3.6.1 POSSIBILITY OF MEASURING GRAVITY-WAVE MOMENTUM FLUX BY SINGLE BEAM OBSERVATION OF MST RADAR

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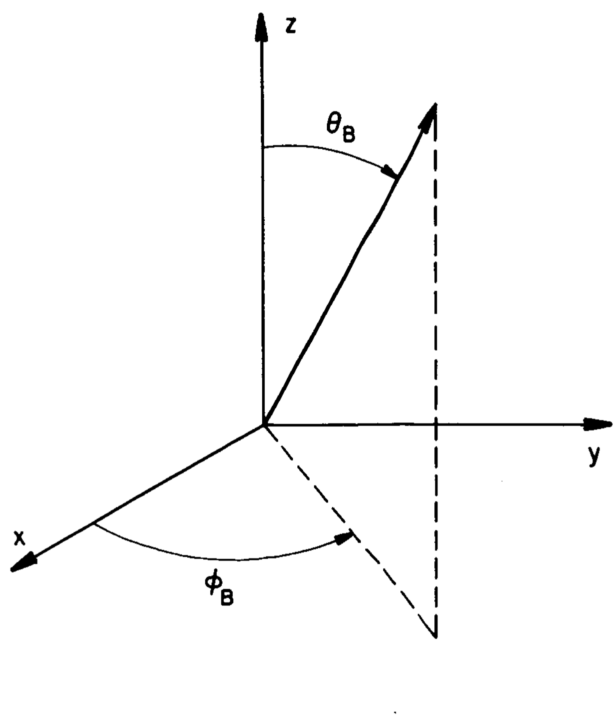
VINCENT and REID (1983) proposed a technique to measure gravity-wave momentum fluxes in the atmosphere by MST radars using two or more radar beams. Since the vertical momentum fluxes are assumed to be due to gravity waves, it appears possible to make use of the dispersion and polarization relations for gravity waves in extracting useful information from the radar data. In particular, for an oblique radar beam, information about both the vertical and the horizontal velocities associated with the waves are contained in the measured Doppler data. Therefore, it should be possible to extract both V_z and V_h from a single beam observational configuration. In this paper, we propose a procedure to perform such an analysis. The basic assumptions are: the measured velocity fluctuations are due to gravity waves and a separable model gravity-wave spectrum of the Garrett-Munk type that is statistically homogeneous in the horizontal plane. Analytical expressions can be derived that relate the observed velocity fluctuations to the wave momentum flux at each range gate. In practice, the uncertainties related to the model parameters and measurement accuracy will affect the results.

Let us consider an MST radar configuration. For an oblique beam, the radial velocity contains information of both $\vec{v}_h$ and v_z. The polarization relations (SCHEFFLER and LIU, 1985)



$$R_x = -\left(\frac{\omega_b^{2}-\omega^2}{\omega^2-\omega_i^{\,2}}\right)\left(\frac{k_x+i\,\frac{\omega_i}{\omega}\,k_y}{k_z}\right) R_z$$

$$R_y = -\left(\frac{\omega_b^{2}-\omega^2}{\omega^2-\omega_i^{\,2}}\right)\left(\frac{k_y-i\,\frac{\omega_i}{\omega}\,k_x}{k_z}\right) R_z$$

$$\therefore\ R_xR_z{*}+R_yR_z{*} = -\left(\frac{\omega_b^{2}-\omega^2}{\omega^2-\omega_i^{\,2}}\right)\frac{(k_x+k_y)-i\,\frac{\omega_i}{\omega}\,(k_x-k_y)}{k_z}\,|R_z|^2 \qquad (1)$$

On the other hand, the polarization for the observed radial velocity is given by

$$R_{0_b} = R_x\cos\phi_B\sin\theta_B+R_y\,\sin\phi_B\sin\theta_B+R_z\cos\theta_B \qquad (2)$$

From (1) to (2), we obtain

$$R_xR_z{*}+R_yR_z{*} = f(\vec{k},\omega,\phi_B)\,|R_{0_b}|^2$$

where

$$f = \frac{\left(\frac{\omega^2-\omega_i^{\,2}}{\omega_b^{\,2}-\omega^2}\right) k_hk_z\left[\cos\phi+\sin\phi-i\,\frac{\omega_i}{\omega}\,(\cos\phi-\sin\phi)\right]}{[k_h\cos(\phi-\phi_B)\sin\theta_B+\frac{\omega^2-\omega_i^{\,2}}{\omega_b^{\,2}-\omega^2}\,k_z\,\cos\theta_B]^2+\frac{\omega_i^{\,2}}{\omega^2}\,k_h\,\sin^2\theta_B\,\sin(\phi-\phi_B)} \qquad (3)$$

Spectral representation of the velocities can be written as a Fourier-Stieltjes integral:

$$v_x = \int R_x(\vec{k},\omega)\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}\,d\nu(\vec{k},\omega)$$

$$v_y = \int R_y(\vec{k},\omega)\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}\,d\nu(\vec{k},\omega)$$

$$v_z = \int R_z(\vec{k},\omega)\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}\,d\nu(\vec{k},\omega)$$

$$v_{0_b} = \int R_{0_b}(\vec{k},\omega)\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}\,d\nu(\vec{k},\omega)$$

Therefore

$$\langle v_xv_z\rangle+\langle v_yv_z\rangle = \int (R_xR_z{*}+R_yR_z{*})E(\vec{k},\omega)d\vec{k}\,d\omega$$

$$= \int f(\vec{k},\omega)\,|R_{0_b}|^2\,E(\vec{k},\omega)\,d\vec{k}\,d\omega. \qquad (4)$$

where the relation

$$\langle d\nu(\vec{k}_1,\omega_1)d\nu(\vec{k}_2,\omega_2)\rangle = E(\vec{k}_1,\omega_1)\ \delta(\vec{k}_1-\vec{k}_2)\delta(\omega_1-\omega_2)d\vec{k}_1 d\vec{k}_2 d\omega_1 d\omega_2$$

has been used.

On the other hand, we have

$$\langle v_{0_b}{}^2\rangle = \int |R_{0_b}|^2\, E(\vec{k},\omega)d\vec{k}\, d\omega$$

$$|R_{0_b}|^2 = [(\frac{\omega_b{}^2-\omega^2}{\omega^2-\omega_i{}^2})^{1/2}\frac{k_h}{k}\cos(\phi-\phi_B)\sin\theta_B - (\frac{\omega^2-\omega_i{}^2}{\omega_b{}^2-\omega^2})^{1/2}\frac{k_z}{k}\cos\theta_B]^2$$

$$+ (\frac{\omega_b{}^2-\omega^2}{\omega^2-\omega_i{}^2})\frac{k_h{}^2}{k^2}(\frac{\omega_i}{\omega})^2 \sin^2(\phi-\phi_B)\sin^2\theta_B \qquad (5)$$

Procedure proposed for computing the momentum flux:

1. Assume a model spectrum $E(\vec{k}, \omega)$ with several parameters, including possible anisotropy.
2. Fit the theoretical spectrum $E_{0_b}(\omega)$ to the observed radial velocity spectrum to determine the best set of parameters for the model. This determines the model.
3. Use this model, compute through equations (4) and (5)

Discussion:

1. This procedure assumes the spectral shape remains approximately the same at different heights and they are due to gravity waves.
2. The spectral fitting procedure depends on how sensitive is the spectrum due to changes of model parameters. This should be studied.
3. It is expected noise in observed spectra due to the uncertainties related to the model parameters and measurement inaccuracy may greatly affect the results.
4. In principle, the procedure should also apply to cases with Doppler shift. However, unless one has information about the background wind vector, the model may contain too many parameters for realistic spectral fitting.

REFERENCES

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