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Q3DG - A COMPUTER PROGRAM FOR STRAIN-ENERGY-RELEASE RATES FOR DELAMINATION GROWTH IN COMPOSITE LAMINATES

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Q3DG - A COMPUTER PROGRAM FOR STRAIN-ENERGY-RELEASE RATES FOR DELAMINATION GROWTH IN COMPOSITE LAMINATES

FORTRAN V APPROXIMATELY 4000 SOURCE STATEMENTS
9 TRACK 1600 BPI CDC NOS INTERNAL FORMAT MAGNETIC TAPE

ABSTRACT

The Q3DG is a computer program developed to perform a quasi-three-dimensional stress analysis for composite laminates which may contain delaminations. The laminates may be subjected to mechanical, thermal and hygroscopic loads. The program uses the finite element method and models the laminates with eight-noded parabolic isoparametric elements. The program computes the strain-energy-release components and the total strain-energy release in all three modes for delamination growth. A rectangular mesh and data file generator, DATGEN, is included. The DATGEN program can be executed interactively and is user friendly. The documentation includes sections dealing with the Q3D analysis theory, derivation of element stiffness matrices and consistent load vectors for the parabolic element. Several sample problems with the input for Q3DG and output from the program are included. The capabilities of the DATGEN program are illustrated with examples of interactive sessions. A microfiche containing all the examples presented in this report is included with the documentation. The Q3DG and DATGEN program have been implemented on CYBER 170 class computers. Q3DG and DATGEN were developed at the Langley Research Center during the early eighties and documented in 1984-1985.

INTRODUCTION

Because of their high strength-to-weight ratios, composite materials are being extensively used in aerospace, mechanical, and automotive structures. As these materials are being considered for heavily loaded primary aircraft structures, increased attention is being focused on understanding and characterizing composite delamination. Delaminations may result from low-velocity impact, eccentricities in load paths that induce out-of-plane loads, or discontinuities in the structure which create out-of-plane stresses.

Fracture mechanics concepts applied to delaminated structural components were found to be useful in understanding and characterizing delaminations. Strain-energy-release rates associated with delamination growth were shown to be useful in predicting the onset of edge delaminations in composites [1, 2]. While the total strain-energy-release rates can be calculated using the classical laminated plate theory (CLT), the separation of the total strain-energy-release rates into the three components associated with the three modes of fracture cannot be accomplished using the CLT approach. To calculate the individual components, one has to resort to numerical methods such as finite element analyses.

The purpose of this report, therefore, is to document the computer program, Q3DG, which uses a finite element analysis to calculate the strain-energy-release rates for long, rectangular composite laminates containing delaminations. The laminates may be subjected to mechanical, thermal and hygroscopic loadings. The Q3DG program uses the quasi-three-dimensional (Q3D) analysis in conjunction with linear fracture mechanics concepts and finite element methods. The program calculates G_I, G_{II}, and G_{III}, the strain-energy-release rate components due to the three fracture modes, and the total strain-energy-release rate G for delamination growth in composite laminates. The program can also be used as a Q3D stress analysis tool for laminates with or without delaminations.

First, the theoretical aspects of the Q3D analysis are presented. Next, the finite element analysis and the strain-energy-release rates are discussed. Then, the program specifications and organization are presented. Finally, the input data for and output of the program are described. Appendix A presents the derivation of the stiffness matrix and thermal vectors for an eight-noded isoparametric element. Appendix B lists the names and functions of subroutines and major program variables and the common blocks and their elements. In Appendix C, three example problems with their output are presented. Appendix D illustrates some conditions which cause the program to terminate and discusses some debug strategies. Next, Appendix E describes DATGEN, an automatic rectangular mesh generator, and shows several example problems using the mesh generator. Appendix F presents the COSMIC documentation requirements. Finally, Appendix G describes the floppy disks for the IBM PC and compatibles. These disks can be easily reproduced and used to upload the source and data files to the mainframe computers.

DESCRIPTION OF THE DELAMINATION PROBLEM

Consider a long, rectangular laminate with an edge delamination as shown in fig. 1. Away from the ends where the loads are applied, the displacements at any x = constant plane are assumed to be given by (refs. 4-8)

$$\begin{aligned} u(x, y, z) &= \varepsilon_o x + U(y, z) \\ v(x, y, z) &= V(y, z) \\ w(x, y, z) &= W(y, z) \end{aligned} \tag{1}$$

The ε_o term is the uniform axial strain and U, V, and W are functions of y and z alone. Equation (1) describes a "quasi-three-dimensional" (Q3D) problem. The modifier "quasi" is used because there are displacements in three directions, but the gradients of U, V, and W with respect to the x-coordinate are zero.

Because of the symmetries inherent in the layups of the fiber reinforced composites, the displacement functions U, V, and W satisfy the following requirements [4-7].

$$
\begin{aligned}
U(y, z) &= -U(-y, -z) \\
V(y, z) &= -V(-y, \ z) \\
W(y, z) &= -W(\ y, -z)
\end{aligned} \tag{2}
$$

For mechanical loading, the uniform axial strain ε_0 in equation (1) is specified. For thermal and hygroscopic loads the laminate is subjected to temperature and moisture changes, ΔT and ΔH, respectively. For these cases the magnitude of ε_0 in equation (1) is unknown and is determined as a part of the solution (see ref. 9, 10).

In the Q3D analyses, each ply is idealized as a homogenous, elastic orthotropic material. The material properties of each ply are defined by Young's modulii (E_{11}, E_{22}, E_{33}), shear modulii (G_{12}, G_{13}, G_{23}), Poisson's ratios (ν_{12}, ν_{13}, ν_{23}), and the ply fiber angle (θ). In addition, the expansion coefficients for thermal and hygroscopic loadings, α_1, α_2, and α_3 and β_1, β_2, and β_3, respectively, are needed. The subscripts, 1, 2, and 3 on the various quantities listed above correspond to the longitudinal, transverse, and thickness directions, respectively, of a zero-degree ply.

Finite Element Analysis

Because of the symmetries in the problem, only one quadrant of the x = constant plane is idealized. This quadrant ($0 < y < b$ and $0 < z < t$) is shown as the shaded region in fig. 1(b). The analysis region is modeled by eight-noded, isoparametric elements [11]. At the delamination tip, no singularity elements are used. Instead, the finite element idealization is usually made much finer near the delamination tip. A typical finite element idealization involving 367 nodes and 102 elements is shown in fig. 2. Due to symmetry the displacement functions, U and V, are prescribed zero at all nodes on the y = 0 line, and the displacement function W is prescribed zero on the z = 0 line.

The derivation of stiffness matrices and the mechanical, thermal and hygroscopic load vectors needed for the finite element analysis is described in Appendix A.

Strain-Energy-Release Rates

Mode I (opening mode), mode II (sliding mode), mode III (tearing mode) components and the total strain-energy-release rates for delamination growth are calculated by a virtual crack extension technique similar to that reported in ref. 12. This technique uses the nodal forces transmitted at and near the delamination tip and the relative displacements just behind the delamination tip to determine the strain-energy-release rates. The nodes at which the forces and relative displacements are evaluated are shown in fig. 3. The strain-energy-release rates are computed as shown below:

$$
\begin{aligned}
G_I &= \left[F_{z_i}(W_l - W_k) + F_{z_j}(W_n - W_m)\right]/(2\Delta) \\
G_{II} &= \left[F_{y_i}(V_l - V_k) + F_{y_j}(V_n - V_m)\right]/(2\Delta) \\
G_{III} &= \left[F_{x_i}(V_l - V_k) + F_{x_j}(U_n - U_m)\right]/(2\Delta) \qquad (3) \\
G &= G_I + G_{II} + G_{III}
\end{aligned}
$$

The forces F_{x_i}, F_{y_i}, and F_{z_i}, are the forces in x-, y-, and z-directions, respectively, acting at node i. The forces at node i, shown in fig. 3, are computed from the elements A and B while the forces at node j are computed from element A alone. This procedure for computing the strain-energy-release rates requires that the finite element mesh be symmetric about the crack tip. Square elements with a side of length $\Delta = h/4$ were found to be the optimum size and, hence, an arrangement of the type shown in fig. 3 is recommended.

The program first computes the forces and displacements individually for a mechanical strain of one micro in/in (m/m), a temperature change of +1°F (°K), and a moisture change of one percent. Then, the program computes the appropriate forces and displacements for the given input values of mechanical strain and temperature and moisture changes and uses the forces and displacements in equation (3) to calculate the strain-energy-release rates. The mode I, II, III components, and the total strain-energy-release rates are computed for the seven possible loading combinations listed below:

1. Mechanical loading acting alone.
2. Thermal loading acting alone.
3. Hygroscopic loading acting alone.
4. Mechanical and thermal loading acting together.
5. Mechanical and hygroscopic loading acting together.
6. Thermal and hygroscopic loading acting together.
7. Mechanical, thermal, and hygroscopic loading acting together.

PROGRAM SPECIFICATIONS

The program Q3DG is written in FORTRAN V and has a core requirement of about 343,000 octal words. The program was compiled and successfully executed on CDC CYBER 170 class computers. The current maximum allowable values of major arrays are given in Appendix B.

The program uses a scratch unit, Tape 10, for storage of the element stiffness matrices and consistent load vectors when they are first computed. This information is later used to check the element and global equilibrium conditions. A scratch unit is used since the recomputation of element stiff-nesses and load vectors is more time-consuming than reading from the scratch unit.

The program uses an in-core equation solver. Hence the maximum size of the finite element model is limited by the memory of the computer being used. Most of the core is required to hold the global stiffness matrix, BIGK. The array BIGK is dimensioned (75,1110), which permits 1110 degrees of freedom (370 nodes) and a bandwidth of 75.

The Q3DG program can be easily modified using a text editor to change any of the problem dimensions. The required changes are given below:

1) Change the string " 75,1110 " to " xx,yyy " everywhere, where xx and yyy are the new number of rows and columns, respectively, of the array BIGK.

2) Change the string (83250) to "zzz" everywhere, where zzz is the product of xx and yyy.

3) Change the string "(1110,3)" to "(yyy,3)" everywhere, where yyy is the number of columns in the array BIGK.

4) Change the string "X(400, 2), NOD(200, 8)" to "X(aaa, 2), NOD(bbb, 8)" everywhere, where aaa is the maximum number of nodes and bbb is the maximum number of elements.

5) Change the string "(400)" to "(aaa)" everywhere, where aaa is, again, the maximum number of nodes.

6) Change the string "(200)" to "(bbb)" everywhere, where bbb, is, again, the maximum number of elements.

7) Change the DATA LX, . . . card which is the 51st line in the source code. The Integers, LX, LNOD, LLIST, LOLD, are computed as follows:

```
LX    = aaa * 2
LNOD  = bbb * 8
LLIST = 200
LOLD  = aaa
```

8) Change the dimensions of the arrays NDUM, NDUD, and XDUM in subprogram ADJUST to the following:

```
NDUM(bbb,8), NDUD(bbb)
XDUM(aaa,2)
```

and change the equivalence string ", (BIGK (1601), NDUD (1))" to ", (BIGK (bbb*8+1), NDUD (1))".

9) Change the string "D(1110)", the working storage for the band-solver, in subprogram ASEMBL, to "D(yyy)".

10) Change four lines in subprogram FORCES as follows.

```
DIMENSION FORCE (aaa,3,3), SP(aaa, 6,10), ND(aaa,10)
EQUIVALENCE (BIGK(1), FORCE (1,1,1)), (BICK (9*aaa+1), SP(1,1,1)),
       (BIGK (aaa*9+aaa*60+1), ND(1,1))
DIMENSION SPA (aaa,6), NDA (aaa)
EQUIVALENCE (BIGK (aaa*9 + aaa*360 + aaa*10 + 1),
     SPA (1)), (BIGK (aaa*9 + aaa*360 + aaa*10 + aaa*6 + 1), NDA (1)).
```

The program is set up for a 3 × 3 Gaussian integration of the element stiffners matrices. The order of integration can be changed by changing the following:

1. NGAUSS in the BLOCK DATA subprogram (currently it reads

DATA NGAUSS |3|

change 3 to n, where n is the order of integration desired ($2 \leq n \leq 8$).

2. If the order of integration n is chosen to be greater than 3, change the dimensions of array PDER everywhere in the program to (n*n*2,8). Current dimension of array PDER are (18,8). Also note that the array PDER is the COMMON block CDER.

PROGRAM ORGANIZATION

In this section the flow of Q3DG is described. Brief descriptions of the subprograms and major program variables are presented in Appendix B.

Fig. 4 presents the flow chart of the program. Most of the input is read in the main program Q3DG, which calls 5 main subprograms. Subprogram MODULUS generates the modulus matrices and their thermal and hygroscopic strain vectors for all the materials. Subprogram CLD calculates the coordinates of the midside nodes (if these are unspecified), reads boundary conditions and creates the boundary condition array, LIST. Subprogram ADJUST reorders the coordinate array and the nodal connectivity array according to the renumbering scheme input by the user. Subprogram ASEMBL calls the relevant routines to evaluate the stiffness matrices and load vectors and assembles the global stiffness matrix. Subprogram FORCES checks equilibrium at the element and global levels, calculates the nodal stresses and calls the G-calculation routines. The functions of the subprograms called by ASEMBL and FORCES are explained in Appendix B.

INPUT DATA

The required input data is described in this section. The data can be created on a file, LFN, equated to TAPE5. An alternate method of creating the input file is to use the data generation program DATGEN described in Appendix E.

CARD SET	NUMBER OF CARDS	COLS	FORMAT	VARIABLE	DESCRIPTION
1	1	1-80	20A4	TITLE	TITLE of problem
2	1	1- 8	A8	POUT	Output option. SHORT-- for short output. XLONG-- for long output.
3	1	1- 5	I5	NPOIN	Total number of nodes
		6-10	I5	NELEM	Number of Elements
		11-15	I5	NFREE	Number of degrees of Freedom per node = 3
		16-20	I5	NNODE	Number of Nodes per element = 8
4	*	1-10	F10.5	YI	Coordinate
		11-15	I5	JCORD(1)	Node numbers with this
		16-20	I5	JCORD(2)	coordinate in columns 11 through 60.
		.	.	.	
		.	.	.	
		56-60	I5	JCORD(10)	
					2 sets of cards of this format, with each set terminated by a zero (or blanks in cols. 11-15). First set is y-coordinates; second set is z-coordinates.

*Input until all y-coordinates are specified. End y-coordinates with a blank card.
Input until all z-coordinates are specified. End z-coordinates with a blank card.

CARD SET	NUMBER OF CARDS	COLS	FORMAT	VARIABLE	DESCRIPTION
5	NELEM	1- 5	I5	I	Element number
		6-10	I5	NOD(I,1)	Element connectivity
		11-15	I5	NOD(I,1)	starting at any corner node in the counter clockwise direction.
		.	.	.	
		.	.	.	
			I5	NOD(I,NNODE)	
					One card for each element hence, NELEM cards.

CARD SET	NUMBER OF CARDS	COLS	FORMAT	VARIABLE	DESCRIPTION
6	1	1- 5	I5	NMAT	Number of materials
7	4*NMAT	1-20	E20.7	EM(1,1)	Specification matrix for
		21-40	E20.7	EM(1,2)	materials.
		.	.	.	
		.	.	.	
			E20.7	EM(NMAT,16)	
					This card occurs 4 times for each material (NMAT materials), containing 4 fields per card: total of 16 elements per material.
8	†				Ply-Material correlation. One card per ply.
		1- 5	I5	NF	Number of first element in ply
		6-10	I5	NL	Number of last element
		11-15	I5	NI	Increment
		16-20	I5	MR	Material number
	e.g.	1	7 2	K	Defines elements 1, 3, 5, and 7 to belong to the k^{th} material.

†Repeat until all elements are defined. Terminate this set of cards with a zero (or blanks) in cols. 1-5.

CARD SET	NUMBER OF CARDS	COLS	FORMAT	VARIABLE	DESCRIPTION
	9	1	1- 5	I5	NPLY Number of plies in the laminate
			6-10	I5	IRHS Number of loading conditions (see card 10).
			11-20	F10.4	WIDTH Width of laminate
10	1	1-20	E20.7	SMECH	Mechanical Strain
		21-40	E20.7	DELT	Temperature change
		41-60	E20.7	DELH	Hygroscopic (moisture) change
11	1	1- 5	I5	LEM(1)	Elements used in the G-
		6-10	I5	LEM(2)	calculation (elements A & B shown in Fig. 3.)
		e.g.	44	45	For the model shown in fig. 5.
12	1	1- 5	I5	NGF(1)	Nodes at which forces are
		6-10	I5	NGF(2)	evaluated and used in the G-calculation (nodes i, j in Fig. 3 and in that order.)
		e.g.	205	196	For the model shown in fig. 5

CARD SET	NUMBER OF CARDS	COLS	FORMAT	VARIABLE	DESCRIPTION
13	1	1- 5	I5	NGD(1)	Nodes at which relative displacement are evaluated and used in the G-calculation (nodes k, l, m, n in fig. 3 and in that order).
		6-10	I5	NGD(2)	
		11-15	I5	NGD(3)	
		16-20	I5	NGD(4)	
		e.g.	226 227	216 217	For the model shown in fig. 5
14	†	1- 5	I5	NN	Node Number
		6-10	I5	NX	Restraint code in the X-direction
		11-15	I5	NY	Restraint code in the Y-direction
		16-20	I5	NZ	Restraint code in the Z-direction. One card for each node with boundary conditions. Terminate this set of cards with a zero (or blanks) in cols. 1-5.

Restraint Code: 0 is free, 1 is fixed

†Repeat until all boundary conditions are defined.

CARD SET	NUMBER OF CARDS	COLS	FORMAT	VARIABLE	DESCRIPTION
15	1	1- 5	I5	IRENUM	Renumbering option: 0 if no renumbering desired; 1 if renumbering scheme follows.
16	*	1-80	16I5	JNEW(NPOIN)	Renumbering scheme: needed only if IRENUM not equal to zero

* NPOIN/16 (rounded up to the next integer when necessary)

Special Cases

a) Delamination at midplane (z = 0)

When the delamination is at the midplane and the laminate is symmetric, the input is slightly different.

Card sets 1-10 and card sets 14-16 remain unchanged. The G-calculation card sets 11-13 need to be input slightly differently. To explain this, consider the modeling near the delamination as shown in fig. 5(b). Card sets 11-13 will be as follows:

Card set 11.	44	45	Element used in the G-calculation.
Card set 12.	205	196	Nodes at which the forces are evaluated and used in the G-calculations
Card set 13.	-227 227	-217 217	Nodes at which relative displacements are evaluated and used in the G-calculations.

Note the negative signs on the integers NGD(1) and NGD (3). The program recognizes that deformation is symmetric about the midplane and calculates the appropriate relative displacements at these nodes. Also note that for this case, only mode I strain-energy-release rate exists, mode II and mode III are identically zero.

b) Quasi-three-dimensional analysis with or without delaminations.

The program can be used to perform edge-stress analyses very easily for laminates without delaminations and stress analysis alone (without G-calculations) for laminates with delaminations. To exercise this option, simply input zeros in card sets 11, 12, and 13 as shown below:

Card set	11.	0	0		
" "	12.	0	0		
" "	13.	0	0	0	0

Note that all three card sets 11, 12 and 13 must be input.

OUTPUT

The output of the Q3DG Program is presented in this section. Two versions of output, long and short, are possible with this program. If the user exercises the long output option (XLONG on the second card in columns 1-5), then the following output will be obtained. (Only the general description of major output sections is presented.)

*Title

*Output option

*Program specifications

*Y-coordinates and corresponding node numbers

*Z-coordinates and corresponding node numbers

*Material Specifications: Modulus matrix, thermal and hygroscopic strain vectors for all materials used in the model.

*Ply-material correlation

*G-calculation variables: Elements and nodes involved in the G-calculations.

*Boundary conditions

*Renumbering options

Element stiffness matrix (24 × 24) and consistent loads for element number 1.

Sum of F_x, F_y, and F_z for all nodes before and after boundary conditions.

*Degrees of freedom and bandwidth: Maximum and current values.

Displacements (U, V, and W) at all nodes for each of the loading conditions.

Nodal forces at all nodes for the elements involved in the G-calculation.

*Sum of F_x forces in the model due to each of the loading conditions.

Strain ε_x required to satisfy $\Sigma F_x = 0$ for thermal and hygroscopic loadings.

Nodal forces at each of the nodes for each of the loading conditions. (If all of the nodal forces, F_{x_i}, F_{y_i} and F_{z_i}, at node i are less than 1.0E-6, then the forces are not listed).

Average nodal stresses at each of the nodes in the model due to combined loading.

Average nodal stresses at each of the nodes in each material due to combined loading.

*Equilibrium checks for each of the loading conditions. (ΣF_{x_i}, ΣF_{y_i}, ΣF_{z_i}, i = 1, Nodes in the model).

†*Strain-energy-release rate calculations -

Forces and displacements involved in the G-calculations for each of the loading conditions.

G_I, G_{II}, G_{III} and G_T for each of the seven possible loading combinations.

†*Summary of the strain-energy-release rates for the seven possible loading combinations.

Element equilibrium. If all elements satisfy equilibrium, i.e., if $\Sigma F_x = \Sigma F_y = \Sigma F_z < 1.0E-6$ for each element, then the program prints "All elements satisfy equilibrium. The solution may be correct." If some elements do not satisfy equilibrium the program prints a warning message that n numbers of elements do not satisfy equilibrium and lists the n element numbers.

†These items do not appear in the output when only Q3D stress analysis is performed (i.e., when card sets 11, 12, and 13 contain zeros).

Only the asterick (*) items appear in the output when the user exercises the short output option (SHORT on the second card in columns 1-5).

APPENDIX A: DERIVATION OF STIFFNESS MATRIX AND CONSISTENT LOAD VECTORS

This appendix presents the derivation of the stiffness matrix and the thermal load vector for the eight-noded element used in the Q3D analysis. The derivation of the hygroscopic vector is very similar to that of the thermal vector and hence is omitted. Additional details on these derivations can be found in refs. 5, 9, and 10.

As previously pointed out, the Q3D analysis determines the displacements, U, V, and W (eq. (1)), at each node in the finite element model. Therefore, the displacements at any interior point in the element,

$$\{u\} = \{U \ V \ W\} \tag{A-1}$$

are interpolated by the standard shape functions [N] (Ref. 9) as

$$\{u\} = [N] \ \{\delta\} \tag{A-2}$$

where $\{\delta\}$ is the vector of element nodal displacements (dimensioned (24,1) for the eight-noded element). The strains in the element are obtained by differentiating eq. (A-2) to obtain the following equation:

$$\{\varepsilon\} = [B] \ \{\delta\} \tag{A-3}$$

where the column vector of the strains $\{\varepsilon\}$ is

$$\{\varepsilon\} = \{\varepsilon_x \ \varepsilon_y \ \varepsilon_z \ \varepsilon_{xy} \ \varepsilon_{yz} \ \varepsilon_{zx}\}^T \tag{A-4}$$

Equation (A-3) is partitioned as shown below:

$$\begin{bmatrix} \varepsilon_x \\ \hline \varepsilon_R \end{bmatrix} = \begin{bmatrix} B_x \\ \hline B_R \end{bmatrix} \{\delta\} \tag{A-5}$$

The strain-displacement relationship is partitioned in this manner because each element has a prescribed strain of $\varepsilon_x = \varepsilon_o$ (see eq. (1)). The remaining strains $\{\varepsilon_R\}$ are the unknown strains in the element and are thus related to nodal displacements $\{\delta\}$ by the following equation:

$$\{\varepsilon_R\} = [B_R]\ \{\delta\} \tag{A-6}$$

The vector $\{\varepsilon_R\}$ is of size (5,1) and the matrix $[B_R]$ is dimensioned (5,24) for the eight-noded element.

The stress-strain matrix of the orthotropic material of the element also can be written in partitioned form as

$$\left[\begin{array}{c} \sigma_x \\ \hline \sigma_y \\ \sigma_z \\ \hline \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{array}\right] = \left[\begin{array}{c|ccccc} D_{11} & D_{12} & D_{13} & D_{14} & D_{15} & D_{16} \\ \hline & D_{22} & D_{23} & D_{24} & D_{25} & D_{26} \\ & & D_{33} & D_{34} & D_{35} & D_{36} \\ & \text{SYMM} & & & & \\ & & & D_{44} & D_{45} & D_{46} \\ & & & & D_{55} & D_{56} \\ & & & & & D_{66} \end{array}\right] \left[\left[\begin{array}{c} \varepsilon_x \\ \hline \varepsilon_y \\ \varepsilon_z \\ \hline \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{zx} \end{array}\right] - \left[\begin{array}{c} \alpha_x \Delta T \\ \hline \alpha_y \Delta T \\ \alpha_z \Delta T \\ \hline \alpha_{xy} \Delta T \\ \alpha_{yz} \Delta T \\ \alpha_{zx} \Delta T \end{array}\right] \right] \tag{A-7}$$

The stress-strain relationship of eq. (A-7) can be concisely rewritten as

$$\{\sigma\} = \left[\begin{array}{c|c} D_{xx} & D_{xR} \\ \hline D_{xR}^T & D_{RR} \end{array}\right] \left(\left[\begin{array}{c} \varepsilon_x \\ \hline \varepsilon_R \end{array}\right] - \left[\begin{array}{c} T_x \\ \hline T_R \end{array}\right] \right) \tag{A-8}$$

The matrices $[D_{xx}]$, $[D_{xR}]$, and $[D_{RR}]$ are of dimensions (1,1), (1,5) and (5,5), respectively, and the column vectors $\{\varepsilon_x\}$, $\{\varepsilon_R\}$, $\{T_x\}$, and $\{T_R\}$ are of dimensions (1,1), (5,1), (1,1), and (5,1), respectively.

The strain energy of the element, then, is written as

$$U' = \frac{1}{2} \int \{\sigma\}^T \{\varepsilon\}\ dvol \tag{A-9}$$

Substituting eqs. (A-8) into (A-9), using the partitioned form of $\{\varepsilon\}$, and rearranging terms produces the following equation.:

$$U' = \frac{1}{2}\int \lfloor \varepsilon_o^T\, D_{xx}\, \varepsilon_o - 2\varepsilon_o^T\, D_{xx}\, T_x - 2\varepsilon_o^T\, D_{xR}\, T_R + T_x^T\, D_{xx}\, T_x$$

$$+ 2T_x^T\, D_{xR}\, T_R + T_R^T\, D_{RR}\, T_R \rfloor\, dvol$$

$$+ \frac{1}{2}\int \lfloor \varepsilon_R^T\, D_{RR}\, \varepsilon_R + 2\varepsilon_R^T\, D_{xR}^T\, \varepsilon_o - 2\varepsilon_R^T\, D_{RR}\, T_R$$

$$- 2\varepsilon_R^T\, D_{xR}^T\, T_x \rfloor\, dvol \qquad \text{(A-10)}$$

The first integral term in eq. (10) is not a function of the unknown strains $\{\varepsilon_R\}$, and hence can be disregarded from further consideration.

The unknown strains are related to the nodal displacements by eq. (A-6). Therefore, the strain energy can be written in terms of the nodal displacements $\{\delta\}$ by substituting eq. (A-6) into eq. (A-10). In the absence of any other external loading the strain-energy U' is equal to the total potential energy π_p. Minimizing π_p with respect to the nodal displacements $\{\delta\}$ yields the following equation:

$$[K]\{\delta\} + \{f\}_{\varepsilon_o} + \{f\}_{\Delta T} = 0 \qquad \text{(A-11)}$$

where

$$[K] = \int [B_R^T][D_{RR}][B_R] \, dvol$$

$$\{f\}_{\varepsilon_o} = \int [B_R^T][D_{xR}^T]\{\varepsilon_o\} \, dvol \qquad \text{(A-12)}$$

$$\{f\}_{\Delta T} = -\int \Big([B_R^T][D_{xR}^T]\{T_x\} + [B_R^T][D_{RR}]\{T_R\} \Big) \, dvol$$

The matrices $[B_R]$, $[D_{RR}]$, and $[D_{xR}]$ are of dimensions (5,24), (5,5) and (1,5), respectively; the column vectors $\{\varepsilon_o\}$, $\{T_x\}$, and $\{T_R\}$ are of dimensions (1,1), (1,1), and (5,1), respectively; the stiffness matrix [K] and the consistent load vectors, $\{f\}_{\varepsilon_o}$, and $\{f\}_{\Delta T}$ are of dimensions (24,24), (24,1) and (24,1), respectively.

APPENDIX B: SUBROUTINES, MAJOR PROGRAM VARIABLES, AND COMMON BLOCKS

This appendix first presents the names and functions of the subroutines and major program variables and, next, gives the common blocks with their elements and the subroutines which use the common blocks.

B.1 Subroutines

Name	Function
1. ADJUST	Reorders the nodal coordinate array, the element connectivity array and the boundary condition array according to the renumbering scheme provided by the user. If no renumbering is given, no reordering of these arrays is performed.
2. ASEMBLE	Processes all elements, obtains element stiffness matrices, load vectors and assembles the global stiffness matrix. Also calls the band-solver.
3. ASTAR	Obtains the transformation relationship between the generalized coordinates and the nodal coordinates. This subprogram is called only once, when the first element is processed.
4. BLOCK DATA	Contains the Gaussian coordinates and weights up to an 8-point integration rule. Also contains the parent coordinates (ξ,η) of the 8-noded element.
5. BMAT	Evaluates the stiffness matrix and the load vectors (see eq. (A-12)) at the integration points.
6. BOUND	Prescribes the boundary conditions.
7. CKBNDW	Calculates the current bandwidth from the element nodal connectivity and checks if the current bandwidth is greater than the maximum bandwidth in the program.
8. CKDEGF	Calculates the degrees of freedom (dof) in the model and checks if the current degrees of freedom are greater than the maximum in the program.
9. CLD	Calculates the coordinates of the midside nodes and reads in and forms the boundary condition array.
10. DERIVE	Obtains the derivatives of the shape functions at any point (ξ,η) in the element.

11. FORCES — Calculates the element forces and checks the element equilibrium. Calculates the model stresses and forces and checks global equilibrium. Calculates the ε_x strains needed to satisfy the condition $F_x = 0$ for the thermal and hygroscopic loadings.

12. GEVAL — Computes the strain-energy-release rates for delamination growth for all seven possible loading conditions.

13. INT — Obtains the stiffness matrix and load vectors of an eight-noded element by Gaussian quadrature.

14. INTSX — Obtains the total force, F_x, in the x-direction in an element due to mechanical strain of $\varepsilon_x = 10^{-6}$, a temperature change of $\Delta T = 1°F$, and a moisture change of $\Delta H = 1\%$. F_x is obtained by integration of σ_x stresses on the element.

15. MATINV — Obtains the inverse of a square matrix.

16. MATMUL — Obtains the product of two matrices.

17. MODULUS — Computes the modulus matrix, and thermal and hygroscopic strains of each material.

18. PARENT — Calculates the parent derivatives $\frac{\partial N_i}{\partial \xi}$, $\frac{\partial N_i}{\partial \eta}$, $i = 1,8$ at each of the $(NGAUSS)^2$ Gaussian points. Also calculates the 2×2 Gaussian-nodal stress transformation matrix.

19. PICKD — Calculates the displacement differences (see eq. (3)) needed in the G-calculations.

20. PICKF — Calculates the forces (see eq.(3)) needed in the G-calculations.

21. PROCESS — Summarizes the strain-energy-release rate results for all the 7 possible loading conditions.

22. Q3DG — Main program of this analysis. Reads most of the input.

23. SBANDPD — Solves a linear system of equations $Ax = B$. A is a banded symmetric positive definite coefficient matrix and B is a matrix of right hand side vectors.

24. SHAPE — Shape functions of the eight-noded element.

25. SMALLK — Calculates the element stiffness matrix.

26. STRESS — Calculates the nodal stresses for an eight-noded element from the stresses at the 2×2 Gaussian points.

27. TRANS — Obtains the transpose of a matrix.

28. ZEROIN — Zeros out an integer array.

29. ZEROLV — Zeros out a real variable array.

B.2 Major Program Variables

VARIABLE NAME	COMMON	DESCRIPTION
AMOD (6,6)	MOD	Modulus matrix of the material being processed
BIGK (75,1110)	ASTIF	Assembled global stiffness matrix. First subscript corresponds to the bandwidth; second subscript corresponds to the degrees of freedom in the model.
CORD (8,8)	GAUSS	Gaussian coordinates up to 8-point Gaussian integration.
DELH	CONFG	Change in moisture content, ΔH, (percentage) of the laminates for hygroscopic loading.
DELT	CONFG	Change in temperature, ΔT, of the laminate for thermal loading.
DIS (1110,3)	DISP	Global displacements. First subscript corresponds to the degree of freedom; second subscript corresponds to the three loading conditions.
EE (6,6,10)	MOD	Modulus matrices for each material. The third subscript corresponds to the material number.
ELDIS (24,3)	-----	Nodal displacements for the three loading conditions of the element being processed.
EM (10,16)	MATER	Material property specifications of each material. The first subscript corresponds to the material number.
EMO (6,1)	MOD	Hygroscopic strains due to $\Delta H = 1\%$ of the material being processed.
EPSM	-----	Strain ε_x needed to satisfy the condition $F_x = 0$, due to $\Delta H = 1\%$.
EPST	-----	Strain ε_x needed to satisfy the condition $F_x = 0$, due to $\Delta T = 1°$.

VARIABLE NAME	COMMON	DESCRIPTION
ET (6,1)	MOD	Thermal strains due to $\Delta T = 1°$ for the material being processed.
ETM (6,10)	MOD	Hygroscopic strains due to $\Delta H = 1°$ for all materials. The second subscript corresponds to the material number.
ETH (6,10)	MOD	Thermal strains due to $\Delta T = 1\%$ for all materials. The second subscript corresponds to the material number.
FEMT	-----	Net force in the x-direction of the laminate due to $\Delta H = 1\%$ and $\varepsilon_x = 0$.
FETT	-----	Net force in the x-direction of the laminate due to $\Delta T = 1°$ and $\varepsilon_x = 0$.
FEXT	-----	Net force in the x-direction of the laminate due to a mechanical strain of $\varepsilon_x = 10^{-6}$ alone.
FOR (24,3)	-----	Nodal forces of the element being processed. The second subscript indicates one of the three loading conditions.
FORCE (400,3,3)	-----	Nodal forces at each node of the model. The first subscript indicates the node number; the second subscript indicates the x-, y-, or z-direction; the third subscript indicates one of the loading conditions. Array equivalenced with BIGK.
GCR (8,2)	GENRL	Parent coordinates (ξ,η) of the 8-noded isoparametric element. Note that $0 \leq \xi, \eta \leq 1$.
GDIS (2,3,3)	GCAL	Relative displacements used in the G-calculation. The first subscript indicates one of the sets of nodes just behind the delamination tip; the second subscript indicates the x-, y-, or z-direction; the third subscript indicates one of the three loading conditions.
GFOR (2,3,3)	GCAL	Forces used in the G-calculation. The first subscript indicates one of the set of nodes at and ahead of the delamination tip; the second subscript indicates the x-, y-, or z-direction; the third subscript indicates one of the three loading conditions.

VARIABLE NAME	COMMON	DESCRIPTION
IMAT (200)	MATER	Array containing material code (number) of all the elements.
IMT	MOD	Material code of the element being processed.
IRENUM	-----	Renumbering option. When IRENUM=0, no renumbering option is exercised. User must provide the renumbering scheme.
IRHS	CONFG	Number of loading conditions. Maximum number of loading conditions that can be specified is 3: mechanical, thermal and hygroscopic loadings.
JNEW (400)	RENUM	Array which relates new node numbers to old node numbers. JNEW (IO) gives the new node number of the old node number IO. This array is complementary to the array JOLD.
JOLD (400)	RENUM	Array which relates numbers to the new node numbers. JOLD (IN) gives the old node number of the new node IN. This array is complementary to the array JNEW.
LEM	CONGF	Elements used in the G-calculation.
LIST (200)	CLIST	Array containing the degrees of freedom with a prescribed zero displacement.
LLIST	-----	Total length of the LIST array, currently LLIST = 200.
LNOD	-----	Total length of the NOD array, currently LNOD = 200 * 8 = 1600.
LOLD	-----	Total length of the JNEW and JOLD arrays, currently LOLD = 400.
LX	-----	Total length of the x array, currently LX = 400 * 2 = 800.
NBAD (200)	-----	Array which stores the numbers of elements that do not satisfy equilibrium conditions.
NBAND	INTGR	Maximum number of rows in the BIGK array. Also corresponds to the maximum allowable bandwith, currently NBAND = 75.

VARIABLE NAME	COMMON	DESCRIPTION
NBOUN	INTGR	Number of boundary conditions. (Degrees of freedom with a prescribed zero displacement). This value should not exceed LLIST.
NBW	INTGR	Current bandwidth of the finite element model.
ND (400,10)	-----	Array containing the number of nodal adjoining a specific node in each material. The first subscript corresponds to a node number; the second subscript indicates the material number. Array is equivalenced with BIGK.
NDA (400)	-----	Array containing the number of nodes adjoining a specific node. Array is equivalenced with BIGK
NDIS	INTGR	Current degrees of freedom of the finite element model, NDIS = NPOIN * NFREE.
NFREE	INTGR	Number of degrees of freedom per node, currently NFREE = 3.
NGAUSS	INTGR	Number of Gaussian integration points in each direction. Currently NGAUSS = 3.
NGD (4)	GCAL	Array containing the node numbers where the relative displacements are calculated and used in the G-calculation.
NGF (2)	GCAL	Numbers of nodes at which forces are calculated and used in the G-calculation.
NMAT	-----	Total number of materials in the problem. Currently up to 10 materials can be specified.
NMAX	INTGR	Maximum number of columns in the BIGK array. Also corresponds to the maximum allowable degrees of freedom in the model. Currently NMAX = 1110.
NNODE	INTGR	Number of nodes on each element. For the parabolic element used in this program, NNODE = 8.
NOD (200,8)	BNOD	Nodal connectivity of each element. The first subscript indicates the element number and the second subscript indicates the 8 nodes on the element.

VARIABLE NAME	COMMON	DESCRIPTION
NPLY	CONFG	Number of plies used in the laminate.
NPOIN	INTGR	Total number of nodes in the finite element model.
POUT	OUT	Output option indicator.
PDER (18,8)	CDER	Array containing the parent derivatives, $\frac{\partial N_i}{\partial \xi}$, $\frac{\partial N_i}{\partial \eta}$ at each of the Gaussian points. Array is stored as, $\begin{bmatrix} \left(\frac{\partial N_1}{\partial \xi}\right)_j & \left(\frac{\partial N_2}{\partial \xi}\right)_j, & \cdots & \left(\frac{\partial N_8}{\partial \xi}\right)_j \\ \left(\frac{\partial N_1}{\partial \eta}\right)_j & \left(\frac{\partial N_2}{\partial \eta}\right)_j & \cdots & \left(\frac{\partial N_8}{\partial \eta}\right)_j, \end{bmatrix}$ $j = 1$, NGAUSS × NGAUSS.
SHORT	OUT	Short output option.
SMECH	CONFG	Mechanical strain ε_x prescribed on the laminate.
SP (400,6,10)	-----	Average nodal stresses at each node in each material due to the combined loading. The first subscript indicates the node number; the second subscript indicates one of the six stresses σ_x, σ_y, σ_z, σ_{xy}, σ_{yz}, σ_{zx}; the third subscript indicates the material number. Array is equivalenced with BIGK.
SPA (400,6)	-----	Average nodal stresses at each node in the model due to combined loading. The first subscript indicates the node number; the second subscript indicates one of the six stresses σ_x, σ_y, σ_z, σ_{xy}, σ_{yz}, σ_{zx}. Array is equivalenced with BIGK.
THETA (10)	MATER	Fiber angle for each material. Currently, up to 10 materials can be specified.
TSTR (8,4)	CDER	2 × 2 Gaussian-Nodal stress transformation matrix.

VARIABLE NAME	COMMON	DESCRIPTION
UEL (24,3)	BLOAD	Consistent nodal loads of an element. The first subscript indicates one of the degrees of freedom of the element; the second subscript indicates one of the three loading conditions of $\varepsilon_x = 10^{-6}$, $\Delta T = 1°$, or $\Delta H = 1\%$.
UMO (24,1)	BLOAD	Consistent hygroscopic vector at the integration point corresponding to $\Delta H = 1\%$.
UO (24,1)	BLOAD	Consistent mechanical vector at the integration point corresponding to a mechanical strain $\varepsilon_x = 10^{-6}$.
UT (24,1)	BLOAD	Consistent thermal vector at the integration point corresponding to $\Delta T = 1°$.
WEIGHT (8,8)	GAUSS	Gaussian weights up to 8-point Gaussian integration.
WIDTH	CONFG	Width of the laminate.
X(400,2)	BNOD	Y- and z-coordinate array of all nodes in the finite element model.
XE (8,2)	-----	Nodal coordinates of the nodes for the element being processed.
XLONG	OUT	Long output option.

B.3 Common Blocks

BLOCK NAME	ELEMENTS	ROUTINE
AINV	A(8,8)	ASEMBL, ASTAR, BMAT
ASTIF	BIGK(75,1110)	ADJUST, ASEMBL, BOUND, FORCES, Q3DG
BLOAD	UO(24,1), UT(24,1) UMO(24,1), UEL(24,3)	BMAT, FORCES, INT
BNOD	X(400,2), NOD(200,8)	ADJUST, ASEMBL, CKBNDW, CLD, GEVAL, PICKF, Q3DG
CDER	PDER (18,8), TSTR (8,4)	ASEMBL, FORCES
CLIST	LIST(200)	ADJUST, ASEMBL, BOUND, CLD, FORCES, Q3DG
CONFG	NPLY, IRHS, WIDTH, SMECH, DELT, DELM	ASEMBL, FORCES, GEVAL, PICKD, PICKF, STRESS, Q3DG
DISP	DIS(1110,3)	ASEMBL, BOUND, FORCES, PICKD, Q3DG
GAUSS	CORD(8,8), WEIGHT(8,8)	BLOCK DATA, INT, INTSX, STRESS
GCAL	LEM(2), NGF(2), NGD (4), GFOR (2,3,3), GDIS (2,3,3)	FORCES, GEVAL, PICKD, PICKF, Q3DG
GENRL	GCR(8,2)	ASTAR, BLOCK DATA, FORCES
INTGR	NPOIN, NBOUN, NELEM, NFREE, NMAX, NBW, NBAND, NNODE, NDIS, NGAUSS	ADJUST, ASEMBL, BOUND, CKBNDW, CKDEGF, CLD, FORCES, GEVAL, Q3DG
MATER	IMAT(200), EMO(10,16), THETA(10)	ASEMBL, FORCES, MODULUS, Q3DG
MOD	EE(6,6,10), AMOD(6,6), ETH(6,10), ET(6,1), IMT, ETM(6,10), EMO(6,1)	ASEMBL, BMAT, FORCES, INTSX, MODULUS, STRESS, Q3DG
OUT	POUT, SHORT, XLONG	ASEMBL, BLOCK DATA, FORCES, GEVAL, Q3DG
RENUM	JOLD(400), JNEW(400)	ADJUST, ASEMBL, FORCES, GEVAL, PICKD, PICKF, Q3DG

APPENDIX C: EXAMPLE PROBLEMS

In this appendix the input data and detailed outputs for three example problems are presented.

Example Problem 1

The first example problem is that of a long rectangular $[\pm 35/0_{\uparrow}90]_s$ laminate with a semi-width b of 0.75 inches. The laminate is edge delaminated at the 0/90 interfaces as indicated by the upward arrow in the stacking sequence. The delamination length a is 0.129 inches. The laminate is subjected to a uniform axial strain ε_x of 0.00254 in/in, a temperature change ΔT of −280°F (cooling from 350°F cure to room temperature of 70°F), and a moisture change ΔH of 0.6%.

The laminate was modeled with 102 eight-noded elements involving 367 nodes as shown in fig. 2. The details of the modeling near the delamination tip are shown in fig. 5 along with the elements and nodes involved in the G-calculation. The input data for this model is presented in fig. 6. The input data of fig. 6 were obtained using the rectangular mesh generation program, DATGEN. This program is described in Appendix E.

The detailed output from Q3DG for the input in fig. 6 is presented in fig. 7. The program calculated the total strain-energy-release rates for M, M + T, M + H, and M + T + H loading conditions as 0.1102, 0.2637, 0.0700, and 0.1332 lb in/in^2, respectively. The CLT analyses of ref. 3 yielded the total strain-energy-release rates of 0.1104, 0.2577, 0.0686, and 0.1314 lb in/in^2 for the same loading cases. The finite element values, thus, agreed very well with CLT values in these cases.

Example Problem 2

The second example problem uses the same laminate configuration and loading as the first problem except that the stacking sequence for this laminate is $[\pm 45/0/90]_s$. The input data is the same as shown in fig. 6 except for the material specification cards. The material specification cards for this laminate are presented in fig. 8. The short output option was exercised and the output obtained from Q3DG is presented in fig. 9.

Example Problem 3

This example is included to demonstrate the input required for the renumbering option. Consider a $[0/90]_s$ laminate with edge delaminations between the 0° and 90° plies with b = 0.75 in. and a = 0.15 in. The laminate is subjected to $\varepsilon_x = 0.00254$ in/in, $\Delta T = -280°F$ and $\Delta H = 0.6\%$.

Two identical very coarse finite element idealizations were used. In the first model, the nodes were numbered in the thickness direction as shown in fig. 10(a). In the second model, the nodes were numbered in the width direction as shown in fig. 10(b). Obviously, the width-wise numbering scheme yields a larger bandwidth (bandwidth = 59) than the thickness-wise numbering scheme (bandwidth = 39). Therefore, the renumbering option was exercised for the model shown in fig. 10(b). The input data for the model of fig. 10(b) is presented in fig. 11. Both models of fig. 10, when executed by the Q3DG program, gave identical results for displacements, stresses, and strain-energy-release rates. The summary of the strain-energy-release rates for these models is shown in fig. 12.

Typical Results

Table 1 presents the strain-energy-release rates for 4 edge-delaminated laminates subjected to $\varepsilon_x = 0.001$, $\Delta T = 280°F$ and $\Delta H = 1.0\%$. These results can be used by the user for verification purposes. These results also compare laminates with symmetric edge delaminations (about midplane) with those with only one edge delamination.

Table 1: Strain-energy-release rate results for edge delaminations under laminate combined loadings: $\varepsilon_x = 0.001$; $\Delta T = -280°F$; $\Delta H = 1.0$. Material properties used here are: $E_{11} = 18.2$ Msi; $E_{22} = E_{33} = 1.23$ Msi; $\nu_{12} = \nu_{13} = \nu_{23} = 0.292$ $G_{12} = G_{13} = G_{23} = 0.832$ Msi

Laminate: $[35/-35/0/90]_s$			
	M	M + T	M + T + H
G (Total)	0.0150	0.0926	0.0098
G_I/G	94.43%	43.95%	88.45%
G_{II}/G	5.50%	56.03%	11.25%
G_{III}/G	0.07%	0.02%	0.30%
Laminate: $[35/-35/0/90_2/0/-35/35]_T$			
	M	M + T	M + T + H
G (Total)	0.0266	0.1192	0.0150
G_I/G	75.87%	36.79%	97.23%
G_{II}/G	24.06%	63.19%	2.40%
G_{III}/G	0	0	0
Laminate: $[30/-30_2/30/90]_s$			
	M	M + T	M + T + H
G (Total)	0.0273	0.1292	0.0110
G_I/G	63.58%	63.58%	63.58%
G_{II}/G	36.90%	36.90%	36.90%
G_{III}/G	-0.48%	-0.48%	-0.48%
Laminate: $[30/-30_2/30/90_2/30/-30_2/30]_T$			
	M	M + T	M + T + H
G (Total)	0.0459	0.2149	0.0186
G_I/G	68.25%	68.25%	68.25%
G_{II}/G	32.03%	32.03%	32.03%
G_{III}/G	-0.28%	-0.28%	-0.28%

APPENDIX D: EXECUTION ERRORS AND DEBUG STRATEGIES

This appendix presents the conditions under which the program will terminate execution and discusses potential debug strategies.

The program will terminate execution and print a self-explanatory diagnostic message under the following conditions:

1. A node or nodes have unspecified coordinates.
2. An element has an unspecified material group number.
3. The degress of freedom in the model is greater than NMAX, the column dimension of BIGK.
4. The bandwidth of the model is greater than NBAND, the row dimension of BIGK.
5. Negative diagonal terms are computed in the element stiffness matrix of any element.
6. The band-solver aborts the program under the following conditions:
   a. The matrix is not positive definite.
   b. The bandwidth is greater than the number of equations.

The first two error conditions can be corrected by checking and modifying the input. The third and fourth error conditions are caused by the size of the model. To correct these errors, two alternatives are available. The first is to increase the dimensions of BIGK and the other associated variables of the program, as explained earlier. The second approach is to reduce the size of the finite element model. Error condition 5 is usually caused by an improper description of one or more elements. Each element's nodal connectivity must be described in the counterclockwise sense starting at any corner node. If this is not done correctly error condition 5 may result. Another possible source of this error may be erroneous nodal coordinates. The band-solver's error condition a is due to failure to specify sufficient restraints to the model to prevent rigid body motion. Therefore, the boundary conditions specified on the model should be checked and corrected. Error condition b of the band-solver results whenever the bandwidth is greater than the total degrees of freedom in the model. This condition usually occurs because of erroneous nodal numbering. This condition, however, will never occur because the program would abort execution before it calls the band-solver.

Plotting the finite element model is recommended since a plot will quickly reveal any input errors in the nodal coordinates and element connectivities, which constitute the bulk of the input. Any other errors which are encountered can usually be debugged using the host computer debug and dump utilities.

As an additional check, the total strain-energy-release rates computed by Q3DG should be compared with those obtained by CLT theory analyses [1-3] which can be performed on microcomputers. The Q3DG results should agree very well with the CLT results even for coarse models. Therefore, if there is a large discrepancy between the Q3DG results and CLT results, both analyses should be examined and the causes for these discrepancies should be determined.

APPENDIX E: DATGEN PROGRAM

This appendix describes a rectangular mesh generation program, DATGEN, which creates a data file that can be used as an input for the Q3DG program. First, the capabilities of this program are explained. Next, a few examples of data generation and types of errors encountered in the use of DATGEN are presented. Finally, the input data for the program is given.

E.1 Capabilities of DATGEN

DATGEN is a program which can be executed interactively on any mainframe computer. The program requires 41,000 octal words of core memory with the current dimensions. With the input of y- and z-coordinates on the y = constant and z = constant lines and the location of the delamination tip, the program generates a rectangular mesh, computes the nodal coordinates of the midside nodes and the nodal connectivities. The user then enters the rest of the data requested by the program. The program formats the input and at the end of the interactive session, a complete data file is ready for use as an input file for Q3DG.

Material properties and thermal expansion coefficients of the T300/5208 graphite/epoxy unidirectional composite lamina are built into the DATGEN program's BLOCK DATA. The user can change these properties very easily using a text editor.

The input variables of DATGEN are listed in section E.4.

E.2 Examples of Interactive Sessions

Consider the $[\pm 35/0/90]_s$ laminate of example 1 in Appendix C. A finite element model for this laminate was shown in fig. 2. The y- and z-coordinates of y = constant and z = constant lines used in this model are presented in fig. 13 and were stored in file YZ. There are 18 y-coordinates and 7 z-coordinates listed in this file which correspond to the y- and z-coordinates of the vertical and horizontal lines, respectively. Therefore, the values of NYP and NZP in the program are 18 and 7, respectively. The delamination tip is located at (0.6210, 0.0054). The y- and z-coordinates of the delamination tip correspond to the 11th y-coordinate and the 3rd z-coordinate in the file YZ. Therefore, the values NYC and NZC in the program are 11 and 3, respectively.

Fig. 14 presents the complete interactive session for this model. All input is echoed onto the terminal screen. The output from DATGEN is written on the file TRIAL. A complete listing of TRIAL is presented in fig. 6.

E.3 Recoverable and Irrecoverable Errors

During an interactive session it is possible that the data may be input incorrectly. Some errors in the data input can lead to recoverable errors while certain others can lead to irrecoverable errors. This section presents examples of each kind.

Fig. 15 presents a partial interactive session for the $[\pm 35/0/90]_s$ laminate discussed in fig. 14, where errors were made during the input of material and thickness specifications. The bottom line of the ply did not correspond to any of the given z-coordinates. As illustrated by fig. 15, the errors are recoverable. The program warns of improper input and recycles back to the point where the error was made and allows the user to enter the correct data.

Fig. 16 illustrates an interactive session that terminated due to an irrecoverable error. If the sum of the thicknesses of all plies does not equal to the maximum z-coordinate input by the user, DATGEN terminates the execution with an error as shown in fig. 16.

E.4 Renumbering Option

Fig. 17 presents an interactive session in which a finite element model, as shown in fig. 10(b), is created for a $[0/90]_s$ laminate with a delamination between the 0° and 90° plies. In this session the renumbering option was exercised. The renumbering scheme was stored in the file REN and the y- and z-coordinate were stored in the file YZC. These two files are shown in fig. 17.

When the renumbering option is exercised, DATGEN checks the renumbering scheme to see if any number is repeated, exceeds the maximum number of nodes in the model, or does not occur at all. If any of the three conditions occurs, DATGEN gives a summary of the error condition and no renumbering scheme is entered on the output data file. An example of DATGEN's warning message for this error is presented in fig. 18. In this example, a node with number 39 was repeated twice and a node with number 30 was not given in the renumbering scheme.

E.5 Description of Input Data for DATGEN

CARD SET	COLUMNS	FORMAT	VARIABLE	DESCRIPTION
1.	1-10	A10	CORCON	Name of file on which output is written. This is the file that will be used as input for Q3DG.
2.	1-80	8A10	TITLE	Title of the problem.
3.		*	NYP, NZP	Number of the corner nodes in y- and z-directions, respectively, of the rectangular mesh.
4.		*	NYC, NZC	Number of the y- and z-coordinates, respectively, of the delamination tip.
5.	1-10	A10	CFILE	File name on which the y- and z-coordinates values are stored.

Note: y- and z-coordinates should be greater than or equal to zero.

CARD SET	COLUMNS	FORMAT	VARIABLE	DESCRIPTION
6.		*	NMAT	Number of materials in the model (each ply with a different fiber angle is classified as a different material).
7.		*	THETA	Fiber angle for each material.

This line occurs once for each material and hence occurs NMAT times.

CARD SET	COLUMNS	FORMAT	VARIABLE	DESCRIPTION
8.		*	NPLY	Number of plies in the laminate.
9.		*	MATC	Material number of each ply.
10.		*	PLH	Ply thickness.

The lines 9 and 10 occur NPLY times. The program assumes that the plies are numbered from the top of the laminate.

CARD SET	COLUMNS	FORMAT	VARIABLE	DESCRIPTION
11.		*	IRHS	Number of loading conditions. Note that the number of loading conditions IRHS $\leq$ 3.

Note: * denotes a free format.

CARD SET	COLUMNS	FORMAT	VARIABLE	DESCRIPTION
12.		*	SMECH	Mechanical strain ε_x.
13.		*	DELT	Temperature change, ΔT.
14.		*	DELH	Moisture (hygroscopic) absorption, ΔH.

If IRHS = 1, Card set 13 and 14 are not needed. If IRHS = 2 Card set 14 is not needed. Also note that input of $\varepsilon_x = 0$ is not permitted by Q3DG.

CARD SET	COLUMNS	FORMAT	VARIABLE	DESCRIPTION
15.	*	*	IRENUM	Renumbering option. IRENUM = 0, no renumbering is attempted. IRENUM = 1, renumbering scheme follows.
16.	1-10	A10	RFILE	Name of file which contains the renumbering scheme. If a blank is input, renumbering scheme should be input manually.
17.	*	*	JNEW (1) . . . JNEW (NPOIN)	Renumbering scheme manual input. Separate numbers by commas, use as many lines as required.

Note: * denotes a free format

Special Cases:

The above input will generate the rectangular mesh and data file for delaminations at any location other than the midplane. When the delamination is at the midplane, input NYC = NYP and NZC = 1. The resulting data is correct except the G-calculation cards, card sets 11, 12, and 13, and the boundary conditions. The user can use the text editor and input the correct integers in card sets 11, 12, and 13 and delete the unnecessary boundary conditions. Also, be sure to use the negative integers on card set 13 as explained in the special cases part of the INPUT DATA section.

If the user wants to perform only a Q3D edge-stress analysis, then as above, use NYC = NYP and NZC = 1. Furthermore, change card sets 11, 12, and 13 to zero values as explained in the special cases part of the INPUT DATA section. The boundary conditions generated by the DATGEN are correct for this case.

APPENDIX F: COSMIC DOCUMENTATION

This appendix describes the program documentation requirements required by COSMIC. Items such as method of solution, user instructions, accuracy of results, sample input and output, and flow chart were already discussed in detail in the text and Appendixes A-E. Therefore, this appendix limits itself to the remaining items described in COSMIC software submittal guidelines.

1. Computer Configuration: The Q3DG program was written, compiled and successfully executed on CDC Computers such as on CYBER 173, 175, 855 using NOS 1.4 and 2.1 operating systems.

2. Memory Required: With the current dimensions in the program, about 343,000 octal words of memory were needed for execution. As pointed out earlier, with the current dimensions, finite elements models of up to 370 nodes with a bandwidth of 75 can be successfully run with the program.

3. Source Language: The Q3DG program was written in FORTRAN V. The compilation showed that, except for one statement in BLOCK DATA subprogram (where the output options are set), the program is machine independent FORTRAN. This statement DATA SHORT, XLONG/ 5HSHORT, 5HXLONG/ can be changed, for example, to DATA SHORT, XLONG/ 1111.11, 9999.99/. However, this was not done because SHORT and LONG clearly are meaningful to describe the output option.

4. Program Timing: Execution times varied widely and are dependent on the size of the finite element models. Small models with less than 80 nodes were executed within about 20 CP seconds interactively. The largest model with 367 nodes presented in Appendix C took about 184 CP seconds on CYBER 855 Computer.

5. <u>Tape Documentation</u>: The tape containing the Q3DG program is a 9 track, 1600 BPI, CDC NOS Internal unlabeled tape containing eight files. The first file contains the source code of the Q3DG program. The second file contains the source code for the rectangular mesh and data generator DATGEN. The third file contains input data for the $[\pm 35/0/90]_s$ laminate, shown in fig. 6, discussed in Appendix C. The fourth file contains the output of Q3DG for the input shown in fig. 6. This output file is presented in fig. 7.

   The fifth file of the tape is the input data for the $[\pm 45/0/90]_s$ laminate, discussed in Appendix C. The sixth file contains the output of Q3DG for the input data in the fifth file. This output file is presented in fig. 9.

   The seventh file of the tape contains the y- and z-coordinate file shown in fig. 13. This file is used in conjunction with the DATGEN program and the resulting interactive session is the last (eighth) file of the tape.

The NOS procedural file which created this tape is shown in fig. 19. A microfiche which contains the compiler listings of Q3DG and DATGEN, the input and output files, and interactive sessions of the DATGEN program is enclosed with this manual.

Finally, a dayfile of a run on the NOS 2.1 operating system is presented in fig. 20. The output is written and saved on a file SOUT. The dayfile is written and saved on a file DAY. The dayfile is also included in this figure.

APPENDIX G: PC FLOPPY DISKS

Because of the popularity of the IBM PC and its compatibles, the Q3DG program was downloaded on to floppy disks. The floppies titled Q3D-1 and Q3D-2 are enclosed with this manual. These disks contain the files listed under Tape Documentation of Appendix F (see page 42). The directories for these files are reproduced below.

Any one of the PC communications packages, such as PC-PLOT, PC-TALK, etc., can be used to upload all the files from the Q3D floppy disks on to the mainframe computers easily. The procedures to do this can be found in the documentation of the communication packages.

```
A:\>DIR/P

 Volume in drive A is Q3D-1
 Directory of  A:\

Q3DG              82688   9-05-86   2:11p
DATGEN            17664   9-09-86  10:03a
TFICH1           240768   9-08-86   3:16p
        3 File(s)      19456 bytes free

A:\>B:

B:\>DIR/P

 Volume in drive B is Q3D-2
 Directory of  B:\

TFICH2           245248   9-08-86   4:00p
TFICH3            32128   9-08-86   4:13p
TFICH4            61312   9-09-86   9:34a
TFICH5             8064   9-09-86   9:55a
        4 File(s)      14336 bytes free
```

APPENDIX H: LIST OF SYMBOLS

a	delamination length
b	half-width of the laminate
$[D]$	stress-strain matrix
E_{ii}	Young's modulus in the i-direction
$F_{x_i}, F_{y_i}, F_{z_i}$	force at node i in the x-, y-, and z- directions, respectively
G, G_I, G_{II}, G_{III}	total, mode I, mode II, and mode III strain-energy release rates, respectively
G_{ij}	shear modulus
h	ply thickness
ΔT	temperature change
t	half-thickness of the laminate
U, V, W	displacement functions
u, v, w	displacements in x-, y-, and z-directions, respectively
x, y, z	Cartesian coordinates
α_i	coefficient of thermal expansion in the i-direction
ε_o	uniform axial strain imposed on the laminate
Δ	length of the elements nearest to the delamination tip
θ	ply fiber angle measured from the x-axis toward the y-axis
ν_{ij}	Poisson's ratio
$\{\sigma\}$	Cartsian stresses, $\{\sigma_x \ \sigma_y \ \sigma_z \ \sigma_{xy} \ \sigma_{yz} \ \sigma_{zx}\}$
$\{\varepsilon\}$	Cartsian strains, $\{\varepsilon_x \ \varepsilon_y \ \varepsilon_z \ \varepsilon_{xy} \ \varepsilon_{yz} \ \varepsilon_{zx}\}$

Subscript

i, j	$i, j = 1,2,3$
1,2,3	longitudinal, transverse, and thickness directions of a zero-degree ply

ACKNOWLEDGEMENTS

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The author is grateful to Dr. C. A. Bigelow for many helpful suggestions and an indepth review of this manuscript. During the summer of 1984, a student from Virginia Polytechnic Institute and State University, Mr. Lee J. Jaap, worked on this program. When he first started working on the program, the output was decipherable only to the author. Mr. Jaap labelled and reformatted the output, and introduced comment cards throughout the program, so that most of the users can understand the program and its output. His efforts greatly improved the program. The author takes this opportunity to thank him for his assistance.

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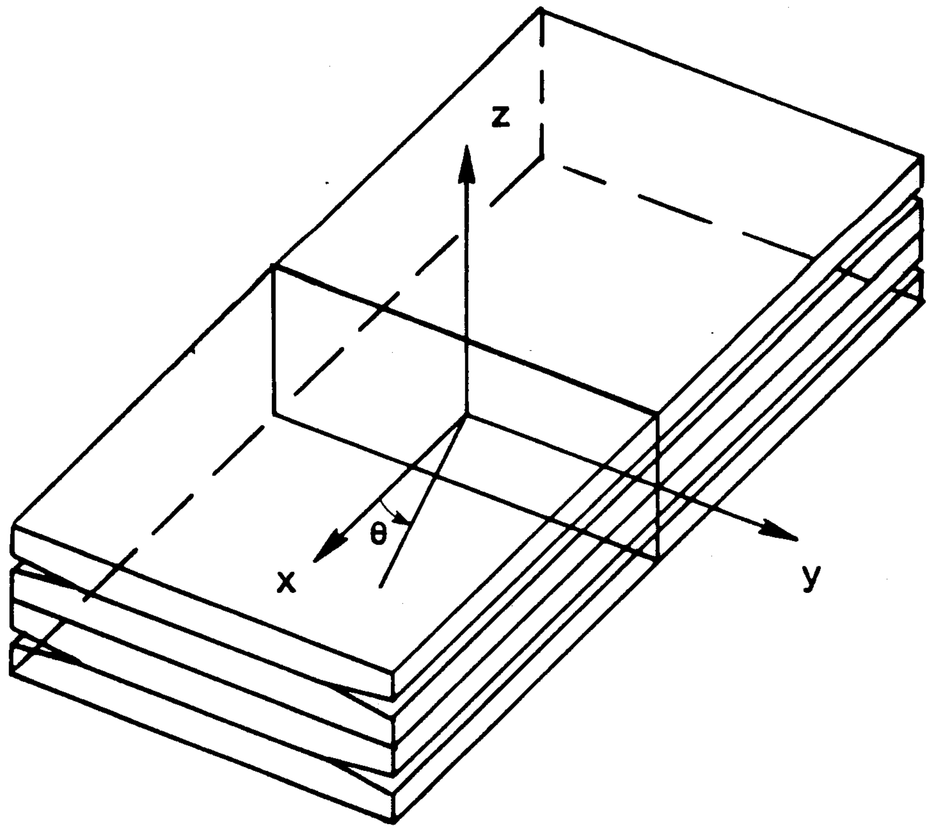
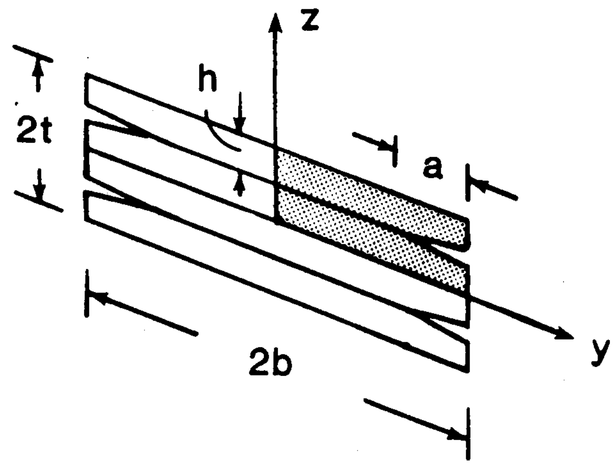
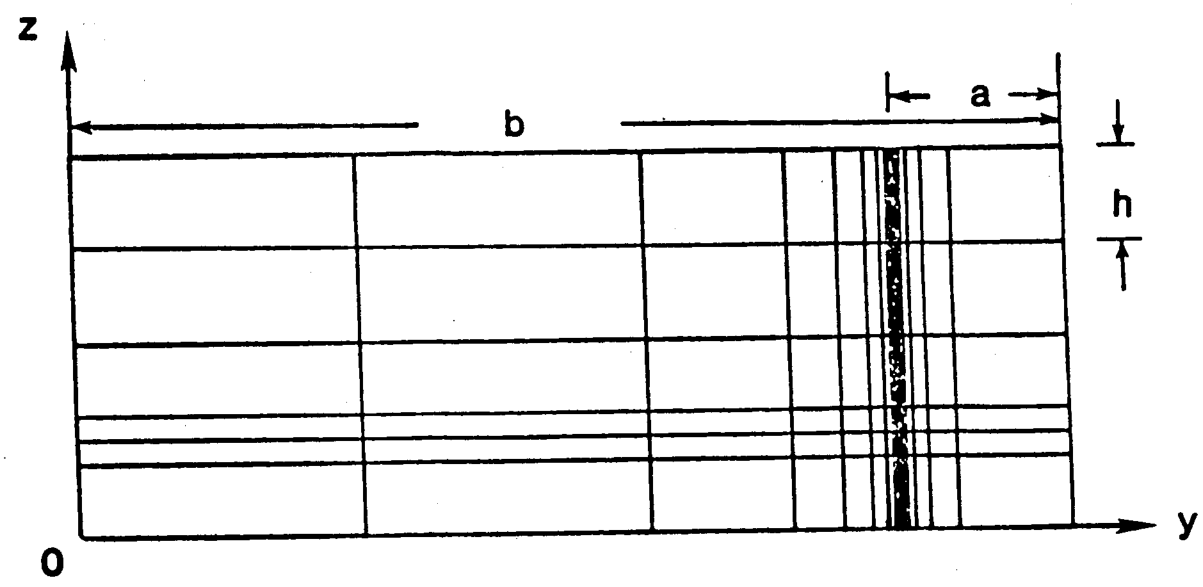


(a) Delaminated four ply laminate



(b) x = constant plane

Fig. 1 Laminate Configuration and Analysis Region



NOT TO SCALE . Z–Coordinates are scaled up.

Fig. 2 A Typical Finite Element Idealization

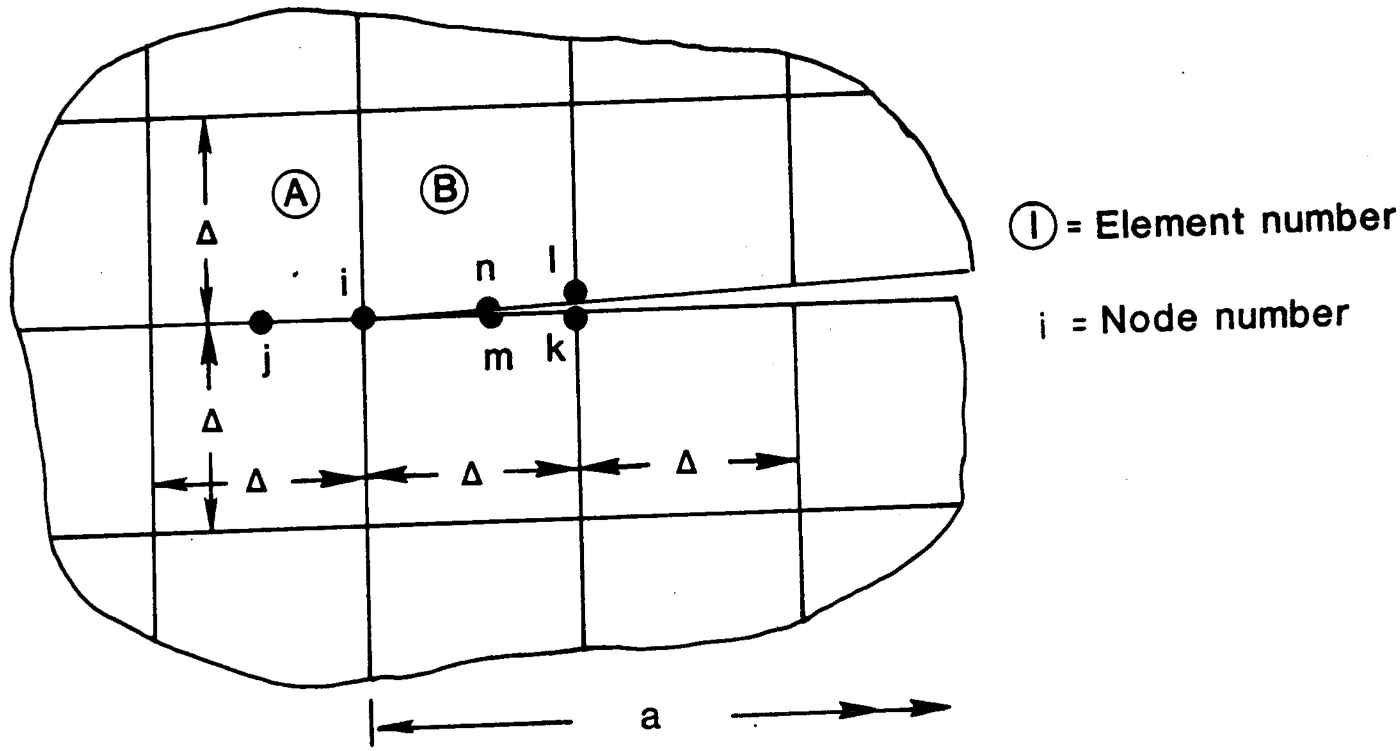


Fig. 3 Elements and Nodes Near the Delamination Tip Used in the G Calculations

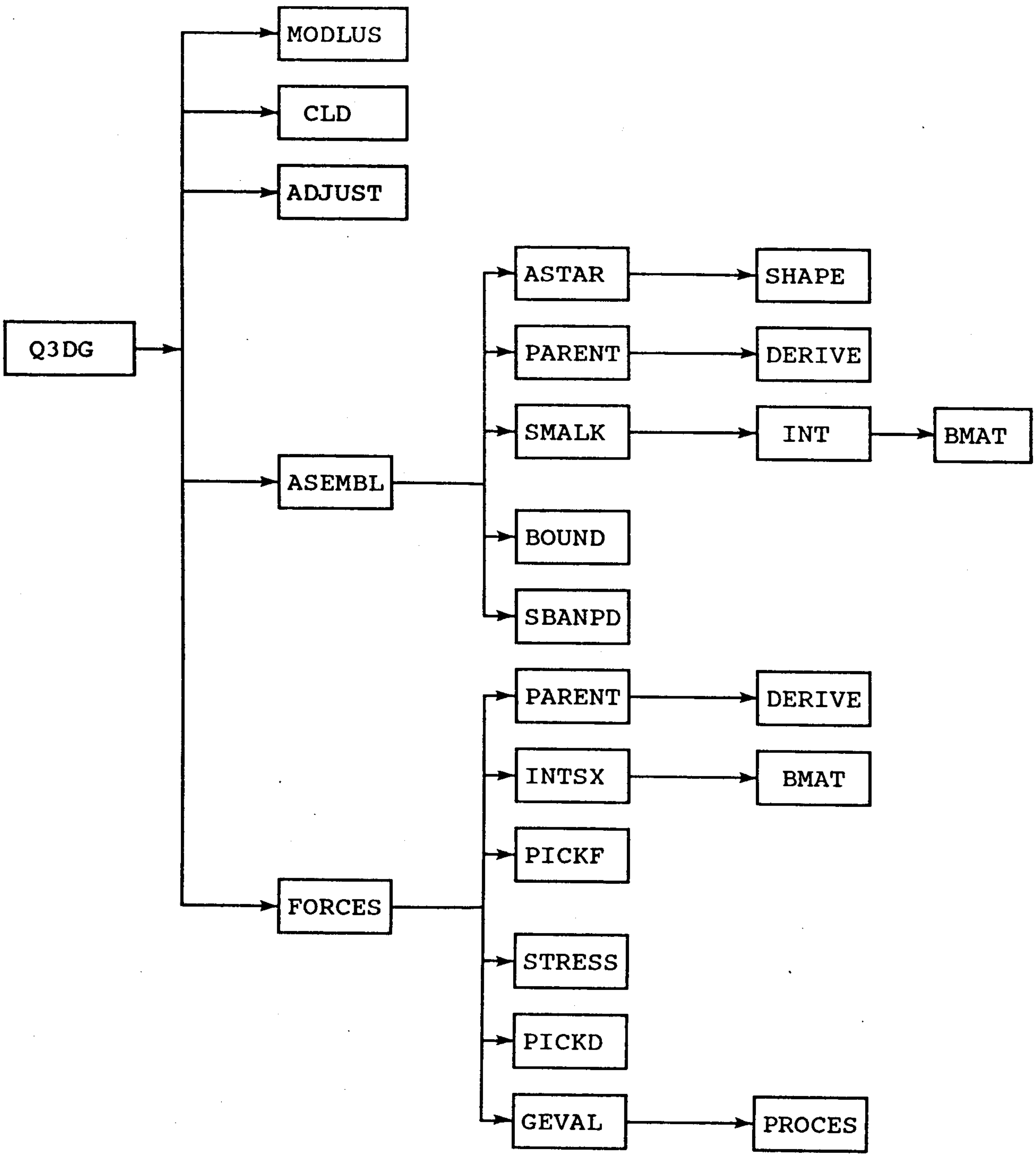
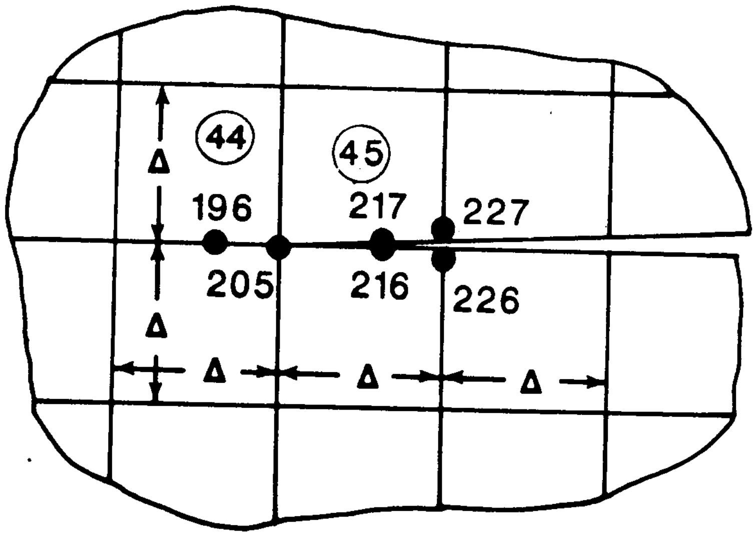
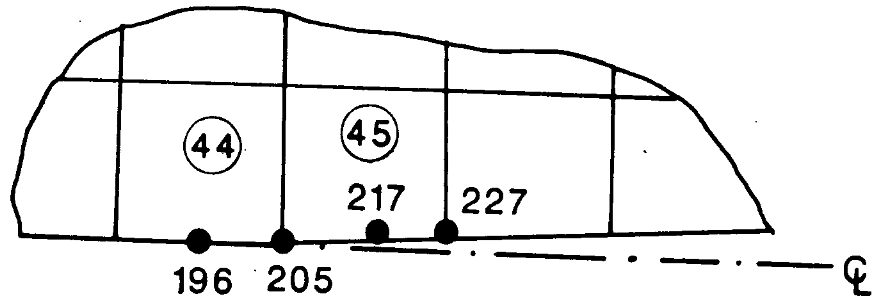


Fig. 4 Flow Chart for Q3DG Program

(Read the flow chart from top to bottom and left to right)



(a) Delamination between plies



(b) Delamination at mid plane

Ⓘ = Element number

i = Node number

Δ = h/4 : h 0.0054 in.

Fig. 5 Detail Near the Delamination Tip for the Model Shown in Fig. 2 and for a Midplane Delamination

(+-35/0/90) SYMM LAMINATE, G-CAL. FOR MECH,THERM,AND HYGRO. LOADINGS
XLONG

267 102	3	8								
.000000	1	2	3	4	5	6	7	8	9	10
.000000	11	12	13	0	0	0	0	0	0	0
.108000	14	15	16	17	18	19	20	0	0	0
.216000	21	22	23	24	25	26	27	28	29	30
.216000	31	32	33	0	0	0	0	0	0	0
.324000	34	35	36	37	38	39	40	0	0	0
.432000	41	42	43	44	45	46	47	48	49	50
.432000	51	52	53	0	0	0	0	0	0	0
.486000	54	55	56	57	58	59	60	0	0	0
.540000	61	62	63	64	65	66	67	68	69	70
.540000	71	72	73	0	0	0	0	0	0	0
.558900	74	75	76	77	78	79	80	0	0	0
.577800	81	82	83	84	85	86	87	88	89	90
.577800	91	92	93	0	0	0	0	0	0	0
.588600	94	95	96	97	98	99	100	0	0	0
.599400	101	102	103	104	105	106	107	108	109	110
.599400	111	112	113	0	0	0	0	0	0	0
.604800	114	115	116	117	118	119	120	0	0	0
.610200	121	122	123	124	125	126	127	128	129	130
.610200	131	132	133	0	0	0	0	0	0	0
.612900	134	135	136	137	138	139	140	0	0	0
.615600	141	142	143	144	145	146	147	148	149	150
.615600	151	152	153	0	0	0	0	0	0	0
.616950	154	155	156	157	158	159	160	0	0	0
.618300	161	162	163	164	165	166	167	168	169	170
.618300	171	172	173	0	0	0	0	0	0	0
.618975	174	175	176	177	178	179	180	0	0	0
.619650	181	182	183	184	185	186	187	188	189	190
.619650	191	192	193	0	0	0	0	0	0	0
.620325	194	195	196	197	198	199	200	0	0	0
.621000	201	202	203	204	205	206	207	208	209	210
.621000	211	212	213	0	0	0	0	0	0	0
.621675	214	215	216	217	218	219	220	221	0	0
.622350	222	223	224	225	226	227	228	229	230	231
.622350	232	233	234	235	0	0	0	0	0	0
.623025	236	237	238	239	240	241	242	243	0	0
.623700	244	245	246	247	248	249	250	251	252	253
.623700	254	255	256	257	0	0	0	0	0	0
.625050	258	259	260	261	262	263	264	265	0	0
.626400	266	267	268	269	270	271	272	273	274	275
.626400	276	277	278	279	0	0	0	0	0	0
.629100	280	281	282	283	284	285	286	287	0	0
.631800	288	289	290	291	292	293	294	295	296	297
.631800	298	299	300	301	0	0	0	0	0	0
.637200	302	303	304	305	306	307	308	309	0	0
.642600	310	311	312	313	314	315	316	317	318	319

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Fig. 6 Input data for the model shown in fig. 2 .

.642600	320	321	322	323	0	0	0	0	0	0
.653400	324	325	326	327	328	329	330	331	0	0
.664200	332	333	334	335	336	337	338	339	340	341
.664200	342	343	344	345	0	0	0	0	0	0
.707100	346	347	348	349	350	351	352	353	0	0
.750000	354	355	356	357	358	359	360	361	362	363
.750000	364	365	366	367	0	0	0	0	0	0
.000000	0									
.000000	1	14	21	34	41	54	61	74	81	94
.000000	101	114	121	134	141	154	161	174	181	194
.000000	201	214	222	236	244	258	266	280	288	302
.000000	310	324	332	346	354	0	0	0	0	0
.002025	2	22	42	62	92	102	122	142	162	182
.002025	202	223	245	267	289	311	333	355	0	0
.004050	3	15	23	35	43	55	63	75	83	95
.004050	103	115	123	135	143	155	163	175	183	195
.004050	203	215	224	237	246	259	266	281	290	303
.004050	312	325	334	347	356	0	0	0	0	0
.004725	4	24	44	64	94	104	124	144	164	184
.004725	204	225	247	269	291	313	335	357	0	0
.005400	5	16	25	36	45	56	65	76	85	96
.005400	105	116	125	136	145	156	165	176	185	196
.005400	205	216	217	226	227	238	239	248	249	260
.005400	261	270	271	282	283	292	293	304	305	314
.005400	315	326	327	336	337	343	349	358	359	0
.006075	6	26	46	66	86	106	126	146	166	186
.006075	206	228	250	272	294	316	338	360	0	0
.006750	7	17	27	37	47	57	67	77	87	97
.006750	107	117	127	137	147	157	167	177	187	197
.006750	207	218	229	240	251	262	273	284	295	306
.006750	317	328	339	350	351	0	0	0	0	0
.008775	8	28	48	68	88	109	129	148	163	188
.008775	208	230	252	274	296	318	340	362	0	0
.010800	9	18	29	38	49	58	69	78	89	98
.010800	109	118	129	134	149	158	169	178	189	198
.010800	209	219	231	241	253	263	275	285	297	307
.010800	319	329	341	351	363	0	0	0	0	0
.013500	10	30	50	70	90	110	130	150	170	190
.013500	210	232	254	276	298	320	342	364	0	0
.016200	11	19	31	39	51	59	71	79	91	99
.016200	111	119	131	139	151	159	171	179	191	199
.016200	211	220	233	242	255	264	277	286	299	308
.016200	321	330	343	352	365	0	0	0	0	0
.018900	12	32	52	72	92	112	132	152	172	192
.018900	212	234	256	278	300	322	344	366	0	0
.021600	13	20	33	40	53	60	73	80	93	100
.021600	113	120	133	140	153	160	173	180	193	200
.021600	213	221	235	243	257	265	279	287	301	309
.021600	323	331	345	353	367	0	0	0	0	0

Fig. 6 (Continued)

.000000	0							
1	3	2	1	14	21	22	23	15
2	23	22	21	34	41	42	43	35
3	43	42	41	54	61	62	63	55
4	63	62	61	74	81	82	83	75
5	83	82	81	94	101	102	103	95
6	103	102	101	114	121	122	123	115
7	123	122	121	134	141	142	143	135
8	143	142	141	154	161	162	163	155
9	163	162	161	174	181	182	183	175
10	183	182	181	194	201	202	203	195
11	203	202	201	214	222	223	224	215
12	224	223	222	236	244	245	246	237
13	246	245	244	258	266	257	268	259
14	268	267	266	280	288	289	290	281
15	290	289	288	302	310	311	312	303
16	312	311	310	324	332	333	334	325
17	334	333	332	346	354	355	356	347
18	5	4	3	15	23	24	25	16
19	25	24	23	35	43	44	45	36
20	45	44	43	55	63	64	65	56
21	65	64	63	75	83	84	85	76
22	85	84	83	95	103	104	105	96
23	105	104	103	115	123	124	125	116
24	125	124	123	135	143	144	145	136
25	145	144	143	155	163	164	165	156
26	165	164	163	175	183	184	185	176
27	185	184	183	195	203	204	205	196
28	205	204	203	215	224	225	226	216
29	226	225	224	237	246	247	248	238
30	248	247	246	259	268	269	270	260
31	270	269	268	281	290	291	292	282
32	292	291	290	303	312	313	314	304
33	314	313	312	325	334	335	336	326
34	336	335	334	347	356	357	358	348
35	7	6	5	16	25	26	27	17
36	27	26	25	36	45	46	47	37
37	47	46	45	55	65	66	67	57
38	67	66	65	76	85	86	87	77
39	87	86	85	96	105	106	107	97
40	107	106	105	116	125	126	127	117
41	127	126	125	136	145	146	147	137
42	147	146	145	156	165	166	167	157
43	167	166	165	176	185	186	187	177
44	187	186	185	196	205	206	207	197
45	207	206	205	217	227	228	229	218
46	229	228	227	239	249	250	251	240
47	251	250	249	261	271	272	273	262
48	273	272	271	283	293	294	295	284

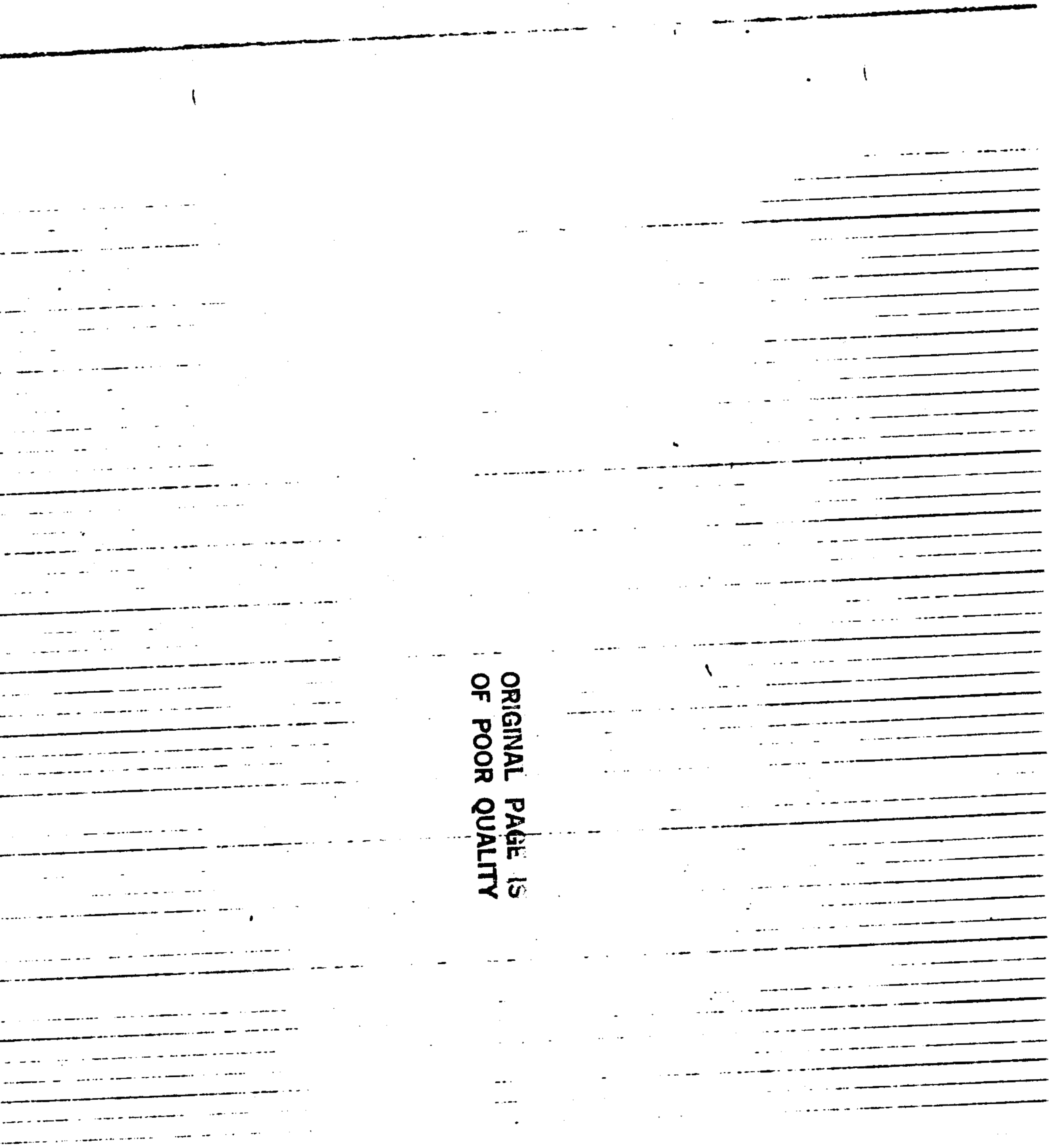

Fig. 6 (Continued)

49	295	294	293	305	315	316	317	306
50	317	316	315	327	337	338	339	328
51	339	338	337	349	359	360	361	350
52	9	8	7	17	27	28	29	18
53	29	28	27	37	47	48	49	38
54	49	48	47	57	67	68	69	58
55	69	68	67	77	87	88	89	78
56	89	88	87	97	107	108	109	98
57	109	108	107	117	127	128	129	118
58	129	128	127	137	147	148	149	138
59	149	148	147	157	167	168	169	158
60	169	168	167	177	187	188	189	178
61	189	188	187	197	207	208	209	198
62	209	208	207	219	229	230	231	219
63	231	230	229	240	251	252	253	241
64	253	252	251	262	273	274	275	263
65	275	274	273	284	295	296	297	285
66	297	296	295	306	317	318	319	307
67	319	318	317	328	339	340	341	329
68	341	340	339	350	361	362	363	351
69	11	10	9	18	29	30	31	19
70	31	30	29	38	49	50	51	39
71	51	50	49	58	69	70	71	59
72	71	70	69	78	89	90	91	79
73	91	90	89	98	109	110	111	99
74	111	110	109	118	129	130	131	119
75	131	130	129	138	149	150	151	139
76	151	150	149	158	169	170	171	159
77	171	170	169	178	189	190	191	179
78	191	190	189	198	209	210	211	199
79	211	210	209	219	231	232	233	220
80	233	232	231	241	253	254	255	242
81	255	254	253	263	275	276	277	264
82	277	276	275	285	297	298	299	286
83	299	298	297	307	319	320	321	308
84	321	320	319	329	341	342	343	330
85	343	342	341	351	363	364	365	352
86	13	12	11	19	31	32	33	20
87	33	32	31	39	51	52	53	40
88	53	52	51	59	71	72	73	60
89	73	72	71	79	91	92	93	80
90	93	92	91	99	111	112	113	100
91	113	112	111	119	131	132	133	120
92	133	132	131	139	151	152	153	140
93	153	152	151	159	171	172	173	160
94	173	172	171	179	191	192	193	180
95	193	192	191	199	211	212	213	200
96	213	212	211	220	233	234	235	221
97	235	234	233	242	255	256	257	243

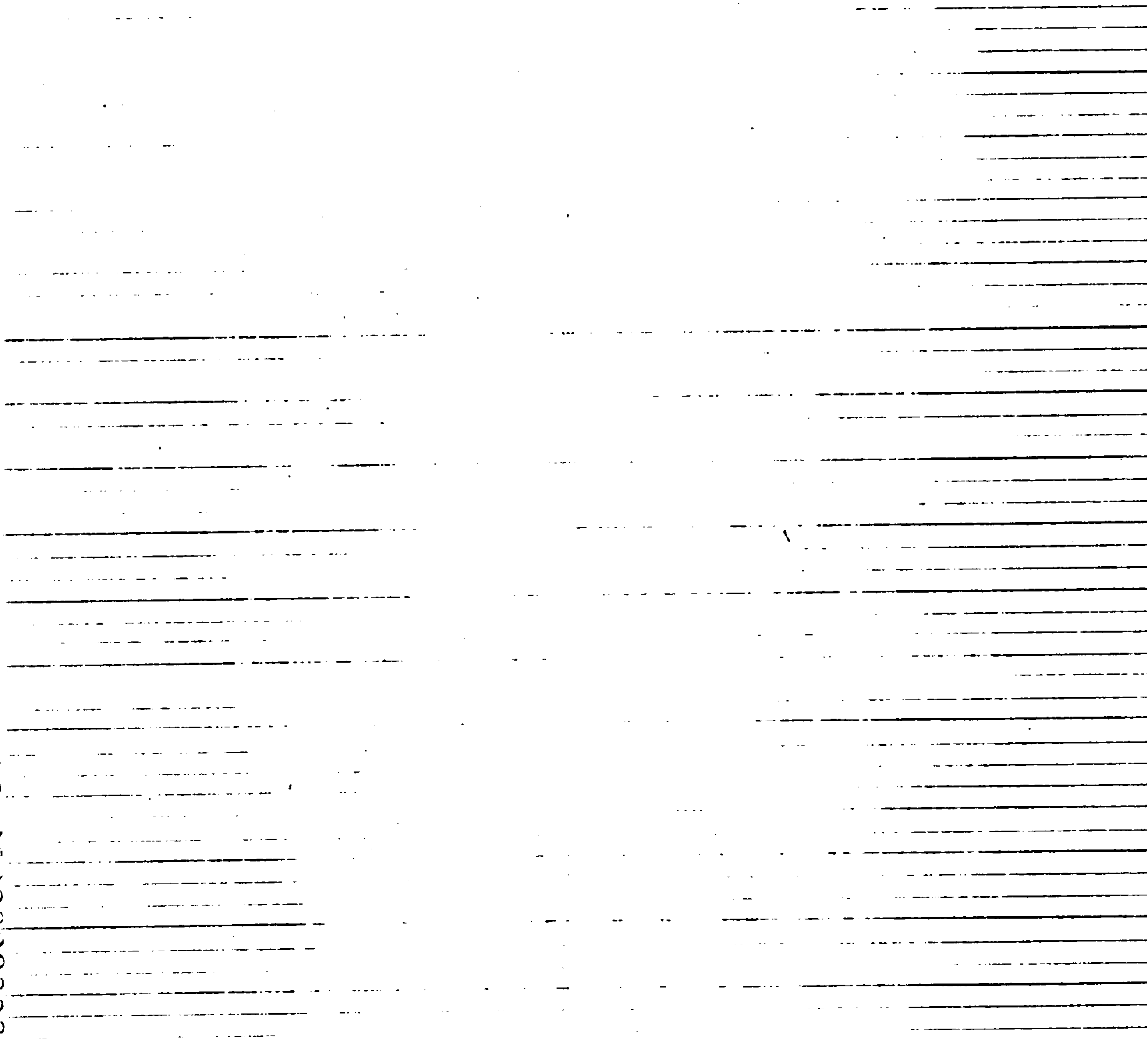

Fig. 6 (Continued)

```
   98   257   256   255   264   277   278   279   [illegible]
   99   279   278   277   256   299   300   301   [illegible]
  100   301   300   299   308   321   322   323   [illegible]
  101   323   322   321   330   343   344   345   [illegible]
  102   345   344   343   352   365   366   367   [illegible]
    4
        .1950000E+08     .1480000E+07     .1480000E+07     .3000000E+00
        .3000000E+00     .3000000E+00     .8000000E+06     .8000000E+06
        .8000000E+06     .3500000E+02    -.2300000E-06     .1490000E-04
        .1490000E-04     .0000000E+00     .5560000E-02     .5560000E-02
        .1950000E+08     .1480000E+07     .1480000E+07     .3000000E+00
        .3000000E+00     .3000000E+00     .8000000E+06     .8000000E+06
        .8000000E+06    -.3500000E+02    -.2300000E-06     .1490000E-04
        .1490000E-04     .0000000E+00     .5560000E-02     .5560000E-02
        .1950000E+08     .1480000E+07     .1480000E+07     .3000000E+00
        .3000000E+00     .3000000E+00     .8000000E+06     .8000000E+06
        .8000000E+06     .0000000E+00    -.2300000E-06     .1490000E-04
        .1490000E-04     .0000000E+00     .5560000E-02     .5560000E-02
        .1950000E+08     .1480000E+07     .1480000E+07     .3000000E+00
        .3000000E+00     .3000000E+00     .8000000E+06     .8000000E+06
        .8000000E+06     .9000000E+02    -.2300000E-06     .1490000E-04
        .1490000E-04     .0000000E+00     .5560000E-02     .5560000E-02
   86   102     1     1
   49    55     1     2
   35    49     1     3
    1    34     1     4
    0     0     0     0
    4     3  .7500
        .2540000E-02    -.2800000E+03     .6000000E+00
   44    45
  205   196
  226   227   216   217
    1     1     1     0
    2     1     1     0
    3     1     1     0
    4     1     1     0
    5     1     1     0
    6     1     1     0
    7     1     1     0
    8     1     1     0
    9     1     1     0
   10     1     1     0
   11     1     1     0
   12     1     1     0
   13     1     1     0
    1     0     0     1
   14     0     0     1
   21     0     0     1
   34     0     0     1
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ORIGINAL PAGE IS
OF POOR QUALITY

Fig. 6 (Continued)

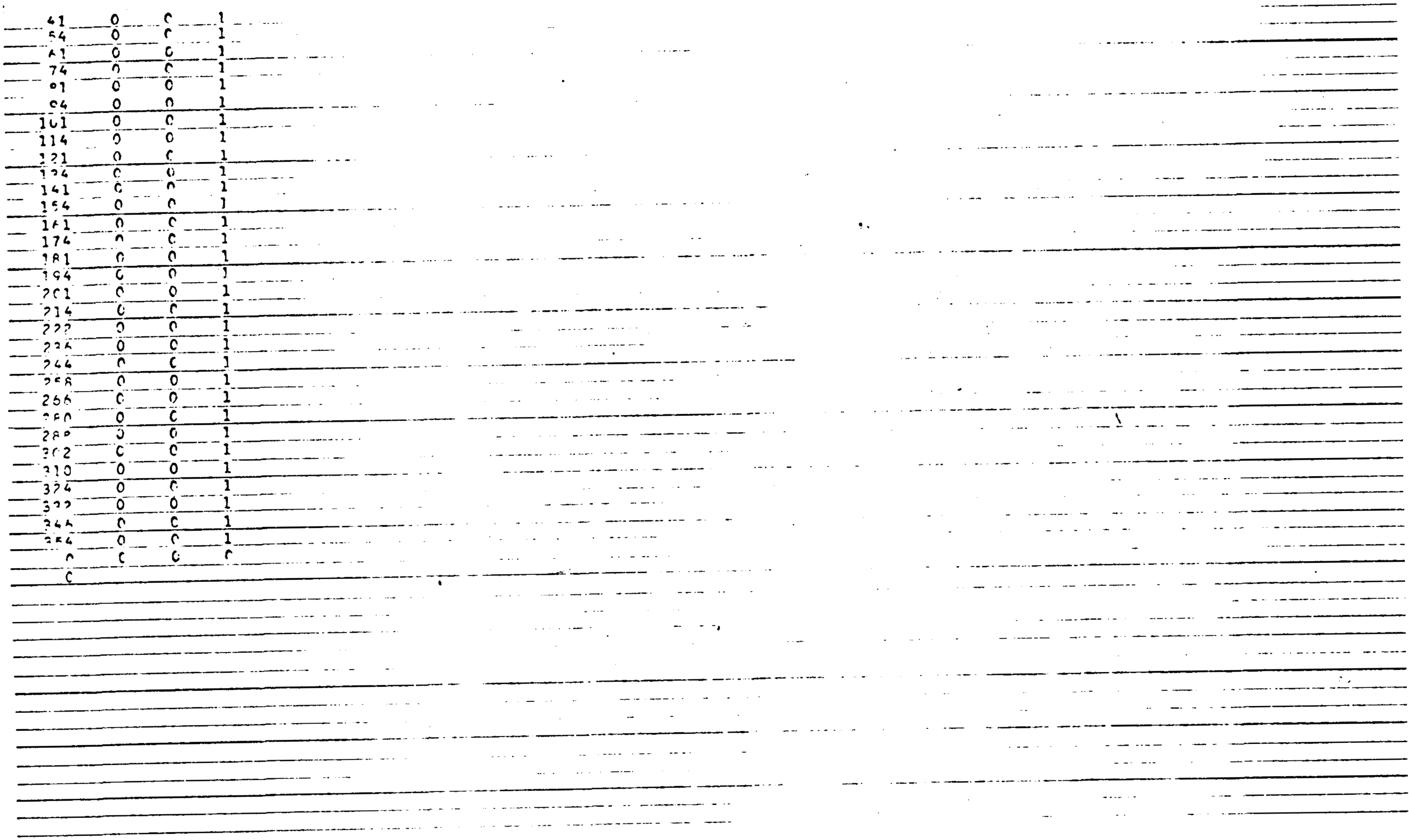

Fig. 6 (Concluded)

```
****************************************************************************************
        (+-35/0/90) SYMM LAMINATE. G-CAL. FOR MECH,THERM,AND HYGRO. LOADINGS
****************************************************************************************
XLONG

                         SOME PROBLEM SPECIFICATIONS

      TOTAL NUMBER OF NODES                    =    367
      NUMBER OF ELEMENTS                       =    102
      NUMBER OF DEGREES OF FREEDOM PER NODE    =      3
      NUMBER OF NODES PER ELEMENT              =      8
```

ORIGINAL PAGE IS
OF POOR QUALITY

Fig. 7 Long output of Q3DG for the input shown in fig. 6.

Y-COORDINATES AND NODE NUMBERS

.00000	1	2	3	4	5	6	7	8	9	10
.00000	11	12	13	0	0	0	0	0	0	0
.10800	14	15	16	17	18	19	20	0	0	0
.21600	21	22	23	24	25	26	27	28	29	30
.21600	31	32	33	0	0	0	0	0	0	0
.32400	34	35	36	37	38	39	40	0	0	0
.43200	41	42	43	44	45	46	47	48	49	50
.43200	51	52	53	0	0	0	0	0	0	0
.48600	54	55	56	57	58	59	60	0	0	0
.54000	61	62	63	64	65	66	67	68	69	70
.54000	71	72	73	0	0	0	0	0	0	0
.55890	74	75	76	77	78	79	80	0	0	0
.57780	81	82	83	84	85	86	87	88	89	90
.57780	91	92	93	0	0	0	0	0	0	0
.58860	94	95	96	97	98	99	100	0	0	0
.59940	101	102	103	104	105	106	107	108	109	110
.59940	111	112	113	0	0	0	0	0	0	0
.60480	114	115	116	117	118	119	120	0	0	0
.61020	121	122	123	124	125	126	127	128	129	130
.61020	131	132	133	0	0	0	0	0	0	0
.61290	134	135	136	137	138	139	140	0	0	0
.61560	141	142	143	144	145	146	147	148	149	150
.61560	151	152	153	0	0	0	0	0	0	0
.61695	154	155	156	157	158	159	160	0	0	0
.61830	161	162	163	164	165	166	167	168	169	170
.61830	171	172	173	0	0	0	0	0	0	0
.61898	174	175	176	177	178	179	180	0	0	0
.61965	181	182	183	184	185	186	187	188	189	190
.61965	191	192	193	0	0	0	0	0	0	0
.62033	194	195	196	197	198	199	200	0	0	0
.62100	201	202	203	204	205	206	207	208	209	210
.62100	211	212	213	0	0	0	0	0	0	0
.62168	214	215	216	217	218	219	220	221	0	0
.62235	222	223	224	225	226	227	228	229	280	231
.62235	232	233	234	235	0	0	0	0	0	0
.62303	236	237	238	239	240	241	242	243	0	0
.62370	244	245	246	247	248	249	250	251	252	253
.62370	254	255	256	257	0	0	0	0	0	0
.62505	258	259	260	261	262	263	264	265	0	0
.62640	266	267	268	269	270	271	272	273	274	275
.62640	276	277	278	279	0	0	0	0	0	0
.62910	280	281	282	283	284	285	286	287	0	0
.63180	288	289	290	291	292	293	294	295	296	297
.63180	298	299	300	301	0	0	0	0	0	0
.63720	302	303	304	305	306	307	308	309	0	0
.64260	310	311	312	313	314	315	316	317	318	319

Fig. 7 (Continued)

.64260	320	321	322	323	0	0	0	0	0	0
.65340	324	325	326	327	328	329	330	331	0	0
.66420	332	333	334	335	336	337	338	339	340	341
.66420	342	343	344	345	0	0	0	0	0	0
.70710	346	347	348	349	350	351	352	353	0	0
.75000	354	355	356	357	358	359	360	361	362	363
.75000	364	365	366	367	0	0	0	0	0	0
.00000	0	0	0	0	0	0	0	0	0	0

Fig. 7 (Continued)

Z-COORDINATE AND NODE NUMBERS

.00000	1	14	21	[illegible]	41	54	61	74	81	94
.00000	101	114	121	[illegible]	141	154	161	174	181	194
.00000	201	214	222	[illegible]	244	258	266	280	288	302
.00000	310	324	332	[illegible]	354	0	0	0	0	0
.00203	2	22	42	62	82	102	122	142	162	182
.00203	202	223	245	267	289	311	333	355	0	0
.00405	3	15	23	35	43	55	63	75	83	95
.00405	103	115	123	135	143	155	163	175	183	195
.00405	203	215	224	237	246	259	268	281	290	303
.00405	312	325	334	347	356	0	0	0	0	0
.00473	4	24	44	64	84	104	124	144	164	184
.00473	204	225	247	269	291	313	335	357	0	0
.00540	5	16	25	35	45	56	65	76	85	96
.00540	105	116	125	135	145	156	165	176	185	196
.00540	205	216	217	226	227	238	239	248	249	260
.00540	261	270	271	282	283	292	293	304	305	314
.00540	315	326	327	336	337	348	349	358	359	0
.00608	6	26	46	66	86	106	126	146	166	186
.00608	206	228	250	272	294	316	338	360	0	0
.00675	7	17	27	37	47	57	67	77	87	97
.00675	107	117	127	137	147	157	167	177	187	197
.00675	207	218	229	240	251	262	273	284	295	306
.00675	317	328	339	350	351	0	0	0	0	0
.00878	8	28	48	68	88	108	128	148	168	188
.00878	208	230	252	274	296	318	340	362	0	0
.01080	9	18	29	38	49	58	69	78	89	98
.01080	109	118	129	138	149	158	169	178	189	198
.01080	209	219	231	241	253	263	275	285	297	307
.01080	319	329	341	[illegible]	353	0	0	0	0	0
.01350	10	30	50	70	90	110	130	150	170	190
.01350	210	232	254	276	298	320	342	364	0	0
.01620	11	19	31	39	51	59	71	79	91	99
.01620	111	119	131	139	151	159	171	179	191	199
.01620	211	220	233	242	255	264	277	286	299	308
.01620	321	330	343	352	355	0	0	0	0	0
.01890	12	32	52	72	92	112	132	152	172	192
.01890	212	234	256	[illegible]	300	322	344	366	0	0
.02160	13	20	33	40	53	60	73	80	93	100
.02160	113	120	133	140	153	160	173	180	193	200
.02160	213	221	235	243	257	265	279	287	301	309
.02160	323	331	345	353	357	0	0	0	0	0
.00000	0	0	0	0	0	0	0	0	0	0

Fig. 7 (Continued)

ELEMENT NUMBER AND CONNECTIVITY

1	3	2	1	14	21	22	23	15
2	23	22	21	34	41	42	43	35
3	43	42	41	54	61	62	63	55
4	63	62	61	74	81	82	83	75
5	83	82	81	94	101	102	103	95
6	103	102	101	114	121	122	123	115
7	123	122	121	134	141	142	143	135
8	143	142	141	154	161	162	163	155
9	163	162	161	174	181	182	183	175
10	183	182	181	194	201	202	203	195
11	203	202	201	214	222	223	224	215
12	224	223	222	236	244	245	246	237
13	246	245	244	258	266	267	268	259
14	268	267	266	280	288	289	290	281
15	290	289	288	302	310	311	312	303
16	312	311	310	324	332	333	334	325
17	334	333	332	346	354	355	356	347
18	5	4	3	15	23	24	25	16
19	25	24	23	35	43	44	45	36
20	45	44	43	55	63	64	65	56
21	65	64	63	75	83	84	85	76
22	85	84	83	95	103	104	105	96
23	105	104	103	115	123	124	125	116
24	125	124	123	135	143	144	145	136
25	145	144	143	155	163	164	165	156
26	165	164	163	175	183	184	185	176
27	185	184	183	195	203	204	205	196
28	205	204	203	215	224	225	226	216
29	226	225	224	237	246	247	248	238
30	248	247	246	259	268	269	270	260
31	270	269	268	281	290	291	292	282
32	292	291	290	303	312	313	314	304
33	314	313	312	325	334	335	336	326
34	336	335	334	347	356	357	358	348
35	7	6	5	16	25	26	27	17
36	27	26	25	36	45	46	47	37
37	47	46	45	56	65	66	67	57
38	67	66	65	76	85	86	87	77
39	87	86	85	96	105	106	107	97
40	107	106	105	116	125	126	127	117
41	127	126	125	136	145	146	147	137
42	147	146	145	156	165	166	167	157
43	167	166	165	176	185	186	187	177
44	187	186	185	196	205	206	207	197
45	207	206	205	217	227	228	229	218
46	229	228	227	239	249	250	251	240

Fig. 7 (Continued)

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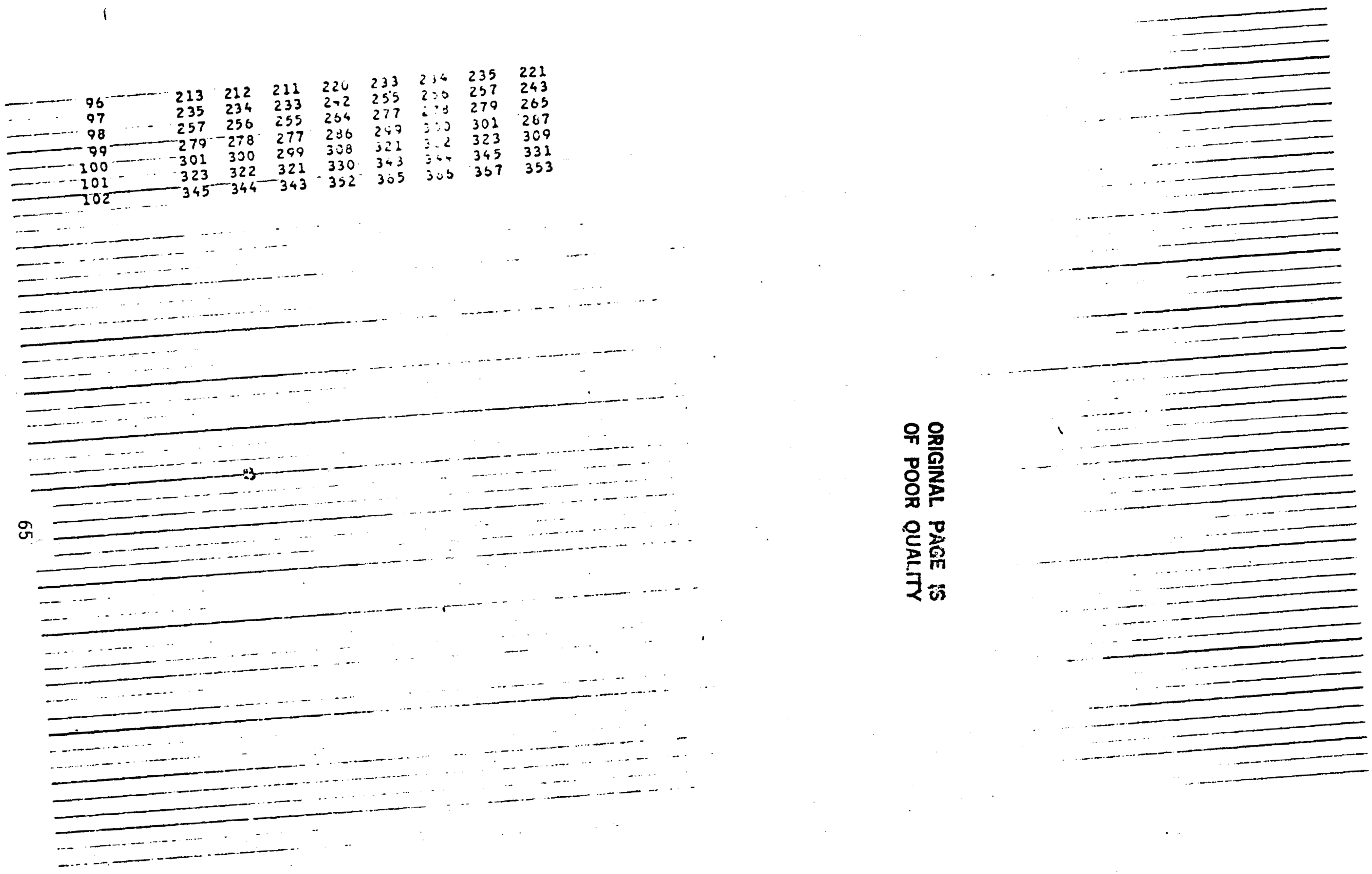
47	251	250	249	261	271	272	273	262
48	273	272	271	283	293	294	295	284
49	295	294	293	305	315	316	317	306
50	317	316	315	327	337	338	339	328
51	339	338	337	349	359	360	361	350
52	9	8	7	17	27	28	29	18
53	29	28	27	37	47	48	49	38
54	49	48	47	57	67	68	69	58
55	69	68	67	77	87	88	89	78
56	89	88	87	97	107	108	109	98
57	109	108	107	117	127	128	129	118
58	129	128	127	137	147	148	149	138
59	149	148	147	157	167	168	169	158
60	169	168	167	177	187	188	189	178
61	189	188	187	197	207	208	209	198
62	209	208	207	218	229	230	231	219
63	231	230	229	240	251	252	253	241
64	253	252	251	262	273	274	275	263
65	275	274	273	284	295	296	297	285
66	297	296	295	306	317	318	319	307
67	319	318	317	328	339	340	341	329
68	341	340	339	350	361	362	363	351
69	11	10	9	18	29	30	31	19
70	31	30	29	38	49	50	51	39
71	51	50	49	58	69	70	71	59
72	71	70	69	78	89	90	91	79
73	91	90	89	98	109	110	111	99
74	111	110	109	118	129	130	131	119
75	131	130	129	138	149	150	151	139
76	151	150	149	158	169	170	171	159
77	171	170	169	178	189	190	191	179
78	191	190	189	198	209	210	211	199
79	211	210	209	219	231	232	233	220
80	233	232	231	241	253	254	255	242
81	255	254	253	263	275	276	277	264
82	277	276	275	285	297	298	299	286
83	299	298	297	307	319	320	321	308
84	321	320	319	329	341	342	343	330
85	343	342	341	351	363	364	365	352
86	13	12	11	19	31	32	33	20
87	33	32	31	39	51	52	53	40
88	53	52	51	59	71	72	73	60
89	73	72	71	79	91	92	93	80
90	93	92	91	99	111	112	113	100
91	113	112	111	119	131	132	133	120
92	133	132	131	139	151	152	153	140
93	153	152	151	159	171	172	173	160
94	173	172	171	179	191	192	193	180
95	193	192	191	199	211	212	213	200

Fig. 7 (Continued)



96	213	212	211	220	233	234	235	221
97	235	234	233	242	255	256	257	243
98	257	256	255	264	277	278	279	265
99	279	278	277	286	299	300	301	287
100	301	300	299	308	321	322	323	309
101	323	322	321	330	343	344	345	331
102	345	344	343	352	365	365	367	353

Fig. 7 (Continued)

```
                         MATERIAL SPECIFICATIONS


                SPECIFICATION MATRIX IS IN THE FOLLOWING FORM


                    +-                                 -+
                    ! E11     E22     E33     NU12      !
                    ! NU13    NU23    G12     G13       !
                    ! G23     ANGLE   ALPHA1  ALPHA2    !
                    ! ALPHA3  BETA1   BETA2   BETA3     !
                    +-                                 -+


      NUMBER OF MATERIALS =     4
```

Fig. 7 (Continued)

MATERIAL NUMBER 1

SPECIFICATIJN MATRIX

.1950000E+08	.1480000E+07	.1480000E+07	.3000000E+00
.3000000E+00	.3000000E+00	.8000000E+06	.8000000E+06
.8000000E+06	.3500000E+02	-.2300000E-06	.1490000E-04
.1490000E-04	.0000000E+00	.5560000E-02	.5560000E-02

MODULUS MATRIX

.1012507E+08	.4408970E+07	.6015248E+06	.5654447E+07	.0000000E+00	.0000000E+00
.4408970E+07	.3886374E+07	.5543408E+06	.2915893E+07	.0000000E+00	.0000000E+00
.6015248E+06	.5543408E+06	.1647416E+07	.6481843E+05	.0000000E+00	.0000000E+00
.5654447E+07	.2915893E+07	.6481843E+05	.4562059E+07	.0000000E+00	.0000000E+00
.0000000E+00	.0000000E+00	.0000000E+00	.0000000E+00	.8000000E+06	.0000000E+00
.0000000E+00	.0000000E+00	.0000000E+00	.0000000E+00	.0000000E+00	.8000000E+06

THERMAL VECTOR

.4747618E-05	.9922382E-05	.1490000E-04	-.1421755E-04	.0000000E+00	.0000000E+00

HYGROSCOPIC VECTOR

.1829184E-02	.3730816E-02	.5560000E-02	-.5224691E-02	.0000000E+00	.0000000E+00

Fig. 7 (Continued)

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MATERIAL NUMBER 2

SPECIFICATION MATRIX

.1950000E+08	.1480000E+07	.1480000E+07	.3000000E+00
.3000000E+00	.3000000E+00	.8000000E+06	.8000000E+06
.8000000E+06	-.3500000E+02	-.2300000E-06	.1490000E-04
.1490000E-04	.0000000E+00	.5560000E-02	.5560000E-02

MODULUS MATRIX

.1012507E+08	.4408970E+07	.6015248E+06	-.5654447E+07	.0000000E+00	.0000000E+00
.4408970E+07	.3885374E+07	.5543408E+06	-.2915893E+07	.0000000E+00	.0000000E+00
.6015248E+06	.5543408E+06	.1647416E+07	-.6481843E+05	.0000000E+00	.0000000E+00
-.5654447E+07	-.2915893E+07	-.6481843E+05	.4562059E+07	.0000000E+00	.0000000E+00
.0000000E+00	.0000000E+00	.0000000E+00	.0000000E+00	.8000000E+06	.0000000E+00
.0000000E+00	.0000000E+00	.0000000E+00	.0000000E+00	.0000000E+00	.8000000E+06

THERMAL VECTOR

.4747618E-05	.9922382E-05	.1490000E-04	.1421755E-04	.0000000E+00	.0000000E+00

HYGROSCOPIC VECTOR

.1829184E-02	.3730816E-02	.5560000E-02	.5224691E-02	.0000000E+00	.0000000E+00

Fig. 7 (Continued)

MATERIAL NUMBER 3

SPECIFICATION MATRIX

.1950000E+08	.1480000E+07	.1480000E+07	.3000000E+00
.3000000E+00	.3000000E+00	.8000000E+06	.8000000E+06
.8000000E+06	.0000000E+00	-.2300000E-06	.1490000E-04
.1490000E-04	.0000000E+00	.5560000E-02	.5560000E-02

MODULUS MATRIX

.1988815E+08	.6469111E+06	.6469111E+06	.0000000E+00	.0000000E+00	.0000000E+00
.6469111E+06	.1647416E+07	.5089545E+06	.0000000E+00	.0000000E+00	.0000000E+00
.6469111E+06	.5089545E+06	.1647416E+07	.0000000E+00	.0000000E+00	.0000000E+00
.0000000E+00	.0000000E+00	.0000000E+00	.8000000E+06	.0000000E+00	.0000000E+00
.0000000E+00	.0000000E+00	.0000000E+00	.0000000E+00	.8000000E+06	.0000000E+00
.0000000E+00	.0000000E+00	.0000000E+00	.0000000E+00	.0000000E+00	.8000000E+06

THERMAL VECTOR

-.2300000E-06	.1490000E-04	.1490000E-04	.0000000E+00	.0000000E+00	.0000000E+00

HYGROSCOPIC VECTOR

.0000000E+00	.5560000E-02	.5560000E-02	.0000000E+00	.0000000E+00	.0000000E+00

Fig. 7 (Continued)

MATERIAL NUMBER 4

SPECIFICATION MATRIX

.1950000E+08	.1480000E+07	.1480000E+07	.3000000E+00
.3000000E+00	.3000000E+00	.8000000E+06	.8000000E+06
.8000000E+06	.9000000E+02	-.2300000E-06	.1490000E-04
.1490000E-04	.0000000E+00	.5560000E-02	.5560000E-02

MODULUS MATRIX

.1647416E+07	.6469111E+06	.5089545E+06	-.1229521E-03	.0000000E+00	.0000000E+00
.6469111E+06	.1988815E+08	.6469111E+06	-.3618090E-02	.0000000E+00	.0000000E+00
.5089545E+06	.6469111E+06	.1647416E+07	-.2829392E-04	.0000000E+00	.0000000E+00
-.1229521E-03	-.3618090E-02	-.2829392E-04	.8000000E+06	.0000000E+00	.0000000E+00
.0000000E+00	.0000000E+00	.0000000E+00	.0000000E+00	.8000000E+06	.0000000E+00
.0000000E+00	.0000000E+00	.0000000E+00	.0000000E+00	.0000000E+00	.8000000E+06

THERMAL VECTOR

.1490000E-04	-.2300000E-06	.1490000E-04	.6206107E-14	.0000000E+00	.0000000E+00

HYGROSCOPIC VECTOR

.5560000E-02	.2338705E-21	.5560000E-02	.2280632E-11	.0000000E+00	.0000000E+00

Fig. 7 (Continued)

SOME MORE PROBLEM SPECIFICATIONS

PLY-MATERIAL CORRELATION

FIRST ELEMENT	LAST ELEMENT	INCREMENT	MATERIAL CODE
86	102	1	1
69	85	1	2
35	68	1	3
1	34	1	4
0	0	0	0

```
NUMBER OF PLIES                 =     4
NUMBER OF RIGHT HAND VECTORS    =     3
WIDTH OF THE LAMINATE           =      .7500

MECHANICAL STRAIN  =        .2540000E-02
DELTA T            =       -.2800000E+03
DELTA M            =        .6000000E+00
```

ELEMENTS IN THE G CALCULATION

44 45

NODES INVOLVED IN THE FORCES IN THE G CALCULATION

205 196

NODES INVOLVED IN THE DISPLACEMENTS IN THE G CALCULATION

226 227 216 217

Fig. 7 (Continued)

BOUNDARY CONDITIONS

1-FIXED 0-FREE

NODE	U	V	W
1	1	1	0
2	1	1	0
3	1	1	0
4	1	1	0
5	1	1	0
6	1	1	0
7	1	1	0
8	1	1	0
9	1	1	0
10	1	1	0
11	1	1	0
12	1	1	0
13	1	1	0
1	0	0	1
14	0	0	1
21	0	0	1
34	0	0	1
41	0	0	1
54	0	0	1
61	0	0	1
74	0	0	1
81	0	0	1
94	0	0	1
101	0	0	1
114	0	0	1
121	0	0	1
134	0	0	1
141	0	0	1
154	0	0	1
161	0	0	1
174	0	0	1
181	0	0	1
194	0	0	1
201	0	0	1
214	0	0	1
222	0	0	1
236	0	0	1
244	0	0	1
258	0	0	1
266	0	0	1
280	0	0	1
288	0	0	1
302	0	0	1

Fig. 7 (Continued)

```
     310     0     0     1
     324     0     0     1
     332     0     0     1
     346     0     0     1
     354     0     0     1
       0     0     0     0

    RENUMBERING OPTION = 0    NO RENUMBERING SCHEME SPECIFIED


***********    INPUT COMPLETED    **********

     1.0000      .0000     1.0000      .0000      .0000     1.0000      .0000      .0000
     1.0000      .0000      .5000      .0000      .0000      .2500      .0000      .0000
     1.0000      .0000      .0000      .0000      .0000      .0000      .0000      .0000
     1.0000      .5000      .0000      .0000      .2500      .0000      .0000      .0000
     1.0000     1.0000      .0000      .0000     1.0000      .0000      .0000      .0000
     1.0000     1.0000      .5000      .5000     1.0000      .0000      .0000      .0000
     1.0000     1.0000     1.0000     1.0000     1.0000      .2500      .5000      .2500
     1.0000      .5000     1.0000      .5000      .2500     1.0000     1.0000     1.0000
                                                            1.0000      .2500      .5000

      .0000      .0000     1.0000      .0000      .0000      .0000      .0000      .0000
      .0000      .0000    -3.0000     4.0000    -1.0000      .0000      .0000      .0000
    -1.0000     4.0000    -3.0000      .0000      .0000      .0000      .0000      .0000
    -1.0000    -4.0000     5.0000    -4.0000    -1.0000     4.0000    -3.0000     4.0000
      .0000      .0000     2.0000    -4.0000     2.0000      .0000      .0000      .0000
     2.0000    -4.0000     2.0000      .0000      .0000      .0000      .0000      .0000
     2.0000      .0000    -2.0000     4.0000    -2.0000      .0000     2.0000    -4.0000
    -2.0000     4.0000    -2.0000      .0000     2.0000    -4.0000     2.0000      .0000
```

Fig. 7 (Continued)

The rest of the output is too long to be reproduced here.

The complete output file can be found on the microfiche and on the PC Floppy Disk-2 supplied with this manual.

Fig. 7 (continued)

SUMMARY OF RESULTS

RESULTS FOR STRAIN= .254E-02 DELTA T= -280.00 DELTA M= .60

STRAIN ENERGY RELEASE RATES ARE AS FOLLOWS

	M	T	H	M+T	M+H	T+H	M+T+H
MODE1	.10352E+00	.86638E-02	.53724E-02	.17251E+00	.61380E-01	.39137E-03	.11673E+00
MODE2	.65995E-02	.48617E-01	.30147E-01	.91146E-01	.84531E-02	.21962E-02	.16432E-01
MODE3	.69244E-04	.45194E-04	.28025E-04	.22986E-05	.18557E-03	.20416E-05	.47451E-04
TOTAL	.11019E+00	.57326E-01	.35547E-01	.26366E+00	.70018E-01	.25896E-02	.13321E+00

PERCENTAGES

	M	T	H	M+T	M+H	T+H	M+T+H
MODE1	93.95	15.11	15.11	65.43	87.66	15.11	87.63
MODE2	5.99	84.81	84.61	34.57	12.07	84.81	12.34
MODE3	.06	.08	.08	.00	.27	.08	.04

CROSS TERMS

	MT	MH	TH
MODE1	.60332E-01	-.47509E-01	-.13645E-01
MODE2	.35930E-01	-.28293E-01	-.76568E-01
MODE3	-.11214E-03	.88306E-04	-.71178E-04

Fig. 7 (Continued)

```
TOTAL          .9[illegible]150E-01     -.75714E-01     -.90283E-01

----------------------------------------------------------------------------------------
ALL ELEMENTS SATISFY EQUILIBRIUM.  THE SOLUTION MAY BE CORRECT.
----------------------------------------------------------------------------------------
```

Fig. 7 (Concluded)

4	.1950000E+08	.1480000E+07	.1480000E+07	.3000000E+00
	.3000000E+00	.3000000E+00	.8000000E+06	.8000000E+06
	.8000000E+06	.4500000E+02	-.2300000E-06	.1490000E-04
	.1490000E-04	.0000000E+00	.5560000E-02	.5560000E-02
	.1950000E+08	.1480000E+07	.1480000E+07	.3000000E+00
	.3000000E+00	.3000000E+00	.8000000E+06	.8000000E+06
	.8000000E+06	-.4500000E+02	-.2300000E-06	.1490000E-04
	.1490000E-04	.0000000E+00	.5560000E-02	.5560000E-02
	.1950000E+08	.1480000E+07	.1480000E+07	.3000000E+00
	.3000000E+00	.3000000E+00	.8000000E+06	.8000000E+06
	.8000000E+06	.0000000E+00	-.2300000E-06	.1490000E-04
	.1490000E-04	.0000000E+00	.5560000E-02	.5560000E-02
	.1950000E+08	.1480000E+07	.1480000E+07	.3000000E+00
	.3000000E+00	.3000000E+00	.8000000E+06	.8000000E+06
	.8020000E+06	.9200000E+02	-.2300000E-06	.1490000E-04
	.1490000E-04	.0000000E+00	.5560000E-02	.5560000E-02
86	102 1 1			

Fig. 8 Partial listing of input data for Example Problem 2 .

```
******************************************************************************************
     (+-45/0/90) SYMM LAMINATE. 3-CAL. FOR M+T+H LOADING SHORT OUTPUT OPTION
******************************************************************************************
SHORT

                         SOME PROBLEM SPECIFICATIONS

   TOTAL NUMBER OF NODES                         =   367
   NUMBER OF ELEMENTS                            =   102
   NUMBER OF DEGREES OF FREEDOM PER NODE         =     3
   NUMBER OF NODES PER ELEMENT                   =     8
```

Fig. 9 Short output of Q3DG for the input shown in fig. 8.

The rest of the output is too long to be reproduced here.

The complete output file can be found on the microfiche and on the PC Floppy Disk-2 supplied with this manual.

Fig. 9 (continued)

SUMMARY OF RESULTS

RESULTS FOR STRAIN= .254E-02 DELTA T= -280.00 DELTA M= .60

STRAIN ENERGY RELEASE RATES ARE AS FOLLOWS

	M	T	H	M+T	M+H	T+H	M+T+H
MODE1	.61288E-01	-.40325E-04	-.25005E-04	.65711E-01	.57748E-01	-.18216E-05	.62235E-01
MODE2	.64599E-02	.38321E-01	.23763E-01	.76316E-01	.53897E-02	.17311E-02	.14893E-01
MODE3	.48217E-04	.14410E-03	.19392E-04	.25357E-04	.26913E-03	.65121E-05	.19231E-04
TOTAL	.67796E-01	.38425E-01	.23627E-01	.14205E+00	.63407E-01	.17358E-02	.77148E-01

PERCENTAGES

	M	T	H	M+T	M+H	T+H	M+T+H
MODE1	90.40	-.10	-.10	46.26	91.08	-.10	80.67
MODE2	9.53	99.73	99.73	53.72	8.50	99.73	19.31
MODE3	.07	.38	.38	.02	.42	.38	.02

CROSS TERMS

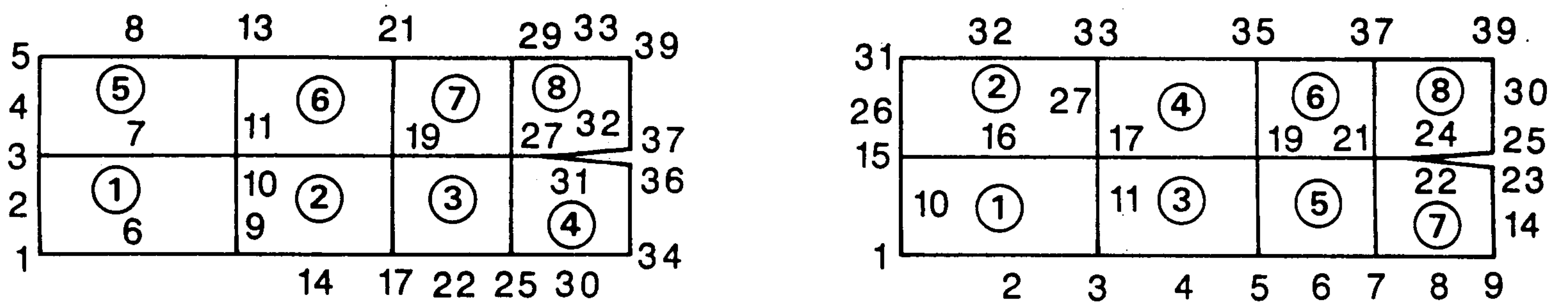
	MT	MH	TH
MODE1	.44035E-02	-.35149E-02	.63508E-04
MODE2	.31535E-01	-.24833E-01	-.60353E-01
MODE3	-.16702E-03	.13152E-03	-.22704E-03

Fig. 9 (Continued)

TOTAL	.356832E-01	-.28216E-01	-.60516E-01

ALL ELEMENTS SATISFY EQUILIBRIUM. THE SOLUTION MAY BE CORRECT.

Fig. 9 (Concluded)



(a) Thickness wise numbering

(b) Width wise numbering

(I) = element #

i = node #

Fig. 10 Very Coarse Models for a $[0/90]_S$ Laminate

```
( 0/90 ) SYMM LAMINATE. WIDTH-WISE NUMBERING SCHEME. RENUMBERED. M+T+H LOADING
XLONG
  39     8     3     8
   .000000     1    15    31     0     0     0     0     0     0     0
   .250000     3    17    33     0     0     0     0     0     0     0
   .450000     5    19    35     0     0     0     0     0     0     0
   .600000     7    21    37     0     0     0     0     0     0     0
   .750000     9    23    25    39     0     0     0     0     0     0
   .000000     0
   .000000     1     3     5     7     9     0     0     0     0     0
   .005400    15    17    19    21    23    25     0     0     0     0
   .010800    31    33    35    37    39     0     0     0     0     0
   .000000     0
    1    15    10     1     2     3    11    17    16
    2    31    26    15    16    17    27    33    32
    3    17    11     3     4     5    12    19    18
    4    33    27    17    18    19    28    35    34
    5    19    12     5     6     7    13    21    20
    6    35    28    19    20    21    29    37    36
    7    21    13     7     8     9    14    23    22
    8    37    29    21    24    25    30    39    38
    2
        .1950000E+08        .1480000E+07        .1480000E+07        .3000000E+00
        .3000000E+00        .3000000E+00        .8000000E+06        .8000000E+06
        .8000000E+06        .0000000E+00       -.2300000E-06        .1490000E-04
        .1490000E-04        .0000000E+00        .5560000E-02        .5560000E-02
        .1950000E+08        .1480000E+07        .1480000E+07        .3000000E+00
        .3000000E+00        .3000000E+00        .8000000E+06        .8000000E+06
        .8000000E+06        .9000000E+02       -.2300000E-06        .1490000E-04
        .1490000E-04        .0000000E+00        .5560000E-02        .5560000E-02
    2     8     2     1
    1     7     2     2
    0     0     0     0
    2     3   .7500
        .2540000E-02       -.2800000E+03        .6000000E+00
    6     8
   21    20
   23    25    22    24
    1     0     0     1
    2     0     0     1
    3     0     0     1
    4     0     0     1
    5     0     0     1
    6     0     0     1
    7     0     0     1
    8     0     0     1
    9     0     0     1
```

Fig. 11 Input data for the model shown in fig. 10(b).

1	1	1	0												
10	1	1	0												
15	1	1	0												
26	1	1	0												
31	1	1	0												
0	0	0	0												
1															
1	6	9	14	17	22	25	32	34	2	10	18	26	35	3	7
11	15	19	23	27	31	36	32	37	4	12	20	28	38	5	8
13	16	21	24	29	33	39									

Fig. 11 (Concluded)

S U M M A R Y   O F   R E S U L T S

RESULTS FOR   STRAIN=   .254E-02   DELTA T=   -280.00   DELTA M=   .60

STRAIN ENERGY RELEASE RATES ARE AS FOLLOWS

	M	T	H	M+T	M+H	T+H	M+T+H
MODE1	.79734E-05	.26255E-03	.16280E-03	.36203E-03	.98719E-04	.11860E-04	.39282E-04
MODE2	.18438E-02	.60713E-01	.37648E-01	.83718E-01	.22829E-01	.27426E-02	.90839E-02
MODE3	-.13262E-22	.48994E-20	.30381E-20	.47805E-20	.31080E-20	.22132E-21	.18560E-21
TOTAL	.18518E-02	.60976E-01	.37811E-01	.84080E-01	.22927E-01	.27545E-02	.91232E-02

P E R C E N T A G E S

MODE1	.43	.43	.43	.43	.43	.43	.43
MODE2	99.57	99.57	99.57	99.57	99.57	99.57	99.57
MODE3	.00	.00	.00	.00	.00	.00	.00

C R O S S   T E R M S

	MT	MH	TH
MODE1	.91507E-04	-.72059E-04	-.41349E-03
MODE2	.21161E-01	-.16663E-01	-.95619E-01

Fig. 12 Strain energy release rates obtained with the models shown in fig. 10.

MODE3	-.10567E-21	.83208E-22	-.77162E-20
TOTAL	21252E-01	-.16735E-01	-.96032E-01

```
-------------------------------------------------------------------------------
   ALL ELEMENTS SATISFY EQUILIBRIUM.  THE SOLUTION MAY BE CORRECT.
-------------------------------------------------------------------------------
```

Fig. 12 (Concluded)

```
0,0.2160,0.432,0.540,0.5778,0.5994,0.610[illegible],0.6156
0.6183,0.61965,0.6210,0.62235,0.6237,0.6[illegible]54,0.6318,0.6426
0.6642,0.7500
0,0.00405,0.0054,0.00675,0.0108,0.0162,0.0216
```

Fig. 13 Listing of the coordinates file, YZ.

```
/ldobj
  FILE NAME FOR WRITING THE DATA (MAXIMUM 6 CHARACTERS)
? trial
TRIAL
  ENTER THE TITLE OF THE PROBLEM
?  (+-35/0/90)symm. laminate. g-cal  mech,therm,and hygro loadings
 (+-35/0/90)SYMM. LAMINATE. G-CAL. MECH,THERM,AND HYGRO LOADINGS
  SPECIFY OUTPUT OPTION
  ENTER   SHORT - FOR SHORT OUTPUT OPTION
          XLONG - FOR LONG OUTPUT OPTION
? xlong
XLONG
  ENTER NUMBER OF Y-COORDS AND Z-COORDS
? 18,7
   18    7
  ENTER Y- AND Z-COORDINATE NUMBER OF THE CRACK TIP
? 11,3
   11    3
  ENTER THE FILE NAME ON WHICH THE COORDS EXIST
? yz
YZ

 THE NUMBER OF Y-COORDINATES IS    18

 THE Y-COORDINATES ARE AS FOLLOWS:

     .0000    .2160    .4320    .5400    .5778    .5994    .6102    .6156
     .6183    .6197    .6210    .6224    .6237    .6264    .6318    .6426
     .6642    .7500


 THE NUMBER OF Z-COORDINATES IS     7

 THE Z-COOORDINATES ARE AS FOLLOWS:

     .0000    .0041    .0054 ,  .0068    .0108    .0162    .0216


  NUMBER OF NODES =   367


  ENTER NUMBER OF MATERIALS
 ? 4
  4
```

Fig. 14 Listing of a DATGEN interactive session for Example Problem 1.

```
 PLY ANGLE FOR MATERIAL   1
? 35
 35.
 PLY ANGLE FOR MATERIAL   2
? -35
 -35.
 PLY ANGLE FOR MATERIAL   3
? 0
 0.
 PLY ANGLE FOR MATERIAL   4
? 90
 90.
 ENTER NUMBER OF PLIES
? 4
 4
 ** PLIES ARE NUMBERED FROM THE TOP OF THE LAMINATE **
 ENTER MATERIAL NUMBER FOR PLY 1
? 1
 1
 ENTER THICKNESS FOR PLY 1
? .0054
 .0054
 ENTER MATERIAL NUMBER FOR PLY 2
? 2
 2
 ENTER THICKNESS FOR PLY 2
? .0054
 .0054
 ENTER MATERIAL NUMBER FOR PLY 3
? 3
 3
 ENTER THICKNESS FOR PLY 3
? .0054
 .0054
 ENTER MATERIAL NUMBER FOR PLY 4
? 4
 4
 ENTER THICKNESS FOR PLY 4
? .0054
 .0054
 LOADING CONDITIONS ARE MECHANICAL,THERMAL,HYGROSCOPIC
 ENTER NUMBER OF LOADING CONDITIONS (1, 2, OR 3)
? 3
 3
 ENTER MECHANICAL STRAIN.
? .00254
 .00254
 ENTER DELTA-T.
? -3280
 -280.
 ENTER DELTA-M
```

Fig. 14 (Continued)

```
? 0.6
 .6

 ------------------------------------------------------------
          CRACK TIP IS AT NODE = 205
               SUMMARY INFORMATION
                FOR G-CALCULATION
 ------------------------------------------------------------
   44   45
  205  196
  226  227  216  217
  DO YOU WISH TO ENTER A RENUMBERING SCHEME?
 ENTER 1 FOR 'YES' ; 0 TO ACCEPT DEFAULT
? 0
 0
 NO RENUMBERING SCHEME ENTERED IN DATA FILE

          CURRENT BANDWIDTH IN THE MODEL =    75
 DATA GENERATION COMPLETE -- FILE TRIAL
     1.182 CP SECONDS EXECUTION TIME.
/
```

Fig. 14 (Concluded)

```
 PLY ANGLE FOR MATERIAL   1
? 35
 35.
 PLY ANGLE FOR MATERIAL   2
? -35
 -35.
 PLY ANGLE FOR MATERIAL   3
? 0
 0.
 PLY ANGLE FOR MATERIAL   4
? 90
 90.
 ENTER NUMBER OF PLIES
? 4
 4
** PLIES ARE NUMBERED FROM THE TOP OF THE LAMINATE **
 ENTER MATERIAL NUMBER FOR PLY 1
? 1
 1
 ENTER THICKNESS FOR PLY 1
? .005
 .005
**ERROR**  BOTTOM OF PLY NOT A Z-COORD.
PLY NUMBER 1  TOP = .0216  BOTTOM = .0166
PLY THICKNESS = .005
 ENTER THICKNESS FOR PLY 1
? .0054
 .0054
 ENTER MATERIAL NUMBER FOR PLY 2
? 2
 2
 ENTER THICKNESS FOR PLY 2
? .008
 .008
**ERROR**  BOTTOM OF PLY NOT A Z-COORD.
PLY NUMBER 2  TOP = .0162  BOTTOM = .0082
PLY THICKNESS = .008
 ENTER THICKNESS FOR PLY 2
? .0054
 .0054
 ENTER MATERIAL NUMBER FOR PLY 3
? 3
 3
 ENTER THICKNESS FOR PLY 3
? .0055
 .0055
**ERROR**  BOTTOM OF PLY NOT A Z-COORD.
PLY NUMBER 3  TOP = .0108  BOTTOM = .0053
PLY THICKNESS = .0055
 ENTER THICKNESS FOR PLY 3
? .0054
```

Fig. 15 Partial listing of an interactive session with recoverable errors.

```
? .0054
 .0054
  ENTER MATERIAL NUMBER FOR PLY 4
? .01
 0
  ENTER MATERIAL NUMBER FOR PLY 4
? 4
 4
  ENTER THICKNESS FOR PLY 4
? .01
 .01
 **ERROR**  PLY THICKNESS NOT CORRECTLY SPECIFIED.
 BOTTOM OF PLY IS LESS THAN LOWEST Z-COORDINATE.
 PLY NUMBER 4  TOP = .0054  BOTTOM = -.0046
 PLY THICKNESS = .01  LEAST Z-COORD = 0.
  ENTER THICKNESS FOR PLY 4
? .02
 .02
 **ERROR**  PLY THICKNESS NOT CORRECTLY SPECIFIED.
 BOTTOM OF PLY IS LESS THAN LOWEST Z-COORDINATE.
 PLY NUMBER 4  TOP = .0054  BOTTOM = -.0146
 PLY THICKNESS = .02  LEAST Z-COORD = 0.
  ENTER THICKNESS FOR PLY 4
? .0055
 .0055
 **ERROR**  PLY THICKNESS NOT CORRECTLY SPECIFIED.
 BOTTOM OF PLY IS LESS THAN LOWEST Z-COORDINATE.
 PLY NUMBER 4  TOP = .0054  BOTTOM = -.00009999999999999
 PLY THICKNESS = .0055  LEAST Z-COORD = 0.
  ENTER THICKNESS FOR PLY 4
? .0054
 .0054
 LOADING CONDITIONS ARE MECHANICAL,THERMAL,HYGROSCOPIC
 ENTER NUMBER OF LOADING CONDITIONS (1, 2, OR 3)
? 2
 2
 ENTER MECHANICAL STRAIN.
? .00254
 .00254
 ENTER DELTA-T.
? -280
 -280.

 ------------------------------------------------------
         CRACK TIP IS AT NODE = 205
              SUMMARY INFORMATION
               FOR G-CALCULATION
 ------------------------------------------------------
  44   45
 205  196
 226  227  216  217
```

Fig. 15 (Continued)

```
 DO YOU WISH TO ENTER A RENUMBERING SCHEME?
 ENTER 1 FOR 'YES' ; 0 TO ACCEPT DEFAULT
? 0
 0
 NO RENUMBERING SCHEME ENTERED IN DATA FILE

        CURRENT BANDWIDTH IN THE MODEL =    75
 DATA GENERATION COMPLETE -- FILE TRIAL
     1.191 CP SECONDS EXECUTION TIME.
/
```

Fig. 15 (Concluded)

```
 THE NUMBER OF Y-COORDINATES IS     18

 THE Y--COORDINATES ARE AS FOLLOWS:

     .0000     .2160     .4320     .5400     .5778     .5994     .6102     .6156
     .6183     .6197     .6210     .6224     .6237     .6264     .6318     .6426
     .6642     .7500


 THE NUMBER OF Z-COORDINATES IS     7

 THE Z-COOORDINATES ARE AS FOLLOWS:

     .0000     .0041     .0054     .0068     .0108     .0162     .0216


  NUMBER OF NODES =   367


  ENTER NUMBER OF MATERIALS
? 4
 4
  PLY ANGLE FOR MATERIAL   1
? 354
 35.
  PLY ANGLE FOR MATERIAL   2
? -35
 -35.
  PLY ANGLE FOR MATERIAL   3
? 0
 0.
  PLY ANGLE FOR MATERIAL   4
? 90
 90.
  ENTER NUMBER OF PLIES
? 4
 4
 ** PLIES ARE NUMBERED FROM THE TOP OF THE LAMINATE **
  ENTER MATERIAL NUMBER FOR PLY 1
? 1
 1
  ENTER THICKNESS FOR PLY 1
? .0054
 .0054
  ENTER MATERIAL NUMBER FOR PLY 2
? 2
 2
  ENTER THICKNESS FOR PLY 2
? .0108
 .0108
```

Fig. 16 Partial listing of an interactive session with irrecoverable errors.

```
 ENTER MATERIAL NUMBER FOR PLY 3
? 3
 3
 ENTER THICKNESS FOR PLY 3
? .0054
 .0054
 ENTER MATERIAL NUMBER FOR PLY 4
? 4
 4
 ENTER THICKNESS FOR PLY 4
? -.0054
 -.0054
 **ERROR**  PLY INPUT INCORRECT
 BOTTOM OF LAST PLY DIFFERENT FROM LEAST Z-COORD.
 PLY NUMBER 4  TOP = -2.775557561563E-17  BOTTOM = .0054
 PLY THICKNESS = -.0054  LEAST Z-COORD = 0.
 PROGRAM TERMINATED
     1.026 CP SECONDS EXECUTION TIME.
/
```

C-2

Fig. 16 (Concluded)

```
xedit,yzc
 XEDIT LARC 1.0
?? p*
0,0.25,0.45,0.6,0.75
0,.0054,.0108
--EOI/TOP--
?? end
YZC     IS A LOCAL FILE
/rewind,*
   8 FILE(S) PROCESSED.
/xedit,ren
 XEDIT LARC 1.0
?? p*
1,10,15,26,31
2,16,32
3,11,17,27,33
4,18,34
5,12,19,28,35
6,20,36
7,13,21,29,37
8,22,24,38
9,14,23,25,30,39
--EOI/TOP--
?? end
REN     IS A LOCAL FILE
/ldobj
  FILE NAME FOR WRITING THE DATA (MAXIMUM 6 CHARACTERS)
? try
TRY
  ENTER THE TITLE OF THE PROBLEM
?  renumbering scheme for a (0/90) symm laminate
 RENUMBERING SCHEME FOR A (0/90) SYMM LAMINATE
  SPECIFY OUTPUT OPTION
  ENTER  SHORT - FOR SHORT OUTPUT OPTION
         XLONG - FOR LONG OUTPUT OPTION
? short
SHORT
  ENTER NUMBER OF Y-COORDS AND Z-COORDS
? 5,3
    5    3
  ENTER Y- AND Z-COORDINATE NUMBER OF THE CRACK TIP
? 4,2
    4    2
  ENTER THE FILE NAME ON WHICH THE COORDS EXIST
? yzc
YZC

 THE NUMBER OF Y-COORDINATES IS     5
```

Fig. 17 Listing of an interactive session with renumbering option.

```
 THE Y-COORDINATES ARE AS FOLLOWS:

     .0000     .2500     .4500     .6000     .7500


 THE NUMBER OF Z-COORDINATES IS     3

 THE Z-COOORDINATES ARE AS FOLLOWS:

     .0000     .0054     .0108


  NUMBER OF NODES =    39


  ENTER NUMBER OF MATERIALS
? 2
 2
  PLY ANGLE FOR MATERIAL    1
? 0
 0.
  PLY ANGLE FOR MATERIAL    2
? 90
 90.
  ENTER NUMBER OF PLIES
? 2
 2
 ** PLIES ARE NUMBERED FROM THE TOP OF THE LAMINATE **
  ENTER MATERIAL NUMBER FOR PLY 1
? 1
 1
  ENTER THICKNESS FOR PLY 1
? .0054
 .0054
  ENTER MATERIAL NUMBER FOR PLY 2
? 2
 2
  ENTER THICKNESS FOR PLY 2
? .0054
 .0054
  LOADING CONDITIONS ARE MECHANICAL,THERMAL,HYGROSCOPIC
  ENTER NUMBER OF LOADING CONDITIONS (1, 2, OR 3)
? 1
 1
  ENTER MECHANICAL STRAIN.
? .00254
 .00254


  -----------------------------------------------------
        CRACK TIP IS AT NODE = 27
```

Fig. 17 (Continued)

```
               SUMMARY INFORMATION
                FOR G-CALCULATION
 -----------------------------------------------------
   7    8
  27   23
  36   37   31   32
  DO YOU WISH TO ENTER A RENUMBERING SCHEME?
 ENTER 1 FOR 'YES'; 0 TO ACCEPT DEFAULT
? 1
 1
 ENTER NAME OF FILE CONTAINING RENUMBERING SCHEME
 IF YOU WISH TO ENTER SCHEME, ENTER A BLANK
? ren
REN
    1   10   15   26   31    2   16   32    3   11   17   27   33    4   18   34
    5   12   19   28   35    6   20   36    7   13   21   29   37    8   22   24
   38    9   14   23   25   30   39

          CURRENT BANDWIDTH IN THE MODEL =    57
 DATA GENERATION COMPLETE -- FILE TRY
     0.452 CP SECONDS EXECUTION TIME.
/
```

Fig. 17 (Concluded)

```
? end
REN      IS A LOCAL FILE
/xedit,ren
 XEDIT LARC 1.0
?? p*
1,10,15,26,31
2,16,32
3,11,17,27,33
4,18,34
5,12,19,28,35
6,20,36
7,13,21,29,37
8,22,24,38
9,14,23,25,39,39
--EOI/TOP--
?? end
REN      IS A LOCAL FILE
/rewind,*
   8 FILE(S) PROCESSED.
/ldobj
  FILE NAME FOR WRITING THE DATA (MAXIMUM 6 CHARACTERS)
? try
TRY
  ENTER THE TITLE OF THE PROBLEM
? error check -- renumberibnng scheme--- node 39 repeated twice
ERROR CHECK -- RENUMBERING SCHEME--- NODE 39 REPEATED TWICE
  SPECIFY OUTPUT OPTION
  ENTER  SHORT - FOR SHORT OUTPUT OPTION
         XLONG - FOR LONG OUTPUT OPTION
? xlong
XLONG
  ENTER NUMBER OF Y-COORDS AND Z-COORDS
? 5,3
    5    3
  ENTER Y- AND Z-COORDINATE NUMBER OF THE CRACK TIP
? 4,2
    4    2
  ENTER THE FILE NAME ON WHICH THE COORDS EXIST
? yzc
YZC

 THE NUMBER OF Y-COORDINATES IS     5

 THE Y-COORDINATES ARE AS FOLLOWS:

     .0000     .2500     .4500     .6000     .7500
```

Fig. 18 Listing of an interactive session with errors in the renumbering scheme.

```
 THE NUMBER OF Z-COORDINATES IS     3

 THE Z-COOORDINATES ARE AS FOLLOWS:

     .0000     .0054     .0108


  NUMBER OF NODES =    39


  ENTER NUMBER OF MATERIALS
? 2
 2
  PLY ANGLE FOR MATERIAL    1
? 0
 0.
  PLY ANGLE FOR MATERIAL    2
? 90
 90.
  ENTER NUMBER OF PLIES
? 2
 2
 ** PLIES ARE NUMBERED FROM THE TOP OF THE LAMINATE **
  ENTER MATERIAL NUMBER FOR PLY 1
? 1
 1
  ENTER THICKNESS FOR PLY 1
? .0054
 .0054
  ENTER MATERIAL NUMBER FOR PLY 2
? 2
 2
  ENTER THICKNESS FOR PLY 2
? .0054
 .0054
  LOADING CONDITIONS ARE MECHANICAL,THERMAL,HYGROSCOPIC
  ENTER NUMBER OF LOADING CONDITIONS (1, 2, OR 3)
? 2
 2
  ENTER MECHANICAL STRAIN.
? .0066
 .0066
  ENTER DELTA-T.
? -200
 -200.

  ----------------------------------------------------
          CRACK TIP IS AT NODE = 27
               SUMMARY INFORMATION
                FOR G-CALCULATION
```

Fig. 18 (Continued)

```
 ---------------------------------------------------
     7     8
    27    23
    36    37    31    32
  DO YOU WISH TO ENTER A RENUMBERING SCHEME?
 ENTER 1 FOR 'YES'; 0 TO ACCEPT DEFAULT
? 1
 1
 ENTER NAME OF FILE CONTAINING RENUMBERING SCHEME
 IF YOU WISH TO ENTER SCHEME, ENTER A BLANK
? ren
REN
     1    10    15    26    31     2    16    32     3    11    17    27    33     4    18    34
     5    12    19    28    35     6    20    36     7    13    21    29    37     8    22    24
    38     9    14    23    25    39    39
 **ERROR**     30 OCCURS    0 TIMES.
 **ERROR**     39 OCCURS    2 TIMES.
 NO RENUMBERING SCHEME ENTERED IN DATA FILE

          CURRENT BANDWIDTH IN THE MODEL =    39
 DATA GENERATION COMPLETE -- FILE TRY
     0.464 CP SECONDS EXECUTION TIME.
/
```

Fig. 18 (Concluded)

```
Q3DGT,T200.                                    B 1205      RAJU
USER,User number,Password.
CHARGE,Charge number,LRC.
GET,Q3DG,DATGEN,D35,D4509,YZ,TINT,SOUT,SOUT2.
REWIND,*.
LABEL(TNAME,NT,PO=W,LB=KU,F=S,VSN=Q3DG)
TCOPY(Q3DG,TNAME,F=E,CC=80)
WRITEF,TNAME.
TCOPY(DATGEN,TNAME,F=E,CC=80)
WRITEF,TNAME.
TCOPY(D35,TNAME,F=E,CC=80)
WRITEF,TNAME.
TCOPY(SOUT,TNAME,F=E,CC=136)
WRITEF,TNAME.
TCOPY(D4509,TNAME,F=E,CC=80)
WRITEF,TNAME.
TCOPY(SOUT2,TNAME,F=E,CC=136)
WRITEF,TNAME.
TCOPY(YZ,TNAME,F=E,CC=80)
WRITEF,TNAME.
TCOPY(TINT,TNAME,F=E,CC=136)
WRITEF,TNAME.
DAYFILE,DTAPE.
REPLACE,DTAPE.
EXIT.
DAYFILE,DTAPE.
REPLACE,DTAPE.
```

FIG. 19   NOS Procedural file used to create the COSMIC tape.

```
    FULL  DAYFILE.  85/08/06. 20.12.09.
          JOB NAME  A3MIHUM ON C-MACHINE.
09.07.04.LQG,T300.                                        B 1205
09.07.04.RAJU
09.07.04.USER,654237E,.
09.07.04.CHARGE,101955,LRC.
09.07.05.GET,Q3DG,TAPE5=D35.
09.08.49.FTN5,I=Q3DG,L=OUT.
09.12.39.     62700 CM STORAGE USED.
09.12.39.    33.685 CP SECONDS COMPILATION TIME.
09.12.40.REWIND,*.
09.12.40.   6 FILE(S) PROCESSED.
09.12.40.DELIVER. B 1205 RAJU.
09.12.40.ROUTE,OUT,DC=LP.
09.12.40. ROUTE COMPLETE.
09.12.42.LDSET(PRESETA=NGINF,MAP=SBEX)
09.12.42.LGO(TAPE5,SOUT)
20.12.06.    STOP
20.12.06.    343600 MAXIMUM EXECUTION FL.
20.12.06.   185.084 CP SECONDS EXECUTION TIME.
20.12.06.REWIND,*.
20.12.06.   7 FILE(S) PROCESSED.
20.12.06.REPLACE,SOUT.
20.12.09.DELIVER. B 1205 RAJU.
20.12.09.ROUTE,SOUT,DC=LP.
20.12.09. ROUTE COMPLETE.
20.12.09.DAYFILE,DAY.
```

FIG. 20 A sample dayfile of a NOS run of Q3DG.

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16. Abstract

The Q3DG is a computer program developed to perform a quasi-three-dimensional stress analysis for composite laminates which may contain delaminations. The laminates may be subjected to mechanical, thermal, and hygroscopic loads. The program uses the finite element method and models the laminates with eight-noded parabolic isoparametric elements. The program computes the strain-energy-release components and the total strain-energy release in all three modes for delamination growth. A rectangular mesh and data file generator, DATGEN, is included. The DATGEN program can be executed interactively and is user friendly. The documentation includes sections dealing with the Q3D analysis theory, derivation of element stiffness matrices and consistent load vectors for the parabolic element. Several sample problems with the input for Q3DG and output from the program are included. The capabilities of the DATGEN program are illustrated with examples of interactive sessions. A microfiche containing all the examples presented in this report is included with the documentation. The Q3DG and DATGEN program have been implemented on CYBER 170 class computers. Q3DG and DATGEN were developed at the Langley Research Center during the early eighties and documented in 1984-1985.

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