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ENGINEERING DESCRIPTION OF THE ASCENT/DESCENT BET PRODUCT

August 1986

Prepared for

CONTRACT NAS9-17554
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LYNDON B. JOHNSON SPACE CENTER
HOUSTON, TEXAS

Prepared by

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1.0 INTRODUCTION AND SCOPE OF DOCUMENT

1.1 ASCENT/DESCENT OUTPUT OVERVIEW

The Ascent/Descent output product is produced in the OPIP routine from three files which constitute its input. One of these, OPIP.IN, contains mission specific parameters. Meteorological data, such as atmospheric wind velocities, temperatures, and density, are obtained from the second file, the Corrected METeorological DATA file (METDATA). The third file is the TRJATTDATA file which contains the time-tagged state vectors that combine trajectory information from the Best Estimate of Trajectory (BET) filter, LRBET5 (Applicable Document 4,Section 3.0), and Best Estimate of Attitude (BEA) derived from IMU telemetry.

Ascent/Descent output products are provided with two files. The major product contains the BET, which is in the file BETDATA. The other product file contains the system parameters which were used to determine the BET. This is The Navigation Block, or NAVBLK, file.

Each of the product files is delivered on three magnetic computer tapes. Two of these are in binary format, one of which is formatted to be read by a CDC Cyber machine, and the other is formatted to be read by a Sperry Univac machine. The third tape is written in ASCII format and is used to produce a microfiche display of the output.

1.2 SCOPE OF DOCUMENT

This engineering description defines each term in the two output data files. The description of the BETDATA file includes an outline of the algorithm used to calculate each term. Most computations are performed in the program OPIP or in simple subroutines called by OPIP. Some subroutines are extensive, however, and their algorithms are described separately in a section following that for OPIP.

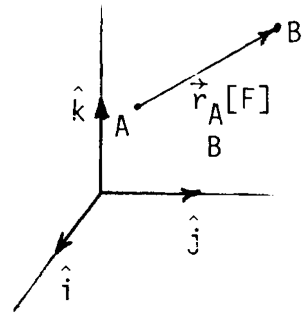
To facilitate describing the algorithms, a nomenclature is defined in Sections 2.1 through 2.4. The description of the nomenclature includes a definition of the coordinate systems used.

The NAVBLK file contains navigation input parameters. Each term in NAVBLK is defined, and its source (e.g., user input) is listed. The production of NAVBLK requires only two computational algorithms. These two algorithms, which compute the terms DELTA and RSUBO, are described in Section 4.2.2 following the definition of NAVBLK terms. Finally, the distribution of data in the NAVBLK records is listed.

The English system of units is used throughout this document. Unless specified otherwise, lengths, velocities, and accelerations are output in feet, feet/sec, and feet/sec^2, respectively. Mass is output in slugs, force (and weight) in pounds, temperature in degrees Rankine, and angular measure (eg., latitude, longitude, Euler angles, angle of attack and sideslip angle) are in degrees. Also, unless specified otherwise, time is output in seconds.

2.0 NOMENCLATURE

2.1 VECTORS



- $\vec{r}_{\substack{A\\B}}[F]$ will represent a three-element position vector from point A to point B; i.e., the position of B with respect to A. The components of the vector are with respect to the reference frame F whose orthogonal basis set is $[\hat{i}_F, \hat{j}_F, \hat{k}_F]$; $\hat{i}_F$, $\hat{j}_F$, and $\hat{k}_F$ are unit vectors. Thus,

$$\vec{r}_{\substack{A\\B}}[F] = x_B\hat{i}_F + y_B\hat{j}_F + z_B\hat{k}_F.$$

- $\hat{r}_{\substack{A\\B}}[F]$ is the unit vector in the direction of $\vec{r}_{\substack{A\\B}}[F]$; $\hat{r}_{\substack{A\\B}}[F] = \dfrac{\vec{r}_{\substack{A\\B}}[F]}{|\vec{r}_{\substack{A\\B}}[F]|}$.

- $\dot{\vec{r}}_{\substack{A\\B}}[F] \triangleq \frac{d}{dt}\vec{r}_{\substack{A\\B}}[F]$ which is a velocity if F is an inertial frame.

- $\ddot{\vec{r}}_{\substack{A\\B}}[F] \triangleq \frac{d^2}{dt^2}\vec{r}_{\substack{A\\B}}[F]$ which is an acceleration if F is an inertial frame.

- $\vec{v}_{\substack{P\\wind}}[F]$ If a relative velocity (say, wind relative to some point, P) is indicated or if it is not useful to express a velocity in terms of a time derivative, then the velocity may be expressed as $\vec{v}_{\substack{P\\wind}}[F]$ where, again, F is the coordinate frame in which the vector components are expressed.

- $\vec{a}_{A_P}[F]$ Likewise, the acceleration of a point P with respect to a point A as expressed in frame F is as shown. In this document, the term "contact acceleration" (which is the term most often used in the software listings) is equivalent to sensed acceleration.

- $\vec{S}[M50]$ is the M50 state vector.

- $\vec{\sigma}(\vec{w})$ will represent the standard deviation of the components of the vector $\vec{w}$. The term "uncertainty" will be equivalent to the standard deviation. If $\vec{w}$ has three components, then so will $\vec{\sigma}(\vec{w})$. That is, if $\vec{w} = w_x\hat{i} + w_y\hat{j} + w_z\hat{k}$, then $\vec{\sigma}(\vec{w}) = [\sigma(w_x), \sigma(w_y), \sigma(w_z)]^T$.

- $\vec{\omega}[F]$ is the angular velocity expressed in the F-frame.

- $\dot{\vec{\omega}}[F]$ is the angular acceleration expressed in the F-frame.

2.2 MATRICES

- $[F1 \rightarrow F2]$ represents a transformation matrix which transforms a vector from frame F1 to frame F2. Thus,

$$\vec{r}_{A_B}[F2] = [F1 \rightarrow F2]\, \vec{r}_{A_B}[F1]$$

In terms of elements, this matrix may be expressed as

$[F1 \rightarrow F2] = \mathbf{M} = [m_{ij}]$ where i represents the row and j represents the column of element m_{ij}.

- $[F1 \rightarrow F2]^T$ represents the transpose of the above matrix. Thus, since

  $[F1 \rightarrow F2]^T = [F2 \rightarrow F1]$,

  then

  $\vec{r}_{A_B}[F1] = [F1 \rightarrow F2]^T \vec{r}_{A_B}[F2]$.

- $₵(\vec{w})$ represents the covariance matrix of the vector $\vec{w}$. If $\vec{w}$ has n elements, then $₵(w)$ will be an n x n matrix.

2.3 SPECIFIC LOCATIONS

The following convention will be used to express the following points, or locations.

- CM = the center of mass (usually equivalent to the center of gravity).

- NB = the Navigation Base (= Nav Base).

- $\oplus$ = the center of the Earth.

- L = origin of Landing Field (Runway) coordinate system.

- S = origin of either Launch Site for Ascent analysis or Landing Site Runway threshold for Descent analysis.

- P = a general, specified point.

2.4 COORDINATE SYSTEMS

The following abbreviations will be used for the indicated coordinate systems. These coordinate systems are discussed in Applicable Document 1 listed in Section 3.0. For all of them, the Y-axis forms a right hand orthogonal triad with the X and Z axes.

- M50 - The Mean of 1950 is the Earth-centered inertial system whose X-axis is in the direction of the equinox at the beginning of the Besselian year 1950, and the X-Y plane is in the mean equator of that epoch. The Z-axis lies along the mean Earth's rotation axis of that epoch.

- TOD - The True Of Date is the Earth-centered inertial system whose X-axis is in the direction of the Vernal Equinox of the midnight prior to launch. The XY plane is in the equitorial plane of that epoch, and the Z-axis lies along the Earth's rotation axis of that epoch.

- ECI - The Earth-Centered Inertial system has its X-axis in the mean equator of the epoch (midnight prior to launch) and remains (i.e., is inertial) in the direction of the Greenwich meridian at that time. The Z-axis lies along the Earth's rotation axis of that epoch.

- GEO - The GEOgraphic frame is Earth-centered and Earth-fixed with the X-axis lying in the equator of date and passing through the Greenwich

Note that $\underline{h}_1$, $\underline{h}_2$, and $\underline{h}_3$ are not all mutually orthogonal. By linear transformation of coordinates, each $\underline{h}_i$ can be given in M50 by $\underline{H}_i$ according to

$$\underline{H}_1 = \underline{F}_3 \ ,$$

$$\underline{H}_2 = (-\sin\psi)\ \underline{F}_1 + (\cos\psi)\ \underline{F}_2 \ , \text{ and}$$

$$\underline{H}_3 = (\cos\psi\ \cos\theta)\ \underline{F}_1 + (\sin\psi\ \cos\ \theta)\ \underline{F}_2 + (-\sin\theta)\ \underline{F}_3 \ .$$

In accordance with foregoing arguments, the topodetic Euler angle rates are given by the projections of the body angular velocity $\underline{U}$ as seen in the topodetic frame onto the axes $\underline{H}_1$, $\underline{H}_2$, and $\underline{H}_3$ used for the Euler angle rotations ψ , θ , and ϕ :

$$\dot{\psi} = \underline{U} \cdot \underline{H}_1 \ ,$$

$$\dot{\theta} = \underline{U} \cdot \underline{H}_2 \ , \text{ and}$$

$$\dot{\phi} = \underline{U} \cdot \underline{H}_3 \ .$$

4. REFERENCE

OFT Ascent/Descent Ancillary Data Requirements Document, Mission Planning and Analysis Division, Johnson Space Center, NASA, 78-FM-40, Rev. 2, JSC-14370, February 1982.

meridian. Its Z-axis lies along the Earth's rotation axis of that date.

- TOP - The TOPodetic system is an inertial system whose origin is on the Navigation Base with the X-axis pointing northward along the local meridian. The Z-axis points downward in the direction normal to the reference ellipsoid (presently, the Fischer Ellipsoid of 1960). Note that the topodetic frame is non-rotating; a new frame is redefined at each time point.

- BOD - The vehicle BODy system has its origin centered at the Navigation Base (not at the Center of Mass). The X-axis points toward the Orbiter's nose, and the Z-axis points down through the Orbiter's bottom.

- PLM - The PLuMbline coordinate system is Earth-centered and inertial. Its axes are parallel to a right handed orthogonal system centered at the launch site and fixed inertially at the time of SRB ignition (i.e., GET=0). Define $\hat{n}$ as the unit vector normal to the tangent plane at the launch site; its direction is along the gravity gradient at that point. Also define a unit vector $\hat{k}$ at the launch site, in the tangent plane, and pointing in the direction of the launch azimuth. The unit vectors $\hat{n}$ and $\hat{k}$ are normal to each other and to the third unit vector, $\hat{j}$, of the triad. Then the plumbline X, Y, and Z axes are parallel and positive in the direction of the unit vectors $\hat{n}$, $\hat{j}$, and $\hat{k}$ at the instant of SRB ignition. The Plumbline system is used only during ascent.

- LF - The Landing Field (or Runway) coordinate system is a rotating coordinate system whose origin is at the intersection of the runway centerline and threshhold. The X-axis lies along the centerline, and is positive in the direction of rollout. The Z-axis is normal to the reference (i.e., Fischer) ellipsoid at the origin and is positive toward the Earth's center.

3.0 APPLICABLE DOCUMENTS

1. Davis, L. D., "Coordinate Systems for the Space Shuttle Program," NASA Technical Memorandum TM X 58153 JSC, October 1974.

2. NOAA, U. S. Standard Atmosphere, 1976, "National Oceanic and Atmosphere Administration, U. S. Government Printing Office, October 1976.

3. Trajectory MCC Level B and C Requirements for Shuttle Volume II, JSC-11028, 5 November 1982, p. 7-2 and following.

4. Lear, W. M., "The Ascent/Entry BET Program, LRBET5", JSC-19310, December 1983.

5. Poritz, D. H., "Definition of Topodetic Euler Angle Rates for the Shuttle Ascent/Descent Ancillary Data Output Products", TRW IOC 83:W482.4-28, 25 March 1983.

6. Brans, H. R.; Seacord, A. W.; Ulmer, J. W., "The Ascent/Descent BET Production Process User's Guide," in preparation.

4.0 THE OUTPUT PRODUCTS

The output product of the Ascent/Descent analysis program consists of two files, BETDATA and NAVBLK. Each is produced by a major program. The purpose of this section is to describe the methods by which these programs calculate the output product files.

4.1 THE BETDATA FILE

The BETDATA file consists of 239 double precision words calculated by the program OPIP and subroutines called by it. The following Section 4.1.1 describes the processing within OPIP, and Section 4.1.2 describes the calculations of subroutines called by OPIP.

4.1.1 OPIP Calculations

The contents of the BETDATA file are discussed on the following pages in the order in which the words are calculated. The OPIP program and its subroutines calculate terms, each of which is represented by a name and an algebraic symbol and may consist of more than one double precision word in the file. For example, the M50 position vector has the term name M50, is designated by the algebraic symbol $\vec{r}_{\underset{NB}{\oplus}}[\mathrm{M50}]$, and consists of three double precision words. The following discussion provides the term name, algebraic symbol, the program, or subroutine in which the terms is computed, and the algorithm by which the term is computed.

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
Time from SRB ignition (in seconds); i.e., Ground Elapsed Time (GET)				
1	GET	t_{GET}	OPIP	Read next output time, t_{NEXT}, from TRJATTDATA file
			OPIP	Read SRB ignition time, t_{SRBI}, from OPIP.IN file.
			OPIP	$t_{GET} = t_{NEXT} - t_{SRBI}$
Ground time in day:hour:min:sec of UT since the start of the year, but where day is the day number. That is, ground time is the ellapsed Astronomical UT since the start of the year + 1 day.				
2	GMTDAY	t_{NEXT}	TELAPSE	Time is maintained internally in seconds from the astronomical UT start of year. The program is initialized with a Launch time (year, day number, seconds from the the UT midnight day of launch), stepped forward in time, and updated with time-tagged observations. Data records are written with GMTSEC as double precision.
3	GMTHR			
4	GMTMIN			
5	GMTSEC			

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Onboard (Shuttle) time in day:hour:min:sec since SRB ignition

word	name	symbol	program	algorithm
6	SGMTDAY	t_{sNEXT}	OPIP	Compute onboard time of next output, t_{sNEXT}
7	SGMTHR			
8	SGMTMIN			
9	SGMTSEC			

$$t_{sNext} = \frac{t_{NEXT} - \varepsilon_t + \Delta t_{drift} * t_{os}}{1 - \Delta t_{drift}}$$

ε_t = Difference (or error) between onboard and ground times.

Δt_{drift} = Drift rate of onboard clock.

t_{os} = time, in seconds, from midnight prior to launch as measured by the onboard clock. A value of ε_t is assumed.

TELAPSE — The output is computed in the same way as are words 2, 3, 4, and 5. The times are onboard times, however, rather than ground times (eg., t_{sNEXT} replaces t_{NEXT}).

Year of present (Ground) time, UT

word	name	symbol	program	algorithm
10	GMTYR		TELAPSE	The present UT year is calculated along with words 2, 3, 4, and 5.

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
	Position of the Nav Base relative to the (inertial) M50 coordinate frame			
11	M50(1)	$\vec{r}_{\substack{\oplus \\ NB}}$ [M50]	OPIP	Extracted from the state vector:
12	(2)			$\vec{r}_{\substack{\oplus \\ NB}}$ [M50] = M50STAT(1,2,3) read in
13	(3)			from the input file TRJATTDATA.
	Velocity of the Nav Base relative to the (inertial) M50 coordinate frame			
14	DM50(1)	$\dot{\vec{r}}_{\substack{\oplus \\ NB}}$ [M50]	OPIP	Extracted from the state vector:
15	(2)			$\dot{\vec{r}}_{\substack{\oplus \\ NB}}$ [M50] = M50STAT(4,5,6) read
16	(3)			in from the input file TRJATTDATA.
	Contact (or sensed) acceleration of the Nav Base relative to the (inertial) M50 coordinate frame			
17	DDM50(1)	$\ddot{\vec{r}}_{\substack{\oplus \\ NB}}$ [M50]	OPIP	Extracted from the state vector:
18	(2)			$\ddot{\vec{r}}_{\substack{\oplus \\ NB}}$ [M50] = M50STAT(7,8,9) read
19	(3)			in from the input file TRJATTDATA.

BET OUTPUT PRODUCTS (continued)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
	Position of the Orbiter Center of Mass (CM) relative to the M50 coordinate frame			
20	M50C(1)		OPIP	Read in $\vec{r}_{\underset{CM}{NB}}$[BOD] from TRJATTDATA file.
21	(2)	$\vec{r}_{\underset{CM}{\oplus}}$ [M50]	M3TX1	$\vec{r}_{\underset{CM}{NB}}[M50] = [M50 \rightarrow BOD]^T \, \vec{r}_{\underset{CM}{NB}}[BOD]$
22	(3)		VADD	$\vec{r}_{\underset{CM}{\oplus}}[M50] = \vec{r}_{\underset{NB}{\oplus}}[M50] + \vec{r}_{\underset{CM}{NB}}[M50]$
	Velocity of the Orbiter CM relative to the M50 coordinate frame			
23	DM50C(1)		OPIP	Read in $\vec{\omega}[BOD]$ = M50STAT(13,14,15)
24	(2)	$\dot{\vec{r}}_{\underset{CM}{\oplus}}$ [M50]	CROSS	$\vec{v}_{\underset{CM}{NB}}[BOD] = \vec{\omega}[BOD] \times \vec{r}_{\underset{CM}{NB}}[BOD]$
25	(3)		M3TX1	$\vec{v}_{\underset{CM}{NB}}[M50] = [M50 \rightarrow BOD]^T \, \vec{v}_{\underset{CM}{NB}}[BOD]$
			VADD	$\dot{\vec{r}}_{\underset{CM}{\oplus}}[M50] = \dot{\vec{r}}_{\underset{NB}{\oplus}}[M50] + \vec{v}_{\underset{CM}{NB}}[M50]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
	Sensed Acceleration of the Orbiter Center of Mass relative to the M50 coordinate frame			
26	DDM50C(1)	$\ddot{\vec{r}}_{\oplus_{CM}}$ [M50]	OPIP	Read in $\dot{\vec{\omega}}$ = M50STAT(16,17,18)
27	(2)		CROSS	$\vec{v}_{NB_{CM}}[BOD] = \vec{\omega}[BOD] \times \vec{r}_{NB_{CM}}[BOD]$
28	(3)		CROSS	$\vec{a}_{NB_{CM}}[BOD] = \vec{\omega}[BOD] \times \vec{v}_{NB_{CM}}$
			VADD	$+ \dot{\vec{\omega}}[BOD] \times \vec{r}_{NB_{CM}}[BOD]$
			M3TX1	$\vec{a}_{NB_{CM}}[M50] = [M50 \rightarrow BOD]^T \vec{a}_{NB_{CM}}[BOD]$
			VADD	$\ddot{\vec{r}}_{\oplus_{CM}}[M50] = \ddot{\vec{r}}_{\oplus_{NB}}[M50] + \vec{a}_{NB_{CM}}[M50]$
	Position of the Nav Base in the GEOgraphic coordinate system			
29	GEO(1)	$\vec{r}_{\oplus_{NB}}$ [GEO]	M3X1	$\vec{r}_{\oplus_{NB}}[ECI] = [M50 \rightarrow ECI]\, \vec{r}_{\oplus_{NB}}[M50]$
30	(2)			$[ECI \rightarrow GEO] = \text{fctn}(\omega_{\oplus}, t_{NEXT})$
			ECIGEO	
31	(3)			$\vec{r}_{\oplus_{NB}}[GEO] = [ECI \rightarrow GEO]\, \vec{r}_{\oplus_{NB}}[ECI]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Velocity of the Nav Base in the GEOgraphics coordinate system

word	name	symbol	program	algorithm
32	DGEO(1)	$\dot{\vec{r}}_{\oplus NB}$ [GEO]	M3X1	$\dot{\vec{r}}_{\oplus NB}$ [ECI] = [M50 $\rightarrow$ ECI] $\dot{\vec{r}}_{\oplus NB}$ [M50]
33	(2)			[ECI $\rightarrow$ GEO] = fctn($\omega_{\oplus}$, t_{NEXT}) $\vec{\omega}_{\oplus} \triangleq$ Earth's rotation rate vector
34	(3)		ECIGEO	$\dot{\vec{r}}_{\oplus NB}$ [GEO] = [ECI $\rightarrow$ GEO] $\dot{\vec{r}}_{\oplus NB}$ [ECI] $- \vec{\omega}_{\oplus} \times \vec{r}_{\oplus NB}$ [GEO]

Sensed Acceleration of the Nav Base in the GEOgraphic coordinate system

word	name	symbol	program	algorithm
35	DDGEO(1)	$\vec{a}_{NB}$[GEO] sensed	M3X1	$\ddot{\vec{r}}_{\oplus NB}$ [ECI] = [M50 $\rightarrow$ ECI] $\ddot{\vec{r}}_{\oplus NB}$ [M50]
36	(2)		M3X1	$\vec{a}_{NB}$[GEO] sensed = [ECI $\rightarrow$ GEO] $\ddot{\vec{r}}_{\oplus NB}$ [ECI]
37	(3)			

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Total Acceleration of the Nav Base expressed in the GEOgraphic Coordinate System

word	name	symbol	program	algorithm
38	DDTGEO(1)	$\ddot{\vec{r}}_{\oplus NB}$ [GEO]	EARP	Calculate the gravitation acceleration at the Nav Base in ECI frame. $\vec{g}_{NB}[ECI] = fctn(\vec{r}_{\oplus NB}[ECI], t_{NEXT})$
39	(2)			
40	(3)		VADD	$\ddot{\vec{r}}_{\oplus NB}[ECI] = \vec{g}_{NB}[ECI] + [M50 \rightarrow ECI]\ \ddot{\vec{r}}_{\oplus NB}[M50]$
			ECIGEO	$\ddot{\vec{r}}_{\oplus NB}[GEO] = [ECI \rightarrow GEO]\ \ddot{\vec{r}}_{\oplus NB}[ECI] - \vec{\omega}_{\oplus} \text{ X } (\vec{\omega}_{\oplus} \text{ X } \vec{r}_{\oplus NB}[ECI]) - 2\vec{\omega}_{\oplus} \text{ X } [ECI \rightarrow GEO]\ \dot{\vec{r}}_{\oplus NB}[ECI]$

Gravitational Acceleration of the Nav Base expressed in the GEOgraphic coordinate system

word	name	symbol	program	algorithm
41	DDGGEO(1)	$\vec{g}_{NB}[GEO]$	EARP	$\vec{g}_{NB}[ECI] = fctn(\vec{r}_{\oplus NB}[ECI], t_{NEXT})$
42	(2)		M3X1	$\vec{g}_{NB}[GEO] = [ECI \rightarrow GEO]\ \vec{g}_{NB}[ECI]$
43	(3)			

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm

Initial computations for all wind-relative velocities which follow

SELMETA (Ascent)	Read the meteorological data
	input file CMETDATA, interpolate
DESCMET (Descent)	to the location (ϕ_{GEOD}, λ_{GEOD}) and generate the array METDATA for that location. Define the following terms from METDATA:

w_H = Horizontal wind speed

$\sigma_S(w_H)$ = Systematic uncertainty in w_H

$\sigma_N(w_H)$ = Noise uncertainty in w_H

θ_{WH} = Direction of horizontal wind (= 0° North and is positive clockwise from that)

$\sigma_S(\theta_{WH})$ = Systematic uncertainty in θ_{WH}

$\sigma_N(\theta_{WH})$ = Noise uncertainty in θ_{WH}

$\sigma_N(w_V)$ = Noise uncertainty in vertical wind speed

The total uncertainty in horizontal wind speed, $\sigma_T(w_H)$, is

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
			$\sigma_T(w_H) = [\sigma_S(w_H)^2 + \sigma_N(w_H)^2]^{1/2}$ The total uncertainty in horizontal wind direction, $\sigma_T(\theta_{WH})$, is $\sigma_T(\theta_{WH}) = [\sigma_S(\theta_{WH})^2 + \sigma_N(\theta_{WH})^2]^{1/2}$

Wind-relative velocity of the Nav Base projected onto the GEOgraphic coordinate system

word name	symbol	program	algorithm
44 DGEOW(1) 45 (2) 46 (3)	$\vec{v}_{\substack{wind\\NB}}$ [GEO]	OPIP	Set conversion constant $C_{D/R}$ =degrees/radian Read $R_{\oplus E}$ Earth's equitorial radius and $R_{\oplus P}$ Earth's polar radius from input file OPIP.IN $\vec{r}_{\oplus NB}$ [GEO] is computed above in words (29-31)
		GEOGEOD	Compute Geodetic latitude (ϕ_{GEOD}), longitude (λ_{GEOD}), height (h_{GEOD}), and declination, δ. (See description of GEOGEOD in Section 4.1.2)
		GEOTOP	$[GEO \rightarrow TOP] = fctn(\lambda_{GEOD}, \phi_{GEOD})$
		M3X1	$\dot{\vec{r}}_{\oplus NB}[TOP] = [GEO \rightarrow TOP]\, \dot{\vec{r}}_{\oplus NB}[GEO]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
DGEOW continued			
		WINDTOP	$\vec{v}_{wind}[TOP] = [-w_H \cos\theta_{WH}, -w_H \sin\theta_{WH}, 0]^T$
			$¢(\vec{v}_{wind}[TOP]) = \mathrm{fctn}(\sigma(w_H), \sigma(\theta_W), w_H, \sigma(w_H))$
			(See the description of WINDTOP in Section 4.1.2 for the expressions for the matrix elements in $¢(\vec{v}_{wind}[TOP])$.)
		VSUB	$\underset{NB}{\vec{v}_{wind}}[TOP] = \underset{NB}{\dot{\vec{r}}_\oplus}\ [TOP] - \vec{v}_{wind}[TOP]$
		M3TX1	$\underset{NB}{\vec{v}_{wind}}[GEO] = [GEO \rightarrow TOP]^T\ \underset{NB}{\vec{v}_{wind}}[TOP]$

Position of the Center of Mass in the GEOgraphic coordinate system

word name	symbol	program	algorithm
47 GEOC(1)	$\underset{CM}{\vec{r}_\oplus}\ [GEO]$	OPIP	Read $[M50 \rightarrow ECI]$ from OPIP.IN
48 (2)		OPIP	Read $\underset{CM}{\vec{r}_{NB}}[BOD]$ and $[M50 \rightarrow BOD]$ from TRJATTDATA
49 (3)			
		ECIGEO	$\underset{NB}{\vec{r}_\oplus}\ [GEO]$ defined previously in words (29-31)
		M3TX1	$\underset{CM}{\vec{r}_{NB}}[M50] = [M50 \rightarrow BOD]^T\ \underset{CM}{\vec{r}_{NB}}[BOD]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name		symbol	program	algorithm
GEOC continued				
			M3X1	$\vec{r}_{\substack{NB\\CM}}[ECI] = [M50 \rightarrow ECI]\ \vec{r}_{\substack{NB\\CM}}[M50]$
			ECIGEO	$\vec{r}_{\substack{NB\\CM}}[GEO] = [ECI \rightarrow GEO]\ \vec{r}_{\substack{NB\\CM}}[ECI]$
				$\vec{r}_{\substack{\oplus\\NB}}[GEO]$ is defined in words (29-31)
			VADD	$\vec{r}_{\substack{\oplus\\CM}}[GEO] = \vec{r}_{\substack{\oplus\\NB}}[GEO] + \vec{r}_{\substack{NB\\CM}}[GEO]$

Velocity of the Center of Mass in the GEOgraphic coordinate system

word name		symbol	program	algorithm
50	DGEOC(1)	$\dot{\vec{r}}_{\substack{\oplus\\CM}}[GEO]$	OPIP	Read in $\vec{\omega}[BOD]$ = M50STAT(13,14,15)
51	(2)		CROSS	$\dot{\vec{r}}_{\substack{NB\\CM}}[BOD] = \vec{\omega}[BOD] \times \vec{r}_{\substack{NB\\CM}}[BOD]$
52	(3)		M3TX1	$\dot{\vec{r}}_{\substack{NB\\CM}}[M50] = [M50 \rightarrow BOD]^T\ \dot{\vec{r}}_{\substack{NB\\CM}}[BOD]$
			M3X1	$\dot{\vec{r}}_{\substack{NB\\CM}}[ECI] = [M50 \rightarrow ECI]\ \dot{\vec{r}}_{\substack{NB\\CM}}[M50]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
DGEOC continued			
		ECIGEO	$\dot{\vec{r}}_{\substack{NB\\CM}}[GEO] = [ECI \rightarrow GEO]\ \dot{\vec{r}}_{\substack{NB\\CM}}[ECI] - \vec{\omega}_{\oplus} \times \vec{r}_{\substack{NB\\CM}}[GEO]$
		VADD	$\dot{\vec{r}}_{\substack{\oplus\\CM}}[GEO] = \dot{\vec{r}}_{\substack{\oplus\\NB}}[GEO] + \dot{\vec{r}}_{\substack{NB\\CM}}[GEO]$

Sensed acceleration of the Center of Mass in the GEOgraphic coordinate system.

word name	symbol	program	algorithm
53 DDGEOC(1)	$\vec{a}_{CM}[GEO]$ sensed	OPIP	Read in $\dot{\vec{\omega}}[BOD]$ = M50STAT(16,17,18)
54 (2)		CROSS	$\dot{\vec{r}}_{\substack{NB\\CM}}[BOD] = \vec{\omega}[BOD] \times \vec{r}_{\substack{NB\\CM}}[BOD]$
55 (3)		CROSS	Centrifugal acceleration: $\vec{a}_{CEN}[BOD] = \vec{\omega}[BOD] \times \dot{\vec{r}}_{\substack{NB\\CM}}[BOD]$
		CROSS	Rotational acceleration: $\vec{a}_{ROT}[BOD] = \dot{\vec{\omega}}[BOD] \times \vec{r}_{\substack{NB\\CM}}[BOD]$
		VADD	$\ddot{\vec{r}}_{\substack{NB\\CM}}[BOD] = \vec{a}_{CEN}[BOD] + \vec{a}_{ROT}[BOD]$
		MT3X1	$\ddot{\vec{r}}_{\substack{NB\\CM}}[M50] = [M50 \rightarrow BOD]^T\ \ddot{\vec{r}}_{\substack{NB\\CM}}[BOD]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
DDGEOC continued			
		M3X1	$\ddot{\vec{r}}_{\substack{NB\\CM}}[ECI] = [M50 \rightarrow ECI]\ \ddot{\vec{r}}_{\substack{NB\\CM}}[M50]$
		ECIGEO	$\vec{a}_{\substack{CM\\sensed}}[GEO] = [ECI \rightarrow GEO]\ \ddot{\vec{r}}_{\substack{NB\\CM}}[ECI] - \vec{\omega}_{\oplus} \times (\vec{\omega}_{\oplus} \times \vec{r}_{\substack{NB\\CM}}[ECI]) - 2\vec{\omega}_{\oplus} \times [ECI \rightarrow GEO]\ \dot{\vec{r}}_{\substack{NB\\CM}}[ECI]$

Wind-relative velocity of the Center of Mass projected into the Geographic system

word name	symbol	program	algorithm
56 DGEOWC(1)	$\vec{v}_{\substack{wind\\CM}}[GEO]$	OPIP	$\vec{v}_{\substack{wind\\NB}}[GEO]$ computed previously, words (44-46)
57 (2)			$\dot{\vec{r}}_{\substack{NB\\CM}}[GEO]$ calculated previously, in order to calculate $\dot{\vec{r}}_{\substack{\oplus\\CM}}$ [GEO], words (50-61)
58 (3)		VADD	$\vec{v}_{\substack{wind\\CM}}[GEO] = \vec{v}_{\substack{wind\\NB}}[GEO] + \dot{\vec{r}}_{\substack{NB\\CM}}[GEO]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Position of the Nav Base relative to the landing runway expressed in Landing Field coordinates

word	name	symbol	program	algorithm
59	LF(1)		GEOLF	First Pass obtain [GEO → LF] and $\vec{r}_{\oplus_L}$[GEO] from Landing site geophysical parameters
60	(2)	$\vec{r}_{L_{NB}}$ [LF]		
61	(3)		OPIP	$\vec{r}_{\oplus_{NB}}$ [GEO] computed previously, words (29-31)
			VSUB	$\vec{r}_{L_{NB}}$ [GEO] = $\vec{r}_{\oplus_{NB}}$ [GEO] - $\vec{r}_{\oplus_L}$[GEO]
			M3X1	$\vec{r}_{L_{NB}}$ [LF] = [GEO → LF] $\vec{r}_{L_{NB}}$ [GEO]

Velocity of the Nav Base relative to the landing runway in Landing Field coordinates

word	name	symbol	program	algorithm
62	DLF(1)		OPIP	$\dot{\vec{r}}_{\oplus_{NB}}$ [GEO] computed previously, words (32-34)
63	(2)	$\vec{v}_{L_{NB}}$ [LF]	M3X1	$\vec{v}_{L_{NB}}$ [LF] = [GEO → LF] $\dot{\vec{r}}_{\oplus_{NB}}$ [GEO]
64	(3)			

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm

Sensed acceleration of the Nav Base relative to landing runway in Landing Field coordinates

word	name	symbol	program	algorithm
65	DDLF(1)	$\vec{a}_{L_{NB}}$ [LF]	OPIP	$\ddot{\vec{r}}_{\oplus_{NB}}$ [GEO] computed previously, words (38-40)
66	(2)		M3X1	$\vec{a}_{L_{NB}}$ [LF] = [GEO → LF] $\ddot{\vec{r}}_{\oplus_{NB}}$ [GEO]
67	(3)			

Geodetic velocity of the Nav Base projected in the TOPodetic coordinate frame

word	name	symbol	program	algorithm
68	DTOP(1)	$\vec{v}_{\oplus_{NB}}$ [TOP]	GEOGEOD	Geodetic latitude and longitude, ϕ_{GEOD} and λ_{GEOD}, respectively, were computed previously (words 44-48) from the geophysical parameters and $\vec{r}_{\oplus_{NB}}$ [GEO] which was computed previously, words 29-31
69	(2)		GEOTOP	[GEO → TOP] = fctn(λ_{GEOD}, ϕ_{GEOD})
70	(3)		M3X1	$\vec{v}_{\oplus_{NB}}$ [TOP] = [GEO → TOP] $\dot{\vec{r}}_{\oplus_{NB}}$ [GEO]

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
		Geodetic sensed acceleration projected in the TOPodetic coordinate frame		
71	DDTOP(1)	$\vec{a}_{\oplus\,\mathrm{NB\ sensed}}$ [TOP]	M3X1	$\vec{a}_{\oplus\,\mathrm{NB\ sensed}}[TOP] = [GEO \rightarrow TOP]\ \ddot{\vec{r}}_{\oplus\,\mathrm{NB}}[GEO]$
72	(2)			
73	(3)			
		Uncertainty (one sigma) in $\vec{v}_{\oplus\,\mathrm{NB}}$ [TOP]		
74	DTOPU(1)	$\vec{\sigma}(\vec{v}_{\oplus\,\mathrm{NB}}[TOP])$	OPIP	Read $¢(\vec{S}[M50])$ from input file TRJATTDATA
75	(2)		OPIP	$¢(\ddot{\vec{r}}_{\oplus\,\mathrm{NB}}[M50])$ is extracted from $¢(\vec{S}[M50])$
76	(3)		M3X3UN2	$¢(\dot{\vec{r}}_{\oplus\,\mathrm{NB}}[ECI]) = [M50 \rightarrow ECI]\ ¢(\dot{\vec{r}}_{\oplus\,\mathrm{NB}}[M50])\ [M50 \rightarrow ECI]^T$
			M3X3UN2	$¢(\dot{\vec{r}}_{\oplus\,\mathrm{NB}}[GEO]) = [ECI \rightarrow GEO]\ ¢(\dot{\vec{r}}_{\oplus\,\mathrm{NB}}[ECI])\ [ECI \rightarrow GEO]^T$
			M3X3UN2	$¢(\vec{v}_{\oplus\,\mathrm{NB}}[TOP]) = [GEO \rightarrow TOP]\ ¢(\dot{\vec{r}}_{\oplus\,\mathrm{NB}}[GEO])\ [GEO \rightarrow TOP]^T$
			SIGMAS	$\vec{\sigma}(\vec{v}_{\oplus\,\mathrm{NB}}[TOP]) = [C_{11}{}^{1/2}, C_{22}{}^{1/2}, C_{33}{}^{1/2}]^T$ where $[C_{ij}] = ¢(\vec{v}_{\oplus\,\mathrm{NB}}[TOP])$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
	Uncertainty (one sigma) in $\vec{a}_{\oplus}$ [TOP] NB sensed			
77	DDTOPU(1)		OPIP	$\ddot{\mathrm{C}}(\vec{r}[M50])$ is extracted from $\mathrm{C}(\vec{S}[M50])$
78	(2)	$\vec{\sigma}(\vec{a}_{\oplus}\ [TOP])$ NB sensed	M3X3UN2	$\mathrm{C}(\ddot{\vec{r}}[ECI]) = [M50 \to ECI]\ \mathrm{C}(\ddot{\vec{r}}[M50])\ [M50 \to ECI]^T$
79	(3)		M3X3UN2	$\mathrm{C}(\ddot{\vec{r}}[GEO]) = [ECI \to GEO]\ \mathrm{C}(\ddot{\vec{r}}[ECI])\ [ECI \to GEO]^T$
			M3X3UN2	$\mathrm{C}(\vec{a}[TOP]) = [GEO \to TOP]\ \mathrm{C}(\ddot{\vec{r}}[GEO])\ [GEO \to TOP]^T$
			SIGMAS	$\vec{\sigma}(\vec{a}_{\oplus}\ [TOP]) = [C_{11}{}^{1/2}, C_{22}{}^{1/2}, C_{33}{}^{1/2}]^T$ (NB sensed) where $[C_{ij}] = \mathrm{C}(\vec{a}_{\oplus}[TOP])$ (NB sensed)
	Wind-relative velocity of the Nav Base projected on to the TOPodetic frame			
80	DTOPW(1)		WINDTOP	$\vec{v}_{wind}\ [TOP] = [-w_H \cos\theta_{wH}, -w_H \sin\theta_{wH}, 0]^T$
81	(2)	$\vec{v}_{wind}\ [TOP]$ NB	WINDTOP	$\mathrm{C}(\vec{v}_{wind}\ [TOP]) = \mathrm{fctn}(\sigma(w_H), \sigma(\theta_{WH}), w_H, \sigma(w_V))$ NB (See description of WINDTOP in Section 4.1.2
82	(3)			for the expressions for the matrix elements)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Velocity of the Nav Base in the GEOgraphic coordinate system, <u>but</u> projected into the vehicle BODy axis coordinate system

word	name	symbol	program	algorithm
83	DBOD(1)	$\vec{v}_{\oplus_{NB}}[BOD]$	ECIGEO	$\dot{\vec{r}}_{\oplus_{NB}}$ [GEO] computed previously, words (32-34)
84	(2)		M3X3T	$[ECI \rightarrow BOD] = [M50 \rightarrow BOD]\,[M50 \rightarrow ECI]^T$
85	(3)		M3X3T	$[GEO \rightarrow BOD] = [ECI \rightarrow BOD]\,[ECI \rightarrow GEO]^T$
			M3X1	$\vec{v}_{\oplus_{NB}}[BOD] = [GEO \rightarrow BOD]\,\dot{\vec{r}}_{\oplus_{NB}}[GEO]$

Sensed acceleration of the Nav Base projected onto the vehicle BODy system

word	name	symbol	program	algorithm
86	DDBOD(1)	$\vec{a}_{\oplus_{NB}}$ [BOD]	OPIP	Read $\ddot{\vec{r}}_{\oplus_{NB}}$ [M50] and $[M50 \rightarrow BOD]$ from the input file TRJATTDATA
87	(2)			
88	(3)		M3X1	$\vec{a}_{\oplus_{NB}}[BOD] = [M50 \rightarrow BOD]\,\ddot{\vec{r}}_{\oplus_{NB}}[M50]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Wind vector projected into the vehicle BODy coordinate system

word	name	symbol	program	algorithm
89	WBODV(1)	$\vec{v}_{\oplus\,wind}$ [BOD]	WINDTOP	$\vec{v}_{\oplus\,wind}[TOP] = [-w_H \cos\theta_{wH}, -w_H \sin\theta_{wH}, 0]^T$
90	(2)		M3X1	$\vec{v}_{\oplus\,wind}[BOD] = [TOP \rightarrow BOD]\,\vec{v}_{\oplus\,wind}[TOP]$
91	(3)			

Wind-relative velocity of the Nav Base expressed in vehicle BODy coordinates

word	name	symbol	program	algorithm
92	DBODW(1)	$\vec{v}_{wind\,NB}$ [BOD]	OPIP	$\vec{v}_{\oplus\,NB}[TOP]$ computed previously, words (68-70)
93	(2)		WINDTOP	$\vec{v}_{\oplus\,wind}[TOP] = [-w_H \cos\theta_{wH}, -w_H \sin\theta_{wH}, 0]^T$
94	(3)		VSUB	$\vec{v}_{wind\,NB}[TOP] = \vec{v}_{\oplus\,NB}[TOP] - \vec{v}_{\oplus\,wind}[TOP]$
			M3X1	$\vec{v}_{wind\,NB}[BOD] = [TOP \rightarrow BOD]\,\vec{v}_{wind\,NB}[TOP]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
				Uncertainty of velocity of Nav Base in GEOgraphics coordinates projected into BOD system
95	DBODU(1)		OPIP	$\not{c}(\dot{\vec{r}}_{\oplus_{NB}}\,[M50])$ extracted from $\not{c}(\vec{S}[M50])$
96	(2)	$\vec{\sigma}(\vec{v}_{\oplus_{NB}}\,[BOD])$	OPIP	$\not{c}(\phi, \theta, \psi)$ extracted from $\not{c}(\vec{S}[M50])$; ϕ = roll, θ = pitch, ψ = yaw
97	(3)		M3X3UN2	$\not{c}(\dot{\vec{r}}_{\oplus_{NB}}\,[ECI]) = [M50 \rightarrow ECI]\ \not{c}(\dot{\vec{r}}_{\oplus_{NB}}\,[M50])\ [M50 \rightarrow ECI]^T$
			ECIGEO	$[ECI \rightarrow GEO]$ computed previously, for words (19-31)
			M3X3UN2	$\not{c}(\dot{\vec{r}}_{\oplus_{NB}}\,[GEO]) = [ECI \rightarrow GEO]\ \not{c}(\dot{\vec{r}}_{\oplus_{NB}}\,[ECI])\ [ECI \rightarrow GEO]^T$
			M3X3T	$[ECI \rightarrow BOD] = [M50 \rightarrow BOD]\ [M50 \rightarrow ECI]^T$
			M3X3T	$[GEO \rightarrow BOD] = [ECI \rightarrow BOD]\ [ECI \rightarrow GEO]^T$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
DBODU continued			
		M3XUN2	$\text{¢}'(\vec{v}_{\underset{NB}{\oplus}}\ [BOD]) = [GEO \rightarrow BOD]\ \text{¢}(\dot{\vec{r}}_{\underset{NB}{\oplus}}\ [GEO])\ [GEO \rightarrow BOD]^T$
		SBCALC	$\text{¢}(\vec{v}_{\underset{NB}{\oplus}}\ [BOD]) = \text{¢}'(\vec{v}_{\underset{NB}{\oplus}}\ [BOD]) + \{$ effects of body axis errors $\}$ (See the description of SBCALC in Section 4.1.2)
		SIGMAS	$\vec{\sigma}(\vec{v}_{\underset{NB}{\oplus}}\ [BOD]) = [C_{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2}]^T$ where $[C_{ij}] = \text{¢}(\vec{v}_{\underset{NB}{\oplus}}\ [BOD])$

Uncertainty of sensed acceleration of Nav Base projected onto the BODy system

word name	symbol	program	algorithm
98 DDBODU(1)	$\vec{\sigma}(\vec{a}_{\underset{NB}{\oplus}}\ [BOD])$	OPIP	$\text{¢}(\ddot{\vec{r}}_{\underset{NB}{\oplus}}\ [M50])$ & $\text{¢}(\phi, \theta, \psi)$ from $\text{¢}(\vec{S}[M50])$
99 (2)		OPIP	$[M50 \rightarrow BOD]$ from TRJATTDATA
100 (3)		M3X3UN2	$\text{¢}'(\vec{a}_{\underset{NB}{\oplus}}\ [BOD]) = [M50 \rightarrow BOD]\ \text{¢}(\ddot{\vec{r}}_{\underset{NB}{\oplus}}\ [M50])\ [M50 \rightarrow BOD]^T$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
DDBODU continued			
		SPBCALC	$ ¢(\vec{a}_{\oplus NB}\,[BOD]) = ¢'(\vec{a}_{\oplus NB}\,[BOD]) + \{\text{effects of body axis errors}\}$ (See the description of SPBCALC in in Section 4.1.2)
		SIGMAS	$\vec{\sigma}(\vec{a}_{\oplus NB}\,[BOD]) = [C_{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2}]^T$ where $[C_{ij}] = ¢(\vec{a}_{\oplus NB}\,[BOD])$

Uncertainty in wind vector projected onto BODy axis coordinates

word name	symbol	program	algorithm
101 WBODVU(1)	$\vec{\sigma}(\vec{v}_{\oplus wind}\,[BOD])$	OPIP	$¢(\phi, \theta, \psi)$ extracted from $¢(\vec{S}[M50])$
102 (2)		OPIP	[M50 → BOD] read from the TRJATTDATA file
103 (3)		OPIP	[M50 → ECI] read from the OPIP.IN file
		M3X3T	$[ECI \to BOD] = [M50 \to BOD]\,[M50 \to ECI]^T$
		GEOTOP	$[GEO \to TOP] = \mathrm{fctn}(\phi_{GEOD}, \lambda_{GEOD})$
		ECIGEO	$[ECI \to GEO] = \mathrm{fctn}(\omega_{\oplus}, t, \vec{r}_{\oplus NB}\,[ECI], \dot{\vec{r}}_{\oplus NB}\,[ECI], \dot{\vec{r}}_{\oplus NB}\,[ECI], \ddot{\vec{r}}_{\oplus NB}\,[ECI])$
		M3X3	$[ECI \to TOP] = [GEO \to TOP]\,[ECI \to GEO]$
		M3X3T	$[TOP \to BOD] = [ECI \to BOD]\,[ECI \to TOP]^T$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
WBODVU continued			
		WINDTOP	$\text{¢}(\vec{v}_{wind}\,[TOP]) = \text{fctn}(\sigma(w_H),\ \sigma(\theta_{WH}),\ w_H,\ \sigma(w_v))$
			(See description of WINDTOP in Section 4.1.2 for expressions for the matrix elements)
		M3X3UN2	$\text{¢}'(\vec{v}_{wind}\,[BOD]) =$
			$[TOP \rightarrow BOD]\ \text{¢}(\vec{v}_{wind}\,[TOP])\ [TOP \rightarrow BOD]^T$
		SPBCALC	$\text{¢}(\vec{v}_{wind}\,[BOD]) = \text{¢}'(\vec{v}_{wind}\,[BOD]) + \begin{Bmatrix}\text{effects of body} \\ \text{axis errors}\end{Bmatrix}$
			(See description of SPBCALC in Section 4.1.2)
		SIGMAS	$\vec{\sigma}(\vec{v}_{\oplus\,wind}\,[BOD]) = [C_{11}{}^{1/2},\ C_{22}{}^{1/2},\ C_{33}{}^{1/2}]^T$
			where $[C_{ij}] = \text{¢}(\vec{v}_{\oplus\,wind}\,[BOD])$

Uncertainty in the wind-relative velocity of the Nav Base expressed in BODy coordinates

word name	symbol	program	algorithm
104 DBODWU(1)	$\vec{\sigma}(\vec{v}_{wind\,NB}\,[BOD])$	OPIP	$\text{¢}(\phi,\ \theta,\ \psi)$ extracted from $\text{¢}(\vec{S}[M50])$
105 (2)		OPIP	$[TOP \rightarrow BOD]$ computed previously, words (101-103)
106 (3)		OPIP	$\text{¢}(\vec{v}_{\oplus\,NB}\,[TOP])$ computed previously, words (74-76)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
DBODWU continued			
		WINDTOP	$\vec{v}_{wind}[TOP] = [-w_H \cos \theta_{wH}, -w_H \sin \theta_{wH}, 0]^T$
		WINDTOP	$¢(\vec{v}_{wind}[TOP]) = fctn(\sigma(w_H), \sigma(\theta_{wH}), w_H, \sigma(w_v))$ (See description of WINDTOP in Section 4.1.2 for the expressions for the matrix elements)
		VSUB	$\vec{v}_{wind\,NB}[TOP] = \vec{v}_{\oplus NB}[TOP] - \vec{v}_{wind}[TOP]$
		OPIP	$¢(\vec{v}_{wind\,NB}[TOP]) = ¢(\vec{v}_{\oplus NB}[TOP]) + ¢(\vec{v}_{wind}[TOP])$
		M3X3UN2	$¢'(\vec{v}_{wind\,NB}[BOD]) = [TOP \to BOD]\, ¢(\vec{v}_{wind\,NB}[TOP])\, [TOP \to BOD]^T$
		SPBCALC	$¢(\vec{v}_{wind\,NB}[BOD]) = ¢'(\vec{v}_{wind\,NB}[BOD]) + \{\text{effects of body axis errors}\}$ (See description of SPBCALC in Section 4.1.2)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
DBODWU continued			
		SIGMAS	$\vec{\sigma}(\vec{v}_{\substack{wind\\NB}}[BOD]) = [C_{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2}]^T$ where $[C_{ij}] = \mathcal{C}(\vec{v}_{\substack{wind\\NB}}[BOD])$

Quaternion for transforming from M50 coordinates of BODy coordinates

word name	symbol	program	algorithm
107 Q50(1)	$\tilde{q}$ [M50 → BOD]	OPIP	Read directly from the input file TRJATTDATA and equivalenced to the output buffer array elements
108 (2)			
109 (3)			
110 (4)			

Index of selected IMU

word name	symbol	program	algorithm
111 PIMU		OPIP	Read directly from the input file TRJATTDATA and equivalenced to the output buffer array element

Transformation from the ECI to launch-site-located Plumbline coordinate system

word name	symbol	program	algorithm
		ECIPLM	[ECI → PLM] is calculated on the first pass for ascent (only) from geophysical parameters, launch site parameters, and the time of SRB ignition

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

Position of the Nav Base relative to the Earth's center expressed in Plumbline coordinates (Ascent only)

word	name	symbol	program	algorithm
112	PLM(1)		OPIP	Read [M50 → ECI] from input file OPIP.IN
113	(2)	$\vec{r}_{\oplus_{NB}}$ [PLM]	OPIP	Read $\vec{r}_{\oplus_{NB}}$ [M50] from the input file TRJATTDATA
114	(3)		M3X1	$\vec{r}_{\oplus_{NB}}$ [ECI] = [M50 → ECI] $\vec{r}_{\oplus_{NB}}$ [M50]
			M3X1	$\vec{r}_{\oplus_{NB}}$ [PLM] = [ECI → PLM] $\vec{r}_{\oplus_{NB}}$ [ECI]

Velocity of the Nav Base relative to the Earth's center expressed in Plumbline coordinates (Ascent only)

word	name	symbol	program	algorithm
115	DPLM(1)		OPIP	Read $\dot{\vec{r}}_{\oplus_{NB}}$ [M50] from TRJATTDATA
116	(2)	$\dot{\vec{r}}_{\oplus_{NB}}$ [PLM]	M3X1	$\dot{\vec{r}}_{\oplus_{NB}}$ [ECI] = [M50 → ECI] $\dot{\vec{r}}_{\oplus_{NB}}$ [M50]
117	(3)		M3X1	$\dot{\vec{r}}_{\oplus_{NB}}$ [PLM] = [ECI → PLM] $\dot{\vec{r}}_{\oplus_{NB}}$ [ECI]

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Sensed acceleration of the Nav Base relative to Earth's center expressed in Plumbline coordinates (Ascent only)

word	name	symbol	program	algorithm
118	DDPLM(1)	$\ddot{\vec{r}}_{\oplus_{NB}}$ [PLM]	OPIP	Read $\ddot{\vec{r}}_{\oplus_{NB}}$ [M50] from TRJATTDATA
119	(2)			$\ddot{\vec{r}}_{\oplus_{NB}}$ [ECI] = [M50 → ECI] $\ddot{\vec{r}}_{\oplus_{NB}}$ [M50]
120	(3)			$\ddot{\vec{r}}_{\oplus_{NB}}$ [PLM] = [ECI → PLM] $\ddot{\vec{r}}_{\oplus_{NB}}$ [ECI]

Uncertainty in the position of the Nav Base relative to the Earth's center expressed in Plumbline coordinates (Ascent only)

word	name	symbol	program	algorithm
121	PLMU(1)	$\vec{\sigma}(\vec{r}_{\oplus_{NB}}$ [PLM])	OPIP	¢($\vec{r}_{\oplus_{NB}}$ [M50]) is extracted from ¢($\vec{S}$[M50])
122	(2)		M3X3UN2	¢($\vec{r}_{\oplus_{NB}}$ [ECI]) = [M50 → ECI] ¢($\vec{r}_{\oplus_{NB}}$[M50]) [M50 → ECI]^T
123	(3)			
			M3X3UN2	¢($\vec{r}_{\oplus_{NB}}$ [PLM]) = [ECI → PLM] ¢($\vec{r}_{\oplus_{NB}}$ [ECI]) [ECI → PLM]^T

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
PLMU continued				
			SIGMAS	$\vec{\sigma}(\vec{r}_{\oplus NB}\,[PLM]) = [C_{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2}]^T$ where $[C_{ij}] = ¢(\vec{r}_{\oplus NB}\,[PLM])$

Uncertainty in the velocity of the Nav Base relative to the Earth's center expressed in Plumbline coordinates (Ascent only)

word	name	symbol	program	algorithm
124	DPLMU(1)	$\vec{\sigma}(\dot{\vec{r}}_{\oplus NB}\,[PLM])$	OPIP	$¢(\dot{\vec{r}}_{\oplus NB}\,[M50])$ is extracted from $¢(\vec{S}[M50])$
125	(2)		M3X3UN2	$¢(\dot{\vec{r}}_{\oplus NB}\,[ECI]) = [M50 \rightarrow ECI]\ ¢(\dot{\vec{r}}_{\oplus NB}\,[M50])\ [M50 \rightarrow ECI]^T$
126	(3)		M3X3UN2	$¢(\dot{\vec{r}}_{\oplus NB}[PLM]) = [ECI \rightarrow PLM]\ ¢(\dot{\vec{r}}_{\oplus NB}[ECI])\ [ECI \rightarrow PLM]^T$
			SIGMAS	$\vec{\sigma}(\dot{\vec{r}}_{\oplus NB}\,[PLM]) = [C_{11}^{1/2}, C_{22}^{1/2}, C_{33}^{1/2}]^T$ where $[C_{ij}] = ¢(\dot{\vec{r}}_{\oplus NB}[PLM])$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
				Uncertainty in the sensed acceleration of the Nav Base relative to the Earth's center expressed in Plumbline coordinates (Ascent only)
127	DDPLMU(1)	$\vec{\sigma}(\ddot{\vec{r}}_{\oplus NB}\ [PLM])$	OPIP	$¢(\ddot{\vec{r}}_{\oplus NB}\ [M50])$ is extracted from $¢(\vec{S}[M50])$
128	(2)		M3X3UN2	$¢(\ddot{\vec{r}}_{\oplus NB}\ [ECI]) = [M50 \rightarrow ECI]\ ¢(\ddot{\vec{r}}_{\oplus NB}\ [M50])\ [M50 \rightarrow ECI]^T$
129	(3)			
			M3X2UN2	$¢(\ddot{\vec{r}}_{\oplus NB}\ [PLM]) = [ECI \rightarrow PLM]\ ¢(\ddot{\vec{r}}_{\oplus NB}\ [ECI])\ [ECI \rightarrow PLM]^T$
			SIGMAS	$\vec{\sigma}(\ddot{\vec{r}}_{\oplus NB}\ [PLM]) = [C_{11}{}^{1/2}, C_{22}{}^{1/2}, C_{33}{}^{1/2}]^T$ where $[C_{ij}] = ¢(\ddot{\vec{r}}_{\oplus NB}\ [PLM])$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

Wind-relative velocity of the Nav Base but projected onto Plumbline coordinates (Ascent only)

word	name	symbol	program	algorithm
130	PPLMW(1)		VSUB	$\vec{v}_{\substack{wind\\NB}}$ [TOP] was computed previously for words (104-106)
131	(2)	$\vec{v}_{\substack{wind\\NB}}$ [PLM]	M3X3T	[ECI → TOP] was computed previously for words (101-103)
			ECIPLM	[ECI → PLM] was computed previously just before words (112-114)
132	(3)			
			M3TX1	$\vec{v}_{\substack{wind\\NB}}[ECI] = [ECI \rightarrow TOP]^T \, \vec{v}_{\substack{wind\\NB}}[TOP]$
			M3X1	$\vec{v}_{\substack{wind\\NB}}[PLM] = [ECI \rightarrow PLM] \, \vec{v}_{\substack{wind\\NB}}[ECI]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
				Uncertainty in the wind-relative velocity of the Nav Base projected onto the Plumbline coordinate system (Ascent only)
133	DPLMWU(1)		WINDTOP	$\not{C}(\vec{v}_{wind}\ [TOP])$ computed previously for words (80-82)
134	(2)	$\vec{\sigma}(\vec{v}_{wind_{NB}}\ [PLM])$	M3X3UN2	$\not{C}(\vec{v}_{\oplus_{NB}}\ [TOP])$ computed previously for words (74-76)
135	(3)		OPIP	$\not{C}(\vec{v}_{wind_{NB}}\ [TOP]) = \not{C}(\vec{v}_{\oplus_{NB}}\ [TOP]) + \not{C}(\vec{v}_{wind}\ [TOP])$
			M3X3UN1	$\not{C}(\vec{v}_{wind_{NB}}\ [ECI]) = [ECI \rightarrow TOP]^T\ \not{C}(\vec{v}_{wind_{NB}}\ [TOP])\ [ECI \rightarrow TOP]$
			M3X3UN2	$\not{C}(\vec{v}_{wind_{NB}}\ [PLM]) = [ECI \rightarrow PLM]\ \not{C}(\vec{v}_{wind_{NB}}\ [ECI])\ [ECI \rightarrow PLM]^T$
			SIGMAS	$\vec{\sigma}(\vec{v}_{wind_{NB}}\ [PLM]) = [C_{11}{}^{1/2},\ C_{22}{}^{1/2},\ C_{33}{}^{1/2}]^T$ where $[C_{ij}] = \not{C}(\vec{v}_{wind_{NB}}\ [PLM])$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Position of the vehicle Center of Mass relative to Earth's center expressed in Plumbline coordinates (Ascent only)

word	name	symbol	program	algorithm
136	PLMC(1)		OPIP	Read $\vec{r}_{\substack{NB\\CM}}$[BOD] from the input file TRJATTDATA
			ECIPLM	[ECI → PLM] computed previously before words (112-114)
137	(2)	$\vec{r}_{\substack{\oplus\\CM}}$ [PLM]		
			M3X1	$\vec{r}_{\substack{NB\\CM}}[M50] = [M50 \rightarrow BOD]^T \vec{r}_{\substack{NB\\CM}}[BOD]$
138	(3)			
			M3X1	$\vec{r}_{\substack{NB\\CM}}[ECI] = [M50 \rightarrow ECI] \vec{r}_{\substack{NB\\CM}}[M50]$
			M3X1	$\vec{r}_{\substack{NB\\CM}}[PLM] = [ECI \rightarrow PLM] \vec{r}_{\substack{NB\\CM}}[ECI]$
			OPIP	$\vec{r}_{\substack{\oplus\\NB}}$ [PLM] is word set (112-114)
			VADD	$\vec{r}_{\substack{\oplus\\CM}}[PLM] = \vec{r}_{\substack{\oplus\\NB}}[PLM] + \vec{r}_{\substack{NB\\CM}}[PLM]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Velocity of the vehicle Center of Mass relative to the Earth's center expressed in Plumbline coordinates (Ascent only)

word	name	symbol	program	algorithm
139	DPLMC(1)	$\dot{\vec{r}}_{\oplus_{CM}}[PLM]$	CROSS	$\vec{v}_{NB_{CM}}[BOD] = \vec{\omega}[BOD] \times \vec{r}_{NB_{CM}}[BOD]$
140	(2)		M3TX1	$\dot{\vec{r}}_{NB_{CM}}[M50] = [M50 \rightarrow BOD]^T \, \vec{v}_{NB_{CM}}[BOD]$
141	(3)		M3X1	$\dot{\vec{r}}_{NB_{CM}}[ECI] = [M50 \rightarrow ECI] \, \dot{\vec{r}}_{NB_{CM}}[M50]$
			M3X1	$\dot{\vec{r}}_{NB_{CM}}[PLM] = [ECI \rightarrow PLM] \, \dot{\vec{r}}_{NB_{CM}}[ECI]$
			OPIP	$\dot{\vec{r}}_{\oplus_{NB}}[PLM]$ is word set (115-117)
			VADD	$\dot{\vec{r}}_{\oplus_{CM}}[PLM] = \dot{\vec{r}}_{\oplus_{NB}}[PLM] + \dot{\vec{r}}_{NB_{CM}}[PLM]$

Sensed acceleration of the vehicle Center of Mass expressed in Plumbline coordinates (Ascent only)

word	name	symbol	program	algorithm
142	DDPLMC(1)	$\ddot{\vec{r}}_{\oplus_{CM}}[PLM]$	CROSS	$\vec{a}_{NB_{CM}}[BOD] = \dot{\vec{\omega}}[BOD] \times \vec{v}_{NB_{CM}}[BOD]$
143	(2)		VADD	$+ \, \dot{\vec{\omega}}[BOD] \times \vec{r}_{NB_{CM}}[BOD]$
144	(3)		M3TX1	$\ddot{\vec{r}}_{NB_{CM}}[M50] = [M50 \rightarrow BOD]^T \, \vec{a}_{NB_{CM}}[BOD]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
DDPLMC continued			
		M3X1	$\ddot{\vec{r}}_{\substack{NB\\CM}}[ECI] = [M50 \rightarrow ECI]\ \ddot{\vec{r}}_{\substack{NB\\CM}}[M50]$
		M3X1	$\ddot{\vec{r}}_{\substack{NB\\CM}}[PLM] = [ECI \rightarrow PLM]\ \ddot{\vec{r}}_{\substack{NB\\CM}}[ECI]$
		OPIP	$\ddot{\vec{r}}_{\substack{\oplus\\NB}}$ [PLM] is word set (118-120)
		VADD	$\ddot{\vec{r}}_{\substack{\oplus\\CM}}[PLM] = \ddot{\vec{r}}_{\substack{\oplus\\NB}}[PLM] + \ddot{\vec{r}}_{\substack{NB\\CM}}[PLM]$

Wind-relative velocity of the vehicle Center of Mass expressed in Plumbline coordinates (Ascent only)

word name	symbol	program	algorithm
145 DPLMCW(1)	$\left.\right\}\ \vec{v}_{\substack{wind\\CM}}[PLM]$	OPIP	$v_{\substack{wind\\NB}}$ [TOP] computed previously for words (80-82)
146 (2)		M3X1	$\dot{\vec{r}}_{\substack{NB\\CM}}$[PLM] computed previously for words (139-141)
147 (3)		M3X3	[ECI $\rightarrow$ TOP] computed previously for words (101-103)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
DPLMCW continued			
		ECIPLM	[ECI $\rightarrow$ PLM] computed previously for words (112-114)
		M3TX1	$\vec{v}_{\substack{wind\\NB}}[ECI] = [ECI \rightarrow TOP]^T \vec{v}_{\substack{wind\\NB}}[TOP]$
		M3X1	$\vec{v}_{\substack{wind\\NB}}[PLM] = [ECI \rightarrow PLM]\, \vec{v}_{\substack{wind\\NB}}[ECI]$
		VADD	$\vec{v}_{\substack{wind\\CM}}[PLM] = \vec{v}_{\substack{wind\\NB}}[PLM] + \dot{\vec{r}}_{\substack{NB\\CM}}[PLM]$

Yaw, and its uncertainty, of the vehicle body with respect to a wind-relative local frame

word name	symbol	program	algorithm
148 YAW	ψ	EULANG	Given in the description of EULANG in Section 4.1.2
149 YAWU	$\sigma(\psi)$		

Pitch, and its uncertainty, of the vehicle body with respect to a wind-relative local frame

word name	symbol	program	algorithm
150 PITCH	θ	EULANG	Given in the description of EULANG in Section 4.1.2
151 PITCHU	$\sigma(\theta)$		

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Roll, and its uncertainty, of the vehicle body with respect to a wind-relative local frame

152	ROLL	ϕ	EULANG	Given in the description of EULANG in Section 4.1.2
153	ROLLU	$\sigma(\phi)$		

Yaw Rate, and its uncertainty, of the vehicle body with respect to a wind-relative local frame

154	YAWD	$\dot{\psi}$	EULANG	Given in the description of EULANG in Section 4.1.2
155	YAWDU	$\sigma(\dot{\psi})$		

Pitch Rate, and its uncertainty, of the vehicle body with respect to a wind-relative local frame

156	PITCHD	$\dot{\theta}$	EULANG	Given in the description of EULANG in Section 4.1.2
157	PITCHDU	$\sigma(\dot{\theta})$		

Roll Rate, and its uncertainty, of the vehicle body with respect to a wind-relative local frame

158	ROLLD	$\dot{\phi}$	EULANG	Given in the description of EULANG in Section 4.1.2
159	ROLLDU	$\sigma(\dot{\phi})$		

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
Bank angle				
160	BA	Φ	BANK	Given in the description of BANK in Section 4.1.2
Longitude of the Nav Base				
161	GEOLONG	λ_{GEOD}	GEOGEOD	Given in the description of GEOGEOD in Section 4.1.2
Geodetic Latitude of the Nav Base				
162	GEODLAT	ϕ_{GEOD}	GEOGEOD	Given in the description of GEOGEOD in Section 4.1.2
Geodetic altitude (height) above the (Fischer) reference ellipsoid of the Nav Base				
163	GEODALT	h_{GEOD}	GEOGEOD	Given in the description of GEOGEOD in Section 4.1.2
Uncertainty in the geodetic altitude (height) of the Nav Base				
164	GEOHU	$\sigma(h_{GEOD})$	OPIP	Let the local vertical unit vector, $\hat{n}$, which is normal to the reference ellipsoid be $\hat{n} = [n_1, n_2, n_3]^T$ where $n_1 = \cos\phi_{GEOD} \cos\lambda_{GEOD}$ $n_2 = \cos\phi_{GEOD} \sin\lambda_{GEOD}$ $n_3 = \sin\phi_{GEOD}$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
GEOHU continued			
		OPIP	Read M50COV from input file TRJATTDATA
		OPIP	$¢(\vec{r}_{\oplus_{NB}}\ [M50]) = MVCOV(i,j);\ i = 1,3;\ j = 1,3$
		M3X3UN2	$¢(\vec{r}_{\oplus_{NB}}\ [ECI]) = [M50 \rightarrow ECI]\ ¢(\vec{r}_{\oplus_{NB}}\ [M50])\ [M50 \rightarrow ECI]^T$
		M3X3UN2	$¢(\vec{r}_{\oplus_{NB}}\ [GEO]) = [ECI \rightarrow GEO]\ ¢(\vec{r}_{\oplus_{NB}}\ [ECI])\ [ECI \rightarrow GEO]^T$
		OPIP	$\sigma(h_{GEOD})^2 = \hat{n}\ ¢(\vec{r}_{\oplus_{NB}}\ [GEO])\ \hat{n}^T$
		OPIP	$\sigma(h_{GEOD}) = [\sigma(h_{GEOD})^2]^{1/2}$

Geodetic altitude rate of the Nav Base

165 DH	$\dot{h}_{GEOD}$	OPIP	$\dot{\vec{r}}_{\oplus_{NB}}$ [GEO] computed previously, words (32-34)
			$\dot{h}_{GEOD} = \hat{n} \cdot \dot{\vec{r}}_{\oplus_{NB}}\ [GEO]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm

Uncertainty in geodetic altitude rate of the Nav Base

166 DHU $\sigma(\dot{h}_{GEOD})$ OPIP $¢(\dot{\vec{r}}_{\oplus NB}\ [M50]) = MVCOV(i+3,j+3);\ i = 1,3;\ j = 1,3$

M3X3UN2 $¢(\dot{\vec{r}}_{\oplus NB}\ [ECI]) =$

$$[M50 \rightarrow ECI]\ ¢(\dot{\vec{r}}_{\oplus NB}\ [M50])\ [M50 \rightarrow ECI]^T$$

M3X3UN2 $¢(\dot{\vec{r}}_{\oplus NB}\ [GEO]) =$

$$[ECI \rightarrow GEO]\ ¢(\dot{\vec{r}}_{\oplus NB}\ [M50])\ [ECI \rightarrow GEO]^T$$

$$\sigma(\dot{h}_{GEOD})^2 = \hat{n}\ ¢(\dot{\vec{r}}_{\oplus NB}\ [GEO])\ \hat{n}^T$$

$$\sigma(\dot{h}_{GEOD}) = [\sigma(\dot{h}_{GEOD})^2]^{1/2}$$

Declination of the Nav Base

167 DELTA δ_{NB} GEOGEOD Let $\vec{r}_{\oplus NB}\ [GEO] = [x_g,\ y_g,\ z_g]^T$; then,

$$\delta_{NB} = \arctan \frac{z_g}{(x_g^2 + y_g^2)^{1/2}}$$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER				
word	name	symbol	program	algorithm

Magnitude of the geocentric position vector of the Nav Base

168	RM	$R_{\oplus_{NB}}$	FPANG2	$R_{\oplus_{NB}} = \lvert \vec{r}_{\oplus_{NB}} [M50] \rvert$

Magnitude of inertial velocity of the Nav Base

169	VM	V_{NB}	FPANG2	$V_{NB} = \lvert \dot{\vec{r}}_{\oplus_{NB}} [M50] \rvert$

Flight path angle as observed in the ECI coordinate frame

170	FPM	α	FPANG2	Given in the description of FPANG2 in Section 4.1.2

Azimuth of the velocity observed in the ECI coordinate frame

171	AZM	A_z	FPANG2	Given in the description of FPANG2 of Section 4.1.2

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm

Slant range of the Nav Base measured from the launch site (Ascent) or Landing Site (Runway threshold, Descent)

word name	symbol	program	algorithm
172 RS	$R_{s_{NB}}$	OPIP	Read geodetic position parameters $[\phi_{GEOD}(s), \lambda_{GEOD}(s), h_{GEOD}(s)]$ for s = Origin of Launch Site (Ascent) or s = Origin of Landing Site (Descent), from input file OPIP.IN
		GEODGEO	$\vec{r}_{\oplus_s}[GEO] = \text{fctn}(\phi_{GEOD}(s), \lambda_{GEOD}(s), h_{GEOD}(s))$ (Algorithm given in the description of GEODGEO in Section 4.1.2)
		VSUB	$\vec{r}_{s_{NB}}[GEO] = \vec{r}_{\oplus_{NB}}[GEO] - \vec{r}_{\oplus_s}[GEO]$
		OPIP	$R_{s_{NB}} = \lvert\vec{r}_{s_{NB}}[GEO]\rvert$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
Transformation matrix, M50 frame to vehicle BODy frame				
173	M502BOD(1,1)	$[M50 \rightarrow BOD]$	OPIP	Read from the input file TRJATTDATA and loaded directly into the output array.
174	(1,2)			
175	(1,3)			
176	(2,1)			
177	(2,2)			
178	(2,3)			
179	(3,1)			
180	(3,2)			
181	(3,3)			
Wind-relative velocity magnitude of the vehicle Center of Mass				
182	VTOP	$V_{wind_{CM}}$	FPANG	Given in the description of FPANG in Section 4.1.2
Uncertainty in the wind-relative velocity magnitude of the vehicle Center of Mass				
183	VTOPU	$\sigma(V_{wind})_{CM}$	FPANG	Given in the description of FPANG in Section 4.1.2
Wind-relative flight path angle of the vehicle Center of Mass				
184	GAMTOP	$\gamma_{wind_{CM}}$	FPANG	Given in the description of FPANG in Section 4.1.2

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Uncertainty in the wind-relative flight path angle of the vehicle Center of Mass

185	GAMTOPU	$\sigma(\gamma_{wind_{CM}})$	FPANG	Given in the description of FPANG in Section 4.1.2

Wind-relative azimuth of the vehicle Center of Mass

186	PSITOP	$\psi_{wind_{CM}}$	FPANG	Given in the description of FPANG in Section 4.1.2

Uncertainty in the wind-relative azimuth of the vehicle Center of Mass

187	PSITOPU	$\sigma(\psi_{wind_{CM}})$	FPANG	Given in the description of FPANG in Section 4.1.2

Earth surface range from subvehicle (Nav Base) point to s, where s is the Launch Site origin (for Ascent analysis) or Runway threshold (for Descent analysis)

188	S	R_{NBS_s}	OPIP	Read geodetic position parameters, $[\phi_{GEOD}(s), \lambda_{GEOD}(s), h_{GEOD}(s)]$ for s = Origin of Launch Site (Ascent) or s = Origin of Landing Site (Descent), from input file OPIP.IN

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
S continued			
		GEODGEO	$\vec{r}_{\oplus_s}[GEO] = fctn(\phi_{GEOD}(s), \lambda_{GEOD}(s), h_{GEOD}(s))$ (Algorithm given in the description of GEODGEO in Section 4.1.2) Let $\vec{r}_{\oplus_s}[GEO] = [x_s, y_s, z_s]^T$ and $\vec{r}_{\oplus_{NB}}[GEO] = [x_{NB}, y_{NB}, z_{NB}]^T$ Then
		OPIP	$R_{\oplus_s}^2 = x_s^2 + y_s^2 + z_s^2$ and $R_{\oplus_{NB}}^2 = x_{NB}^2 + y_{NB}^2 + z_{NB}^2$. Then,
		OPIP	$R_{NBS_s} = R_{\oplus_s} \beta$ β is the angle, in radians, between the geocentric vectors to s and to the Nav Base and is calculated from
		OPIP	$\beta = \arc\tan \dfrac{R_{\oplus_s}^2 R_{\oplus_{NB}}^2 - (R_{\oplus_s} R_{\oplus_{NB}} \cos\beta)^2}{R_{\oplus_s} R_{\oplus_{NB}} \cos\beta}$ where $R_{\oplus_s} R_{\oplus_{NB}} \cos\beta = \vec{r}_{\oplus_s}[GEO] \cdot \vec{r}_{\oplus_{NB}}[GEO]$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Wind-relative angle of attack for the vehicle Center of Mass

189	ALPHA	α_W	AERODYN	Given in the description of AERODYN in Section 4.1.2

Uncertainty in the wind-relative angle of attack for the vehicle Center of Mass

190	ALPHAU	$\sigma(\alpha_W)$	AERODYN	Given in the description of AERODYN in Section 4.1.2

Wind-relative guidance sideslip angle for the vehicle Center of Mass

191	BETAP	β_{WG}	AERODYN	Given in the description of AERODYN in Section 4.1.2

Uncertainty in the wind-relative guidance sideslip angle for the Center of Mass

192	BETAPU	$\sigma(\beta_{WG})$	AERODYN	Given in the description of AERODYN in Section 4.1.2

Wind-relative dynamic pressure for the total Center of Mass velocity, in lb/ft^2

193	QAERO	q	AERODYN	Given in the description of AERODYN in Section 4.1.2

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
Uncertainty in the dynamic pressure for the velocity of the Center of Mass				
194	QAEROU	$\sigma(q)$	AERODYN	Given in the description of AERODYN in Section 4.1.2
Wind-relative pitch dynamic pressure for the Center of Mass, in lb-deg/ft^2				
195	QALPHA	q_α	AERODYN	Given in the description of AERODYN in Section 4.1.2
Wind-relative yaw dynamic pressure for the Center of Mass, in lb-deg/ft^2				
196	QBETA	q_β	AERODYN	Given in the description of AERODYN in Section 4.1.2
Wind-relative Mach number at the Center of Mass				
197	MACH	M	AERODYN	Given in the description of AERODYN in Section 4.1.2
Uncertainty in the wind-relative Mach number at the Center of Mass				
198	MACHU	$\sigma(M)$	AERODYN	Given in the description of AERODYN in Section 4.1.2

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
				Wind-relative viscous parameter affecting the motion of the Center of Mass
199	VOO	ν	AERODYN	Given in the description of AERODYN in Section 4.1.2
				Uncertainty in the wind-relative viscous parameter for the Center of Mass
200	VOOU	$\sigma(\nu)$	AERODYN	Given in the description of AERODYN in Section 4.1.2
				Ambient atmospheric temperature in °R
201	METDATA(3)	T_A	OPIP	Placed in output buffer directly
				Uncertainty in ambient atmospheric temperature in °R
202	METDATA(7)	$\sigma(T_A)$	OPIP	Placed in output buffer directly
				Ambient atmospheric pressure in lb/ft^2
203	METDATA(4)	P_A	OPIP	Placed in output buffer directly

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Uncertainty in ambient atmospheric pressure in lb/ft^2

204	METDATA(8)	$\sigma(P_A)$	OPIP	Placed in output buffer directly

Atmospheric mass density in $slugs/ft^2$

205	METDATA(5)	ρ_{AM}	OPIP	Placed in output buffer directly

Wind-relative equivalent air speed at the Center of Mass

206	EAS	$V_{EA_{CM}}$	AERODYN	Given in the description of AERODYN in Section 4.1.2

Uncertainty in the wind-relative equivalent air speed at the Center of Mass

207	EASU	$\sigma(V_{EA_{CM}})$	AERODYN	Given in the description of AERODYN in Section 4.1.2

Load factor at the Center of Mass, in g's [ft/sec^2]

208	L	n	AERODYN	Given in the description of AERODYN in Section 4.1.2

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER				
word	name	symbol	program	algorithm
Wind-relative drag over mass for the Center of Mass				
209	DOM	$F_{D/M}$	AERODYN	Given in the description of AERODYN in Section 4.1.2
Wind-relative lift over drag for the Center of Mass				
210	LOD	L_D	AERODYN	Given in the description of AERODYN in Section 4.1.2
Wind-relative aerodynamic side slipangle for the Center of Mass				
211	BETA	β_{WA}	AERODYN	Given in the description of AERODYN in Section 4.1.2
Uncertainty in the wind-relative aerodynamic sideslip angle				
212	BETAU	$\sigma(\beta_{WA})$	AERODYN	Given in the description of AERODYN in Section 4.1.2
Euler angles, yaw (ψ_T), pitch (θ_T), and roll (ϕ_T), expressed in TOPodetic coordinates				
213	TOPEUL(1)	ψ_T	OPIP	Let [TOP $\rightarrow$ BOD] = $[m_{ij}]$ which was calculated previously for words (101-103)

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
214	TOPEUL(2)	θ_T	OPIP	(1) Pitch not vertical $\psi = \arctan(m_{12}/m_{11})$ $\theta = -\arctan(m_{13})$
215	TOPEUL(3)	ϕ_T		$\phi = \arctan(m_{23}/m_{33})$
			OPIP	(2) Pitch vertical $\psi = 0$ $\theta = \pm 90^0 = -\text{sign}(90, m_{13})$ $\phi = \arctan(m_{21}/m_{31})$
			OPIP	(3) For Ascent and if $\phi < 0^0$, $\phi = \phi + 360^0$

Euler angle rates relative to the TOPodetic frame

word	name	symbol	program	algorithm
216	TOPRATE(1)	$\vec{\omega}$[TOP]	OPIP	[ECI $\rightarrow$ TOP] was computed previously for words (101-103)
217	(2)		OPIP	[M50 $\rightarrow$ ECI] was read from input file OPIP.IN
			M3X3	[M50 $\rightarrow$ TOP] = [ECI $\rightarrow$ TOP] [M50 $\rightarrow$ ECI]
218	(3)		EULRATE	$\vec{\omega}[TOP] = [\dot{\psi}_T, \dot{\theta}_T, \dot{\phi}_T]^T$ calculated by algorithm given in the description of EULRATE in Section 4.1.2

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm
				Inertial body angular rates around the BODy axes
219	IBRATES(1)	$\vec{\omega}[BOD]$	OPIP	Read M50 state vector from input file TRJATTDATA: $\vec{S} = [S_1 \ldots S_{18}]^T$ and convert from radians to degrees
220	(2)			
221	(3)			
			OPIP	$\omega_{xB} = S_{13} * C_{D/R}$ $\omega_{yB} = S_{14} * C_{D/R}$ $\omega_{zB} = S_{15} * C_{D/R}$
			OPIP	$\vec{\omega}[BOD] = [\omega_{xB}, \omega_{yB}, \omega_{zB}]^T$
				Uncertainties inertial body angular rates around the BODy axes
222	IBRATEU(1)	$\vec{\sigma}(\vec{\omega}[BOD])$	OPIP	Read $\text{¢}(\vec{S}[M50]) = [C_{ij}]_{18x18}$ from the input file TRJATTDATA and convert from radians to degrees
223	(2)			
224	(3)			

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm
IBRATEU continued			
		OPIP	$\left\{ \begin{array}{l} \sigma(\omega_{xB}) = (C_{13,13})^{1/2} * C_{D/R} \\ \sigma(\omega_{yB}) = (C_{14,14})^{1/2} * C_{D/R} \\ \sigma(\omega_{zB}) = (C_{15,15})^{1/2} * C_{D/R} \end{array} \right.$
		OPIP	$\vec{\sigma}(\vec{\omega}[BOD]) = [\sigma(\omega_{xB}), \sigma(\omega_{yB}), \sigma(\omega_{zB})]^T$

Inertial body angular accelerations around the BODy axes

word name	symbol	program	algorithm
225 IBANGA(1)	$\dot{\vec{\omega}}[BOD]$	OPIP	Read M50 state vector from input file TRJATTDATA: $\vec{S} = [S_1 \ldots S_{18}]^T$ convert from radians to degrees
226 (2)			
227 (3)			
		OPIP	$\left\{ \begin{array}{l} \dot{\omega}_{xB} = S_{16} * C_{D/R} \\ \dot{\omega}_{yB} = S_{17} * C_{D/R} \\ \dot{\omega}_{zB} = S_{18} * C_{D/R} \end{array} \right.$
		OPIP	$\dot{\vec{\omega}} = [\dot{\omega}_{xB}, \dot{\omega}_{yB}, \dot{\omega}_{zB}]^T$

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word name	symbol	program	algorithm

Uncertainties in the inertial body accelerations around the BODy axes

word name	symbol	program	algorithm
228 IBANGAU(1)	$\vec{\sigma}(\dot{\vec{\omega}})$	OPIP	Read $\text{¢}(\vec{S}[M50]) = [C_{ij}]_{18x18}$ from the input file TRJATTDATA and convert from radians to degrees
229 (2)			
230 (3)		OPIP	$\sigma(\dot{\omega}_{xB}) = (C_{16,16})^{1/2} * C_{D/R}$ $\sigma(\dot{\omega}_{yB}) = (C_{17,17})^{1/2} * C_{D/R}$ $\sigma(\dot{\omega}_{zB}) = (C_{18,18})^{1/2} * C_{D/R}$
		OPIP	$\vec{\sigma}(\dot{\vec{\omega}}) = [\dot{\omega}_{xB}, \dot{\omega}_{yB}, \dot{\omega}_{zB}]^T$

Sensed acceleration of the vehicle Center of Mass expressed in BODy axes coordinates

word name	symbol	program	algorithm
231 DDBODC(1)	$\ddot{\vec{r}}_{\oplus\,CM}$ [BOD]	M3X1	$\vec{a}_{\oplus\,NB}$ [BOD] = [M50 → BOD] $\ddot{\vec{r}}_{\oplus\,NB}$ [M50]
232 (2)		VADD	$\ddot{\vec{r}}_{NB\,CM}$[BOD] = $\vec{a}_{CEN}$ [BOD] + $\vec{a}_{ROT}$ [BOD]
233 (3)		CROSS	= $\vec{\omega}$[BOD] X $\dot{\vec{r}}_{NB\,CM}$[BOD] + $\dot{\vec{\omega}}$[BOD] X $\vec{r}_{NB\,CM}$[BOD]
		VADD	$\ddot{\vec{r}}_{\oplus\,CM}$ [BOD] = $\vec{a}_{\oplus\,NB}$ [BOD] + $\ddot{\vec{r}}_{NB\,CM}$[BOD]

DEFINITION OR DESCRIPTION OF ANCILLARY DATA PARAMETER

word	name	symbol	program	algorithm

Uncertainty in the sensed acceleration of the vehicle Center of Mass in BODy coordinates

word	name	symbol	program	algorithm
234	DDBODCU(1)	$\vec{\sigma}(\ddot{\vec{r}}_{\substack{\oplus \\ CM}}[BOD])$	OPIP	Read $\vec{S}[M50]$, $\vec{r}_{\substack{NB \\ CM}}[BOD]$, $[M50 \rightarrow BOD]$ and ¢$(\vec{S}[M50])$ from TRJATTDATA
235	(2)			
236	(3)		OPIP	Read $\vec{\sigma}(\vec{r}_{\substack{NB \\ CM}}[BOD])^2$ from OPIP.IN
			DDBCU	¢$(\ddot{\vec{r}}_{\substack{\oplus \\ CM}}[BOD])$ = fctn$(\vec{S}[M50]$, $\vec{r}_{\substack{NB \\ CM}}[BOD]$, $[M50 \rightarrow BOD]$, ¢$(\vec{S}[M50])$, $\sigma(\vec{r}_{\substack{NB \\ CM}}[BOD])^2)$
			OPIP	$\vec{\sigma}(\ddot{\vec{r}}_{\substack{\oplus \\ CM}}[BOD]) = [C_{11}{}^{1/2}, C_{22}{}^{1/2}, C_{33}{}^{1/2}]^T$ where $[C_{ij}]$ = ¢$(\ddot{\vec{r}}_{\substack{\oplus \\ CM}}[BOD])$

Position of the vehicle Center of Mass relative to the Nav Base in BODy coordinates

word	name	symbol	program	algorithm
237	BODCG(1)	$\vec{r}_{\substack{NB \\ CM}}[BOD]$	OPIP	Read from the input file TRJATTDATA and placed directly into the output buffer array.
238	(2)			
239	(3)			

4.1.2 Subroutines Called by OPIP

This section contains the description of subroutines called by OPIP and which contain significant algorithms needed for computing of Ascent/Descent output products. The subsection for each subroutine starts with a list of terms in the subroutine calling argument. Each output term is indicated in the text by the symbol *>> next to the left-hand margin.

The list of subroutines documented in this section is as follows:

AERODYN

Calling Argument List (in list order)

Input

L_1	First-pass flag.
$C_{D/R}$	Conversion constant equal to degrees per radian.
$\vec{v}^{\,wind}_{CM}[BOD]$	Velocity of vehicle Center of Mass with respect to the wind. This vector is expressed in vehicle BODy coordinates.
$¢(\vec{v}^{\,wind}_{CM}[BOD])$	Covariance matrix of $\vec{v}^{\,wind}_{CM}[BOD]$.
$\vec{a}_{\theta\,CM}[BOD]$	Sensed acceleration of the Center of Mass expressed in BODy coordinates.
ρ_{AM}	Atmospheric mass density (slugs/ft^3).
$\sigma(\rho_{AM})$	Uncertainty in atmospheric mass density.
T_{AR}	Atmospheric temperature in degrees Rankine.
$\sigma(T_{AR})$	Uncertainty in the atmospheric temperature, T_{AR}.

Output

α_W	Wind-relative angle of attack (degrees).
$\sigma(\alpha_W)$	Uncertainty in angle of attack.
β_{WG}	Wind-relative guidance sideslip angle (degrees).

$\sigma(\beta_{WG})$	Uncertainty in β_{WG}.
β_{WA}	Wind-relative aerodynamic sideslip angle.
$\sigma(\beta_{WA})$	Uncertainty in β_{WA}.
q	Total dynamic pressure.
$\sigma(q)$	Uncertainty in the total dynamic pressure.
q_α	Pitch dynamic pressure.
q_β	Yaw dynamic pressure.
M	Wind-relative Mach number.
$\sigma(M)$	Uncertainy in M.
$V_{EA_{CM}}$	Equivalent air speed of the Center of Mass (knots).
$\sigma(V_{EA_{CM}})$	Uncertainty in $V_{EA_{CM}}$.
n	Wind-relative load factor (in units of g, the sea-level acceleration of gravity).
$\vec{a}_{WD}$	Wind-relative drag acceleration (magnitude).
L_D	Wind-relative lift over drag.
ν	Wind-relative viscous parameter.
$\sigma(\nu)$	Uncertainty in the wind-relative viscous parameter.

The Algorithms

Constants (set on first pass through the subroutine)

$g = 32.174$ Earth-surface gravitational acceleration.

$k_{ew} = 12.1527$ Constant for computing the equivalent air speed, $V_{EA_{CM}}$.

$k_M = 4289.05$ Constant for computing the Mach number, M.

$\hat{y} = [0,1,0]^T$ The (column) unit vector used to compute lift over drag, L_D.

Preliminary computations: Magnitude of wind-relative velocity and its variance

Let the magnitude of $\vec{v}_{wind_{CM}}[BOD]$ be V;

$$V = (v_x^2 + v_y^2 + v_z^2)^{1/2}.$$

To compute the variance of V, first $[\frac{\partial V}{\partial \vec{v}}] = [\frac{\partial V}{\partial v_x}, \frac{\partial V}{\partial v_y}, \frac{\partial V}{\partial v_z}]^T$ is computed.

$$\frac{\partial V}{\partial v_x} = \frac{\partial}{\partial v_x}(v_x^2 + v_y^2 + v_z^2)^{1/2} = \frac{v_x}{V}.$$

Likewise,

$$\frac{\partial V}{\partial v_y} = \frac{v_y}{V} \text{ and } \frac{\partial V}{\partial v_z} = \frac{v_z}{V}.$$

The variance of V is computed from this and the covariance matrix:

$$\sigma(V)^2 = [\frac{\partial V}{\partial v_x}, \frac{\partial V}{\partial v_y}, \frac{\partial V}{\partial v_z}]\ \text{¢}(v_{wind_{CM}}[BOD])\ [\frac{\partial V}{\partial v_x}, \frac{\partial V}{\partial v_y}, \frac{\partial V}{\partial v_z}]^T.$$

The uncertainty, which will be used later, used here is the standard deviation given by

$$\sigma(V) = [\sigma(V)^2]^{1/2}.$$

Wind-relative angle of attack, α_W

Let $$V_{xz} = (v_x^2 + v_z^2)^{1/2}.$$

Then

$$\sin \alpha_W = \frac{v_z}{V_{xz}} \text{ and } \cos \alpha_W = \frac{v_x}{V_{xz}}$$

So,

*>> $$\alpha_W = \arcsin(\frac{v_z}{V_{xz}})$$

and α_W is converted to degrees and adjusted so that $0 \lessgtr \alpha_W \lessgtr 180^0$.

Uncertainty in α_W

First, the partial derivative matrix is computed:

$$[\frac{\partial \alpha_W}{\partial \vec{v}_{\substack{wind\\CM}}\ [BOD]}] = [\frac{\partial \alpha_W}{\partial v_x}, \frac{\partial \alpha_W}{\partial v_y}, \frac{\partial \alpha_W}{\partial v_z}]^T = [\frac{-\sin \alpha_W}{V_{13}}, 0, \frac{\cos \alpha_W}{V_{13}}]^T.$$

Using this and the covariance matrix, the variance is calculated from

$$\sigma(\alpha_W)^2 = [\frac{\partial \alpha_W}{\partial v_x}, \frac{\partial \alpha_W}{\partial v_y}, \frac{\partial \alpha_W}{\partial v_z}]\ ¢(v_{\substack{wind\\CM}}\ [BOD])\ [\frac{\partial \alpha_W}{\partial v_x}, \frac{\partial \alpha_W}{\partial v_y}, \frac{\partial \alpha_W}{\partial v_z}]^T.$$

The uncertainty, or standard deviation, in α_W is then

*>> $$\sigma(\alpha_W) = [\sigma(\alpha_W)^2]^{1/2}.$$

Wind-relative aerodynamic side slip angle, β_{WA}

As before,

$$V = |\vec{v}_{\substack{wind\\CM}}[BOD]| = (v_x^2 + v_y^2 + v_z^2)^{1/2}.$$

Then,

$$\sin \beta_{WA} = \frac{v_y}{V} \text{ and } \cos \beta_{WA} = \frac{(v_x^2 + v_z^2)^{1/2}}{V}.$$

So,

*>> $$\beta_{WA} = \arcsin(\frac{v_y}{V})$$

and β_{WA} is converted to degrees and adjusted so that $0 \lessgtr \beta_{WA} \lessgtr 180^0$.

Uncertainty in β_{WA}

First, the partial derivative matrix is computed:

$$[\frac{\partial \beta_{WA}}{\partial \vec{v}_{\substack{wind\\CM}}[BOD]}] = [\frac{\partial \beta_{WA}}{\partial v_x}, \frac{\partial \beta_{WA}}{\partial v_y}, \frac{\partial \beta_{WA}}{\partial v_z}]^T$$

$$= [\frac{- \cos \alpha_W \sin \beta_{WA}}{V}, \frac{\cos \beta_{WA}}{V}, \frac{- \sin \alpha_W \sin \beta_{WA}}{V}]^T.$$

Using this and the covariance matrix, the variance is calculated from

$$\alpha(\beta_{WA})^2 = [\frac{\partial \beta_{WA}}{\partial v_x}, \frac{\partial \beta_{Wa}}{\partial v_y}, \frac{\partial \beta_{WA}}{\partial v_z}] ¢(\vec{v}_{\substack{wind\\CM}}[BOD]) [\frac{\partial \beta_{WA}}{\partial v_x}, \frac{\partial \beta_{WA}}{\partial v_y}, \frac{\partial_{WA}}{a v_z}]^T.$$

The uncertainty, or standard deviation, in β_{WA} is then

*>> $$\sigma(\beta_{WA}) = [\sigma(\beta_{WA})^2]^{1/2}.$$

Wind-relative guidance side slip angle, β_{WG}

Let $V_{xy} = (v_x^2 + v_y^2)^{1/2}$.

Then,

$$\sin \beta_{WG} = \frac{v_y}{V_{xy}} \text{ and } \cos \beta_{WG} = \frac{v_x}{V_{xy}}.$$

So,

*>> $$\beta_{WG} = \arcsin(\frac{v_y}{V_{xy}})$$

and β_{WG} is converted to degrees and adjusted so that $0 \lessgtr \beta_{WG} \lessgtr 180^0$.

Uncertainty in β_{WG}

First compute the partial derivative matrix

$$[\frac{\partial\beta_{WG}}{\partial\vec{v}\,[BOD]_{\substack{wind\\CM}}}] = [\frac{\partial\beta_{WG}}{\partial v_x}, \frac{\partial\beta_{WG}}{\partial v_y}, \frac{\partial\beta_{WG}}{\partial v_z}]^T = [\frac{-\sin\beta_{WG}}{V_{xy}}, \frac{\cos\beta_{WG}}{V_{xy}}, 0]^T.$$

Using this and the covariance matrix, the variance is calculated from

$$\sigma(\beta_{WG})^2 = [\frac{\partial\beta_{WG}}{\partial v_x}, \frac{\partial\beta_{WG}}{\partial v_y}, \frac{\partial\beta_{WG}}{\partial v_z}]\ \text{¢}(\vec{v}\,[BOD]_{\substack{wind\\CM}})\ [\frac{\partial\beta_{WG}}{\partial v_x}, \frac{\partial\beta_{WG}}{\partial v_y}, \frac{\partial\beta_{WG}}{\partial v_z}]^T.$$

The uncertainty, or standard deviation, in β_{WG} is then

*>> $$\alpha(\beta_{WG}) = [\sigma(\beta_{WG})^2]^{1/2}.$$

Dynamic pressures and the uncertainty in q

The (total) dynamic pressure, q, is calculated by

*>> $$q = \frac{1}{2}\rho_{AM}V$$

where ρ_{AM} is the atmospheric mass density.

The uncertainty in q is calculated from

*>> $$\sigma(q) = [\rho_{AM}^{2} V \sigma(V)^{2} + \frac{1}{4} V^{2} \sigma(\rho_{AM})^{2}]^{1/2}$$

where $\sigma(\rho_{AM})$ is the uncertainty in ρ_{AM}. Note that both ρ_{AM} and $\sigma(\rho_{AM})$ are brought in through the argument list.

The <u>pitch</u> dynamic pressure, q_α, is calculated from

*>> $$q_\alpha = \alpha_W q.$$

The <u>yaw</u> dynamic pressure, q_β, is calculated from

*>> $$q_\beta = \beta_{WG} q.$$

<u>The equivalent air speed, $V_{\underset{CM}{EA}}$, and its uncertainty, $\sigma(V_{\underset{CM}{EA}})$</u>

The equivalent air speed, in knots, of the Center of Mass is calculated by

*>> $$V_{\underset{CM}{EA}} = k_{ew} \rho_{AM}^{1/2} V.$$

Its uncertainty is calculated from

*>> $$\sigma(V_{\underset{CM}{EA}}) = k_{ew}[\rho_{AM} V + \frac{1}{4} V \sigma(\rho_{AM})^{2}]^{1/2}.$$

<u>The load factor, n</u>

The load factor is computed by dividing the sensed acceleration of the Center of Mass, $\vec{a}_{\underset{CM}{\oplus}}[BOD]$ brought in through the calling argument, by the Earth's (surface) gravitational acceleration, g. Thus,

*>> $$n = \frac{|\vec{a}_{\oplus}[BOD]_{CM}|}{g}.$$

The wind-relative drag acceleration, a_{WD}

First, the unit vector of the wind-relative velocity of the Center of Mass is calculated; let it be $\hat{v}_W$. Then

$$\hat{v}_W = \frac{\vec{v}_{wind}[BOD]_{CM}}{V}.$$

And then the magnitude of the drag acceleration is computed from

*>> $$a_{WD} = - \hat{v}_W \bullet \vec{a}_{\oplus}[BOD]_{CM}.$$

The wind-relative lift-over-drag, L_D

Lift is computed as follows. $\hat{y} = [0,1,0]^T$ was set up during the first pass through the program along with other constants. The direction, $\hat{L}$, of the lift force is determined from the cross product

$$\vec{L} = \hat{y} \times \hat{v}_W.$$

The unit vector in this direction is

$$\hat{L} = \frac{\vec{L}}{|\vec{L}|}.$$

The lift acceleration, a_L, is the component of $\vec{a}_{\oplus}[BOD]_{CM}$ along $\hat{L}$; thus,

$$a_L = \vec{a}_{\oplus}[BOD]_{CM} \bullet \hat{L}.$$

The lift-over-drag is the ratio of a_L to a_{WD} if $a_{WD} \neq 0$. Thus,

*>> $$L_D = \begin{cases} a_L / a_{WD} & \text{if } a_{WD} \neq 0 \\ 0 & \text{if } a_{WD} = 0 \end{cases}.$$

Mach number, M, viscous parameter, ν, and their uncertainties

First, the ambient atmospheric temperature and its uncertainty is converted from degrees Rankine to degrees Kelvin:

$$T_{AK} = \frac{5}{9} T_{AR} = T_A$$

and

$$\sigma(T_{AK}) = \frac{5}{9} \sigma(T_{AR}) = \sigma(T_A).$$

Next, the stagnation temperature, T_S, and its uncertainty are computed using V, the magnitude of the wind-relative Center of Mass velocity. The algorithm is

$$T_S = 726.97 + 0.468\ T_A + 3.4098447*10^{-6}\ V^2.$$

Its uncertainty is computed from

$$\sigma(T_S) = [(0.468\ \sigma(T_A)\ )^2 + (4.650816351*10^{-11}\ V\ \sigma(V)\ )^2]^{1/2}.$$

Next, the "intermediate coefficient of viscosity", C_0, and its uncertainty, $\sigma(C_0)$, are calculated using the following algorithms:

$$C_0 = \frac{\left(\frac{T_S}{T_A}\right)^{1/2} [T_A + 122.1(10^{-5/T_A})]}{[T_S + 122.1(10^{-5/T_S})]}$$

and its uncertainty,

$$\sigma(C_0) = \left[\frac{\sigma(T_S)^2}{4T_S T_A} + \frac{T_S\ \sigma(T_A)^2}{4T_A^3}\right]^{1/2}.$$

Next, the coefficient of dynamic viscosity, μ, and its uncertainty, $\sigma(\mu)$, are calculated using the following algorithms based on the U. S. Standard Atmosphere of 1976:

$$\mu = 3.0449939*10^{-8} \left(\frac{T_A^{3/2}}{T_A + 110.4}\right)$$

and its uncertainty,

$$\sigma(\mu) = 3.0449939*10^{-8}\ T_A^{1/2}\ \sigma(T_A)\ \frac{(\frac{T_A}{2} + 165.6)}{(T_A + 110.4)^2}.$$

Finally, the wind-relative Mach number and its uncertainty are computed using the following algorithms:

*>> $$M = V(k_M\ T_A)^{-1/2}$$

and its uncertainty,

*>> $$\sigma(M) = \left[\frac{\sigma(v)^2}{4289.05\ T_A} + \frac{V^2\ \sigma(T_A)^2}{17156.2\ T_A^3}\right]^{1/2}$$

where, as before, V is the magnitude of $\vec{v}_{\oplus_{CM}}$[BOD] and $\sigma(V)$ is its uncertainty. The constant k_M is set during the first pass through the routine.

The last output terms to be computed are the wind-relative "viscous parameter", ν, and its uncertainty, $\sigma(\nu)$. They are computed with the following algorithms based on the U. S. Standard Atmosphere of 1976:

*>> $$\nu = M\left[\frac{C_o\ \mu}{107.5\ V\ \rho_{AM}}\right]^{1/2}.$$

Its uncertainty is computed from the following series.

These terms are computed:

$$M_{pss} = \frac{\sigma(M)^2\ C_o\ \mu}{107.5\ V\ \rho_{AM}}$$

$$C_{pss} = \frac{\sigma(C_o)^2 \ M^2 \ \mu}{(107.5 \ V\rho_{AM}) \ (4 \ C_o)}$$

$$\mu_{pss} = \frac{\sigma(\mu)^2 \ M^2 \ C_o}{(107.5 \ V \ \rho_{AM}) \ (4 \ \mu)}$$

$$V_{pss} = \frac{V^2 \ M^2 \ C_o \ \mu}{(107.5 \ \rho_{AM}) \ (4 \ V^3)}$$

$$\rho_{pss} = \frac{\sigma(\rho_{AM})^2 \ M^2 \ C_o \ \mu}{(107.5 \ V) \ (4 \ \rho^3)}$$

With these, the uncertainty in the "viscous parameter" is

*>> $$\sigma(\nu) = (M_{pss} + C_{pss} + \mu_{pss} + V_{pss} + \rho_{pss})^{1/2}.$$

BANK

Calling Argument List (in list order)

Input

$C_{D/R}$	Conversion constant equal to degrees per radian.
L_{ASC}	Flag which is .TRUE. if ascent analysis is being done.
$[TOP \rightarrow BOD]$	Transformation matrix, TOPodetic to BODy frame.
$\vec{v}_{\substack{wind\\CM}}[TOP]$	Wind-relative velocity of the Center of Mass expressed in the topodetic frame.

Output

Φ_W	Wind-relative bank angle in degrees.

The Algorithm

First, the unit vector

$$\hat{v}_W = \frac{\vec{v}_{\substack{wind\\CM}}[TOP]}{|\vec{v}_{\substack{wind\\CM}}[TOP]|},$$

is calculated. Next, the body pitch and roll axes are extracted from the transformation matrix. Let

$$[TOP \rightarrow BOD] = \begin{bmatrix} r_x & r_y & r_z \\ p_x & p_y & p_z \\ y_x & y_y & y_z \end{bmatrix} = \begin{bmatrix} \hat{r}^T \\ \hat{p}^T \\ \hat{y}^T \end{bmatrix}$$

This means that each row of the TOP to BOD matrix is the row unit vector representation of a rotation axis expressed in the topodetic frame. Expressed as the column vectors, they are

$\hat{r}$ = roll body axis expressed in the topodetic frame;
$\hat{p}$ = pitch body axis expressed in the topodetic frame; and
$\hat{y}$ = yaw body axis expressed in the topodetic frame.

Let $\hat{n}$ be the unit vector normal to both $\hat{r}$ and $\hat{v}_W$. The normal vector, $\vec{n}$ is found from the cross product,

$$\vec{n} = \hat{r} \times \hat{v}.$$

Its unit vector, which has the components n_1, n_2, n_3, is

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = [n_1, n_2, n_3]^T.$$

The angle, α, between $\hat{r}$ and $\hat{v}_W$ is found from

$$\alpha = \arccos(\hat{r} \cdot \hat{v}_W).$$

Then the rotation matrix, [R] which rotates $\hat{r}$ about the axis $\hat{n}$ through the angle α is:

$$[R] = \begin{bmatrix} [n_1^2 + (1-n_1^2)\cos\alpha] & [n_1 n_2(1-\cos\alpha) - n_3 \sin\alpha] & [n_1 n_3(1-\cos\alpha) + n_2 \sin\alpha \\ [n_1 n_2(1-\cos\alpha) + n_3 \sin\alpha] & [n_2^2 + (1-n_2^2)\cos\alpha] & [n_2 n_3(1-\cos\alpha) - n_1 \sin\alpha] \\ [n_1 n_3(1-\cos\alpha) - n_2 \sin\alpha] & [n_2 n_3(1-\cos\alpha) + n_1 \sin\alpha] & [n_3^2 + (1-n_3^2)\cos\alpha] \end{bmatrix}$$

Next, determine the new pitch axis, $\hat{P}$, after the rotation. It is found by the following logic:

If $\vec{v}_{wind}$ [TOP]$_{CM}$ does NOT lie along $\hat{r}$; $\hat{P}$ = [R] $\hat{p}$

If $\vec{v}_{wind}$ [TOP]$_{CM}$ DOES lie along $\hat{r}$; $\hat{P} = \hat{p}$.

Define $\hat{w}$ as the unit vector in the topodetic x-y plane which is also normal to $\hat{v}_W$. If $\hat{v}_W$ is expressed in terms of its components,

$$\hat{v}_W = [v_x, v_y, v_z]^T,$$

then

$$\vec{w} = [-v_y, v_x, 0]^T$$

and its unit vector is

$$\hat{w} = \frac{\vec{w}}{|\vec{w}|}.$$

Finally, the wind-relative bank angle, Φ_W, is calculated by

*>> $$\Phi_W = |\arccos(\hat{w} \cdot \hat{P})|.$$

This, being in radians, must be converted to degrees.

Finally, adjustments must be made. Given that $\hat{P} = [P_x, P_y, P_z]^T$, then

if $P_z < 0$, change the sign of Φ_W: $\Phi_W = -\Phi_W$.

Also, if this is an ascent analysis (L_{ASC} = .TRUE.) and $\Phi_W < 0$, then add 360° to Φ_W.

Calling Argument List (in list order)

Input

$C_{D/R}$ — Conversion constant equal to degrees per radian.

L_{ASC} — Flag which is .TRUE. if ascent analysis is being done.

L_1 — Flag which is .TRUE. for the first pass through the routine.

N_{err} — Error counter (This can be incremented within the routine, so it also can become an output).

$\vec{S}$ — The 15-element state vector = $[S_1 \cdots S_{15}]^T$.

$[S_1, S_2, S_3]^T = \vec{r}_{\oplus NB}[BOD]$	Geocentric position vector of the Nav Base in body coordinates.
$[S_4, S_5, S_6]^T = \vec{v}_{wind\,NB}[BOD]$	Wind-relative velocity of the Nav Base in body coordinates.
$[S_7, S_8, S_9]^T = \vec{a}_{wind\,NB}[BOD]$	Wind-relative acceleration of the Nav Base in body coordinates.
S_{10}, S_{11}, S_{12}	Not used.
$[S_{13}, S_{14}, S_{15}]^T = \vec{\omega}_B[ECI]$	Inertial (ECI) angular rates of the body axes.

$\mathbb{C}(S)$ — Covariance matrix of the state vector, $\vec{S}$.

$\vec{w}$[BOD]	Wind velocity expressed in body axis coordinates.
¢($\vec{w}$[BOD])	Covariance matrix of $\vec{v}_W$ [BOD].

Output

ψ_W	Wind-relative yaw angle in degrees.
θ_W	Wind-relative pitch angle in degrees.
ϕ_W	Wind-relative roll angle in degrees.
$\sigma(\psi_W)$	Uncertainty (standard deviation) in ψ_W.
$\sigma(\theta_W)$	Uncertainty (standard deviation) in θ_W.
$\sigma(\phi_W)$	Uncertainty (standard deviation) in ϕ_W.
ψ_W	Wind-relative yaw rate in degrees/sec.
θ_W	Wind-relative pitch rate in degrees/sec.
ϕ_W	Wind-relative roll rate in degrees/sec.
$\sigma(\psi_W)$	Uncertainty (standard deviation) in ψ_W.
$\sigma(\theta_W)$	Uncertainty (standard deviation) in θ_W.
$\sigma(\phi_W)$	Uncertainty (standard deviation) in ϕ_W.

Algorithms

Determination of a wind-related coordinate system

First, obtain the relative velocity $\vec{v}_{REL}[BOD] = \vec{v}_R$ which is

$$\vec{v}_R \triangleq \vec{v}_{\substack{wind\\NB}}[BOD] - \vec{w}[BOD] = [v_{Rx}, v_{Ry}, v_{Rz}]^T$$

Then, define a coordinate system $\hat{x}$, $\hat{y}$, $\hat{z}$ such that

$\hat{y}_W$ is normal to $\vec{v}_R$ and $\vec{r}_{\oplus NB}[BOD]$: the "right" vector;

$\hat{z}_W$ is opposite in direction to $\vec{r}_{\oplus NB}[BOD]$: the "down" vector;

and $\hat{x}_W$ is normal to $\hat{y}_W$ and $\hat{z}_W$: the "forward" vector (which completes the triad).

For notational simplicity, let $\vec{r}_{\oplus NB}[BOD]$ be represented by $\vec{r}_B = [x_B, y_B, z_B]^T$.

Since uncertainties of the wind-relative Euler angles are to be calculated, it is necessary to calculate partial derivative matrices. They are set up in subroutines P3UV and PCROSS. The following notation will be used.
If $\vec{a} = [a_1, a_2, a_3]^T$ and $\vec{b} = [b_1, b_2, b_3]^T$ are two vectors, then the partial derivative matrices will be indicated by

$$\left[\frac{\partial \vec{a}}{\partial \vec{b}}\right] = \begin{bmatrix} \frac{\partial a_1}{\partial b_1} & \frac{\partial a_1}{\partial b_2} & \frac{\partial a_1}{\partial b_3} \\ \frac{\partial a_2}{\partial b_1} & \frac{\partial a_2}{\partial b_2} & \frac{\partial a_2}{\partial b_3} \\ \frac{\partial a_3}{\partial b_1} & \frac{\partial a_3}{\partial b_2} & \frac{\partial a_3}{\partial b_3} \end{bmatrix} .$$

Compute the "right" unit vector, $\hat{y}$, and its partial derivative matrices

The subroutine PCROSS is called to compute the cross product

$$\hat{y} = \vec{v}_R \times \vec{r}_B = \vec{v}_{REL}[BOD] \times \vec{r}_{\oplus NB}[BOD].$$

Then it computes the skew-symmetric partial derivative matrices of $\vec{y}$ with respect to the two input vectors, $\vec{v}_R$ and $\vec{r}_B$, of which $\vec{y}$ is the cross product,

$$[\frac{\partial \vec{y}}{\partial \vec{v}_R}] \text{ and } [\frac{\partial \vec{y}}{\partial \vec{r}_B}].$$

Another subroutine, P3UV, is called which computes the unit vector of $\vec{y}$

$$\hat{y} = \frac{\vec{y}}{|\vec{y}|} = [y_1, y_2, y_3]^T$$

and the partial derivative matrix of $\hat{y}$ with respect to $\vec{y}$: $[\frac{\partial \hat{y}}{\partial \vec{y}}]$.

Then, the following partial derivative matrices are computed:

$$[P(\frac{\hat{y}}{\vec{r}_B})] = [\frac{\partial \hat{y}}{\partial \vec{y}}] [\frac{\partial \vec{y}}{\partial \vec{r}_B}]$$

and

$$[P(\frac{\hat{y}}{\vec{v}_R})] = [\frac{\partial \hat{y}}{\partial \vec{y}}] [\frac{\partial \vec{y}}{\partial \vec{v}_R}].$$

Compute the "down" unit vector, $\hat{z}$, and its partial derivative matrix

The subroutine P3UV is called to calculate

$$\hat{r}_B = \frac{\vec{r}_B}{|\vec{r}_B|} \text{ and } [\frac{\partial \hat{r}_B}{\partial \vec{r}_B}]$$

Since the "down" direction is opposite to that of $\vec{r}_B$,

$$\hat{z} = - \hat{r}_B = [z_1, z_2, z_3]^T$$

and its partial derivative matrix is

$$[P(\frac{\hat{z}}{\vec{r}_B})] = - [\frac{\partial \hat{r}_B}{\partial \vec{r}_B}].$$

<u>Compute the "forward" unit vector, $\hat{x}$, and its partial derivative matrices</u>

Since the "forward" vector $\hat{x}$ completes the triad, the subroutine PCROSS is called to calculate

$$\hat{x} = \hat{y} \times \hat{z} = [x_1, x_2, x_3]^T,$$

$$[P(\frac{\hat{x}}{\hat{y}})] = [\frac{\partial \hat{x}}{\partial \hat{y}}]$$

and

$$[P(\frac{\hat{x}}{\hat{z}})] = [\frac{\partial \hat{x}}{\partial \hat{z}}].$$

<u>Compute the wind-relative pitch angle, θ_W, and its partial derivatives</u>

*>> $$\theta_W = - C_{D/R} \arcsin \left(\frac{z_1}{(x_1^2 + z_1^2)^{1/2}}\right).$$

θ_W is constrained so that $-180 \lessapprox \theta_W \lessapprox 180$.

The partial derivative matrices are expressed as row matrices:

$$[\frac{\partial \theta_W}{\partial \hat{x}}] = [\frac{\partial \theta_W}{\partial x_1}, \frac{\partial \theta_W}{\partial x_2}, \frac{\partial \theta_W}{\partial x_3}] = [\frac{z_1}{(x_1^2 + z_1^2)^{1/2}}, 0, 0]$$

and

$$[\frac{\partial \theta_W}{\partial \hat{z}}] = [\frac{\partial \theta_W}{\partial z_1}, \frac{\partial \theta_W}{\partial z_2}, \frac{\partial \theta_W}{\partial z_3}] = [\frac{- x_1}{(x_1^2 + z_1^2)^{1/2}}, 0, 0].$$

These will be used to determine the angle rates and uncertainties.

Compute the wind-relative yaw angle, ψ_W, and its partial derivatives

*>> $$\psi_W = C_{D/R} \arcsin (y_1)$$

Its partial derivatives with respect to $\hat{y}$ are

$$\left[\frac{\partial \psi_W}{\partial \hat{y}}\right] = \left[\frac{\partial \psi_W}{\partial y_1}, \frac{\partial \psi_W}{\partial y_2}, \frac{\partial \psi_W}{\partial y_3}\right] = \left[\frac{1}{(y_2^2 + y_3^2)^{1/2}}, 0, 0\right]$$

Compute the wind-relative roll angle, ϕ_W, and its partial derivatives

*>> $$\phi_W = - C_{D/R} \arcsin \left(\frac{y_3}{(y_2^2 + y_3^2)^{1/2}}\right)$$

ϕ_W is constrained so that $-180^0 \lessgtr \phi_W \lessgtr 180^0$.

For ascent and $\phi_W < 0$, ϕ_W is made positive: $\phi_W = \phi_W + 360^0$.

Its partial derivatives are

$$\left[\frac{\partial \phi_W}{\partial \hat{y}}\right] = \left[\frac{\partial \phi_W}{\partial y_1}, \frac{\partial \phi_W}{\partial y_2}, \frac{\partial \phi_W}{\partial y_3}\right] = \left[0, \frac{-y_2}{y_2^2 + y_3^2}, \frac{y_3}{y_2^2 + y_3^2}\right].$$

Compute partial derivative matrices used to compute angle rates and their uncertainties

The matrix of the partial derivatives of the wind-relative Euler angles, E_{Wi}, with respect to the unit vectors $\hat{y}$ and $\hat{z}$ is calculated to be

$$\left[\frac{\partial E_{Wi}}{\partial(\hat{y},\ \hat{z})}\right] = \begin{bmatrix} \frac{\partial\psi_W}{\partial y_1} & \frac{\partial\psi_W}{\partial y_2} & \frac{\partial\psi_W}{\partial y_3} & \frac{\partial\psi_W}{\partial z_1} & \frac{\partial\psi_W}{\partial z_2} & \frac{\partial\psi_W}{\partial z_3} \\ \frac{\partial\theta_W}{\partial y_1} & \frac{\partial\theta_W}{\partial y_2} & \frac{\partial\theta_W}{\partial y_3} & \frac{\partial\theta_W}{\partial z_1} & \frac{\partial\theta_W}{\partial z_2} & \frac{\partial\theta_W}{\partial z_3} \\ \frac{\partial\phi_W}{\partial y_1} & \frac{\partial\phi_W}{\partial y_2} & \frac{\partial\phi_W}{\partial y_3} & \frac{\partial\phi_W}{\partial z_1} & \frac{\partial\phi_W}{\partial z_2} & \frac{\partial\phi_W}{\partial z_3} \end{bmatrix}$$

The individual derivatives, or elements, of this matrix are as follows:

$$\frac{\partial\psi_W}{\partial y_1} = \frac{1}{(y_1^2 + y_2^2)^{1/2}};\quad \frac{\partial\psi_W}{\partial y_2} = \frac{\partial\psi_W}{\partial y_3} = \frac{\partial\psi_W}{\partial z_1} = \frac{\partial\psi_W}{\partial z_2} = \frac{\partial\psi_W}{\partial z_3} = 0$$

$$\frac{\partial\theta_W}{\partial y_i} = \frac{\partial\theta_W}{\partial x_1}\ \frac{\partial x_1}{\partial y_i} + \frac{\partial\theta_W}{\partial x_2}\ \frac{\partial x_2}{\partial y_i} + \frac{\partial\theta_W}{\partial x_3}\ \frac{\partial x_3}{\partial y_i};\quad i = 1,2,3$$

$$\frac{\partial\theta_W}{\partial z_i} = \frac{\partial\theta_W}{\partial x_1}\ \frac{\partial x_1}{\partial z_i} + \frac{\partial\theta_W}{\partial x_2}\ \frac{\partial x_2}{\partial z_i} + \frac{\partial\theta_W}{\partial x_3}\ \frac{\partial x_3}{\partial z_i};\quad i = 1,2,3$$

$$\frac{\partial\phi_W}{\partial y_1} = 0;\quad \frac{\partial\phi_W}{\partial y_2} = \frac{-\ y_2}{y_2^2 + y_3^2};\quad \frac{\partial\phi_W}{\partial y_3} = \frac{y_3}{y_2^2 + y_3^2}$$

and

$$\frac{\partial\phi_W}{\partial z_1} = \frac{\partial\phi_W}{\partial z_2} + \frac{\partial\phi_W}{\partial z_3} = 0.$$

The matrices of partial derivatives of the state vector position, $\vec{r}_B$, and wind-relative velocity, $\vec{v}_R$, with respect to the wind-relative Euler angles, E_{Wi}, are as follows. With $\vec{r}_B = [x_B,\ y_B,\ z_B]^T$,

$$\left[\frac{\partial \vec{r}_B}{\partial E_i}\right] = \begin{bmatrix} \frac{\partial x_B}{\partial \psi_W} & \frac{\partial x_B}{\partial \theta_W} & \frac{\partial x_B}{\partial \phi_W} \\ \frac{\partial y_B}{\partial \psi_W} & \frac{\partial y_B}{\partial \theta_W} & \frac{\partial y_B}{\partial \phi_W} \\ \frac{\partial z_B}{\partial \psi_W} & \frac{\partial z_B}{\partial \theta_W} & \frac{\partial z_B}{\partial \phi_W} \end{bmatrix} = \begin{bmatrix} 0 & -z_B & y_B \\ z_B & 0 & -x_B \\ -y_B & x_B & 0 \end{bmatrix}.$$

With $v_R = [v_{Rx}, v_{Ry}, v_{Rz}]^T$,

$$\left[\frac{\partial \vec{v}_R}{\partial E_i}\right] = \begin{bmatrix} \frac{\partial v_{Rx}}{\partial \psi_W} & \frac{\partial v_{Rx}}{\partial \theta_W} & \frac{\partial v_{Rx}}{\partial \phi_W} \\ \frac{\partial v_{Ry}}{\partial \psi_W} & \frac{\partial v_{Ry}}{\partial \theta_W} & \frac{\partial v_{Ry}}{\partial \phi_W} \\ \frac{\partial v_{Rz}}{\partial \psi_W} & \frac{\partial v_{Rz}}{\partial \theta_W} & \frac{\partial v_{Rz}}{\partial \phi_W} \end{bmatrix} = \begin{bmatrix} 0 & -v_{Rz} & v_{Ry} \\ v_{Rz} & 0 & -v_{Rx} \\ -v_{Ry} & v_{Rx} & 0 \end{bmatrix}.$$

Another matrix that is used for the determination of Euler angle rates is that containing the partial derivatives of $\hat{y}$ and $\hat{z}$ with respect to $\vec{r}_B$, $\vec{v}_R$, ψ_W θ_W, ϕ_W. With $\vec{r}_B = [x_B, y_B, z_B]^T$ and $\vec{v} = [v_{Rx}, v_{Ry}, v_{Rz}]^T$,

$$\left[\frac{\partial(\hat{y}, \hat{z})}{\partial(\vec{r}_B, \vec{v}_R, \psi_W, \theta_W, \phi_W)}\right] = [p_{ij}]$$

$$
[p_{ij}] = \begin{bmatrix}
\frac{\partial y_1}{\partial x_B} & \frac{\partial y_2}{\partial x_B} & \frac{\partial y_3}{\partial x_B} & \frac{\partial y_1}{\partial v_x} & \frac{\partial y_2}{\partial v_x} & \frac{\partial y_3}{\partial v_x} & p_{17} & p_{18} & p_{19} \\
\frac{\partial y_1}{\partial y_B} & \frac{\partial y_2}{\partial y_B} & \frac{\partial y_3}{\partial y_B} & \frac{\partial y_1}{\partial v_y} & \frac{\partial y_2}{\partial v_y} & \frac{\partial y_3}{\partial v_y} & p_{27} & p_{28} & p_{29} \\
\frac{\partial y_1}{\partial z_B} & \frac{\partial y_2}{\partial z_B} & \frac{\partial y_3}{\partial z_B} & \frac{\partial y_1}{\partial v_z} & \frac{\partial y_2}{\partial v_z} & \frac{\partial y_3}{\partial v_z} & p_{37} & p_{38} & p_{39} \\
\frac{\partial z_1}{\partial x_B} & \frac{\partial z_2}{\partial x_B} & \frac{\partial z_3}{\partial x_B} & 0 & 0 & 0 & p_{47} & p_{48} & p_{49} \\
\frac{\partial z_1}{\partial y_B} & \frac{\partial z_2}{\partial y_B} & \frac{\partial z_3}{\partial y_B} & 0 & 0 & 0 & p_{57} & p_{58} & p_{59} \\
\frac{\partial z_1}{\partial z_B} & \frac{\partial z_2}{\partial z_B} & \frac{\partial z_3}{\partial z_B} & 0 & 0 & 0 & p_{67} & p_{68} & p_{69}
\end{bmatrix}
$$

The elements p_{ij} $i = 1 - 6$ and $j = 7, 8, 9$ are computed as follows. Let $E_{W1} = \psi_W$, $E_{W2} = \theta_W$, and $E_{W3} = \phi$, then for $i = 1,2,3$ and $j = 7,8,9$

$$
p_{ij} = \sum_{k=1}^{3} \left(\frac{\partial y_i}{\partial r_k} \frac{\partial E_{Wk}}{\partial r_j} + \frac{\partial y_i}{\partial v_k} \frac{\partial E_{Wk}}{\partial v_j}\right)
$$

and for $i = 4,5,6$ with $j = 7,8,9$

$$p_{ij} = \sum_{k}^{3} \left(\frac{\partial z_i}{\partial r_k} \quad \frac{\partial E_{Wk}}{\partial r_j}\right).$$

The index k is used as follows: for k=1, $r_k = x_B$ and $v_k = v_{Rx}$; for k=2, $r_k = y_B$ and $v_k = v_{Ry}$; and for k=3, $r_k = z_B$ and $v_k = v_{Rz}$.

The next step is to calculate the partial derivative matrix

$$\left[\frac{\partial(\psi_W, \theta_W, \phi_W)}{\partial(\vec{r}_B, \vec{v}_R, \psi_W, \theta_W, \phi_W)}\right] = \frac{\partial(\psi_W, \theta_W, \phi_W)}{\partial(\hat{y}, \hat{z})} \left[\frac{\partial(\hat{y}, \hat{z})}{\partial(\vec{r}_B, \vec{v}_R, \psi_W, \theta_W, \phi_W)}\right]$$

$$= [P_{ij}]$$

where

$$P_{i1} = \frac{\partial E_{Wi}}{\partial \hat{y}} \quad \frac{\partial y_1}{\partial \vec{r}_B} + \frac{\partial E_{Wi}}{\partial \hat{z}} \quad \frac{\partial z_1}{\partial \vec{r}_B};$$

$$P_{i2} = \frac{\partial E_{Wi}}{\partial \hat{y}} \quad \frac{\partial y_2}{\partial \vec{r}_B} + \frac{\partial E_{Wi}}{\partial \hat{z}} \quad \frac{\partial z_2}{\partial \vec{r}_B};$$

$$P_{i3} = \frac{\partial E_{Wi}}{\partial \hat{y}} \quad \frac{\partial y_3}{\partial \vec{r}_B} + \frac{\partial E_{Wi}}{\partial \hat{z}} \quad \frac{\partial z_3}{\partial \vec{r}_B};$$

$$P_{i4} = \frac{\partial E_{Wi}}{\partial \hat{y}} \quad \frac{\partial y_1}{\partial \vec{v}_R} + \frac{\partial E_{Wi}}{\partial \hat{z}} \quad \frac{\partial x_1}{\partial \vec{v}_R};$$

$$P_{i5} = \frac{\partial E_{Wi}}{\partial \hat{y}} \quad \frac{\partial y_2}{\partial \vec{v}_R} + \frac{\partial E_{Wi}}{\partial \hat{z}} \quad \frac{\partial z_2}{\partial \vec{v}_R};$$

$$P_{i6} = \frac{\partial E_{Wi}}{\partial \hat{y}} \quad \frac{\partial y_3}{\partial \vec{v}_R} + \frac{\partial E_{Wi}}{\partial \hat{z}} \quad \frac{\partial z_B}{\partial \vec{v}_R}.$$

The remaining elements are taken directly from $[p_{ij}]$; that is,

$$P_{ij} = p_{ij} \quad \text{for } i = 1,2,3 \text{ and } j = 7,8,9.$$

The angle rates, $\dot{\psi}_W$, $\dot{\theta}_W$, and $\dot{\phi}_W$, are calculated from

$$\vec{\omega}[BOD] = [\psi_W, \theta_W, \phi_W]^T$$

$$= [\frac{\partial(\psi_W, \theta_W, \phi_W)}{\partial(\vec{r}_B, \vec{v}_R, \psi_W, \theta_W, \phi_W)}] [v_x, v_y, v_z, \dot{v}_x, \dot{v}_y, \dot{v}_z, \dot{\psi}_I, \dot{\theta}_I, \dot{\psi}_I]^T$$

Note the right hand matrix is a portion of the state vector, $\vec{\xi}$, which includes words 4 through 9 and 13 through 15.

Specifically in terms of the elements P_{ij}

$$[P_{ij}] = [\frac{\partial(\psi_W, \theta_W, \mu_W)}{\partial(\vec{r}_B, \vec{v}_R, \psi_W, \theta_W, \phi_W)}]:$$

the angle rates are

*>> $$\psi_W = P_{11}v_x + P_{12}v_y + P_{13}v_z + P_{14}v_x + P_{15}v_z + P_{16}v_z + P_{17}\psi_I + P_{18}\theta_I + P_{19}\phi_I$$

*>> $$\theta_W = P_{21}v_x + P_{22}v_y + P_{23}v_z + P_{24}v_x + P_{25}v_y + P_{26}v_z + P_{27}\psi_I + P_{28}\theta_I + P_{29}\psi_I$$

*>> $$\phi_W = P_{31}v_x + P_{32}v_y + P_{33}v_z + P_{34}v_x + P_{35}v_y + P_{36}v_z + P_{37}\psi_I + P_{38}\theta_I + P_{39}\phi_I$$

These are converted from radians per sec to degrees per sec.

<u>Compute the wind-relative Euler angle rate uncertainties</u>

Consider the [P] matrix as a three-element (column) matrix whose elements are nine-element row vectors; that is,

$$\left[\frac{\partial(\psi_W, \theta_W, \phi_W)}{\partial(\vec{r}_B, \vec{v}_R, \psi_W, \theta_W, \phi_W)}\right] = [\vec{P}_1, \vec{P}_2, \vec{P}_3]^T$$

where

$$\vec{P}_i = [P_{i1}, \ldots P_{i9}].$$

The state vector $\vec{\xi}$, is dimensioned as a 15-element array, and, therefore, its covariance matrix, $\not{C}(S)$, is dimensioned 15 x 15. However, elements 10, 11, and 12 are not used, so, for the Euler angle and angle rate uncertainties, terms of $\vec{\xi}$ and $\not{C}(\vec{\xi})$ involving position, $\vec{r}_B$, are not used. The elements of $\vec{\xi}$ and $\not{C}(\vec{\xi})$ are accessed in such a way as to effectively make $\vec{\xi}$ a nine element vector and $\not{C}(\vec{\xi})$ a 9 x 9 matrix.

Then consider $\not{C}(\vec{\xi})$ as a nine element column matrix of nine element row vectors,

$$\not{C}(\vec{\xi}) = [C_{ij}] = [C_1, C_2 \ldots C_9]^T = [\vec{C}_j],$$

where

$$\vec{C}_j = [C_{j1}, \ldots C_{j9}].$$

Now, define R_{ij} as the dot product

$$R_{ij} = \vec{C}_j \cdot \vec{P}_i^{\,T}$$

Nine of these, $1 \leq j \leq 9$, are arranged to form a vector, $\vec{R}_i$.

The variance, $\sigma(E_{Wi})^2$, of the i^{th} wind-relative Euler angle rate is computed from

$$\sigma(E_{Wi})^2 = \vec{P}_i \cdot \vec{R}_i;\ i = 1,2,3.$$

In other words,

$$\sigma(\psi_W)^2 = \vec{P}_1 \cdot \vec{R}_1;\ \sigma(\theta_W)^2 = \vec{P}_2 \cdot \vec{R}_2;\ \sigma(\phi_W)^2 = \vec{P}_3 \cdot \vec{R}_3.$$

All three variances are tested for being positive. Any which is found to be negative is set to zero, and a message is written to an output file.

Finally, the uncertainties (standard deviations) of the wind-relative Euler angle rates are computed from

*>> $$\sigma(\psi_W) = [\sigma(\psi_W)^2]^{1/2};$$

*>> $$\sigma(\theta_W) = [\sigma(\theta_W)^2]^{1/2}; \text{ and}$$

*>> $$\sigma(\phi_W) = [\sigma(\phi_W)^2]^{1/2}.$$

These are all converted from radian/sec to degrees sec.

<u>Compute the wind-relative Euler angle uncertainties</u>

Again, considering $\text{¢}(\vec{S})$ as a 9 X 9 matrix, the wind velocity covariance matrix, $\text{¢}(\vec{v}_{wind}[BOD])$, which is a 3 X 3 matrix, is added to it. If

$$\text{¢}(\vec{v}_{wind}[BOD]) = [v_{ij}],$$

the sum, $\text{¢}(\vec{W})$, a 9 X 9 matrix, is formed as follows:

$$\text{¢}(\vec{W}) = \text{¢}(\vec{S}) + \text{¢}(\vec{v}_{wind}[BOD])$$

$$
\Phi(\vec{W}) = \begin{bmatrix}
C_{11} & \vdots & & & \\
 & C_{34} & C_{35} & C_{36} & \\
\cdots & C_{44} + v_{11} & C_{45} + v_{12} & C_{46} + v_{13} & \cdots \\
\cdots & C_{54} + v_{21} & C_{55} + v_{22} & C_{56} + v_{23} & \cdots \\
\cdots & C_{64} + v_{31} & C_{65} + v_{32} & C_{66} + v_{33} & \cdots \\
 & C_{74} & C_{75} & C_{76} & \\
 & \vdots & \vdots & \vdots & C_{99}
\end{bmatrix} .
$$

$\Phi(\vec{W})$ is considered as a column vector with each of the nine elements being a nine-element vector. Let

$$
\Phi(\vec{W}) = [\vec{W}_1, \vec{W}_2, \ldots \vec{W}_9]^T = [\vec{W}_j]
$$

where each $\vec{W}_j$ is

$$\vec{W}_j = [w_{j1}, \ldots w_{j9}].$$

Then, as before, using

$$\left[\frac{\partial(\psi_W, \theta_W, \phi_W)}{\partial(\vec{r}_B, \vec{v}_R, \psi_W, \theta_W, \phi_W)}\right] = [\vec{P}_1, \vec{P}_2, \vec{P}_3]^T = [\vec{P}_i],$$

the dot product R_{ij} is computed from

$$R_{ij} = \vec{W}_j \cdot \vec{P}_i^{\,T}.$$

There are nine $(1 \leq j \leq 9)$ elements, R_{ij}, which form a vector $\vec{R}_i$. The variance, $\sigma(E_i)^2$, of the i^{th} wind-relative Euler angle is computed from

$$\sigma(E_{Wi})^2 = \vec{P}_i \cdot \vec{R}_i;\ i = 1,2,3.$$

All three variances are tested for being positive. Any which is found to be negative is set to zero, and a message is written to the output file.

Finally, the uncertainties (standard deviations) of the wind-relative Euler angles are computed from

*>> $\sigma(\psi_W) = [\sigma(\psi_W)^2]^{1/2}$;

*>> $\sigma(\theta_W) = [\sigma(\theta_W)^2]^{1/2}$; and

*>> $\sigma(\phi_W) = [\sigma(\phi_W)^2]^{1/2}$.

These are all converted from radians to degrees.

EULRATE

Caling Argument List (in list order)

Input

$\vec{r}_{\oplus_{NB}}[M50]$	Geocentric position of the Navigation Base in M50 coordinates.
$\vec{r}_{\oplus_{NB}}[M50]$	Geocentric velocity of the Navigation Base in M50 coordinates.
$\vec{\omega}[BOD]$	Orbiter angular velocity in body axis coordinates.
$[M50 \rightarrow BOD]$	Transformation matrix from M50 to BODy axis coordinates (The Navigation Base is the origin of the BODy system.).
$[M50 \rightarrow TOP]$	Transformation matrix from M50 to TOPodetic coordinates.
$[TOP \rightarrow BOD]$	Transformation matrix from TOPodetic to BODy coordinates.

Output

$\vec{\omega}[TOP]$	Topodetic Euler angle rates for yaw, pitch, and roll: $\vec{\omega}[TOP] = [\psi_T, \theta_T, \phi_T]^T$

Input

C_{DR}	Conversion constant equal to degrees per radian.

Algorithm

The algorithm is described in TRW IOC 83:W482.4-28, dated 25 March 1983 and written by Darwin H. Poritz. This document is included in the Attachment.

Calling Argument List (in list order)

Input

$C_{D/R}$	Conversion constant equal to degrees per radian.
$\vec{v}_{\substack{wind\\CM}}[TOP]$	Wind-relative velocity of the Center of Mass in the TOPodetic frame.
$\sigma(\vec{v}_{\substack{wind\\CM}}[TOP])$	Uncertainty in $\vec{v}_{\substack{wind\\CM}}[TOP]$.

Output

V_{WR}	Magnitude of $\vec{v}_{\substack{wind\\CM}}[TOP]$.
$\sigma(V_{WR})$	Uncertainty in V_{WR}.
γ_W	Wind-relative ("local") flight path angle in degrees.
$\sigma(\gamma_W)$	Uncertainty in γ_W.
Ψ_W	Wind-relative ("local") azimuth, or heading angle, in degrees.
$\sigma(\Psi_W)$	Uncertainty in Ψ_W.

Algorithms

Compute the magnitude, V_{WR}, and its uncertainty, $\sigma(V_{WR})$

Let $\vec{v}_{\substack{wind\\CM}}[TOP] = [v_{Rx}, v_{Ry}, v_{Rz}]^T$

Then, the magnitude of this velocity is

*>> $$V_{WR} = (v_{Rx}^2 + v_{Ry}^2 + v_{Rz}^2)^{1/2}.$$

Let $\sigma(\vec{v}_{\substack{wind\\CM}}[TOP]) = [\sigma_x(\vec{v}_{WR}), \sigma_y(\vec{v}_{WR}), \sigma_z(\vec{v}_{WR})]^T$

Then, the uncertainty, or standard deviation, in V_{WR}, the magnitude of the wind-relative velocity, is

*>> $$\sigma(V_{WR}) = \left[\frac{v_{Rx}\ \sigma_x(\vec{v}_{WR})^2 + v_{Ry}\ \sigma_y(\vec{v}_{WR})^2 + v_{Rz}\ \sigma_z(\vec{v}_{WR})^2}{V_{WR}}\right]^{1/2}$$

Compute the wind-relative flight path angle and its uncertainty

The wind-relative flight path angle is computed from

*>> $$\gamma_W = \arcsin\left(\frac{-\ v_{Rx}}{V_{WR}}\right).$$

which is converted from radians to degrees.

Its uncertainty, or standard deviation, is computed from

$\sigma(\gamma_W) =$

*>> $$\left[\frac{\sigma_x(\vec{v}_{WR})^2\ v_{Rx}^2\ v_{Ry}^2 + \sigma_y(\vec{v}_{WR})^2\ v_{Ry}^2\ v_{Rz}^2 + \sigma_z(\vec{v}_{WR})\ (v_{Rx}^2 + v_{Ry}^2)}{V_{WR}^4\ (v_{Rx}^2 + v_{Ry}^2)}\right]^{1/2}$$

Compute the aximuth, or heading, and its uncertainty

The wind-relative aximuth angle is computed from

$$*>> \qquad \Psi_W = \arctan\left(\frac{v_{Ry}}{v_{Rx}}\right)$$

which is converted from radians to degrees. Its uncertainty, or standard deviation, is computed with

$$*>> \qquad \sigma(\Psi_W) = \frac{[\sigma_x(\vec{v}_{WR})^2\ v_{Ry}^2 + \sigma_y(\vec{v}_{WR})^2\ v_{Rx}^2]^{1/2}}{v_{R_x}^2 + v_{R_y}^2}$$

Calling Argument List (in list order)

Input

$C_{D/R}$	Conversion constant equal to degrees per radian.
$\vec{r}_{\oplus_{NB}}[M50]$	Geocentric position of the Navigation Base in the M50 frame.
$\vec{r}_{\oplus_{NB}}[M50]$	Geocentric velocity of the Navigation Base in the M50 frame.

Output

R_{NB}	Magnitude of $\vec{r}_{\oplus_{NB}}[M50]$.
V_{NB}	Magnitude of $\vec{r}_{\oplus_{NB}}[M50]$.
γ_I	Inertial flight path angle measured, in degrees, in horizon plane
Ψ_I	Inertial azimuth, or heading angle, measured in degrees, from the North.

Algorithms

Computation of the magnitudes of position and velocity

Let $\vec{r}_{\oplus_{NB}}[M50] = [x_I,\ y_I,\ z_I]^T$ and $\vec{r}_{\oplus_{NB}}[M50] = [v_x,\ v_y,\ v_z]^T$.

Then,

*>> $$R_{NB} = (x_I^2 + y_I^2 + z_I^2)^{1/2}$$

and

*>> $$V_{NB} = (v_x^2 + v_y^2 + v_z^2)^{1/2}$$

<u>Computation of the inertial flight path angle, γ_I</u>

First, the unit vectors $\hat{r}_{NB}$ and $\hat{v}_{NB}$ for the position and velocity vectors, respectively, are computed:

$$\hat{r}_{NB} = [\frac{x_I}{R_{NB}}, \frac{y_I}{R_{NB}}, \frac{z_I}{R_{NB}}]^T$$

and

$$\hat{v}_{NB} = [\frac{v_x}{V_{NB}}, \frac{v_y}{V_{NB}}, \frac{v_z}{V_{NB}}]^T.$$

The inertial flight path angle is computed from

*>> $$\gamma_I = \arcsin (\hat{r}_{NB} \cdot \hat{v}_{NB})$$

which is converted from radians to degrees.

<u>Computation of the inertial azimuth or heading, Ψ_I</u>

First, the cross product of velocity and position is calculated;

$$\vec{L} = \vec{r}_{\underset{NB}{\theta}}[M50] \times \vec{r}_{\underset{NB}{\theta}}[M50].$$

Then, the horizontal velocity, $\vec{v}_H$, is calculated from

$$\vec{v}_H = \vec{r}_{\underset{NB}{\theta}}[M50] \times \vec{L}$$

and its unit vector, $\hat{v}_H$, is determined;

$$\hat{v}_H = \frac{\vec{v}_H}{|\vec{v}_H|}.$$

The vector, $\vec{N}$, pointing North is computed from

$$\vec{N} = \vec{r}_{\oplus}[M50]_{NB} \times (\hat{k} \times \vec{r}_{\oplus}[M50]_{NB})$$

where $\hat{k}$ is the (unit) vector $[0,0,1]^T$.

Then,

$$\vec{N} = (x_I\hat{i} + y_I\hat{j} + z_I\hat{k}) \times [\hat{k} \times (x_I\hat{i} + y_I\hat{j} + z_I\hat{k})]$$

$$= - zx\,\hat{i} - zy\hat{j} + (x^2 + y^2)\hat{k}$$

$$= [- zx, - zy, (x^2 + y^2)]^T.$$

The unit vector is calculated from

$$\hat{N} = \frac{\vec{N}}{|\vec{N}|}.$$

Next, the unit vector, $\hat{E}$' pointing East is computed from

$$\hat{E} = \hat{N} \times \hat{r}_{NB}.$$

Then,

$$\cos \Psi_I = \hat{N} \cdot \hat{v}_H \quad \text{and} \quad \sin \Psi_I = \hat{E} \cdot \hat{v}_H.$$

To avoid a singularity when computing Ψ_I, $\cos \Psi_I$ is checked.

If $\cos \Psi_I = 0$ and $\sin \Psi_I > 0$, set $\Psi_I = 0^0$

If $\cos \Psi_I = 0$ and $\sin \Psi_I < 0$, set $\Psi_I = 270^0$

If $\cos \Psi_I \neq 0$, then Ψ is computed from

*>> $$\Psi_I = \arctan\left(\frac{\sin \Psi_I}{\cos \Psi_I}\right)$$

which is converted from radians to degrees.

Calling Argument List (in list order)

Input

$C_{D/R}$ Conversion constant equal to degrees per radian.

P_E Array of Earth geophysical parameters. Those used here are as follows:

$R_{\oplus E}$ = Earth's equitorial radius = $P_E(1)$
$R_{\oplus P}$ = Earth's polar radius = $P_E(6)$

ϕ_D Geodetic latitude, in degrees, of the desired location, P.

λ_D Geodetic longitude, in degrees, of the desired location, P.

h_D Geodetic height, in feet, of the desired location, P.

Output

$\vec{r}_{\oplus P}[GEO]$ Geocentric position vector of the desired location, P, in the GEOgraphic ("Earth-Fixed-Greenwich") coordinate system.

Algorithm

First ϕ_D and λ_D are converted from degrees to radians. Then

$$d_{xy} = \frac{R_{\oplus E}}{[1 + (\frac{R_{\oplus P}}{R_{\oplus E}} \tan \phi_E)^2]^{1/2}}$$

and

$$d_z = \left(\frac{R_{\oplus P}}{R_{\oplus E}}\right)^2 d_{xy} \tan \phi_D .$$

Then, the geocentric position vector is

$$\vec{r}_{\underset{P}{\oplus}}[GEO] = \begin{bmatrix} x_G \\ y_G \\ z_G \end{bmatrix} = \begin{bmatrix} (d_{xy} + h_D \cos \phi_D) \cos \lambda_D \\ (d_{xy} + h_D \sin \phi_D) \sin \lambda_D \\ d_z + h_D \sin \lambda_D \end{bmatrix} .$$

GEOGEOD

Calling Argument List (in list order)

Input

$C_{D/R}$	Conversion constant equal to degrees per radian.
$R_{\oplus E}$	Equitorial radius of the Earth.
$R_{\oplus P}$	Polar radius of the Earth.
$\vec{r}_{\oplus P}[GEO]$	Position vector of a point, P, (eg., for the vehicle, the Center of Mass, or the Navigation Base) expressed in GEOgraphic coordinates.

Outputs

ϕ_D	Geodetic latitude, in degrees, of point P.
λ_D	Geodetic longitude, in degrees, of point P.
h_D	Geodetic height, in feet, of point P.
δ	Declination, in degrees, of point P.

Algorithms

Preliminary calculations

First, the Earth's flattening, f, is calculated from its definition:

$$f = \frac{R_{\oplus E} - R_{\oplus P}}{R_{\oplus E}}$$

The components of the geographic position vector are x_G, y_G, z_G; that is,

$$\vec{r}_{\underset{P}{\oplus}}[GEO] = [x_G, y_G, z_G]^T.$$

The following parameters are calculated from f and the geocentric position:

$$A = x_G^2 + y_G^2$$

and

$$D = (1 - f)^2 \, z_G^2 .$$

Now, a parameter, B_{P1}, is calculated by means of a five-pass iteration. The iteration is initialized by setting

$$B_0 = 0.0067$$

Then, for i = 0,1,2,3, and 4

(1) $$B_{P1} = B_i + 1$$

(2) $$C = \frac{A}{B_{P1}^2}$$

(3) $$B_i = \frac{f(2 - f)\, R_{\oplus E}}{(C + D)^{1/2}}$$

Steps 1 through 3 are repeated until i = 4.

Then,

$$B_{P1} = B_4 + 1$$

Calculate the geodetic latitude, ϕ_D, and longitude, λ_D

The latitude is calculated from

*>> $$\phi_D = \arctan\left[\frac{z_G}{(A/B_{p1}^2)^{1/2}}\right]$$

which is converted from radans to degrees.

The longitude is

*>> $$\lambda_D = \arctan\left(\frac{y_G}{x_G}\right)$$

which is converted from radians to degrees.

Calculalate the geodetic height, h_D

The height is calculated from

*>> $$h_D = \frac{\left[1 - \frac{B_4(1 - f)^2}{f(2 - f)}\right]}{\left[\frac{A}{B_{p1}{}^2} + z_G\right]^{1/2}}$$

Calculate the declination, δ

The declination is calculated from

*>> $$\delta = \arctan\left[\frac{z_G}{(x_G^2 + y_G^2)^{1/2}}\right]$$

which is converted from radians to degrees.

Calling Argument List (in list order)

Input

$\text{₵}(\vec{\varepsilon}[BOD])$ Covariance matrix of errors in body axis orientation which are errors in the body axis Euler angles in radians. $\vec{\varepsilon}[BOD] = [\varepsilon_\psi, \varepsilon_\theta, \varepsilon_\phi]^T$ where ψ = yaw, θ = pitch, and φ = roll.

$\vec{u}$ Vector for which the covariance matrix is to be corrected for body axis orientation errors. The vector has the elements $\vec{u} = [u_1, u_2, u_3]^T$.

Input and Output

$\text{₵}(\vec{u})$ Covariance matrix of the vector $\vec{u}$. It is input without the effect of body axis errors and output with these effects added to it.

Algorithm

First, the partial derivative matrix is constructed as follows:

$$\left[\frac{\partial u_i}{\partial \varepsilon_j}\right] = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix} .$$

Then, the contribution of the axis uncertainty is calculated by

$$\text{₵}(\vec{\varepsilon}[\vec{u}]) = \left[\frac{\partial u_i}{\partial \varepsilon_j}\right] \text{₵}(\vec{\varepsilon}[BOD]) \left[\frac{\partial u_i}{\partial \varepsilon_j}\right]^T .$$

The axis uncertainty matrix is then added to $\text{₵}(\vec{u})$ to obtain the output;

*>> $\cent(\vec{u}) = \cent(\vec{u})_{IN} + \cent(\vec{\varepsilon}[\vec{u}])$

WINDTOP

Calling Argument List

Input

$C_{D/R}$	Conversion constant equal to degrees per radian.
W_H	Horizontal wind speed.
θ_H	Direction angle of horizontal wind velocity in degrees.
$\sigma(W_H)$	Uncertainty in W_H.
$\sigma(\theta_H)$	Uncertainty in θ_H (θ_H in degrees).
$\sigma(W_v)$	Uncertainty in vertical wind speed.

Output

$\vec{v}_{wind}[TOP]$	Wind velocity expressed in TOPodetic coordinates.
$¢(\vec{v}_{wind}[TOP])$	Covariance matrix of $\vec{v}_{wind}[TOP]$.

Algorithm

Calculation of wind velocity expressed in topodetic coordinates

First, θ_H and $\sigma(\theta_H)$ are converted to radians by dividing by $C_{D/R}$. Then

*>> $$\vec{v}_{wind}[TOP] = \begin{bmatrix} -W_H \cos \theta_H \\ -W_H \sin \theta_H \\ 0 \end{bmatrix} .$$

<u>Calculation of the covariance matrix of $\vec{v}_{wind}[TOP]$</u>

The matrix elements are set as follows:

*>> $\quad ¢(\vec{v}_{wind}[TOP]) =$

$$\begin{bmatrix} \{[\sigma(W_H)\cos\theta_H]^2 + [W_H\,\sigma(\theta_H)\sin\theta_H]^2\} & \{\cos\theta_H \sin\theta_H[\sigma(W_H)^2 + W_H^2\,\sigma(\theta_H)^2]\} & 0 \\ \{\cos\theta_H \sin\theta_H[\sigma(W_H)^2 + W_H^2\,\sigma(\theta_H)^2]\} & \{[\sigma(W_H)\sin\theta_H]^2 + [W_H\,\sigma(\theta_H)\cos\theta_H]^2\} & 0 \\ 0 & 0 & \sigma(W_V) \end{bmatrix}$$

4.2 THE NAVBLK FILE

The purpose of the NAVBLK file is to provide the user with a magnetic tape file which contains a list of input parameter values used by the trajectory estimation (BET) system. This file is produced by the program I2IIPC. The contents of the NAVBLK file are gathered by the program INPUT whose inputs are mainly user inputs. The only calculated parameters are the declination (DELTA) and geocentric distance (RSUBO) of the Nav Base as the vehicle sits on the launch pad before launch. These two terms, DELTA and RSUBO, are calculated in INPUT; the algorithms for them are given in Section 4.2.2 of this document.

4.2.1 Definition of Terms

The NAVBLK file contains constants required for a BET solution. The terms in this file are defined in Table 1 on the next page. Section 4.2.3 shows how the contents of the NAVBLK are arranged in the file records.

Table 1. Definition of Terms

Term Name	Description	Source
IHEADR [180]	NAVBLocK header from MET tape	See CMET[20]
GRR [4]	Guidance [Stable Member] Release times	User input via terminal editor
CGT [200, 4]	Center of Mass (CG) history timeline	User input via terminal editor
DPSET [20, 200]	Special Event Timeline with description	User input via terminal editor
REF [3, 3, 3]	REFerence to Stable Member transformation MATrix (REFSMMAT)	User input via terminal editor
TRAC [6, 120]	Tracking Station data	Stored on file RADAR
CMET [20]	Corrected (units conversion) METeorological data	Entered by means of MET (data) tapes Ascent from NOAA station at KSC Descent from MSC
KAPPA	Earth-fixed launch azimuth	User input via terminal editor
PHIO	Geodetic latitude of launch site	User input via terminal editor
DELTA	Declination of Nav Base on launch pad	Computed in program INPUT
LAMO	Geodetic longitude of launch site	User input via terminal editor
RSUBO	Geocentric distance of Nav Base on launch pad	Computed in program INPUT
XSUBO	Geodetic height of Nav Base above Fischer ellipsoid	User input via terminal editor
NAVO [3]	Position of Nav Base with respect to structure frame	User input via terminal editor

4.2.2 Computation of the Terms DELTA and RSUBO

These terms are computed for ascent analysis in the program INPUT. RSUBO is the geocentric distance of the Navigation Base as the Shuttle sits on the launch pad. DELTA is the declination (or geocentric latitude) of the Navigation Base before launch. The following symbols will be used:

$R_{\oplus E}$	Equitorial radius of the Earth.
$R_{\oplus P}$	Polar radius of the Earth.
ϕ_o	Geodetic latitude of the launch site, in degrees.
X_o	Geodetic height of the Nav Base, at the launch site, above the Fischer ellipsoid.
r_o	RSUBO
δ	DELTA

The latitude is converted to radians within the algorithm. The algorithm is as follows. Define

$$S_{xy} = \frac{R_{\oplus E}}{\left[1 + \left(\frac{R_{\oplus P}}{R_{\oplus E}} \tan \phi_o\right)^2\right]^{1/2}}$$

and

$$S_z = \frac{R_{\oplus P} \tan \phi_o}{\left[1 + \left(\frac{R_{\oplus P}}{R_{\oplus E}} \tan \phi_o\right)^2\right]^{1/2}}.$$

Also define

$$V_{xy} = \frac{S_{xy}}{R_{\oplus E}^2} = \frac{1}{R_{\oplus E}\left[1 + \left(\frac{R_{\oplus P}}{R_{\oplus E}} \tan \phi_o\right)^2\right]^{1/2}}$$

and

$$V_z = \frac{S_z}{R_\oplus^2} = \frac{\tan\phi_o}{R_{\oplus P}[1 + (\frac{R_{\oplus P}}{R_{\oplus E}}\tan\phi_o)^2]^{1/2}}.$$

Let

$$V = (V_{xy}^2 + V_z^2)^{1/2}.$$

Then, let

$$u_{xy} = \frac{V_{xy}}{V} \text{ and } u_z = \frac{V_z}{V}.$$

Then, the declination is

*>> $$\delta = \arctan(\frac{S_z + u_z X_o}{S_{xy} + u_{xy} X_o}).$$

The geocentric distance of the Nav Base is

*>> $$r_o = [(S_z + u_z X_o)^2 + (S_{xy} + u_{xy} X_o)^2]^{1/2}.$$

4.2.3 NAVBLK File Format

Each record written by the program I2IIPC is 1000 words long. The file format is shown in the following table.

Table 2. NAVBLK File Format

	START		END	
TERM	RECORD	WORD	RECORD	WORD
IHEADR	1	1	1	180
GRR	2	1	2	4
CGT	2	5	2	804
DPSET	2	805	6	4
REF	6	5	6	31
TRAC	6	32	6	631
CMET	6	632	26	631
KAPPA	6	632	6	632
PHIO	6	633	6	633
DELTA	6	634	6	634
LAMO	6	635	6	635
RSUBO	6	636	6	636
XSUBO	6	637	6	637
NAVO	6	638	6	640

APPENDIX A

TOPODETIC EULER ANGLE RATES

(Applicable Document 5, Section 3.0)

1. INTRODUCTION

The Ascent/Descent Best Estimated Trajectory (BET) which is produced by the OPIP program provides space in words 216, 217, and 218 of each record for rates of Orbiter Euler angles relative to the instantaneous topodetic axes. This memorandum proposes a method for computing these angular rates and presents this method in Section 3 below. The definition of the Euler angles is given in Section 2.

2. TOPODETIC EULER ANGLES: YAW, PITCH, AND ROLL

The topodetic coordinate system is defined and displayed in a figure in Appendix A of Reference 1. The origin of the system is the point of interest, e.g., the Orbiter's center of gravity or navigation base. The unit vector $\underline{f}_3$ representing the topodetic z axis points down (in the direction of Earth's gravity) from the origin along the local perpendicular to the Fischer-ellipsoid model of the Earth's surface. The unit vector $\underline{f}_1$ representing the topodetic x axis is perpendicular to $\underline{f}_3$ and points northward in the origin's meridian plane. The unit vector $\underline{f}_2$ representing the topodetic y axis completes the right-handed, orthogonal system.

The Orbiter's body-axis coordinate system is also defined and displayed in a figure in Appendix A of Reference 1. The origin of the system is the Orbiter's center of gravity. The unit vector $\underline{e}_1$ representing the body x axis is parallel to the Orbiter structural body X_0 axis and positive toward the Orbiter's nose. The unit vector $\underline{e}_3$ representing the body z axis is parallel to the Orbiter's plane of symmetry, is perpendicular to $\underline{e}_1$, and points downward with respect to the Orbiter's fuselage. The unit vector $\underline{e}_2$ representing the body y axis completes the right-handed, orthogonal system.

The topodetic Euler angles of yaw ψ , pitch θ , and roll ϕ are defined by fictitiously aligning the Orbiter's body axes x , y , and z with the topodetic unit vectors $\underline{f}_1$, $\underline{f}_2$, and $\underline{f}_3$, respectively, and then rotating the Orbiter in a particular way until the body axes x , y , and z are aligned with the body-axis unit vectors $\underline{e}_1$, $\underline{e}_2$, and $\underline{e}_3$, respectively. The first rotation is about the fictitiously aligned body z axis through the angle ψ . The second rotation is through the angle θ about the fictitiously aligned by y axis after the first rotation through

ψ . The third rotation is through the angle ϕ about the fictitiously aligned x axis after the rotations through ψ and θ .

Let $I(\psi)$, $J(\theta)$, and $K(\phi)$ denote the orthogonal transformations for the successive rotations of yaw, pitch, and roll by ψ , θ , and ϕ radians respectively. From simple figure drawings, it can be shown that

$$I(\psi) = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$J(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}, \text{ and}$$

$$K(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}.$$

The complete rotation $L(\psi,\theta,\phi)$ is given by the product

$$L(\psi,\theta,\phi) = K(\phi)\ J(\theta)\ I(\psi) \quad .$$

3. TOPODETIC EULER ANGLE RATES

It is convenient to calculate the Euler angle rates $\dot{\psi}$, $\dot{\theta}$, and $\dot{\phi}$ from the perspective of an inertial coordinate system. The mean-of-1950 (M50) inertial coordinate system is defined and displayed in a figure in Appendix A of Reference 1. Let $\underline{g}_1$, $\underline{g}_2$, and $\underline{g}_3$ be the unit vectors in the directions of the M50 x , y , and z axes, respectively. Let

$$A = (\underline{e}_i \cdot \underline{g}_j) = (\underline{E}_1, \underline{E}_2, \underline{E}_3)^t$$

be the orthognal transformation from M50 to body-axis coordinates. It follows that the $\underline{E}_j$'s are the body axes $\underline{e}_1$, $\underline{e}_2$, and $\underline{e}_3$ expressed in M50 coordinates. Let

$$B = (\underline{f}_i \cdot \underline{g}_j) = (\underline{F}_1, \underline{F}_2, \underline{F}_3)^t$$

be the orthogonal transformation from M50 to topodetic coordinates. The $\underline{F}_j$'s are the topodetic axes $\underline{f}_1$, $\underline{f}_2$, and $\underline{f}_3$ expressed in M50 coordinates. The matrices A and B are already computed within the OPIP program.

The body angular velocity $\underline{w}$ expressed in body-axis coordinates is supplied as an input to the OPIP program. Since the body axes are fixed in the Orbiter, the angular velocity of the body axes relative to an inertial frame is the same as the angular velocity of the body. Let

$$\underline{W} = A^t \, \underline{w}$$

be the body angular velocity expressed in M50 coordinates.

Define $\underline{Q}$ as the angular velocity of the topodetic axes as seen from the M50 inertial frame and as expressed in M50 coordinates. The body angular velocity as seen in the topodetic frame is found by removing the rotational motion of the topodetic frame from the inertial body angular velocity. Therefore

$$\underline{U} = \underline{W} - \underline{Q}$$

is the body angular velocity as seen in the topodetic frame and as expressed in M50 coordinates. The next step is to compute $\underline{Q}$.

$\underline{Q}$ can be decomposed into rotational motion about $\underline{f}_2$ due to northward motion of the Orbiter and into rotational motions about $\underline{f}_1$ and $\underline{f}_3$ due to eastward motion of the Orbiter. Let $\underline{R}$ and $\underline{V}$ be respectively the inertial position and velocity of the Orbiter in M50 coordinates. An approximate value for $\underline{Q}$ is

$$\underline{Q} = \frac{(\underline{F}_2 \times \underline{R}) \cdot \underline{V}}{|\underline{F}_2 \times \underline{R}| \; |\underline{R}|} \underline{F}_2 + \frac{(\underline{F}_1 \times \underline{R}) \cdot \underline{V}}{|\underline{F}_1 \times \underline{R}| \; |\underline{R}|} \underline{F}_1$$

$$+ \frac{(\underline{F}_3 \times \underline{R}) \cdot \underline{V}}{|\underline{F}_3 \times \underline{R}| \; |\underline{R}|} \underline{F}_3$$

The first term above is approximate because it neglects the additional rotation about $\underline{f}_2$ due to the increase in flattening of the Fischer ellipsoid with motion toward the pole. This effect is believed to be negligible.

All rotational motion of the Orbiter as seen in the topodetic frame is due entirely to the Orbiter's angular velocity as seen in the topodetic frame. Therefore, the topodetic Euler angle rates $\dot{\psi}$, $\dot{\theta}$, and $\dot{\phi}$ are the components of body angular velocity along the instantaneous axes of rotation used for the topodetic Euler angles ψ , θ , and ϕ , respectively. The next step is to compute the axes of rotation which will be denoted by $\underline{h}_1$, $\underline{h}_2$, and $\underline{h}_3$.

The yaw is about $\underline{f}_3$ and thus

$$\underline{h}_1 = \underline{f}_3 \; .$$

The pitch is about $\underline{f}_2$ after rotation about $\underline{h}_1$ by ψ . Since the columns of $I^t(\psi)$ are the rotated axes in topodetic coordinates, the second column of $I^t(\psi)$ gives $\underline{h}_2$, the rotated $\underline{f}_2$, in topodetic coordinates:

$$\underline{h}_2 = (-\sin\psi)\, \underline{f}_1 + (\cos\psi)\, \underline{f}_2 \; .$$

The roll is about $\underline{f}_1$ after rotation about $\underline{h}_1$ by ψ followed by rotation about $\underline{h}_2$ by θ . By a similar argument, the first column of $[J(\theta)\; I(\psi)]^t$ gives $\underline{h}_3$, the rotated $\underline{f}_1$, in topodetic coordinates:

$$\underline{h}_3 = (\cos\psi \, \cos\theta)\, \underline{f}_1 + (\sin\psi \, \cos\theta)\, \underline{f}_2 + (-\sin\theta)\, \underline{f}_3 \; .$$