

# Swept Frequency Technique for Dispersion Measurement of Microstrip Lines

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SWEPT FREQUENCY TECHNIQUE FOR DISPERSION  
MEASUREMENT OF MICROSTRIP LINES

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1. SUMMARY

Microstrip lines used in microwave integrated circuits are dispersive. Because a microstrip line is an open structure, the dispersion can not be derived with pure TEM, TE, or TM mode analysis. Dispersion analysis has commonly been done using a spectral domain approach, and dispersion measurement has been made with high Q microstrip ring resonators. Since the dispersion of a microstrip line is fully characterized by the frequency dependent phase velocity of the line, dispersion measurement of microstrip lines requires the measurement of the line wavelength as a function of frequency. In this paper, a swept frequency technique for dispersion measurement is described. The measurement was made using an automatic network analyzer with the microstrip line terminated in a short circuit. Experimental data for two microstrip lines on a 10 and 30 mil. Cufion substrates were recorded over a frequency range of 2 to 20 GHz. Agreement with theoretical results computed by the spectral domain approach is

good. Possible sources of error for the discrepancy are discussed.

## 2. INTRODUCTION

Microstrip lines used in microwave integrated circuit network and device designs are very attractive because of low cost and easy fabrication. Unlike ordinary transmission lines of TEM mode, the analysis of a microstrip line is very complex and requires the hybrid mode expansion of both the TE and TM modes to account for the fringing. Furthermore, a microstrip line is more difficult to design because of its dispersion characteristics. The dispersion characteristics of microstrip lines have been investigated experimentally and theoretically.<sup>1,2</sup> Dispersion analysis has commonly been done using a spectral domain approach,<sup>3</sup> and dispersion measurements have been made with a high Q microstrip ring resonator.<sup>2</sup> This paper describes a swept frequency technique for dispersion measurements. The technique is applied to analyze the dispersion characteristics of an open microstrip structure as shown in Fig. 2. The measured results are compared to the analytical results computed by a spectral domain method.

## 3. DISPERSION MEASUREMENT

Since the dispersion of an open microstrip line is fully characterized by its frequency dependent phase velocity, dispersion measurement of the microstrip line requires the measurement of the line wavelength as a function of frequency. Troughton<sup>2</sup>

first proposed a technique for measuring wavelengths and dispersion characteristics using a microstrip ring resonator. By capacitively coupling RF power into and out of the ring resonator with 50  $\Omega$  transmission test probes, the line wavelength is determined by measuring the transmitted power as a function of frequency. The effective dielectric constant at each resonance frequency,  $f$ , is obtained from

$$\epsilon_{\text{eff}} = \left( \frac{nc}{2\pi rf} \right)^2$$

where  $c$  is the velocity of light and  $n = 1, 2, 3, \dots$ . The mean radius,  $r$ , of the ring is approximated from Wheeler's results<sup>4</sup> which are valid for frequencies below 6 GHz. Although the ring resonator approach eliminates errors in determining the electrical length due to fringing, the curvature of the ring and the coupling probes may introduce errors in the resonance frequency measurements. At high frequencies, the validity of Wheeler's theory, for approximating the mean radius  $r$  is also questionable. In fact, large discrepancies between calculated and measured results have been reported.<sup>5</sup> The swept frequency technique shown in Fig. 2 is based on the transmission line theory. The dispersion measurement was made using an automatic network analyzer with the microstrip line terminated in a short circuit. Treating the microstrip line like a quarter-wavelength resonant stub, the line wavelength,  $\lambda$ , may be calculated by measuring the input impedance as a function of frequency. The

effective dielectric constant,  $\epsilon_{\text{eff}}$ , at each resonance can be determined from the following relations:

$$l = \frac{n\lambda}{4} \quad (n = 1, 2, 3, \dots)$$

$$\epsilon_{\text{eff}} = \left( \frac{nc}{4lf} \right)^2$$

where  $l$  is the electrical length of the microstrip line. All of the measurements were made on 1 in. long microstrip lines on a Cufion substrate ( $\epsilon_r = 2.1$ ). The line width is chosen to provide a low-frequency characteristic impedance of approximately 50  $\Omega$ . A SMA/MIC flange mount connector was used to connect the microstrip line to the automatic network analyzer and the short circuit termination.

#### 4. NUMERICAL ANALYSIS

A spectral domain method as outlined in Ref. 3 is utilized to analyze the dispersion of an open microstrip structure. In brief, the formulation of the problem yields a hybrid mode solution of linear combinations of TE and TM fields. The hybrid-mode fields in the spectral domain are given by

$$\tilde{E}_{zi}(\alpha, y, z) = \frac{k_1^2 - k_z^2}{jk_z} \tilde{\phi}_1^e(\alpha, y) e^{-jk_z z}$$

$$\tilde{H}_{zi}(\alpha, y, z) = \frac{k_1^2 - k_z^2}{jk_z} \tilde{\phi}_1^h(\alpha, y) e^{-jk_z z}$$

$$\tilde{E}_{x1}(\alpha, y, z) = \left( -j\alpha\tilde{\phi}_1^e(\alpha, y) - \frac{\omega\mu_0\mu_r}{k_z} \frac{\partial\tilde{\phi}_1^h(\alpha, y)}{\partial y} \right) e^{-jk_z z}$$

$$\tilde{H}_{x1}(\alpha, y, z) = \left( \frac{\omega\epsilon_0\epsilon_r}{k_z} \frac{\partial\tilde{\phi}_1^e(\alpha, y)}{\partial y} - j\alpha\tilde{\phi}_1^h(\alpha, y) \right) e^{-jk_z z}$$

The subscripts  $1 = 1, 2$  designate the regions 1 (substrate) and 2 (air). The Fourier transforms of scalar potentials satisfy the homogeneous Helmholtz equation

$$\nabla^2 \tilde{\phi}_1 + k_1^2 \tilde{\phi}_1 = 0$$

or

$$\frac{\partial^2 \tilde{\phi}_2}{\partial y^2} - \gamma_1^2 \tilde{\phi}_1 = 0$$

where

$$\gamma_1^2 = \alpha^2 + k_z^2 - k_0^2 \epsilon_r \mu_r$$

$$\gamma_2^2 = \alpha^2 + k_z^2 - k_0^2$$

$$k_0^2 = \omega^2 \mu_0 \epsilon_0$$

By matching tangential components (x and z) of the fields at the air-dielectric interface, and satisfying the radiation condition and appropriate boundary conditions at the conducting strip, two coupled equations of the unknown transformed currents ( $\tilde{J}_x, \tilde{J}_z$ ) and the transformed longitudinal fields ( $\tilde{E}_{z2}, \tilde{H}_{z2}$ ) are obtained.

$$G_{11}(\alpha, k_z) \tilde{J}_x(\alpha) + G_{12}(\alpha, k_z) \tilde{J}_z(\alpha) = \tilde{E}_{z2}(\alpha, d)$$

$$G_{21}(\alpha, k_z) \tilde{J}_x(\alpha) + G_{22}(\alpha, k_z) \tilde{J}_z(\alpha) = \tilde{H}_{z2}(\alpha, d)$$

where the expressions for the Green's functions  $G_{11}(\alpha, k_z)$ ,  $G_{21}(\alpha, k_z)$  etc. are given in Ref. 3. To find the unknown propagation constant  $k_z$ , Galerkin's method in the spectral domain is applied to reduce the coupled equations to a matrix equation. By expanding the transformed current  $\tilde{J}_x$  and  $\tilde{J}_z$  in terms of some known basis functions  $\tilde{J}_{xn}$  and  $\tilde{J}_{zn}$ , i.e.,

$$\tilde{J}_x(\alpha) = \sum_{n=1}^M c_n \tilde{J}_{xn}(\alpha)$$

$$\tilde{J}_z(\alpha) = \sum_{n=1}^M d_n \tilde{J}_{zn}(\alpha)$$

A matrix equation of the unknown constants  $c_n$  and  $d_n$  is obtained.

$$\sum_{n=1}^M K_{mn}^{(1,1)} c_n + \sum_{n=1}^N K_{mn}^{(1,2)} d_n = 0 \quad m = 1, 2, \dots, N$$

$$\sum_{n=1}^M K_{mn}^{(2,1)} c_n + \sum_{n=1}^N K_{mn}^{(2,2)} d_n = 0 \quad m = 1, 2, \dots, M$$

where

$$K_{mn}^{(1,1)} = \int_{-\infty}^{\infty} \tilde{J}_{zm}(\alpha) G_{11}(\alpha, k_z) \tilde{J}_{xn}(\alpha) d\alpha$$

$$K_{mn}^{(1,2)} = \int_{-\infty}^{\infty} \tilde{J}_{zm}(\alpha) G_{12}(\alpha, k_z) \tilde{J}_{zn}(\alpha) d\alpha$$

$$K_{mn}^{(2,1)} = \int_{-\infty}^{\infty} \tilde{J}_{xm}(\alpha) G_{21}(\alpha, k_z) \tilde{J}_{xn}(\alpha) d\alpha$$

$$K_{mn}^{(2,2)} = \int_{-\infty}^{\infty} \tilde{J}_{xm}(\alpha) G_{22}(\alpha, k_z) \tilde{J}_{zn}(\alpha) d\alpha$$

The unknown propagation constant,  $k_z$ , is calculated numerically by finding the root to the determinant of the matrix equation

$$[K] = 0$$

The root to the determinant is computed by iteration technique. Romberg's numerical integration algorithm has been used to compute  $K_{mn}$ . The numerical result is strongly dependent on the choice of the basis functions. Different bases function for  $\tilde{J}_x$  and  $\tilde{J}_z$  have been reported in the open literature.<sup>1,6</sup> For the work here, the following basis functions have been chosen.

$$J_z(x) = \sqrt{\frac{2}{\pi}} \frac{J_{z0}}{\sqrt{\left(\frac{W}{2}\right)^2 - x^2}} - \frac{W}{2} < x < \frac{W}{2}$$

$$= 0 \quad \text{otherwise}$$

$$J_x(X) = J_{x0} \sin \frac{\pi X}{0.8W} \quad |x| \leq 0.8 \frac{W}{2}$$

$$= J_{x0} \cos \frac{\pi X}{0.2X} \quad 0.8 \frac{W}{2} < |X| \leq \frac{W}{2}$$

##### 5. COMPARISON OF CALCULATED AND MEASURED VALUES

Figure 3 shows the measured and computed values of the effective dielectric constant,  $\epsilon_{eff}$ , as a function of frequency for a microstrip line of 0.89 mm line width on a 10 mil Cuflon substrate ( $\epsilon_r = 2.1$ ). The measured curve exhibits the



usual oscillatory characteristics of the coaxial microstrip transition.<sup>2</sup> If the substrate thickness is increased, the transition effect is enhanced as clearly indicated in Fig. 4 for the 30 mil substrate. The transition effect, if not removed, will obscure the small deviations in impedance and phase velocity due to the dispersion of the microstrip line. To remove the oscillations experimentally requires a broadband, well matched coaxial-microstrip transition. However, by fitting the measured data to an interpolating polynomial to smooth out the transition characteristics, the measured curves show good agreements with the computed curves. The maximum deviation is approximately 5 percent over a frequency range of 2 to 20 GHz. The discrepancy between the measured and the calculated values can be traced to different sources of error. Since the physical line length is used to determine the line wavelength, the effective dielectric constant,  $\epsilon_{eff}$ , computed from the measured line wavelength generally has a higher value. The fringing at the end of the line tends to increase the electrical length of the line, and thus, lowers the value of the measured  $\epsilon_{eff}$ . By including the fringing effect, a closer agreement between the measured and the computed values can be obtained. Also, it has been observed experimentally that the type of short termination has a profound effect on  $\epsilon_{eff}$ . By terminating the microstrip line with a SMA coaxial short and by wire bonding the microstrip line to its ground

plane, the measurements can be changed by a few percent. For the latter case, the impact of fringing on the electrical length is significant. Better agreement between the analytical and experimental results are also possible if a more accurate basis function for the unknown current or a better analytical model for the microstrip line is used. Finally, the impedance variation along a short-circuit microstrip line can be obtained by measuring the input impedance as a function of frequency or electrical lengths. Figures 5 and 6 show the variation of the normalized resistance and reactance with frequency. The resistance curve shows an impulse type of response at resonant frequencies, while the reactance curve demonstrates similar characteristics of a lossless short-circuited transmission line of TEM modes. It is important to point out that because of the dispersion of the microstrip line, the resonant frequencies for the higher order modes do not always occur at exactly the multiple of the fundamental frequency. As a consequence, the phase velocity and the effective dielectric constant are frequency-dependent.

## 7. CONCLUSION

A swept frequency technique for dispersion measurement of microstrip lines has been described. The technique is easy to apply, and yet, yields fairly accurate results for both thin and thick substrate over a frequency range of 2 to 20 GHz. From this technique, one is able to obtain the impedance variation along a

dispersive line, and a better physical understanding of the dispersive characteristics of an open microstrip line.

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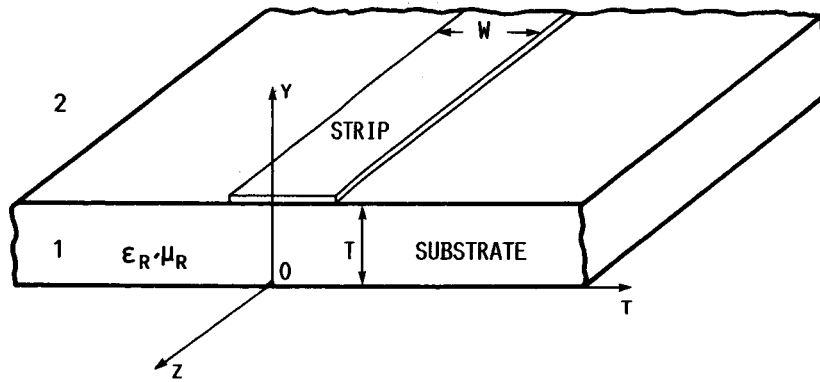


FIGURE 1. - MICROSTRIP CONFIGURATION.

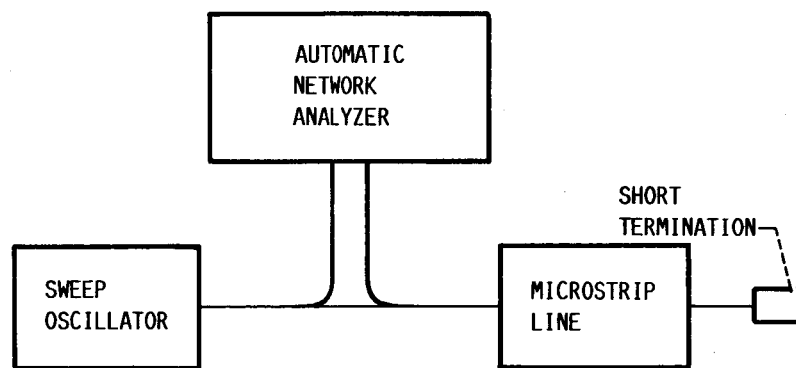


FIGURE 2. - EXPERIMENTAL SETUP.

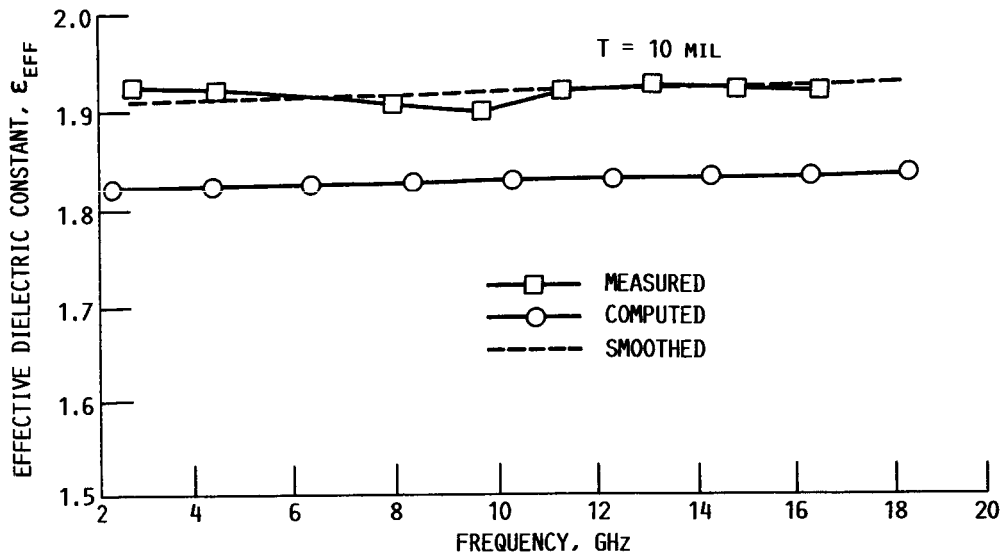


FIGURE 3. - EFFECTIVE DIELECTRIC CONSTANT,  $\epsilon_{EFF}$ , VERSUS FREQUENCY.

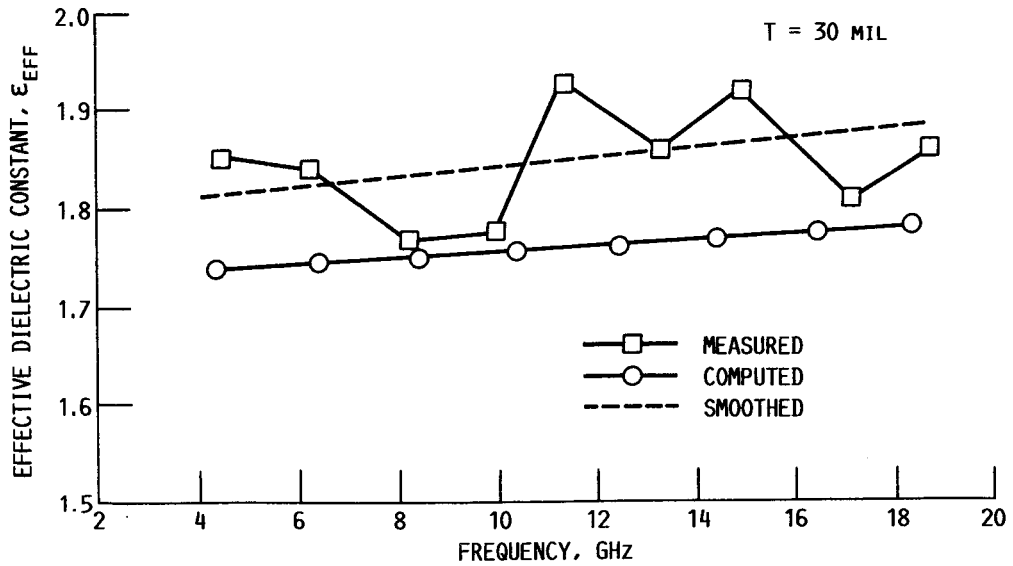


FIGURE 4. - EFFECTIVE DIELECTRIC CONSTANT,  $\epsilon_{EFF}$ , VERSUS FREQUENCY.

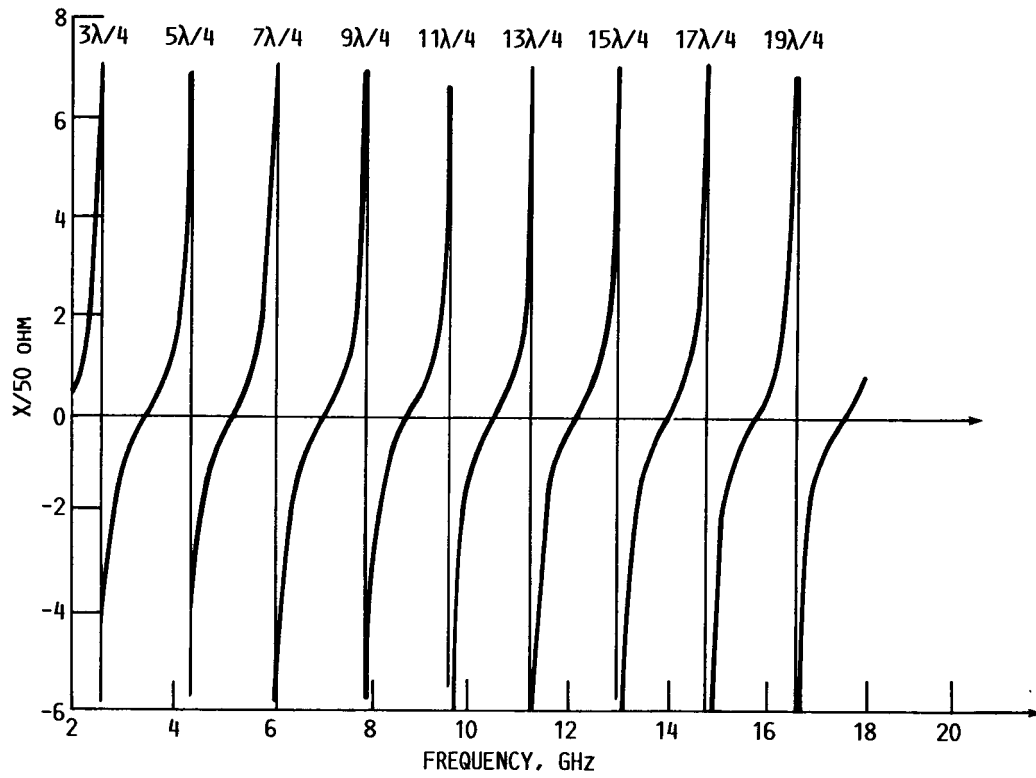


FIGURE 5. - REACTANCE VARIATION ALONG A DISPERSIVE LINE TERMINATION IN A SHORT.

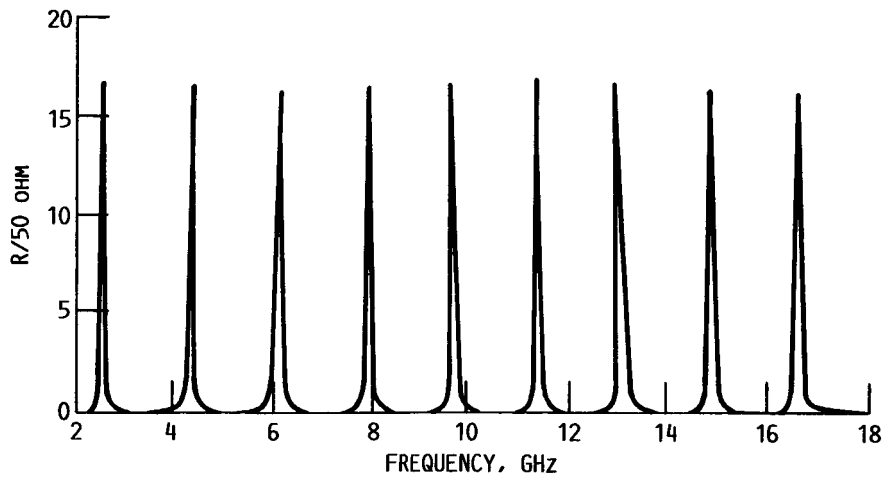


FIGURE 6. - RESISTANCE VARIATION ALONG A DISPERSIVE LINE TERMINATED IN A SHORT.

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