000

---

* FLOWS *

PUMPING	BACK	XCAV	NET FLOW	STARVED NET FLOW
0.106219D+01	0.488604D+00	0.999770D+00	0.573582D+00	0.296125D+00

****

```
* PUMPING RING ANALYSIS PROGRAM *

INPUTS
*************************************************************************
STRING = BABBIT RING A-1-A-1  PO=750
*************************************************************************
DIMENSIONAL INPUTS

ELI    = 0.26700000D+00; EL1I  = 0.29800000D+00;
EI     = 0.11500000D+00; CI    = 0.50000000D-03;
C1I    = 0.00000000D+00; TI    = 0.47000000D-01;
SI     = 0.20000000D+01; RI    = 0.37500000D+00;
RMUI   = 0.88500000D-05; NI    = 0.35000000D+02;

POI    = 0.75000000D+03; PFI   = 0.00000000D+00;
EMOD   = 0.75000000D+07; POIS  = 0.36000000D+00;
*************************************************************************
IDIM  = 1;
AL    = 0.66085654D-02; BET   = 0.71535454D+01;
EPS   = 0.43071161D+00; PO    = 0.94464244D-01;
EL1   = 0.11161049D+01; PF    = 0.00000000D+00;
STARV = 0.32041966D+00; C1    = 0.00000000D+00;

NDX   =   40,   25,   25;
NDP   =   10,   25,   25;

NMAX  =   10; NSIG  =    4;
IPR   =    0; ICAV  =    2;
XF    =    0.81814D+00   -0.69559D+00
XB    =    0.48845D+00   -0.67281D+00
*************************************************************************

OUTPUTS
*************************************************************************
-------------------------------------------------------------------------
                         * ELASTIC SOLUTION *

    X        H        H'        H''       H'''      PRES        FORCE
 1.0000   0.23267  -1.00922   0.00000   0.00000   0.00000     0.00000
-------------------------------------------------------------------------
                      * PUMPING FLOW SOLUTION *

      X          H          H'          H''          H'''          PRES

   1.0000     0.71873    -0.93560     0.00000      0.00000      0.00000

-------------------------------------------------------------------------
                        * BACK FLOW SOLUTION *

      X          H          H'          H''          H'''          PRES

   1.0000     0.23267    -1.00922     0.00000      0.00000      0.00000

-------------------------------------------------------------------------
                             * FLOWS *

   PUMPING          BACK           XCAV          NET FLOW    STARVED NET FLOW
 0.100210D+01   0.232747D+00   0.999927D+00   0.769350D+00   0.329915D+00
*************************************************************************
```

```
* PUMPING RING ANALYSIS PROGRAM *

INPUTS
*************************************************************************
STRING = BABBIT RING A-1-A-1  P0=1000
*************************************************************************
DIMENSIONAL INPUTS

ELI    = 0.26700000D+00; EL1I  = 0.29800000D+00;
EI     = 0.11500000D+00; CI    = 0.50000000D-03;
C1I    = 0.00000000D+00; TI    = 0.47000000D-01;
SI     = 0.20000000D+01; RI    = 0.37500000D+00;
RMUI   = 0.88500000D-05; NI    = 0.35000000D+02;

POI    = 0.10000000D+04; PFI   = 0.00000000D+00;
EMOD   = 0.75000000D+07; POIS  = 0.36000000D+00;
*************************************************************************
IDIM  = 1;
AL    = 0.66085654D-02; BET   = 0.71535454D+01;
EPS   = 0.43071161D+00; P0    = 0.12595233D+00;
EL1   = 0.11161049D+01; PF    = 0.00000000D+00;
STARV = 0.32041966D+00; C1    = 0.00000000D+00;

NDX   =   40,   25,   25;
NDP   =   10,   25,   25;

NMAX  =   10; NSIG  =    4;
IPR   =    0; ICAV  =    2;
XF    =    0.71873D+00   -0.93560D+00
XB    =    0.23267D+00   -0.10092D+01
*************************************************************************

OUTPUTS
*************************************************************************
-------------------------------------------------------------------------
                       * ELASTIC SOLUTION *

    X         H         H'        H''       H'''      PRES       FORCE
 1.0000  -0.02310  -1.34562   0.00000   0.00000   0.00000     0.00000
 1.0000   0.00000  -1.28833   0.00000  -0.70479   0.00000     0.00065
-------------------------------------------------------------------------
                    * PUMPING FLOW SOLUTION *

      X          H          H'          H''         H'''        PRES

   1.0000     0.63082    -1.13187     0.00000     0.00000     0.00000

-------------------------------------------------------------------------
                      * BACK FLOW SOLUTION *

    X         H         H'        H''       H'''      PRES       FORCE
 1.0000   0.00000  -1.28833   0.00000  -0.70479   0.00000     0.00065
-------------------------------------------------------------------------
                            * FLOWS *

   PUMPING           BACK            XCAV          NET FLOW   STARVED NET FLOW
 0.929129D+00    0.000000D+00    0.100000D+01    0.929129D+00    0.333707D+00
*************************************************************************
```

B.4 FORTRAN LISTING


```
      PROGRAM RING
C$ VS
C>-----------------------------------------------------------------------
CNOTE:  XLAM=.739*ELOS**-.585 IS NOW BUILT INTO PROGRAM
C
C
C      FUNCTION              - PUMPING RING ANALYSIS PROGRAM
C
C      RESTRICTIONS          - STEADY-STATE ANALYSIS ONLY
C
C      REMARKS               - VS FORTRAN (FORTRAN 77)
C                              IMPLICIT DOUBLE PRECISION REAL*8
C
C      EXTERNAL REFERENCES -
C        FORTRAN ROUTINES
C                              DATAN
C        IMSL ROUTINES
C                              ZSCNT ; SOLVES THE SYSTEM OF NON-LINEAR
C                              EQUATIONS ; LISTING PROVIDED HEREIN BY
C                              PERMISSION OF IMSL.
C        USER ROUTINES
C                              AIN ; COMPUTE INVERSE OF 2X2 MATRIX
C                              AMU ; MULTIPLY 2 2X2 MATRICES
C                              CALCD ; CALCULATE ELASTIC INFLUENCE
C                                      COEFFICIENTS C,D
C                              CHECK ; FOR MULTIPLE RUNS CHECK INPUTS
C                                      FOR RECALCULATION OF C,D
C                              CONST ; CALCULATE SLIDER BEARING PRESSURE
C                                      CONSTANTS
C                              CONST2 ; SAME AS ABOVE
C                              DFN1 ; DERIVATIVE FUNCTION USED BY RUK
C                              DFN2 ;    "         "      "   "   "
C                              DFN3 ;    "         "      "   "   "
C                              ELAS ; DETERMINE ELASTIC SOLUTION
C                                     (NO HYDRODYNAMICS)
C                              ERRMSG ; PRINT ERROR MESSAGE IF *ZSCNT* NOT
C                                       CONVERGED
C                              EVAL ; DEFINE NON-LINEAR SYSTEM IN H AND H'
C                                     TO BE SOLVED BY *ZSCNT*
C                              P ; PRESSURE FUNCTION
C                              PRT ; CONVERT FROM W TO H AND PRINT
C                              PRTOUT ; PRINT OUT RESULTS
C                              RUK ; RUNGE-KUTTA
C
C      INPUT/OUTPUT:
C      UNIT         DESCRIPTION
C        4            TERMINAL I/O
C        5            INPUT FILE IN NAMELIST FORMAT
C        6            OUTPUT FILE
C
C      INPUT VARIABLE DEFINITIONS
C        * NOTE : (D) INDICATES VARIABLE HAS A DEFAULT VALUE
C
C      NAME         DESCRIPTION
C       NAMELIST /INPUTS/
```

```
C
C     STRING      CHARACTER STRING TO IDENTIFY JOB (MAX. 60 CHARS.)
C     IDIM (D)    TYPE OF INPUT
C                  0 - DIMENSIONLESS
C                  1 - DIMENSIONAL (ANY CONSISTENT SET OF UNITS)
C
C  DIMENSIONAL INPUTS (NOTE: U0=2.*NI*SI)
C
C     NI          FREQUENCY (HZ)
C     SI          STROKE (L)
C     CI          CLEARANCE (L)
C     ELI         BEARING LENGTH (L)
C     EL1I        LENGTH OF NON-BEARING PORTION OF RING (L)
C     EI          LENGTH FROM END OF BEARING PRELOADED WITH P0I (L)
C     RI          RING RADIUS (L)
C     TI          RING THICKNESS (L)
C     C1I (D)     BEARING SLOPE
C     P0I         PRELOAD PRESSURE (F/L**2)
C     PFI         RESERVOIR PRESSURE (F/L**2)
C     RMUI        LUBRICANT VISCOSITY (FT/L**2)
C     EMOD        RING MODULUS OF ELASTICITY (F/L**2)
C     POIS        RING POISSON'S RATIO
C
C  NON-DIMENSIONAL INPUTS
C
C     AL          (TI*RI)**2/(12*ELI**4*(1.-POIS)**2))
C     BET         (T*RMUI*U0*RI**2*ELI)/(CI**3*TI*EMOD)
C     EPS         LENGTH FROM END OF PRELOAD P0 (EI/ELI)
C     EL1         LENGTH OF NON-BEARING PORTION OF RING (EL1I/ELI)
C     ELOS (D)    L/S LAND TO STROKE RATIO FOR STARVATION CALCULATION
C     XLAM (D)    MULTIPLIES ELOS FOR INCREASED STARVATION
C     C1 (D)      SLOPE OF BEARING
C     P0          PRELOAD PRESSURE (CI**2/(6*RMUI*U0*ELI))*P0I
C     PF          RESERVOIR PRESSURE (CI**2/(6*RMUI*U0*ELI))*PFI
C
C     NDX(3) (D)  DELTA X INCREMENTS FOR RUK
C     NDP(3) (D)   "   "     "      "   "
C
C     ICAV (D)    FLAG FOR CALCULATION OF CAVITATION (BACK STROKE)
C                  0 - NO CAVITATION (NEG. PRESSURES ALLOWED)
C                  1 - CAVITATION (NO NEG. PRESSURES)
C                  2 - FIND ITS OWN SOLUTION
C     IPR  (D)    PRINT FLAG
C                  0 - SHORT OUTPUT
C                  1 - LONGER OUTPUT (PRINT PRESSURE PROFILE)
C     NMAX (D)    MAX. ITERATIONS FOR *ZSCNT*
C     NSIG (D)    NO. OF SIGNIFICANT DIGITS IN ACCURACY
C                  OF SOLUTION USING *ZSCNT*
C     XF(2)       INITIAL GUESS FOR H AND H' FOR FORWARD STROKE
C     XB(2)       INITIAL GUESS FOR H AND H' FOR BACKWARD STROKE
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
C
```

```
      COMMON/BDIM  /CI,ELI,CON1,IDIM
      COMMON/BPAR  /AL,BET,P0,EPS,PF
      COMMON/BCD   /XLOC(100),C(100),D(100),NDX(3),NDP(3),DX(3),DP(3)
      COMMON/BCOEFF/RK,RC,R1,R2,XCAV,ICAV
      COMMON/BINT  /IELAS,IBACK,IPR
      COMMON/BLAST /X1LAST,X2LAST,F1LAST,F2LAST
      COMMON/BELAS /FORCE,HELAS,H1ELAS,H2ELAS,H3ELAS,W1(4),W2(4)
      COMMON/BPROF /C1
      COMMON/BFLAG /IFLAG,IPLOAD
C
      DIMENSION PAR(1),XX(2),XF(2),XB(2),WK(68)
      REAL*8    NFLOW,NI
      CHARACTER*60 STRING
      LOGICAL   RECALC
      EXTERNAL  EVAL,EVAL3,EVAL4
      NAMELIST  /INPUTS/
C
C     DIMENSIONAL INPUTS
C
     + STRING,NI,SI,C1I,CI,RMUI,ELI,P0I,PFI,EI,EL1I,RI,TI,EMOD,POIS
C
C     NON-DIMENSIONAL INPUTS
C
     +,BET,P0,EPS,AL,EL1,ELOS,XLAM,
     +NDX,NDP,PF,NMAX,IPR,NSIG,ICAV,XF,XB,C1,IDIM
C
      PI=4.*DATAN(1.D0)
C
C     DEFAULTS
C
      IDIM=0
      ELOS=0.
      XLAM=1.
      C1I=0.
      C1=0.
      NDX(1)=40
      NDX(2)=25
      NDX(3)=25
      NDP(1)=10
      NDP(2)=25
      NDP(3)=25
      NMAX=10
      NSIG=4
      ICAV=2
      IPR=0
C
      IRUN=0
2001  READ(05,INPUTS,END=2000)
C
      IRUN=IRUN+1
      WRITE(6,'(''1* PUMPING RING ANALYSIS PROGRAM *'',/)')
      WRITE(6,'('' INPUTS'')')
      WRITE(6,'(1X,72(''*''))')
      WRITE(6,'('' STRING = '',A60)') STRING
      WRITE(6,'(1X,72(''*''))')
```

```
      IF(IDIM.NE.0)THEN
         WRITE(6,'('' DIMENSIONAL INPUTS'',/)')
         WRITE(6,'('' ELI    ='',E15.8,''; EL1I  ='',E15.8,'';'')')
     +ELI,EL1I
         WRITE(6,'('' EI     ='',E15.8,''; CI    ='',E15.8,'';'')')EI,CI
         WRITE(6,'('' C1I    ='',E15.8,''; TI    ='',E15.8,'';'')')
     +C1I,TI
         WRITE(6,'('' SI     ='',E15.8,''; RI    ='',E15.8,'';'')')SI,RI
         WRITE(6,'('' RMUI   ='',E15.8,''; NI    ='',E15.8,'';'')')
     +RMUI,NI
         WRITE(6,'('' '')')
         WRITE(6,'('' POI    ='',E15.8,''; PFI   ='',E15.8,'';'')')
     +POI,PFI
         WRITE(6,'('' EMOD   ='',E15.8,''; POIS  ='',E15.8,'';'')')
     +EMOD,POIS
         WRITE(6,'(1X,72(''*''))')
         U0=2.*NI*SI
         CON1=6.*RMUI*U0*ELI/CI**2
         ELOS=ELI/SI
         AL=(TI*(RI+TI*.5))**2/12./ELI**4/(1.-POIS**2)
         BET=CON1*(RI+TI*.5)**2/CI/TI/EMOD
         EPS=EI/ELI
         EL1=EL1I/ELI
         C1=C1I/CI*ELI
         P0=POI/CON1
         PF=PFI/CON1
      END IF
      IF(ELOS.GT.1.D-5)XLAM=.739/ELOS**.585
      STARV=ELOS*XLAM
      WRITE(6,'('' IDIM  ='',I2,'';'')')IDIM
      WRITE(6,'('' AL    ='',E15.8,''; BET   ='',E15.8,'';'')')AL,BET
      WRITE(6,'('' EPS   ='',E15.8,''; P0    ='',E15.8,'';'')')EPS,P0
      WRITE(6,'('' EL1   ='',E15.8,''; PF    ='',E15.8,'';'')')EL1,PF
      WRITE(6,'('' STARV ='',E15.8,''; C1    ='',E15.8,'';'')')STARV,C1
      WRITE(6,'('' '')')
      WRITE(6,'('' NDX   ='',I5,'','',I5,'','',I5,'';'')')NDX
      WRITE(6,'('' NDP   ='',I5,'','',I5,'','',I5,'';'')')NDP
      WRITE(6,'('' '')')
      WRITE(6,'('' NMAX  ='',I5,''; NSIG  ='',I5,'';'')')NMAX,NSIG
      WRITE(6,'('' IPR   ='',I5,''; ICAV  ='',I5,'';'')')IPR,ICAV
      WRITE(6,'('' XF    ='',2E15.5)')XF(1),XF(2)
      WRITE(6,'('' XB    ='',2E15.5)')XB(1),XB(2)
      WRITE(6,'(1X,72(''*''),/)')
      WRITE(6,'('' OUTPUTS'')')
      WRITE(6,'(1X,72(''*''))')
C
      IFLAG=0
      IF(0..LT.EPS.AND.EPS.LT.1.)THEN
         IFLAG=1
         DX(1)=EL1/FLOAT(NDX(1))
         DP(1)=EL1/FLOAT(NDP(1))
         DX(2)=(1.-EPS)/FLOAT(NDX(2))
         DP(2)=(1.-EPS)/FLOAT(NDP(2))
         DX(3)=EPS/FLOAT(NDX(3))
         DP(3)=EPS/FLOAT(NDP(3))
```

```
      ELSE IF(EPS.EQ.1.)THEN
         IFLAG=2
         DX(1)=EL1/FLOAT(NDX(1))
         DP(1)=EL1/FLOAT(NDP(1))
         DX(2)=1./FLOAT(NDX(2))
         DP(2)=1./FLOAT(NDP(2))
         DX(3)=0.
         DP(3)=0.
      ELSE IF(1..LT.EPS.AND.EPS.LT.EL1+1)THEN
         IFLAG=3
         DX(1)=(EL1+1.-EPS)/FLOAT(NDX(1))
         DP(1)=(EL1+1.-EPS)/FLOAT(NDP(1))
         DX(2)=(EPS-1.)/FLOAT(NDX(2))
         DP(2)=(EPS-1.)/FLOAT(NDP(2))
         DX(3)=1./FLOAT(NDX(3))
         DP(3)=1./FLOAT(NDP(3))
      ELSE IF(EPS.EQ.1.+EL1)THEN
         IFLAG=4
         DX(1)=EL1/FLOAT(NDX(1))
         DP(1)=EL1/FLOAT(NDP(1))
         DX(2)=1./FLOAT(NDX(2))
         DP(2)=1./FLOAT(NDP(2))
         DX(3)=0.
         DP(3)=0.
      END IF
      IF(IFLAG.EQ.0)STOP
C
      CALL CHECK(AL,BET,EPS,EL1,NDX,NDP,IRUN,RECALC)
      IF(RECALC)THEN
         IF(IFLAG.LE.2)THEN
            CALL CALCD
         ELSE IF(IFLAG.EQ.3)THEN
            CALL CALCD3
         ELSE IF(IFLAG.EQ.4)THEN
            CALL CALCD4
         END IF
      ELSE
C        WRITE(6,'(20X,'' * NOTE: C,D NOT RECALCULATED ! *'')')
      END IF
C
C     ELASTIC ANALYSIS
C
      WRITE(6,'('' '',72(''-''))')
      WRITE(6,'(26X,'' * ELASTIC SOLUTION *'',/)')
      CALL ELAS
C
C     STEADY STATE ANALYSIS
C
      IELAS=0
C
C     FORWARD FLOW SOLUTION
C
      IBACK=0
      WRITE(6,'('' '',72(''-''))')
      WRITE(6,'(23X,'' * PUMPING FLOW SOLUTION *'',/)')
```

```
      IF(XF(1)**2+XF(2)**2.LT.1.D-5)THEN
         XX(1)=1.-HELAS+.01
         XX(2)=C1-H1ELAS+.01
      ELSE
         XX(1)=1.-XF(1)
         XX(2)=C1-XF(2)
      END IF
      IF(IFLAG.LE.2)THEN
         CALL ZSCNT(EVAL,NSIG,2,NMAX,PAR,XX,FNORM,WK,IER)
      ELSE IF(IFLAG.EQ.3)THEN
         CALL ZSCNT(EVAL3,NSIG,2,NMAX,PAR,XX,FNORM,WK,IER)
      ELSE IF(IFLAG.EQ.4)THEN
         CALL ZSCNT(EVAL4,NSIG,2,NMAX,PAR,XX,FNORM,WK,IER)
      END IF
      IF(IER.NE.0)CALL ERRMSG
      CALL PRTOUT(XX)
      XF(1)=1.-XX(1)
      XF(2)=C1-XX(2)
      FFLOW=RK
C
C     BACK FLOW SOLUTION
C
      IBACK=1
      WRITE(6,'('' '',72(''-''))')
      WRITE(6,'(25X,'' * BACK FLOW SOLUTION *'',/)')
      IF(XB(1)**2+XB(2)**2.LT.1.D-5)THEN
         XX(1)=1.-HELAS
         XX(2)=C1-H1ELAS
      ELSE
         XX(1)=1.-XB(1)
         XX(2)=C1-XB(2)
      END IF
      BFLOW=0.
      IF(FORCE.LT.1.D-8)THEN
         IF(IFLAG.LE.2)THEN
            CALL ZSCNT(EVAL,NSIG,2,NMAX,PAR,XX,FNORM,WK,IER)
         ELSE IF(IFLAG.EQ.3)THEN
            CALL ZSCNT(EVAL3,NSIG,2,NMAX,PAR,XX,FNORM,WK,IER)
         ELSE IF(IFLAG.EQ.4)THEN
            CALL ZSCNT(EVAL4,NSIG,2,NMAX,PAR,XX,FNORM,WK,IER)
         END IF
         IF(IER.NE.0)CALL ERRMSG
         BFLOW=RK
      END IF
      CALL PRTOUT(XX)
      XB(1)=1.-XX(1)
      XB(2)=C1-XX(2)
C
      WRITE(6,'('' '',72(''-''))')
3000  CONTINUE
      NFLOW=FFLOW-BFLOW
      WRITE(6,'(32X,''* FLOWS *'',/)')
      WRITE(6,'(5X,''PUMPING'',10X,''BACK'',11X,''XCAV'',9X
     +   ,''NET FLOW'',3X,''STARVED NET FLOW'')')
      SFLOW=NFLOW
```

```
      HSTV=XB(1)
      DHSTV=XB(2)
         IF(IBACK.EQ.1.AND.FORCE.GT.1.D-8)THEN
         XCAV=1.
         HSTV=HELAS
         DHSTV=H1ELAS
         END IF
      IF(XCAV.GT.1.D-6.AND.XCAV.LT.1.000001)SFLOW=FFLOW*(1.+
     +2.*ELOS*XLAM*XCAV**2*DHSTV/(2.*HSTV-DHSTV*(2.-XCAV)))-BFLOW
      WRITE(6,'(5E15.6)')FFLOW,BFLOW
     +   ,XCAV,NFLOW,SFLOW
      WRITE(6,'(1X,72(''*''),/)')
      GOTO 2001
2000  CALL EXIT
      END

      SUBROUTINE AIN(A,B)
C>-----------------------------------------------------------------------
C
C
C      FUNCTION             - CALCULATE INVERSE OF 2X2 MATRIX
C
C      RESTRICTIONS         -
C
C      REMARKS              -
C
C      EXTERNAL REFERENCES - NONE
C
C      ARGUMENT DEFINITION:
C      NAME         DESCRIPTION
C        A            INPUT MATRIX
C        B            OUTPUT MATRIX (INVERSE OF A)
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(2,2),B(2,2)
      D=A(1,1)*A(2,2)-A(2,1)*A(1,2)
      B(1,1)=A(2,2)/D
      B(2,2)=A(1,1)/D
      B(1,2)=-A(1,2)/D
      B(2,1)=-A(2,1)/D
      RETURN
      END
```

```
      SUBROUTINE AMU(A,B,C)
C>-----------------------------------------------------------------------
C
C
C      FUNCTION              - PERFORM MATRIX MULTIPICATION OF 2X2 MATRIX
C
C      RESTRICTIONS          -
C
C      REMARKS               -
C
C      EXTERNAL REFERENCES - NONE
C
C      ARGUMENT DEFINITION:
C      NAME         DESCRIPTION
C        A            INPUT MATRIX
C        B            INPUT MATRIX
C        C            OUTPUT MATRIX (C=AXB)
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION A(2,2),B(2,2),C(2,2)
      C(1,1)=A(1,1)*B(1,1)+A(1,2)*B(2,1)
      C(2,1)=A(2,1)*B(1,1)+A(2,2)*B(2,1)
      C(1,2)=A(1,1)*B(1,2)+A(1,2)*B(2,2)
      C(2,2)=A(2,1)*B(1,2)+A(2,2)*B(2,2)
      RETURN
      END
```

```
      SUBROUTINE CALCD
C>-----------------------------------------------------------------------
C
C
C      FUNCTION                - CALCULATE INFLUENCE COEFFICIENTS C,D
C                                FOR H AND H' AT PSI=1
C
C      RESTRICTIONS            - FOR MULTIPLE RUNS, ONLY COMPUTED WHEN
C                                CERTAIN PARAMETERS CHANGE.  ALWAYS
C                                COMPUTED AT LEAST ONCE.
C
C      REMARKS                 - NOTE VARIABLES PASSED IN COMMON
C
C      EXTERNAL REFERENCES - AIN,AMU,RUK
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BCD    /XLOC(100),C(100),D(100),NDX(3),NDP(3),DX(3),DP(3)
      COMMON/BELAS /FORCE,HELAS,H1ELAS,H2ELAS,H3ELAS,W1(4),W2(4)
      COMMON/BPAR   /AL,BET,PO,EPS,PF,S,U,DT
      COMMON/BFLAG /IFLAG,IPLOAD
      DIMENSION YO(4),YT(4),DJ(4),CKJ(4,4)
      DIMENSION XLEN(3)
      DIMENSION A1(2,2,100),A2(2,2,100)
      DIMENSION B1(2,2,100),B2(2,2,100)
      DIMENSION A1I(2,2),AT(2,2)
      EXTERNAL DFN1
C
      XLEN(1)=DX(1)*NDX(1)
      XLEN(2)=DX(2)*NDX(2)
      XLEN(3)=DX(3)*NDX(3)
      EL1=XLEN(1)
      YO(1)=0.
      YO(2)=0.
      YO(3)=1.
      YO(4)=0.
      CALL RUK(DX(1),XLEN(1),-EL1,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      CALL RUK(DX(2)/2.,DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      XLOC(1)=XNN
      A1(1,1,1)=YO(1)
      A1(2,1,1)=YO(2)
      A2(1,1,1)=YO(3)
      A2(2,1,1)=YO(4)
      DO 100 I=2,NDP(2)
      CALL RUK(DX(2),DP(2),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      XLOC(I)=XNN
      A1(1,1,I)=YO(1)
      A1(2,1,I)=YO(2)
      A2(1,1,I)=YO(3)
100   A2(2,1,I)=YO(4)
      CALL RUK(DX(2)/2.,DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      IF(IFLAG.EQ.2)GOTO 111
      CALL RUK(DX(3)/2.,DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      XLOC(NDP(2)+1)=XNN
```

```
      A1(1,1,NDP(2)+1)=YO(1)
      A1(2,1,NDP(2)+1)=YO(2)
      A2(1,1,NDP(2)+1)=YO(3)
      A2(2,1,NDP(2)+1)=YO(4)
      DO 101 I=2,NDP(3)
      CALL RUK(DX(3),DP(3),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      XLOC(NDP(2)+I)=XNN
      A1(1,1,NDP(2)+I)=YO(1)
      A1(2,1,NDP(2)+I)=YO(2)
      A2(1,1,NDP(2)+I)=YO(3)
101   A2(2,1,NDP(2)+I)=YO(4)
      CALL RUK(DX(3)/2.,DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
111   W1(1)=YO(1)
      W1(2)=YO(2)
      W1(3)=YO(3)
      W1(4)=YO(4)
C
      YO(1)=0.
      YO(2)=0.
      YO(3)=0.
      YO(4)=1.
      CALL RUK(DX(1),XLEN(1),-EL1,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      CALL RUK(DX(2)/2.,DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      A1(1,2,1)=YO(1)
      A1(2,2,1)=YO(2)
      A2(1,2,1)=YO(3)
      A2(2,2,1)=YO(4)
      DO 200 I=2,NDP(2)
      CALL RUK(DX(2),DP(2),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      A1(1,2,I)=YO(1)
      A1(2,2,I)=YO(2)
      A2(1,2,I)=YO(3)
200   A2(2,2,I)=YO(4)
      CALL RUK(DX(2)/2.,DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      IF(IFLAG.EQ.2)GOTO 112
      CALL RUK(DX(3)/2.,DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      A1(1,2,NDP(2)+1)=YO(1)
      A1(2,2,NDP(2)+1)=YO(2)
      A2(1,2,NDP(2)+1)=YO(3)
      A2(2,2,NDP(2)+1)=YO(4)
      DO 201 I=2,NDP(3)
      CALL RUK(DX(3),DP(3),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      A1(1,2,NDP(2)+I)=YO(1)
      A1(2,2,NDP(2)+I)=YO(2)
      A2(1,2,NDP(2)+I)=YO(3)
201   A2(2,2,NDP(2)+I)=YO(4)
      CALL RUK(DX(3)/2.,DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
112   W2(1)=YO(1)
      W2(2)=YO(2)
      W2(3)=YO(3)
      W2(4)=YO(4)
C
      YO(1)=1.
      YO(2)=0.
      YO(3)=0.
```

```
      YO(4)=0.                                                          RIN05120
      XNN=1.                                                            RIN05130
      IF(IFLAG.EQ.2)GOTO 113                                            RIN05140
      CALL RUK(-DX(3)/2.,-DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)    RIN05150
      B1(1,1,NDP(2)+NDP(3))=YO(1)                                       RIN05160
      B1(2,1,NDP(2)+NDP(3))=YO(2)                                       RIN05170
      B2(1,1,NDP(2)+NDP(3))=YO(3)                                       RIN05180
      B2(2,1,NDP(2)+NDP(3))=YO(4)                                       RIN05190
      DO 300 II=2,NDP(3)                                                RIN05200
      I=NDP(2)+NDP(3)-II+1                                              RIN05210
      CALL RUK(-DX(3),-DP(3),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)          RIN05220
      B1(1,1,I)=YO(1)                                                   RIN05230
      B1(2,1,I)=YO(2)                                                   RIN05240
      B2(1,1,I)=YO(3)                                                   RIN05250
300   B2(2,1,I)=YO(4)                                                   RIN05260
      CALL RUK(-DX(3)/2.,-DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)    RIN05270
113   CALL RUK(-DX(2)/2.,-DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)    RIN05280
      B1(1,1,NDP(2))=YO(1)                                              RIN05290
      B1(2,1,NDP(2))=YO(2)                                              RIN05300
      B2(1,1,NDP(2))=YO(3)                                              RIN05310
      B2(2,1,NDP(2))=YO(4)                                              RIN05320
      DO 301 II=2,NDP(2)                                                RIN05330
      I=NDP(2)-II+1                                                     RIN05340
      CALL RUK(-DX(2),-DP(2),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)          RIN05350
      B1(1,1,I)=YO(1)                                                   RIN05360
      B1(2,1,I)=YO(2)                                                   RIN05370
      B2(1,1,I)=YO(3)                                                   RIN05380
301   B2(2,1,I)=YO(4)                                                   RIN05390
C                                                                       RIN05400
      YO(1)=0.                                                          RIN05410
      YO(2)=1.                                                          RIN05420
      YO(3)=0.                                                          RIN05430
      YO(4)=0.                                                          RIN05440
      XNN=1.                                                            RIN05450
      IF(IFLAG.EQ.2)GOTO 114                                            RIN05460
      CALL RUK(-DX(3)/2.,-DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)    RIN05470
      B1(1,2,NDP(2)+NDP(3))=YO(1)                                       RIN05480
      B1(2,2,NDP(2)+NDP(3))=YO(2)                                       RIN05490
      B2(1,2,NDP(2)+NDP(3))=YO(3)                                       RIN05500
      B2(2,2,NDP(2)+NDP(3))=YO(4)                                       RIN05510
      DO 400 II=2,NDP(3)                                                RIN05520
      I=NDP(2)+NDP(3)-II+1                                              RIN05530
      CALL RUK(-DX(3),-DP(3),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)          RIN05540
      B1(1,2,I)=YO(1)                                                   RIN05550
      B1(2,2,I)=YO(2)                                                   RIN05560
      B2(1,2,I)=YO(3)                                                   RIN05570
400   B2(2,2,I)=YO(4)                                                   RIN05580
      CALL RUK(-DX(3)/2.,-DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)    RIN05590
114   CALL RUK(-DX(2)/2.,-DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)    RIN05600
      B1(1,2,NDP(2))=YO(1)                                              RIN05610
      B1(2,2,NDP(2))=YO(2)                                              RIN05620
      B2(1,2,NDP(2))=YO(3)                                              RIN05630
      B2(2,2,NDP(2))=YO(4)                                              RIN05640
      DO 401 II=2,NDP(2)                                                RIN05650
      I=NDP(2)-II+1                                                     RIN05660
```

```
      CALL RUK(-DX(2),-DP(2),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      B1(1,2,I)=YO(1)
      B1(2,2,I)=YO(2)
      B2(1,2,I)=YO(3)
401   B2(2,2,I)=YO(4)
C
      DO 500 I=1,NDP(2)
      CALL AIN(A1(1,1,I),A1I)
      CALL AMU(A2(1,1,I),A1I,AT)
      CALL AMU(AT,B1(1,1,I),A1(1,1,I))
      DO 21 II=1,2
      DO 21 JJ=1,2
   21 AT(II,JJ)=B2(II,JJ,I)-A1(II,JJ,I)
      CALL AIN(AT,B2(1,1,I))
      C(I)=B2(1,2,I)/AL*DP(2)
      D(I)=B2(2,2,I)/AL*DP(2)
500   CONTINUE
      IF(IFLAG.EQ.2)RETURN
      INDEX1=NDP(2)+1
      INDEX2=NDP(2)+NDP(3)
      DO 501 I=INDEX1,INDEX2
      CALL AIN(A1(1,1,I),A1I)
      CALL AMU(A2(1,1,I),A1I,AT)
      CALL AMU(AT,B1(1,1,I),A1(1,1,I))
      DO 22 II=1,2
      DO 22 JJ=1,2
   22 AT(II,JJ)=B2(II,JJ,I)-A1(II,JJ,I)
      CALL AIN(AT,B2(1,1,I))
      C(I)=B2(1,2,I)/AL*DP(3)
      D(I)=B2(2,2,I)/AL*DP(3)
501   CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE CALCD3                                                 RIN06010
C>----------------------------------------------------------------------RIN06020
C                                                                       RIN06030
C                                                                       RIN06040
C       FUNCTION            - CALCULATE INFLUENCE COEFFICIENTS C,D      RIN06050
C                             FOR H AND H' AT PSI=1                     RIN06060
C                                                                       RIN06070
C       RESTRICTIONS        - FOR MULTIPLE RUNS, ONLY COMPUTED WHEN     RIN06080
C                             CERTAIN PARAMETERS CHANGE.  ALWAYS        RIN06090
C                             COMPUTED AT LEAST ONCE.                   RIN06100
C                                                                       RIN06110
C       REMARKS             - NOTE VARIABLES PASSED IN COMMON           RIN06120
C                                                                       RIN06130
C       EXTERNAL REFERENCES - AIN,AMU,RUK                               RIN06140
C                                                                       RIN06150
C                                                                       RIN06160
C>----------------------------------------------------------------------RIN06170
      IMPLICIT REAL*8 (A-H,O-Z)                                         RIN06180
      COMMON/BCD   /XLOC(100),C(100),D(100),NDX(3),NDP(3),DX(3),DP(3)   RIN06190
      COMMON/BELAS /FORCE,HELAS,H1ELAS,H2ELAS,H3ELAS,W1(4),W2(4)        RIN06200
      COMMON/BPAR  /AL,BET,PO,EPS,PF,S,U,DT                             RIN06210
      COMMON/BFLAG /IFLAG,IPLOAD                                        RIN06220
      DIMENSION YO(4),YT(4),DJ(4),CKJ(4,4)                              RIN06230
      DIMENSION XLEN(3)                                                 RIN06240
      DIMENSION A1(2,2,100),A2(2,2,100)                                 RIN06250
      DIMENSION B1(2,2,100),B2(2,2,100)                                 RIN06260
      DIMENSION A1I(2,2),AT(2,2)                                        RIN06270
      EXTERNAL DFN1                                                     RIN06280
C                                                                       RIN06290
      XLEN(1)=DX(1)*NDX(1)                                              RIN06300
      XLEN(2)=DX(2)*NDX(2)                                              RIN06310
      XLEN(3)=DX(3)*NDX(3)                                              RIN06320
      EL1=XLEN(1)+XLEN(2)                                               RIN06330
      YO(1)=0.                                                          RIN06340
      YO(2)=0.                                                          RIN06350
      YO(3)=1.                                                          RIN06360
      YO(4)=0.                                                          RIN06370
      CALL RUK(DX(1),XLEN(1),-EL1,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)         RIN06380
      CALL RUK(DX(2)/2.,DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)      RIN06390
      XLOC(1)=XNN                                                       RIN06400
      A1(1,1,1)=YO(1)                                                   RIN06410
      A1(2,1,1)=YO(2)                                                   RIN06420
      A2(1,1,1)=YO(3)                                                   RIN06430
      A2(2,1,1)=YO(4)                                                   RIN06440
      DO 100 I=2,NDP(2)                                                 RIN06450
      CALL RUK(DX(2),DP(2),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)            RIN06460
      XLOC(I)=XNN                                                       RIN06470
      A1(1,1,I)=YO(1)                                                   RIN06480
      A1(2,1,I)=YO(2)                                                   RIN06490
      A2(1,1,I)=YO(3)                                                   RIN06500
100   A2(2,1,I)=YO(4)                                                   RIN06510
      CALL RUK(DX(2)/2.,DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)      RIN06520
      CALL RUK(DX(3)/2.,DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)      RIN06530
      XLOC(NDP(2)+1)=XNN                                                RIN06540
      A1(1,1,NDP(2)+1)=YO(1)                                            RIN06550
```

```
      A1(2,1,NDP(2)+1)=YO(2)
      A2(1,1,NDP(2)+1)=YO(3)
      A2(2,1,NDP(2)+1)=YO(4)
      DO 101 I=2,NDP(3)
      CALL RUK(DX(3),DP(3),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      XLOC(NDP(2)+I)=XNN
      A1(1,1,NDP(2)+I)=YO(1)
      A1(2,1,NDP(2)+I)=YO(2)
      A2(1,1,NDP(2)+I)=YO(3)
101   A2(2,1,NDP(2)+I)=YO(4)
      CALL RUK(DX(3)/2.,DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      W1(1)=YO(1)
      W1(2)=YO(2)
      W1(3)=YO(3)
      W1(4)=YO(4)
C
      YO(1)=0.
      YO(2)=0.
      YO(3)=0.
      YO(4)=1.
      CALL RUK(DX(1),XLEN(1),-EL1,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      CALL RUK(DX(2)/2.,DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      A1(1,2,1)=YO(1)
      A1(2,2,1)=YO(2)
      A2(1,2,1)=YO(3)
      A2(2,2,1)=YO(4)
      DO 200 I=2,NDP(2)
      CALL RUK(DX(2),DP(2),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      A1(1,2,I)=YO(1)
      A1(2,2,I)=YO(2)
      A2(1,2,I)=YO(3)
200   A2(2,2,I)=YO(4)
      CALL RUK(DX(2)/2.,DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      CALL RUK(DX(3)/2.,DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      A1(1,2,NDP(2)+1)=YO(1)
      A1(2,2,NDP(2)+1)=YO(2)
      A2(1,2,NDP(2)+1)=YO(3)
      A2(2,2,NDP(2)+1)=YO(4)
      DO 201 I=2,NDP(3)
      CALL RUK(DX(3),DP(3),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      A1(1,2,NDP(2)+I)=YO(1)
      A1(2,2,NDP(2)+I)=YO(2)
      A2(1,2,NDP(2)+I)=YO(3)
201   A2(2,2,NDP(2)+I)=YO(4)
      CALL RUK(DX(3)/2.,DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      W2(1)=YO(1)
      W2(2)=YO(2)
      W2(3)=YO(3)
      W2(4)=YO(4)
C
      YO(1)=1.
      YO(2)=0.
      YO(3)=0.
      YO(4)=0.
      XNN=1.
```

```
      CALL RUK(-DX(3)/2.,-DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)   RIN07110
      B1(1,1,NDP(2)+NDP(3))=YO(1)                                      RIN07120
      B1(2,1,NDP(2)+NDP(3))=YO(2)                                      RIN07130
      B2(1,1,NDP(2)+NDP(3))=YO(3)                                      RIN07140
      B2(2,1,NDP(2)+NDP(3))=YO(4)                                      RIN07150
      DO 300 II=2,NDP(3)                                               RIN07160
      I=NDP(2)+NDP(3)-II+1                                             RIN07170
      CALL RUK(-DX(3),-DP(3),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)         RIN07180
      B1(1,1,I)=YO(1)                                                  RIN07190
      B1(2,1,I)=YO(2)                                                  RIN07200
      B2(1,1,I)=YO(3)                                                  RIN07210
300   B2(2,1,I)=YO(4)                                                  RIN07220
      CALL RUK(-DX(3)/2.,-DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)   RIN07230
      CALL RUK(-DX(2)/2.,-DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)   RIN07240
      B1(1,1,NDP(2))=YO(1)                                             RIN07250
      B1(2,1,NDP(2))=YO(2)                                             RIN07260
      B2(1,1,NDP(2))=YO(3)                                             RIN07270
      B2(2,1,NDP(2))=YO(4)                                             RIN07280
      DO 301 II=2,NDP(2)                                               RIN07290
      I=NDP(2)-II+1                                                    RIN07300
      CALL RUK(-DX(2),-DP(2),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)         RIN07310
      B1(1,1,I)=YO(1)                                                  RIN07320
      B1(2,1,I)=YO(2)                                                  RIN07330
      B2(1,1,I)=YO(3)                                                  RIN07340
301   B2(2,1,I)=YO(4)                                                  RIN07350
C                                                                      RIN07360
      YO(1)=0.                                                         RIN07370
      YO(2)=1.                                                         RIN07380
      YO(3)=0.                                                         RIN07390
      YO(4)=0.                                                         RIN07400
      XNN=1.                                                           RIN07410
      CALL RUK(-DX(3)/2.,-DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)   RIN07420
      B1(1,2,NDP(2)+NDP(3))=YO(1)                                      RIN07430
      B1(2,2,NDP(2)+NDP(3))=YO(2)                                      RIN07440
      B2(1,2,NDP(2)+NDP(3))=YO(3)                                      RIN07450
      B2(2,2,NDP(2)+NDP(3))=YO(4)                                      RIN07460
      DO 400 II=2,NDP(3)                                               RIN07470
      I=NDP(2)+NDP(3)-II+1                                             RIN07480
      CALL RUK(-DX(3),-DP(3),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)         RIN07490
      B1(1,2,I)=YO(1)                                                  RIN07500
      B1(2,2,I)=YO(2)                                                  RIN07510
      B2(1,2,I)=YO(3)                                                  RIN07520
400   B2(2,2,I)=YO(4)                                                  RIN07530
      CALL RUK(-DX(3)/2.,-DP(3)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)   RIN07540
114   CALL RUK(-DX(2)/2.,-DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)   RIN07550
      B1(1,2,NDP(2))=YO(1)                                             RIN07560
      B1(2,2,NDP(2))=YO(2)                                             RIN07570
      B2(1,2,NDP(2))=YO(3)                                             RIN07580
      B2(2,2,NDP(2))=YO(4)                                             RIN07590
      DO 401 II=2,NDP(2)                                               RIN07600
      I=NDP(2)-II+1                                                    RIN07610
      CALL RUK(-DX(2),-DP(2),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)         RIN07620
      B1(1,2,I)=YO(1)                                                  RIN07630
      B1(2,2,I)=YO(2)                                                  RIN07640
      B2(1,2,I)=YO(3)                                                  RIN07650
```

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401    B2(2,2,I)=YO(4)
C
       DO 500 I=1,NDP(2)
       CALL AIN(A1(1,1,I),A1I)
       CALL AMU(A2(1,1,I),A1I,AT)
       CALL AMU(AT,B1(1,1,I),A1(1,1,I))
       DO 21 II=1,2
       DO 21 JJ=1,2
  21   AT(II,JJ)=B2(II,JJ,I)-A1(II,JJ,I)
       CALL AIN(AT,B2(1,1,I))
       C(I)=B2(1,2,I)/AL*DP(2)
       D(I)=B2(2,2,I)/AL*DP(2)
500    CONTINUE
       INDEX1=NDP(2)+1
       INDEX2=NDP(2)+NDP(3)
       DO 501 I=INDEX1,INDEX2
       CALL AIN(A1(1,1,I),A1I)
       CALL AMU(A2(1,1,I),A1I,AT)
       CALL AMU(AT,B1(1,1,I),A1(1,1,I))
       DO 22 II=1,2
       DO 22 JJ=1,2
  22   AT(II,JJ)=B2(II,JJ,I)-A1(II,JJ,I)
       CALL AIN(AT,B2(1,1,I))
       C(I)=B2(1,2,I)/AL*DP(3)
       D(I)=B2(2,2,I)/AL*DP(3)
501    CONTINUE
C
       RETURN
       END
```

```
      SUBROUTINE CALCD4
C>-----------------------------------------------------------------------
C
C
C      FUNCTION               - CALCULATE INFLUENCE COEFFICIENTS C,D
C                               FOR H AND H' AT PSI=1
C
C      RESTRICTIONS           - FOR MULTIPLE RUNS, ONLY COMPUTED WHEN
C                               CERTAIN PARAMETERS CHANGE.  ALWAYS
C                               COMPUTED AT LEAST ONCE.
C
C      REMARKS                - NOTE VARIABLES PASSED IN COMMON
C
C      EXTERNAL REFERENCES - AIN,AMU,RUK
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BCD    /XLOC(100),C(100),D(100),NDX(3),NDP(3),DX(3),DP(3)
      COMMON/BELAS /FORCE,HELAS,H1ELAS,H2ELAS,H3ELAS,W1(4),W2(4)
      COMMON/BPAR   /AL,BET,PO,EPS,PF,S,U,DT
      COMMON/BFLAG /IFLAG,IPLOAD
      DIMENSION YO(4),YT(4),DJ(4),CKJ(4,4)
      DIMENSION XLEN(3)
      DIMENSION A1(2,2,100),A2(2,2,100)
      DIMENSION B1(2,2,100),B2(2,2,100)
      DIMENSION A1I(2,2),AT(2,2)
      EXTERNAL DFN1
C
      XLEN(1)=DX(1)*NDX(1)
      XLEN(2)=DX(2)*NDX(2)
      XLEN(3)=DX(3)*NDX(3)
      EL1=XLEN(1)
      YO(1)=0.
      YO(2)=0.
      YO(3)=1.
      YO(4)=0.
      CALL RUK(DX(1)/2.,DP(1)/2.,-EL1,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      XLOC(1)=XNN
      A1(1,1,1)=YO(1)
      A1(2,1,1)=YO(2)
      A2(1,1,1)=YO(3)
      A2(2,1,1)=YO(4)
      DO 100 I=2,NDP(1)
      CALL RUK(DX(1),DP(1),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      XLOC(I)=XNN
      A1(1,1,I)=YO(1)
      A1(2,1,I)=YO(2)
      A2(1,1,I)=YO(3)
100   A2(2,1,I)=YO(4)
      CALL RUK(DX(1)/2.,DP(1)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      CALL RUK(DX(2)/2.,DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      XLOC(NDP(1)+1)=XNN
      A1(1,1,NDP(1)+1)=YO(1)
      A1(2,1,NDP(1)+1)=YO(2)
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      A2(1,1,NDP(1)+1)=YO(3)                                            RIN08500
      A2(2,1,NDP(1)+1)=YO(4)                                            RIN08510
      DO 101 I=2,NDP(2)                                                 RIN08520
      CALL RUK(DX(2),DP(2),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)            RIN08530
      XLOC(NDP(1)+I)=XNN                                                RIN08540
      A1(1,1,NDP(1)+I)=YO(1)                                            RIN08550
      A1(2,1,NDP(1)+I)=YO(2)                                            RIN08560
      A2(1,1,NDP(1)+I)=YO(3)                                            RIN08570
101   A2(2,1,NDP(1)+I)=YO(4)                                            RIN08580
      CALL RUK(DX(2)/2.,DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)      RIN08590
      W1(1)=YO(1)                                                       RIN08600
      W1(2)=YO(2)                                                       RIN08610
      W1(3)=YO(3)                                                       RIN08620
      W1(4)=YO(4)                                                       RIN08630
C                                                                       RIN08640
      YO(1)=0.                                                          RIN08650
      YO(2)=0.                                                          RIN08660
      YO(3)=0.                                                          RIN08670
      YO(4)=1.                                                          RIN08680
      CALL RUK(DX(1)/2.,DP(1)/2.,-EL1,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)     RIN08690
      A1(1,2,1)=YO(1)                                                   RIN08700
      A1(2,2,1)=YO(2)                                                   RIN08710
      A2(1,2,1)=YO(3)                                                   RIN08720
      A2(2,2,1)=YO(4)                                                   RIN08730
      DO 200 I=2,NDP(1)                                                 RIN08740
      CALL RUK(DX(1),DP(1),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)            RIN08750
      A1(1,2,I)=YO(1)                                                   RIN08760
      A1(2,2,I)=YO(2)                                                   RIN08770
      A2(1,2,I)=YO(3)                                                   RIN08780
200   A2(2,2,I)=YO(4)                                                   RIN08790
      CALL RUK(DX(1)/2.,DP(1)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)      RIN08800
      CALL RUK(DX(2)/2.,DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)      RIN08810
      A1(1,2,NDP(1)+1)=YO(1)                                            RIN08820
      A1(2,2,NDP(1)+1)=YO(2)                                            RIN08830
      A2(1,2,NDP(1)+1)=YO(3)                                            RIN08840
      A2(2,2,NDP(1)+1)=YO(4)                                            RIN08850
      DO 201 I=2,NDP(2)                                                 RIN08860
      CALL RUK(DX(2),DP(2),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)            RIN08870
      A1(1,2,NDP(1)+I)=YO(1)                                            RIN08880
      A1(2,2,NDP(1)+I)=YO(2)                                            RIN08890
      A2(1,2,NDP(1)+I)=YO(3)                                            RIN08900
201   A2(2,2,NDP(1)+I)=YO(4)                                            RIN08910
      CALL RUK(DX(2)/2.,DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)      RIN08920
      W2(1)=YO(1)                                                       RIN08930
      W2(2)=YO(2)                                                       RIN08940
      W2(3)=YO(3)                                                       RIN08950
      W2(4)=YO(4)                                                       RIN08960
C                                                                       RIN08970
      YO(1)=1.                                                          RIN08980
      YO(2)=0.                                                          RIN08990
      YO(3)=0.                                                          RIN09000
      YO(4)=0.                                                          RIN09010
      XNN=1.                                                            RIN09020
      CALL RUK(-DX(2)/2.,-DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)    RIN09030
      B1(1,1,NDP(1)+NDP(2))=YO(1)                                       RIN09040
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      B1(2,1,NDP(1)+NDP(2))=YO(2)
      B2(1,1,NDP(1)+NDP(2))=YO(3)
      B2(2,1,NDP(1)+NDP(2))=YO(4)
      DO 300 II=2,NDP(2)
      I=NDP(1)+NDP(2)-II+1
      CALL RUK(-DX(2),-DP(2),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      B1(1,1,I)=YO(1)
      B1(2,1,I)=YO(2)
      B2(1,1,I)=YO(3)
300   B2(2,1,I)=YO(4)
      CALL RUK(-DX(2)/2.,-DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      CALL RUK(-DX(1)/2.,-DP(1)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      B1(1,1,NDP(1))=YO(1)
      B1(2,1,NDP(1))=YO(2)
      B2(1,1,NDP(1))=YO(3)
      B2(2,1,NDP(1))=YO(4)
      DO 301 II=2,NDP(1)
      I=NDP(1)-II+1
      CALL RUK(-DX(1),-DP(1),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      B1(1,1,I)=YO(1)
      B1(2,1,I)=YO(2)
      B2(1,1,I)=YO(3)
301   B2(2,1,I)=YO(4)
C
      YO(1)=0.
      YO(2)=1.
      YO(3)=0.
      YO(4)=0.
      XNN=1.
      CALL RUK(-DX(2)/2.,-DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      B1(1,2,NDP(1)+NDP(2))=YO(1)
      B1(2,2,NDP(1)+NDP(2))=YO(2)
      B2(1,2,NDP(1)+NDP(2))=YO(3)
      B2(2,2,NDP(1)+NDP(2))=YO(4)
      DO 400 II=2,NDP(2)
      I=NDP(1)+NDP(2)-II+1
      CALL RUK(-DX(2),-DP(2),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      B1(1,2,I)=YO(1)
      B1(2,2,I)=YO(2)
      B2(1,2,I)=YO(3)
400   B2(2,2,I)=YO(4)
      CALL RUK(-DX(2)/2.,-DP(2)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      CALL RUK(-DX(1)/2.,-DP(1)/2.,XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      B1(1,2,NDP(1))=YO(1)
      B1(2,2,NDP(1))=YO(2)
      B2(1,2,NDP(1))=YO(3)
      B2(2,2,NDP(1))=YO(4)
      DO 401 II=2,NDP(1)
      I=NDP(1)-II+1
      CALL RUK(-DX(1),-DP(1),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
      B1(1,2,I)=YO(1)
      B1(2,2,I)=YO(2)
      B2(1,2,I)=YO(3)
401   B2(2,2,I)=YO(4)
C
```

```
      DO 500 I=1,NDP(1)
      CALL AIN(A1(1,1,I),A1I)
      CALL AMU(A2(1,1,I),A1I,AT)
      CALL AMU(AT,B1(1,1,I),A1(1,1,I))
      DO 21 II=1,2
      DO 21 JJ=1,2
   21 AT(II,JJ)=B2(II,JJ,I)-A1(II,JJ,I)
      CALL AIN(AT,B2(1,1,I))
      C(I)=B2(1,2,I)/AL*DP(1)
      D(I)=B2(2,2,I)/AL*DP(1)
500   CONTINUE
      INDEX1=NDP(1)+1
      INDEX2=NDP(1)+NDP(2)
      DO 501 I=INDEX1,INDEX2
      CALL AIN(A1(1,1,I),A1I)
      CALL AMU(A2(1,1,I),A1I,AT)
      CALL AMU(AT,B1(1,1,I),A1(1,1,I))
      DO 22 II=1,2
      DO 22 JJ=1,2
   22 AT(II,JJ)=B2(II,JJ,I)-A1(II,JJ,I)
      CALL AIN(AT,B2(1,1,I))
      C(I)=B2(1,2,I)/AL*DP(2)
      D(I)=B2(2,2,I)/AL*DP(2)
501   CONTINUE
C
      RETURN
      END
```

```
      SUBROUTINE CHECK(AL,BET,EPS,EL1,NDX,NDP,IRUN,RECALC)
C>-------------------------------------------------------------------------
C
C
C      FUNCTION             - CHECK INPUT VARIABLES FOR RECALCULATION
C                             INFLUENCE COEFFICIENTS C,D
C
C      RESTRICTIONS         - ONLY USED FOR MULTIPLE RUNS WITH IRUN.GE.2
C
C      REMARKS              -
C
C      EXTERNAL REFERENCES -
C
C      ARGUMENT DEFINITION:
C      NAME         DESCRIPTION
C       AL,BET,...   INPUT VARIABLES, ARE THEY CHANGED ?
C       RECALC       LOGICAL VARIABLE, IF .TRUE., RECALCULATE C,D
C
C
C>-------------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BOLD  /ALOLD,BETOLD,EPSOLD,EL1OLD,NDXOLD(3),NDPOLD(3)
      DIMENSION NDX(3),NDP(3)
      LOGICAL RECALC,TEMP
      RECALC=.TRUE.
      IF(IRUN.GT.1)THEN
         TEMP=.TRUE.
         TEMP=TEMP.AND.(AL .EQ. ALOLD)
         TEMP=TEMP.AND.(EPS.EQ.EPSOLD).AND.(EL1.EQ.EL1OLD)
         DO 1 I=1,3
            TEMP=TEMP.AND.(NDX(I).EQ.NDXOLD(I))
            TEMP=TEMP.AND.(NDP(I).EQ.NDPOLD(I))
1        CONTINUE
         IF(TEMP)RECALC=.FALSE.
      END IF
      ALOLD =AL
      EPSOLD=EPS
      EL1OLD=EL1
      DO 2 I=1,3
         NDXOLD(I)=NDX(I)
         NDPOLD(I)=NDP(I)
2     CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CONST(H0,H1,PF)
C>-----------------------------------------------------------------------------
C
C
C      FUNCTION            - CALCULATE CONSTANTS RK,RC,R1,R2,XCAV,ICAV
C                            FOR SLIDER BEARIN PRESSURES
C
C      RESTRICTIONS        -
C
C      REMARKS             - CALLED BY SUBROUTINE EVAL
C                            NOTE OTHER VARIABLES PASSED IN COMMON
C
C      EXTERNAL REFERENCES - DABS,DSQRT
C
C      ARGUMENT DEFINITION:
C      NAME        DESCRIPTION
C       H0          FILM THICKNESS AT PSI=0
C       H1          FILM THICKNESS AT PSI=1
C       PF          RESERVOIR PRESSURE
C
C
C>-----------------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BCOEFF/RK,RC,R1,R2,XCAV,ICAV
      COMMON/BINT  /IELAS,IBACK,IPR
      R0=H1+H0
      R1=H0
      R2=H1-H0
      IF(IBACK.EQ.0)THEN
         RK=2.*H0*H1/R0*(1.-H0*H1*PF)
         RC=1./R0*(1.+H1**2*PF)
         XCAV=0.
      ELSE IF(IBACK.EQ.1)THEN
         ALF=H1*(-R2)*PF
         RK=H1*(1.+ALF+DSQRT(DABS(ALF**2+2.*ALF+1.E-7)))
         IF(DABS(R2).GT.1.D-10)THEN
            XCAV=(RK-H1)/R2+1.
         ELSE
            XCAV=1.0
         END IF
         IF((RK.LE.H0.AND.ICAV.NE.0).OR.ICAV.EQ.1)THEN
            RC=1./2./RK
         ELSE
            RK=2.*H0*H1/R0*(1.+H0*H1*PF)
            RC=1./R0*(1.-H1**2*PF)
         END IF
      END IF
      RETURN
      END
```

```
      SUBROUTINE CONST2(H0,H1,PF)
C>----------------------------------------------------------------------
C
C
C      REMARKS                - SEE CONST
C
C
C>----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BCD   /XLOC(100),C(100),D(100),NDX(3),NDP(3),DX(3),DP(3)
      COMMON/BCOEFF/RK,RC,R1,R2,XCAV,ICAV
      COMMON/BINT  /IELAS,IBACK,IPR
      R0=H1+H0
      R1=H0
      R2=H1-H0
      CTEMP=(-1)**(IBACK+1)
      RK=2.*H0*H1/R0*(1.+CTEMP*H0*H1*PF)
      RC=1./R0*(1.-CTEMP*H1**2*PF)
      XCAV=0.
      IF(IBACK.EQ.1)THEN
C        IF(IPR.EQ.1)WRITE(4,*)' '
         PMIN=100.
         DO 10 I=1,NDP(2)+NDP(3)
C           IF(IPR.EQ.1)WRITE(4,*)P(XLOC(I))
            PMIN=DMIN1(PMIN,P(XLOC(I)))
10       CONTINUE
         IF(PMIN.GE.0.D0)RETURN
         H11=H1-H0
         XCAV=0.5
         HCAV=H1+H11*(XCAV-1.)
         ICOUNT=0
20       ICOUNT=ICOUNT+1
         HCVOLD=HCAV
         FHCAV=PF+1./H11*(-1./H1+HCAV/2./H1**2+1./2./HCAV)
         F1HCAV=1./H11*(1./2./H1**2-1./2./HCAV**2)
         HCAV=HCAV-FHCAV/F1HCAV
         XCAV=(HCAV-H1)/H11+1.
C        IF(IPR.EQ.1)WRITE(4,*)XCAV
         IF(ICOUNT.LT.10.AND.DABS(HCAV-HCVOLD).GT.1.D-7)GOTO 20
         RK=HCAV
         RC=1./2./HCAV
      END IF
      RETURN
      END
```

```
      SUBROUTINE DFN1(X,Y,D)
C>-----------------------------------------------------------------------
C
C
C      FUNCTION              - DERIVATIVE ROUTINE USED BY RUK
C                              (NO HYDRODYNAMICS)
C
C      RESTRICTIONS          -
C
C      REMARKS               - NOTE VARIABLES PASSED IN COMMON
C
C      EXTERNAL REFERENCES -
C
C      ARGUMENT DEFINITION:
C      NAME         DESCRIPTION
C        X            VALUE OF X FOR DERIVATIVES
C        Y            VALUES OF INDEPENDENT VARIABLES
C        D            VALUES OF DERIVATIVES
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION Y(4),D(4)
      COMMON/BPAR  /AL,BET,PO,EPS,PF,S,U,DT
      D(1)=Y(2)
      D(2)=Y(3)
      D(3)=Y(4)
      D(4)=-Y(1)/AL
      RETURN
      END
```

```
      SUBROUTINE DFN2(X,Y,D)
C>---------------------------------------------------------------------
C
C
C      FUNCTION            - DERIVATIVE ROUTINE USED BY RUK
C                            (HYDRODYNAMICS INCLUDED)
C
C      RESTRICTIONS        -
C
C      REMARKS             - NOTE VARIABLES PASSED IN COMMON
C
C      EXTERNAL REFERENCES - P
C
C      ARGUMENT DEFINITION:
C      NAME        DESCRIPTION
C        X           VALUE OF X FOR DERIVATIVES
C        Y           VALUES OF INDEPENDENT VARIABLES
C        D           VALUES OF DERIVATIVES
C
C
C>---------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION Y(4),D(4)
      COMMON/BPAR  /AL,BET,PO,EPS,PF,S,U,DT
      COMMON/BFLAG /IFLAG,IPLOAD
      D(1)=Y(2)
      D(2)=Y(3)
      D(3)=Y(4)
      H=0
      IF(IPLOAD.EQ.1)H=PO
      IF(X.GT.O.)H=H-P(X)
      H=H*BET
      D(4)=(H-Y(1))/AL
      RETURN
      END
```

```
      SUBROUTINE DFN3(X,Y,D)
C>-----------------------------------------------------------------------
C
C
C       FUNCTION             - DERIVATIVE ROUTINE USED BY RUK
C                              (HYDRODYNAMICS INCLUDED, NO PRELOAD P0)
C
C       RESTRICTIONS         -
C
C       REMARKS              - NOTE VARIABLES PASSED IN COMMON
C
C       EXTERNAL REFERENCES - P
C
C       ARGUMENT DEFINITION:
C       NAME         DESCRIPTION
C         X           VALUE OF X FOR DERIVATIVES
C         Y           VALUES OF INDEPENDENT VARIABLES
C         D           VALUES OF DERIVATIVES
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION Y(4),D(4)
      COMMON/BPAR  /AL,BET,P0,EPS,PF,S,U,DT
      COMMON/BINT  /IELAS,IBACK,IPR
      D(1)=Y(2)
      D(2)=Y(3)
      D(3)=Y(4)
      H=BET*P0
      D(4)=(H-Y(1))/AL
      RETURN
      END
```

```
      SUBROUTINE ELAS
C>----------------------------------------------------------------------
C
C
C      FUNCTION            - DETERMINE ELASTIC SOLUTION
C                            (NO HYDRODYNAMICS)
C                            HELAS,H1ELAS,H2ELAS,H3ELAS,FORCE
C
C      RESTRICTIONS        -
C
C      REMARKS             - NOTE VARIABLES PASSED IN COMMON
C
C      EXTERNAL REFERENCES - NONE
C
C
C>----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION YO(4)
      COMMON/BPAR  /AL,BET,PO,EPS,PF
      COMMON/BCD   /XLOC(100),C(100),D(100),NDX(3),NDP(3),DX(3),DP(3)
      COMMON/BELAS /FORCE,HELAS,H1ELAS,H2ELAS,H3ELAS,W1(4),W2(4)
      COMMON/BFLAG /IFLAG,IPLOAD
      YO(1)=0.
      YO(2)=0.
      YO(3)=0.
      YO(4)=0.
      WRITE(6,1009)
1009  FORMAT(5X,1HX,8X,1HH,9X,2HH',7X,3HH'',7X,4HH''',6X,4HPRES
     $,8X,5HFORCE)
      IF(IFLAG.EQ.1)THEN
         IMIN=NDP(2)+1
         IMAX=NDP(2)+NDP(3)
      ELSE IF(IFLAG.EQ.2)THEN
         IMIN=1
         IMAX=NDP(2)
      ELSE IF(IFLAG.EQ.3)THEN
         IMIN=1
         IMAX=NDP(2)+NDP(3)
      ELSE IF(IFLAG.EQ.4)THEN
         IMIN=1
         IMAX=NDP(1)+NDP(2)
      END IF
      DO 600 I=IMIN,IMAX
         YO(1)=YO(1)+C(I)
         YO(2)=YO(2)+D(I)
600   CONTINUE
      YO(1)=YO(1)*PO*BET
      YO(2)=YO(2)*PO*BET
      HELAS=1.-YO(1)
      H1ELAS=-YO(2)
      H2ELAS=-YO(3)
      H3ELAS=-YO(4)
      WRITE(6,'(F8.4,5F10.5,F13.5)')1.,HELAS,H1ELAS,H2ELAS,H3ELAS,0.,0.
      FORCE=0.D0
      IF(YO(1).GT.1.)THEN
```

```
         CHI=YO(1)-1.0
         YO(1)=1.0
         B=CHI/(W2(3)*W1(1)/W1(3)-W2(1))
         A=-B*W2(3)/W1(3)
         YO(2)=A*W1(2)+B*W2(2)+YO(2)
         YO(3)=0.
         YO(4)=A*W1(4)+B*W2(4)
         FORCE=YO(4)*AL/BET
         HELAS=1.0-YO(1)
         H1ELAS=-YO(2)
         H2ELAS=-YO(3)
         H3ELAS=-YO(4)
         WRITE(6,'(F8.4,5F10.5,F13.5)')1.0,HELAS,H1ELAS,H2ELAS
     $           ,H3ELAS,0.,FORCE
      END IF
      RETURN
      END

      SUBROUTINE ERRMSG
C>-----------------------------------------------------------------------
C
C
C       FUNCTION              - PRINT ERROR TERMS UPON EXIT FROM *ZSCNT*
C
C       RESTRICTIONS          - CALLED ONLY IF IER.NE.0 FROM *ZSCNT*
C
C       REMARKS               - FINAL VALUES OF X AND F FROM EVAL
C                               ARE PRINTED
C
C       INPUT/OUTPUT:
C       UNIT         DESCRIPTION
C         4            TERMINAL OUTPUT
C         6            OUTPUT FILE
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BLAST /X1,X2,F1,F2
      WRITE(4,'(8X,''W(1)'',10X,''W(2)'',10X,''F(1)'',10X,''F(2)''
     +   ,/)')
      WRITE(4,'(1X,4(2X,E12.5))')X1,X2,F1,F2
      WRITE(6,'(8X,''W(1)'',10X,''W(2)'',10X,''F(1)'',10X,''F(2)''
     +     )')
      WRITE(6,'(1X,4(2X,E12.5))')X1,X2,F1,F2
      RETURN
      END
```

```
      SUBROUTINE EVAL(X,F,N,PAR)
C>----------------------------------------------------------------------
C
C
C      FUNCTION              - DEFINES EQUATIONS FOR H AND H' WHICH ARE
C                              SOLVED BY SECANT METHOD *ZSCNT*
C
C      RESTRICTIONS          -
C
C      REMARKS               - NOTE VARIABLE PASSED IN COMMON
C
C      EXTERNAL REFERENCES - CONST
C
C      INPUT/OUTPUT:
C      UNIT         DESCRIPTION
C
C      ARGUMENT DEFINITION:
C      NAME         DESCRIPTION
C        X           X(1) IS DISPLACEMENT AT 1
C                    X(2) IS X' AT 1
C        F           EQUATIONS (2) FOR H AND H'
C        N           NO. OF EQUATIONS (N=2)
C       PAR          NOT USED
C
C
C>----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(N),F(N),PAR(1)
      COMMON/BPAR  /AL,BET,P0,EPS,PF
      COMMON/BCD   /XLOC(100),C(100),D(100),NDX(3),NDP(3),DX(3),DP(3)
      COMMON/BCOEFF/RK,RC,R1,R2,XCAV,ICAV
      COMMON/BINT  /IELAS,IBACK,IPR
      COMMON/BLAST /X1LAST,X2LAST,F1LAST,F2LAST
      COMMON/BPROF /C1
      COMMON/BFLAG /IFLAG,IPLOAD
      H1=1.0-X(1)
      H11=C1-X(2)
      H0=H1-H11
      CALL CONST(H0,H1,PF)
      SUM1=X(1)
      SUM2=X(2)
      INDEX=0
5     CONTINUE
      INDEX=INDEX+1
      IF(IFLAG.EQ.1.AND.INDEX.GT.NDP(2)+NDP(3))GOTO 6
      IF(IFLAG.EQ.2.AND.INDEX.GT.NDP(2))GOTO 6
      IF(XLOC(INDEX).LT.DMIN1(1.-EPS,XCAV))GOTO 5
      IF(XLOC(INDEX).LT.XCAV)THEN
         PRES=0
      ELSE
         PRES=P(XLOC(INDEX))
      END IF
      IF(XLOC(INDEX).GT.1.-EPS) PRES=PRES-P0
7     CONTINUE
      SUM1=SUM1+C(INDEX)*PRES*BET
```

```
      SUM2=SUM2+D(INDEX)*PRES*BET                                       RIN13760
      GOTO 5                                                            RIN13770
6     CONTINUE                                                          RIN13780
      F(1)=SUM1                                                         RIN13790
      F(2)=SUM2                                                         RIN13800
      X1LAST=X(1)                                                       RIN13810
      X2LAST=X(2)                                                       RIN13820
      F1LAST=F(1)                                                       RIN13830
      F2LAST=F(2)                                                       RIN13840
      RETURN                                                            RIN13850
      END                                                               RIN13860
```

```
      SUBROUTINE EVAL3(X,F,N,PAR)
C>----------------------------------------------------------------------
C
C
C      FUNCTION              - DEFINES EQUATIONS FOR H AND H' WHICH ARE
C                              SOLVED BY SECANT METHOD *ZSCNT*
C
C      RESTRICTIONS          -
C
C      REMARKS               - NOTE VARIABLE PASSED IN COMMON
C
C      EXTERNAL REFERENCES - CONST
C
C      INPUT/OUTPUT:
C      UNIT         DESCRIPTION
C
C      ARGUMENT DEFINITION:
C      NAME         DESCRIPTION
C        X           X(1) IS DISPLACEMENT AT 1
C                    X(2) IS X' AT 1
C        F           EQUATIONS (2) FOR H AND H'
C        N           NO. OF EQUATIONS (N=2)
C       PAR          NOT USED
C
C
C>----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(N),F(N),PAR(1)
      COMMON/BPAR  /AL,BET,P0,EPS,PF
      COMMON/BCD   /XLOC(100),C(100),D(100),NDX(3),NDP(3),DX(3),DP(3)
      COMMON/BCOEFF/RK,RC,R1,R2,XCAV,ICAV
      COMMON/BINT  /IELAS,IBACK,IPR
      COMMON/BLAST /X1LAST,X2LAST,F1LAST,F2LAST
      COMMON/BPROF /C1
      COMMON/BFLAG /IFLAG,IPLOAD
      H1=1.0-X(1)
      H11=C1-X(2)
      H0=H1-H11
      CALL CONST(H0,H1,PF)
      SUM1=X(1)
      SUM2=X(2)
      INDEX=0
5     CONTINUE
      INDEX=INDEX+1
      IF(INDEX.GT.NDP(2)+NDP(3))GOTO 6
      IF(XLOC(INDEX).LT.DMIN1(1.-EPS,XCAV))GOTO 5
      IF(XLOC(INDEX).LT.XCAV)THEN
         PRES=0
      ELSE
         PRES=P(XLOC(INDEX))
      END IF
      PRES=PRES-P0
7     CONTINUE
      SUM1=SUM1+C(INDEX)*PRES*BET
      SUM2=SUM2+D(INDEX)*PRES*BET
```

```
      GOTO 5                                                    RIN14420
6     CONTINUE                                                  RIN14430
      F(1)=SUM1                                                 RIN14440
      F(2)=SUM2                                                 RIN14450
      X1LAST=X(1)                                               RIN14460
      X2LAST=X(2)                                               RIN14470
      F1LAST=F(1)                                               RIN14480
      F2LAST=F(2)                                               RIN14490
      RETURN                                                    RIN14500
      END                                                       RIN14510
```

```
      SUBROUTINE EVAL4(X,F,N,PAR)
C>-----------------------------------------------------------------------
C
C
C      FUNCTION             - DEFINES EQUATIONS FOR H AND H' WHICH ARE
C                             SOLVED BY SECANT METHOD *ZSCNT*
C
C      RESTRICTIONS         -
C
C      REMARKS              - NOTE VARIABLE PASSED IN COMMON
C
C      EXTERNAL REFERENCES - CONST
C
C      INPUT/OUTPUT:
C      UNIT         DESCRIPTION
C
C      ARGUMENT DEFINITION:
C      NAME         DESCRIPTION
C        X           X(1) IS DISPLACEMENT AT 1
C                    X(2) IS X' AT 1
C        F           EQUATIONS (2) FOR H AND H'
C        N           NO. OF EQUATIONS (N=2)
C       PAR          NOT USED
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION X(N),F(N),PAR(1)
      COMMON/BPAR  /AL,BET,PO,EPS,PF
      COMMON/BCD   /XLOC(100),C(100),D(100),NDX(3),NDP(3),DX(3),DP(3)
      COMMON/BCOEFF/RK,RC,R1,R2,XCAV,ICAV
      COMMON/BINT  /IELAS,IBACK,IPR
      COMMON/BLAST /X1LAST,X2LAST,F1LAST,F2LAST
      COMMON/BPROF /C1
      COMMON/BFLAG /IFLAG,IPLOAD
      H1=1.0-X(1)
      H11=C1-X(2)
      H0=H1-H11
      CALL CONST(H0,H1,PF)
      SUM1=X(1)
      SUM2=X(2)
      INDEX=0
5     CONTINUE
      INDEX=INDEX+1
      IF(INDEX.GT.NDP(1)+NDP(2))GOTO 6
      IF(XLOC(INDEX).LT.DMIN1(1.-EPS,XCAV))GOTO 5
      IF(XLOC(INDEX).LT.XCAV)THEN
         PRES=0
      ELSE
         PRES=P(XLOC(INDEX))
      END IF
      PRES=PRES-P0
7     CONTINUE
      SUM1=SUM1+C(INDEX)*PRES*BET
      SUM2=SUM2+D(INDEX)*PRES*BET
```

```
      GOTO 5
6     CONTINUE
      F(1)=SUM1
      F(2)=SUM2
      X1LAST=X(1)
      X2LAST=X(2)
      F1LAST=F(1)
      F2LAST=F(2)
      RETURN
      END

      FUNCTION P(PSI)
C>-----------------------------------------------------------------------
C
C
C       FUNCTION             - RETURN HYDRODYNAMIC PRESSURE AT PSI
C
C       RESTRICTIONS         -
C
C       REMARKS              - NOTE VARIABLES PASSED IN COMMON
C
C       EXTERNAL REFERENCES - DABS
C
C       ARGUMENT DEFINITION:
C       NAME         DESCRIPTION
C        PSI          VALUE OF X WHERE PRESSURE IS DESIRED
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BCOEFF/RK,RC,R1,R2,XCAV,ICAV
      COMMON/BINT  /IELAS,IBACK,IPR
      H=R1+R2*PSI
      HINV=1./H
      P=0.
      IF(PSI.GE.XCAV.AND.DABS(R2).GT.1.D-10)THEN
         P=1./R2*(HINV*(-1.+RK/2.*HINV)+RC)
      END IF
      IF(IBACK.EQ.1)THEN
         P=-P
      END IF
      RETURN
      END
```

```
      SUBROUTINE PRT(X,Y,ITERM)
C>-----------------------------------------------------------------------
C
C
C      FUNCTION            - USING DISPLACEMENTS, CONVERT TO FILM
C                            THICKNESSES AND PRINT OUT
C
C      RESTRICTIONS        -
C
C      REMARKS             -
C
C      EXTERNAL REFERENCES - P
C
C      INPUT/OUTPUT:
C      UNIT         DESCRIPTION
C       ITERM        OUTPUT UNIT
C
C      ARGUMENT DEFINITION:
C      NAME         DESCRIPTION
C       X            DIMENSIONLESS LENGTH VARIABLE
C       Y            DISPLACEMENT AND ITS DERIVATIVES
C       ITERM        OUTPUT UNIT
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BINT  /IELAS,IBACK,IPR
      COMMON/BPROF /C1
      DIMENSION Y(4)
      H=1.-Y(1)+C1*(X-1.)
      H1=C1-Y(2)
      H2=-Y(3)
      H3=-Y(4)
      PRES=0.
      IF(IELAS.EQ.0.AND.X.GE.0.)PRES=P(X)
      WRITE(ITERM,5)X,H,H1,H2,H3,PRES
    5 FORMAT(F10.4,5F12.5)
      RETURN
      END
```

```
      SUBROUTINE PRTOUT(XX)
C>-----------------------------------------------------------------------
C
C
C       FUNCTION             - PRINT FILM THICK., PRES., ETC.
C
C       RESTRICTIONS         -
C
C       REMARKS              - NOTE VARIABLES PASSED IN COMMON
C
C       EXTERNAL REFERENCES  - DFN1,DFN2,DFN3,PRT,RUK
C
C       INPUT/OUTPUT:
C       UNIT         DESCRIPTION
C        6            OUTPUT FILE
C
C       ARGUMENT DEFINITION:
C       NAME         DESCRIPTION
C        XX(2)        DISPLACEMENT, SLOPE OF DISPLACEMENT
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BINT  /IELAS,IBACK,IPR
      COMMON/BELAS /FORCE,HELAS,H1ELAS,H2ELAS,H3ELAS,W1(4),W2(4)
      COMMON/BCD   /XLOC(100),C(100),D(100),NDX(3),NDP(3),DX(3),DP(3)
      COMMON/BFLAG /IFLAG,IPLOAD
      DIMENSION     XX(2),YO(4),YT(4),DJ(4),CKJ(4,4)
      EXTERNAL      DFN1,DFN2,DFN3
C
      XNN=1.0
      YO(1)=XX(1)
      YO(2)=XX(2)
      YO(3)=0.
      YO(4)=0.
      IF(IBACK.EQ.1.AND.FORCE.GT.1.D-8)THEN
         WRITE(6,1009)
         WRITE(6,'(F8.4,5F10.5,F13.5)')1.0,HELAS,H1ELAS,H2ELAS
     $          ,H3ELAS,0.,FORCE
      RETURN
      ELSE
         WRITE(6,1007)
         IF(IPR.EQ.1)THEN
            WRITE(6,'(''@'')')
         ELSE
            WRITE(6,'('' '')')
         END IF
         CALL PRT(XNN,YO,6)
      END IF
      IF(IPR.EQ.1)THEN
         IF(IFLAG.EQ.2.OR.IFLAG.EQ.4)GOTO 111
         IPLOAD=1
         CALL RUK(-DX(3)/2.,-DP(3)/2.,XNN,XNN,YO,YO,4,DFN2,YT,DJ,CKJ,4)
         CALL PRT(XNN,YO,6)
         DO 9 JJ=2,NDP(3)
```

```
         J=NDP(3)+1-JJ
         CALL RUK(-DX(3),-DP(3),XNN,XNN,YO,YO,4,DFN2,YT,DJ,CKJ,4)
         CALL PRT(XNN,YO,6)
  9      CONTINUE
         CALL RUK(-DX(3)/2.,-DP(3)/2.,XNN,XNN,YO,YO,4,DFN2,YT,DJ,CKJ,4)
         CALL PRT(XNN,YO,6)
111      CONTINUE
         IF(IFLAG.EQ.1)THEN
            IPLOAD=0
         ELSE IF(IFLAG.EQ.2)THEN
            IPLOAD=1
         ELSE IF(IFLAG.EQ.3)THEN
            IPLOAD=1
         ELSE IF(IFLAG.EQ.4)THEN
            IPLOAD=1
         END IF
         CALL RUK(-DX(2)/2.,-DP(2)/2.,XNN,XNN,YO,YO,4,DFN2,YT,DJ,CKJ,4)
         CALL PRT(XNN,YO,6)
         DO 8 JJ=2,NDP(2)
         J=NDP(2)+1-JJ
         CALL RUK(-DX(2),-DP(2),XNN,XNN,YO,YO,4,DFN2,YT,DJ,CKJ,4)
         CALL PRT(XNN,YO,6)
  8      CONTINUE
         CALL RUK(-DX(2)/2.,-DP(2)/2.,XNN,XNN,YO,YO,4,DFN2,YT,DJ,CKJ,4)
         CALL PRT(XNN,YO,6)
         IF(IFLAG.EQ.1.OR.IFLAG.EQ.2)THEN
            WRITE(6,'(''@'')')
            RETURN
         END IF
         DO 7 J=1,NDP(1)
         IF(IFLAG.EQ.4)THEN
            CALL RUK(-DX(1),-DP(1),XNN,XNN,YO,YO,4,DFN3,YT,DJ,CKJ,4)
         ELSE
            CALL RUK(-DX(1),-DP(1),XNN,XNN,YO,YO,4,DFN1,YT,DJ,CKJ,4)
         END IF
         CALL PRT(XNN,YO,6)
  7      CONTINUE
      END IF
      WRITE(6,'(''@'')')
      RETURN
C
1007  FORMAT(7X,1HX,10X,1HH,11X,2HH',9X,3HH'',9X,4HH''',8X,4HPRES)
1009  FORMAT(5X,1HX,8X,1HH,9X,2HH',7X,3HH'',7X,4HH''',6X,4HPRES
     $,8X,5HFORCE)
C
      END
```

```
      SUBROUTINE RUK(DX,DP,XO,XN,YO,YN,NO,DFN,YT,D,CK,ID)
C>-----------------------------------------------------------------------
C
C
C       FUNCTION             - RUNGE-KUTTA INTEGRATION
C
C       RESTRICTIONS         -
C
C       REMARKS              -
C
C       EXTERNAL REFERENCES - DFN
C
C       ARGUMENT DEFINITION:
C       NAME          DESCRIPTION
C
C
C>-----------------------------------------------------------------------
      IMPLICIT REAL*8 (A-H,O-Z)
      DIMENSION YO(ID),YN(ID),D(ID),YT(ID),CK(4,ID)
      N=DP/DX+.001
      XN=XO
      DO 2 J=1,NO
    2 YN(J)=YO(J)
      DO 5 I=1,N
      CALL DFN(XN,YN,D)
      DO 6 J=1,NO
      CK(1,J)=D(J)*DX
    6 YT(J)=YN(J)+CK(1,J)/2.
      CALL DFN(XN+DX/2.,YT,D)
      DO 7 J=1,NO
      CK(2,J)=D(J)*DX
    7 YT(J)=YN(J)+CK(2,J)/2.
      CALL DFN(XN+DX/2.,YT,D)
      DO 8 J=1,NO
      CK(3,J)=D(J)*DX
    8 YT(J)=YN(J)+CK(3,J)
      CALL DFN(XN+DX,YT,D)
      DO 9 J=1,NO
      CK(4,J)=D(J)*DX
      YN(J)=YN(J)+(CK(1,J)+2.*(CK(2,J)+CK(3,J))+CK(4,J))/6.
9     CONTINUE
    5 XN=XN+DX
      RETURN
      END
```

```
      SUBROUTINE ZSCNT  (FCN,NSIG,N,ITMAX,PAR,X,FNORM,WK,IER)
C   IMSL ROUTINE NAME   - ZSCNT
C   THIS PROGRAM SHOULD BE COMPILED IN FORTRAN 4 (FORTHX)
C-----------------------------------------------------------------------
C
C   COMPUTER            - IBM/DOUBLE
C
C   LATEST REVISION     - JUNE 1, 1980
C
C   PURPOSE             - SOLVE A SYSTEM OF NONLINEAR EQUATIONS
C
C   USAGE               - CALL ZSCNT (FCN,NSIG,N,ITMAX,PAR,X,FNORM,
C                           WK,IER)
C
C   ARGUMENTS    FCN    - THE NAME OF A USER-SUPPLIED SUBROUTINE WHICH
C                           EVALUATES THE SYSTEM OF EQUATIONS TO BE
C                           SOLVED. FCN MUST BE DECLARED EXTERNAL IN
C                           THE CALLING PROGRAM AND MUST HAVE THE
C                           FOLLOWING FORM,
C                             SUBROUTINE FCN(X,F,N,PAR)
C                             DIMENSION X(N),F(N),PAR(1)
C                             F(1)=
C                              .
C                             F(N)=
C                             RETURN
C                             END
C                           GIVEN X(1)...X(N), FCN MUST EVALUATE THE
C                           FUNCTIONS F(1)...F(N) WHICH ARE TO BE MADE
C                           ZERO. X SHOULD NOT BE ALTERED BY FCN. THE
C                           PARAMETERS IN VECTOR PAR (SEE ARGUMENT
C                           PAR BELOW) MAY ALSO BE USED IN THE
C                           CALCULATION OF F(1)...F(N).
C                NSIG   - THE NUMBER OF DIGITS OF ACCURACY DESIRED
C                           IN THE COMPUTED ROOT (INPUT).
C                N      - THE NUMBER OF EQUATIONS TO BE SOLVED AND
C                           THE NUMBER OF UNKNOWNS (INPUT).
C                ITMAX  - THE MAXIMUM ALLOWABLE NUMBER OF ITERATIONS
C                           (INPUT).
C                PAR    - PAR CONTAINS A PARAMETER SET WHICH IS
C                           PASSED TO THE USER SUPPLIED FUNCTION FCN.
C                           PAR MAY BE USED TO PASS ANY AUXILIARY
C                           PARAMETERS NECESSARY FOR COMPUTATION OF
C                           THE FUNCTION FCN. (INPUT)
C                X      - A VECTOR OF LENGTH N. (INPUT/OUTPUT) ON INPUT,
C                           X IS THE INITIAL APPROXIMATION TO THE ROOT.
C                           ON OUTPUT, X IS THE BEST APPROXIMATION TO
C                           THE ROOT FOUND BY ZSCNT.
C                FNORM  - ON OUTPUT, FNORM IS EQUAL TO
C                           F(1)**2+...F(N)**2 AT THE POINT X.
C                WK     - WORK VECTOR OF LENGTH (N+1)*(3*N+8)
C                IER    - ERROR PARAMETER. (OUTPUT)
C                         TERMINAL ERROR
C                           IER = 129 INDICATES THAT ZSCNT FAILED TO
C                             CONVERGE WITHIN ITMAX ITERATIONS. THE
C                             USER MAY INCREASE ITMAX OR TRY A NEW
```

```
C                                INITIAL GUESS.
C                              IER = 130 INDICATES THE ALGORITHM WAS
C                                UNABLE TO IMPROVE ON THE RETURNED VALUE
C                                OF X. THIS SITUATION ARISES WHEN THE
C                                SOLUTION CANNOT BE DETERMINED TO NSIG
C                                DIGITS DUE TO ERRORS IN THE FUNCTION
C                                VALUES. IT MAY ALSO INDICATE THAT THE
C                                ROUTINE IS TRAPPED IN THE AREA OF A
C                                LOCAL MINIMUM. THE USER MAY TRY A NEW
C                                INITIAL GUESS.
C
C   PRECISION/HARDWARE  - SINGLE AND DOUBLE/H32
C                       - SINGLE/H36,H48,H60
C
C   REQD. IMSL ROUTINES - SINGLE/GGUBFS,LEQT2F,LUDATF,LUELMF,LUREFF,
C                           UERSET,UERTST,UGETIO,ZSCNU
C                       - DOUBLE/GGUBFS,LEQT2F,LUDATF,LUELMF,LUREFF,
C                           UERSET,UERTST,UGETIO,VXADD,VXMUL,VXSTO,
C                           ZSCNU
C
C   NOTATION            - INFORMATION ON SPECIAL NOTATION AND
C                           CONVENTIONS IS AVAILABLE IN THE MANUAL
C                           INTRODUCTION OR THROUGH IMSL ROUTINE UHELP
C
C   COPYRIGHT           - 1980 BY IMSL, INC. ALL RIGHTS RESERVED.
C
C   WARRANTY            - IMSL WARRANTS ONLY THAT IMSL TESTING HAS BEEN
C                           APPLIED TO THIS CODE. NO OTHER WARRANTY,
C                           EXPRESSED OR IMPLIED, IS APPLICABLE.
C
C-----------------------------------------------------------------------
C
C     SUBROUTINE ZSCNT  (FCN,NSIG,N,ITMAX,PAR,X,FNORM,WK,IER)
C                                  SPECIFICATIONS FOR ARGUMENTS
      INTEGER            IER,ITMAX,N,NSIG
      DOUBLE PRECISION   FNORM,PAR(1),WK(1),X(N)
C                                  SPECIFICATIONS FOR LOCAL VARIABLES
      INTEGER            I1,I2,I3,I4,I5,LNEW,LOLD,N1
      EXTERNAL           FCN
C                                  FIRST EXECUTABLE STATEMENT
      N1=N+1
      I1 = N1*N1+1
      I2 = I1+N*N1
      I3 = I2+N1
      I4 = I3+N1
      I5 = I4+N1
      CALL UERSET(0,LOLD)
      CALL ZSCNU(X,N,FCN,NSIG,N1,WK(1),WK(I1),WK(I2),WK(I3),WK(I4)
     * ,WK(I5),ITMAX,PAR,IER)
      CALL FCN(X,WK,N,PAR)
      FNORM = 0.0D0
      DO 5 I = 1,N
         FNORM = FNORM + WK(I)*WK(I)
    5 CONTINUE
      CALL UERSET(LOLD,LNEW)
```

```
      IF (IER.EQ.0) GO TO 9005
 9000 CONTINUE
      CALL UERTST(IER,6HZSCNT )
 9005 RETURN
      END

      FUNCTION GGUBFS (DSEED)
      DOUBLE PRECISION   DSEED
      DOUBLE PRECISION   D2P31M,D2P31
      DATA               D2P31M/2147483647.D0/
      DATA               D2P31 /2147483648.D0/
      DSEED = DMOD(16807.D0*DSEED,D2P31M)
      GGUBFS = DSEED / D2P31
      RETURN
      END
```

```
      SUBROUTINE LEQT2F (A,M,N,IA,B,IDGT,WKAREA,IER)
      DIMENSION          A(IA,1),B(IA,1),WKAREA(1)
      DOUBLE PRECISION   A,B,WKAREA,D1,D2,WA
      IER=0
      JER=0
      J = N*N+1
      K = J+N
      MM = K+N
      KK = 0
      MM1 = MM-1
      JJ=1
      DO 5 L=1,N
         DO 5 I=1,N
            WKAREA(JJ)=A(I,L)
            JJ=JJ+1
    5 CONTINUE
      CALL LUDATF (WKAREA(1),A,N,N,IDGT,D1,D2,WKAREA(J),WKAREA(K),
     1   WA,IER)
      IF (IER.GT.128) GO TO 25
      IF (IDGT .EQ. 0 .OR. IER .NE. 0) KK = 1
      DO 15 I = 1,M
         CALL LUELMF (A,B(1,I),WKAREA(J),N,N,WKAREA(MM))
         IF (KK .NE. 0)
     *   CALL LUREFF (WKAREA(1),B(1,I),A,WKAREA(J),N,N,WKAREA(MM),IDGT,
     *                WKAREA(K),WKAREA(K),JER)
         DO 10 II=1,N
            B(II,I) = WKAREA(MM1+II)
   10    CONTINUE
         IF (JER.NE.0) GO TO 20
   15 CONTINUE
      GO TO 25
   20 IER = 131
   25 JJ=1
      DO 30 J = 1,N
         DO 30 I = 1,N
            A(I,J)=WKAREA(JJ)
            JJ=JJ+1
   30 CONTINUE
      IF (IER .EQ. 0) GO TO 9005
 9000 CONTINUE
      CALL UERTST (IER,6HLEQT2F)
 9005 RETURN
      END
```

```
      SUBROUTINE LUDATF (A,LU,N,IA,IDGT,D1,D2,IPVT,EQUIL,WA,IER)
      DIMENSION          A(IA,1),LU(IA,1),IPVT(1),EQUIL(1)
      DOUBLE PRECISION   A,LU,D1,D2,EQUIL,WA,ZERO,ONE,FOUR,SIXTN,SIXTH,
     *                   RN,WREL,BIGA,BIG,P,SUM,AI,WI,T,TEST,Q
      DATA               ZERO,ONE,FOUR,SIXTN,SIXTH/0.D0,1.D0,4.D0,
     *                   16.D0,.0625D0/
      IER = 0
      RN = N
      WREL = ZERO
      D1 = ONE
      D2 = ZERO
      BIGA = ZERO
      DO 10 I=1,N
         BIG = ZERO
         DO 5 J=1,N
            P = A(I,J)
            LU(I,J) = P
            P = DABS(P)
            IF (P .GT. BIG) BIG = P
    5    CONTINUE
         IF (BIG .GT. BIGA) BIGA = BIG
         IF (BIG .EQ. ZERO) GO TO 110
         EQUIL(I) = ONE/BIG
   10 CONTINUE
      DO 105 J=1,N
         JM1 = J-1
         IF (JM1 .LT. 1) GO TO 40
         DO 35 I=1,JM1
            SUM = LU(I,J)
            IM1 = I-1
            IF (IDGT .EQ. 0) GO TO 25
            AI = DABS(SUM)
            WI = ZERO
            IF (IM1 .LT. 1) GO TO 20
            DO 15 K=1,IM1
               T = LU(I,K)*LU(K,J)
               SUM = SUM-T
               WI = WI+DABS(T)
   15       CONTINUE
            LU(I,J) = SUM
   20       WI = WI+DABS(SUM)
            IF (AI .EQ. ZERO) AI = BIGA
            TEST = WI/AI
            IF (TEST .GT. WREL) WREL = TEST
            GO TO 35
   25       IF (IM1 .LT. 1) GO TO 35
            DO 30 K=1,IM1
               SUM = SUM-LU(I,K)*LU(K,J)
   30       CONTINUE
            LU(I,J) = SUM
   35    CONTINUE
   40    P = ZERO
         DO 70 I=J,N
            SUM = LU(I,J)
            IF (IDGT .EQ. 0) GO TO 55
```

```
            AI = DABS(SUM)
            WI = ZERO
            IF (JM1 .LT. 1) GO TO 50
            DO 45 K=1,JM1
               T = LU(I,K)*LU(K,J)
               SUM = SUM-T
               WI = WI+DABS(T)
   45       CONTINUE
            LU(I,J) = SUM
   50       WI = WI+DABS(SUM)
            IF (AI .EQ. ZERO) AI = BIGA
            TEST = WI/AI
            IF (TEST .GT. WREL) WREL = TEST
            GO TO 65
   55       IF (JM1 .LT. 1) GO TO 65
            DO 60 K=1,JM1
               SUM = SUM-LU(I,K)*LU(K,J)
   60       CONTINUE
            LU(I,J) = SUM
   65       Q = EQUIL(I)*DABS(SUM)
            IF (P .GE. Q) GO TO 70
            P = Q
            IMAX = I
   70    CONTINUE
         IF (RN+P .EQ. RN) GO TO 110
         IF (J .EQ. IMAX) GO TO 80
         D1 = -D1
         DO 75 K=1,N
            P = LU(IMAX,K)
            LU(IMAX,K) = LU(J,K)
            LU(J,K) = P
   75    CONTINUE
         EQUIL(IMAX) = EQUIL(J)
   80    IPVT(J) = IMAX
         D1 = D1*LU(J,J)
   85    IF (DABS(D1) .LE. ONE) GO TO 90
         D1 = D1*SIXTH
         D2 = D2+FOUR
         GO TO 85
   90    IF (DABS(D1) .GE. SIXTH) GO TO 95
         D1 = D1*SIXTN
         D2 = D2-FOUR
         GO TO 90
   95    CONTINUE
         JP1 = J+1
         IF (JP1 .GT. N) GO TO 105
         P = LU(J,J)
         DO 100 I=JP1,N
            LU(I,J) = LU(I,J)/P
  100    CONTINUE
  105 CONTINUE
      IF (IDGT .EQ. 0) GO TO 9005
      P = 3*N+3
      WA = P*WREL
      IF (WA+10.D0**(-IDGT) .NE. WA) GO TO 9005
```

```
      IER = 34
      GO TO 9000
  110 IER = 129
      D1 = ZERO
      D2 = ZERO
 9000 CONTINUE
      CALL UERTST(IER,6HLUDATF)
 9005 RETURN
      END

      SUBROUTINE LUELMF (A,B,IPVT,N,IA,X)
      DIMENSION          A(IA,1),B(1),IPVT(1),X(1)
      DOUBLE PRECISION   A,B,X,SUM
      DO 5 I=1,N
    5 X(I) = B(I)
      IW = 0
      DO 20 I=1,N
         IP = IPVT(I)
         SUM = X(IP)
         X(IP) = X(I)
         IF (IW .EQ. 0) GO TO 15
         IM1 = I-1
         DO 10 J=IW,IM1
            SUM = SUM-A(I,J)*X(J)
   10    CONTINUE
         GO TO 20
   15    IF (SUM .NE. 0.D0) IW = I
   20 X(I) = SUM
      DO 30 IB=1,N
         I = N+1-IB
         IP1 = I+1
         SUM = X(I)
         IF (IP1 .GT. N) GO TO 30
         DO 25 J=IP1,N
            SUM = SUM-A(I,J)*X(J)
   25   CONTINUE
   30 X(I) = SUM/A(I,I)
      RETURN
      END
```

```
      SUBROUTINE LUREFF (A,B,UL,IPVT,N,IA,X,IDGT,RES,DX,IER)
      DIMENSION          A(IA,1),UL(IA,1),B(1),X(1),RES(1),DX(1),IPVT(1)
      DIMENSION          ACCXT(2)
      DOUBLE PRECISION   A,ACCXT,B,UL,X,RES,DX,ZERO,XNORM,DXNORM
      DATA               ITMAX/75/,ZERO/0.D0/
      IER=0
      XNORM = ZERO
      DO 10 I=1,N
         XNORM = DMAX1(XNORM,DABS(X(I)))
   10 CONTINUE
      IF (XNORM .NE. ZERO) GO TO 20
      IDGT = 50
      GO TO 9005
   20 DO 45 ITER=1,ITMAX
         DO 30 I=1,N
      ACCXT(1) = 0.0D0
      ACCXT(2) = 0.0D0
            CALL VXADD(B(I),ACCXT)
            DO 25 J=1,N
               CALL VXMUL(-A(I,J),X(J),ACCXT)
   25       CONTINUE
            CALL VXSTO(ACCXT,RES(I))
   30    CONTINUE
         CALL LUELMF (UL,RES,IPVT,N,IA,DX)
         DXNORM = ZERO
         XNORM = ZERO
         DO 35 I=1,N
            X(I) = X(I) + DX(I)
            DXNORM = DMAX1(DXNORM,DABS(DX(I)))
            XNORM = DMAX1(XNORM,DABS(X(I)))
   35    CONTINUE
         IF (ITER .NE. 1) GO TO 40
         IDGT = 50
         IF (DXNORM .NE. ZERO) IDGT = -DLOG10(DXNORM/XNORM)
   40    IF (XNORM+DXNORM .EQ. XNORM) GO TO 9005
   45 CONTINUE
      IER = 129
 9000 CONTINUE
      CALL UERTST(IER,6HLUREFF)
 9005 RETURN
      END

      SUBROUTINE UERSET (LEVEL,LEVOLD)
      INTEGER            LEVEL,LEVOLD
      LEVOLD = LEVEL
      CALL UERTST (LEVOLD,6HUERSET)
      RETURN
      END
```

```
      SUBROUTINE UERTST (IER,NAME)
      INTEGER            IEQ
      INTEGER*2           NAME(3),NAMSET(3),NAMEQ(3)
      DATA                NAMSET/2HUE,2HRS,2HET/
      DATA                NAMEQ/2H  ,2H  ,2H  /
      DATA                LEVEL/4/,IEQDF/0/,IEQ/1H=/
      IF (IER.GT.999) GO TO 25
      IF (IER.LT.-32) GO TO 55
      IF (IER.LE.128) GO TO 5
      IF (LEVEL.LT.1) GO TO 30
      CALL UGETIO(1,NIN,IOUNIT)
      IF (IEQDF.EQ.1) WRITE(IOUNIT,35) IER,NAMEQ,IEQ,NAME
      IF (IEQDF.EQ.0) WRITE(IOUNIT,35) IER,NAME
      GO TO 30
    5 IF (IER.LE.64) GO TO 10
      IF (LEVEL.LT.2) GO TO 30
      CALL UGETIO(1,NIN,IOUNIT)
      IF (IEQDF.EQ.1) WRITE(IOUNIT,40) IER,NAMEQ,IEQ,NAME
      IF (IEQDF.EQ.0) WRITE(IOUNIT,40) IER,NAME
      GO TO 30
   10 IF (IER.LE.32) GO TO 15
      IF (LEVEL.LT.3) GO TO 30
      CALL UGETIO(1,NIN,IOUNIT)
      IF (IEQDF.EQ.1) WRITE(IOUNIT,45) IER,NAMEQ,IEQ,NAME
      IF (IEQDF.EQ.0) WRITE(IOUNIT,45) IER,NAME
      GO TO 30
   15 CONTINUE
      DO 20 I=1,3
         IF (NAME(I).NE.NAMSET(I)) GO TO 25
   20 CONTINUE
      LEVOLD = LEVEL
      LEVEL = IER
      IER = LEVOLD
      IF (LEVEL.LT.0) LEVEL = 4
      IF (LEVEL.GT.4) LEVEL = 4
      GO TO 30
   25 CONTINUE
      IF (LEVEL.LT.4) GO TO 30
      CALL UGETIO(1,NIN,IOUNIT)
      IF (IEQDF.EQ.1) WRITE(IOUNIT,50) IER,NAMEQ,IEQ,NAME
      IF (IEQDF.EQ.0) WRITE(IOUNIT,50) IER,NAME
   30 IEQDF = 0
      RETURN
   35 FORMAT(19H *** TERMINAL ERROR,10X,7H(IER = ,I3,
     1       20H) FROM IMSL ROUTINE ,3A2,A1,3A2)
   40 FORMAT(36H *** WARNING WITH FIX ERROR  (IER = ,I3,
     1       20H) FROM IMSL ROUTINE ,3A2,A1,3A2)
   45 FORMAT(18H *** WARNING ERROR,11X,7H(IER = ,I3,
     1       20H) FROM IMSL ROUTINE ,3A2,A1,3A2)
   50 FORMAT(20H *** UNDEFINED ERROR,9X,7H(IER = ,I5,
     1       20H) FROM IMSL ROUTINE ,3A2,A1,3A2)
   55 IEQDF = 1
      DO 60 I=1,3
   60 NAMEQ(I) = NAME(I)
   65 RETURN
```

```
      END
      SUBROUTINE UGETIO(IOPT,NIN,NOUT)
      INTEGER            IOPT,NIN,NOUT
      INTEGER            NIND,NOUTD
      DATA               NIND/5/,NOUTD/6/
      IF (IOPT.EQ.3) GO TO 10
      IF (IOPT.EQ.2) GO TO 5
      IF (IOPT.NE.1) GO TO 9005
      NIN = NIND
      NOUT = NOUTD
      GO TO 9005
    5 NIND = NIN
      GO TO 9005
   10 NOUTD = NOUT
 9005 RETURN
      END
      SUBROUTINE VXADD(A,ACC)
      DOUBLE PRECISION   A,ACC(2)
      DOUBLE PRECISION   X,Y,Z,ZZ
      X = ACC(1)
      Y = A
      IF (DABS(ACC(1)).GE.DABS(A)) GO TO 1
      X = A
      Y = ACC(1)
    1 Z = X+Y
      ZZ = (X-Z)+Y
      ZZ = ZZ+ACC(2)
      ACC(1) = Z+ZZ
      ACC(2) = (Z-ACC(1))+ZZ
      RETURN
      END
```

```
      SUBROUTINE VXMUL (A,B,ACC)
      DOUBLE PRECISION   A,B,ACC(2)
      DOUBLE PRECISION   X,HA,TA,HB,TB
      INTEGER            IX(2),I
      LOGICAL*1          LX(8),LI(4)
      EQUIVALENCE        (X,LX(1),IX(1)),(I,LI(1))
      DATA               I/0/
      X = A
      LI(4) = LX(5)
      IX(2) = 0
      I = (I/16)*16
      LX(5) = LI(4)
      HA=X
      TA=A-HA
      X = B
      LI(4) = LX(5)
      IX(2) = 0
      I = (I/16)*16
      LX(5) = LI(4)
      HB = X
      TB = B-HB
      X = TA*TB
      CALL VXADD(X,ACC)
      X = HA*TB
      CALL VXADD(X,ACC)
      X = TA*HB
      CALL VXADD(X,ACC)
      X = HA*HB
      CALL VXADD(X,ACC)
      RETURN
      END

      SUBROUTINE VXSTO (ACC,D)
      DOUBLE PRECISION   ACC(2),D
      D = ACC(1)+ACC(2)
      RETURN
      END
```

```
      SUBROUTINE ZSCNU  (X,N,FCN,NDIGIT,N1,A,Z,Y,XNORM,B,WK,            ZSC04850
     1 MAXIT,PAR,IER)                                                   ZSC04860
      INTEGER            IER,MAXIT,N,NDIGIT,N1                          ZSC04870
      DOUBLE PRECISION  A(N1,1),B(1),PAR(1),WK(1),X(1),XNORM(1),        ZSC04880
     1                  Y(1),Z(N,1)                                     ZSC04890
      INTEGER            I,IBMAX,IBNORM,IDGT,IEVAL,ITER,J,JER,JI,JS,    ZSC04900
     1                   MI,NRS,NSTART                                  ZSC04910
      DOUBLE PRECISION   BIG,BNORM,CFACT,DX,EPS,HALF,HLMAX,RACC,REPS,   ZSC04920
     1                   RRX,RX,SFACT,SMALL,TEST,TN,TR                  ZSC04930
      DOUBLE PRECISION  DSEED                                           ZSC04940
      DATA               SMALL/Z3410000000000000/                       ZSC04950
      IER = 0                                                           ZSC04960
      DSEED = 12345.0D0                                                 ZSC04970
      CFACT = 0.99D0                                                    ZSC04980
      BIG = 5.0D5                                                       ZSC04990
      SFACT = 0.1D0                                                     ZSC05000
      IBMAX = 50                                                        ZSC05010
      NRS = 2                                                           ZSC05020
      RACC = DMIN1(DMAX1(SMALL,10.0D0**(-NDIGIT)),0.1D0)                ZSC05030
      REPS = DSQRT(SMALL)                                               ZSC05040
      HLMAX = 3.0D0                                                     ZSC05050
      ITER = 0                                                          ZSC05060
      IEVAL = 0                                                         ZSC05070
      NSTART = 0                                                        ZSC05080
      IBNORM = 0                                                        ZSC05090
      RX = 1.0D0                                                        ZSC05100
      RRX = 0.0D0                                                       ZSC05110
      EPS = 0.0D0                                                       ZSC05120
      DO 5 I=1,N                                                        ZSC05130
         B(I) = 0.D0                                                    ZSC05140
         A(N1,I) = 1.0D0                                                ZSC05150
         Z(I,N1) = X(I)                                                 ZSC05160
    5 CONTINUE                                                          ZSC05170
      B(N1) = 1.D0                                                      ZSC05180
      A(N1,N1) = 1.0D0                                                  ZSC05190
      JI = N1                                                           ZSC05200
      IEVAL = IEVAL+1                                                   ZSC05210
      CALL FCN (Z(1,N1),A(1,N1),N,PAR)                                  ZSC05220
      BNORM = 0.0D0                                                     ZSC05230
      DO 10 I=1,N                                                       ZSC05240
         BNORM = BNORM+A(I,N1)*A(I,N1)                                  ZSC05250
   10 CONTINUE                                                          ZSC05260
      XNORM(N1) = BNORM                                                 ZSC05270
   15 IF (NSTART.EQ.NRS) EPS = EPS*10.0D0                               ZSC05280
      IF (NSTART.EQ.0) EPS = DMIN1(RX,REPS)                             ZSC05290
      IF (EPS.GT.BIG) GO TO 120                                         ZSC05300
      NSTART = MOD(NSTART,NRS)+1                                        ZSC05310
      DO 30 J=1,N                                                       ZSC05320
         DO 25 I=1,N                                                    ZSC05330
   20       TR = (GGUBFS(DSEED)-0.5D0)*2.0D0                            ZSC05340
            IF (DABS(TR).LT.0.1D0) GO TO 20                             ZSC05350
            Z(I,J) = X(I)+DMAX1(DABS(X(I)),0.1D0)*TR*EPS                ZSC05360
   25    CONTINUE                                                       ZSC05370
         IEVAL = IEVAL+1                                                ZSC05380
         CALL FCN (Z(1,J),A(1,J),N,PAR)                                 ZSC05390
```

```
   30 CONTINUE                                                          ZSC05400
      DO 35 J=1,N                                                       ZSC05410
         XNORM(J) = 0.D0                                                ZSC05420
      DO 35 I=1,N                                                       ZSC05430
         XNORM(J) = XNORM(J)+A(I,J)*A(I,J)                              ZSC05440
   35 CONTINUE                                                          ZSC05450
   40 JI = N1                                                           ZSC05460
      JS = JI                                                           ZSC05470
      DO 45 J=1,N                                                       ZSC05480
         IF (XNORM(J).GT.XNORM(JS)) JS = J                              ZSC05490
         IF (XNORM(J).LT.XNORM(JI)) JI = J                              ZSC05500
   45 CONTINUE                                                          ZSC05510
      IF (XNORM(JI).EQ.0.D0) GO TO 125                                  ZSC05520
      IF (XNORM(JI).GT.SFACT*BNORM) GO TO 50                            ZSC05530
      BNORM = XNORM(JI)                                                 ZSC05540
      IBNORM = ITER                                                     ZSC05550
   50 IF ((ITER-IBNORM).GT.IBMAX) GO TO 120                             ZSC05560
      ITER = ITER+1                                                     ZSC05570
      IF (ITER.GE.MAXIT) GO TO 115                                      ZSC05580
      DO 55 MI=1,N1                                                     ZSC05590
         Y(MI) = B(MI)                                                  ZSC05600
   55 CONTINUE                                                          ZSC05610
      IDGT = 0                                                          ZSC05620
      CALL LEQT2F (A,1,N1,N1,Y,IDGT,WK,JER)                             ZSC05630
      IF (JER.NE.0) GO TO 85                                            ZSC05640
      DO 65 I=1,N                                                       ZSC05650
         DX = 0.0D0                                                     ZSC05660
         DO 60 J=1,N1                                                   ZSC05670
            DX = DX + Y(J)*Z(I,J)                                       ZSC05680
   60    CONTINUE                                                       ZSC05690
         X(I) = DX                                                      ZSC05700
   65 CONTINUE                                                          ZSC05710
      HALF = 0.D0                                                       ZSC05720
   70 IEVAL = IEVAL+1                                                   ZSC05730
      CALL FCN (X,Y,N,PAR)                                              ZSC05740
      TN = 0.D0                                                         ZSC05750
      DO 75 I=1,N                                                       ZSC05760
         TN = TN+Y(I)*Y(I)                                              ZSC05770
   75 CONTINUE                                                          ZSC05780
      IF (TN.LT.XNORM(JS)) GO TO 95                                     ZSC05790
      HALF = HALF+1.D0                                                  ZSC05800
      IF (HALF.GT.HLMAX) GO TO 85                                       ZSC05810
      DO 80 I=1,N                                                       ZSC05820
         X(I) = (X(I)+HALF*Z(I,JI))/(HALF+1.0D0)                        ZSC05830
   80 CONTINUE                                                          ZSC05840
      GO TO 70                                                          ZSC05850
   85 IF (JI.EQ.N1) GO TO 15                                            ZSC05860
      XNORM(N1) = XNORM(JI)                                             ZSC05870
      DO 90 I=1,N                                                       ZSC05880
         Z(I,N1) = Z(I,JI)                                              ZSC05890
         A(I,N1) = A(I,JI)                                              ZSC05900
   90 CONTINUE                                                          ZSC05910
      GO TO 15                                                          ZSC05920
   95 IF ((HALF.NE.0.D0).OR.(ITER.EQ.1)) GO TO 105                      ZSC05930
      RX = SMALL                                                        ZSC05940
```

```
      DO 100 I=1,N
         RX = DMAX1(RX,DABS(X(I)-Z(I,JI))/DMAX1(DABS(X(I)),0.1D0))
  100 CONTINUE
      RRX = DMAX1(-DLOG10(RX),0.0D0)
      IF (RX.LE.RACC) GO TO 125
  105 IF (TN.LT.CFACT*XNORM(JI)) NSTART = 0
      XNORM(JS) = TN
      DO 110 I=1,N
         Z(I,JS) = X(I)
         A(I,JS) = Y(I)
  110 CONTINUE
      GO TO 40
  115 IER = 129
      GO TO 125
  120 IER = 130
  125 DO 130 I=1,N
         X(I) = Z(I,JI)
  130 CONTINUE
      RETURN
      END
```

APPENDIX C

RESULTS OF EXPERIMENTS - NEW SERIES

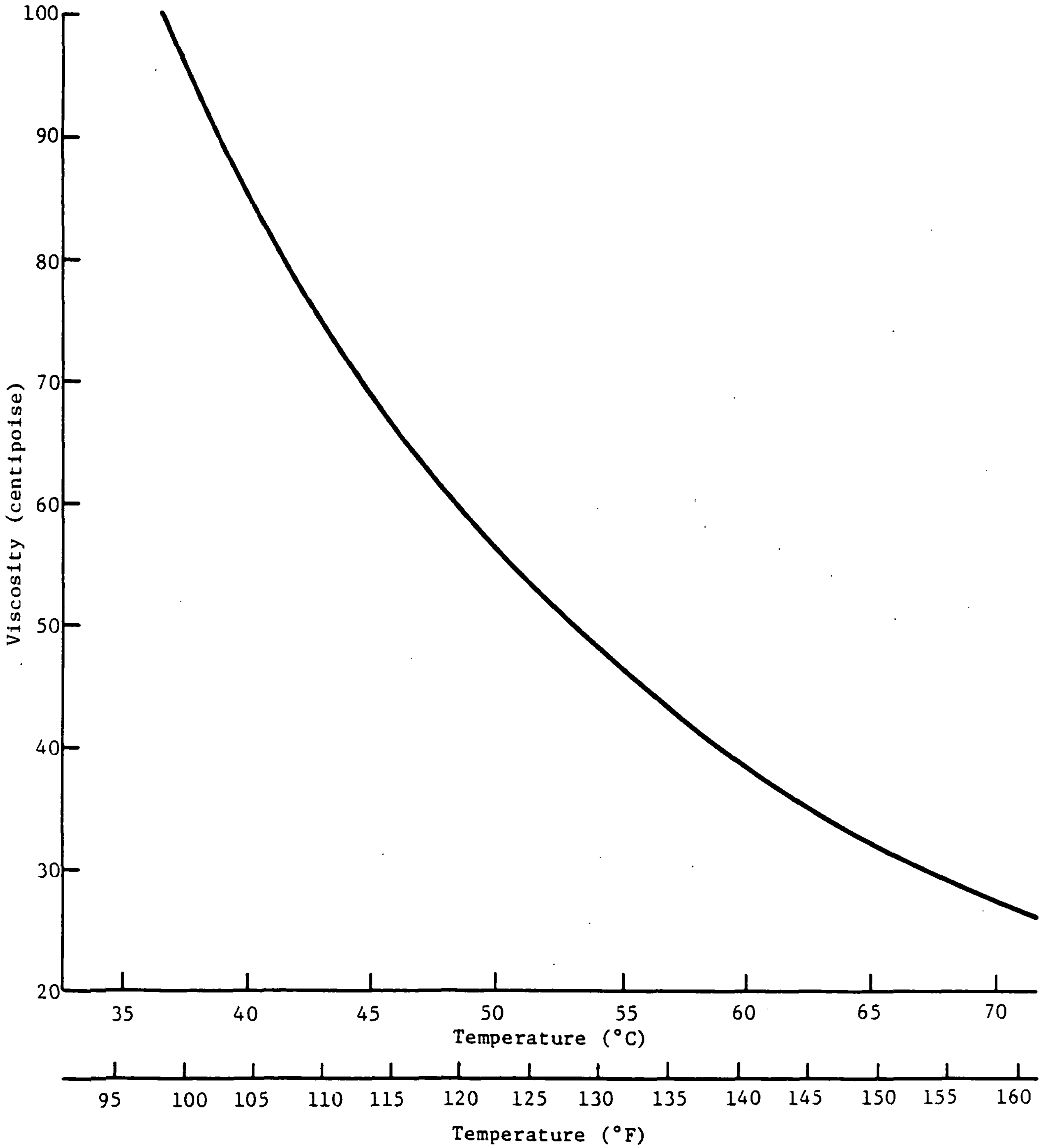

Fig. C-1 Viscosity of 20W40 Oil

TABLE C-1 PUMPING RING DESIGN A-1-A-1 – SMALL CLEARANCE; TIN BASED BABBITT SAE-11 : STROKE = 2 in

DATA POINT	FREQUENCY Hz	OIL INLET TEMPERATURE °C	(°F)	CLAMPING PRESSURE MPa	(lb/in²)	PUMPING PRESSURE MPa	(lb/in²)	PUMPED FLOW g/min
1	10	47.8	(118)	8.62	(1250)	8.87	(1287)	0
		47.2	(117)			7.31	(1060)	2.91
		47.8	(118)			4.79	(695)	3.40
		47.8	(118)			2.09	(303)	3.45
		47.8	(118)			0.04	(6)	3.67
2	35	50.0	(122)	8.62	(1250)	9.18	(1332)	0
		50.0	(122)			8.51	(1235)	15.6
		50.0	(122)			7.58	(1100)	16.2
		50.0	(122)			5.52	(800)	18.8
		48.9	(120)			2.76	(400)	20.4
		47.8	(118)			0.21	(30)	20.8
3	60	48.9	(120)	8.62	(1250)	9.10	(1320)	0
		49.4	(121)			8.81	(1278)	33.3
		50.0	(122)			6.23	(1005)	38.4
		48.9	(120)			5.52	(800)	40.3
		48.9	(120)			2.76	(400)	42.5
		49.4	(121)			0.33	(48)	43.0
4	10	47.8	(118)	6.89	(1000)	7.25	(1052)	0
		47.8	(118)			6.40	(929)	3.2
		47.8	(118)			5.19	(844)	3.5
		47.8	(118)			3.47	(504)	4.0
		47.8	(118)			-0-	(0)	4.3
5	35	47.8	(118)	6.89	(1000)	7.36	(1068)	0
		47.8	(118)			6.76	(980)	19.8
		47.8	(118)			5.52	(800)	21.5
		47.2	(117)			1.38	(200)	24.5
		46.7	(116)			0.12	(18)	14.9

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CALCULATION SHEET

MTI-4 (3-70)

STROKE = 2 in

TABLE C-1 CONT'D PUMPING RING DESIGN – A·1·A·1 SMALL CLEARANCE MAT'L: TIN BASED BABBITT (SAE·11)

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	OIL INLET TEMP (°F)	CLAMPING PRESS MPa	CLAMPING PRESS (lb/in²)	PUMPED PRESS MPa	PUMPED PRESS (lb/in²)	PUMPED FLOW g/min
6	60	50.0	(122)	6.89	(1000)	7.39	(1072)	0
		49.4	(121)	↓	↓	6.89	(1000)	42.0
		49.4	(121)	↓	↓	4.82	(700)	46.5
		49.4	(121)	↓	↓	2.75	(399)	49.4
		50.0	(122)	↓	↓	0.14	(20)	50.1
7	10	47.8	(118)	3.45	(500)	3.85	(558)	0
		47.8	(118)	↓	↓	3.41	(495)	3.02
		47.8	(118)	↓	↓	2.44	(354)	3.97
		47.8	(118)	↓	↓	1.38	(200)	2.92
		47.8	(118)	↓	↓	-0-	(0)	3.83
8	35	48.3	(119)	3.45	(500)	3.86	(560)	0
		48.9	(120)	↓	↓	3.35	(486)	20.9
		50.0	(122)	↓	↓	2.37	(345)	22.7
		49.4	(121)	↓	↓	1.38	(200)	25.6
		50.0	(122)	↓	↓	0.10	(14)	28.0
9	60	48.9	(120)	3.45	(500)	3.79	(550)	0
		50.0	(122)	↓	↓	3.45	(500)	47.2
		49.4	(121)	↓	↓	2.07	(300)	51.5
		48.9	(120)	↓	↓	1.38	(200)	54.2
		48.9	(120)	↓	↓	0.18	(26)	59.0
10	10	47.8	(118)	8.62	(1250)	8.62	(1250)	2.42
		47.8	(118)	6.89	(1000)	6.89	(1000)	3.15
		47.2	(117)	5.52	(800)	5.52	(800)	3.05
		47.2	(117)	4.31	(625)	4.31	(625)	2.00
		47.8	(118)	1.33	(193)	1.33	(193)	0

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MTI-4 (3-79)

MTI-10196

TABLE C-1 CONT'D PUMPING RING DESIGN – A-1-A-1 MAT'L:

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW ℓ/min	
11	35	49.4	(121)	8.69	(1260)	8.69	(1260)	17.9	
		49.4	(121)	6.99	(1014)	6.99	(1014)	21.7	
		48.9	(120)	5.62	(816)	5.62	(816)	24.3	
		50.0	(122)	4.27	(620)	4.27	(620)	23.2	
		50.0	(122)	2.76	(400)	2.76	(400)	19.0	
		50.0	(122)	1.38	(200)	1.38	(200)	17.0	
		50.0	(122)	0.29	(42)	0.29	(42)	14.5	
12	60	51.1	(124)	8.65	(1254)	8.65	(1254)	33.8	
		50.0	(122)	6.87	(997)	6.87	(997)	41.0	
		48.9	(120)	5.67	(822)	5.67	(822)	44.8	
		48.9	(120)	4.22	(612)	4.22	(612)	44.8	
		48.9	(120)	2.76	(400)	2.76	(400)	40.6	
		48.9	(120)	1.40	(203)	1.40	(203)	36.8	
		49.4	(121)	0.35	(51)	0.35	(51)	33.6	
13	60	59.4	(139)	8.62	(1250)	8.69	(1261)	-0-	25.4mm (1.00in.) Stroke
		58.9	(138)	↓	↓	7.45	(1080)	5.45	
		60.6	(141)			4.96	(720)	6.43	
		60.0	(140)			2.76	(400)	6.69	
		60.0	(140)			1.08	(156)	6.74	
14	60	47.8	(118)	8.62	(1250)	9.18	(1331)	-0-	25.4 (1.00in.) Stroke
		48.9	(120)	↓	↓	8.60	(1248)	4.83	
		50.0	(122)			4.96	(720)	6.71	
		48.9	(120)			2.76	(400)	7.26	
		48.9	(120)			1.21	(176)	7.43	

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CALCULATION SHEET

MTI-4 (3-70)

TABLE C-2 PUMPING RING DESIGN – A·2·A·1 LARGE CLEARANCE, MAT'L: TIN BASED BABBITT (SAE-11)

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW ℓ/min
1	10	47.8	(118)	10.3	(1500)	10.5	(1517)	-0-
		47.8	(118)			8.64	(1254)	2.08
		47.8	(118)			7.57	(1099)	2.46
		47.8	(118)			4.82	(700)	2.85
		47.8	(118)			-0-	(0)	3.05
2	35	50.0	(122)	10.3	(1500)	10.8	(1572)	-0-
		50.0	(122)			8.62	(1250)	16.8
		48.3	(119)			7.78	(1128)	17.5
		47.8	(118)			4.14	(600)	18.6
		47.8	(118)			0.14	(20)	18.3
3	60	48.9	(120)	10.3	(1500)	10.8	(1575)	-0-
		48.9	(120)			10.2	(1488)	31.9
		50.0	(122)			9.06	(1315)	34.0
		50.0	(122)			5.88	(853)	35.6
		51.1	(124)			3.42	(496)	35.7
		50.0	(122)			.24	(35)	35.9
4	10	48.9	(120)	8.62	(1250)	8.79	(1275)	-0-
		48.9	(120)			7.58	(1100)	2.93
		48.9	(120)			5.51	(800)	3.46
		48.9	(120)			2.67	(390)	3.64
		48.3	(119)			-0-	(0)	3.71
5	35	48.9	(120)	8.62	1250	8.98	(1302)	-0-
		48.9	(120)			8.35	(1224)	18.8
		48.9	(120)			6.76	(980)	20.2
		48.9	(120)			3.45	(500)	21.4
		48.3	(119)			0.23	(34)	21.8

CHARGE NO.	PROJECT TITLE:	ENGINEER:	DATE:	NOTES:	MTI

CALCULATION SHEET

MTI-4 (3-70)

TABLE C-2 CONT'D PUMPING RING DESIGN – A-2-A-1 LARGE CLEARANCE MAT'L: TIN BASED BABBITT (SAE-11)

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW ℓ/min
6	60	48.3	(119)	8.62	(1250)	9.13	(1324)	-0-
		49.4	(121)			8.77	(1272)	31.8
		50.0	(122)			7.80	(1131)	36.3
		50.0	(122)			6.02	(874)	39.7
		48.9	(120)			3.45	(500)	41.8
		49.4	(121)			0.14	(20)	41.9
7	10	50.0	(122)	6.89	(1000)	6.99	(1014)	-0-
		48.9	(120)			6.13	(889)	3.50
		48.9	(120)			4.81	(698)	4.32
		48.9	(120)			2.73	(396)	4.58
		48.9	(120)			0	-0-	4.69
8	35	47.8	(118)	6.89	(1000)	7.24	(1050)	-0-
		48.9	(120)			6.43	(933)	23.2
		48.9	(120)			6.33	(918)	20.8
		50.0	(122)			6.02	(873)	22.4
		48.3	(119)			3.45	(500)	23.5
		47.8	(118)			0.10	(14)	26.2
9	60	48.9	(120)	6.89	(1000)	7.25	(1052)	-0-
		48.9	(120)			7.20	(1044)	37.1
		48.9	(120)			6.09	(884)	44.2
		49.4	(121)			3.45	(500)	49.5
		48.9	(120)			0.15	(22)	50.6
10	10	48.9	(120)	10.3	(1500)	10.3	(1500)	1.70
		48.9	(120)	8.61	(1249)	8.61	(1249)	2.84
		48.3	(119)	6.91	(1002)	6.91	(1002)	4.25
		47.8	(118)	4.84	(702)	4.84	(702)	6.34
		48.3	(119)	2.76	(400)	2.76	(400)	5.16
		48.3	(119)	1.38	(200)	1.38	(200)	4.70
		47.8	(118)	0.10	(14)	0.10	(14)	7.96

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MTI-18198

TABLE C-2 CONT'D PUMPING RING DESIGN – A-2-A-1 MAT'L:

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW ℓ/min	
11	35		(120)	10.3	(1500)	10.3	(1500)	15.2	
			(118)	8.98	(1303)	8.98	(1303)	18.0	
			(118)	7.58	(1100)	7.58	(1100)	20.9	
			(117)	6.20	(900)	6.20	(900)	22.8	
			(120)	4.85	(703)	4.85	(703)	22.4	
			(121)	3.45	(500)	3.45	(500)	19.3	
			(120)	2.07	(300)	2.07	(300)	18.8	
			(120)	0.69	(100)	0.69	(100)	18.6	
			(120)	0.23	(34)	0.23	(34)	18.3	
12	60		(118)	10.3	(1500)	10.3	(1500)	34.1	
			(119)	8.99	(1304)	8.99	(1304)	40.2	
			(120)	7.58	(1099)	7.58	(1099)	45.2	
			(121)	6.29	(912)	6.29	(912)	52.1	
			(119)	4.84	(702)	4.84	(702)	55.6	
			(119)	3.45	(500)	3.45	(500)	51.7	
			(120)	2.07	(300)	2.07	(300)	46.0	
			(120)	0.67	(97)	0.67	(97)	43.1	
			(122)	0.42	(60)	0.42	(60)	40.7	
13	60		(138)	8.62	(1250)	9.33	(1354)	-0-	
			(137)	↓	↓	8.60	(1248)	5.71	
			(137)			4.96	(720)	7.50	25.4 mm (1.00 in) Stroke
			(136)			2.76	(400)	7.60	
			(138)			0.83	(120)	7.57	

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MTI-4 (3-70)

MTI-10194

TABLE C-3 PUMPING RING DESIGN - A-1-B-1 SHORT LENGTH MAT'L: TIN BASED BABBITT (SAE-11)

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	OIL INLET TEMP (°F)	CLAMPING PRESS MPa	CLAMPING PRESS (lb/in²)	PUMPED PRESS MPa	PUMPED PRESS (lb/in²)	PUMPED FLOW l/min
1	10	47.8	(118)	10.3	(1500)	9.00	(1305)	-0-
		48.3	(119)			6.97	(982)	.659
		48.3	(119)			4.49	(651)	1.14
		48.3	(119)			2.23	(323)	1.62
		46.1	(115)			0.02	(4)	2.39
2	35	49.4	(121)	10.3	(1500)	9.82	(1424)	-0-
		50.0	(122)			7.39	(1072)	6.55
		50.0	(122)			4.91	(712)	10.3
		49.4	(121)			2.48	(360)	12.9
		49.4	(121)			0.04	(6)	15.1
3	60	50.6	(123)	10.3	(1500)	11.5	(1659)	-0-
		50.6	(123)			8.60	(1248)	22.9
		49.4	(121)			5.73	(831)	28.1
		48.9	(120)			2.91	(422)	32.1
		50.6	(123)			0.09	(13)	31.7
4	10	49.4	(121)	6.89	(1000)	6.95	(1008)	-0-
		50.0	(122)			5.20	(754)	1.42
		50.0	(122)			3.47	(503)	1.95
		50.0	(122)			1.74	(253)	2.30
		48.9	(120)			0.03	(5)	2.74
5	35	48.3	(119)	6.89	(1000)	7.69	(1116)	-0-
		47.2	(117)			5.76	(835)	9.08
		48.9	(120)			3.84	(557)	12.6
		49.4	(121)			1.93	(280)	14.8
		52.2	(126)			0.08	(11)	16.6

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TABLE C-3 PUMPING RING DESIGN - A-1-B-1 SHORT LENGTH MAT'L: TIN BASED BABBITT (SAE-11)

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW g/min
6	60	50.0	(122)	6.89	(1000)	7.69	(1116)	0
		50.0	(122)			5.76	(835)	22.6
		50.0	(122)			3.86	(560)	27.8
		50.6	(123)			1.93	(280)	32.0
		50.0	(122)			0.11	(16)	34.6
7	10	51.7	(125)	3.45	(500)	1.72	(250)	0
		51.1	(124)			1.24	(188)	.809
		51.7	(125)			0.86	(125)	2.02
		51.1	(124)			0.43	(63)	2.78
		52.2	(126)			0	0	3.24
8	35	51.1	(124)	3.45	(500)	3.52	(510)	-0-
		51.1	(124)			2.64	(383)	5.46
		51.7	(125)			1.76	(256)	12.4
		52.2	(126)			0.90	(129)	14.6
		51.1	(124)			0.08	(12)	18.6
9	60	52.8	(127)	3.45	(500)	3.71	(538)	-0-
		50.6	(123)			2.78	(404)	190
		51.1	(124)			1.87	(271)	25.9
		52.2	(126)			0.94	(137)	31.0
		53.3	(128)			0.11	(16)	36.1
10	10	47.2	(117)	10.3	(1500)	10.3	(1500)	-0-
		47.2	(117)	8.62	(1250)	8.62	(1250)	.693
		47.2	(117)	6.89	(1000)	6.89	(1000)	1.24
11	35	47.2	(117)	10.3	(1500)	10.3	(1500)	6.08
		48.9	(120)	8.62	(1250)	8.62	(1250)	9.40
		48.3	(119)	6.89	(1000)	6.89	(1000)	12.8

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MTI 4 (3-70)

TABLE C-3 PUMPING RING DESIGN – A-1-B-1 SHORT LENGTH MAT'L:

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW l/min	
12	60	48.9	(120)	10.3	(1500)	10.3	(1500)	18.5	
		49.4	(121)	8.62	(1250)	8.62	(1250)	30.1	
		50.0	(122)	6.89	(1000)	6.89	(1000)	38.8	
13	60	58.3	(137)	8.62	(1250)	7.90	(1146)	-0-	
		58.9	(138)	↓	↓	7.03	(1020)	1.6	
		58.3	(137)	↓	↓	4.96	(720)	4.68	25.4 mm (1.00 in) STROKE
		57.7	(136)	↓	↓	2.76	(400)	5.38	
		58.3	(137)	↓	↓	0.59	(86)	5.55	
14	0	51.1	(124)	0	-0-	0	-0-	1.42	
	10	51.1	(124)	0	-0-	0	-0-	2.34	INLET P = 0.076 MPa (11 lb/in²)
	35	51.1	(124)	0	-0-	0	-0-	12.1 (6.9)	
	60	51.1	(124)	0	-0-	0	-0-	17.8	
15	0	53.3	(128)	0	-0-	0	-0-	4.50	
	10	53.3	(128)	0	-0-	0	-0-	9.74	INLET P = 0.276 MPa (40 lb/in²)
	35	53.3	(128)	0	-0-	0	-0-	11.7	
	60	53.3	(128)	0	-0-	0	-0-	77.4	
16	0	50.6	(123)	0	-0-	0	-0-	2.00	
	10	50.6	(123)	0	-0-	0	-0-	5.99	INLET P = 0.138 MPa (20 lb/in²)
	35	50.0	(122)	0	-0-	0	-0-	9.88	
	60	51.7	(125)	0	-0-	0	-0-	20.7	
17	0	50.0	(122)	6.89	(1000)	0	-0-	—	
	10	50.0	(122)	↓	↓	0	-0-	2.69	
	35	50.0	(122)	↓	↓	0	-0-	15.0	INLET P = 0.138 MPa (20 lb/in²)
	60	50.6	(123)	↓	↓	0	-0-	43.4	

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TABLE C-3 PUMPING RING DESIGN - A-1-B-1 MAT'L:

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	OIL INLET TEMP (°F)	CLAMPING PRESS MPa	CLAMPING PRESS (lb/in²)	PUMPED PRESS MPa	PUMPED PRESS (lb/in²)	PUMPED FLOW g/min	
18	0	50.0	(122)	6.89	(1000)	0.03	(5)	-0-	
	.10	50.0	(122)			0.08	(12)	3.33	INLET P = 0.200 MPa (29 lb/in²)
	35	50.0	(122)			0.10	(15)	20.8	
	60	51.7	(125)			0.12	(18)	47.0	

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MTI-4 (3-70)

TABLE C-4. PUMPING RING DESIGN – A-1-A-2 CONTOURED MAT'L: TIN BASED BABBITT (SAE-11)

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW ℓ/min
1	10	47.8	(118)	8.62	(1250)	8.98	(1303)	-0-
		48.3	(119)			8.62	(1250)	2.64
		47.8	(118)			4.96	(720)	2.52
		47.8	(118)			2.76	(400)	2.40
		47.8	(118)			0.43	(60)	2.69
2	35	50.6	(123)	8.62	(1250)	9.45	(1372)	-0-
		50.0	(122)			8.62	(1250)	18.0
		47.2	(117)			4.96	(720)	17.2
		48.3	(119)			2.76	(400)	17.3
		48.9	(120)			0.78	(114)	17.2
3	60	50.6	(123)	8.62	1250	9.51	(1380)	-0-
		48.9	(120)			8.62	(1250)	39.0
		47.2	(117)			6.70	(972)	38.0
		47.2	(117)			2.71	(393)	36.8
		49.4	(121)			1.03	(150)	33.0
4	10	47.2	(117)	6.89	(1000)	7.16	(1039)	-0-
		48.3	(119)			6.54	(949)	1.30
		48.3	(119)			4.96	(720)	1.40
		48.3	(119)			2.76	(400)	1.50
		48.9	(120)			0.29	(42)	1.60
5	35	49.4	(121)	6.89	(1000)	7.51	(1090)	-0-
		48.9	(120)			6.89	(1000)	18.9
		49.4	(121)			4.96	(720)	18.6
		48.9	(120)			2.55	(370)	18.8
		48.3	(119)			1.36	(198)	18.9

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MTI-4 (3-70)

TABLE C-4 PUMPING RING DESIGN – A-1-A-2 CONTOURED MAT'L: TIN BASED BABBITT (SAE-11)

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	OIL INLET TEMP (°F)	CLAMPING PRESS MPa	CLAMPING PRESS (lb/in²)	PUMPED PRESS MPa	PUMPED PRESS (lb/in²)	PUMPED FLOW ℓ/min
6	60	50.6	(123)	6.89	(1000)	7.58	(1092)	-0-
		50.0	(122)			6.70	(972)	47.2
		48.9	(120)			5.09	(738)	48.2
		47.2	(117)			2.76	(400)	46.8
		50.6	(123)			1.74	(252)	45.2
7	10	49.4	(121)	3.45	500	3.96	(574)	-0-
		50.0	(122)			2.90	(420)	4.02
		50.0	(122)			1.65	(240)	4.00
		50.0	(122)			0.36	(53)	4.00
8	35	47.8	(118)	3.45	500	4.09	(594)	-0-
		48.9	(120)			3.31	(480)	36.5
		50.0	(122)			2.07	(300)	38.2
		50.0	(122)			1.65	(240)	38.4
9	60	48.9	(120)	3.45	500	4.30	(624)	-0-
		48.9	(120)			3.31	(480)	80.8
10	10	47.2	(117)	9.10	(1320)	9.10	(1320)	0.456
		47.8	(118)	6.89	(1000)	6.89	(1000)	1.74
		47.8	(118)	5.51	(800)	5.52	(800)	2.78
		47.2	(117)	4.14	(600)	4.14	(600)	4.20
		47.8	(118)	2.76	(400)	2.76	(400)	6.00
11	35	47.8	(118)	9.10	(1320)	9.10	(1320)	13.4
		49.4	(121)	6.89	(1000)	6.89	(1000)	26.3
		49.4	(121)	5.51	(800)	5.52	(800)	25.8
		47.8	(118)	4.14	(600)	4.14	(600)	33.2
		47.2	(117)	2.76	(400)	2.76	(400)	41.2

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TABLE C-4 PUMPING RING DESIGN — A-1-A-2 CONTOURED MAT'L: TIN BASED BABBITT (SAE-11)

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW ℓ/min	
12	60	48.9	(120)	9.10	(1320)	9.10	(1320)	31.2	
		49.4	(121)	7.02	(1018)	7.02	(1018)	46.4	
		50.0	(122)	5.58	(810)	5.58	(810)	59.2	
		48.9	(120)	4.14	(600)	4.14	(600)	74.0	
		48.9	(120)	3.07	(446)	3.07	(446)	95.8	
13	60	57.7	(136)	8.62	(1250)	9.20	(1335)	-0-	
		58.3	(137)	↓	↓	8.60	(1248)	2.31	
		58.3	(137)			4.96	(720)	2.72	25.4 mm (1.00 in) Stroke
		58.9	(138)			2.76	(400)	3.85	
		57.7	(136)			0.52	(76)	4.29	

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TABLE C-5 POMPING RING DESIGN - B-1-A-1 MAT'L: RULON J

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW l/min
1	10	48.3	(119)	5.17	(750)	4.92	(714)	-0-
		47.8	(118)			3.31	(480)	4.08
		48.9	(120)			1.65	(240)	4.48
		48.3	(119)			0.83	(120)	4.40
		48.3	(119)			0.13	(19)	4.30
2	35	47.8	(118)	5.17	(750)	5.20	(755)	-0-
		47.2	(117)			3.31	(480)	25.7
		49.4	(121)			1.65	(240)	28.2
		48.3	(119)			0.83	(120)	27.8
		47.8	(118)			0.26	(37)	27.0
3	60	48.9	(120)	5.17	(750)	5.18	(751)	-0-
		48.9	(120)			5.38	(480)	59.9
		48.9	(120)			1.65	(240)	65.4
		48.3	(119)			0.83	(120)	63.0
		48.3	(119)			0.63	(91)	61.7
4	10	47.2	(117)	3.45	(500)	3.71	(578)	-0-
		47.8	(118)			2.48	(360)	3.63
		48.9	(120)			1.65	(240)	5.77
		48.9	(120)			0.83	(120)	5.48
		48.3	(119)			0.14	(20)	6.05
5	35	48.3	(119)	3.45	(500)	3.57	(518)	-0-
		48.3	(119)			2.48	(360)	32.5
		48.3	(119)			1.65	(240)	41.6
		48.9	(120)			0.83	(120)	39.7
		48.3	(119)			0.32	(47)	39.7

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TABLE C-5 PUMPING RING DESIGN – B-1-A-1 MAT'L: RULON J

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW g/min
6	60	48.9	(120)	3.45	(500)	3.52	(510)	-0-
		48.9	(120)			2.48	(360)	77.1
		49.4	(121)			1.65	(240)	101
		48.9	(120)			0.83	(120)	113
		50.0	(122)			0.51	(74)	89.2
7	10	48.9	(120)	1.72	(250)	1.98	(288)	-0-
		48.9	(120)			1.65	(240)	6.38
		48.3	(119)			1.24	(180)	7.37
		48.3	(119)			0.83	(120)	7.87
		48.9	(120)			0.30	(43)	11.5
8	35	49.4	(121)	1.72	(250)	2.08	(301)	-0-
		50.6	(123)			1.65	(240)	48.6
		49.4	(121)			1.24	(180)	53.7
		49.4	(121)			1.00	(145)	61.8
9	60	50.0	(122)	1.72	(250)	2.03	(295)	-0-
		48.3	(119)			1.65	(240)	95.4
		49.4	(121)			1.24	(180)	140
		48.9	(120)			1.13	(164)	136
10	10	48.3	(119)	4.14	(600)	4.14	(600)	-0-
		49.4	(121)	3.45	(500)	3.45	(500)	1.24
		48.3	(119)	1.72	(250)	1.72	(250)	6.49
11	35	48.9	(120)	4.14	(600)	4.14	(600)	-0-
		48.9	(120)	3.45	(500)	3.45	(500)	8.97
		48.9	(120)	1.72	(250)	1.72	(250)	43.2
12	60	47.8	(118)	3.79	(550)	3.79	(550)	-0-
		49.4	(121)	3.45	(500)	3.45	(500)	16.5
		47.8	(118)	1.72	(250)	1.72	(250)	105

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TABLE C-5 PUMPING RING DESIGN - B-1-A-1 MAT'L: RULON J

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW ℓ/min
13	60	58.3	(137)	5.17	(750)		(847)	-0-
		58.9	(138)				(800)	9.72
		59.4	(139)				(600)	11.2
		58.9	(138)				(396)	10.8
		58.3	(137)				(118)	10.1
14	35	48.3	119	0	-0-	0.50	72	8.37
		48.3	119	0.34	50	0.58	85	10.9
		49.4	121	0.69	100	0.87	126	18.5
		49.4	121	1.03	150	0.90	130	58.6
		48.9	120	1.38	200	0.98	142	67.8
		48.3	119	1.72	250	1.11	161	66.6
		48.3	119	2.07	300	1.06	154	73.8
		48.3	119	2.41	350	0.78	113	87.6
		48.9	120	2.76	400	0.68	99	64.7
		48.9	120	3.10	450	0.63	91	43.7
		49.4	121	1.72	250	1.17	170	62.1
		48.9	120	1.38	200	1.23	179	63.6
15	35	47.8	118	0	-0-	0	-0-	90.9
		48.3	119	0.34	50			75.3
		48.9	120	0.69	100			87.8
		48.9	120	1.03	150			87.7
		48.9	120	1.38	200			78.9
		48.3	119	1.72	250			82.4
		48.3	119	2.07	300			86.0
		48.9	120	2.41	350			53.7
		48.9	120	2.76	400			49.4

CHARGE NO.	PROJECT TITLE:	ENGINEER:	DATE:	NOTES:	MTI

CALCULATION SHEET

MTI 4 (3-70)

TABLE C-6 PUMPING RING DESIGN - C·1·A·1 MAT'L: CARBON GRAPHITE (CNF-J)

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW l/min
1	10	48.9	(120)	6.89	(1000)	7.04	(1022)	-0-
		49.4	(121)			6.62	(961)	1.51
		48.9	(120)			5.50	(798)	4.27
		49.4	(121)			2.76	(400)	5.58
		48.9	(120)			1.36	(198)	5.20
		48.9	(120)			0.37	(54)	5.0
2	35	50.6	(123)	6.89	(1000)	7.35	(1066)	-0-
		51.1	(124)			6.89	(1000)	8.16
		48.9	(120)			5.50	(798)	13.14
		49.4	(121)			2.76	(400)	29.9
		48.9	(120)			1.36	(198)	29.3
		48.9	(120)			0.23	(34)	26.2
3	60	47.8	(118)	6.89	(1000)	7.44	(1080)	-0-
		47.2	(117)			6.89	(1000)	30.2
		48.3	(119)			5.50	(798)	44.6
		49.4	(121)			2.76	(400)	57.0
		49.4	(121)			1.36	(198)	55.7
		49.4	(121)			0.50	(72)	53.4
4	10	47.2	(117)	5.17	(750)	3.98	(578)	-0-
		47.8	(118)			3.72	(540)	4.95
		48.9	(120)			2.48	(360)	6.92
		48.9	(120)			1.24	(180)	6.77
		48.9	(120)			0.22	(32)	6.38
5	35	48.9	(120)	5.17	(750)	5.09	(738)	-0-
		50.0	(122)			3.72	(540)	27.8
		48.9	(120)			2.07	(300)	35.6
		48.9	(120)			1.24	(180)	36.0
		50.0	(122)			0.35	(51)	35.6

CHARGE NO. | PROJECT TITLE: | ENGINEER: | DATE: | NOTES: | MTI

CALCULATION SHEET MTI-4 (3-70)

TABLE C-6 PUMPING RING DESIGN – C-1-A-1 MAT'L: CARBON GRAPHITE (CNF-J)

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW ℓ/min
6	60	50.6	(123)	5.17	(750)	5.41	(785)	-0-
		50.6	(123)	↓	↓	3.72	(540)	54.6
		49.4	(121)			2.07	(300)	68.8
		49.4	(121)			1.24	(180)	71.2
		47.8	(118)			0.47	(68)	71.6
7	10	48.3	(119)	3.45	(500)	2.15	(312)	-0-
		48.3	(119)	↓	↓	1.65	(240)	4.11
		48.3	(119)			1.24	(180)	7.00
		48.9	(120)			0.83	(120)	8.08
		48.3	(119)			0.16	(24)	9.23
8	35	49.4	(121)	3.45	(500)	2.96	(429)	-0-
		49.4	(121)	↓	↓	2.48	(360)	13.4
		49.4	(121)			1.65	(240)	26.8
		48.9	(120)			0.83	(120)	36.0
		50.0	(122)			0.41	(60)	38.4
9	60	47.2	(117)	3.45	(500)	3.41	(495)	-0-
		47.2	(117)	↓	↓	2.48	(360)	35.2
		47.8	(118)			1.65	(240)	64.0
		47.8	(118)			0.96	(140)	64.0
10	10	WILL NOT SELF SUSTAIN						
11	35	49.4	(121)	7.38	(1070)	7.38	(1070)	10.2
		50.0	(122)	6.89	(1000)	6.89	(1000)	10.6
		WILL NOT SELF SUSTAIN AT LOWER PRESSURES						
12	60	51.1	(124)	7.73	(1121)	7.73	(1121)	24.6
		49.4	(121)	6.89	(1000)	6.89	(1000)	26.9
		50.0	(122)	6.20	(900)	6.20	(900)	27.5
		WILL NOT SELF SUSTAIN AT LOWER PRESSURES						

CHARGE NO. | PROJECT TITLE: | ENGINEER: | DATE: | NOTES: | MTI

CALCULATION SHEET

MTI 4 (3-70)

TABLE C-7 PUMPING RING DESIGN D-1-A-1 (12mm BORE) MAT'L: TIN BASED BABBITT (SAE-11)

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESS MPa	(lb/in²)	PUMPED FLOW ml/min
1	10	48.3	(119)	8.62	(1250)	9.02	(1308)	-0-
		49.4	(121)			7.44	(1080)	1.88
		48.3	(119)			4.93	(720)	1.82
		48.3	(119)			3.10	(450)	1.75
		48.3	(119			0.12	(17)	1.77
2	35	49.4	(121)	8.62	(1250)	9.38	(1361)	-0-
		50.0	(122)			7.44	(1080)	14.6
		50.0	(122)			4.93	(720)	14.4
		48.3	(119)			3.10	(450)	14.2
		48.3	(119)			0.97	(41)	13.4
3	60	49.4	(121)	8.62	(1250)	9.31	(1350)	-0-
		48.3	(119)			7.44	(1080)	31.3
		48.3	(119)			4.93	(720)	29.7
		49.4	(121)			3.10	(450)	27.6
		49.4	(121)			0.21	(31)	28.7
4	10	48.9	(120)	6.89	(1000)	7.43	1077	-0-
		48.9	(120)			5.38	780	2.65
		48.3	(119)			4.14	600	2.60
		47.8	(118)			1.65	240	2.40
		49.4	(121)			0.12	17	2.30
5	35	47.8	(118)	6.89	(1000)	7.51	(1090)	-0-
		48.3	(119)			6.62	(960)	17.1
		50.0	(122)			5.17	(750)	16.6
		48.3	(119)			3.10	(450)	16.6
		48.3	(119)			0.28	(41)	15.8

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CALCULATION SHEET

MTI-4 (3-70)

TABLE C-7 PUMPING RING DESIGN – D1-A-1 (12mm BORE) MAT'L: TIN BASED BABBITT (SAE-11)

DATA POINT	FREQ. Hz	OIL INLET TEMP °C	OIL INLET TEMP (°F)	CLAMPING PRESS MPa	CLAMPING PRESS (lb/in²)	PUMPED PRESS MPa	PUMPED PRESS (lb/in²)	PUMPED FLOW ml/min
6	60	47.8	(118)	6.89	(1000)	7.49	(1086)	-0-
		47.8	(118)			6.20	(900)	40.2
		50.0	(122)			4.14	(600)	37.7
		49.4	(121)			1.65	(240)	35.1
		49.4	(121)			0.41	(59)	33.1
7	10	48.3	(119)	3.45	(500)	3.72	(540)	-0-
		48.9	(120)			2.89	(420)	2.88
		48.3	(119)			1.65	(240)	8.67
		48.3	(119)			0.12	(18)	4.20
8	35	49.4	(121)	3.45	(500)	4.01	(582)	-0-
		49.4	(121)			2.90	(420)	20.8
		49.4	(121)			1.65	(240)	21.7
		49.4	(121)			0.30	(43)	22.2
9	60	50.0	(122)	3.45	(500)	4.01	(582)	-0-
		47.8	(118)			2.90	(420)	39.6
		47.8	(118)			1.65	(240)	43.0
		50.0	(122)			0.32	(47)	44.2
10	10	45.6	(114)	8.62	(1250)	8.62	(1250)	1.35
		47.2	(117)	6.89	(1000)	6.89	(1000)	1.74
		48.3	(119)	5.51	(800)	5.51	(800)	2.23
		48.9	(120)	4.14	(600)	4.14	(600)	2.76
		48.3	(119)	2.76	(400)	2.76	(400)	3.54
		48.3	(119)	1.39	(200)	1.39	(200)	3.80

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MTI-4 (3-78)

TABLE C-7 PUMPING RING DESIGN -- D-1-A-1 (12mm BORE) MAT'L: TIN BASED BABBITT (SAE 11)

DATA POINT	FREQ Hz	OIL INLET TEMP °C	(°F)	CLAMPING PRESS MPa	(lb/in²)	PUMPED PRESSURE MPa	(lb/in²)	PUMPED FLOW g/min	
11	35	48.3	(119)	8.62	(1250)	8.62	(1250)	13.0	
		48.3	(119)	6.89	(1000)	6.89	(1000)	15.4	
		49.4	(121)	5.51	(800)	5.51	(800)	17.6	
		48.9	(120)	4.14	(600)	4.14	(600)	20.0	
		49.4	(121)	2.76	(400)	2.76	(400)	21.0	
		48.9	(120)	1.39	(200)	1.39	(200)	19.3	
12	60	48.3	(119)	8.62	(1250)	8.62	(1250)	23.3	
		48.9	(120)	6.89	(1000)	6.89	(1000)	36.1	
		47.8	(118)	5.51	(800)	5.51	(800)	40.6	
		47.8	(118)	4.14	(600)	4.14	(600)	43.2	
		47.8	(118)	2.76	(400)	2.76	(400)	40.1	
		47.8	(118)	1.39	(200)	1.39	(200)	37.2	
13	60	48.3	(119)	8.62	(1250)	9.75	(1415)	0	
		47.8	(118)			8.27	(1200)	10.4	
		48.3	(119)			6.62	(960)	10.1	25.4 mm (1.00 in) Stroke
		50.6	(123)			4.14	(600)	9.8	
		48.3	(119)			2.48	(360)	9.0	
		49.4	(121)			0.17	(25)	8.7	
14	60	58.9	(138)	8.62	1250	9.35	(1356)	-0-	
		59.4	(139)			8.27	(1200)	7.71	
		60.0	(140)			6.62	(960)	7.56	25.4 mm (1.00 in) Stroke
		59.4	(139)			4.14	(600)	7.20	
		59.4	(139)			2.48	(360)	6.73	
		60.0	(140)			0.12	(17)	6.43	

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CALCULATION SHEET

MTI-4 (3-76)

APPENDIX D

FIGURES SHOWING COMPARISON WITH EXPERIMENTAL DATA AND UNCORRECTED THEORY

TABLE D-1

PARAMETERS USED IN CALCULATIONS

RUN	MATERIAL	R mm (in.)	$C \cdot 10^3$ mm (in.)	t mm (in.)	E $MPa \cdot 10^3$ ($psi \cdot 10^6$)	ν	L_1 mm (in.)	L mm (in.)	e mm (in.)
I	Babbitt	9.52 (0.375)	11.43 (0.45)	1.19 (.047)	51.7 (7.5)	0.36	7.56 (0.298)	6.78 (0.267)	2.92 (.115)
II	Babbitt	9.52 (0.375)	19.05 (0.45)	1.19 (.047)	51.7 (7.5)	0.36	7.56 (0.298)	6.78 (0.267)	2.92 (.115)
V	Rulon J	9.52 (0.375)	42.55 (1.675)	2.44 (.096)	1.72 (.25)	0.46	7.56 (0.298)	6.78 (0.267)	2.92 (.115)
VI	Carbon	9.52 (0.375)	20.35 (0.80)	1.44 (.056)	21.5 (3.11)	0.29	7.56 (0.298)	6.78 (0.267)	2.92 (.115)
VII	Babbitt	6.00 (0.236)	8.89 (0.35)	0.89 (.035)	51.7 (7.5)	0.36	5.59 (0.22)	5.08 (0.2)	1.93 (.076)

*The viscosity used is that corresponding to test temperature plus an assumed 20°F used in the oil film.

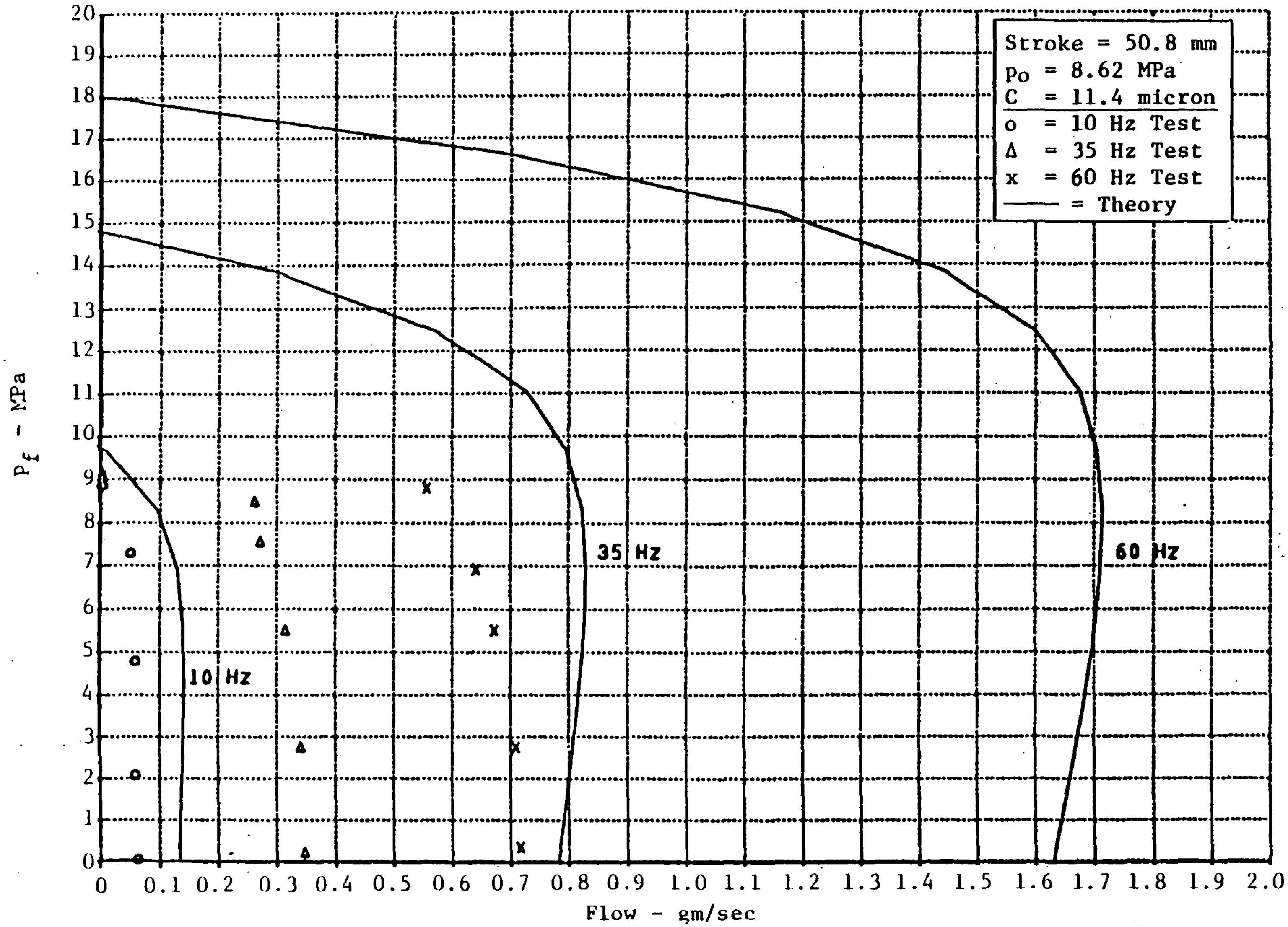


Fig. D-1 Performance of Babbitt Pumping Ring (I)

832932

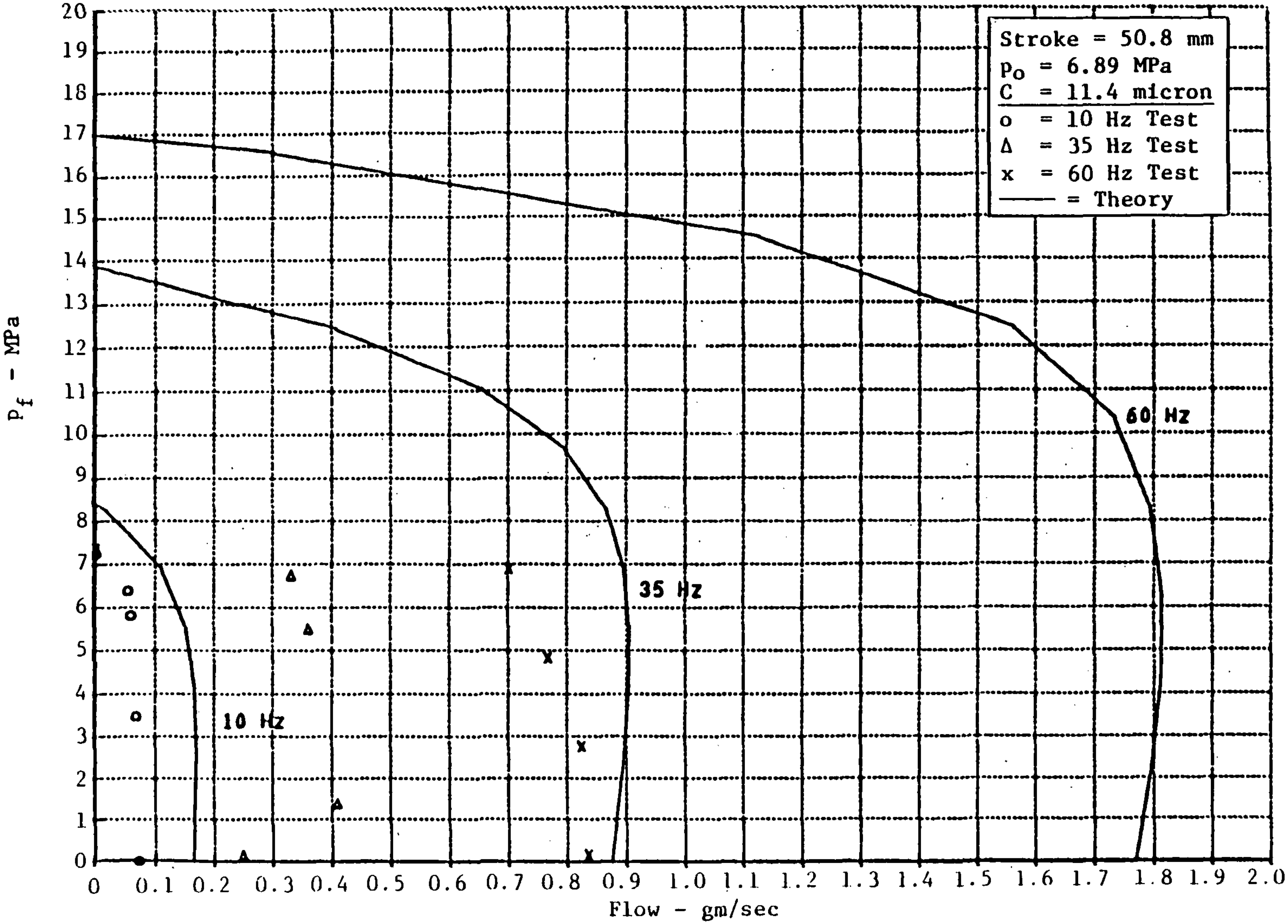


Fig. D-2 Performance of Babbitt Pumping Ring (I)

832933

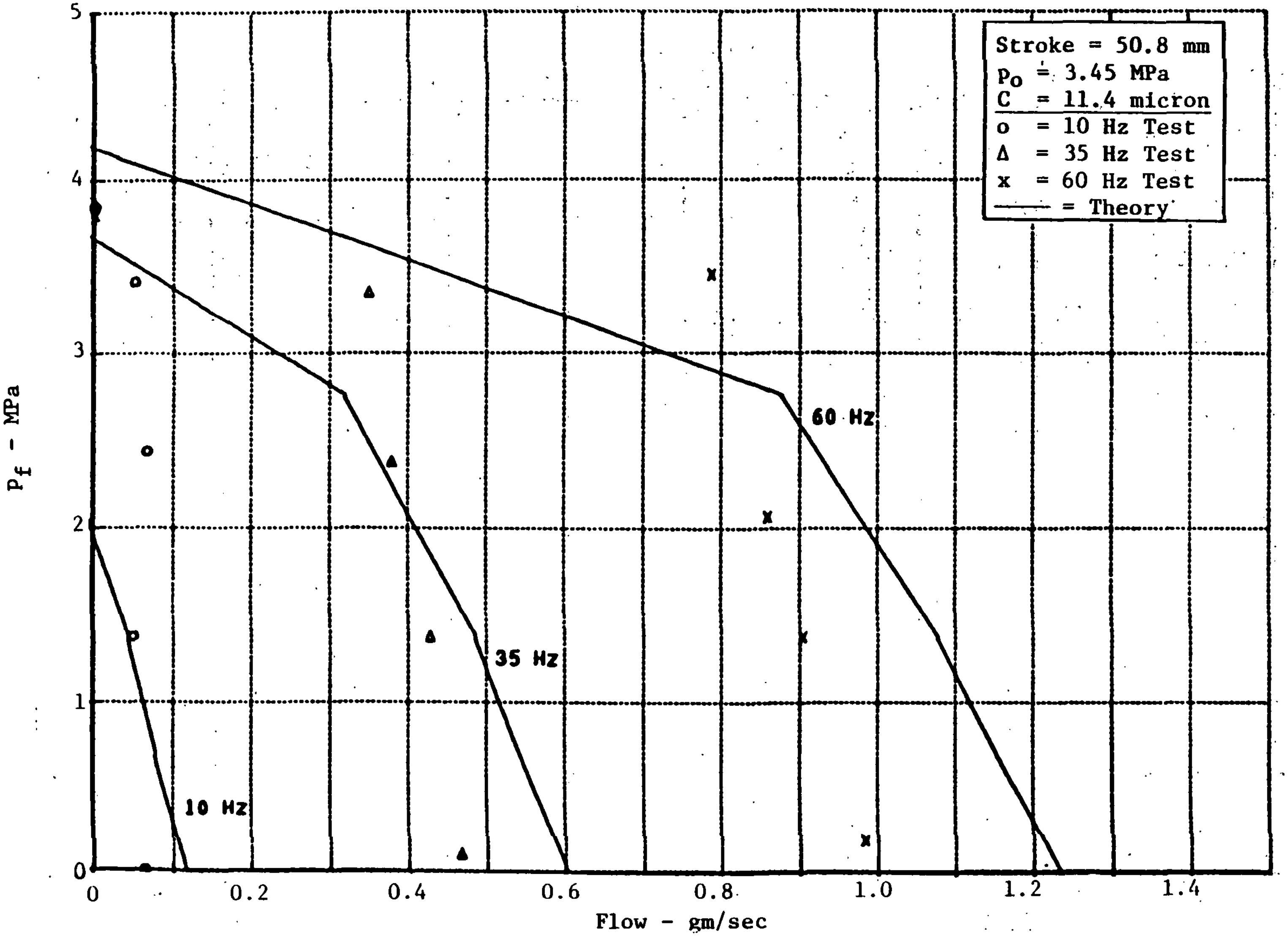


Fig. D-3 Performance of Babbitt Pumping Ring (I)

832934

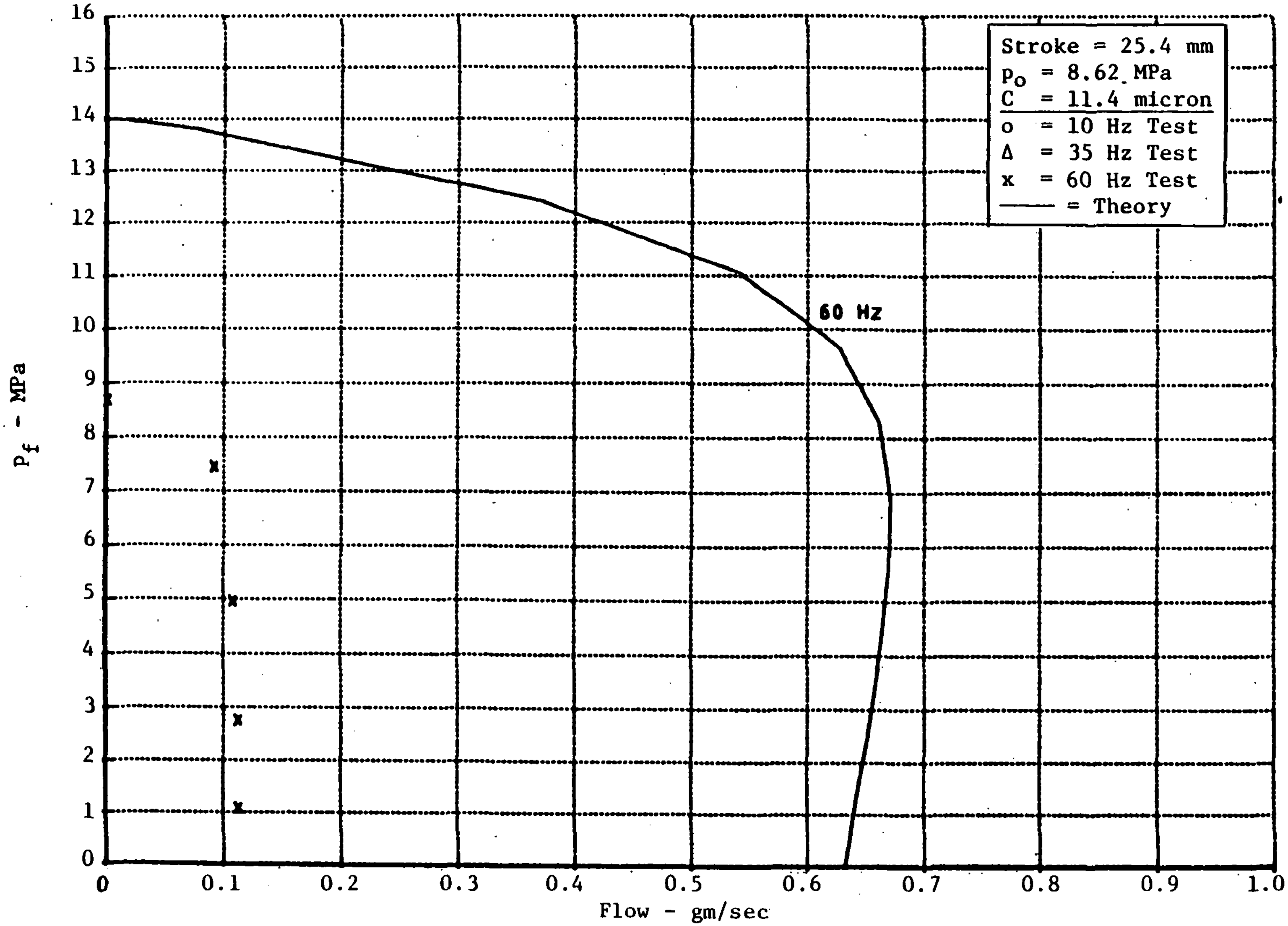


Fig. D-4 Performance of Babbitt Pumping Ring (I)

832935

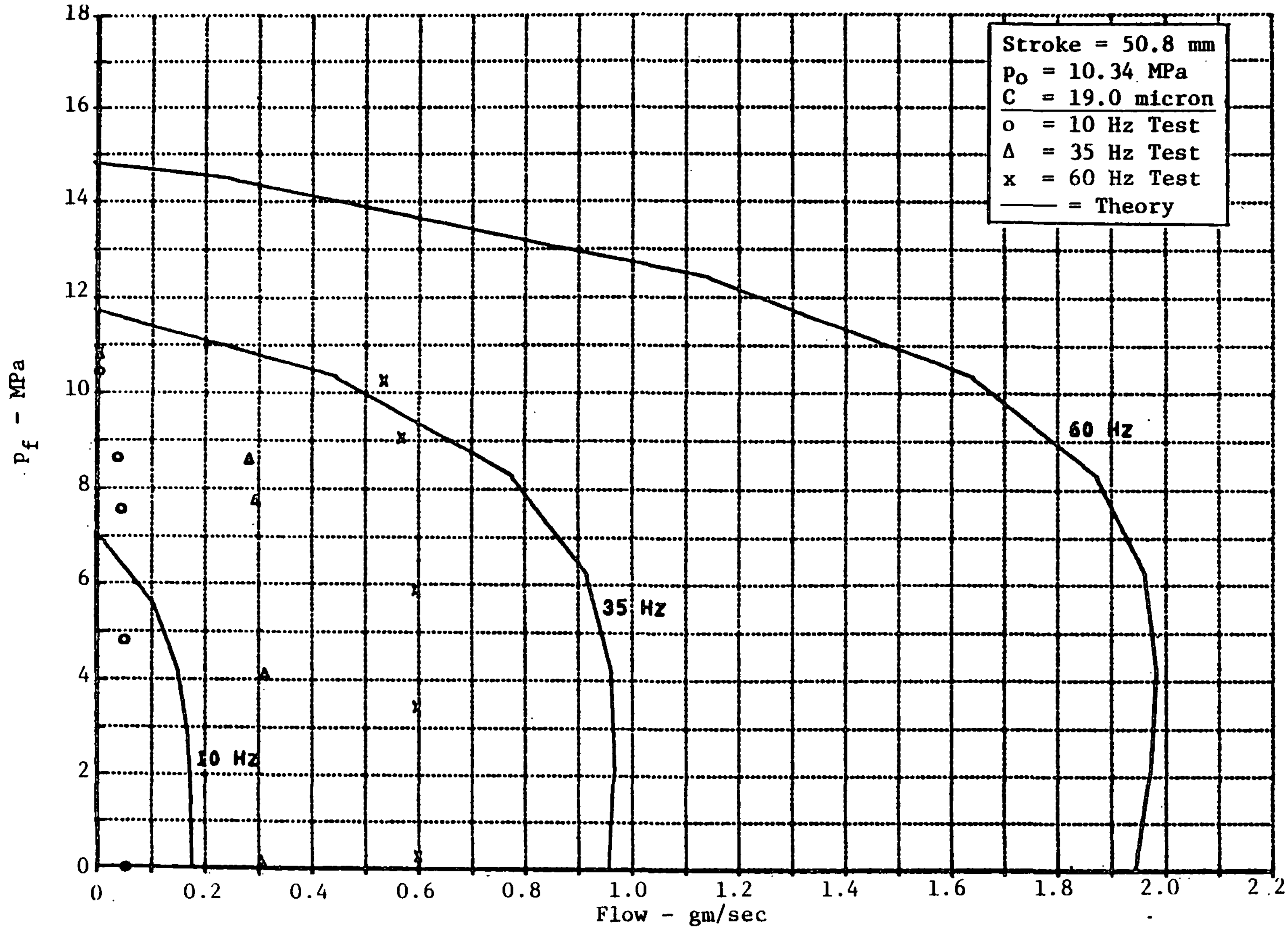


Fig. D-5 Performance of Babbitt Pumping Ring (II)

832936

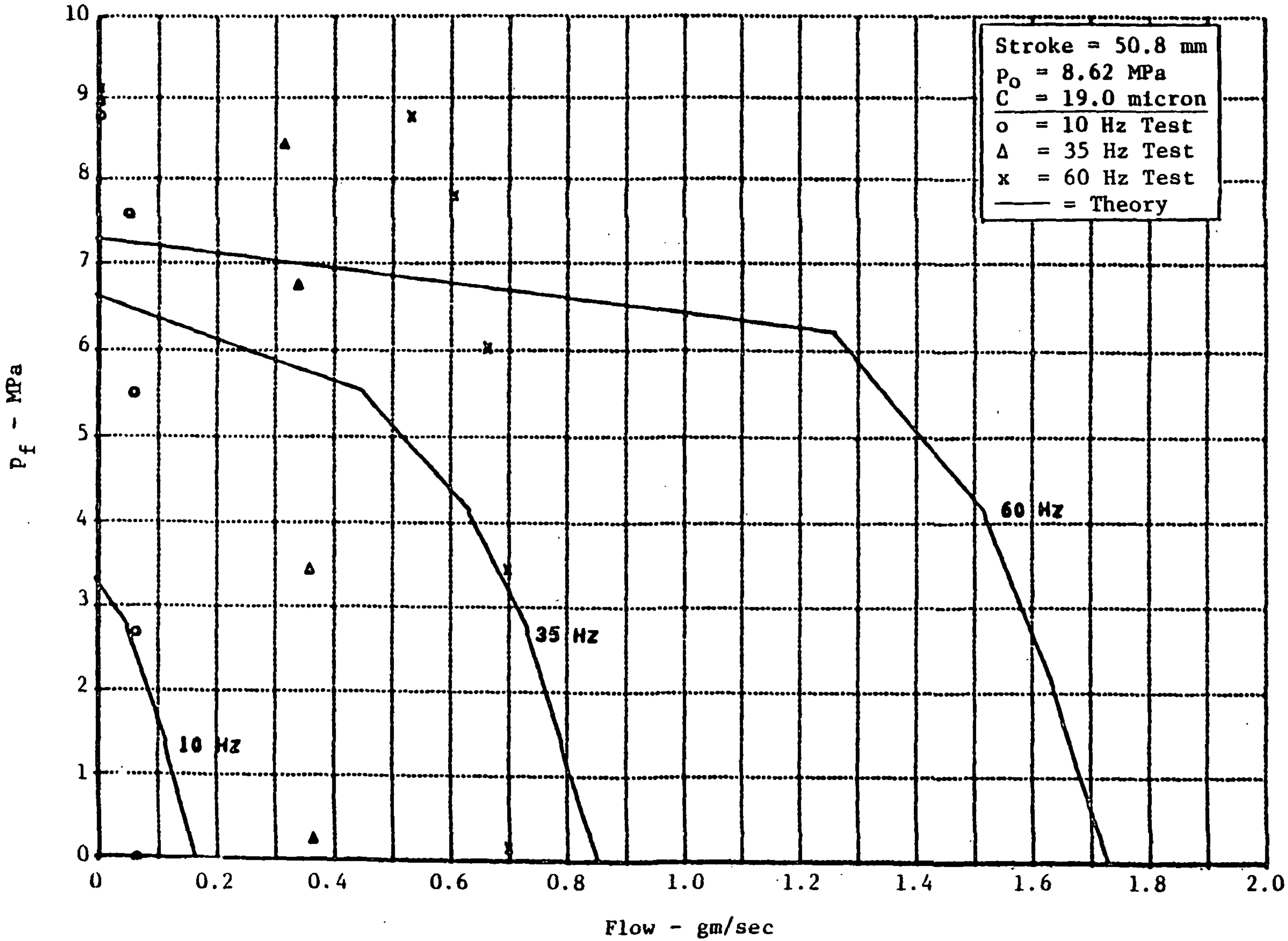


Fig. D-6 Performance of Babbitt Pumping Ring (II)

832937

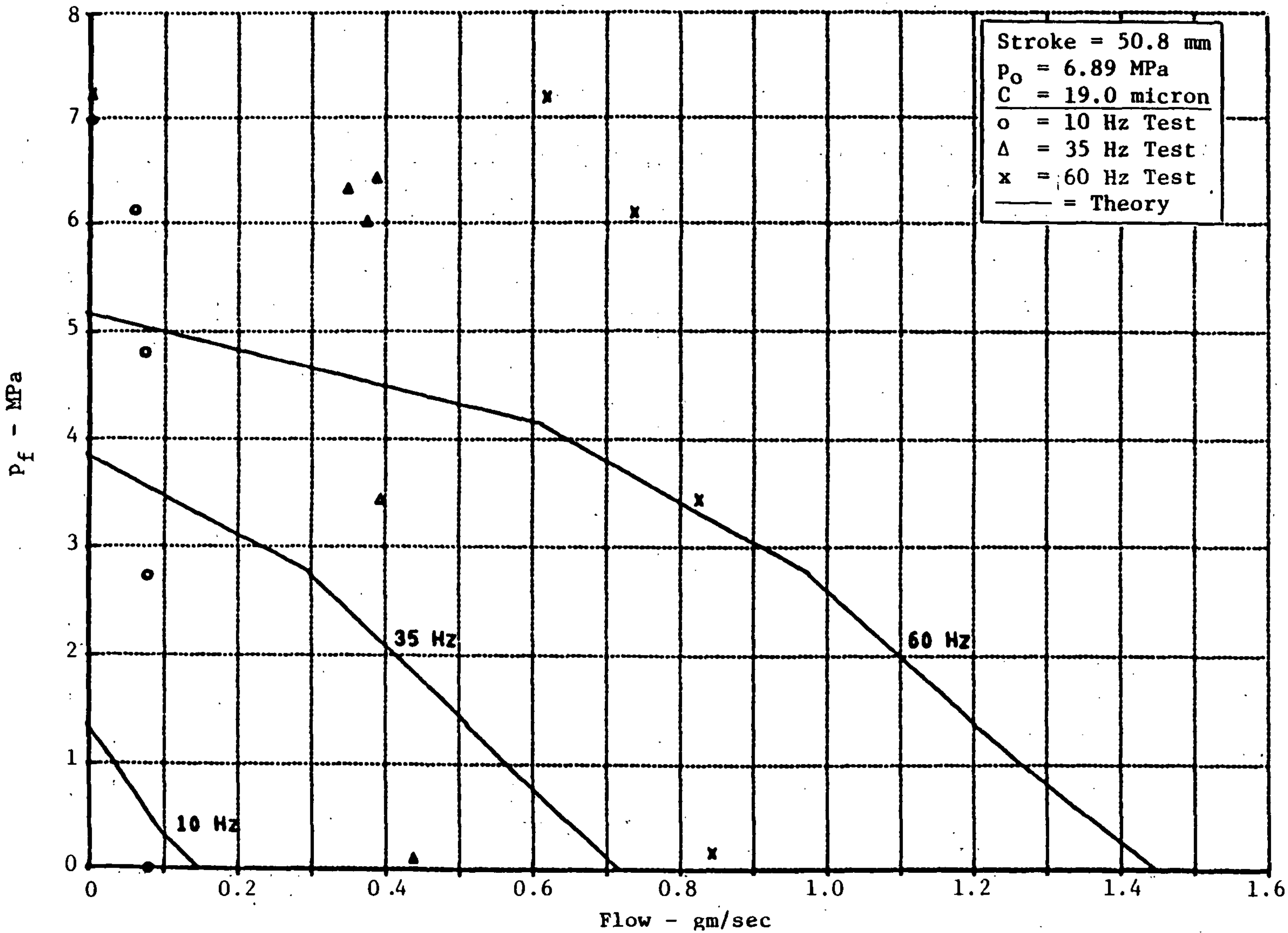


Fig. D-7 Performance of Babbitt Pumping Ring (II)

832938

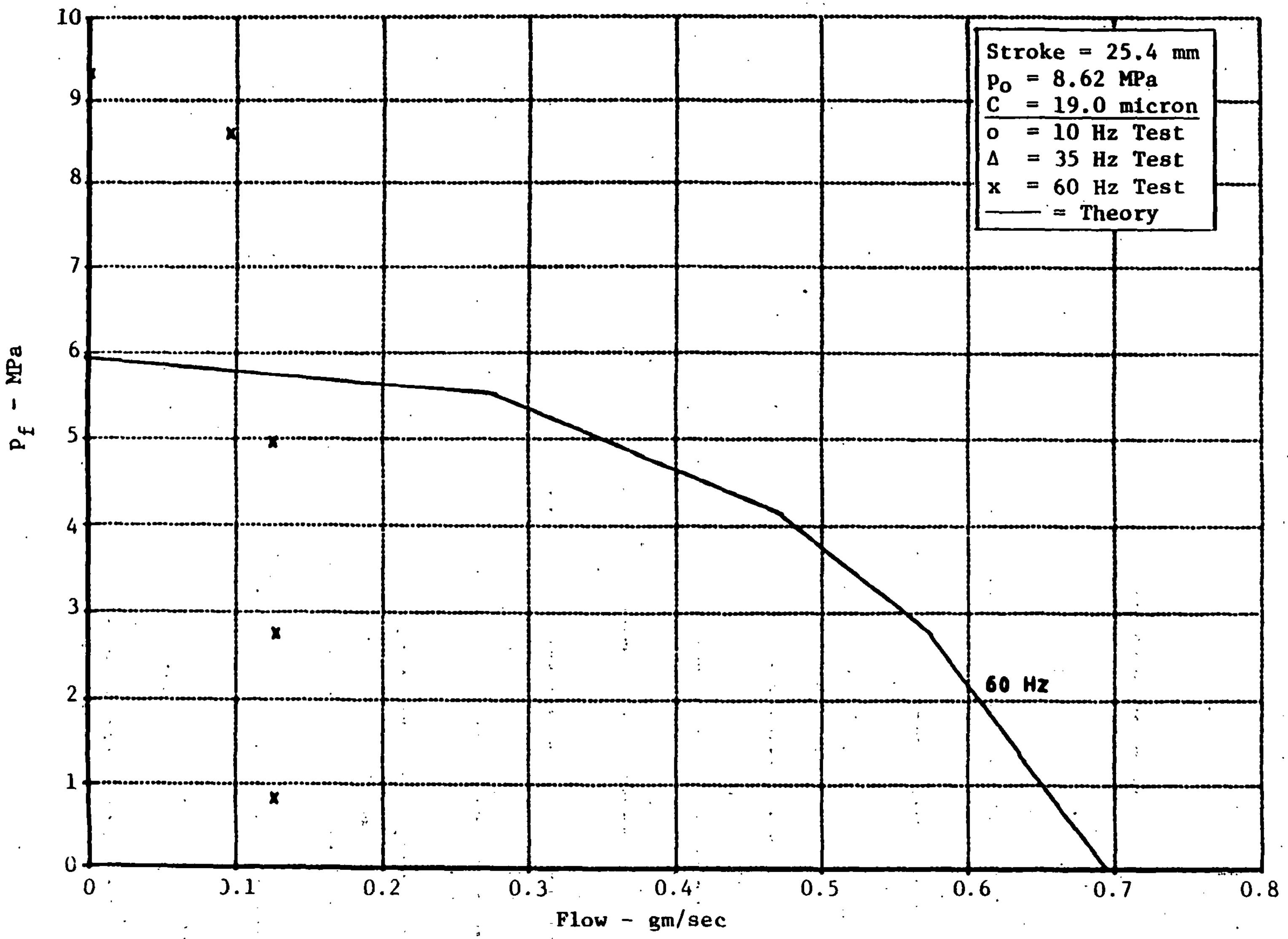


Fig. D-8 Performance of Babbitt Pumping Ring (II)

832939

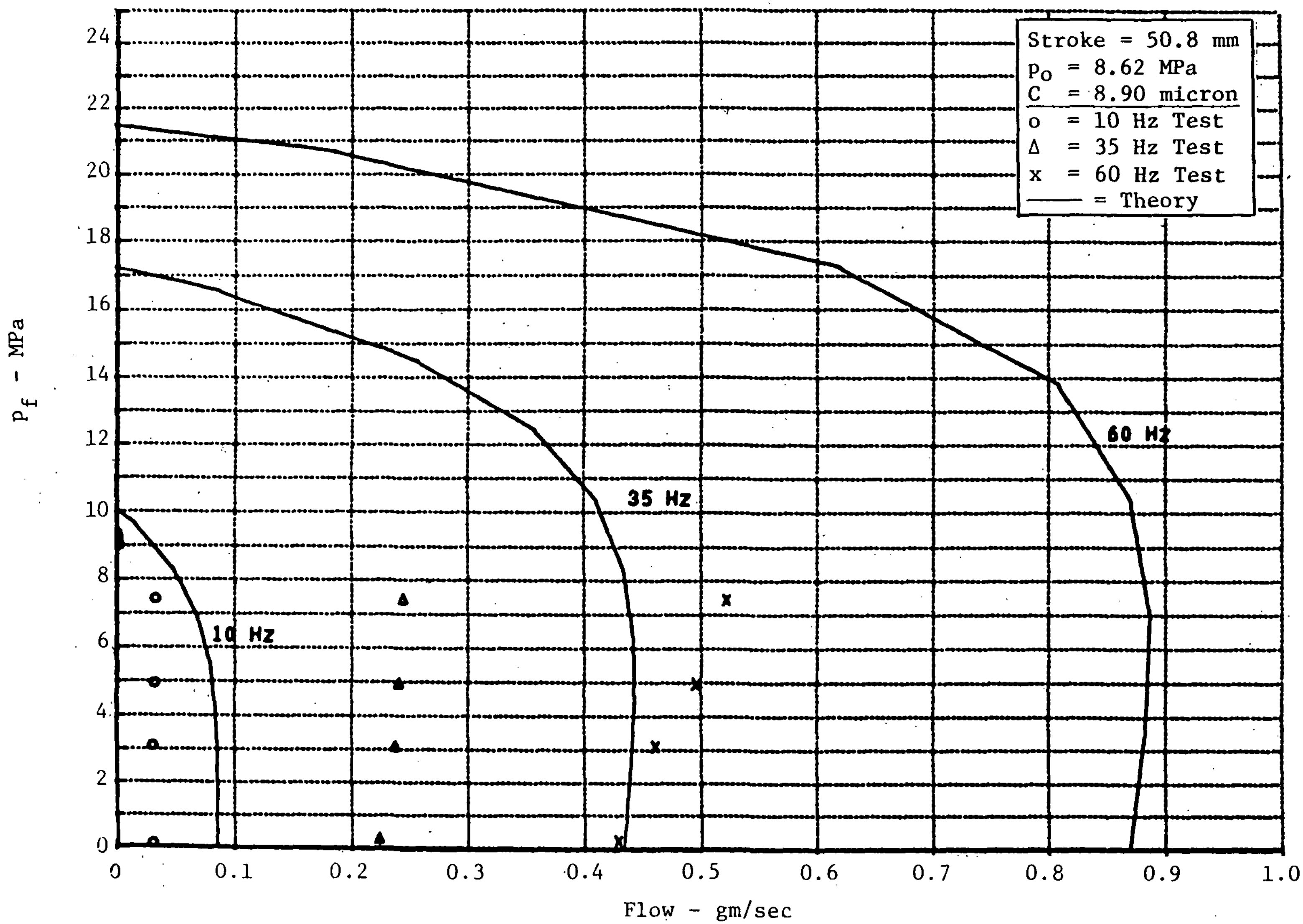


Fig. D-9 Performance of Small Babbitt Pumping Ring (VII)

832940

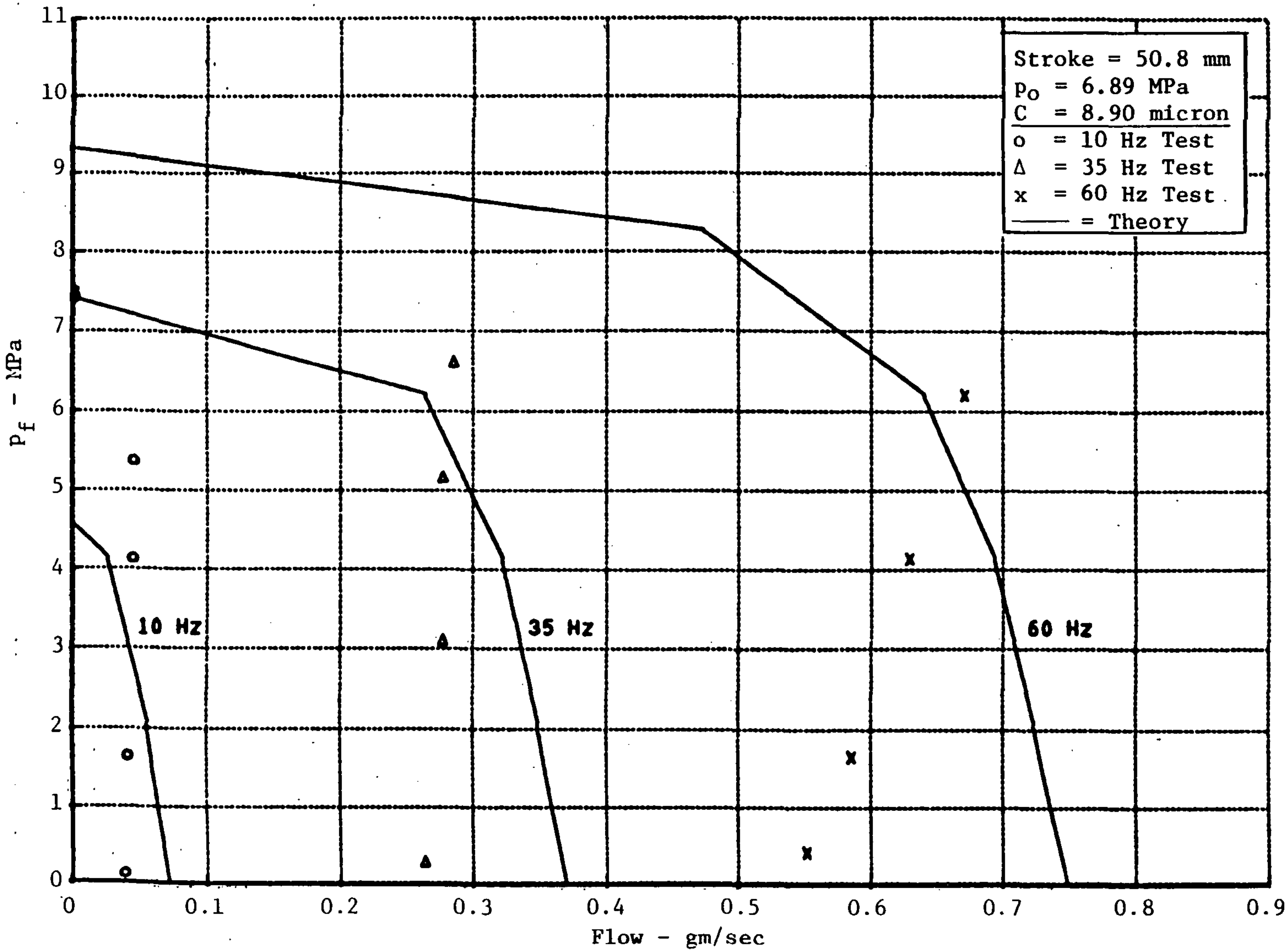


Fig. D-10 Performance of Small Babbitt Pumping Ring (VII)

832941

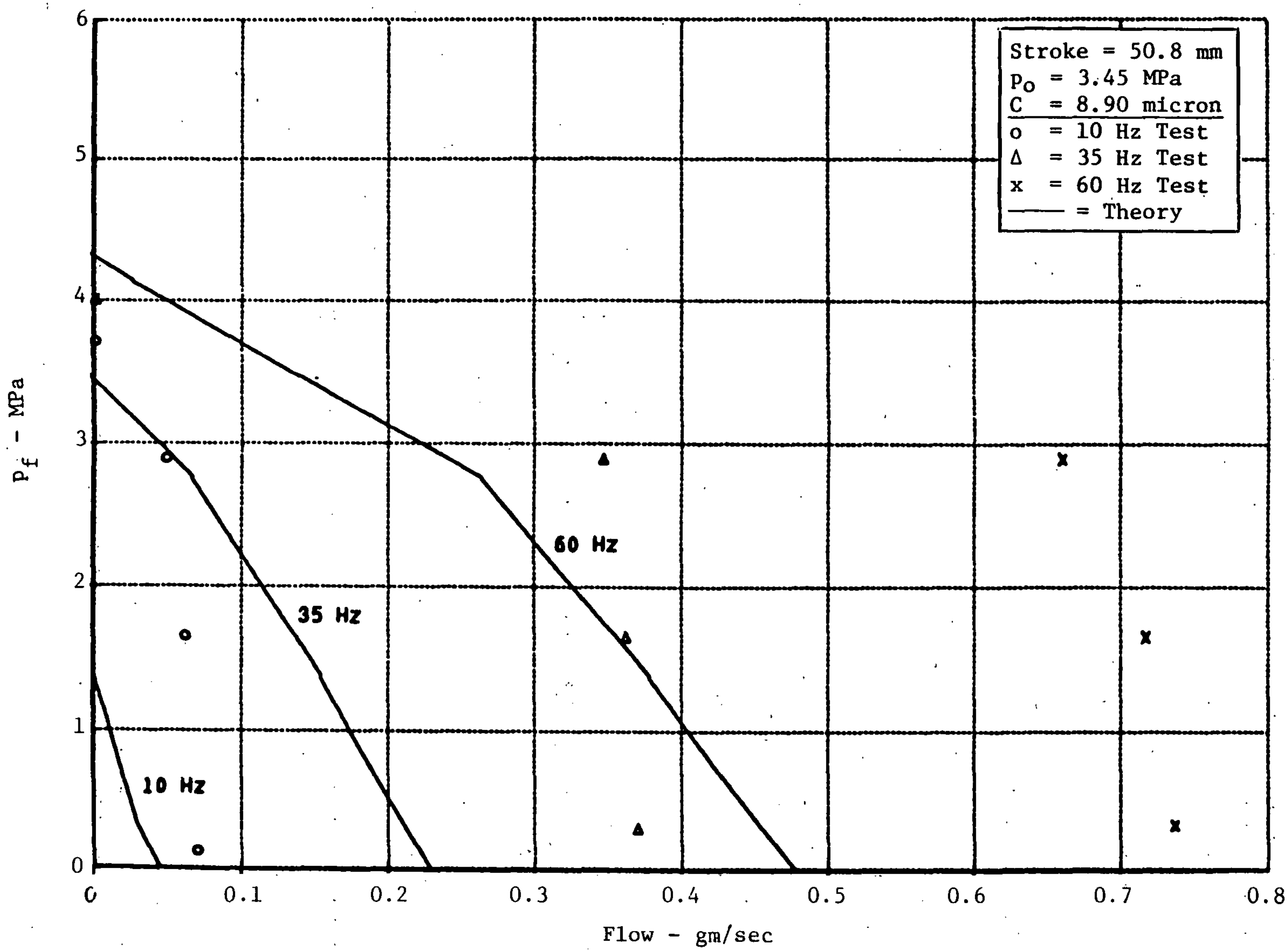


Fig. D-11 Performance of Small Babbitt Pumping Ring (VII)

832942

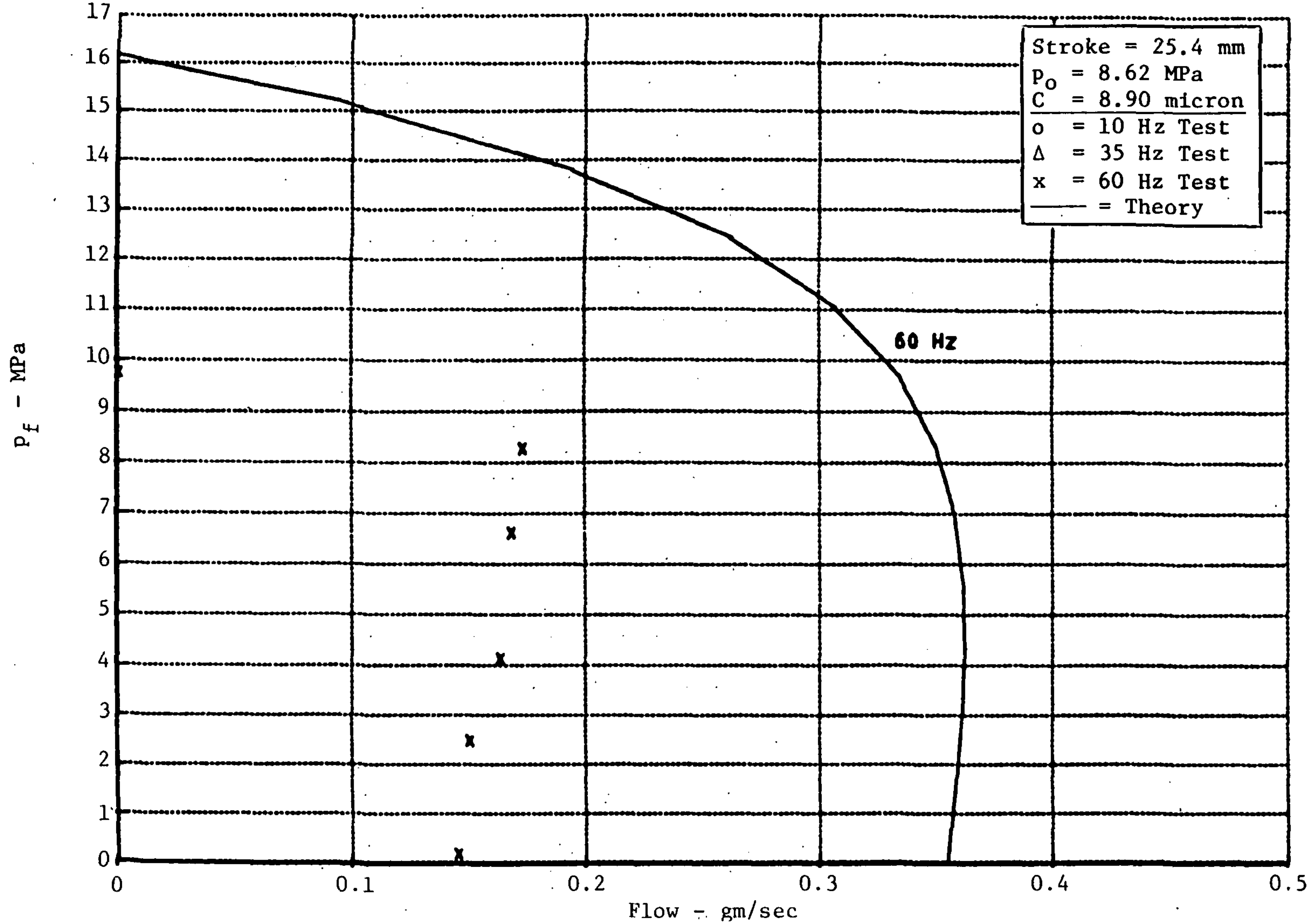


Fig. D-12 Performance of Small Babbitt Pumping Ring (VII)

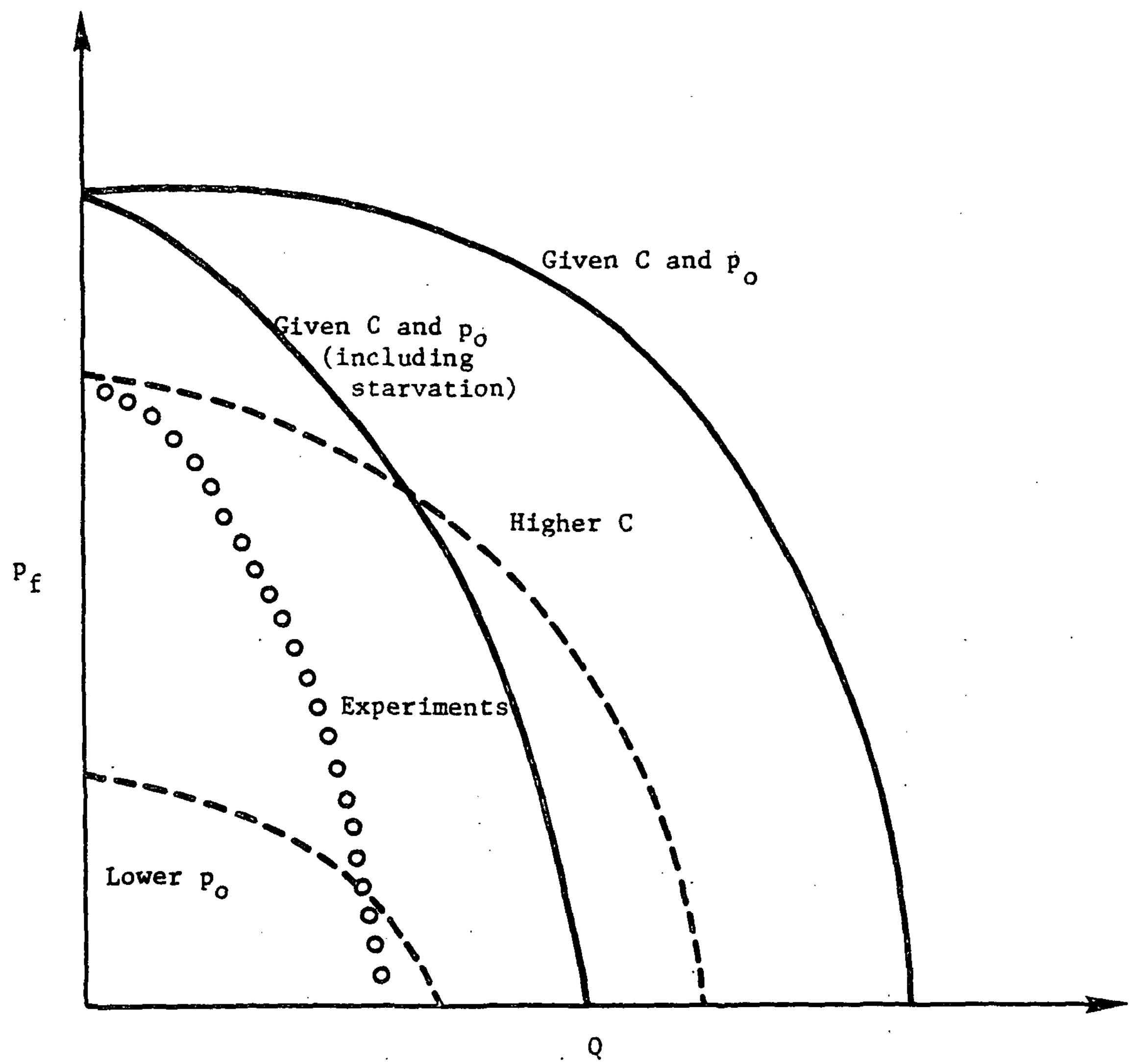
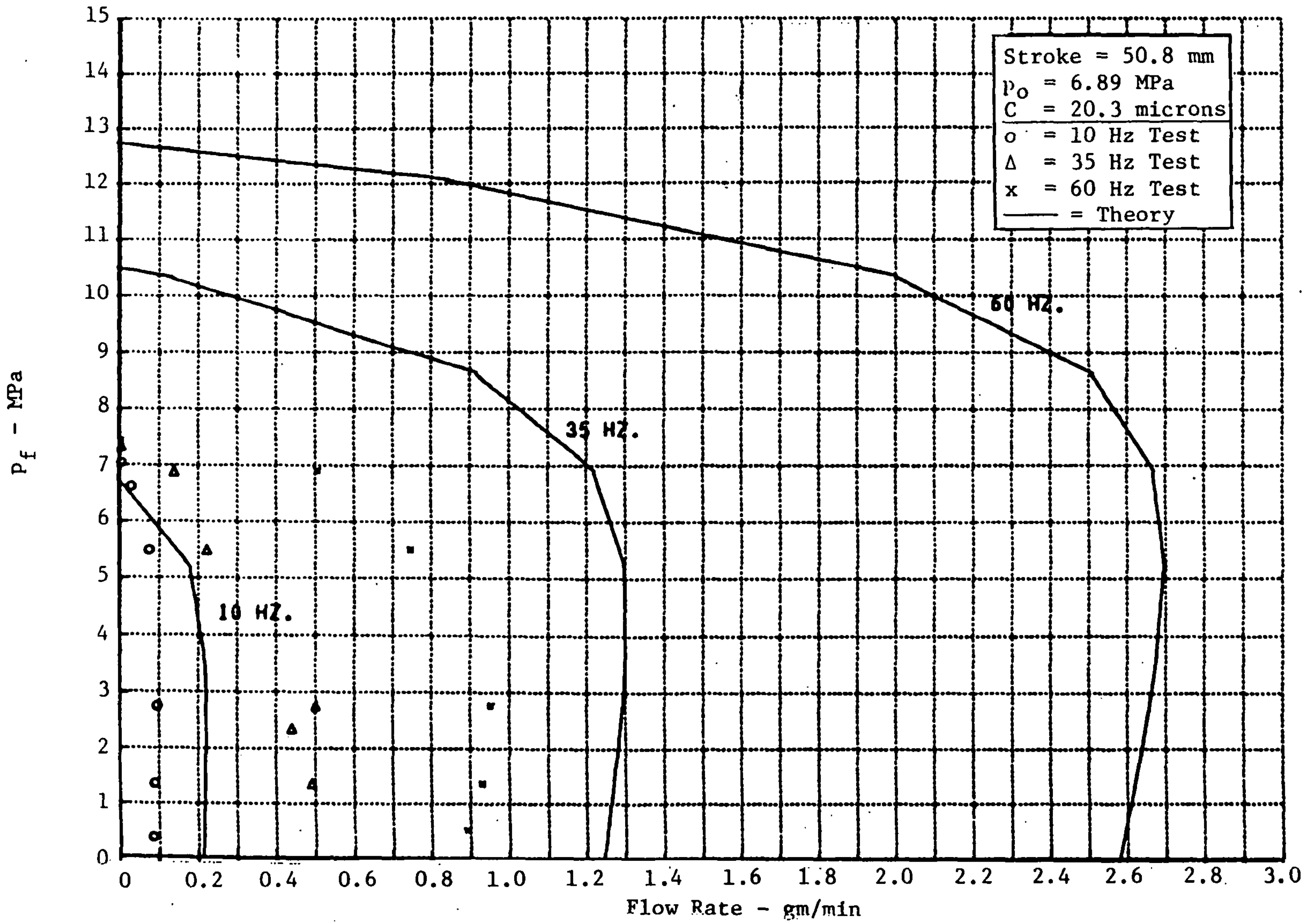


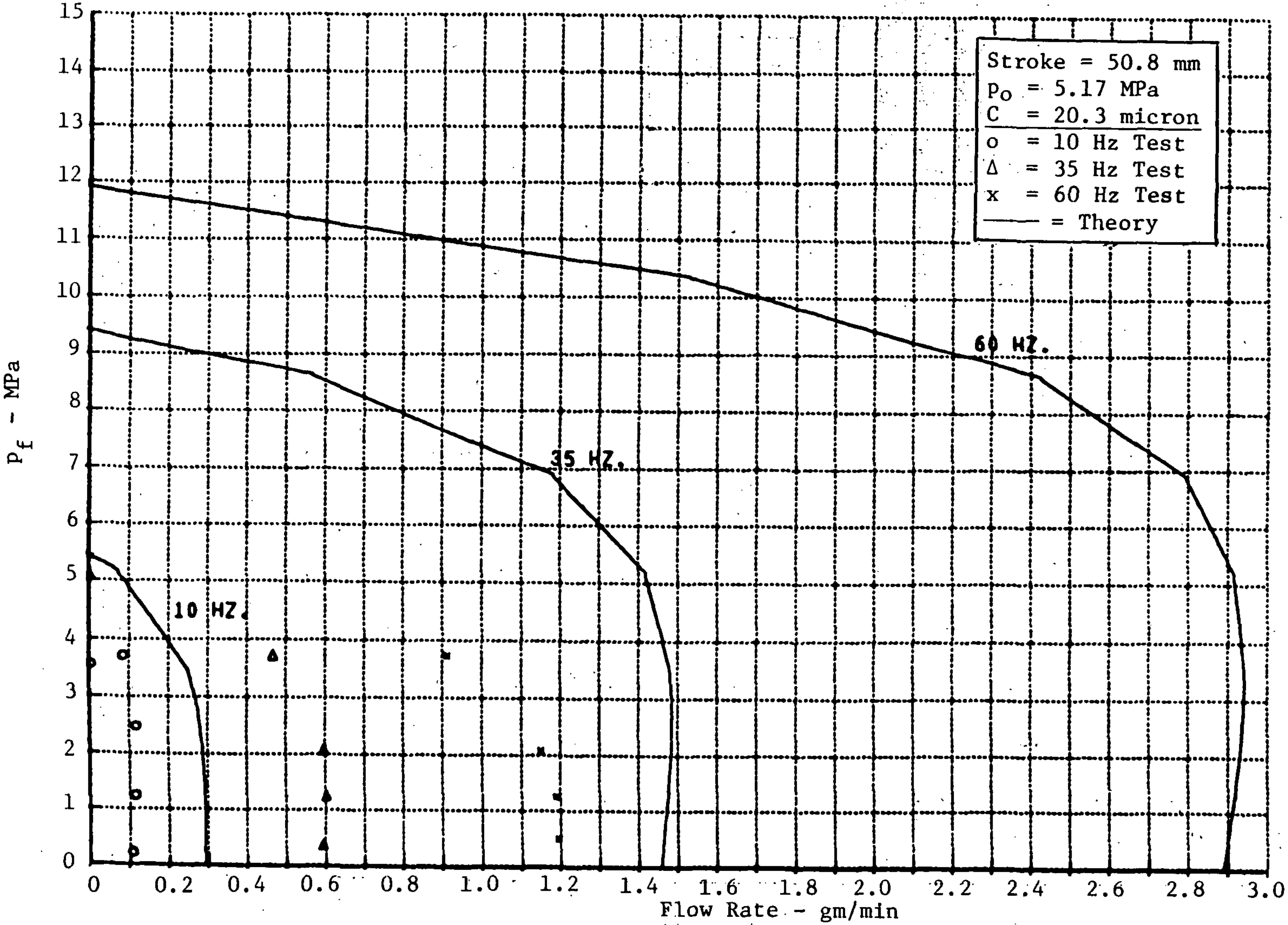
Fig. D-13 Qualitative Comparisons of Theoretical and Experimental Results

831236



832944

Fig. D-14 Performance of Carbon Pumping Ring



832945

Fig. D-15 Performance of Carbon Pumping Ring

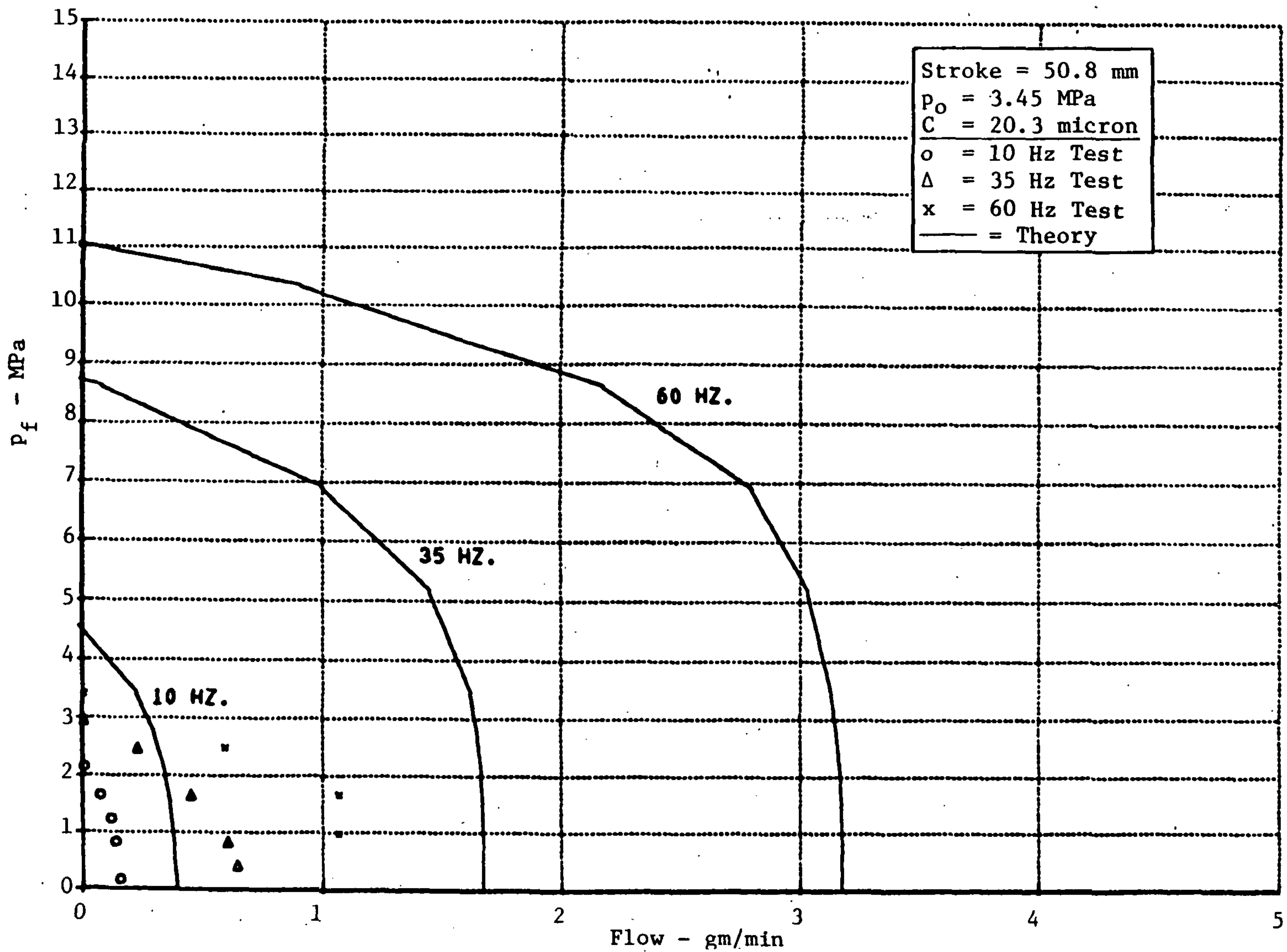


Fig. D-16 Performance of Carbon Graphite Pumping Ring

832946

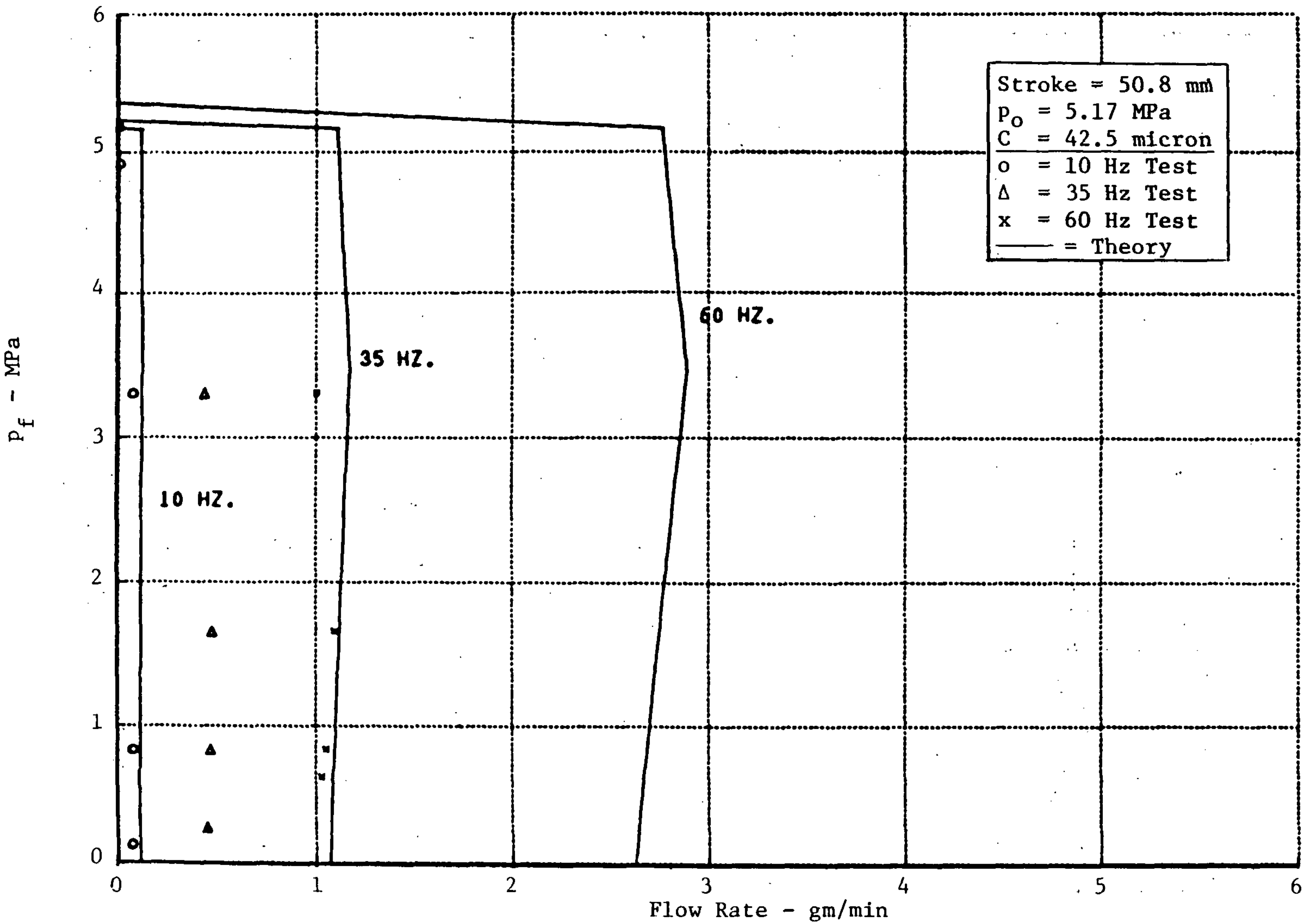
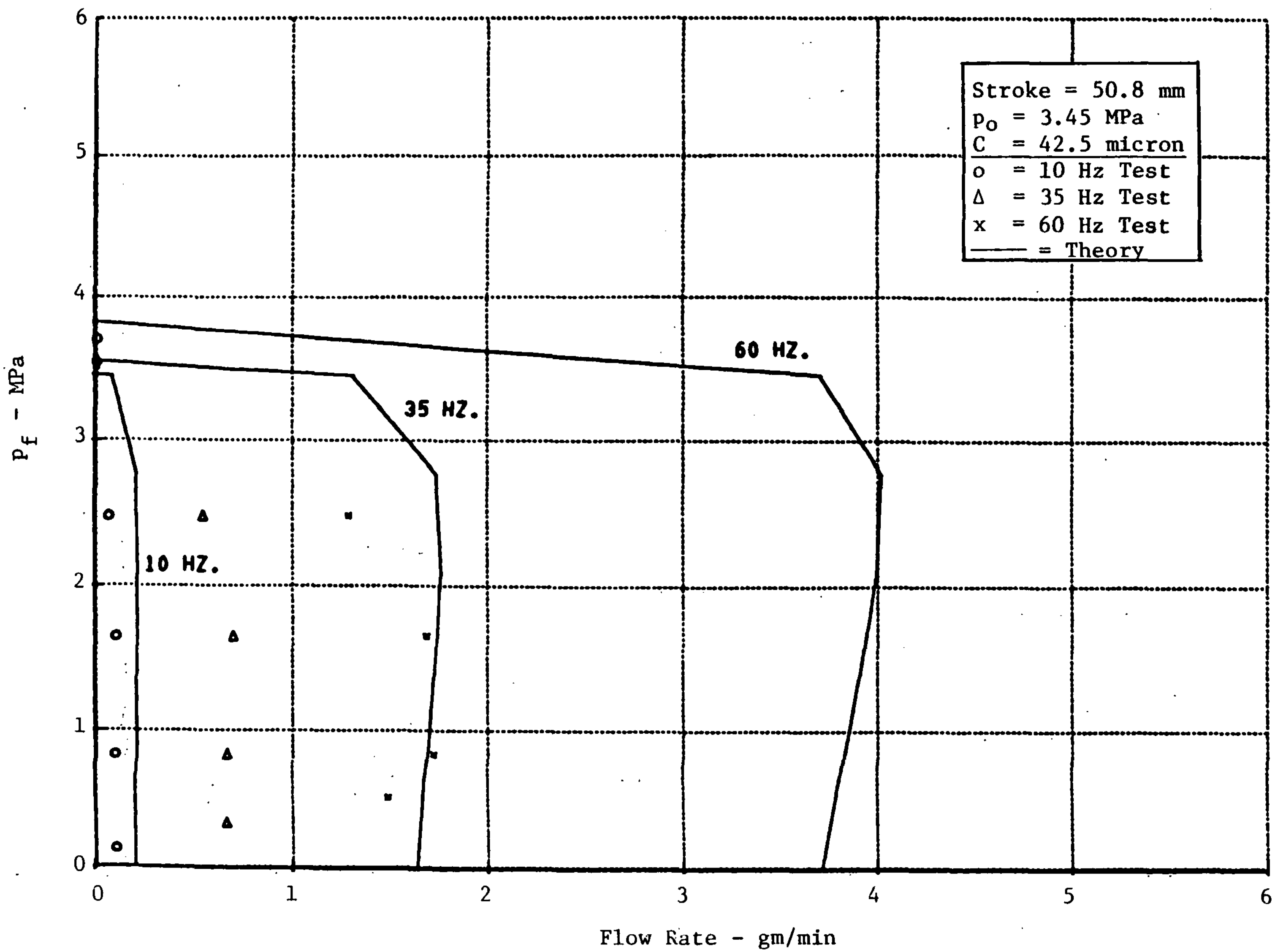


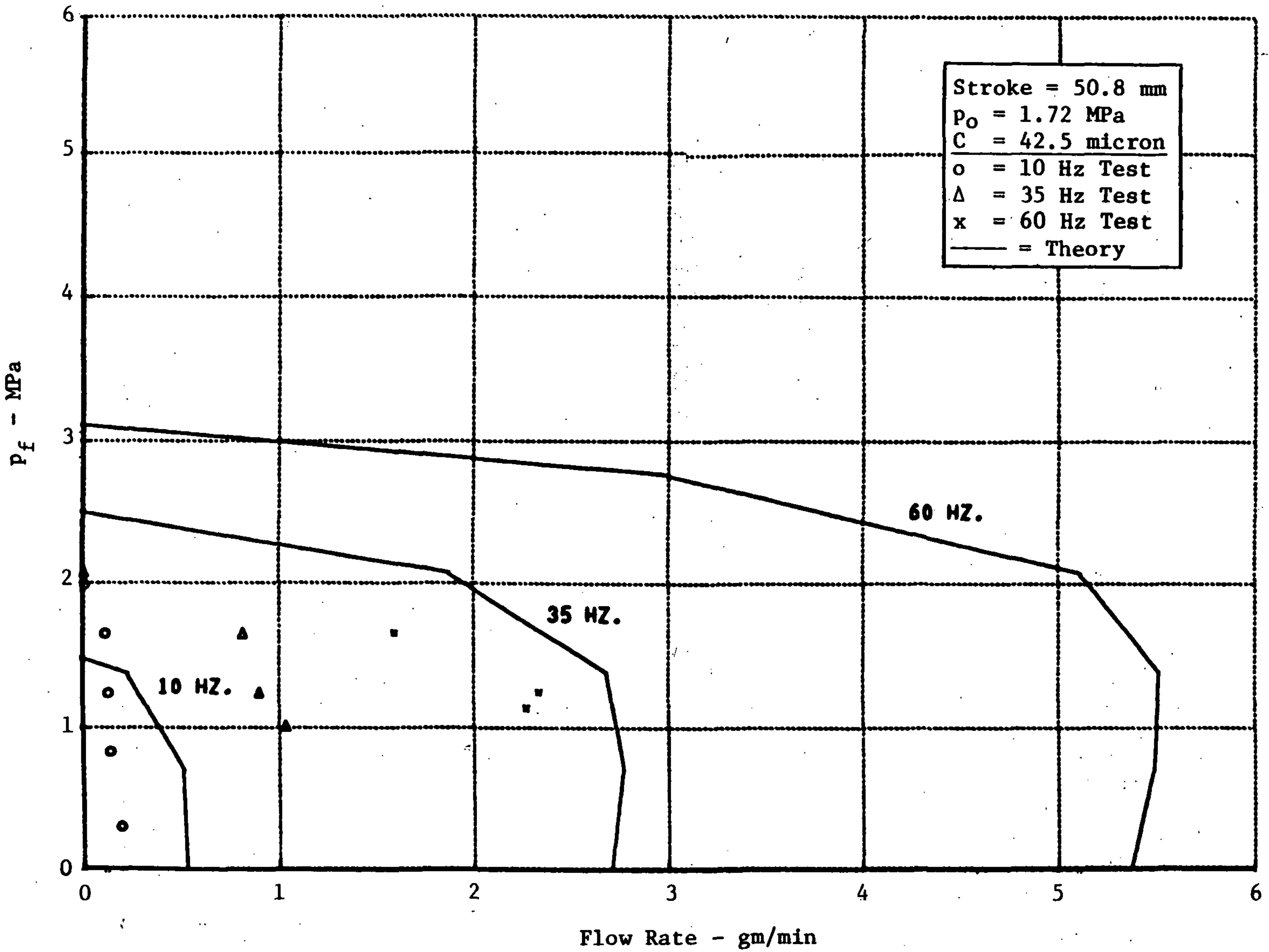
Fig. D-17 Performance of Rulon Pumping Ring

832947



832948

Fig. D-18 Performance of Rulon Pumping Ring



832949

Fig. D-19 Performance of Rulon Pumping Ring

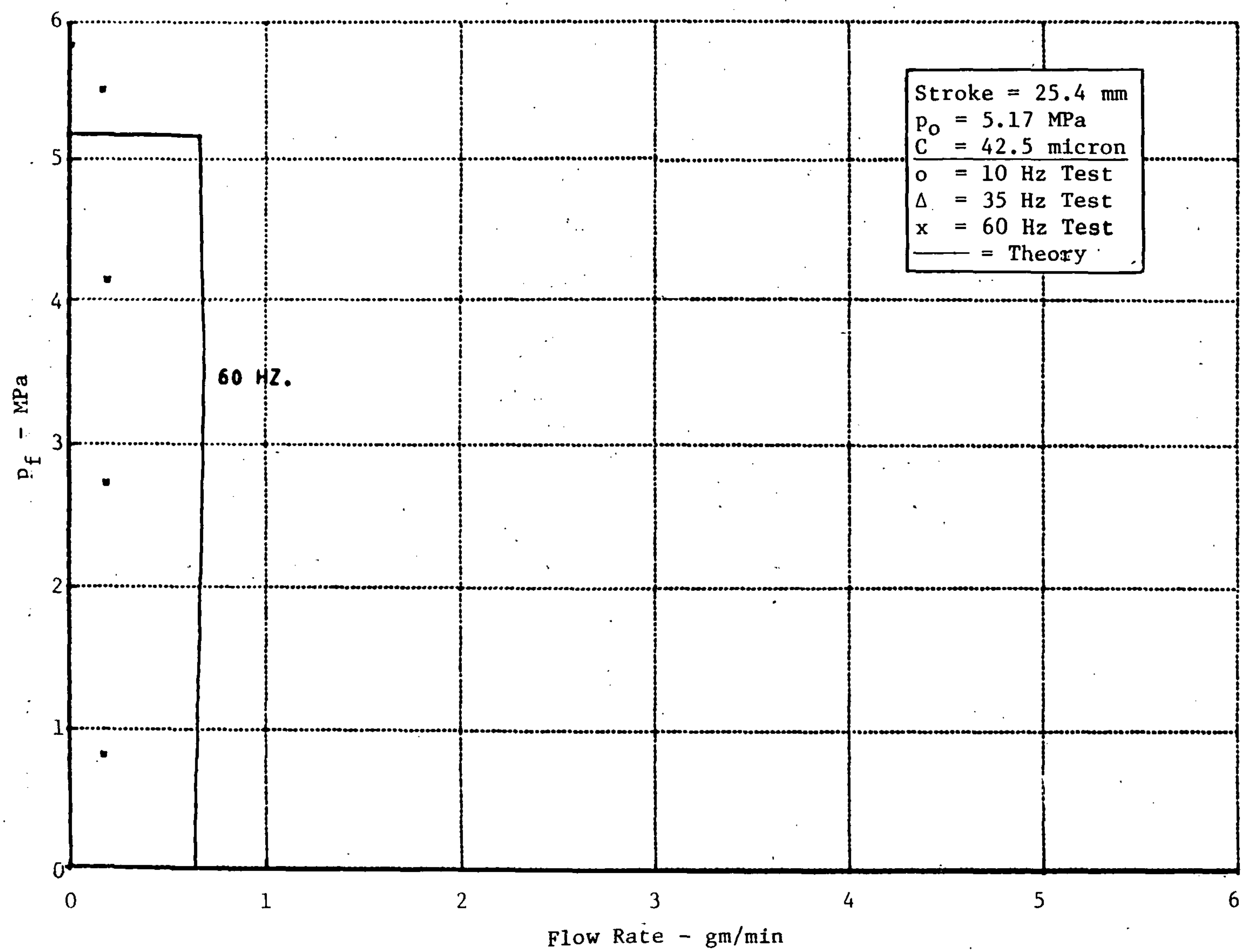


Fig. D-20 Performance of Rulon Pumping Ring

832950

TABLE D-2

QUALITATIVE SUMMARY OF COMPARISON

BETWEEN ANALYTICAL RESULTS AND TESTS

RING		$(Q_{o\ THEO}/Q_{o\ EXP})$		$(p_{fm\ THEO}/p_{fm\ EXP})$	
		High p_o	Low p_o	High p_o	Low p_o
Babbitt	Small C	2	1	2	1
	Large C	3	1.5	1.5 - 0.5	0.7 - 0.2
	Small R	2	0.5	2	1 - 0.3
Carbon Graphite		2.5	3 - 2	1 - 2	1.5 - 4
Rulon		2.5	2.5	1	1.5 - 0.5

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16. Abstract

An analysis, computer program and experimental data have been generated to provide design data for pumping rings. Although some disagreement has been found to occur in flow prediction, a procedure has been used to design pumping rings that performed well experimentally. A preliminary analysis has also been developed for the prediction of film thickness and pressure distribution in a pumping Leningrader seal.

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