

ON THE INTERPRETATION OF THE  $B - \rho$  RELATION  
IN INTERSTELLAR CLOUDS

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Troland and Heiles (1986) have recently presented an updated compilation of observational data concerning the relationship between the interstellar magnetic-field strength  $B$  and the gas density  $\rho$  (or, equivalently, the particle density  $n$ ). One of the main findings of their survey was that  $B$  remains constant (at a value of  $\sim 5 \mu G$ ) over the density range  $0.1 - \sim 100 \text{ cm}^{-3}$  and shows evidence for increase only at higher densities. They compared this result with theoretical predictions based on the Parker-instability scenario for the formation and evolution of interstellar clouds in the presence of the galactic magnetic field. In this picture (reviewed, e.g., by Mouschovias 1985), low-density gas is driven by the magnetic Rayleigh-Taylor instability into magnetic "valleys," where it accumulates into denser concentrations. The gas initially flows along the magnetic field lines and there is little increase of the field strength with density;  $B$  only starts to rise when  $n$  becomes large enough for self-gravity to begin competing with the magnetic stresses. For a cloud mass of  $\sim 10^3 M_{\odot}$  and the measured background field strength, the critical density for contraction is  $\sim 75 \text{ cm}^{-3}$ . Troland and Heiles therefore concluded that this scenario is basically consistent with the observations.

Why should, however, the cloud mass  $M$  in the expression for the critical density  $n_{crit}$  be  $\sim 10^3 M_{\odot}$ ? In fact, since  $n_{crit} \propto B^{1.5} M^{-0.5}$ , why doesn't the total mass in the magnetic "valleys," which is of the order of  $10^5 - 10^6 M_{\odot}$ , cause the field strength to start increasing already at much lower densities? The relevant mass scale for the estimate of  $n_{crit}$  is not specified in the theoretical models considered by Troland and Heiles and must be determined with the help of additional physical input. In fact, the question of the appropriate mass only enters because the relevant observable quantity is  $n$ . By using simple balance-of-forces arguments or the virial theorem, one can readily see that  $B$  is directly related not to the density but rather to the *column density*, or, equivalently, to the surface mass density  $\Sigma$ . Thus, for example, in order for a cold, spheroidal cloud in virial equilibrium to contract, the surface density must satisfy  $\Sigma > \epsilon B / \sqrt{G}$ , where  $G$  is the gravitational constant and  $\epsilon$  is a geometry-dependent factor  $\gtrsim 0.1$  (Strittmatter 1966).

In order to determine the behavior of the gas that accumulates in the magnetic-field "valleys," one must consider the equilibrium configurations of the gas-field system that are reached at the conclusion of the Parker instability. Mouschovias (1974) has constructed explicit solutions to this problem for the case where the gravitational field is dominated by the background galaxy. However, in the present case it is the self-gravity of the inflowing gas that is most relevant. Elmegreen (1982a,b) has carried out a linear analysis of the Parker instability in a self-gravitating gas, drawing useful conclusions about the formation of large-scale cloud complexes. However, his results do not extend to the late stages of the evolution that are of interest here. The influence of self-gravity in the last phase of the Parker instability is a difficult problem that has not yet been addressed in the literature. Here I illustrate, in a very simplified manner, one possible aspect of the final configuration that could be relevant to the observed  $B - \rho$  relation.

In a simple representation of the Parker instability (e.g., Blitz and Shu 1980), the galactic magnetic field is pictured as lying originally in the plane of the galaxy and being "frozen" into an isothermal gas layer whose initial scale height normal to the galactic plane is  $H_i$ . If the galactic gravitational acceleration  $g$  is taken to be a constant ( $\sim 3 \times 10^{-9} \text{ cm s}^{-2}$ ) and the thermal, magnetic, and cosmic-ray contributions to the pressure are approximately equal, then  $H_i \approx 3C_i^2/g$  (where  $C_i$  is the isothermal speed of sound in the initial state) and the critical wavelength along the field for the onset of the instability is  $\lambda_{P,crit} \approx 1.2\pi H_i$ . The mass that ultimately gathers in the magnetic "valleys" is assumed to come from a region of volume  $\sim 2H_i \times 2H_i \times \lambda_{P,crit}$  and can be estimated by multiplying this volume by the initial midplane density  $\rho_i$ .

In order to consider the effect of self-gravity on the magnetic-field strength in the final configuration, we adopt the following idealized picture. We approximate the field as lying in the plane of the galaxy and having a constant magnitude equal to its initial midplane value  $B_i$ . We further assume that the accumulated gas is in the form of a uniform, self-gravitating, isothermal disk of area  $2H_i \times 2H_i$  and scale height (along the field)  $H_f \approx (C_f^2/2\pi G\rho_f)^{1/2}$  (where  $C_f$  is the isothermal sound speed in the final configuration and  $\rho_f$  is the midplane density in the disk). The surface density of the disk is given by  $\Sigma \approx 2H_f \rho_f$  and is equal to  $\sim \lambda_{P,crit} \rho_i$ . To evaluate the Jeans-stability of this configuration, we adopt the results of the infinite-slab fragmentation analysis of Nakano and Nakamura (1978). According to this analysis, the disk will fragment only if  $\Sigma > B_i/2\pi G^{1/2}$  (consistent with the above-mentioned virial-equilibrium results) and if the wavelength *in the plane of the disk* exceeds  $\lambda_{J,crit} \approx 2\pi H_f (1 - B_i/2\pi G^{1/2}\Sigma)^{-1}$ . Now, the first condition is marginally satisfied for representative galactic parameters ( $\rho_i \approx 3 \times 10^{-24} \text{ g cm}^{-3}$ ,  $C_i \approx 7 \text{ km s}^{-1}$ ,  $B_i \approx 5 \mu\text{G}$ ), and so is also the self-consistency requirement that the gravitational acceleration due to the disk be  $\geq g$ . However, the second condition is *not* satisfied for these parameters so long as  $C_f \approx C_i$  because  $\lambda_{J,crit}$  then exceeds the assumed diameter ( $\sim 2H_i$ ) of the disk. This inequality is only strengthened if one considers the effect of the finite size of the disk on the above condition, and it holds also if one substitutes the wavelength of maximum growth for the marginal-stability wavelength in the preceding discussion. Physically, the disk does not fragment in this case because its diameter remains smaller than the relevant "magnetic" Jeans length.

In view of the highly simplified nature of the foregoing arguments and the marginal values obtained in the numerical estimates, it is unclear whether one can draw any firm conclusions from the above result. However, if the Jeans-length effect is at all applicable, then this offers a possible clue to the interpretation of the observed constancy of  $B$  for  $n \lesssim 100 \text{ cm}^{-3}$ . In this picture, the field strength can only start increasing after  $C_f$  has dropped sufficiently below  $C_i$  for  $\lambda_{J,crit}$  to be less than  $2H_i$ . Such a reduction in  $C_f$  could be the result of a thermal instability that might develop in the gathering H I gas; in fact, an instability of this type could produce a phase transition (cf. Field, Goldsmith, and Habing 1969) that would raise the particle density in the disk to  $\sim 100 \text{ cm}^{-3}$  and would thus fix the value of  $n$  at which  $B$  starts to increase. Thermal instabilities might, therefore, trigger the formation of dense interstellar clouds even though they could not by themselves give rise to the observed large-scale condensations (cf. Mouschovias 1978). Other possible implications of this scenario (e.g., to the interpretation of "turbulence" in interstellar clouds) could also be explored; however, one should first verify the validity of the basic idea by calculating self-consistently the effects of self-gravity on the evolution and the end states of the Parker instability.

## References

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