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Received Optical Power Calculations for Optical Communications Link Performance Analysis

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The factors affecting optical communication link performance differ substantially from those at microwave frequencies, due to the drastically differing technologies, modulation formats, and effects of quantum noise in optical communications. In addition, detailed design control table calculations for optical systems are less well developed than corresponding microwave system techniques, reflecting the relatively less mature state of development of optical communications. This article describes detailed calculations of received optical signal and background power in optical communications systems, with emphasis on analytic models for accurately predicting transmitter and receiver system losses.

I. Introduction

In a recent article (Ref. 1), a "simple" method for optical communications link analysis was described. That method provides a means for making quite accurate first-order predictions of optical communication link performance, without the need for involved calculation. In contrast, this article describes the detailed calculations that are necessary to more accurately predict the received optical signal and background power via the range equation. Direct-detection and pulse-position modulation (PPM) are assumed, although many of the results are independent of this assumption.

In this article, signal power is tracked from the output of the transmitter laser to the face of the detector in the receiver. The received optical background power is calculated from the background radiance. Detector sensitivity and required signal calculations are not considered here, but will be addressed in a future article.

Although some empirical factors in the calculation remain, the received power is determined here primarily from analytic models of the system components. The calculations shown here are substantially more detailed than those in earlier reports (Refs. 1–3). Future work is expected to further improve the accuracy of the calculations, and, as experience is gained with optical communications system design, to address additional issues such as design uncertainties and statistical design models (Ref. 4).

II. Basic Parameters

In calculating link performance, we start with a set of basic component values and system parameters, which are assumed to be known and fixed in value. (As the accuracy of the link performance model is improved, some "basic" component values can, in fact, be broken down and computed from other factors. For example, with specific laser hardware, output power can be predicted as a function of the operating conditions.)

A block diagram of an optical communications transceiver is shown in Fig. 1. A system is typically composed of two such transceivers, similar in design, though often with different component values (e.g., telescope sizes, laser powers, etc.). Additional components which are necessary for a working system but which do not directly affect communications performance (for example, the spatial acquisition system) are not shown.

The component values required to determine link performance are as follows:

(1) Laser average power output, P_ℓ (Watts)

(2) Laser wavelength, λ (meters)

(3) Transmit telescope aperture diameter, D_t

(4) Transmit telescope obscuration ratio, γ_t

(5) Transmitter optics throughput, η_t

(6) Transmitter pointing bias error, ϵ_t, and jitter, σ_t

(7) Receiver telescope aperture diameter, D_r

(8) Receiver telescope obscuration ratio, γ_r

(9) Receiver optics throughput, η_r

(10) Narrow band filter transmission, η_λ

(11) Receiver pointing bias error, ϵ_r, and jitter, σ_r

(12) Narrow band filter spectral bandwidth, $\Delta\lambda$ (angstroms)

(13) Detector diametrical field of view, θ

The transmitter and receiver optics throughput, items (5) and (9) above, can in fact be calculated from the reflection and transmission coefficients of the components in the optical system.[1] Table 1 gives a list of sample component values which will be used below in calculating a sample link design control table.

Besides hardware characteristics, system operational parameters are needed to determine link performance. These factors are as follows:

(14) Data rate, R (bits/second)

(15) PPM alphabet size, M

(16) PPM slot time, τ_s (seconds)

(17) Link range, L (meters)

(18) Atmospheric transmission loss factor, ℓ_a

(19) Background radiance, B (Watts/meter^2/steradian/angstrom)

Both the data rate, item (14), and the PPM slot time, item (16), must be specified since there may be a "dead time" between each PPM word during which the laser is always "off." This is typical in systems using Nd:YAG or other crystalline lasers. The data rate, R, slot time, τ_s, and dead time, τ_d, are related by the formula

$$\tau_d + M\tau_s = \frac{\log_2 M}{R} \tag{1}$$

In systems which are average power limited (such as those using Nd:YAG lasers), the addition of dead time improves performance, by reducing the number of background counts per slot (since the slot time is reduced by τ_d/M). In systems which are peak power limited (e.g., those using semiconductor lasers), dead time degrades performance and is usually not used, i.e., $\tau_d = 0$ for these systems.

Item (18), the atmospheric transmission loss factor, includes transmission losses due to Earth or planetary atmospheres between the transmitter and receiver. It is assumed that both signal and background are subject to this loss. Item (19) is the average background radiance over the field of view of the detector. Table 2 gives a sample list of values for the operational parameters.

III. Received Signal Power Calculation

Table 3 shows part of an optical link design control table (DCT) summarizing received signal power calculations for a system using the components and parameters described in Tables 1 and 2. In general, a DCT takes the form of an initial signal power (here the laser average power, P_ℓ), a number of multiplicative gain or loss factors leading to a received signal power, a required signal power (for the desired error rate), and a resulting system operating margin. The DCT is just a formal method for calculating the received signal via the range equation and comparing it to the signal required for some given error rate. Here, the range equation is given by

$$P_r = P_\ell\, g_t\, \eta_t\, \eta_{tp}\, \ell_s \ell_a\, g_r\, \eta_r\, \eta_\lambda\, \eta_D \tag{2}$$

where P_r is the received optical signal power, g_t is the transmitter telescope gain, η_{tp} is the transmitter pointing efficiency,

[1] See, for example, Lambert, S. G., et al., "Design and Analysis Study of a Spacecraft Optical Transceiver Package," Appendix C, McDonnell Douglas Astronautics Co. (final report under JPL contract No. 957061), JPL document No. 9950-1240, Jet Propulsion Laboratory, Pasadena, Calif., Aug. 1985.

ℓ_s is the space loss factor, g_r is the receiver telescope gain, η_D is the detector truncation loss factor, and the remaining factors are defined among items (1)–(19) above. In sections IV to VI below we consider calculation of the various gain and loss factors which appear in Eq. (2). Section VII describes the received background power calculation.

IV. Transmitter Factors

The first two factors in Eq. (2), g_t and η_t, together give the net on-axis gain of the transmitter optical system. The first factor, g_t, is the telescope gain, including effects of nonuniform aperture illumination, beam truncation, and a central beam obscuration, but not including transmission and reflection losses (see below). For telescopes with Gaussian-beam illumination and a central obscuration, the on-axis gain is given (Ref. 5) by

$$g_t = \left(\frac{\pi D_t}{\lambda}\right)^2 \frac{2}{\alpha_t} \left\{ e^{-\alpha_t^2} - e^{-\alpha_t^2 \gamma_t^2} \right\}^2 \tag{3}$$

where D_t is the transmitter aperture diameter, γ_t is the obscuration ratio (defined as the ratio of the central obscuration diameter to the main aperture diameter), and α_t is the truncation ratio, given optimally (Ref. 5) by

$$\alpha_t = 1.12 - 1.30\, \gamma_t^2 + 2.12\, \gamma_t^4 \tag{4}$$

valid for $\gamma_t \leqslant 0.4$. The truncation ratio is the ratio of the main aperture diameter to the Gaussian-beam spot size. Equation (3) ignores the effect of the secondary element support struts, which for some telescopes can cause losses comparable to those due to the obscuration.

The transmitter optics efficiency, η_t, is given by item (5) above, i.e., it is to be specified as a basic component value. This term takes into account transmission and reflection losses in the transmitter, i.e., those in the relay optics, steering mirrors, and in the telescope. As noted above, η_t can in fact be readily calculated from the transmission and reflection coefficients of the individual optical components once the optical system has been specified in detail. For typical systems, η_t is in the range of 0.4 to 0.7.

The next factor in Eq. (2) is the transmitter pointing efficiency, η_{tp}. The pointing loss depends on the telescope gain (i.e., the narrowness of the transmitted optical beam) and on the statistics of the transmitter pointing system. Static errors in determining the desired pointing angle and in achieving that actual pointing angle lead to pointing bias errors; dynamic effects in measuring the desired angle and in maintaining a fixed pointing angle in the face of spacecraft base motion (or apparent image motion in a turbulent atmosphere) lead to pointing jitter errors. Jitter errors are often assumed to be radially symmetric and Rayleigh distributed (Ref. 3). Quasistatic errors, such as "pointing control errors" which can occur in radio-frequency systems on deadband-controlled deep-space spacecraft, would normally be completely removed by the optical communications fine-pointing system. (For a known spacecraft, it would be possible to model some sources of base motion, such as attitude control jets, to produce a better estimate of optical system pointing errors.)

Pointing losses can be accounted for in several ways. Here, we assume that the mean value of the pointing loss factor η_{tp} is desired, and that dynamic errors are in fact Rayleigh distributed (Fig. 2 shows pointing error geometry). The overall pointing error (bias plus jitter) will then be given (Ref. 3) by the well-known Rice density so that

$$\eta_{tp} = \int_0^\infty \eta_{tp}(\phi) \frac{\phi}{\sigma_t^2} \exp\left\{-\frac{\phi_t^2 + \epsilon_t^2}{2\sigma_t^2}\right\} I_o\left(\frac{\phi\epsilon_t}{\sigma_t^2}\right) d\phi \tag{5}$$

where ϵ_t is the root-sum-square (RSS) two-axis pointing bias error, σ_t is the RSS two-axis jitter, I_o is the modified Bessel function of order zero, and $\eta_{tp}(\phi)$ is the instantaneous pointing loss as a function of off-axis pointing angle, ϕ. The term $\eta_{tp}(\phi)$ is given for telescopes with Gaussian beam illumination and a central obscuration by the formula (Ref. 5)

$$\eta_{tp}(\phi) = \left[\frac{\int_{\gamma_t}^1 e^{-\alpha_t^2 u^2} J_o\left(\pi \frac{D_t}{\lambda}\phi u\right) u\, du}{\int_{\gamma_t}^1 e^{-\alpha_t^2 u^2} u\, du}\right]^2 \tag{6}$$

where J_o is the Bessel function of order zero. (The dummy variable u in the integral is the radial position in the transmitter aperture.) For reasonable pointing errors, i.e., for $\phi \lessapprox \lambda/D_t$, $\eta_{tp}(\phi)$ can be approximated (within 0.1 dB) by the series

$$\eta_{tp}(\phi) \cong \frac{1}{f_0^2(\gamma_t)} \left[f_0(\gamma_t) + \frac{f_2(\gamma_t)}{2!} x^2 + \frac{f_4(\gamma_t)}{4!} x^4 + \frac{f_6(\gamma_t)}{6!} x^6\right]^2 \tag{7}$$

where $x = \pi(D_t/\lambda)\phi$ and the coefficients f_0, f_2, f_4, and f_6 are given for several values of γ_t (and α_t as given by Eq. 4) in Table 4.

Since performance can degrade rapidly with increasing pointing errors, it is sometimes preferable to use a burst-error method of determining pointing losses. In this case it is assumed that whenever pointing errors exceed some threshold value, a "burst error" occurs, i.e., that the link is essentially lost momentarily at the time. The pointing loss is calculated as the loss at the threshold value of the pointing error, so that acceptable communications performance is guaranteed except during the occasional "bursts." In practice, an acceptable probability of burst error is chosen (e.g., one acceptable to the spatial tracking system and the synchronization systems), and a corresponding threshold pointing error (which is statistically exceeded at the burst error rate) is then calculated. The threshold pointing error then appears as a static (bias) error in the DCT, and jitter is ignored.

V. Transmission Loss Factors

As in radio frequency systems, the largest loss in an optical communication system is usually the "space loss," i.e., the loss associated with propagation through free space. The space loss factor, ℓ_s, is given for all systems by

$$\ell_s = \left(\frac{\lambda}{4\pi L}\right)^2 \tag{8}$$

Due to the dependence on wavelength, the space loss incurred by optical systems is much larger (i.e., the factor ℓ_s is much smaller) than in RF systems, but this is usually more than offset by higher optical antenna (telescope) gains.

Besides the space loss, there may be additional propagation losses if the signal passes through a lossy medium, e.g., a planetary atmosphere. Many optical links (like the one described in the Tables 1–3) are space-to-space links, i.e., they don't enter an atmosphere and hence experience no such additional transmission losses. Models for determining optical losses due to transmission through the Earth's atmosphere exist (see, for example, Ref. 6), but will not be considered here. Losses of 2 to 6 dB (depending on horizon angle) are typical for transmission from space to the Earth's surface.

VI. Receiver Factors

The next factor, receiver telescope gain, g_r, is the gain for an ideal receiving aperture with area equal to the unobscured part of the telescope, i.e., with area $\pi D_r^2(1-\gamma_r^2)/4$. Hence g_r is given by

$$g_r = \left(\frac{\pi D_r}{\lambda}\right)^2 (1-\gamma_r^2) \tag{9}$$

Transmission and reflection losses in the receiver are taken into account by η_r, the receiver optics efficiency. As in the transmitter, the optics throughput is assumed here to be a basic parameter, but in fact may be calculated from the knowledge of the optical layout in the receiver.

Although it can be considered as part of the overall receiver optical system, the narrow band filter transmission, η_λ, is usually treated as a separate factor. The narrow band filter is an important component in a direct-detection optical communications system, and its properties greatly affect the receiver's sensitivity and rejection of background noise. Ideally, the filter should have 100% transmission in the passband and a very narrow (e.g., 1 angstrom or less) spectral bandwidth, $\Delta\lambda$. Practically, the achievable peak transmission is a function of the spectral bandwidth, with narrower filters usually having lower transmission factors. For this reason, performance is often maximized with filters having relatively wide bandwidths, e.g., $\gtrsim$ 10 angstroms.

The final factor in Eq. (2) is η_D, the detector truncation loss factor. Detector truncation loss occurs because not all of the light collected by the receiver aperture can be focused onto a finite-sized detector, especially when there may also be receiver pointing errors. The detector truncation loss is analogous to the transmitter pointing loss, and is given by

$$\eta_D = \int_0^\infty \eta_D(\phi)\,\frac{\phi}{\sigma_r^2}\exp\left\{-\frac{\phi^2+\epsilon_r^2}{2\sigma_r^2}\right\} I_o\left(\frac{\phi\epsilon_r}{\sigma_r^2}\right) d\phi \tag{10}$$

where ϵ_r is the receiver RSS two-axis pointing bias error, σ_r is the RSS two-axis jitter, and $\eta_D(\phi)$ is the instantaneous detector truncation loss as a function of off-axis pointing error, given by

$$\eta_D(\phi) = \frac{2}{1-\gamma_r^2} \times \frac{1}{2\pi}\int_0^{2\pi}\int_0^{\theta}\left[J_1\left(\pi\frac{D_r}{\lambda}\sqrt{\phi^2+\xi^2-2\phi\xi\cos\psi}\right) - \gamma_r J_1\left(\pi\frac{\gamma_r D_r}{\lambda}\sqrt{\phi^2+\xi^2-2\phi\xi\cos\psi}\right)\right]^2 \frac{d\xi}{\xi}\,d\psi \tag{11}$$

where θ is the receiver field-of-view, γ_r is the receiver obscuration ratio, and J_1 is the Bessel function of order one. (Here the integral is over the solid angle viewed by the detector.) Calculation of η_D using Eqs. (10) and (11) is rather difficult, requiring computation of a triple numerical integral. As a result, it is often assumed that $\eta_D = 1$. (Realistically, $\eta_D \cong 1$ only in cases where no attempt is made to optimize background light rejection. In other cases η_D is in the range $0.5 < \eta_D < 1$.)

It is convenient to normalize the received power by the PPM word rate and by the energy per photon to obtain the number of received signal photons per PPM word (i.e., per laser pulse), N_s:

$$N_s = P_r \frac{\log_2 M}{R} \frac{\lambda}{hc}$$

Here $hc = 1.986 \times 10^{-25}$ joule-meters, Planck's constant times the speed of light.

VII. Received Background Power Calculation

In direct-detection systems, it is also necessary to determine the received background power (due to background radiance) in order to determine the required signal power and hence the operating margin. The received background power, P_b, is given by

$$P_b = B \, \ell_a \frac{\pi D_r^2}{4} \left(1 - \gamma_r^2\right) \frac{\pi \theta^2}{4} \Delta\lambda \, \eta_r \, \eta_\lambda \tag{13}$$

where all the factors on the right-hand side have been defined above. Often the background consists of some point sources (e.g., stars) and some extended sources (e.g., planets, off-axis light scattered in the telescope, light scattered by an atmosphere), in which case the average radiance B, is given by

$$B = \frac{4}{\pi\theta^2} \sum_i A_i + \sum_i B_i \tag{14}$$

where A_i are the spectral irradiances (Watts/meter^2/angstrom) of the point sources and B_i are the radiances (Watts/meter^2/steradian/angstrom) of the extended sources. Table 5 gives a DCT-like summary of the background power calculation.

Noise power is most usefully normalized by the PPM slot rate, not the word rate, since in systems with dead time not all the collected background power in a word time is relevant. The number of received background photons per PPM slot time is given (using Eq. (1)) by

$$N_b = P_b \, \tau_s \frac{\lambda}{hc} \tag{15}$$

Systems using crystalline lasers (e.g., Nd:YAG) typically have fixed slot times in the range of 10 to 100 ns. Systems using semiconductor lasers typically have zero dead time, in which case the slot time is given (using Eq. (1)) by

$$\tau_s = \tau_s^{\max} = \frac{\log_2 M}{MR} \tag{16}$$

VIII. Conclusion

In the preceding sections the optical signal and background power at the detector have been calculated from the transmitter laser output power, the background radiance, and a set of parameters that characterize the transmitter and receiver systems. The remaining steps in calculating communications performance involve only characterization of detector sensitivity, i.e., calculating the required signal power for a desired level of performance, and determining the resulting system margin.

In practice, communications link received power calculations must often be done repetitively. Besides the need to analyze many different potential optical link applications, it is usually necessary to design a given link iteratively in order to meet the desired requirements. For this reason, all the calculations described in this report (with the exception of Eqs. (10) and (11)), a set of predefined background sources (e.g., planets, stars, etc.), and required signal calculations for an "ideal" PPM receiver have all been incorporated into a computer program suitable for use on an IBM personal computer. The program generates an output equivalent to that in Tables 3 and 5, and can also automatically adjust any system parameter to fit a given margin. The program is an extremely useful tool for the design and analysis of optical communications links.

References

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Table 1. Sample communications link component values

	Component	Value
1.	Laser Power (Average)	0.200 Watts
2.	Laser Wavelength	0.532 micrometers
3.	Transmit Telescope Diameter	0.1 meters
4.	Transmitter Obscuration Ratio	0.2
5.	Transmitter Optics Efficiency	0.45
6a.	Transmitter Pointing Bias Error	0.4 microradians
6b.	Transmitter Pointing Jitter	0.8 microradians
7.	Receive Telescope Diameter	1.0 meters
8.	Receiver Obscuration Ratio	0.35
9.	Receiver Optics Efficiency	0.7
10.	Narrowband Filter Transmission	0.7
11a.	Receiver Pointing Bias Error	0 microradians
11b.	Receiver Pointing Jitter	0 microradians
12.	Spectral Filter Bandwidth	10 Angstroms
13.	Detector Diametrical Field of View	5 microradians

Table 2. Sample communications link parameters

	Parameter	Value
14.	Data Rate	30 kbps
15.	PPM Alphabet Size	256
16.	PPM Slot Time	10 nanoseconds
17.	Link Range	2.3×10^{11} meters
18.	Atmospheric Loss Factor	1.0
19.	Background Radiance	0.2 $W/m^2/sr/A$

Table 3. Optical communications received signal power calculation summary

Received Signal Power	Factor	dB
Laser Power (Average)	0.200 W	23.0 dBm
Transmitter Telescope Gain 0.1 meter primary 0.02 meter obscuration $\lambda/D_t = 5.3$ μrad.	2.47×10^{11}	113.9
Transmitter Optics Efficiency	0.45	−3.5
Transmitter Pointing Efficiency 0.4 μrad bias error 0.8 μrad jitter	0.9	−0.5
Space Loss	3.42×10^{-38}	−374.7
Atmospheric Loss	1.0	0.0
Receiver Telescope Gain 10 meter primary 3.5 meter obscuration	3.06×10^{13}	134.9
Receiver Optics Efficiency	0.7	−1.5
Spectral Filter Transmission 10 A bandwidth	0.7	−1.5
Detector Truncation Loss	1.0	0.0
Received Signal Power	1.03×10^{-14} W	−109.9 dBm
Symbol Time	2.67×10^{-4} s	−35.7 dB/Hz
Photons/Joule	2.68×10^{19}/J	154.3 dB/mJ
Received Signal Photons/Pulse	7.34	8.7 dB

Table 4. Values of series coefficients for pointing loss calculation in Eq. (7)

Transmitter Obscuration Ratio, γ_t	f_0	f_2	f_4	f_6
0.0	0.569797	−0.113421	0.0503535	−0.0292921
0.1	0.566373	−0.115327	0.0513655	−0.0299359
0.2	0.555645	−0.120457	0.0542465	−0.0317773
0.3	0.535571	−0.126992	0.0584271	−0.0344978
0.4	0.501381	−0.131777	0.0626752	−0.0374276

Table 5. Received background power calculation summary

Received Background Power	Factor
Background Radiance	0.02 $W/m^2/sr/Å$
Atmospheric Loss Factor	1.0
Receiver Area 1.0 meter primary 0.35 meter obscuration	0.689 m^2
Receiver Solid-Angle Field of View 5.0 μrad diameter FOV	1.96×10^{-11} sr
Spectral Filter Bandwidth	10 Å
Receiver Optics Efficiency	0.7
Spectral Filter Transmission	0.7
Received Background Power	1.32×10^{-12} W
Slot Time	1×10^{-8} s
Photons/Joule	2.68×10^{19}/J
Received Background Photons/Slot	0.354

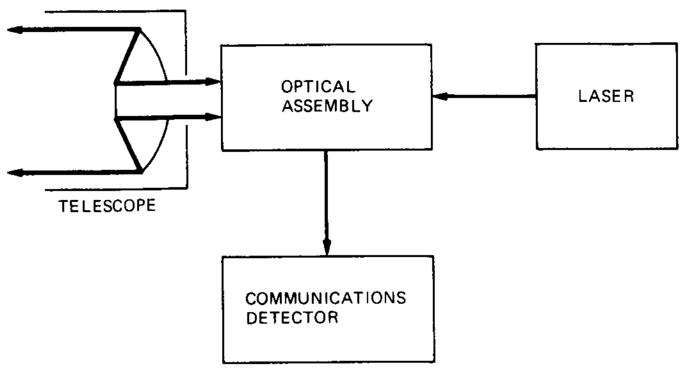

Fig. 1. Block diagram of an optical communications transceiver

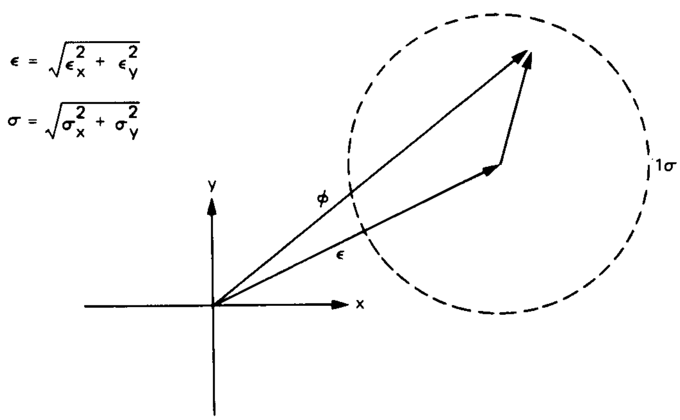

Fig. 2. Pointing error geometry