Modes of Vibration on Square Fiberglass Epoxy Composite Thick Plate

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INTRODUCTION

The problem of vibration of thin plates has long been a subject of interest and study, primarily for isotropic and homogeneous materials. The documented experimental study of the vibration of plates began with Chladni in 1787. By the simple procedure of sprinkling fine sand on the surface of a vibrating plate, the positions of the nodal lines were determined. When the plate was set into vibration, the sand grains gradually migrated to the regions of minimum motion, revealing the nodal patterns.

More recently Waller [1] investigated the vibration of free circular and square isotropic plates. She presented a detailed classification of normal and compounded modes, measured frequencies and photographically recorded many nodal patterns. A normal mode is the usual designation denoting a pure resonant mode of the structure. A compounded mode is the combination of two or more normal modes of equal or nearly equal frequency, occurring simultaneously due to effects of damping in the material or nonuniformities of the structure. In the study of nodal patterns, Grinsted [2] used the principle of compounded mode classification, first explored by Waller, to analyze blade structures in impellers and turbines. He demonstrated that for geometrically nonsymmetric plates irregular mode patterns can be resolved as emanating from consistent series of simpler modes, the frequencies of which can be plotted as families of curves. The construction of the nodal patterns is accomplished via a graphical procedure based on the number of nodal lines in "normal" modes. (Grinsted [2] used

the phrase "normal modes" to denote resonant modes in which the nodal lines were parallel to the edges of the plate. Per the definition above, this is a nonstandard use of the phrase "normal modes".) The composition was especially valuable for geometrically nonsymmetric shapes.

The results obtained by Grinsted were confirmed theoretically by Warburton [4] who derived a simple approximate frequency expression for the modes of vibration of rectangular plates having various boundary conditions. Mode shapes similar to those in beams were assumed. The resulting frequency equation was dependent on the plate nodal patterns, dimensions, material and boundary conditions.

Hearmon [5] conducted vibration analyses for anisotropic plates, deriving expressions for the frequency of rectangular laminated plates. Theoretical and experimental results were compared for wood and plywood plates.

With the development of finite element techniques, a new approach was introduced for the determination of the vibrational characteristics of isotropic and anisotropic plates. Studies of correlations between classical theoretical methods, experimentation and finite element methods can be found in the studies of Thornton and Clary [6,7] for thin plates and by Bert [8] for thick composite plates.

The purpose of this work is to investigate the natural modes of transverse vibration of a thick anisotropic square plate by the experimental measurement of its nodal patterns. For the vibrational analysis of thick anisotropic plates, experimental and finite element methods seem to be the only currently viable approaches. For this study an experimental method [9] is used. The specimen is excited

at its resonant frequencies and the nodal lines are measured. Note that a specific specimen is tested, thus this is not a general treatment on the subject of anisotropic plates. Nevertheless, these results are expected to be very useful in subsequent acoustic emission and ultrasonic non-destructive evaluation studies.

TEST SPECIMEN, EQUIPMENT AND EXPERIMENTAL PROCEDURES

Test Specimen

The test specimen was a square plate with 27.94 cm (11 in) sides and 2.54 cm (1 in) thickness of Scotchply 1002 unidirectional fiber-glass epoxy composite. The plate was set on a 2.54 cm (1 in) thick, 30 x 30 cm (11.81 x 11.81 in) square soft (estimated stiffness of 15 N/cm) rubber base for support and insulation.

Equipment

A schematic of the experimental system is shown in Fig. 1. The equipment for the measurement of the nodal patterns and the output voltage amplitude of the receiving transducer included a signal generator (Tektronics Model FG 501); an oscilloscope (Tektronics Type 502 A dual-beam); a digital frequency counter (Hewlett Packard Model 5381 A 80 MHz); a source transducer (Acoustic Emission Technology Model FC-500 longitudinal type); a receiving transducer (Panametrics Model V109 longitudinal type); and a transducer-specimen interface couplant (AET SC-6 for longitudinal waves).

Procedures for Measurement of Nodal Patterns

The nodal patterns were measured by the following procedure. For spatial reference, a grid of 2.54 cm (1 in) squares was drawn on the top of the plate starting from the center. The top of the plate was then covered with a thin layer of couplant (estimated thickness of 0.0005 cm). The source transducer was positioned at the center of the grid and pressed against the plate for better adherence. To avoid

damage of the surface, no adhesive was used. The receiving transducer was placed as close as possible to the transmitting transducer without touching it. The signal generator was set to produce an 80 V peak-to-peak sinusoidal signal to the source transducer. The signal generator was tuned initially at 1 kHz and the frequency was slowly increased, as the oscilloscope was observed. For each maximum amplitude at the fixed receiving point, the frequency was recorded. The frequencies so obtained corresponded to the resonant frequencies of the plate and are listed in Table 1. The range of frequencies obtained was limited by the attenuation of the specimen material and the diameter of the receiving transducer.

Beginning from the lowest observable resonant frequency, the generator was again tuned and the output of the receiving transducer displayed on the oscilloscope was adjusted such that a peak of the sinusoid was centered on the vertical centerline of the screen. For this position, a positive phase was arbitrarily assigned. The receiving transducer was then slowly moved across the surface and the variations on the oscilloscope observed. If a trough of the sinusoid shifted to the center of the screen, a change of phase was marked on a sketch of the grid. Each phase change, from positive to negative or vice versa, indicated that a nodal line was crossed. The complete scanning of the surface indicated all points of phase change. These points were connected by smooth curves, resulting in a nodal lines sketch.

The process was repeated for all noticeable resonant frequencies until the distance between the nodal lines was comparable to the diameter of the receiving transducer face. At that point, the position

of the changes of phase were so close that individual nodal lines could be missed. By this limitation, the nodal patterns could be determined only for frequencies up to 21.73 kHz. These patterns are shown on Figs. 2 through 13.

The resonant measurements were repeated for other supporting conditions of the plate. The original base was replaced by four supports of cubic shape (1.27 cm sides) at the corners. The supports were cut from the same material used for the original base. These resonant frequencies and the nodal patterns were almost the same as those found before. Examples are shown in Figs. 14, 15 and 16 where the corresponding earlier resonant modes are shown in Figs. 2, 3 and 4, respectively. Thus, it appears that the supports used had very little influence on the resonant behavior of the plate, indicating that the plate can be considered as a freely suspended body. This would not be true if the material of the base possessed a stiffness comparable to the plate itself.

RESULTS

The nodal patterns obtained at the frequencies listed in Table 1 are shown in Figs. 2 through 13, in order of increasing frequency. The heavy lines on the plate top face represent the nodal lines. Note that all nodal patterns are symmetrical, at least about one axis. Also, in general, the number of nodal lines increases with increasing frequency, resulting in a smaller nodal line spacing.

The first nodal pattern obtained (see Fig. 2), corresponding to the first observable resonant frequency, shows four lines running more or less parallel to the fiber direction. As the smallest elastic modulus occurs in the direction perpendicular to the fibers, it is expected that the plate will bend in that direction at lower frequencies, compared with the fiber direction. This is confirmed by Hearmon's frequency expression [5].

By the mode classification for square plates discussed in [1,3], lines running parallel to the plate edges indicate the plate vibrates approximately as a beam. The more these lines approach straight lines, the closer the modal behavior is to the behavior of the beam. This type of pattern can be observed in Figs. 2 and 4.

CONCLUSIONS

The nodal patterns are an important auxiliary set of information in the design of ultrasonic and acoustic emission experiments. If such NDE tests are to be performed, the resonant frequencies and the corresponding nodal patterns of the specimens should be well characterized within the frequency range to be the studied. After the resonances and mode shapes have been determined, a decision should be made as to whether the resonant frequencies should be avoided or sought during a test. If the frequency content of the generated signal is known to include resonant tones, the nodal pattern indicates the positions of vanishing displacement amplitudes corresponding to these resonant tones.

The fiberglass epoxy tested here required a high peak-to-peak input voltage for the determination of nodal paterns because the attenuation of the material is high. Similar tests on an aluminum block [9] of comparable dimensions required much less input voltage for an adequate output level at the receiving transducer (about one-fifth of the voltage required for these tests).

The presence of similar mode shapes with lines approximately parallel along the principal directions suggests that some composition construction can be done using a technique similar to Grinsted's graphical construction for isotropic materials. For the anisotropic material, however, the differences in the elastic properties along the principal axes introduce, as yet undetermined, complexity to this process.

REFERENCES

[1] M.D. Waller, "Vibrations of Free Square Plates: Part I Normal Vibrating Modes", Proceedings of the Physical Society of London, Vol. 51, Jun. 1939, pp. 831-844.

[2] M.D. Waller, "Vibrations of Free Square Plates: Part II, Compounded Normal Modes", Proceedings of the Physical Society of London, Vol. 52, Feb. 1940, pp. 452-455.

[3] B. Grinsted, "Nodal Pattern Analysis", Proceedings of the Institution of Mechanical Engineers, Vol. 166, Jan-Dec 1952, pp. 309-326.

[4] G.B. Warburton, "The Vibration of Rectangular Plates", Proceedings of the Institution of Mechanical Engineers, Vol. 168, No. 12, Jan-Dec 1954, pp. 371-384.

[5] R.F.S. Hearmon, "The Fundamental Frequency of Vibration of Rectangular Wood and Plywood Plates", Proceedings of the Physical Society of London, Vol. 58, Jan. 1946, pp. 78-92.

[6] E.A. Thornton and R.R. Clary, "A Correlation Study of Finite Element Modeling for Vibration of Composite Material Panels", Composite Materials: Testing and Design (Third Conference), American Society for Testing and Materials Special Technical Publication 546, March 1973, pp. 111-129.

[7] R.R. Clary, "Vibration Characteristics of Unidirectional Filamentary Composite Materials Panels", Composite Materials: Testing and Design (Second Conference), American Society for Testing and Materials Special Publication 497, 1971, pp. 415-438.

[8] C.W. Bert, J.N. Reddy, W.C. Chao and V.S. Reddy, "Vibration of Thick Rectangular Plates of Bimodulus Composite Material", Journal of Applied Mechanics, Vol. 48, No. 7, Jun. 1981, pp. 371-376.

[9] J.H. Williams, Jr., E.B. Kahn and S.S. Lee, "Effects of Specimen Resonances on Acoustic-Ultrasonic Testing", Materials Evaluation, Vol. 41, No. 13, Dec. 1983, pp. 1502-1510.

TABLE 1   Experimental resonant frequencies for unidirectional
          fiberglass epoxy plate.

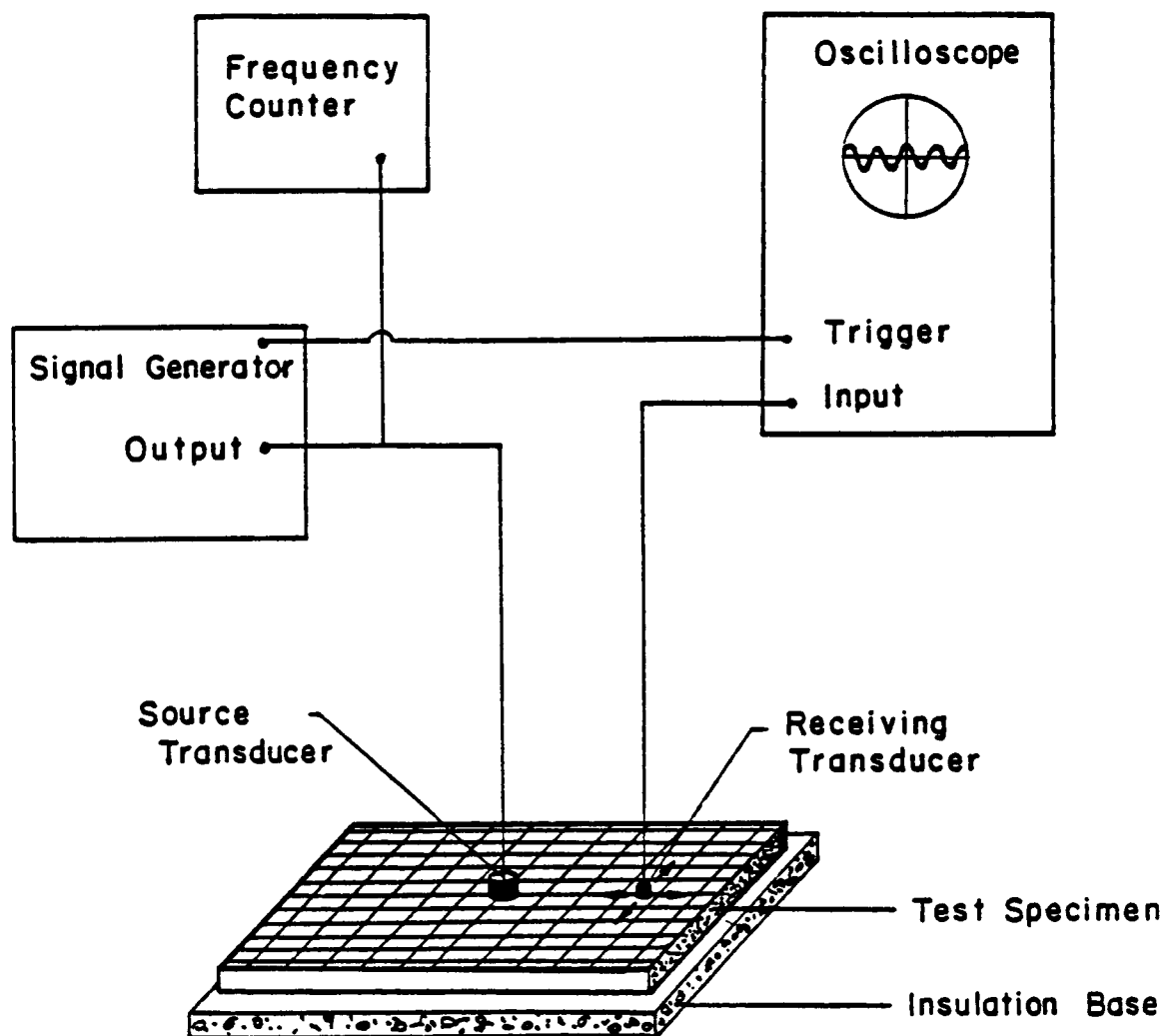| Resonant Frequencies (kHz) |
| --- |
| 3.585 |
| 4.676 |
| 5.745 |
| 6.640 |
| 7.259 |
| 8.426 |
| 9.610 |
| 11.999 |
| 13.820 |
| 16.510 |
| 18.520 |
| 21.730 |

Fig. 1   Schematic of experimental system for measurement of
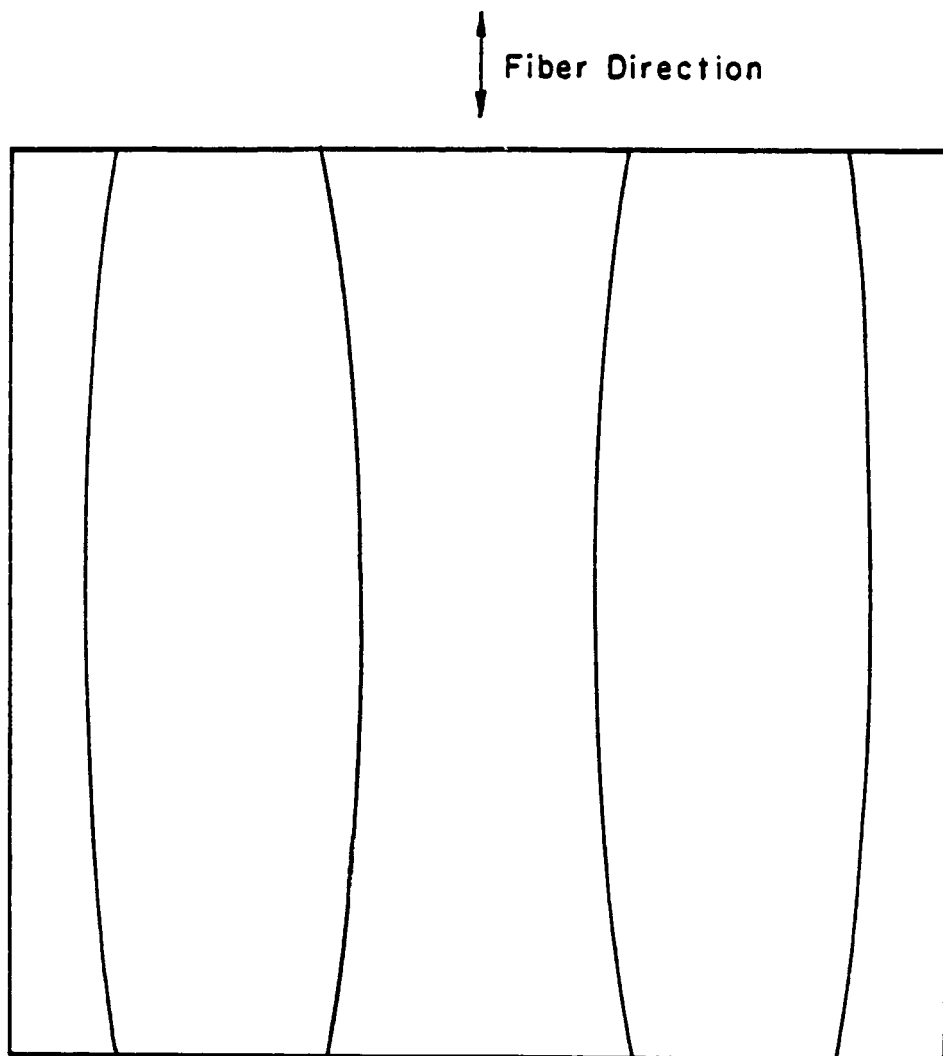   nodal patterns.

Fig. 2   Nodal pattern measured at resonant frequency of
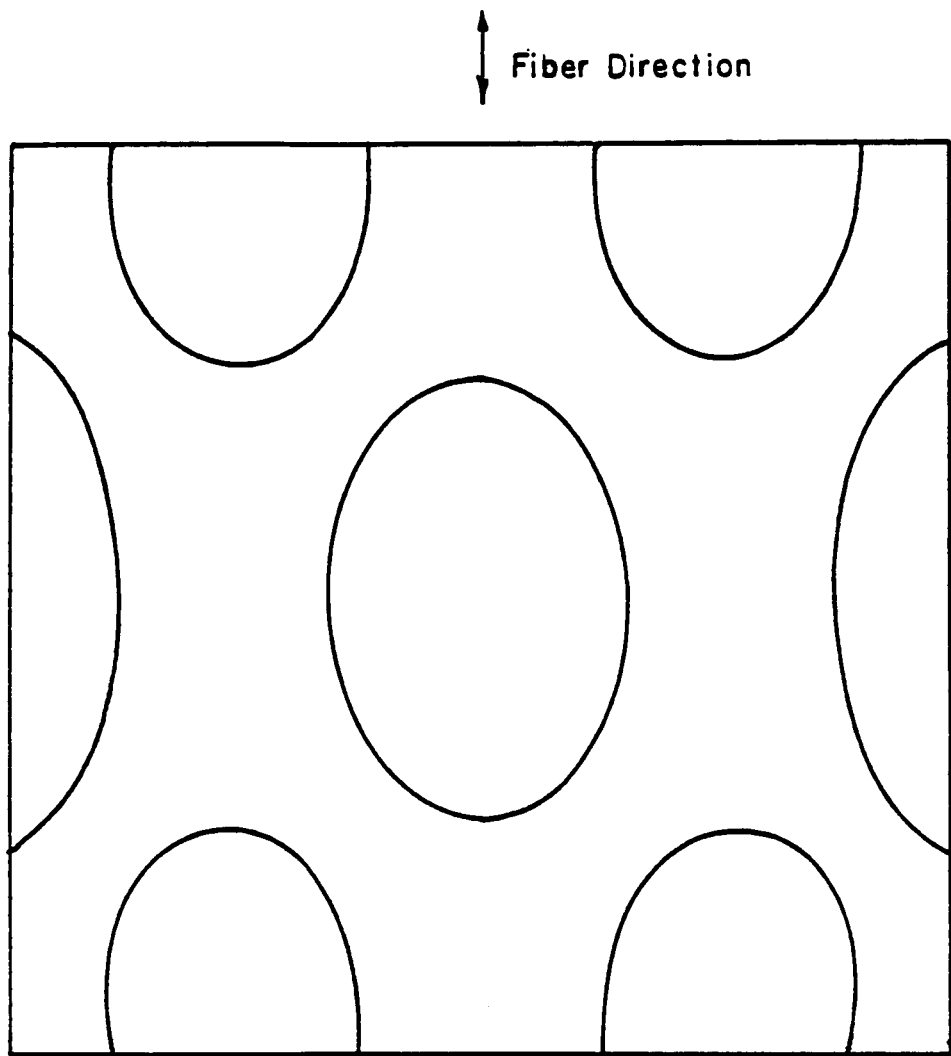   3.585 kHz (top view).
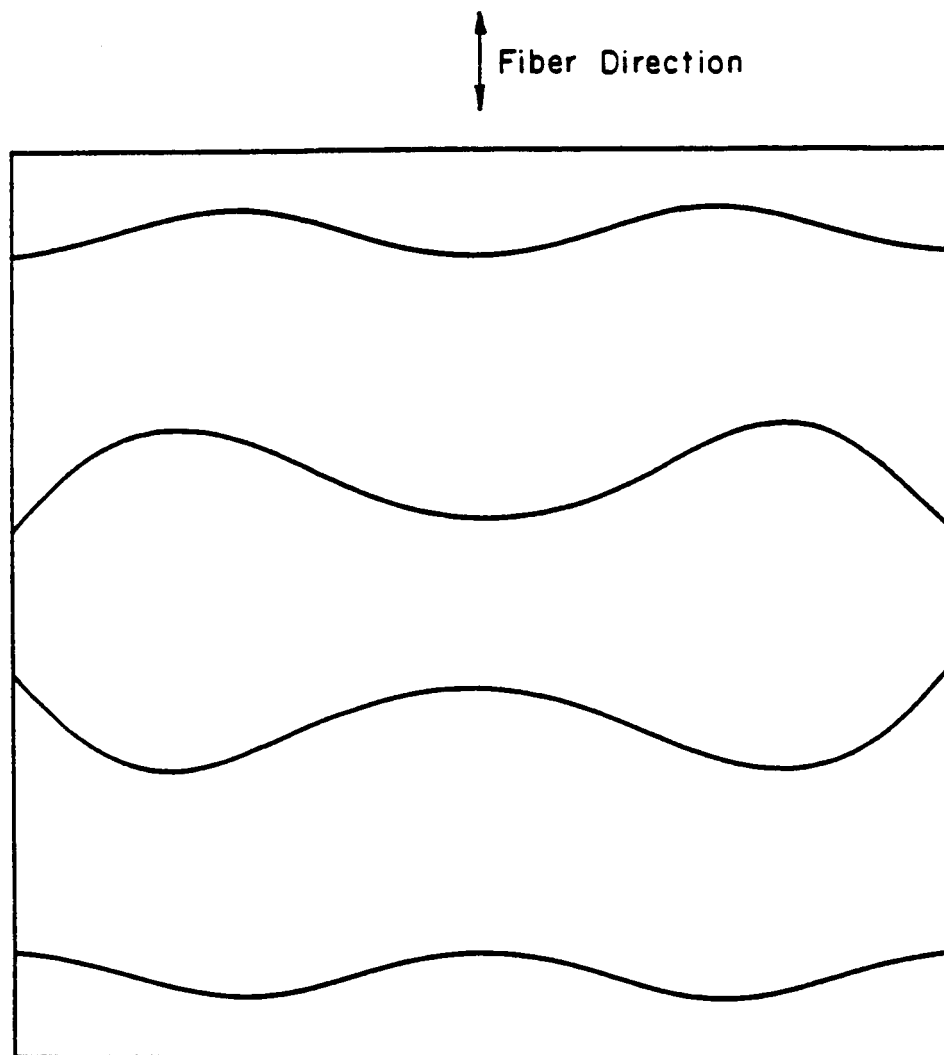
Fig. 3   Nodal pattern measurement at resonant frequency of
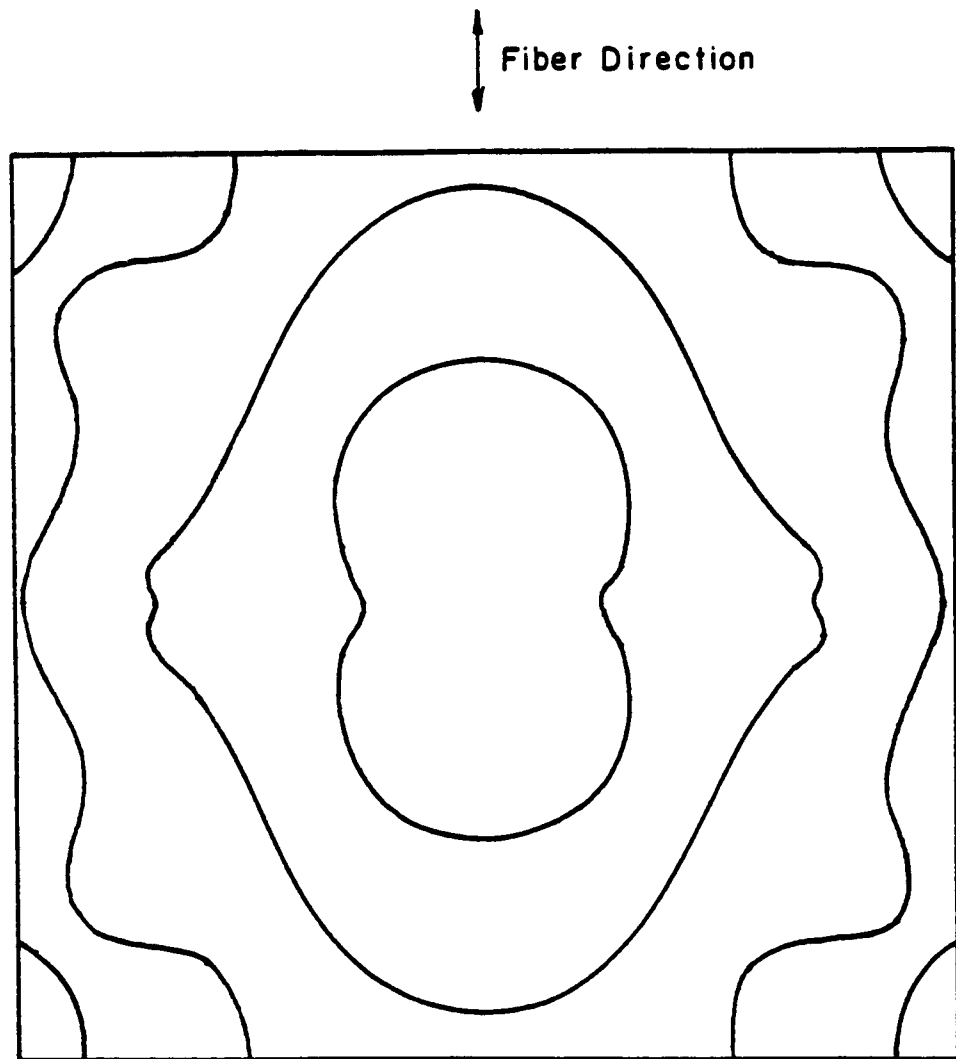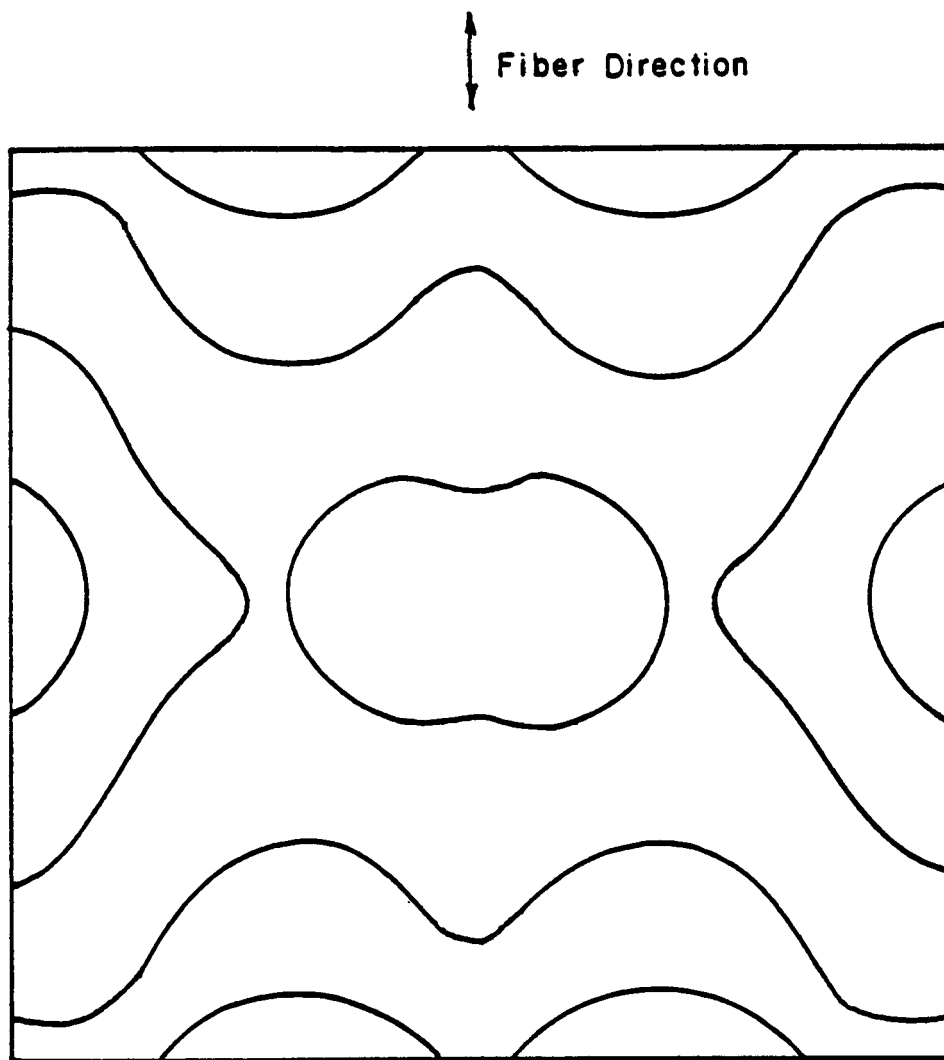4.767 kHz (top view).

Fiber Direction

Fig. 4   Nodal pattern measured at resonant frequency of
          5.745 kHz (top view).

Fiber Direction
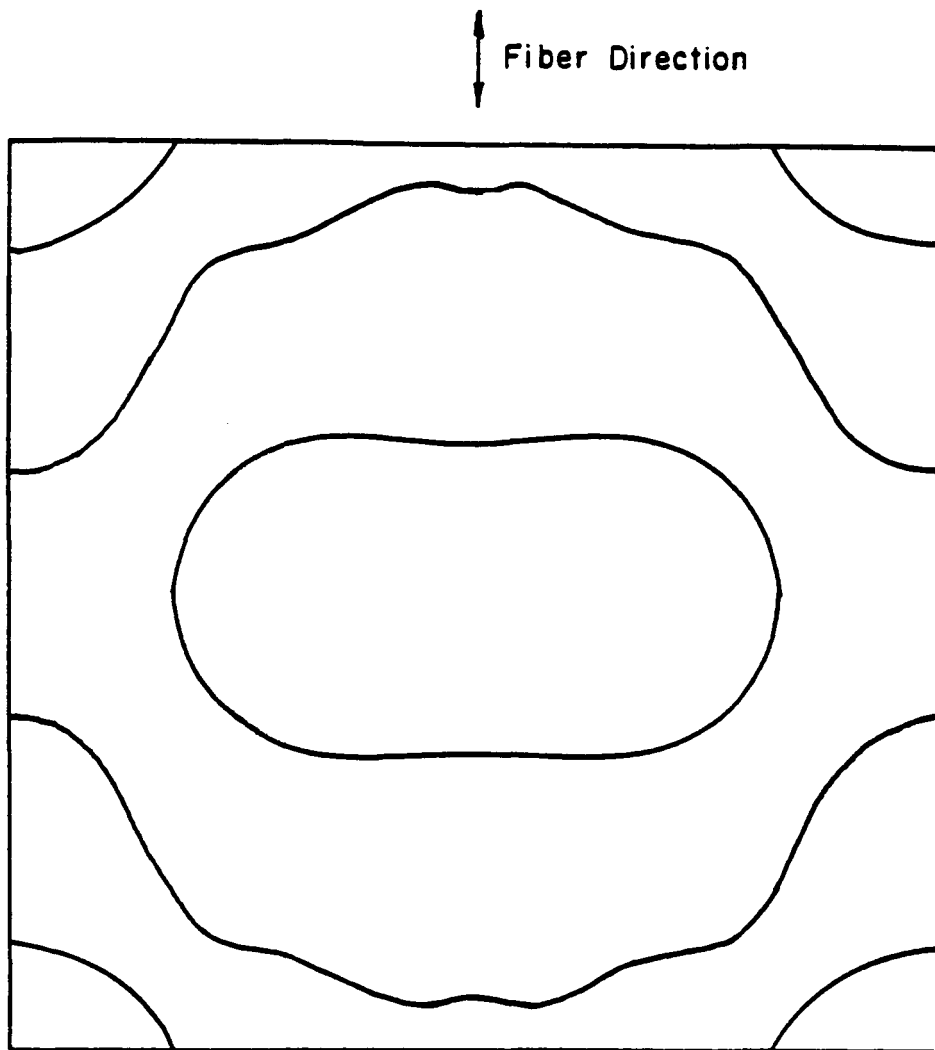
Fig. 5   Nodal pattern measured at resonant frequency of
6.640 kHz (top view).
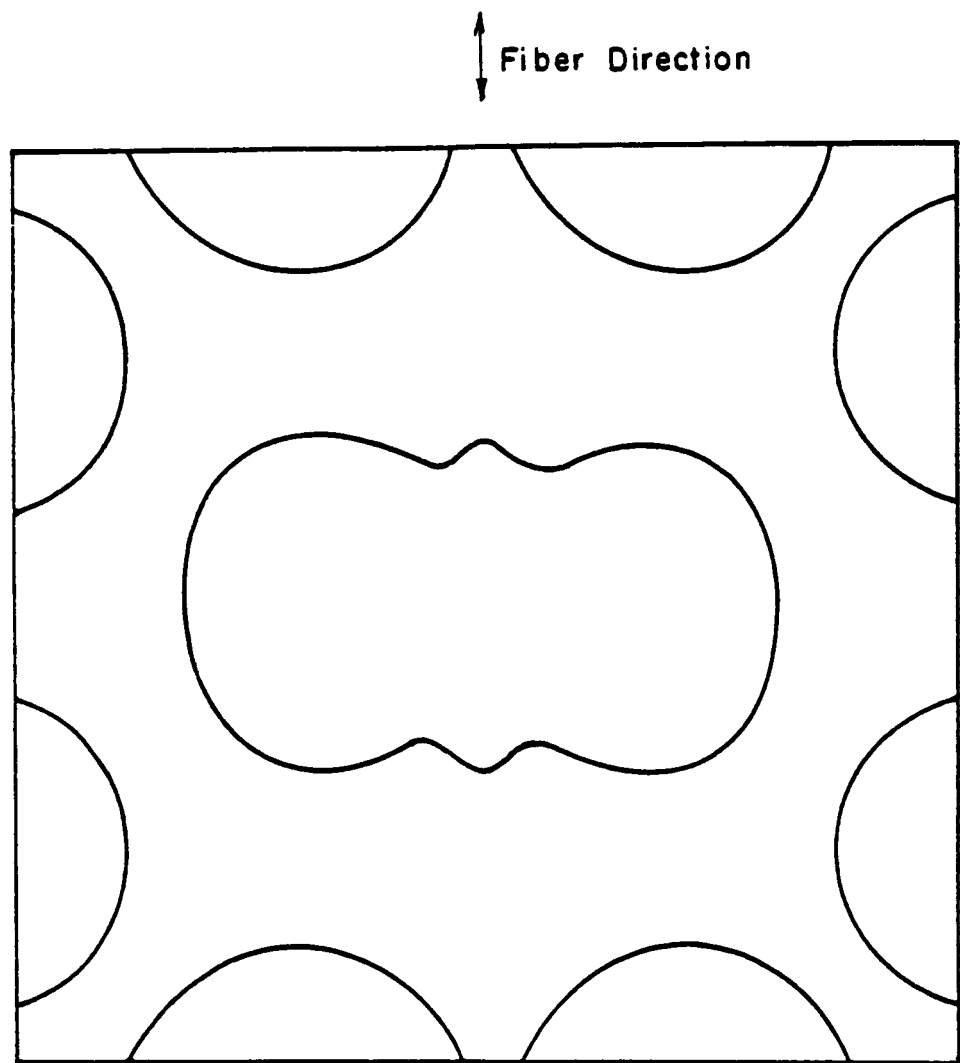
Fig. 6   Nodal pattern measured at resonant frequency of
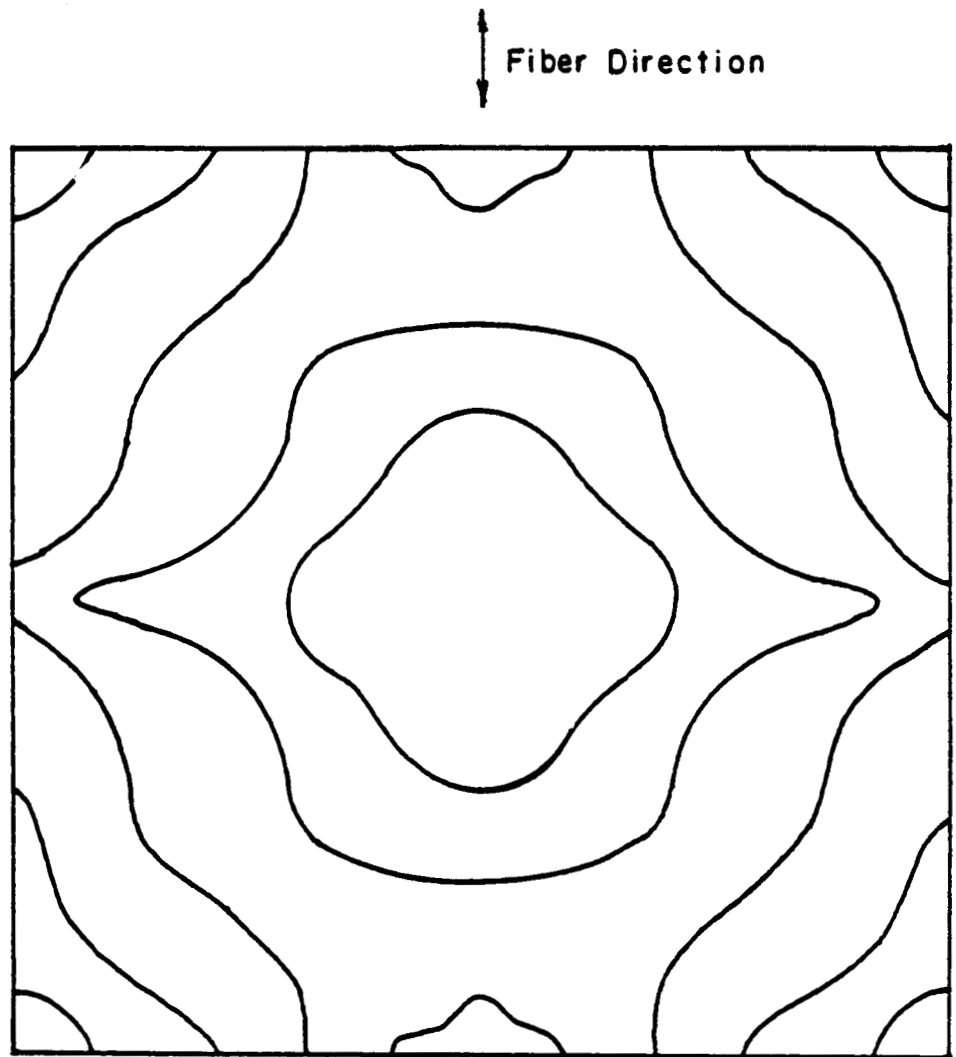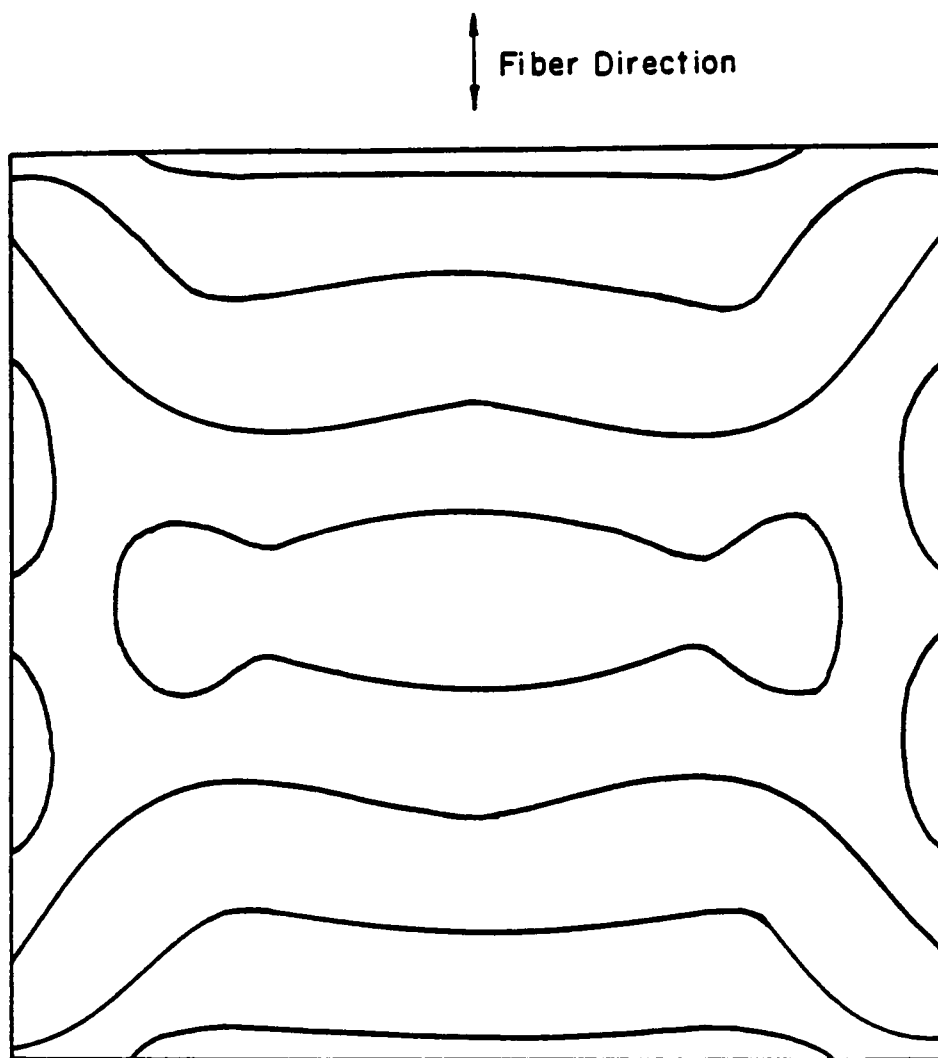   7.258 kHz (top view).
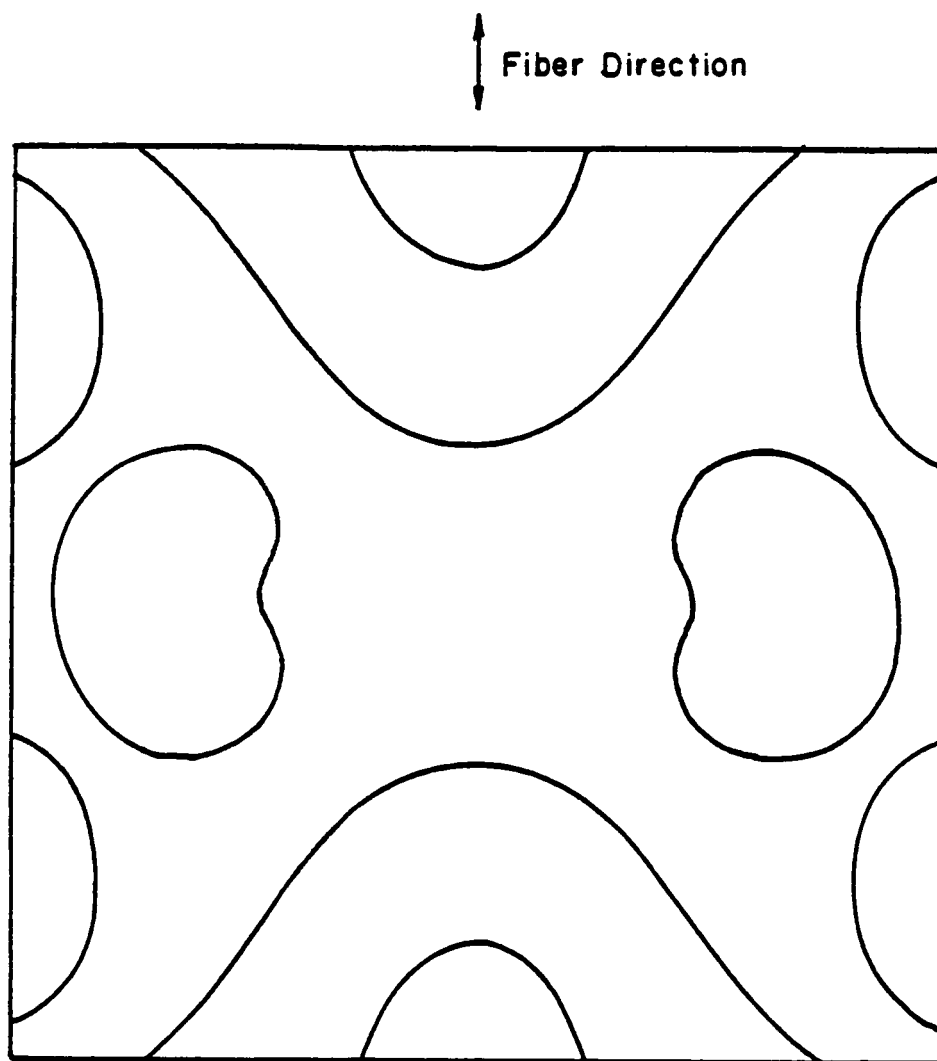
Fig. 7   Nodal pattern measured at resonant frequency of 8.426 kHz
          (top view).

Fiber Direction

Fig. 8   Nodal pattern measured at resonant frequency of 9.610 kHz
(top view).

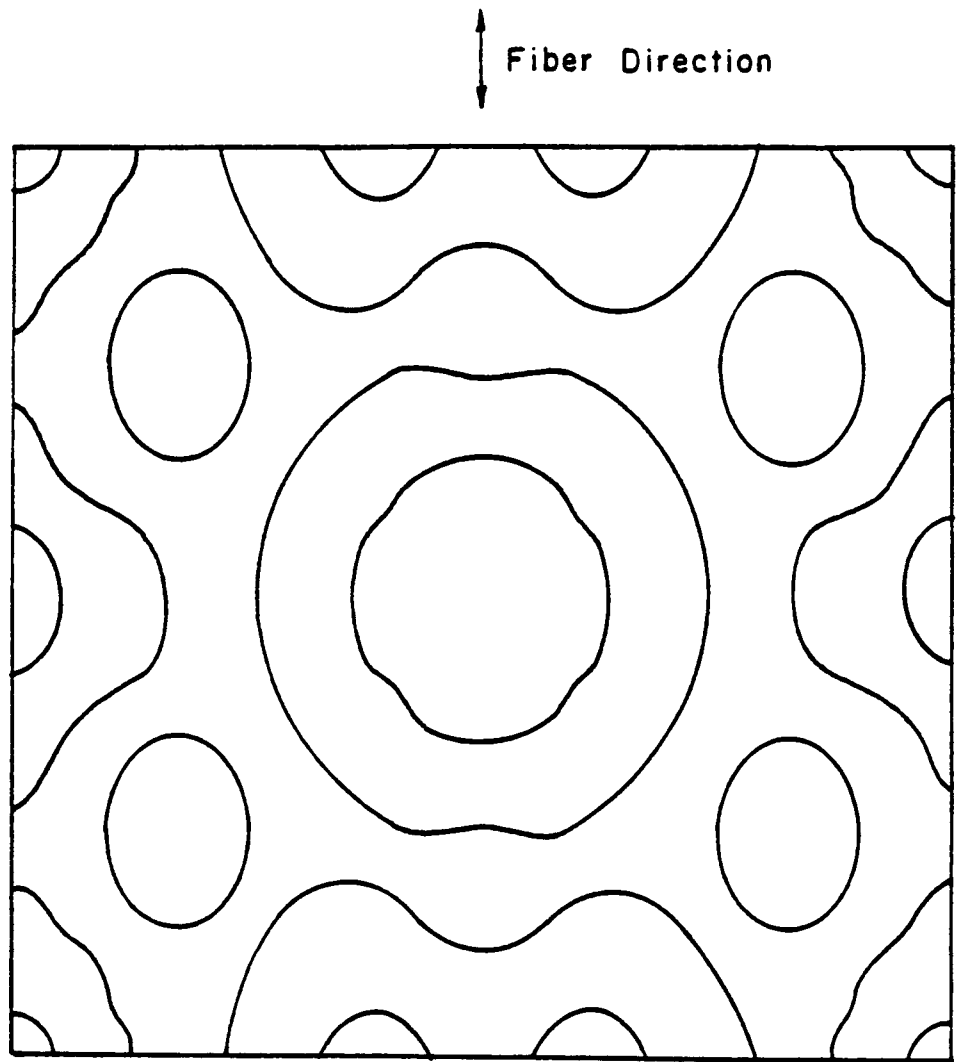Fig. 9   Nodal pattern measured at resonant frequency of 11.999 kHz
          (top view).

Fiber Direction

Fig. 10   Nodal pattern measured at resonant frequency of 13.820 kHz
           (top view).

Fiber Direction

Fig. 11   Nodal pattern measured at resonant frequency of 16.510 kHz
           (top view).

Fiber Direction

Fig. 12   Nodal pattern measured at resonant frequency of 18.520 kHz
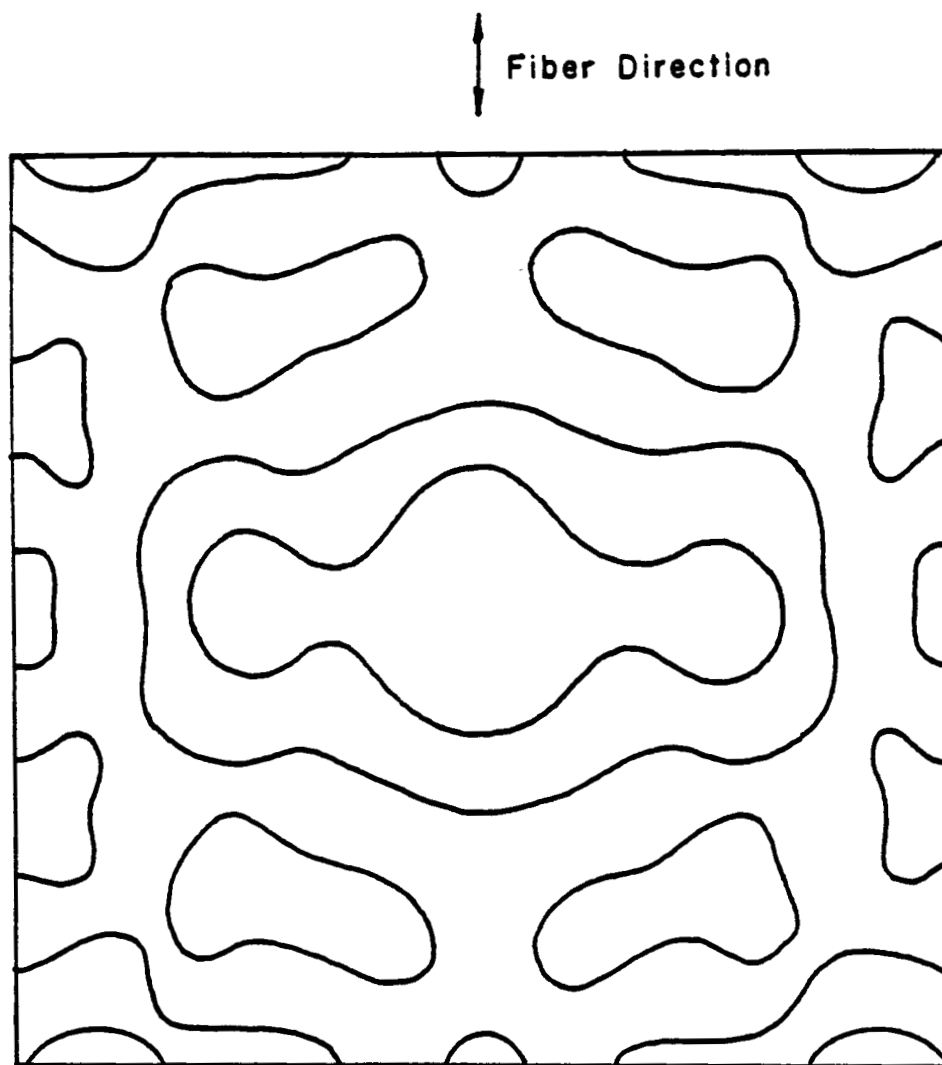   (top view).

Fiber Direction

Fig. 13   Nodal pattern measured at resonant frequency of 21.73 kHz
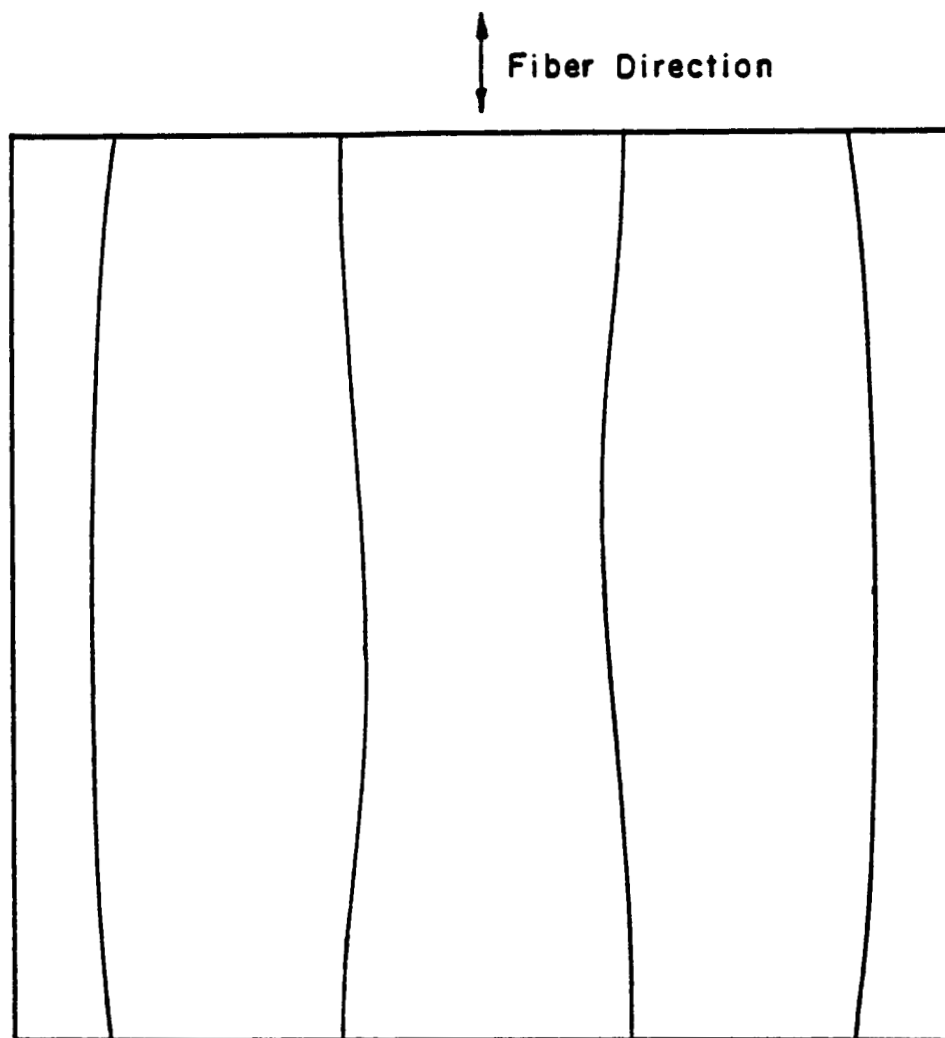   (top view).

Fig. 14   Nodal pattern measured at resonant frequency of 3.550 kHz
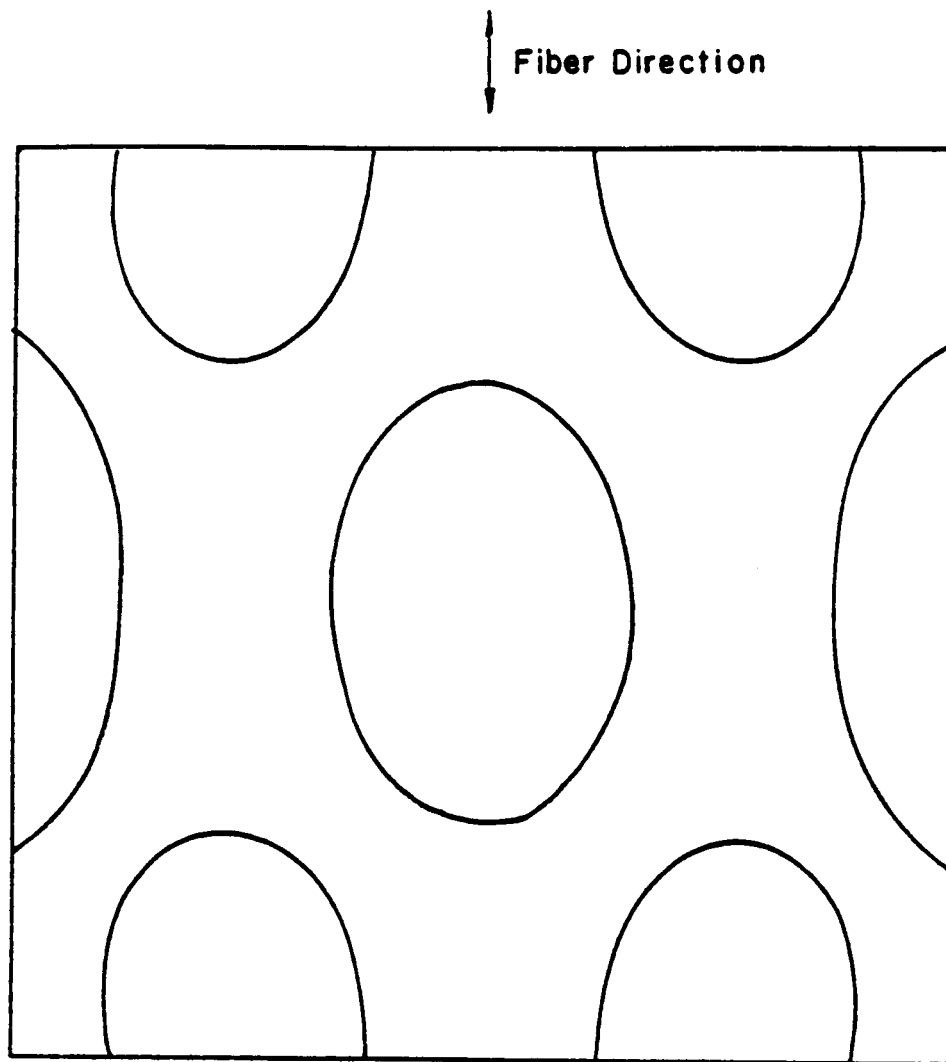for support at four corners (top view).

Fig. 15   Nodal pattern measured at resonant frequency of 4.721 kHz
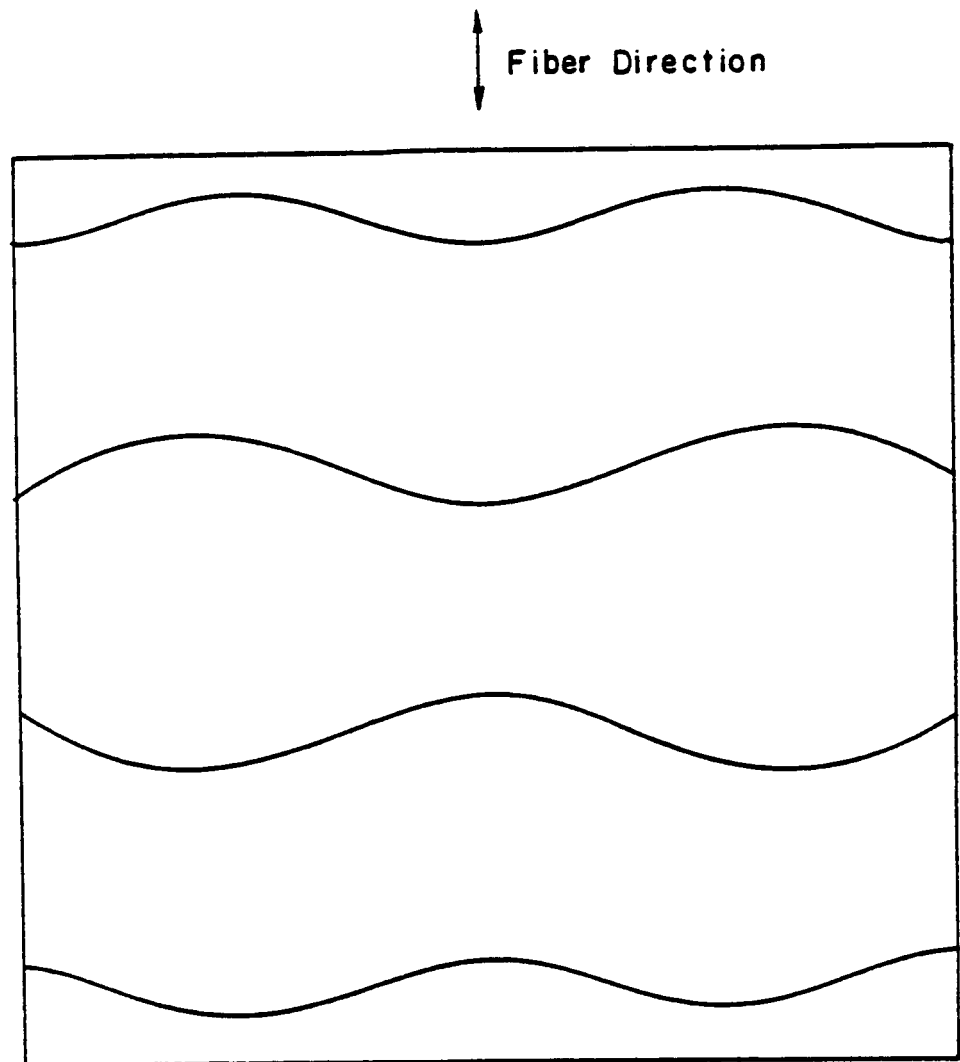for support at four corners (top view).

Fig. 16   Nodal pattern measured at resonant frequency of 5.766 kHz
           for support at four corners (top view).

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15. Supplementary Notes

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16. Abstract

The frequencies and nodal patterns of a square thick plate of unidirectional fiberglass epoxy composite are measured experimentally. The constituent material is transversely isotropic. The plate is transversely excited at the center of the upper face, its resonant frequencies in the frequency range of 3 kHz to 21.73 kHz are detected and the measured nodal patterns are sketched.

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