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SUMMARY

The determination of mean streamline patterns by moving the test point in the direction of the measured velocity is shown to produce cumulative errors that are unacceptable. A two-dimensional algorithm that minimizes these errors is presented and is analytically validated using simple potential flows. The algorithm is extended to three-dimensional flows and is again validated analytically. Finally, as an example of a typical application of the algorithm, mean streamlines are measured in a complex, turbulent flow with a three-dimensional laser anemometer.

INTRODUCTION

Complex, highly turbulent fluid flows are difficult to measure or visualize experimentally. In general, flow surveys are conducted along straight-line paths to create a grid, or mesh, of locations where velocities have been measured. Using those data, vector plots or arrow diagrams on the grid can provide pictorial representations of the flow patterns from which the scientist, it is hoped, can gain insight into the fluid mechanics of the flow.

Where streamline patterns are desired, an alternative approach to using vector plots is to measure the mean values of the three orthogonal components of the velocity at a given location, compute the flow direction, move the measurement probe a short distance in that direction, and repeat the procedure. However, in order to accomplish such measurements, the velocity-measuring device must be capable of accurately measuring all three velocity components, even in regions of stagnation and reversed or recirculating flow. Whereas hot-wire anemometers and pressure-based devices are not acceptable in this kind of flow, recent advances (refs. 1 and 2) in three-dimensional, laser-Doppler anemometry (3D LDA) now make such measurements feasible.

The present paper will briefly review a two-dimensional algorithm for streamline tracing that has been developed by Orloff and Snyder (ref. 3). This algorithm reduces the systematic "drift" of the LDA test point away from the correct streamline that occurs when the above tangential motion technique is used. Then, the two-dimensional algorithm is extended to fully three-dimensional flows, and experimental results are presented of a 3D LDA streamline tracing in a complex turbulent flow in the Ames 7- by 10-Foot (Low Speed) Wind Tunnel.

TWO-DIMENSIONAL LDA STREAMLINE TRACING

Orloff and Snyder (ref. 4) and Snyder et al. (ref. 5) have reported a 3D LDA measurement technique based on statistics and sampling theory that allows measurement to a prescribed accuracy of either a mean velocity component or the mean flow direction. Having applied the technique, they have shown it to be accurate for constructing vector or arrow plots of the mean flow directions along a survey line in a turbulent flow (ref. 5). Also, as shown in figure 1(a), they attempted to follow a mean streamline in a turbulent flow by using the stepping method described above, and the resulting path of the LDA test point was shown (by using reverse traces) to be other than a true fluid streamline (ref. 5).

To prove that the discrepancy is not a result of measurement inaccuracies associated with the 3D LDA, Orloff and Snyder (ref. 3) have used simple two-dimensional potential flows with variable curvature and inflection to simulate the LDA tracing technique. Their results demonstrate that the errors occur because the step segments are always straight lines that are tangent to the local streamline. Figure 1b demonstrates both that a cumulative error is incurred when straight-line motion in the direction of the measured velocity is used and that the numerical trace tends to cross streamlines. More importantly, the numerical reverse-path segments (moving opposite to the velocity direction starting from a point on the forward path) show precisely the same divergence to the outside of the forward trace as is evidenced in the experimental reverse-path traces in figure 1(a). A more exact motion algorithm that minimizes the error is described in detail in reference 3 and is summarized below.

Figure 2(a) depicts a local streamline section in the YZ plane that is concave upward ($d^2Z/dY^2 > 0$). The measured velocity is assumed tangent to the streamline at the current measurement location (point 5). Because the flow may be in either direction along the path, two opposing velocity vectors are shown at point 5, and the forward vector is denoted at an angle θ_f and the reverse vector at an angle θ_r. The angle δ is the change in the step direction that is necessary to correct for the error incurred by moving tangentially.

General expressions for the Cartesian corrections $\Delta Y'$ and $\Delta Z'$ (as indicated in figure 2(b) that describe all possible combinations of concave-upward or concave-downward flow, and positive or negative slope locations, are developed in reference 3 and are given by

$$\Delta Y' = -2(\mathrm{STEP})\sin \delta/2 \, \sin(\delta/2 + \alpha\beta\theta) \tag{1}$$

$$\Delta Z' = 2\alpha\beta(\mathrm{STEP})\sin \delta/2 \, \cos(\delta/2 + \alpha\beta\theta) \tag{2}$$

where

$$\alpha = \text{sign of } (\cos \theta) \tag{3}$$

$$\beta = \text{sign of } \left(\frac{\dot{Y}\ddot{Z} - \dot{Z}\ddot{Y}}{\dot{Y}^3}\right) \tag{4}$$

$$\delta = \sin^{-1}(\mathrm{STEP}/2R) \tag{5}$$

and θ may be either θ_f or θ_r and $\theta = \tan^{-1}(\dot{Z}/\dot{Y})$. The derivatives shown in the above expression for β denote a parametric differentiation with respect to the point number $n = 1, \ldots, 5$, as indicated in figure 2(a). The parametric description must be used to prevent the curve-fitting routine from encountering double-valued functions in the calculation of the radius of curvature (double-valued functions can occur as the streamline transitions between concave-upward and concave-downward where Z is the dependent variable and Y is the independent variable).

The distance STEP is chosen between minimum and maximum values, SMIN and SMAX, respectively, that are appropriate to the flow being studied. The value is calculated within this interval based on the radius of curvature and inflection of the streamline. For the first five points along a streamline trace, SMIN is used, and the motion is in the direction of the measured velocity. For subsequent points, the radius of curvature R is computed using a local parametric quadratic fit through the current test-point location and the previous four points. In regions of local streamline inflection, the step size must not abruptly increase to SMAX because large deviations from the streamline may occur. To prevent this, the radius of curvature at the previous location, ROLD, is compared with the radius of curvature at the current location, R, and the ratio ROLD/R is computed. As the test point moves into a region of inflection, the ratio becomes less than one, and the step size is decreased to prevent significant deviation from the streamline. STEP is therefore given by

$$\mathrm{STEP} = \mathrm{SMIN} + (\mathrm{SMAX} - \mathrm{SMIN})\sqrt{\mathrm{R/RINF}}(\mathrm{RAT}) \tag{6}$$

where RAT is a quantity defined by

$$\mathrm{RAT} = (\mathrm{ROLD/R})^2 \quad \text{if} \quad \mathrm{ROLD/R} < 1$$

$$\mathrm{RAT} = 1 \quad \text{if} \quad \mathrm{ROLD/R} \geq 1$$

RINF is a constant radius that influences the relative rate of variation of the step size with changing radius of curvature. RINF is usually chosen to be a radius at which the curvature is effectively "flat" relative to the scale of the flow.

The ability of the foregoing correction scheme to compensate for the error inherent in tangential stepping is shown, in figure 3, for a simple potential flow wherein the path goes through moderate curvature and two points of inflection. Although the corrected path is not exact, it is significantly better than the path that results from tangential stepping. Most noticeable is the deviation that occurs for the uncorrected path along the second half of the trace. This is a result of the convergence of the streamlines as the flow accelerates over the top of the cylinder. In this region, any small spatial error in the stepping motion is amplified because it represents a large percentage of the distance between streamlines; subsequently, as the trace proceeds downstream (where the streamlines are farther apart) the spatial extent of the error becomes more pronounced. In view of this, the accuracy of the corrected stepping is quite reasonable.

THREE-DIMENSIONAL CORRECTION ALGORITHM

Streamline tracing in three dimensions is more involved than for the two-dimensional case just described. Figure 4 depicts the five most-recent locations of the LDA test point along a measured three-dimensional streamline. The LDA provides a measurement of the velocity vector $\vec{V}$ at point 5. The projections of $\vec{V}$ into the XY, YZ, and ZX planes are denoted by V_{XY}, V_{YZ}, and V_{ZX}, respectively, and the orientations of the projections in the planes are given by the angles γ, θ, and α, respectively. From this information, the next test point location must be determined. Notice, however, that the stepping motion is completely specified using only two planes, and the motion, when viewed in the third plane, is uniquely determined. In order to explain which of the three planes are used for the three-dimensional algorithm that accomplishes the correction for curvature, a flow chart (fig. 5) is presented.

A parabolic least-squares parametric curve fit through the five most-recent locations yields the first derivatives $\dot{X}$, $\dot{Y}$, and $\dot{Z}$, and the second derivatives $\ddot{X}$, $\ddot{Y}$, and $\ddot{Z}$. The radii of curvature for the streamline projections into the three planes are given by

$$\left.\begin{aligned} R_{XY} &= \left|\frac{(\dot{X}^2 + \dot{Y}^2)^{3/2}}{\dot{X}\ddot{Y} - \dot{Y}\ddot{X}}\right| \\ R_{YZ} &= \left|\frac{(\dot{Y}^2 + \dot{Z}^2)^{3/2}}{\dot{Y}\ddot{Z} - \dot{Z}\ddot{Y}}\right| \\ R_{ZX} &= \left|\frac{(\dot{Z}^2 + \dot{X}^2)^{3/2}}{\dot{Z}\ddot{X} - \dot{X}\ddot{Z}}\right| \end{aligned}\right\} \tag{7}$$

Using these radii, step sizes are computed for each plane according to equation (6), and are denoted by STPXY, STPYZ, and STPZX.

To determine which two of the three planes should be used to best describe and predict the motion to the next location, the magnitudes of the projected velocities V_{XY}, V_{YZ}, and V_{ZX} are compared, and the plane containing the smallest projection is the one that is disregarded. Consider, for example, a streamline that lies very nearly along the Y-axis. Here, the projection of the streamline in the ZX plane provides very little path information because the points will be close together in that plane. Figure 5 indicates that the procedure will be similar for any two planes; as an example of the subsequent procedure, the case wherein the ZX plane is the non-critical (disregarded) plane is followed through.

The two remaining planes (XY and YZ) are designated as primary and secondary; the plane with the smallest radius of curvature is the primary plane. The step size is chosen to be the smallest of STPXY and STPYZ, and it is labeled simply $STEP_1$. The flow chart shows the procedure to be followed when the local radius of curvature in the YZ plane is smaller than the radius of curvature in the XY plane. The two-dimensional correction equations (1) through (5) are applied (as shown in fig. 6) to the YZ plane using R_{YZ} and $STEP_1$; the results give $Y_{NEW} = Y + \Delta Y'$ and $Z_{NEW} = Z + \Delta Z'$. Now, the motion in the secondary plane (XY plane) must agree with the motion in the primary plane (YZ plane) to the extent that the Y-location is the same (point P). To compute the remaining correction, $\Delta X'$, the coordinates (Y_0, X_0) are given by

$$X_0 = X_5 + R_{XY}\alpha'\beta' \cos\gamma \tag{8}$$

$$Y_0 = Y_5 - R_{XY}\alpha'\beta' \sin\gamma \tag{9}$$

where

$$\gamma = \tan^{-1}(\dot{X}/\dot{Y}) \tag{10}$$

$$\alpha' = \text{sign of } (\cos \gamma) \tag{11}$$

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$$\beta' = \text{sign of } \left(\frac{\dot{Y}\ddot{X} - \dot{X}\ddot{Y}}{\dot{Y}^3}\right) \tag{12}$$

It can be shown that the ordinate value of point P is

$$X_{NEW} = X_0 - \alpha'\beta'[R_{XY}^2 - (Y_{NEW} - Y_0)^2]^{1/2} \tag{13}$$

It should be noted that the directions of V_{YZ} and V_{XY} will be precisely tangent to the predicted curvature only where the streamlines are circular. Generally, as the curvature varies, the two-dimensional or three-dimensional corrections will cause the streamline trace to lag slightly behind, thereby keeping the test-point motion from reacting too quickly to changes in curvature. Also, for experimental measurements, statistical variations in the velocity direction will be somewhat damped by the corrections. Therefore, if equation (13) is used to determine the X-coordinate, X_{NEW}, then point P will always be located on the circle of radius R_{XY}, and the damping effect will be lost. To maintain the effect, a correction $\Delta X'$ must be computed (as was done for the two-dimensional case) and applied to the vector V_{XY} that may not be precisely tangent to the estimated radius-of-curvature circle. To accomplish this, notice that the step size in the secondary plane is constrained to take on the value

$$STEP_2 = [(X_{NEW} - X_5)^2 + (Y_{NEW} - Y_5)^2]^{1/2} \tag{14}$$

In other words, using $STEP_2$ and R_{XY} in the two-dimensional correction equations for the XY plane will give a correction $\Delta Y'$ that results in the same value Y_{NEW} as was obtained for the YZ plane. Additionally, $\Delta X'$ is given by

$$\Delta X' = 2\alpha'\beta'(STEP_2)\sin(\delta'/2)\cos(\delta'/2 + \alpha'\beta'\gamma) \tag{15}$$

where

$$\delta' = \sin^{-1}(STEP_2/2R_{xy})$$

To verify that the three-dimensional algorithm provides acceptable corrections, the method was checked by simulation, again using a simple analytical flow. Streamlines were generated for a potential flow consisting of a vortex of strength Γ, a sink of strength Q, and a uniform axial velocity U. Figure 7(a) shows the projection of several streamlines into the YZ plane; for clarity, figure 7(b) shows the projection into the ZX plane of only the streamline that passes through the starting location, (X,Y,Z) = (0,4,0). The YZ and XZ planes are presented in figure 7(a) and 7(b) because they are convenient for displaying the three-dimensional nature of the flow; they are not necessarily the primary and/or secondary planes. Instead of indicating the primary and secondary planes, it is easier to indicate on the figure the plane that has been disregarded (labeled between heavy dots in the figure). This flow is a good test case because the streamlines have varying degrees of curvature in the three planes. Hence, the primary and secondary planes will not be the same in all regions of the flow, and the algorithm must transition smoothly when the planes change as the test point moves along the streamlines. The only deviation from the true streamline occurs near the end of the trace where the minimum step size (0.1 cm) is too large (relative to the local radius of curvature) to provide an accurate trace. Experimentally, this problem is solved because the engineer can reduce the minimum step size as the radius of curvature is observed to decrease.

THREE-DIMENSIONAL LDA STREAMLINE TRACING—EXPERIMENTAL RESULTS

In conjunction with V/STOL basic aerodynamics research, the 3D LDA instrumentation described in reference 2 was used to measure the flow associated with a 5-cm-diam cold jet issuing normally from a surface plate into the wind tunnel cross-flow. The jet velocity (V_{jet}) was 85 m/sec and the velocity ratio (V_{jet}/V_{∞}) was 8. Using the criterion that the angles obtained by projecting the measured velocity into the three planes (fig. 4) be statistically accurate to within 10° (as discussed in ref. 5), the streamline traces presented in figure 8 were obtained as real-time, on-line displays during the course of the experiment. Each of the streamlines (except for those labeled F and R) were begun at 5 cm above the surface plate and at various Y-values, as indicated. For clarity, not all of the traces shown in the XY plane are also shown in the ZX plane. It should be emphasized that near the jet and in its wake the local turbulent intensity reaches values in excess of 40%; hence, a streamline represents the mean fluid trajectory.

The accuracy of the experimentally determined trajectories was tested by conducting a reverse-path measurement along a streamline that had been previously measured in the forward direction. Because statistical variations caused by measurement variations could induce random deviation from the streamline, as explained in reference 3, a streamline section was chosen where the turbulence intensity was lower, and the projected vectors could thereby be experimentally determined to better than 2° over the entire trace. The forward trace is labeled F in figure 8, and the reverse-path trace, labeled R, begins where the forward trace ends. Notice that the reverse trace deviates slightly from the forward trace for the first five steps before the curve-fitting routine becomes effective; this is a result of the cumulative error due to tangential stepping. Subsequently, however, no further systematic error is incurred. The reverse streamline remains nearly parallel to the forward trace in the ZX plane, and the reverse trace actually converges to the forward trace in the XY plane (probably due to statistical variations, as noted in reference 3).

CONCLUDING REMARKS

An algorithm has been presented that can be used with a three-dimensional laser-Doppler anemometry system to determine accurately the mean streamline patterns in a complex, turbulent, three-dimensional flow. Cumulative, systematic errors caused by tangential stepping can be successfully removed by using the formalism that has been presented. Also, when the algorithm is made an integral part of the on-line software, it provides a powerful diagnostic tool for gaining insight into the fluid dynamics of the flow.

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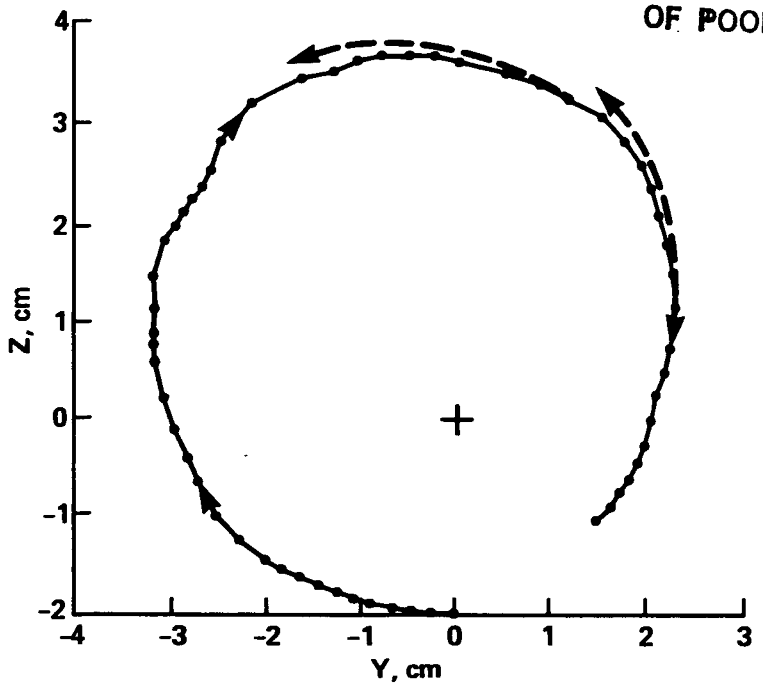
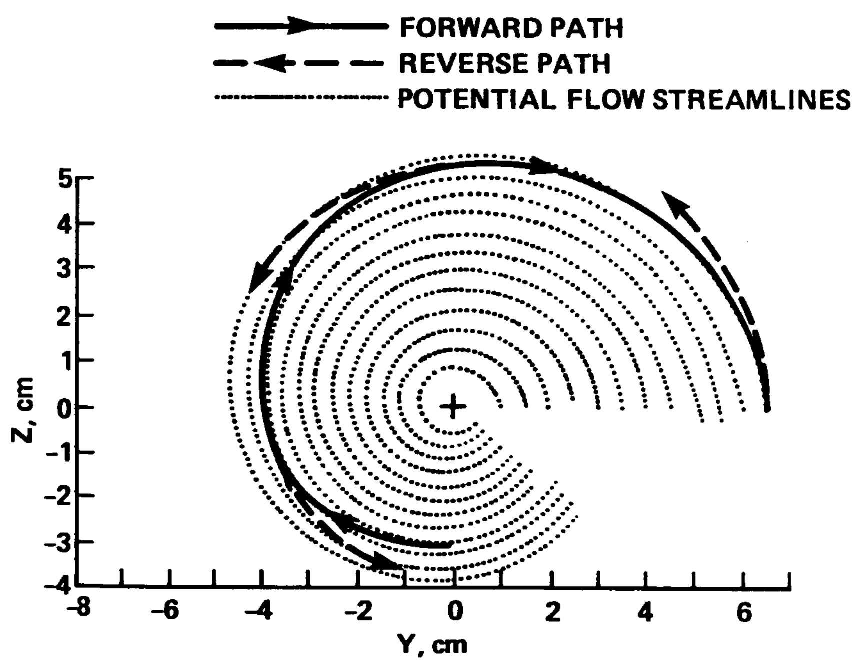


(a) Experimental 3D LDA results from reference 5 showing divergence toward outside of curvature for reverse paths.



(b) Numerical simulation of LDA streamline tracing showing crossing of streamlines and divergence of reverse paths due to cumulative errors.

Figure 1.- Results of straight-line stepping in the direction of the velocity vector (from ref. 3).

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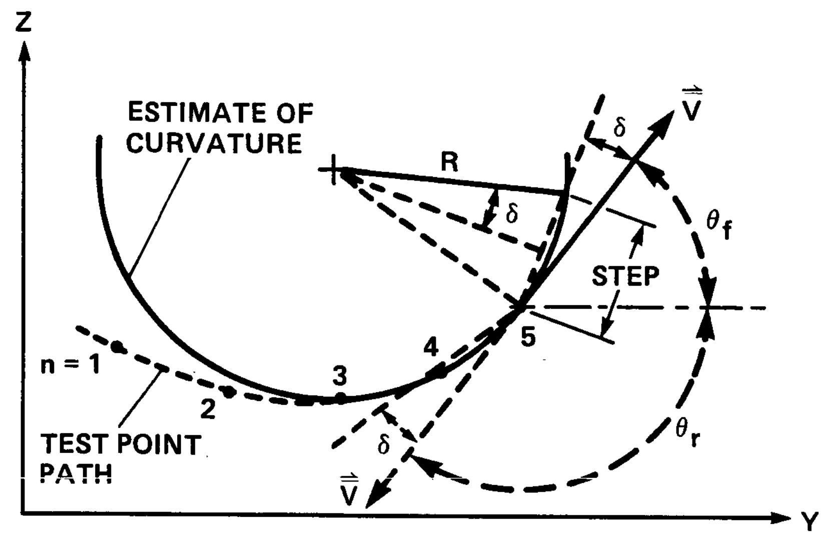
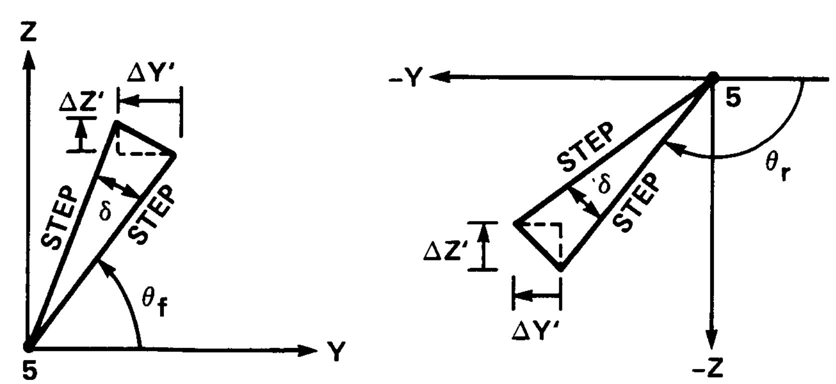


(a) Concave-upward flow; δ is correction angle, R is estimated radius of curvature at point 5.



(b) Cartesian corrections ΔY' and ΔZ' for forward- and reverse-flow directions at point 5.

Figure 2.- Correction scheme used to reduce error created by tangential stepping.

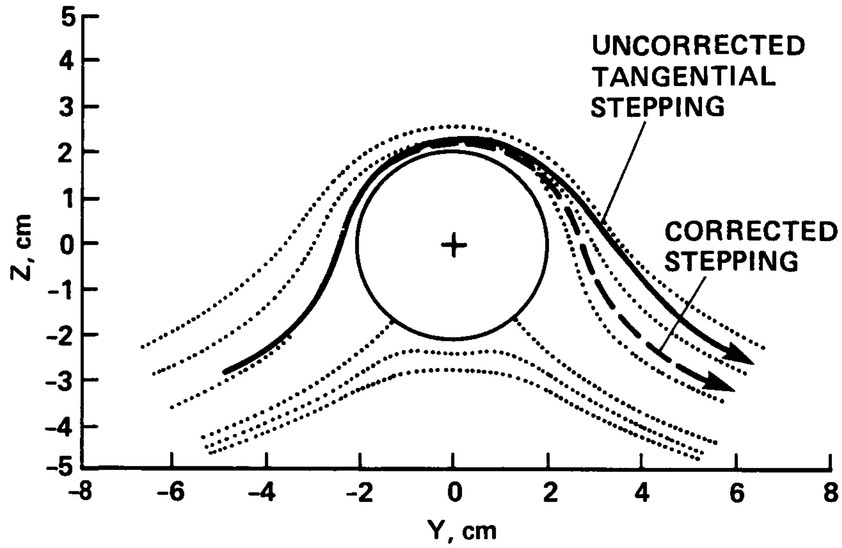


Figure 3.- Using radius-of-curvature algorithm to follow streamlines in a simulated potential flow with local inflection; uncorrected result is shown for comparison SMIN = 0.2 cm; SMAX = 0.6 cm.

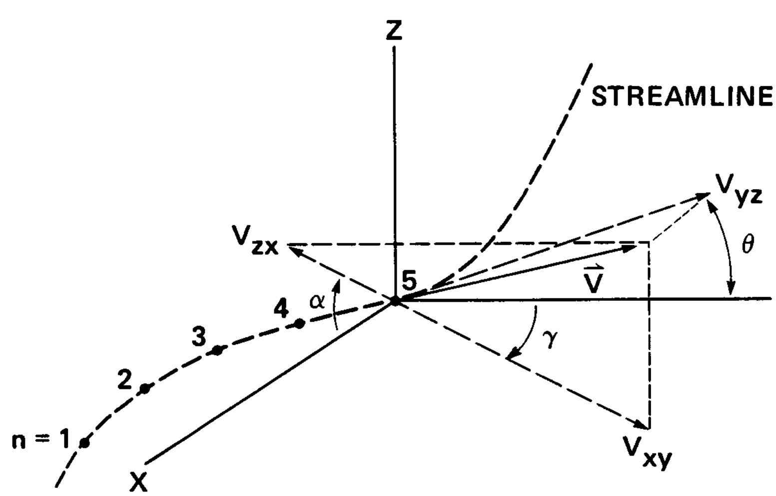


Figure 4.- Velocity vector V at point 5 along the streamline is projected into three planes to define magnitudes and angles.

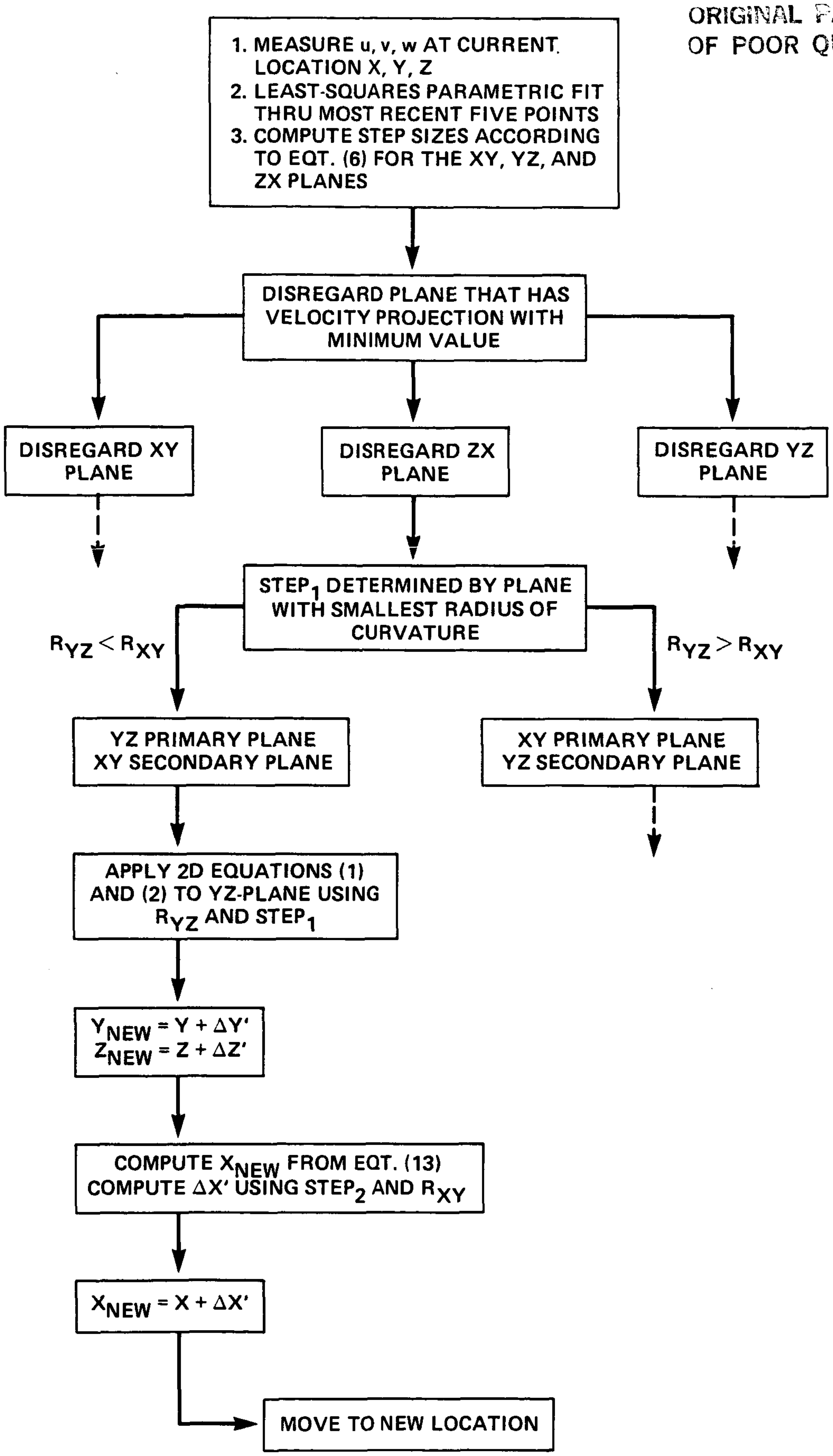


Figure 5.- Flow chart for three-dimensional correction algorithm.

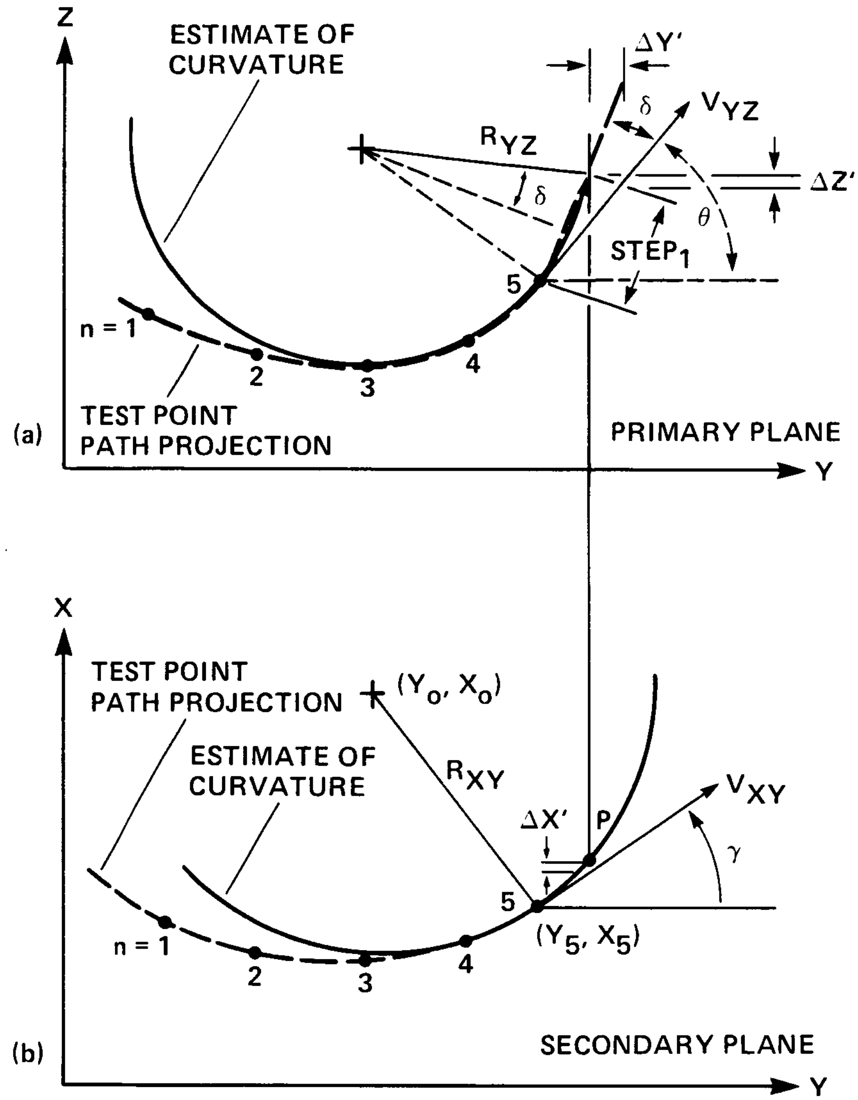


Figure 6.- Example of three-dimensional corrections for tangential stepping.

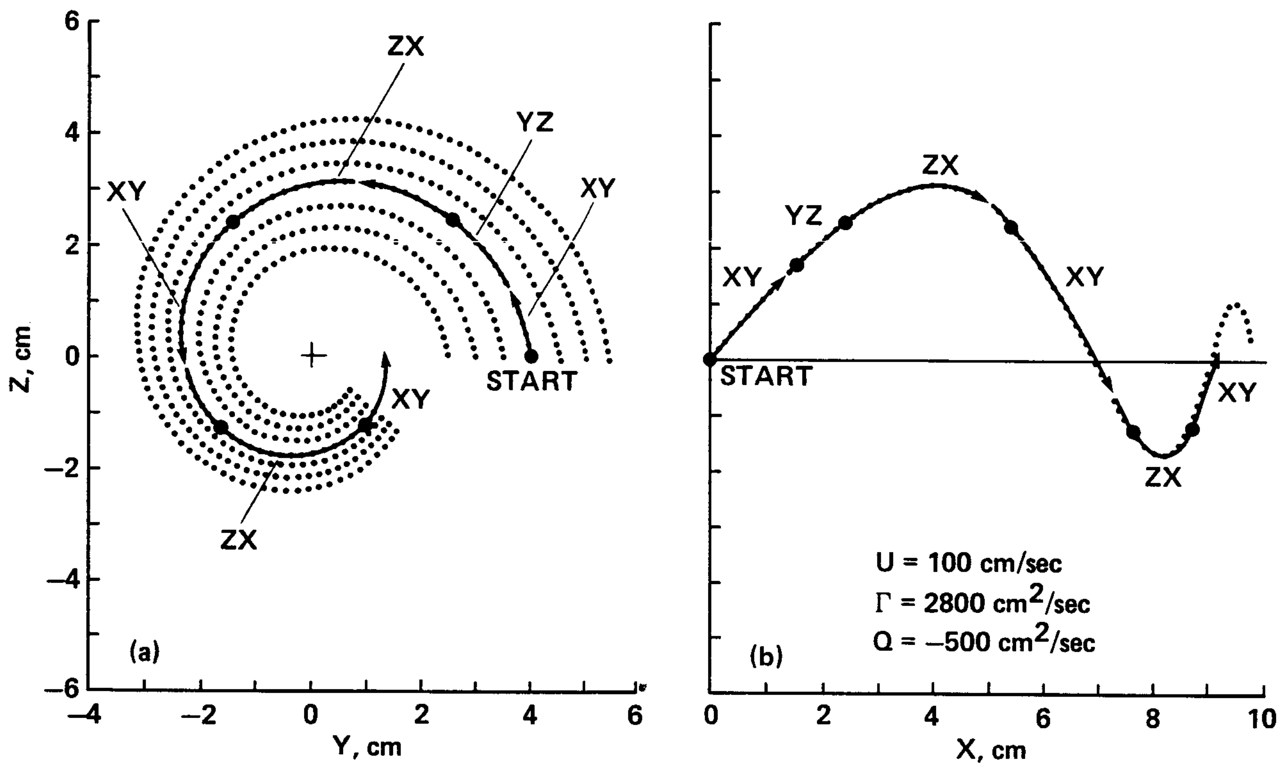


Figure 7.- Simulated streamline tracing in three dimensions showing the plane that is disregarded along each segment of the measured path.

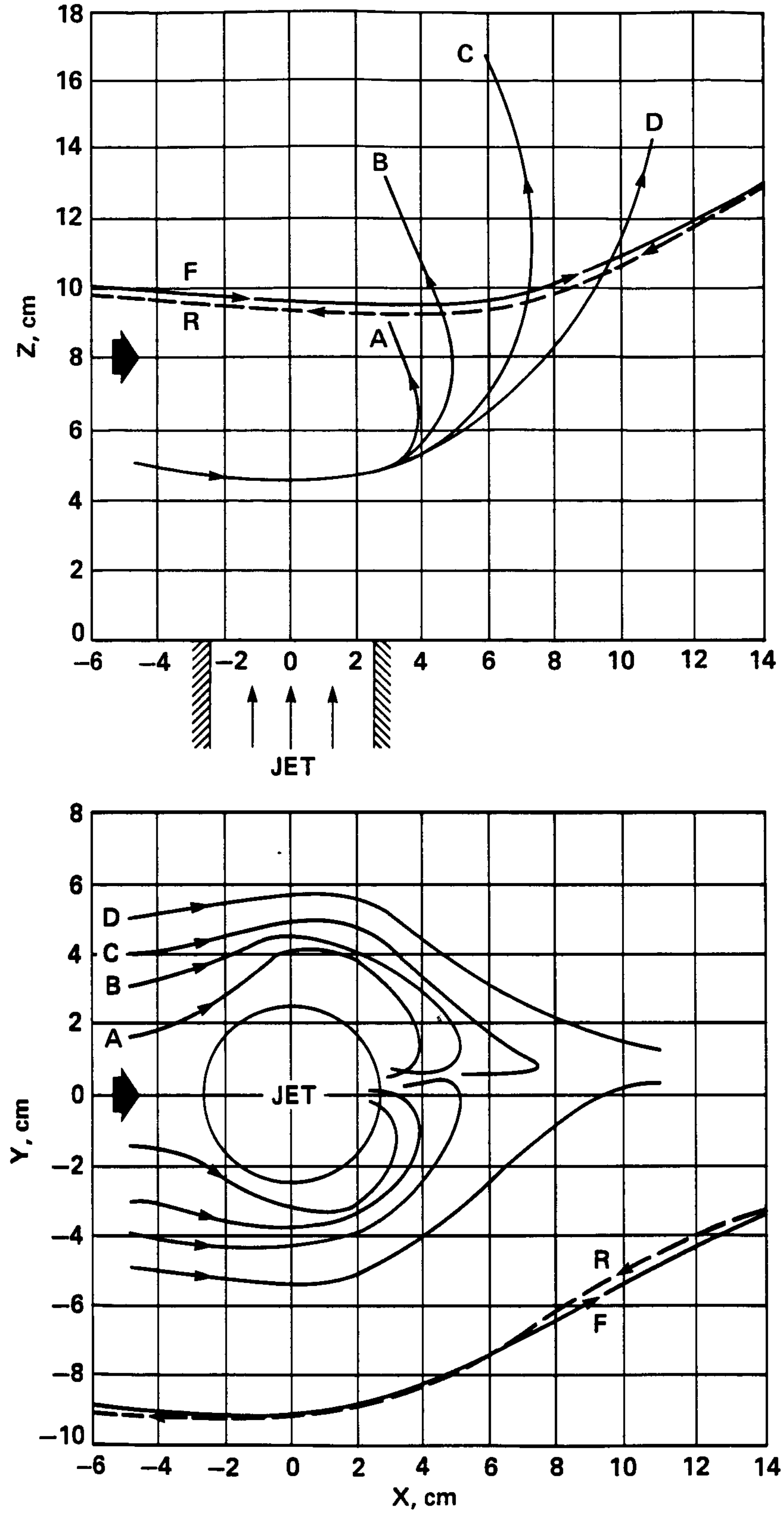


Figure 8.- Measured 3D LDA streamline traces for jet-in-a-crossflow experiment.

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16. Abstract
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