NASA HQ / GRANT / IN-46

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Anual Status Report

for

NASA Grant NAGW-863

NONLINEAR WAVE VACILLATION IN THE ATMOSPHERE

For the period

January 1, 1986 to December 31, 1986

Submitted to:

Atmospheric Dynamics and Radiation Branch

NASA Headquarters

Washington, DC

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(NASA-CR-179985) NONLINEAR WAVE VACILLATION IN THE ATMOSPHERE Annual Status Report, 1 Jan. - 31 Dec. 1986 (Tennessee Univ. Space Inst., Tullahoma.) 11 p CSCL 04A

N87-15659

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1. Theoretical and Numerical Analyses

The problem of vacillation in a baroclinically unstable flow field is being studied as a task for this grant through the time evolution of a single nonlinearly unstable wave. To this end a computer code is being developed to solve numerically for the time evolution of the amplitude of such a wave. The final working code will be the end product resulting from the development of a hierarchy of codes with increasing complexity. The first code in this series has already been completed and is presently undergoing several diagnostic analyses to verify its validity. The development of this first code is detailed below.

The first model to be studied here is what is commonly known as the "Eady problem". This problem is concerned with the baroclinic instablity of an infinite fluid on an f-plane and without potential vorticity. The basic state flow for this model is for constant shear. By assuming the potential vorticity to be initially zero it must then remain so for all time $t > 0$. With these assumptions the equation governing the pressure perturbation, φ, for the flow in the interior of the layer takes the following form:

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{1}{S}\frac{\partial^2 \varphi}{\partial z^2} = 0. \qquad (1)$$

It was decided to begin the hierarchy of codes by first tackling the Eady problem propper. i.e. the baroclinic instability problem without Ekman dissipation. This is done with the knowledge that such a model might not develop the vacillation phenomenon being investigated. However, by supressing the Ekman damping graeter simplicity in coding is achieved without altering any of the fundamentals of the numerical techniques employed in the computer code. Thus, with these considerations in mind Equation (1) was then solved subject to the following boundary conditions:

$$\frac{\partial\varphi}{\partial x} = 0 \quad @ \quad y = 0, 1 \tag{2}$$

and

$$\left(\frac{\partial}{\partial t} + z\frac{\partial}{\partial x}\right)\frac{\partial\varphi}{\partial z} - \frac{\partial\varphi}{\partial x} + \left\{\frac{\partial\varphi}{\partial x}\frac{\partial^2\varphi}{\partial y\partial z} - \frac{\partial\varphi}{\partial y}\frac{\partial^2\varphi}{\partial x\partial z}\right\} = 0. \quad @ \quad z = 0, 1 \tag{3}$$

Note, that in this case the governing equation, (1), is linear while the upper and lower boundary conditions, (3), are nonlinear. This fact then allows for the pressure perturbation function to be expressed in the follwing way:

$$\varphi(x, y) = \sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty} e^{i(kx+ly)}\{A_{k,l}(t)\ cosh\ \mu_{k,l}z + B_{k,l}(t)\ sinh\ \mu_{k,l}z\}, \tag{4}$$

where $\mu = [(k^2 + l^2\pi^2)S]^{1/2}$, and the coefficients $(A_{k,l}, B_{k,l})$ are equal to $(A^*_{-k,-l}, B^*_{-k,-l})$to insure the pressure perturbation, φ is real. Note that the vertical structure of each component of expression (4) depends on the horizontal wavenumber, $(k^2 + l^2\pi^2)^{1/2}$, and on the stratification parameter, S. The amplitude coefficients, $A_{k,l}$ and $B_{k,l}$, are functions of time and their evolution from some prescribed initial conditions is the subject of this study.

Upon substituting the solution form (4) into the boundary conditions (3) and performing the necessary algebraic manipulations, an infinite set of nonlinear, ordinary differential equations in time for the amplitude functions results. Note, that the solution form (4) satisfies the governing equation exactly. These equations take the following form:

$$\frac{dA_{k,l}}{dt} = F_{k,l} \quad , \tag{5a}$$

$$\frac{dB_{k,l}}{dt} = G_{k,l} \quad , \tag{5b}$$

where $F_{k,l}$ and $G_{k,l}$ are complicated nonlinear functions of all the amplitude coefficients $A_{k,l}$ and $B_{k,l}$. Clearly the resulting infinite systemof ordinary differential equations must be truncated to some finite value of $K, (-K \leq k \leq K)$ and $L, (-L \leq l \leq L)$ in order to obtain a numerical solution to the original system. The code developed for this period allows for the solution for any arbitrary value of both K and L and which need not be the same.

The solution representation of the pressure perturbation function, (4), lends itself to the spectral form of numerical solutions to partial differential equations. Consequently this will be the numerical method employed in the development of the code. Furthermore, the pseudo-spectral method is used in the code developed for this problem. This means that the products and sums needed for generating the functions $F_{k,l}$ and $G_{k,l}$ were performed in the physical space, (x, y), while the result is obtained in the wavenumber space, (k, l). A computer code for solving the coupled truncated set of ordinary differential equations was written. The integration is started at $t = 0$ with some known intial values for all $A_{k,l}$ and $B_{k,l}$. the forward integration in time is accomplished via a second order Adams-Bashforth time stepping scheme. The basic algorithm followed in the code is the following. First, a value for both K and L is chosen. Knowing the initial values for all the $A_{k,l}$ and $B_{k,l}$; ρ and its spatial derivatives are then computed from the expansion (4) at the physical grid points. The grid points multiplications and additions are then calculated and Fourier transformed to obtain the right hand sides of Equations (5). Finally, the integration is stepped once in time and the new values of all $A_{k,l}$ and

$B_{k,l}$ at the new time are obtained. The process is then repeated untill the final time is reached.

This code is undergoing several diagnostic proceedures at the present time. All of the subroutines have been independently verified and checked. However, when the code was run in its entirety a stiffness problem was encountered. This problem was manifested by having overflow errors whenever the final integration time exceeded a specific value. This value was found to depend also on the number of terms retained in the expansion (4), i.e. the value of both K and L. The longest final time that we were able to run the code to was 14 days and that was for values of K and L both equal to 5. Figures 1 and 2 show the time evolution of the amplitudes $A_{1,1}$ and $B_{1,1}$ belonging to a single linearly unstable wave and for initial values of .015 and .023 respectively. These figures show that after approximatly nine days the amplitudes of the higher harmonics for both components of the wave begin to exert substantial influence. However, these amplitudes do not show any vacillation tendencies for the time period shown. This is expected since it is well established that the period of vacillation brought about through baroclinic instablity is roughly between two and three weeks. Thus in order to observe vacillation in our analysis integration to a much longer final time is necessary. Certainly the integration time of 14 days is nowhere sufficient.

This problem of integration blow-up for larger times has not been resolved yet, although quite considerable effort have been expended towards resolving it. The plan for the second year of funding is to first resolve this problem satisfactorily and then to introduce Ekman damping conditions on the upper and lower boundaries. It should be kept in mind, however, that experience from annulus experiments suggest that vacillation occurs only when the upper lid of the annulus is removed. Thus if vacillation is not found with Ekman damping on both boundaries then that condition will be relaxed on the upper boundary alone.

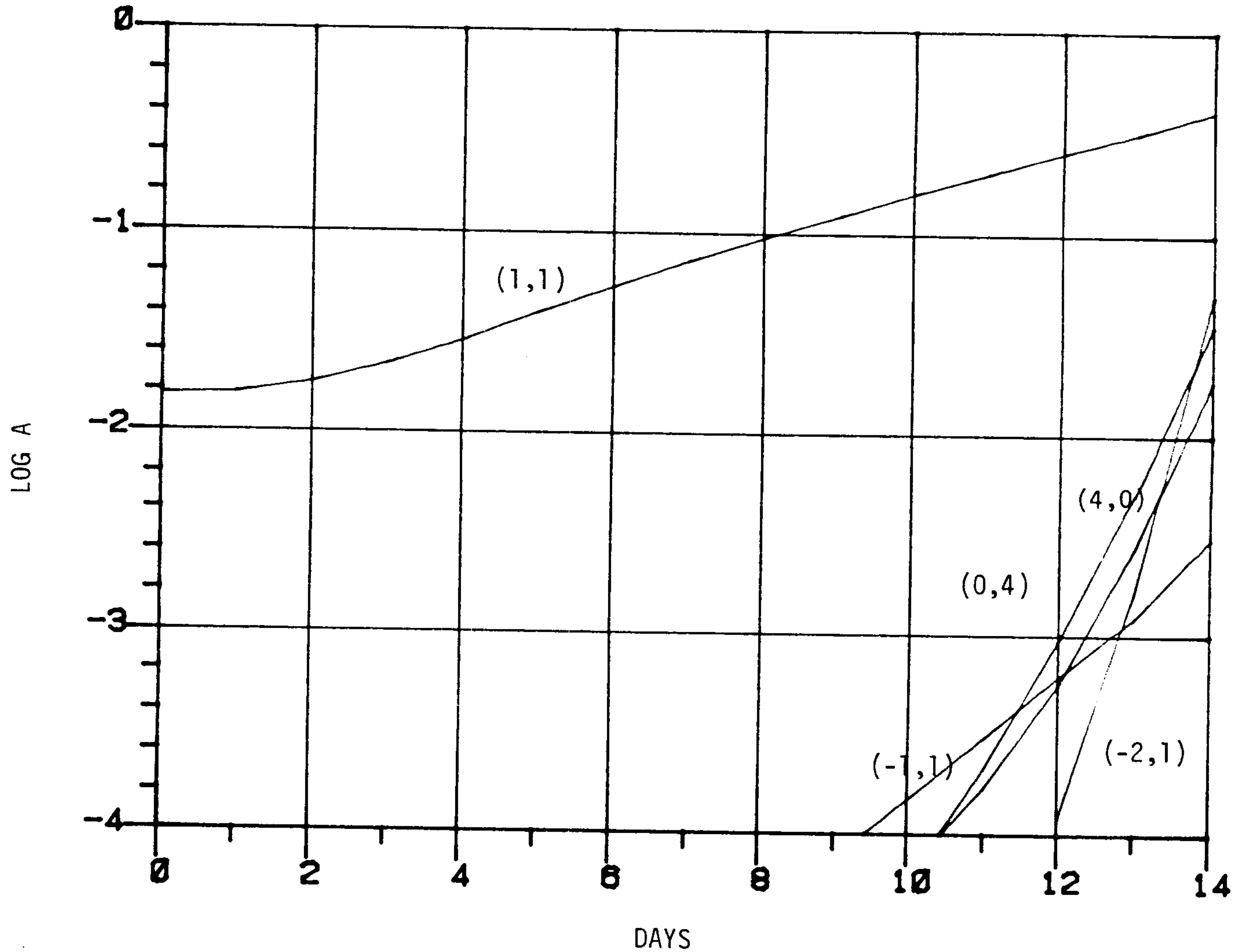

FIGURE 1

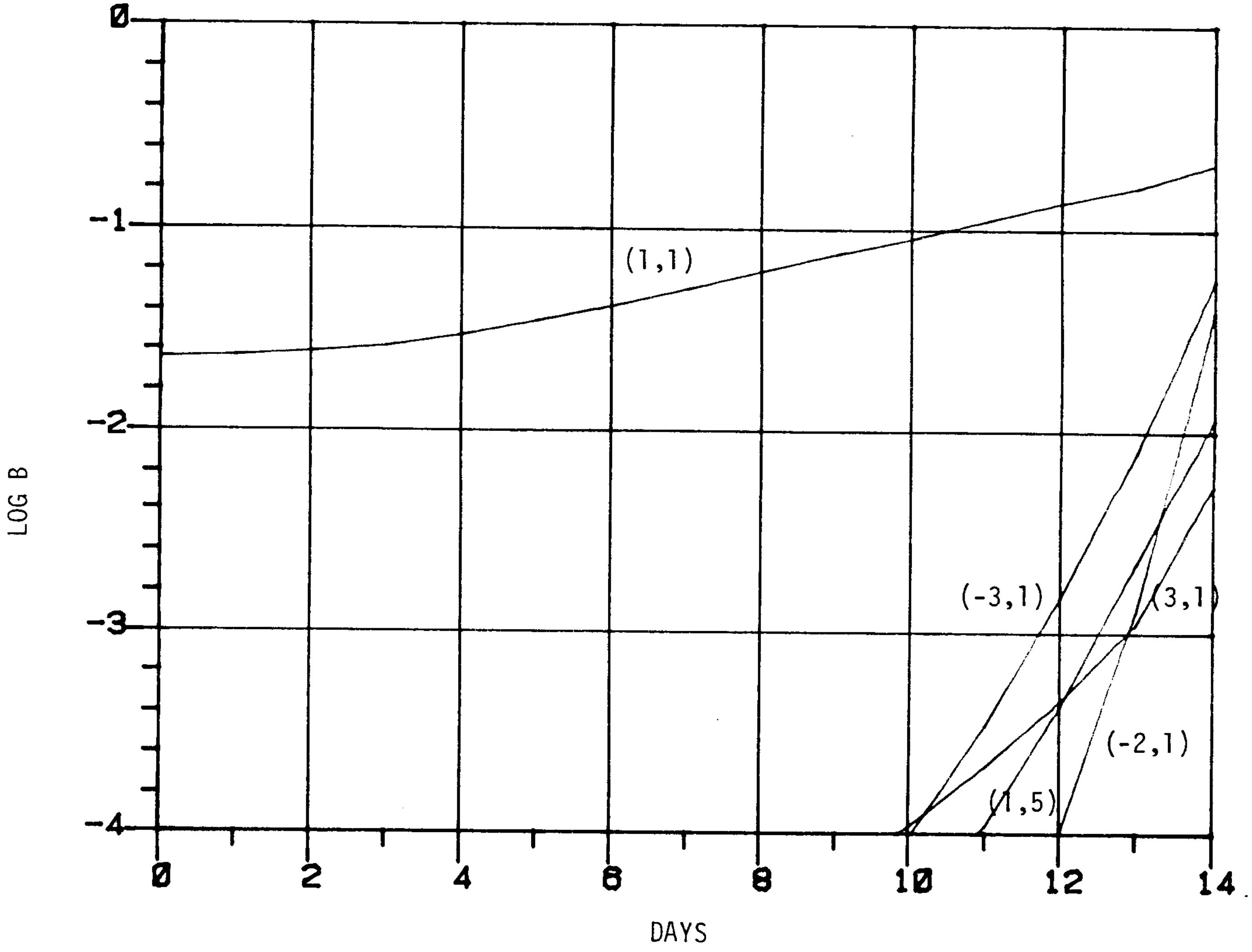

FIGURE 2

2. Atomspheric Data Analysis

The question of whether a vacillation phonomenon, similar to that observed in the annulus, exists in the atmosphere has not yet been satisfactorily answered. There is reasonable evidence on the existance of a two to three week cycle in the zonal index calculations at mid-latitudes in both the northern and southern hemispheres. However, the origin of these cycles, i.e. whether barotropically or baroclinically induced is still being investigated. An important task undertaken in the present contract is to address the existance and the origin of the 2-3 week cycle in the atmosphere. The originaaly proposed task called for the study of this oscillation phenomenon through analyzing available zonal indicies.

During this first year of funding an agreement has been established between the P.I. and the Laboratory for Atmospheres, (GLA), at the Goddard Sapce Flight Center to use the FGGE data sets available at the GLA. With this agreement the GLA is providing for the use of all tapes containing the needed data as well as supplying the P.I. with the necessary software and compuetr time needed to complete the proposed analyses. As an initial step in gaining sufficient working fluency in handling the FGGE data sets it was decided to first experiment with a small portion of these data. This portion is the first Special Observing Period, (SOP 1), which covers the period between Jan. 1 - Feb. 28, 1979. This data set is gridded and well organized making it convenient for the specific use required here. Consequently, the necessary computer codes required for generating the zonal indicies were written and verified.

In generating the zonal index the established conventioal methods were followed. Briefly, the zonal index, R, in this work will be defined in the following manner:

$$R \quad K' \quad K$$

$$K' = \frac{1}{2cos\phi} \int_0^{2\pi} (V - \bar{V})^2 d\lambda$$

$$\bar{V} = \frac{1}{2cos\phi} \int_0^{2\pi} V d\lambda$$

$$\bar{K} = \bar{V}^2/2$$

where λ denotes longitude, ϕ latitude and V the horizontal velocity vector at a given grid point and pressure surface with u and v representing the eastward and northward components respectively.

Figures 3 and 4 show the zonal index variation at the 500mb surface over a period of 40 days starting with Jan. 5,1979 and calculated for latitude positions of 26 degrees north and 38 degrees north respectively. The time perod shown in these figures is appropriate for the SOP 1 experiment. Note, that since the level IIIb of the FGGE data are given at 6 hours intervals the records shown in these figures are plotted for that same interval in time. It is possible to detect a 2-3 week cycle in this time record as well as several higher frequency cycles. These figures are shown here specifically for demonstration purposes and no further comments will be made with regards to thier meaning or utility.

For the next funding year zonal indicies for the entire FGGE year will be generated at several latitude positions covering the region between 30 and 60 degrees of latitude for both the northern and southern hemispheres. These indicies will then be subjected to several numerical filters to establish the occurance of the 2-3 week period cycles. For the time periods exhibiting the strongest signal of such a cycle full energy budgets will be evaluated and analyzed to help in establishing the origin of such cycles and their relationship to the vacillation phnomenon in question.

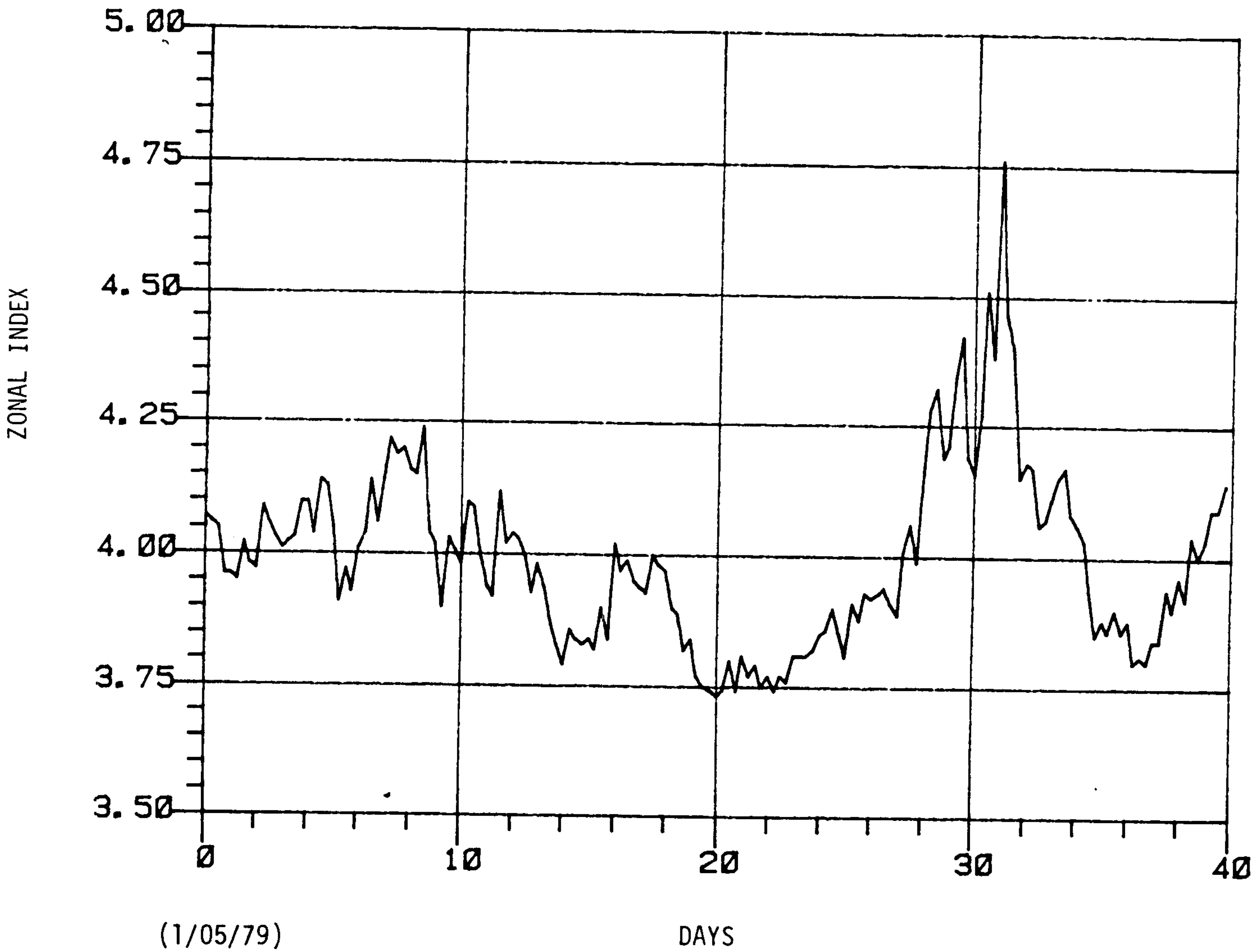

FIGURE 3

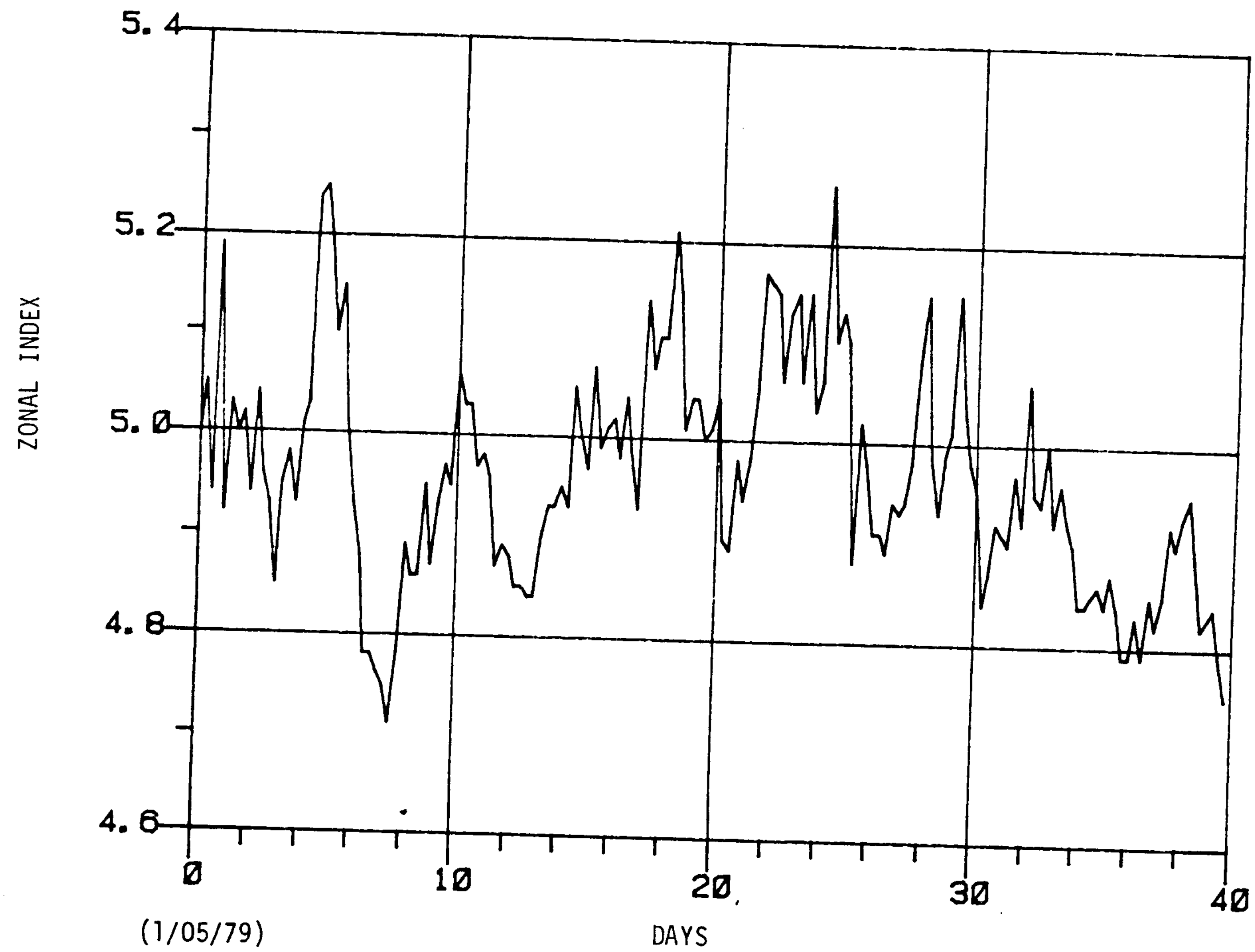

FIGURE 4