NASA
Technical Memorandum 88862

USAAVSCOM
Technical Report 86-C-27

New Generation Methods for Spur, Helical, and Spiral-Bevel Gears

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Prepared for the
Rotary Wing Propulsion System Specialist Meeting
sponsored by the American Helicopter Society
Williamsburg, Virginia, November 12–14, 1986

NASA

E-3216

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SUMMARY

New methods for generating spur, helical, and spiral-bevel gears are proposed. These methods provide the gears with conjugate tooth surfaces, localized bearing contact, and reduced sensitivity to gear misalignment. Computer programs have been developed for simulating gear meshing and bearing contact.

INTRODUCTION

Research was directed at finding new methods for generating spur, helical, and spiral-bevel gears that would provide conjugate gear tooth surfaces and localized bearing contact and would reduce kinematic errors (gear noise) caused by gear misalignment. Tooth contact analysis (TCA) computer programs have been developed for these gears. These programs make it possible to simulate gear meshing and bearing contact and to investigate the influence of gear misalignment on kinematic errors. The proposed generation methods can be applied to the spiral-bevel gears by using current machinery and tools, but new tool shapes are needed for the spur and helical gears.

SPUR GEARS WITH CROWNED PINION TOOTH SURFACES[1]

As is well known, spur gears are very sensitive to axes misalignment, and the pinion gear tooth surface must be crowned to compensate for this disadvantage. Figure 1(a) shows a regular involute surface for a spur pinion, and

[1]The research has been performed with the participation of J. Zhang, Department of Mechanical Engineering, University of Illinois at Chicago, Chicago, IL.

figure 1(b) shows a crowned pinion surface. The problem is developing the topology of the pinion tooth surface. It is the deviation from a regular involute surface that will provide a low level of kinematic errors (i.e., a low level of noise and good bearing contact).

The Maag Co. has proposed a method for grinding deviated pinion tooth surfaces that is based on a point contact between the grinding wheel and the pinion tooth surface being manufactured. However, their method does not solve the main problem, finding the optimal topology of the pinion tooth surface that will provide a low level of kinematic errors.

We propose a method for generating crowned pinion surfaces that can provide this optimal topology. This method can be used for either grinding or cutting. The basic principle is based on two rigidly connected generating surfaces. One surface, a plane (the surface of a regular involute rack cutter), generates the gear tooth surface (fig. 2). The other surface, a cone, generates the pinion tooth surface. Both generating surfaces are tangent along a straight line - the generatrix of the cone. The generating surfaces perform a translational motion like two rack cutters while the pinion and gear being generated rotate about their axes. Figure 3 shows the generation of the pinion tooth surface by a grinding wheel. A similar tool, a "shaver" with slots, can be used for cutting and lapping.

Figure 4 shows how the pinion tooth surface deviates from a regular involute surface. The middle section of the pinion tooth surface obtained by a cutting plane that is perpendicular to the pinion axis is a regular involute curve. The involute gear tooth surface and the pinion tooth surface are conjugate, and their bearing contact is localized. Considering the criterion of bearing contact, it can be stated that the proposed gears are adjusted to the misalignment. The remaining concern is to what level will the kinematic errors be due to gear misalignment. We developed a computer program to perform the tooth contact analysis necessary to determine this.

The entire adjustment of the proposed gears to the misalignment must be based on the evaluation of two criteria - the bearing contact and the induced level of kinematic errors. We have found that a very low level of kinematic errors can be achieved, as indicated by the results of the TCA program, if a surface of revolution that is slightly deviated from the cone surface is used as the generating surface. The optimal radii of curvature for the surface of revolution were determined by the method proposed by Litvin (refs. 1 and 2). Figure 5 shows the kinematic error caused by a 5 percent change in the center distance and a 5-arc-min misalignment of the gear axes for 20 and 40 gear teeth. The function is similar to a parabolic function, and the maximum error is approximately 0.2 arc-sec. Figure 6 shows the bearing contact of the proposed gears on the pinion tooth surface. Another application example with results is given in table I.

HELICAL GEARS WITH PINION CIRCULAR-ARC TEETH AND GEAR SCREW INVOLUTE TEETH

As is well known, regular involute helical gears are sensitive to axes misalignment, and Novikov-Wildhaber (conformal) gears are sensitive to changes in center distance (ref. 3). The proposed helical gears may be considered as a bridge between regular involute helical gears and Novikov-Wildhaber gears. These gears are less sensitive to misalignment since the bearing contact is localized and are less sensitive to the change in center distance since the gear is a regular screw involute. Thus the disadvantages of current helical or conformal gears operating under less than ideal conditions can be overcome.

Figure 7 illustrates the proposed method for generating helical gears. One of the generating surfaces, Σ_P, is a plane and represents the surface of a regular rack cutter. Plane Σ_P will generate a regular screw involute surface. The other generating gear tooth surface, Σ_F, is a cylindrical surface and will generate the pinion tooth surface. The normal section of Σ_F is a circular arc, and the normal section of Σ_P is a straight line. The deviation of Σ_F with respect to Σ_P depends on the radius of the circular arc that is chosen for Σ_F. By controlling this radius we can control the deviation of the pinion tooth surface Σ_1 from a regular screw involute surface. The generating surfaces Σ_F and Σ_P are tangent along the straight line a-a (fig. 7). The line of action of the generated gear tooth surfaces Σ_1 and Σ_2 is represented in the fixed coordinate system S_f by a straight line. Coordinate system S_f is rigidly connected to the gear housing. The line of action is parallel to the gear axes. In the meshing process the point of contact moves along the line of action. The instantaneous point of the contact is the center of the contact ellipse. The dimensions of the contact ellipse depend on the radius of the circular arc of the generating surface Σ_F and on the number of pinion and gear teeth.

We developed a TCA program for the proposed gears. Figures 8 and 9 show the transverse shapes of the pinion and gear. Changing the center distance by 5 percent did not substantially change the location of the bearing contact (fig. 10).

SPIRAL-BEVEL GEARS WITH CONJUGATE GEAR TOOTH SURFACES

For many years the Gleason Works (ref. 4) has provided machinery for the manufacture of spiral-bevel gears. There are several important advantages to the Gleason methods of manufacture. The machines are rigid and produce gears of high quality and consistency. The cutting methods may be used for both milling and grinding. Grinding is especially important for producing hardened high-quality aircraft gears. Both milling and grinding are possible with Gleason's method since the velocity of the cutting wheel does not have to be related in any way to the machine's generating motions.

Gleason's method for generating spiral-bevel gears provides only approximate conjugation of gear tooth surfaces. Because the gear ratio is not constant during the tooth engagement cycle, kinematic errors occur as rotation is transformed from the driving gear to the driven gear. The kinematic errors in spiral-bevel gears are a major source of noise and vibration in transmissions. Special machine-tool settings must be used to reduce the level of kinematic errors in the generated gears. Gleason developed a tooth contact analysis (TCA) program to numerically determine the required machine-tool settings.

We propose a new method for generating spiral-bevel gears that provides exact conjugation of the gear tooth surfaces and no kinematic errors. This method can be applied in practice by using the existing Gleason equipment and tools. The proposed approach results in the direct determination of machine-tool settings. The TCA program developed also simulates the bearing contact and the influence of assembly and manufacturing errors.

The theory of spiral-bevel and hypoid gears was a subject of intensive research by many authors (refs. 2 and 5 to 12). Litvin and his colleagues have addressed the analysis and synthesis of spiral-bevel gears (refs. 1, 2, 5, and 8). Computer-aided simulations of tooth meshing and bearing contact have been worked out by Litvin and Gutman (ref. 6) and by the Gleason Works (refs. 11 to 13).

The existing method for generating Gleason spiral-bevel gears is based on application of the head cutter shown in figure 11. The blades of the head cutter are straight lines that generate two cones while the head cutter rotates about axis C-C. The angular velocity about axis C-C depends not on the generating motion but only on the desired cutting velocity. The flanks of adjacent gear teeth are cut simultaneously (duplex method). Two head cutters are used for pinion generation; they are provided with one-sided blades and cut the pinion tooth sides separately.

In generating the spiral-bevel gear (fig. 12) the head cutter is mounted to the cradle of the cutting machine. The cradle with the tool cone surface represents the generating gear, which is in mesh with the gear that is being cut. The axes of rotation of the cradle (generating gear) $X_m^{(2)}$ and the gear Z_2 intersect. Coordinate system $S_m^{(2)}$ is rigidly connected to the frame of the gear cutting machine, and coordinate system $S_c^{(P)}$ is rigidly connected to the cradle; φ_p is the angle of cradle rotation. Coordinate system S_f is a fixed coordinate system rigidly connected to the gear housing.

In generating the spiral-bevel pinion (fig. 13), the axes of rotation of the pinion cradle $X_m^{(1)}$ and the pinion Z_1 do not intersect but cross. Figure 13 shows the specific corrections of machine-tool settings, designated by ΔE_1 and ΔL_1. The determination of ΔE_1 and ΔL_1 is the subject of Gleason's TCA program and is directed at reducing the kinematic errors of the generated gears.

Our proposed generation method is based on the manufacturing process described here. The main principles of the new method are as follows:

(1) The two generating surfaces, Σ_P and Σ_F, generate the gear tooth surface Σ_2 and the pinion tooth surface Σ_1, respectively. Surfaces Σ_P and Σ_2 (and accordingly, Σ_F and Σ_1) are in mesh in the process of cutting and contact each other at a <u>line</u> at every instant. The generated gear tooth surfaces, Σ_2 and Σ_1, contact each other at a <u>point</u> at every instant (fig. 14). The four surfaces - the two generating surfaces (Σ_P and Σ_F) and the pinion and gear tooth surfaces (Σ_1 and Σ_2) - are tangent at every instant. The ratio of the angular velocities of motion of surfaces Σ_P, Σ_F, Σ_1, and Σ_2 must satisfy the following requirements: (a) the surfaces must be in continuous tangency and (b) the generated pinion and gear must transform rotation with no kinematic errors.

(2) The point of contact of the above-mentioned four surfaces moves in a plane π that is rigidly connected to the gear and cutting machine housing. The normal to the contacting surfaces lies in plane π and performs a parallel motion.

It is necessary to emphasize some specific features of the contact of generating surfaces Σ_P and Σ_F. It is a contact of two cones with crossed axes. Thus the cones have a common normal at the point of contact, but their surfaces interfere with each other near the contact point.

Figure 15 illustrates the location and orientation of the plane of normals - plane π. Surfaces Σ_P and Σ_2 are in contact at point M, and their common normal intersects the pitch line, the tangency line of pitch cones of spiral-bevel gears. The normal also intersects the head-cutter axis (at point A in fig. 15) since the tool surface is a cone. The four surfaces (Σ_P, Σ_F, Σ_1, and Σ_2) must have a common normal at every instant (fig. 14); and this normal will in the process of meshing perform a parallel motion in plane π, keeping its original orientation. The plane of normals π is a plane that passes through the pitch line and $\overline{MA}$ (fig. 15), where $\overline{MA}$ is the instantaneous normal to surfaces Σ_P and Σ_2.

On the plane of normals π (fig. 16) point A is the point where the gear head-cutter axis intersects plane π, and $\overline{MA}$ is the initial position of the normal. Point D is the point where the gear cradle axis intersects plane π. Simultaneously point D is the pitch point (the point where the pinion and gear axes intersect). The axes of the gear cradle and gear head cutter are parallel, but they are not perpendicular to plane π. Recall that the cradle axis $X_m^{(1)}$ is perpendicular to the root cone (fig. 12) and that plane π is drawn through the pitch line.

To provide conjugate gear tooth surfaces, specific machine-tool settings ΔE_1 and ΔL_1 (fig. 13) are used, and the appex of the pinion pitch cone is offset with respect to the pinion cradle axis $X_m^{(1)}$. Thus point O does not coincide with point D (fig. 16). Here point O is the point where the pinion cradle axis intersects plane π (the plane of normals). For reasons given below, the machine-tool settings ΔE_1 and ΔL_1 must provide the colinearity of vectors $\overline{DO}$ and $\overline{MA}$.

At point C (fig. 16) the pinion tool cone axis intersects plane π. The machine-tool settings for the pinion tool cone must provide that points C, A, and M lie on the same straight line and that point M is the tangency point of three surfaces: Σ_P, Σ_F, and Σ_2. We emphasize again that the pinion cradle axis $X_m^{(1)}$ and the gear cradle axis $X_m^{(2)}$ are not parallel since they have to be perpendicular to the pinion and gear root cones, respectively. The orientation of axes $X_m^{(2)}$ and $X_m^{(1)}$ depends on the gear and pinion dedendum angles Δ_2 and Δ_1 (figs. 12 and 13). However, the pinion cradle and pinion head-cutter axes are parallel (if the pinion head cutter is not tilted), but they are not perpendicular to plane π.

During surface generation the head cutter axis $O_s^{(j)}$ rotates about the cradle axis $O_m^{(i)}$; φ_j (fig. 17) is the angle of cradle rotation ($i = 1,2$; $j = F,P$). Thus the cutter axis traces out a cylinder of radius b_j, where b_j is the distance between the head-cutter and cradle axes. The plane of normals π is inclined with respect to the cylinder axis, and the intersection of the cylinder by plane π is an ellipse. Thus points A and C (fig. 16) each trace out an ellipse. The common surface normal $\overline{AC}$ slides along those ellipses while performing the required parallel motion. This parallel motion becomes possible if the dimensions and orientation of both ellipses are related in a specific way. Figure 18 shows two such related ellipses.

We have developed a computer program for determining machine-tool settings for the proposed generation method and for the TCA of the generated gears. The results of this TCA program indicate that kinematic errors would be reduced to zero. Figure 19 shows the bearing contact on the gear tooth convex side for spiral-bevel gears with the proposed gearing. An example with results is given in table II.

CONCLUDING REMARKS

The basic principles of new gear generation methods have been discussed. These methods provide conjugate gear tooth surfaces for three types of gears: (1) spur gears with crowned pinion tooth surfaces that have misaligned axes, (2) helical gears with pinion circular-arc teeth and gear regular screw involute teeth that have a change in center distance, and (3) spiral-bevel gears whose common surface normal performs a parallel motion in the process of meshing.

Litvin
E-3216 J.O. YOM3984
Paper-AHS

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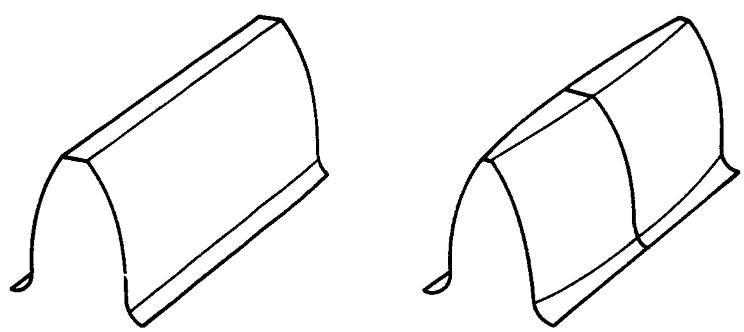
TABLE I. - NUMERICAL EXAMPLE AND RESULTS FOR CROWNED SPUR PINION AND REGULAR INVOLUTE GEAR

Number of teeth on pinion, gear	20, 40
Diametral pitch	10
Face width, in.	0.4
Generating cone angle, deg	80
Cone radius, in.	0.985
Curvature radius of generating surface of revolution, in.	500
Misalignment of gear axes, arc-min	10
Change in center distance, percent	1
Pinion surface deviation in transverse section at midpoint across face width, in.	0.0036-0.0052
Kinematic error, arc-sec	0.35-0.4

TABLE II. - NUMERICAL EXAMPLE AND RESULTS FOR SPIRAL-BEVEL GEAR

Number of teeth on pinion, gear	10, 41
Gear pressure angle, deg	20
Diametral pitch	5.55
Mean pitch cone distance, in.	3.226
Cutter diameter, in.	6.0
Corrections of machine-tool settings, convex side of pinion, in.:	
ΔE_1	-0.0499
ΔL_1	-0.0368
Maximum kinematic error,[a] arc-sec	0.2

[a]These results for kinematic error are based on available cutter angles. To reduce the kinematic error to zero would require non-standard blade cutter angles.



(a) Regular involute. (b) Crowned.

Figure 1. - Spur pinion surfaces.

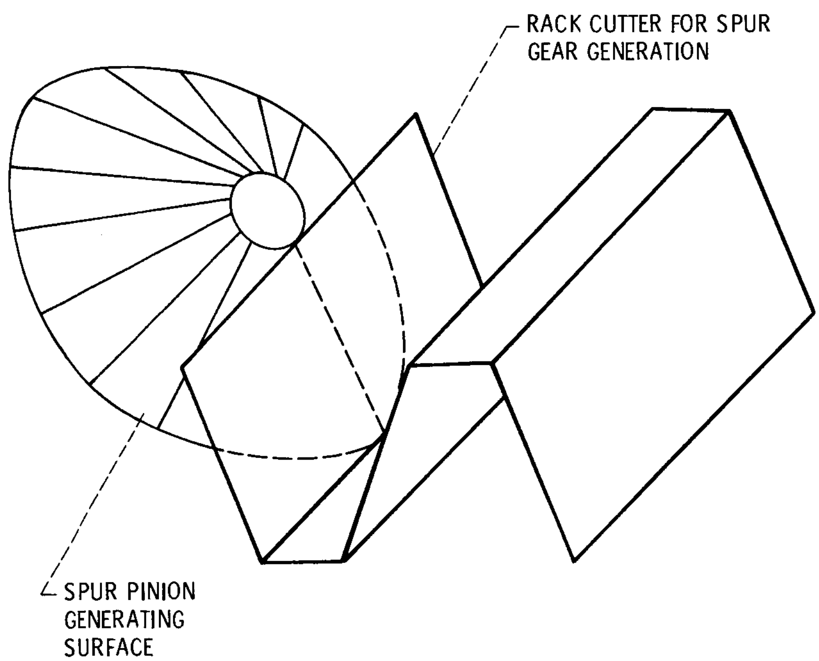


Figure 2. - Orientation of spur pinion (crowned) and gear-generating surfaces.

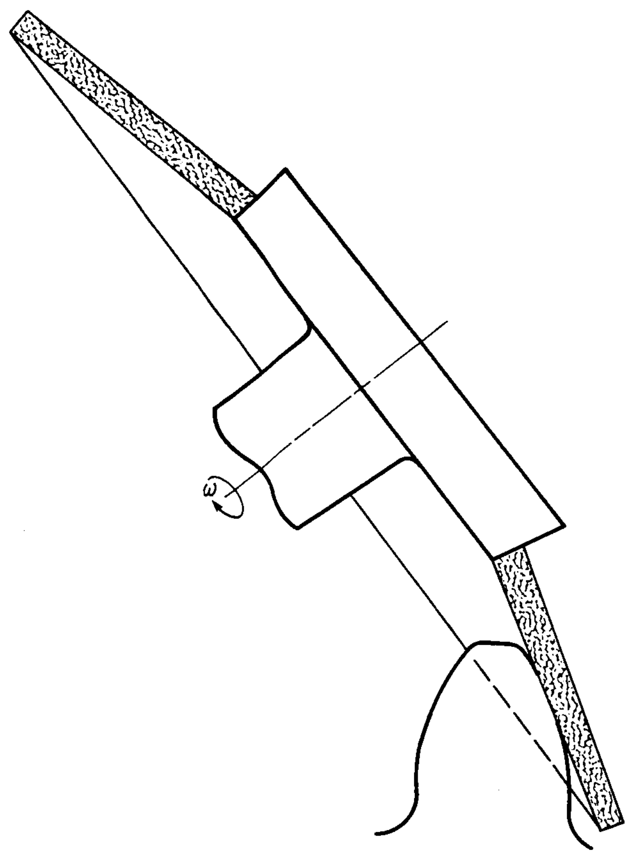


Figure 3. - Grinding wheel generation of crowned spur pinion.

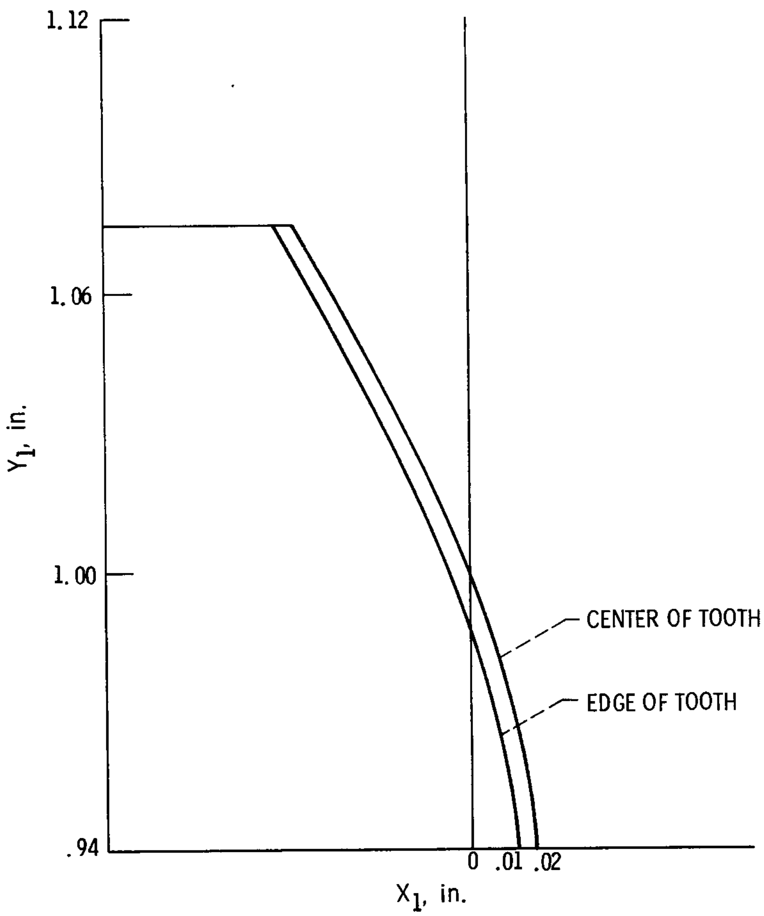


Figure 4. - Deviation of crowned spur pinion surface.

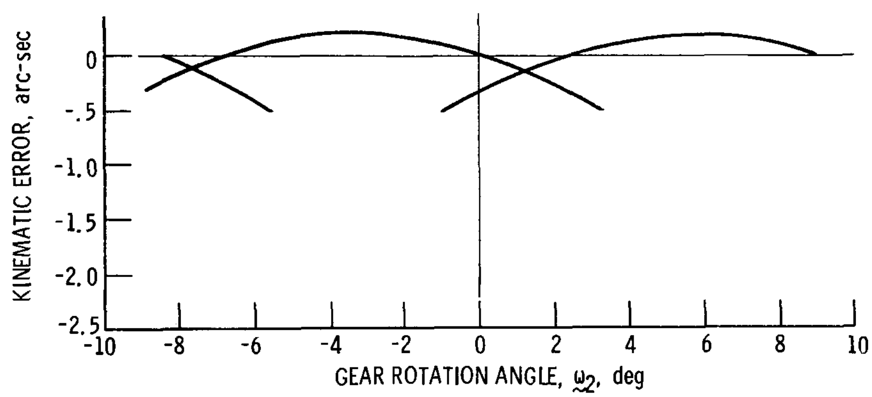


Figure 5. - Kinematic error as a function of gear rotation angle. Change of center distance, 5 percent; axes misalignment, 5 arc-min.

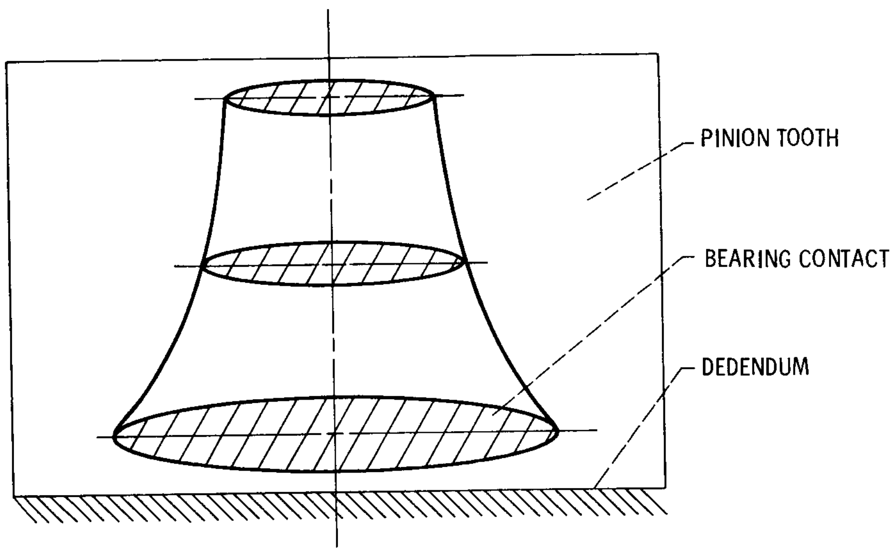


Figure 6. - Location and size of bearing contact as pinion moves through mesh.

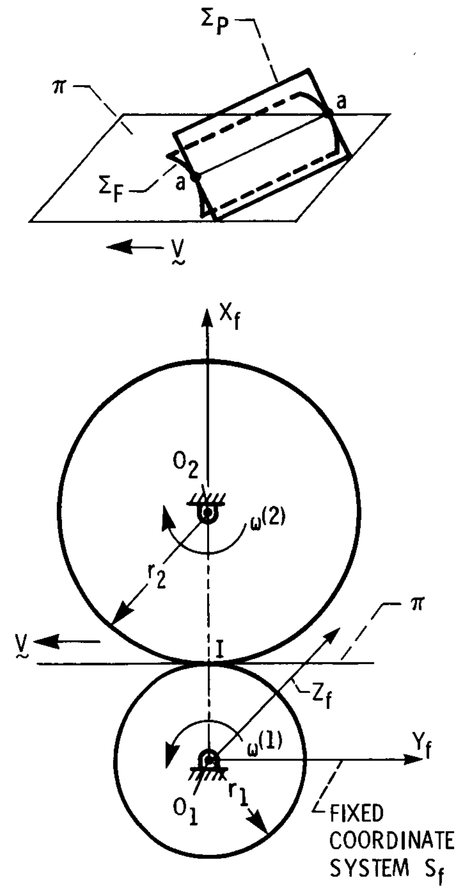


Figure 7. - Generating surfaces for circular-arc helical pinion and regular screw involute.

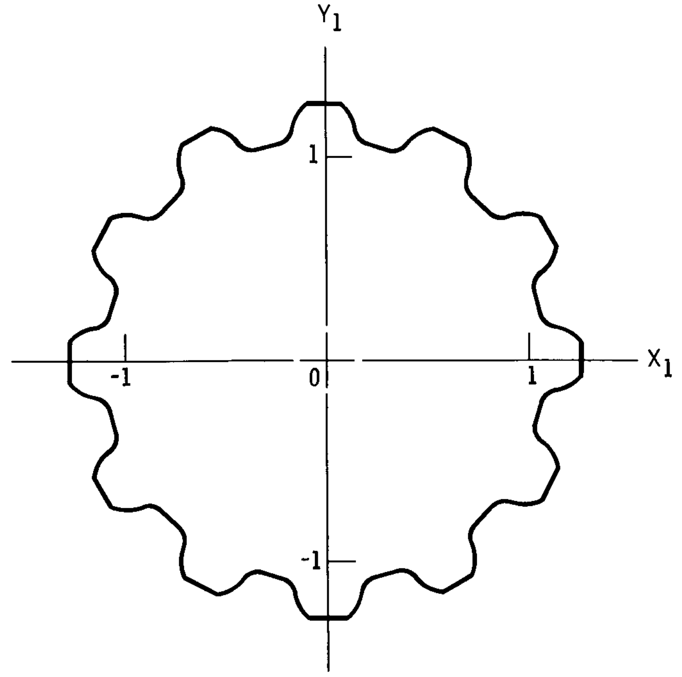


Figure 8. - Cross section of circular-arc helical pinion. (Axes scales are in inches.)

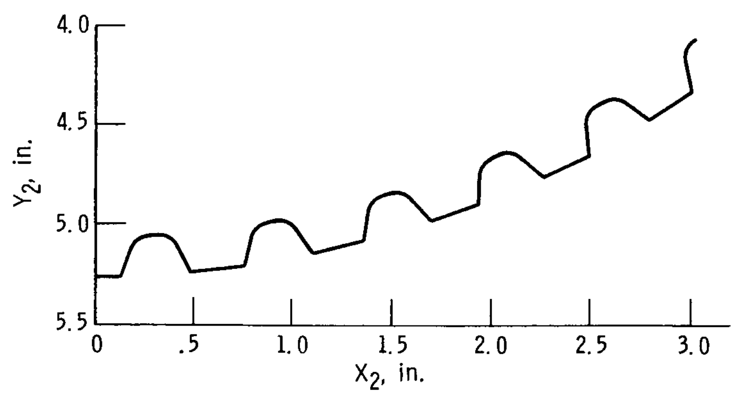


Figure 9. - Cross section of regular helical involute gear.

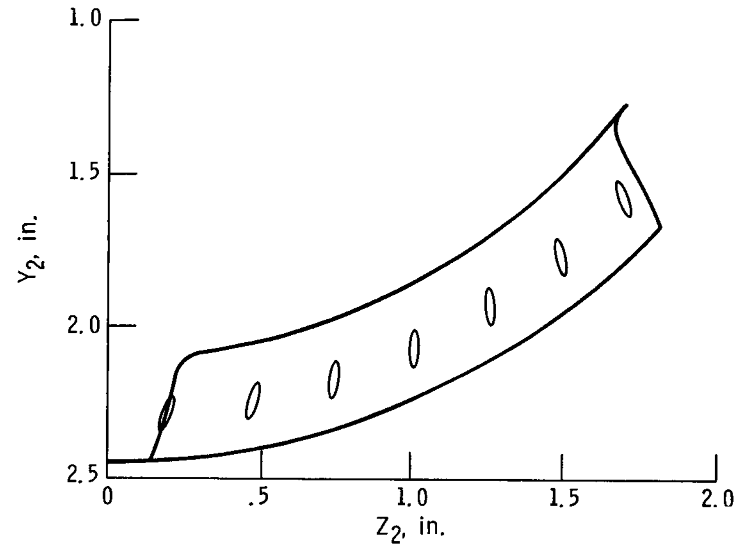


Figure 10. - Bearing contact after change in center distance of 0.04 inch. Number of teeth, 24; diametral pitch, 6.

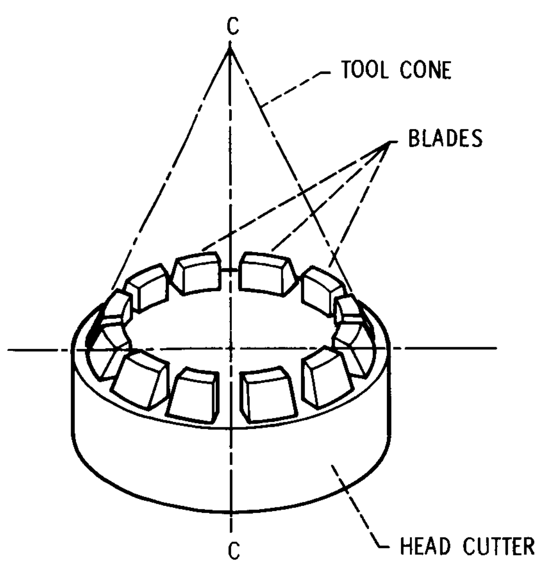


Figure 11. - Spiral-bevel gear cutter.

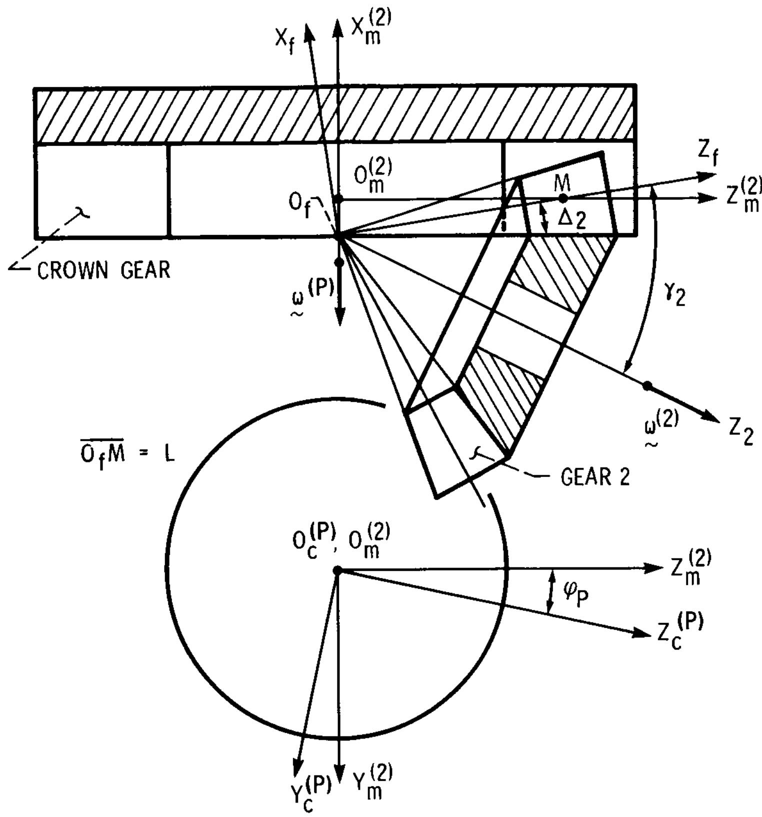


Figure 12. - Coordinate system of generating machinery and spiral-bevel gear (gear 2).

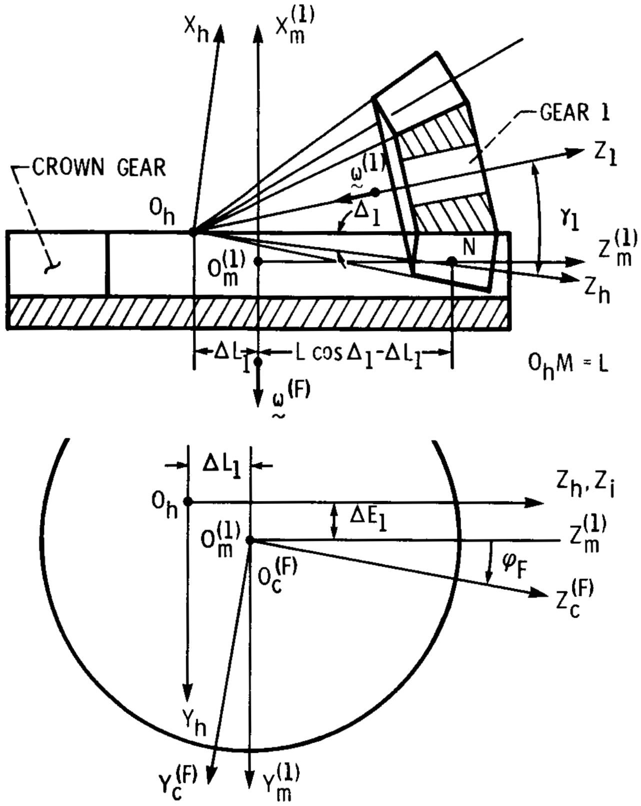


Figure 13. - Coordinate system of generating machinery and spiral-bevel pinion (gear 1).

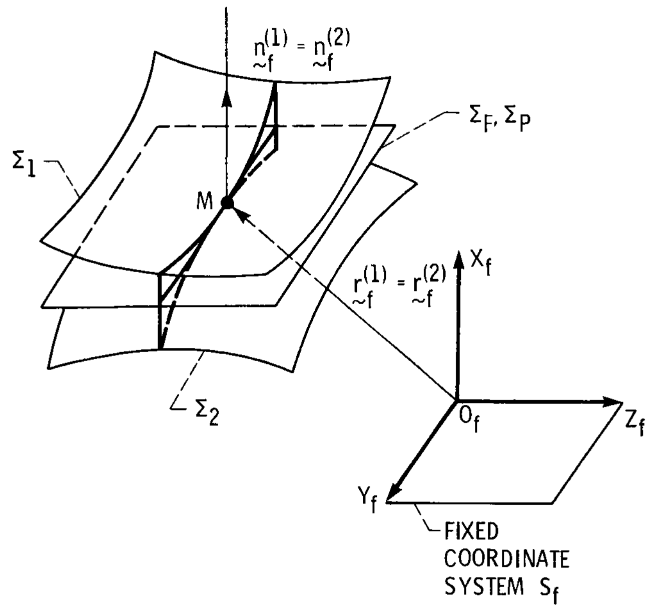


Figure 14. - Gear tooth surfaces in contact.

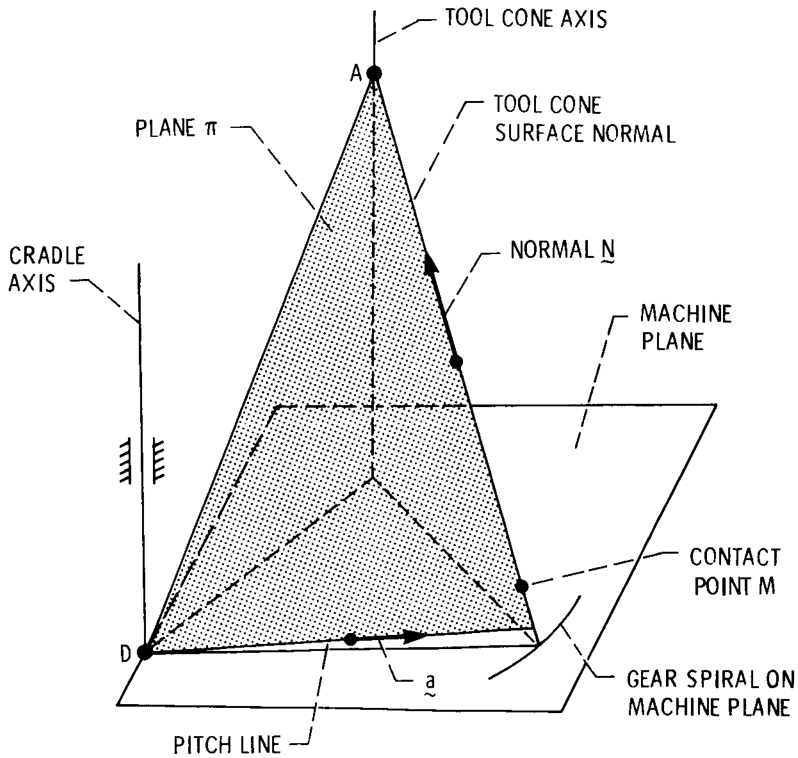


Figure 15. - Orientation of plane of normals, plane π.

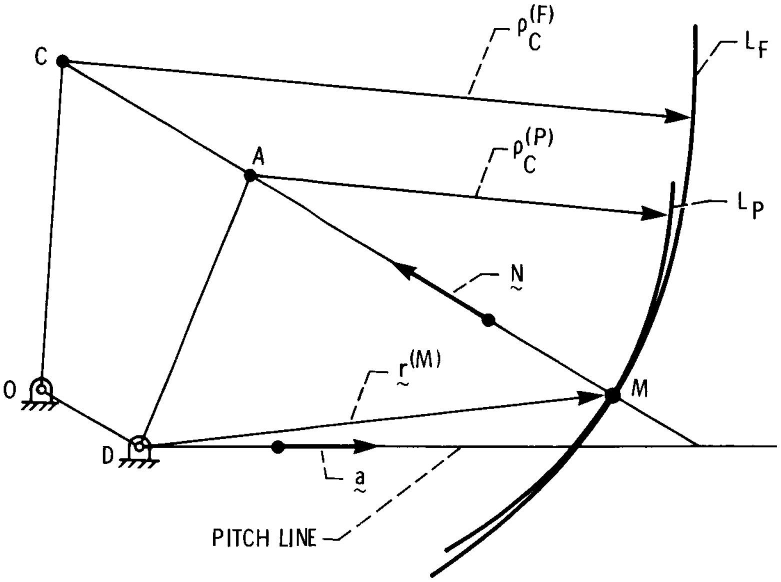


Figure 16. - Plane of normals π.

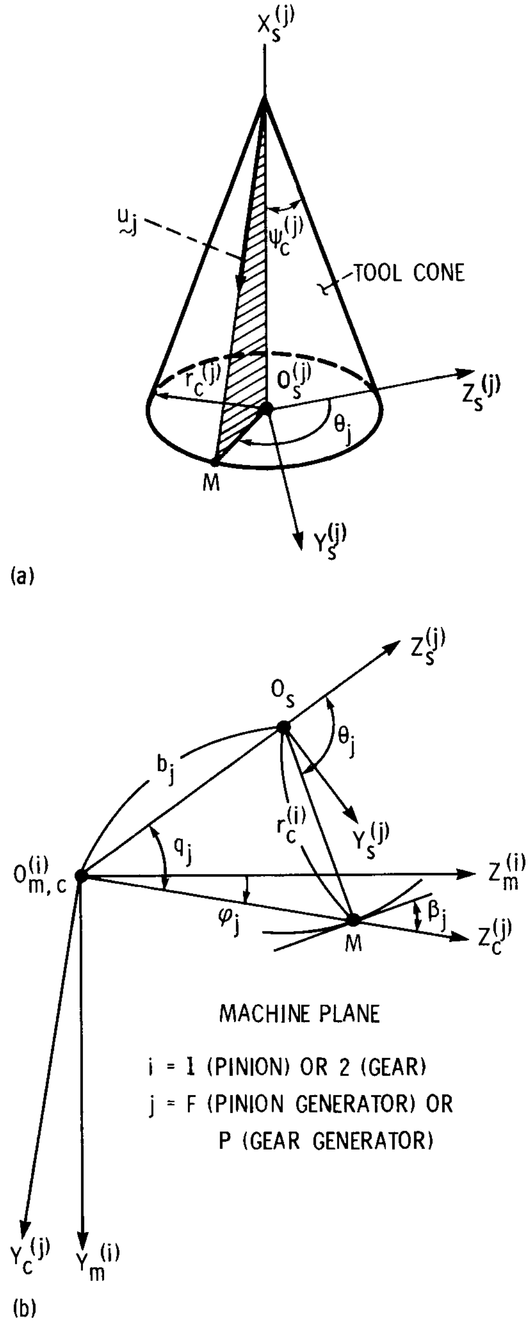


Figure 17. - Coordinate system orientation of cutter for spiral-bevel gears with respect to gear-generating machine.

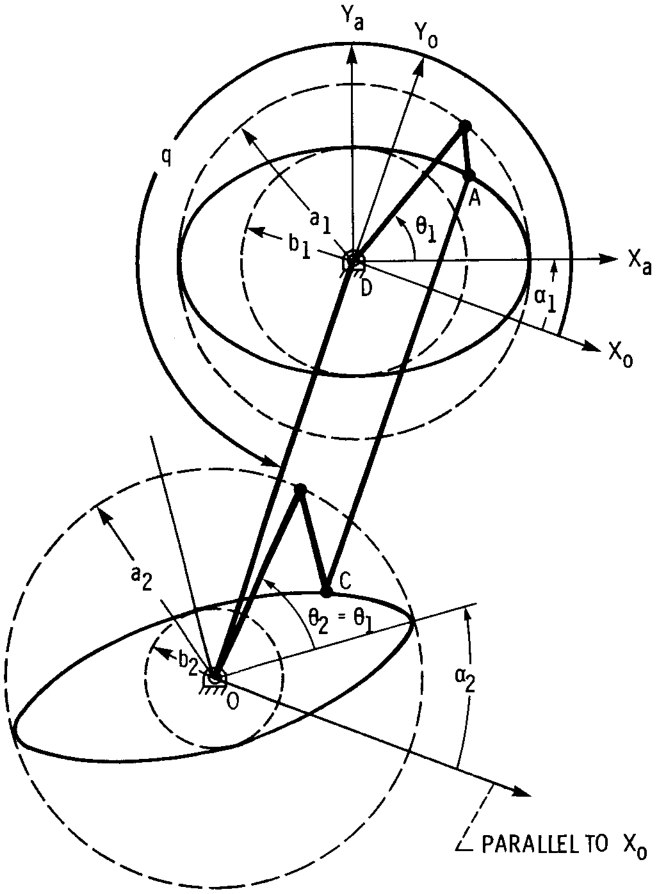


Figure 18. - Sketch showing parallel motion along related ellipses in plane π.

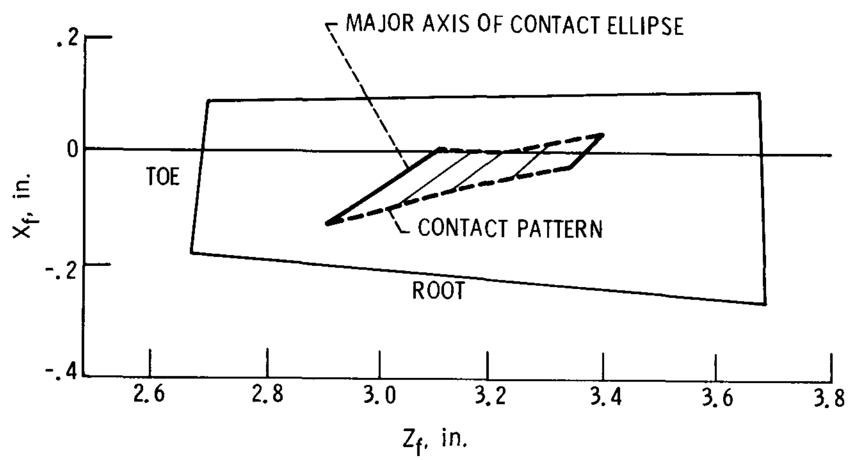


Figure 19. - Location and size of bearing contact on spiral-bevel gear tooth surface, convex side.

1. Report No. NASA TM-88862 USAAVSCOM-TR-86-C-27	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle New Generation Methods for Spur, Helical, and Spiral-Bevel Gears		5. Report Date
		6. Performing Organization Code 505-62-51
7. Author(s) F.L. Litvin, W.-J. Tsung, J.J. Coy, R.F. Handschuh, C.-B.P. Tsay		8. Performing Organization Report No. E-3216
		10. Work Unit No.
9. Performing Organization Name and Address NASA Lewis Research Center and Propulsion Directorate, U.S. Army Aviation Research and Technology Activity - AVSCOM, Cleveland, Ohio 44135		11. Contract or Grant No.
		13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546 and U.S. Army Aviation Systems Command, St. Louis, Mo. 63120		14. Sponsoring Agency Code

15. Supplementary Notes

Prepared for the Rotary Wing Propulsion System Specialist Meeting, sponsored by the American Helicopter Society, Williamsburg, Virginia, November 12-14, 1986. F.L. Litvin and W.-J. Tsung, University of Illinois at Chicago, Chicago, Illinois; J.J. Coy and R.F. Handschuh, Propulsion Directorate, U.S. Army Aviation Research and Technology Activity - AVSCOM, Lewis Research Center; C.-B.P. Tsay, National Chiao Tung University, Taiwan, Republic of China.

16. Abstract

New methods for generating spur, helical, and spiral-bevel gears are proposed. These methods provide the gears with conjugate gear tooth surfaces, localized bearing contact, and reduced sensitivity to gear misalignment. Computer programs have been developed for simulating gear meshing and bearing contact.

17. Key Words (Suggested by Author(s)) Machine design Gears Transmission Helicopter gears	18. Distribution Statement Unclassified - unlimited STAR Category 37

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of pages	22. Price*

*For sale by the National Technical Information Service, Springfield, Virginia 22161