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# EVALUATION OF A NONLINEAR PARAMETER EXTRACTION MATHEMATICAL MODEL INCLUDING THE TERM $C_{m_{\delta e^2}}$

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## SUMMARY

Shuttle flight test data have been used to determine values for the short-period parameters. The best identified, as judged by its estimated standard deviation, was the elevon effectiveness parameter $C_{m_{\delta e}}$. However, the scatter about the preflight prediction of $C_{m_{\delta e}}$ was large. Other investigators have suggested that adding nonlinear terms to the mathematical model used to identify $C_{m_{\delta e}}$ could reduce the scatter.

The results of this investigation show that $C_{m_{\delta e^2}}$ is the only identifiable nonlinear parameter applicable and that the changes in $C_{m_{\delta e}}$ values when $C_{m_{\delta e^2}}$ is included are in the order of ten percent for the data estimated.

## INTRODUCTION

The longitudinal parameters that describe up to 90 percent of the short-period motion of the shuttle vehicle have been determined using flight test data. These parameters are shown plotted versus Mach number in figure 1 taken from reference 1. Of the parameters shown, $C_{m_{\delta e}}$ has the smallest standard deviations and the largest sensitivities, and it was considered the parameter best identified. The trends of the parameter with Mach number matched those of the preflight predictions; however, the scatter of the values extracted was large and reduced the confidence in the actual values determined.

Investigators at Johnson Space Center have found that by using a nonlinear model and including the term $C_{m_{\delta e^2}}$, the identifiability of $C_{m_{\delta e}}$ was improved, and its value was close to that of the preflight predictions. The purpose of this paper is to use Shuttle Discovery flight test data to determine the identifiability of the $C_{m_{\delta e^2}}$ parameter, to check the possibility of including other nonlinear terms in the model, and to compare the values of $C_{m_{\delta e}}$ determined using the linear and nonlinear extraction models.

## SYMBOLS

| | |
|---|---|
| $a_Z$ | acceleration measured along the Z body axis, g units |
| $\bar{c}$ | wing mean geometric chord, m (ft) |
| $C_m$ | pitching-moment coefficient, $M_Y/\bar{q}S\bar{c}$ |
| $C_X$ | axial-force coefficient, $F_X/\bar{q}S$ |
| $C_Z$ | vertical-force coefficient, $F_Z/\bar{q}S$ |
| $F_X$, $F_Z$ | force along X and Z body axes, respectively, N (lb) |

$g$ — acceleration due to gravity, $m/sec^2$ ($ft/sec^2$)

$I_X$, $I_Y$, $I_Z$ — moment of inertia about X, Y, and Z body axes, respectively, $kg\text{-}m^2$ ($slug\text{-}ft^2$)

$I_{XZ}$ — product of inertia, $kg\text{-}m^2$ ($slug\text{-}ft^2$)

$M_Y$ — pitching moments, N-m (ft-lb)

$m$ — mass, kg (slugs)

$p$, $q$, $r$ — rate of roll, pitch and yaw, rad/sec or degrees/sec

$\bar{q}$ — dynamic pressure, $N/m^2$ ($slug/ft^2$)

$S$ — wing area, $m^2$ ($ft^2$)

$u$, $v$, $w$ — velocity along X, Y, and Z body axes, respectively, m/sec (ft/sec)

$V$ — airplane total velocity, m/sec (ft/sec)

$\alpha$ — angle of attack, rad or degrees

$\delta_e$ — elevator, deflection, rad or degrees

$\theta$, $\phi$ — pitch angle, roll angle, rad or degrees

$\rho$ — air density, $kg/m^3$ ($slug/ft^3$)

Stability and control derivatives referenced to a system of body axes with the origin at the airplane center of gravity:

$$C_{m_q} = \frac{\partial C_m}{\partial \frac{q\bar{c}}{2V}} \qquad C_{m_\alpha} = \frac{\partial C_m}{\partial \alpha} \qquad C_{m_{\delta e}} = \frac{\partial C_m}{\partial \delta_e} \qquad C_{X_\alpha} = \frac{\partial C_X}{\partial \alpha}$$

$$C_{Z_q} = \frac{\partial C_Z}{\partial \frac{q\bar{c}}{2V}} \qquad C_{Z_\alpha} = \frac{\partial C_Z}{\partial \alpha} \qquad C_{Z_{\delta e}} = \frac{\partial C_Z}{\partial \delta_e} \qquad C_{m_{\delta e}2} = \frac{\partial C_m}{\partial \delta_e^2}$$

$$C_{m_{\alpha\delta_e}} = \frac{\partial C_m}{\partial \alpha \partial \delta_e} \qquad C_{m_\alpha 2} = \frac{\partial C_m}{\partial \alpha^2}$$

A dot over a symbol signifies a derivative with respect to time.

## MODEL VERIFICATION

In order to verify that the assumed nonlinear model best describes the vehicle motion several questions must be answered:

1. Is $C_{m_{\delta e}}$ identifiable?

2. Does the addition of $C_{m_{\delta e^2}}$ improve the identifiability of the other parameters in the model, especially $C_{m_{\delta e}}$?

3. Is $C_{m_{\delta e^2}}$ the best parameter for an extension of the mathematical model with linear aerodynamic parameters?

4. Do the nonlinear terms added make sense physically?

5. Does the nonlinear aerodynamic model have better prediction capabilities than the linear one?

In the discussion that follows these questions will be answered.

## EQUATIONS

The equations used in this study are standard equations used to describe the longitudinal motion of a vehicle in the atmosphere. For this study, the aerodynamic model equations are made nonlinear by the addition of terms such as $C_{m_{\delta e^2}}\delta e^2$, $C_{m_{\alpha\delta e}}\alpha\delta e$ or $C_{m_{\alpha^2}}\alpha^2$. The basic equations can be found in many references such as reference 2 and are repeated here for convenience.

$$\dot{u} = -qw + rv - g\sin\theta + \frac{1}{2}\frac{\rho V^2 S}{m}\left[ C_{x_0} + C_{x_\alpha}(\alpha - \alpha_t) \right]$$

$$\dot{w} = -pv + qu + g\cos\theta\cos\phi + \frac{1}{2}\frac{\rho V^2 S}{m}\left[ C_{Z_0} + C_{Z_\alpha}(\alpha - \alpha_t) + C_{Z_{\delta e}}(\delta_e - \delta_{e_t}) \right]$$

$$\dot{q} = pr\frac{I_X - I_Z}{I_Y} + \frac{I_{XZ}}{I_Y}(r^2 - p^2) + \frac{1}{2}\frac{\rho V^2 S\bar{c}}{I_y}\Big[ C_{m_0} + C_{m_\alpha}(\alpha - \alpha_t) + C_{m_{\alpha^2}}(\alpha - \alpha_t)^2 + C_{m_q}\frac{q\bar{c}}{2V}$$

$$+ C_{m_{\delta e}}(\delta_e - \delta_{e_t}) + C_{m_{\delta e^2}}(\delta_e - \delta_{e_t})^2 + C_{m_{\alpha_{\delta e}}}(\alpha - \alpha_t)(\delta - \delta_{e_t}) \Big]$$

## DISCUSSION

Longitudinal maneuvers from the first two Shuttle Discovery flights were examined using a maximum likelihood parameter extraction algorithm (ref. 3). Thirteen runs were selected covering a Mach number range from 22 to 1. The results of processing these data for several assumed mathematical models are shown as figure 2. The values determined for $C_{m_{\delta e^2}}$ and $C_{m_{\delta e}}$ at various Mach numbers are shown in Table I. The addition of the $C_{m_{\delta e^2}}$ term seemed to have the greatest effect on the values determined for the other parameters at the highest Mach numbers. The $C_{m_\alpha}$ parameter had the greatest variations when $C_{m_{\delta e^2}}$ was added to the mathematical model. The $C_{m_{\delta e}}$ parameter also showed some change when $C_{m_{\delta e^2}}$ was added to the model, but below Mach 15, this change was generally 10 percent or less.

The question of identifiability can be discussed by considering Table II. The values of sensitivity and standard deviation given are typical of the values seen for all thirteen runs and are a reasonable basis for the discussion of identifiability. The sensitivity referred to is the variation in the output states with respect to a perturbation in a specified parameter assuming the other parameters are fixed. The greater the sensitivity the more identifiable the parameter. An examination of the sensitivities implies the $C_{m_{\delta e^2}}$ is less identifiable than $C_{m_{\delta e}}$, as identifiable as $C_{m_\alpha}$, and more identifiable than $C_{Z_\alpha}$. The estimated standard deviations indicate that the values for $C_{m_{\delta e^2}}$ were well determined since the estimated standard deviation was less than one-tenth of the extracted value. Based on this assessment, $C_{m_{\delta e^2}}$ was considered identifiable. In general, the addition of $C_{m_{\delta e^2}}$ resulted in $C_{m_{\delta e}}$ variations of about 10 percent.

The addition of $C_{m_{\delta e^2}}$ to the model appeared to improve the identifiability of $C_{m_{\delta e}}$. The sensitivities increased, and the estimated standard deviations as a percent of the extracted parameter did not degrade, implying an improved identifiability of $C_{m_{\delta e}}$. Also, the change in the values determined for $C_{m_{\delta e}}$ were generally in a direction so that they were closer to the preflight estimates, although this improvement was small for most runs.

Since the shuttle maneuvers resulted in small amplitude responses, the values of even the best determined parameters show considerable scatter. In particular, in figure 1 $C_{m_{\delta e}}$ was well determined for all runs, but showed significant run-to-run scatter. In this case, when a system is poorly excited, any additional parameter can many times improve fit and identifiability (see ref. 4). To demonstrate that $C_{m_{\delta e^2}}$ was the best parameter to add to the mathematical model, several other parameters were tried in the model, specifically, $C_{m_{\alpha\delta e}}$ and $C_{m_{\alpha^2}}$. The effect on $C_{m_{\delta e}}$ of adding $C_{m_{\alpha\delta e}}$ is shown in figure 2 for several runs. As can be seen, the value of $C_{m_{\delta e}}$ was not significantly changed when $C_{m_{\alpha\delta e}}$ was added to the model. Also, in the course of this analysis it was found that $C_{m_{\alpha\delta e}}$ proved to be much less identifiable than $C_{m_{\delta e^2}}$ when the sensitivity and standard deviations were compared for the same flight data run with $C_{m_{\delta e^2}}$ replaced in the model by $C_{m_{\alpha\delta e}}$. The parameter $C_{m_{\alpha^2}}$ proved to be totally unidentifiable and resulted in no changes in any of the parameter values. The improvement seen when $C_{m_{\delta e^2}}$ was added to the model is not an artifact of the identification procedure but the legitimate result of adding a term that actually describes part of the vehicle motion, as evidenced by the identifiability discussed earlier in this section.

The next question that must be answered is whether or not the parameter to be added makes sense physically. The values determined for $C_{m_{\delta e^2}}$ were negative (see Table II). This implies a $C_m$ versus $\delta_e$ relation as shown in figure 3. The result of this relation would be that a negative $\delta e$ would result in less response than a positive $\delta_e$. Figures 4 and 5 show that this is indeed the case for the shuttle vehicle. For elevon inputs that are essentially equal in magnitude but opposite in sign, the figures show that the responses are at least 40 percent greater for the positive $\delta e$ input.

Finally, since $C_{m_{\delta e2}}$ is the only nonlinear term identifiable, is the predictive capability of the model including $C_{m_{\delta e2}}$ better than the model where $C_{m_{\delta e2}}$ is not included? Figures 6 and 7 show that using the nonlinear model including $C_{m_{\delta e2}}$ does result in a slightly better fit to a set of responses that were not used to determine parameter values.

## CONCLUDING REMARKS

The inclusion of $C_{m_{\delta e2}}$ in the mathematical model describing the longitudinal motions of the shuttle vehicle was evaluated using flight test data from the Discovery vehicle. The $C_{m_{\delta e2}}$ parameter was found to be identifiable, and the values determined were well defined. The parameters can be justified physically by considering the increased responses of the vehicle to a positive elevon deflection when compared to the response from a negative deflection. Inclusion of the $C_{m_{\delta e2}}$ parameter caused $C_{m_{\delta e}}$ values to move closer to the preflight predictions, but the changes seen were small. The model that included $C_{m_{\delta e2}}$ was used to predict the response of a run not used for identification, and the fit was slightly better with $C_{m_{\delta e2}}$ in the model then when $C_{m_{\delta e2}}$ was not included. These results imply that incorporating $C_{m_{\delta e2}}$ produces an improved model for the identification of the longitudinal aerodynamic parameters of the shuttle vehicle.

## REFERENCES

1. Suit, W. T.; and Schiess, J. R.: Supplement to the Shuttle Aerodynamic Database Using Discovery Flight Tests. AIAA Paper No. 85-1765.

2. Cannaday, R. L.; and Suit, W. T.: Effects of Control Inputs on the Estimation of Stability and Control Parameters of a Light Airplane. NASA TP-1043, December 1977.

3. Murphy, P. C.: An Alogrithm for Maximum Likelihood Estimation Using an Efficient Method for Approximating Sensitivities. NASA TP-2311, June 1984.

4. Suit, W. T.: Aerodynamic Parameters of the Navion Airplane Extracted from Flight Data. NASA TN D-6643, 1972.

**TABLE I $C_{m_{\delta e^2}}$ AND $C_{m_{\delta e}}$ VALUES AT SPECIFIC MACH NUMBERS**

| Mach | $C_{m_{\delta e^2}}$ | $C_{m_{\delta e}}$ |
|---|---|---|
| 22 | -.34 | -.170 |
| 18 | -.35 | -.176 |
| 18 | -.69 | -.235 |
| 11 | -.26 | -.16 |
| 8 | -.315 | -.142 |
| 8 | -.32 | -.17 |
| 6 | -.21 | -.117 |
| 5.5 | -.21 | -.110 |
| 4 | -.13 | -.087 |
| 4 | -.09 | -.087 |
| 1.7 | -.57 | -.15 |
| 1.6 | -.051 | -.17 |
| 1.0 | -1.05 | -.42 |

**TABLE II SENSITIVITIES AND STANDARD DEVIATIONS FOR SELECTED SHUTTLE RUNS WITH $C_{m\delta_e 2}$ AND WITHOUT $C_{m\delta_e 2}$ IN THE EXTRACTION MODEL**

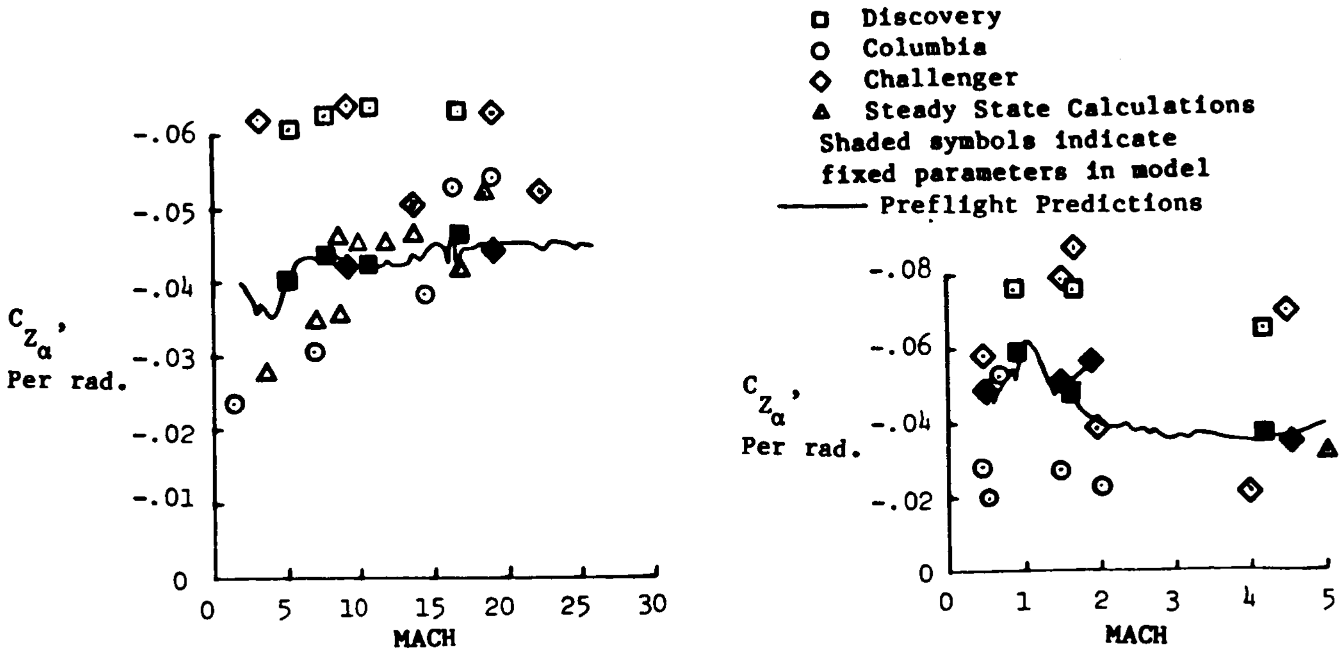
| Mach = 8 | | | | Flight Number = 19 | | |
|---|---|---|---|---|---|---|
| Parameter | Value | Standard Deviation | Sensitivity | Value | Standard Deviation | Sensitivity |
| $C_{X_0}$ | -.027 | --- | --- | -.025 | --- | --- |
| $C_{X_\alpha}$ | .18 | --- | --- | .26 | --- | --- |
| $C_{Z_0}$ | -.96 | .0012 | .19E+8 | -.96 | .0012 | .19E+8 |
| $C_{Z_\alpha}$ | -.78 | .15 | .22E+8 | -.88 | .15 | .34E+8 |
| $C_{Z_{\delta e}}$ | -.39 | --- | --- | -.38 | --- | --- |
| $C_{m_0}$ | -.001 | --- | --- | 0.0 | --- | --- |
| $C_{m_\alpha}$ | -.15 | .004 | .85E+4 | -.072 | .0053 | .44E+4 |
| $C_{m_q}$ | -2.2 | --- | --- | -2.2 | --- | --- |
| $C_{m\delta_e}$ | -1.3 | .0008 | .52E+5 | -.142 | .00083 | .13E+6 |
| $C_{m\delta_e 2}$ | --- | --- | --- | -.315 | .019 | .6E+5 |

| Mach = 8 | | | | Flight Number = 14 | | |
|---|---|---|---|---|---|---|
| Parameter | Value | Standard Deviation | Sensitivity | Value | Standard Deviation | Sensitivity |
| $C_{X_0}$ | -.086 | --- | --- | -.094 | --- | --- |
| $C_{X_\alpha}$ | -.248 | --- | --- | 1.7 | --- | --- |
| $C_{Z_0}$ | -1.03 | .0023 | .12E+7 | -1.03 | .0058 | .2E+6 |
| $C_{Z_\alpha}$ | -4.58 | .27 | .18E+4 | -4.66 | .62 | .34E+3 |
| $C_{Z_{\delta e}}$ | -.130 | --- | --- | -.13 | --- | --- |
| $C_{m_0}$ | 0.0 | --- | --- | .0004 | --- | --- |
| $C_{m_\alpha}$ | -.108 | .018 | .15E+4 | -.13 | .0085 | .96E+4 |
| $C_{m_q}$ | -2.2 | --- | --- | -2.2 | --- | --- |
| $C_{m\delta_e}$ | -1.6 | .0038 | .17E+5 | -.17 | .0019 | .67E+5 |
| $C_{m\delta_e 2}$ | --- | --- | --- | -.32 | .016 | .8E+4 |

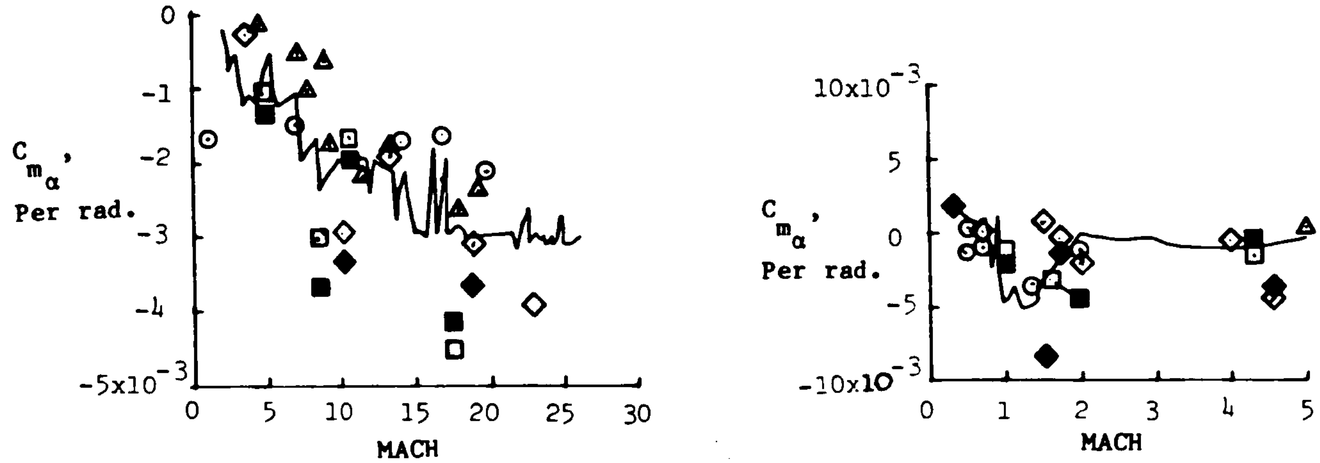
**TABLE III SENSITIVITIES AND STANDARD DEVIATIONS FOR SELECTED SHUTTLE RUNS WITH $C_{m\delta_e 2}$ AND WITHOUT $C_{m\delta_e 2}$ IN THE EXTRACTION MODEL (CONTINUED)**

| Mach = 6 | | | | Flight Number = 19 | | |
|---|---|---|---|---|---|---|
| Parameter | Value | Standard Deviation | Sensitivity | Value | Standard Deviation | Sensitivity |
| $C_{X_0}$ | -.06 | --- | --- | -.056 | --- | --- |
| $C_{X_\alpha}$ | .17 | --- | --- | .45 | --- | --- |
| $C_{Z_0}$ | -.69 | .00061 | .13E+9 | -.70 | .00066 | .15E+9 |
| $C_{Z_\alpha}$ | -1.17 | .055 | .42E+4 | -1.0 | .11 | .98E+4 |
| $C_{Z_{\delta_e}}$ | -.29 | --- | --- | -.31 | --- | --- |
| $C_{m_0}$ | 0.0 | --- | --- | | --- | --- |
| $C_{m_\alpha}$ | -.086 | .0013 | .85E+4 | -.033 | .002 | .37E+4 |
| $C_{m_q}$ | -2.2 | --- | --- | -2.2 | --- | --- |
| $C_{m_{\delta_e}}$ | -.10 | .00035 | .16E+6 | -.11 | .00036 | .37E+6 |
| $C_{m_{\delta_e 2}}$ | --- | --- | --- | -.21 | .011 | .15E+6 |

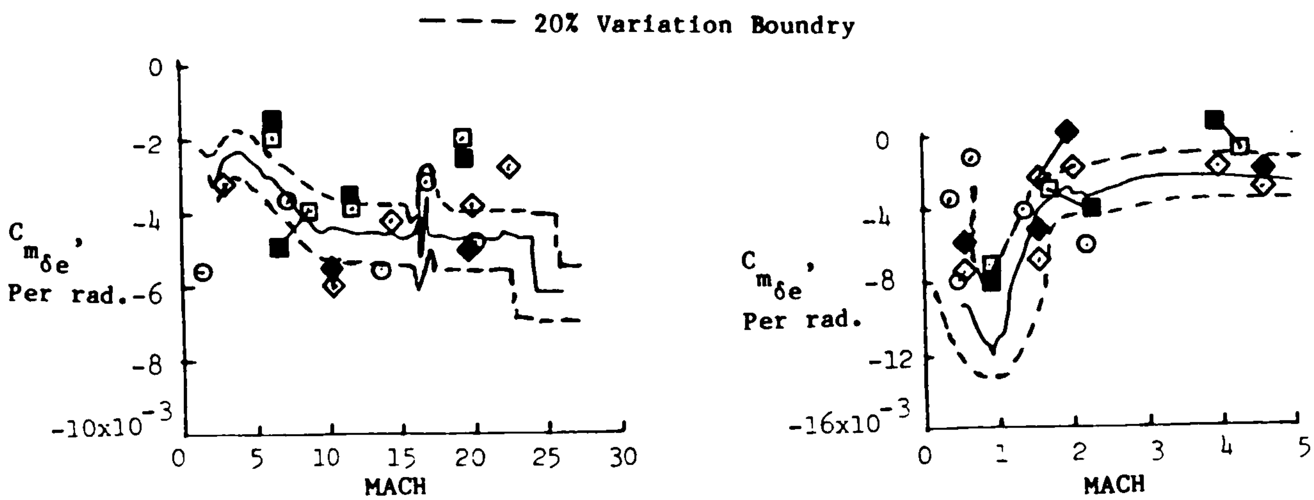
| Mach = 6 | | | | Flight Number = 14 | | |
|---|---|---|---|---|---|---|
| Parameter | Value | Standard Deviation | Sensitivity | Value | Standard Deviation | Sensitivity |
| $C_{X_0}$ | -.033 | --- | --- | -.033 | --- | --- |
| $C_{X_\alpha}$ | 1.28 | --- | --- | 1.26 | --- | --- |
| $C_{Z_0}$ | -.715 | .0014 | .21E+7 | -.71 | .0015 | .19E+7 |
| $C_{Z_\alpha}$ | -4.69 | .23 | .28E+4 | -4.80 | .26 | .25E+4 |
| $C_{Z_{\delta_e}}$ | -.066 | --- | --- | -.061 | --- | --- |
| $C_{m_0}$ | .0004 | --- | --- | 0.0 | --- | --- |
| $C_{m_\alpha}$ | -.20 | .016 | .52E+4 | -.11 | .01 | .41E+4 |
| $C_{m_q}$ | -2.2 | --- | --- | -2.2 | --- | --- |
| $C_{m_{\delta_e}}$ | -.095 | .0025 | .33E+5 | -.117 | .002 | .10E+6 |
| $C_{m_{\delta_e 2}}$ | --- | --- | --- | -.21 | .016 | .84E+4 |




(a) Change in normal force with alpha




(b) Static stability




(c) Elevon effectiveness

Figure 1. - Dominant longitudinal derivatives affecting the short period motion of the Shuttle versus Mach number (derivatives per degree).

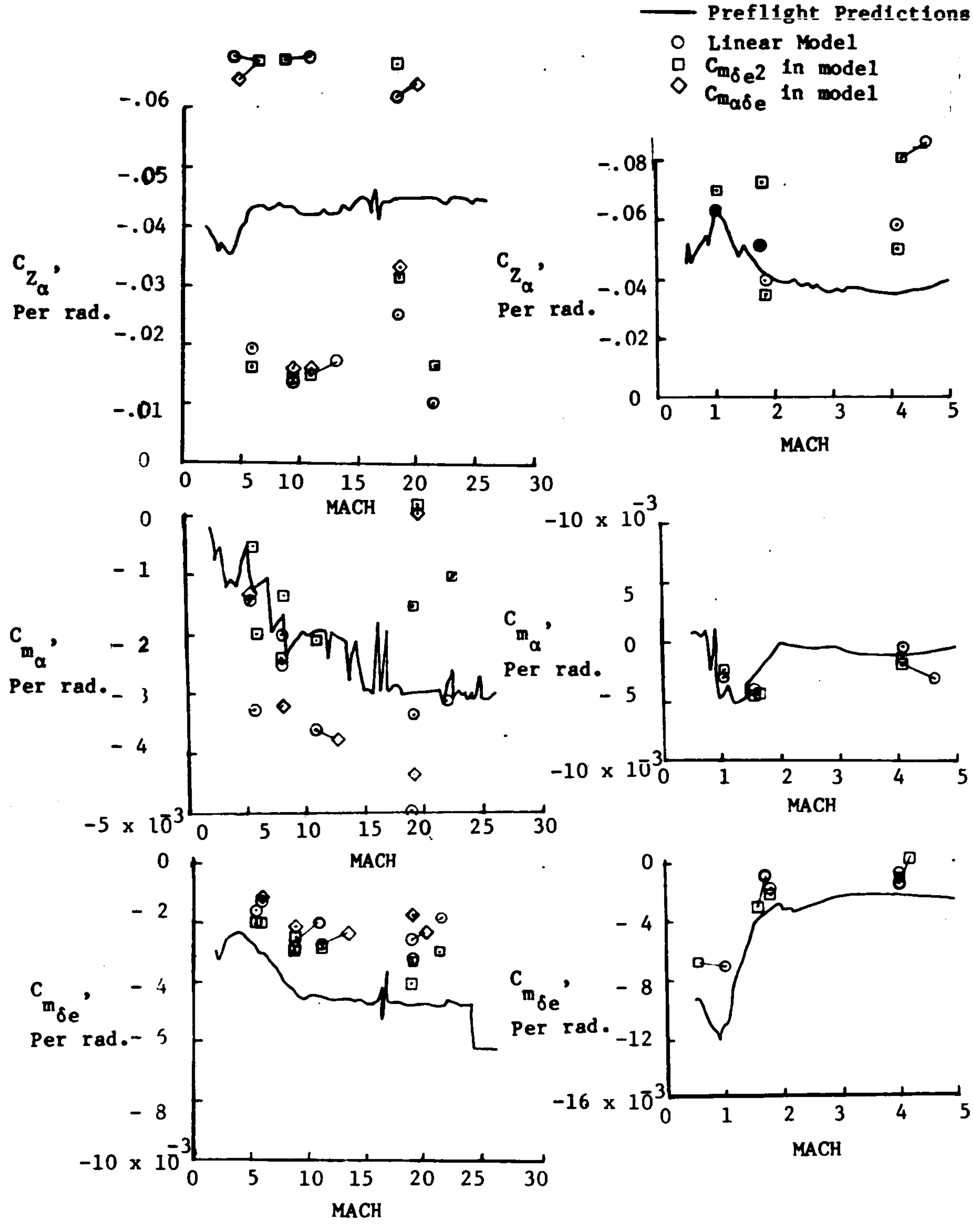



Figure 2. - Dominant longitudinal derivatives affecting the short period motion of the Shuttle versus Mach number from Discovery flight data and using three mathematical models.

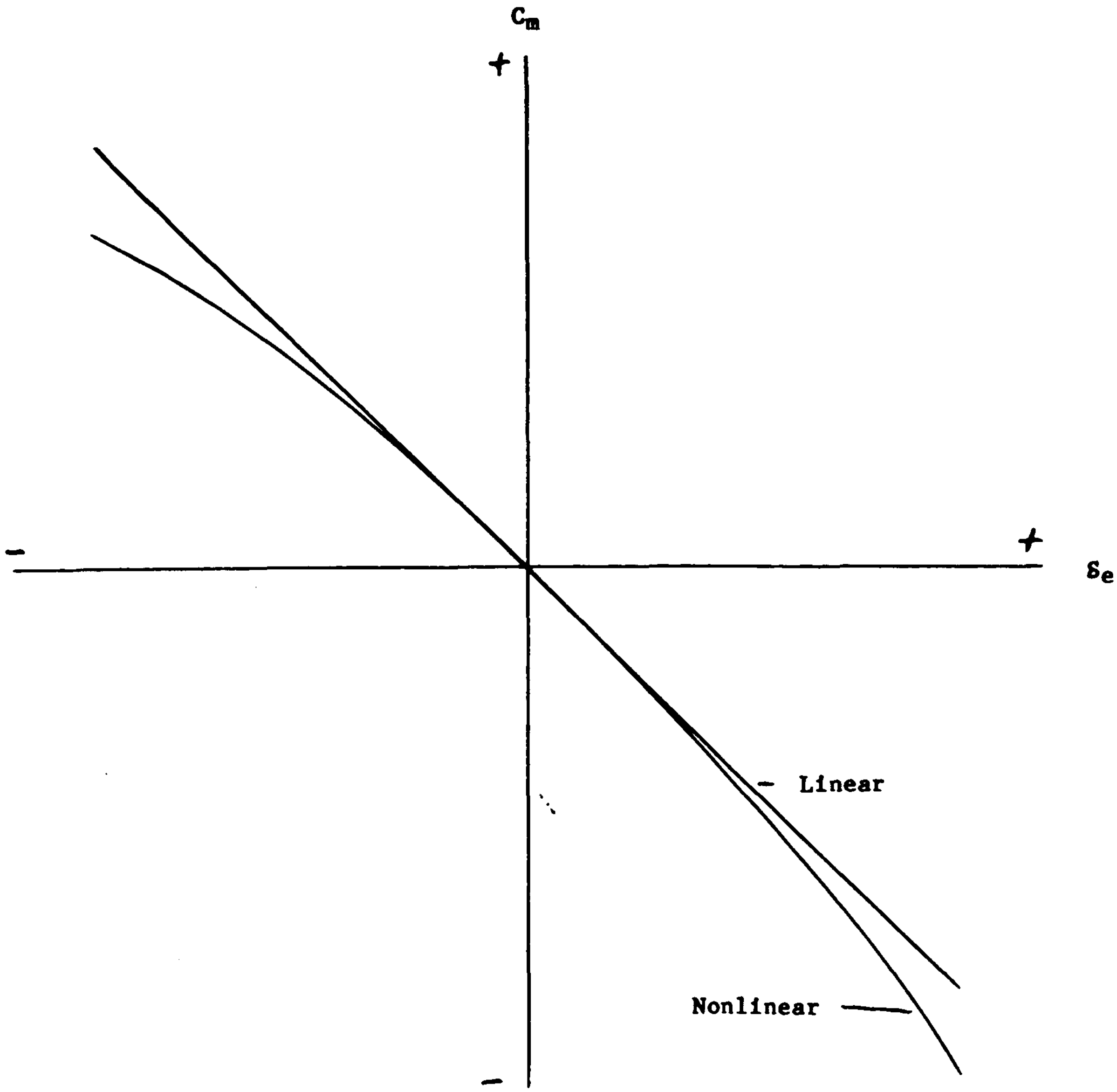



Figure 3. – $C_m$ versus $\delta_e$ curves for linear model and non-linear model including $C_{m_{\delta e}} \delta_e + C_{m_{\delta e^2}} \delta_e^2$ terms

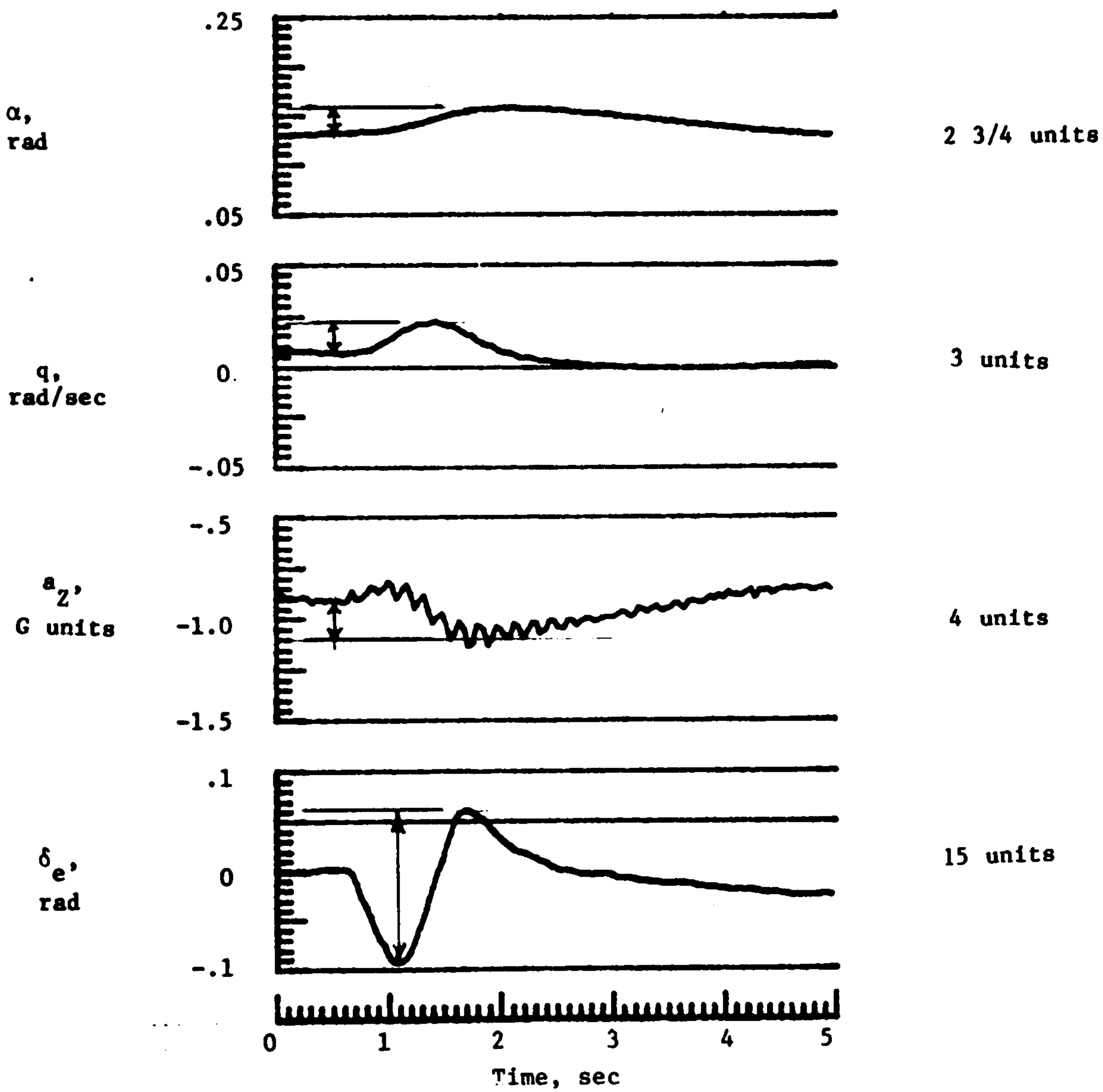



Figure 4. - Time history of doublet maneuver with elevon initially negative

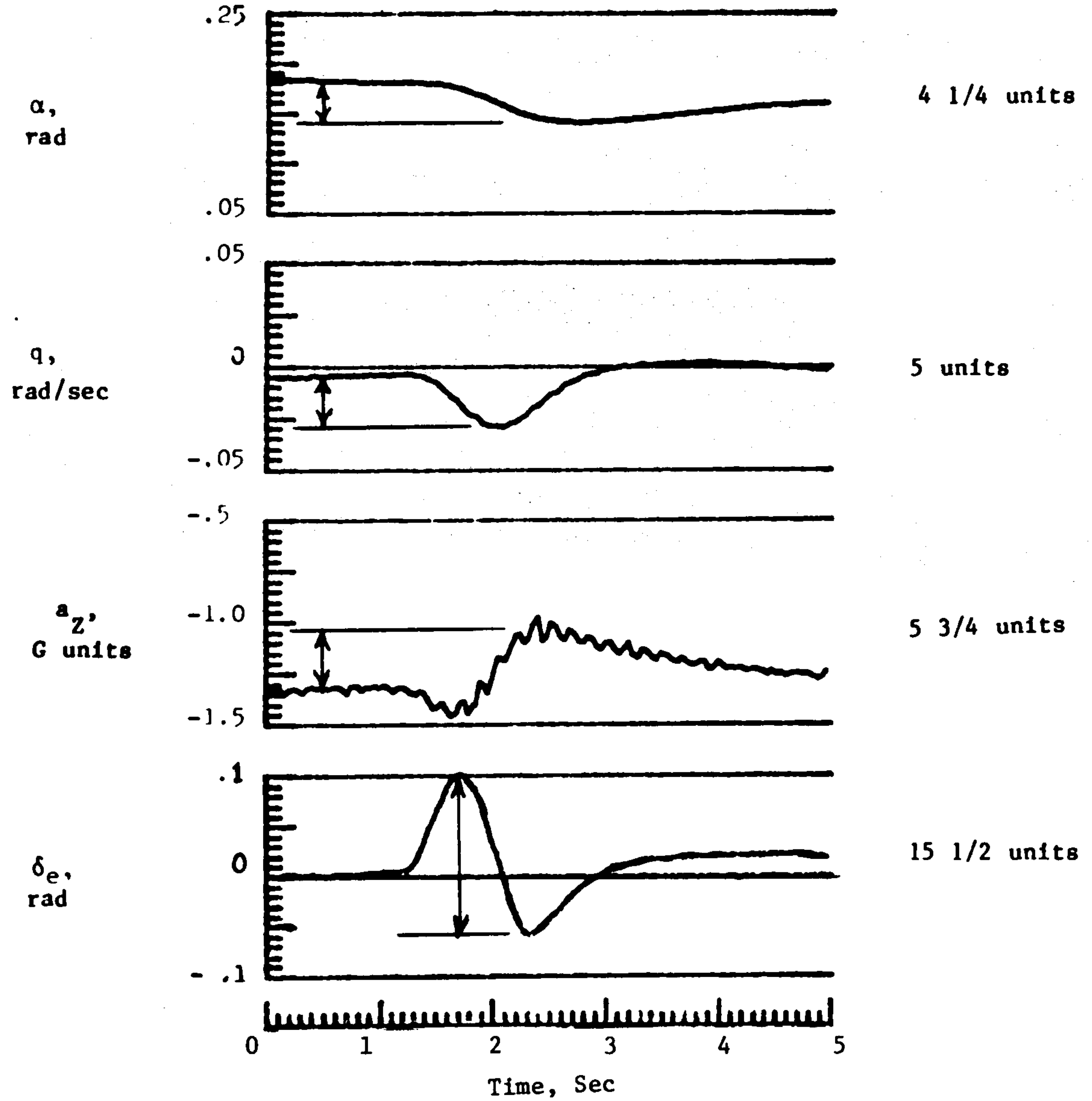



**Figure 5.** - Time history of doublet maneuver with elevon deflection initially positive.

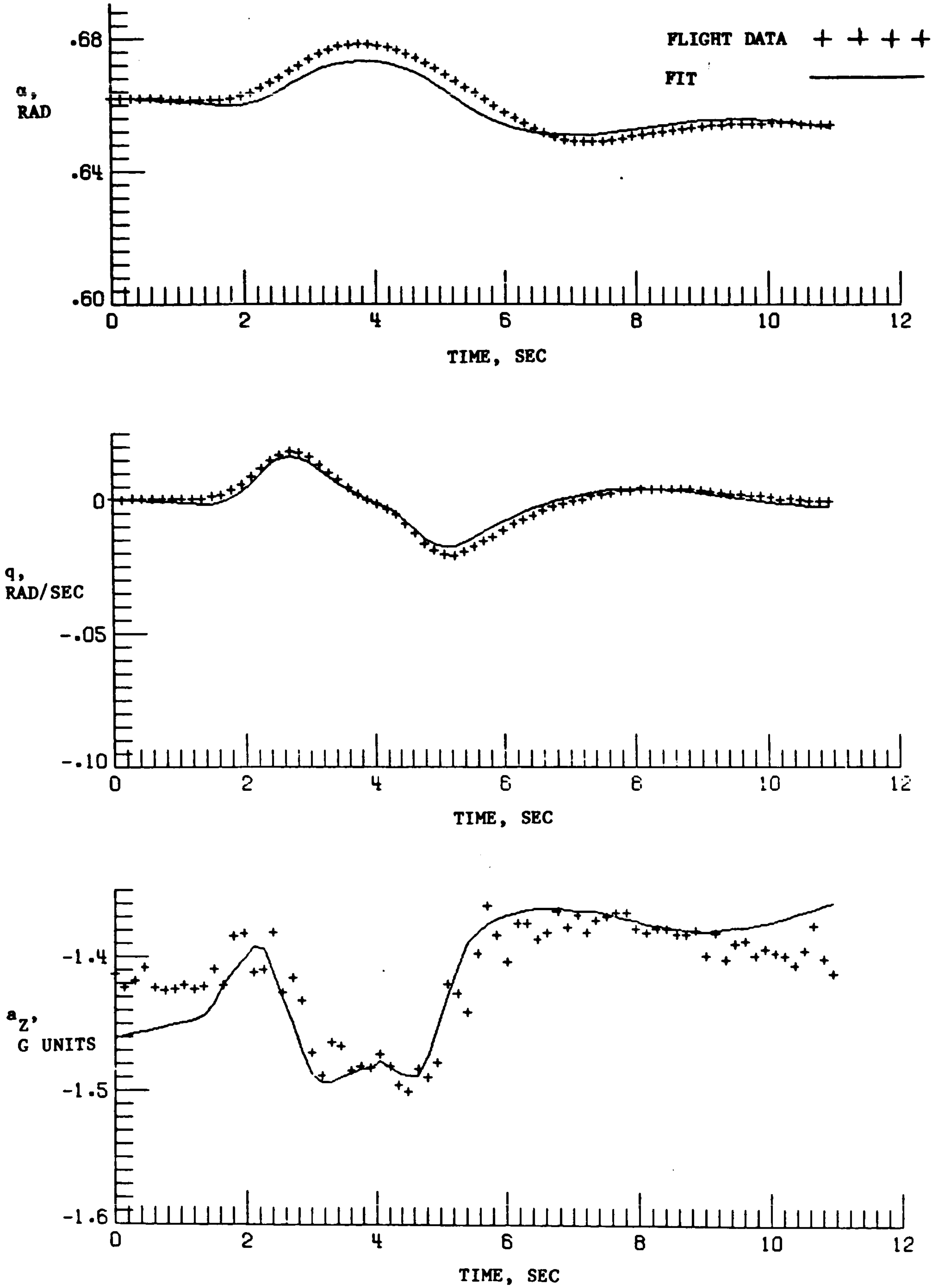



Figure 6. - Prediction of vehicle motion using linear model (only $C_{m_{\delta e}}$ included in model).

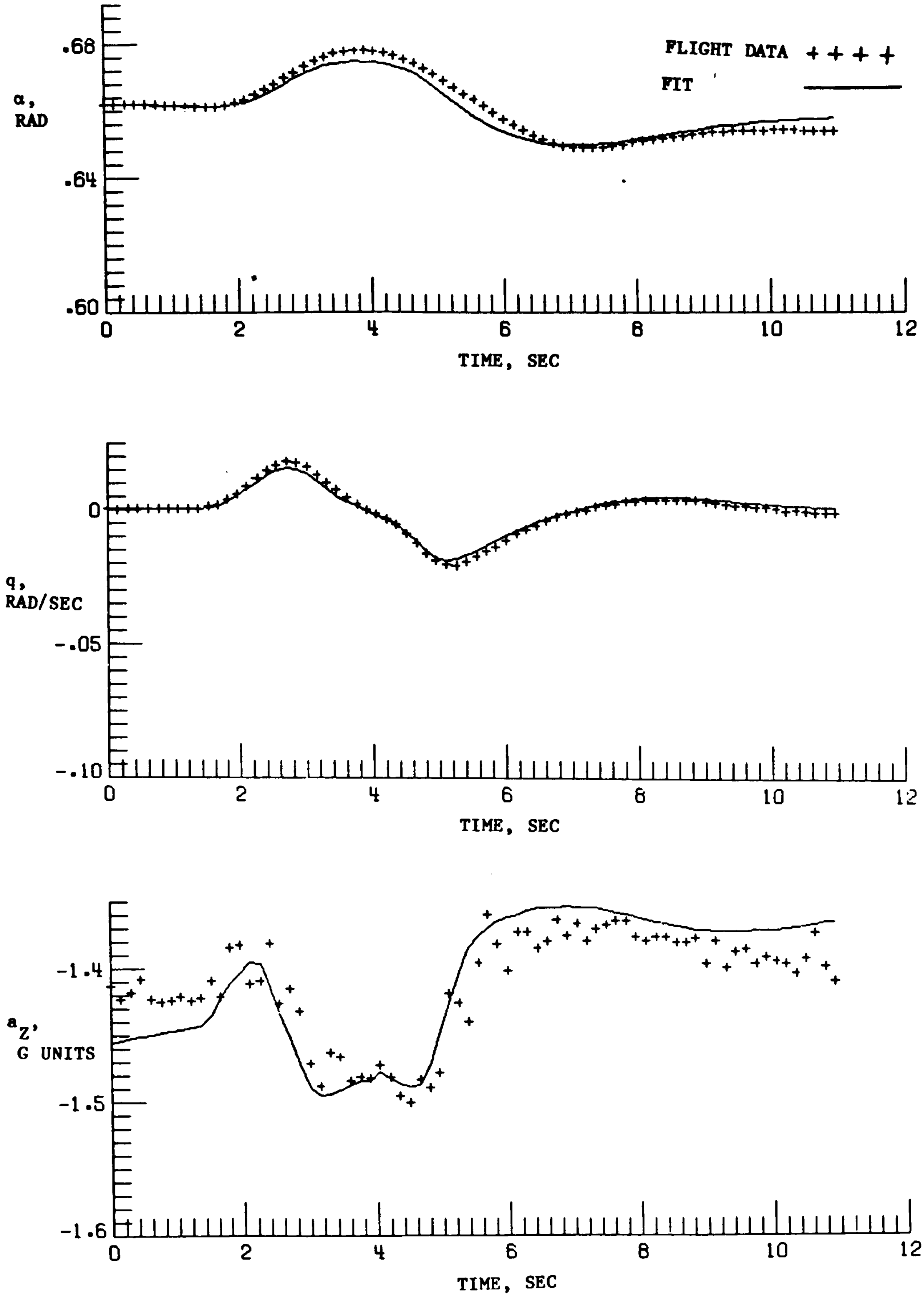



Figure 7. - Prediction of vehicle motion using non-linear model (including $C_{m_{\delta e}} S_e + C_{m_{\delta e^2}} S_e^2$ Term).

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16. Abstract

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