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LAGEOS ORBIT DECAY DUE TO INFRARED RADIATION FROM EARTH

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INTRODUCTION

The Lageos satellite is in a high-altitude (5900 km), almost circular orbit about the earth. The orbit is retrograde: the orbital plane is tipped by about 110 degrees to the earth's equatorial plane. The satellite itself consists of two aluminum hemispheres bolted to a cylindrical beryllium copper core. Its outer surface is studded with laser retroreflectors. For more information about Lageos and its orbit see Smith and Dunn (1980), Johnson et al. (1976), and the Lageos special issue (Journal of Geophysical Research, 90, B11, September 30, 1985). For a photograph see Rubincam and Weiss (1986) and a structural drawing see Cohen and Smith (1985). Note that the core is beryllium copper (Johnson et al., 1976), and not brass as stated by Cohen and Smith (1985) and Rubincam (1982). See Table 1 of this paper for other parameters relevant to Lageos and the study presented here.

Lageos' orbit is the most accurately modeled of any satellite (Cohen and Smith, 1985). However, after subtracting out most of the known forces acting on the satellite, such as the gravitational attraction of the sun and moon, direct solar radiation pressure, etc., there is still a residual along-track acceleration which remains to be explained. That acceleration is the subject of this paper.

The residual is given as a function of time in Figure 1. The figure shows the average monthly values of S, the unexplained along-track acceleration. It clearly acts like drag and has a mean value of -3.33×10^{-12} m s^{-2}. It brings Lageos closer to earth by 1.2 mm per day. Moreover, there are fluctuations in the acceleration which can be as large as the mean value. At times S drops almost to zero, as in March of 1983. Most of the fluctuations are obviously correlated with the sun-orbit geometry: the largest ones occur when Lageos spends time in the earth's shadow.

The problem is to account for the curve shown in Figure 1. Afonso et al. (1980), Mignard (1981), Rubincam (1982), Afonso et al. (1985), and Barlier et al. (1986) all invoke charged particle drag to explain the average decay. Neutral atmospheric drag appears to explain only about 10 percent (Rubincam, 1982; Afonso et al., 1985). Anselmo et al. (1983) and Barlier et al., (1986) invoke earth-reflected sunlight to account for the fluctuations. The reflecting surface cannot be Lambertian (Barlier et al., 1986; Rubincam and Weiss, 1986). Morgan (1984) invokes earth-reflected sunlight to explain both the average drag and the fluctuations.

The purpose of this paper is to evaluate a new mechanism to explain the observed average drag: thermal drag. It is probably best thought of as a variant of the Yarkovsky effect (Opik, 1951; Burns et al., 1979; and Rubincam, 1982).

The basic idea is this: the earth emits infrared radiation which is ultimately due to solar heating. The exitance is a uniform 232 $\mathrm{W\,m^{-2}}$ over the earth's surface, to a first approximation (Stephens et al., 1981, Table 4a; Sehnal, 1981, p. 169). Naturally, Lageos will intercept some of this radiation.

Assume Lageos' spin axis is in the plane of the orbit, as shown in Figure 2; the orbit is taken to be circular. Because Lageos' spin axis is fixed in inertial space, first the retroreflectors on one hemisphere and then the other will be alternately heated by the earth's infrared radiation as the satellite circles the earth. (So will the aluminum surface, but this will be ignored here.) The alternate heating of hemispheres causes a temperature asymmetry between them. The hotter hemisphere radiates away more energy and thus more momentum than the cooler. The satellite therefore feels a force along the spin axis directed away from the hotter hemisphere, due to momemtum conservation.

If the retroreflectors had no thermal inertia, then a hemisphere would be hottest when the earth is directly over its pole. In this case the net along-track component of the force is zero because of cancellations when averaging over one revolution. However, since the retroreflectors do have thermal inertia, a hemisphere becomes hottest after the earth has passed over its pole; there is a delay. It is then easy to show that the force does have a net along-track component when averaging over one revolution (and is best envisioned when the lag is 90 degrees; see Figure 2). Moreover, the average along-track acceleration is always opposite to the direction of motion; in other words, it acts like drag and causes the orbit to decay.

This paper estimates the magnitude of the along-track acceleration due to the thermal drag by modeling the thermal behavior of the retroreflectors. The result is that the thermal drag accounts for about 47 per cent of the observed average drag. A rough estimate of the aluminum surface's contribution is put at less than 5 per cent. In view of these results, thermal drag may be the dominant drag mechanism operating on Lageos; the other proposed mechanisms mentioned above may have to be reassessed. However, thermal drag apparently cannot explain the large fluctuations in the observed along-track acceleration. Some other mechanism must be invoked to account for them, such as earth-reflected sunlight or perhaps some unthought-of phenomemon.

LAGEOS ALONG-TRACK ACCELERATION RESIDUALS

(Units: 10^{-12} ms^{-2})

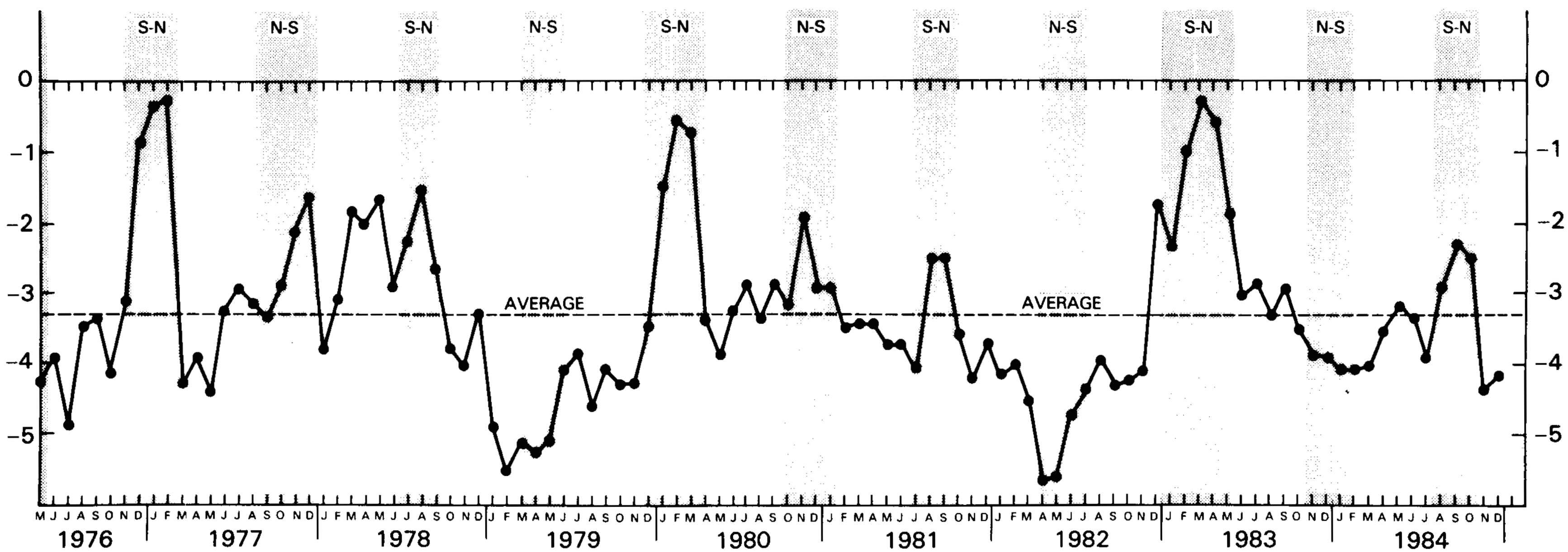

Figure 1. Monthly averages for the unexplained along-track acceleration of Lageos. The month and year are shown along the bottom. The shaded areas show when the orbit intersects the earth's shadow. The letters at the top indicate shadow entry and exit: "S-N", for instance, means Lageos entered the shadow from the south and exited from the north. The dashed line gives the average acceleration: -3.33×10^{-12} m s^{-2}.

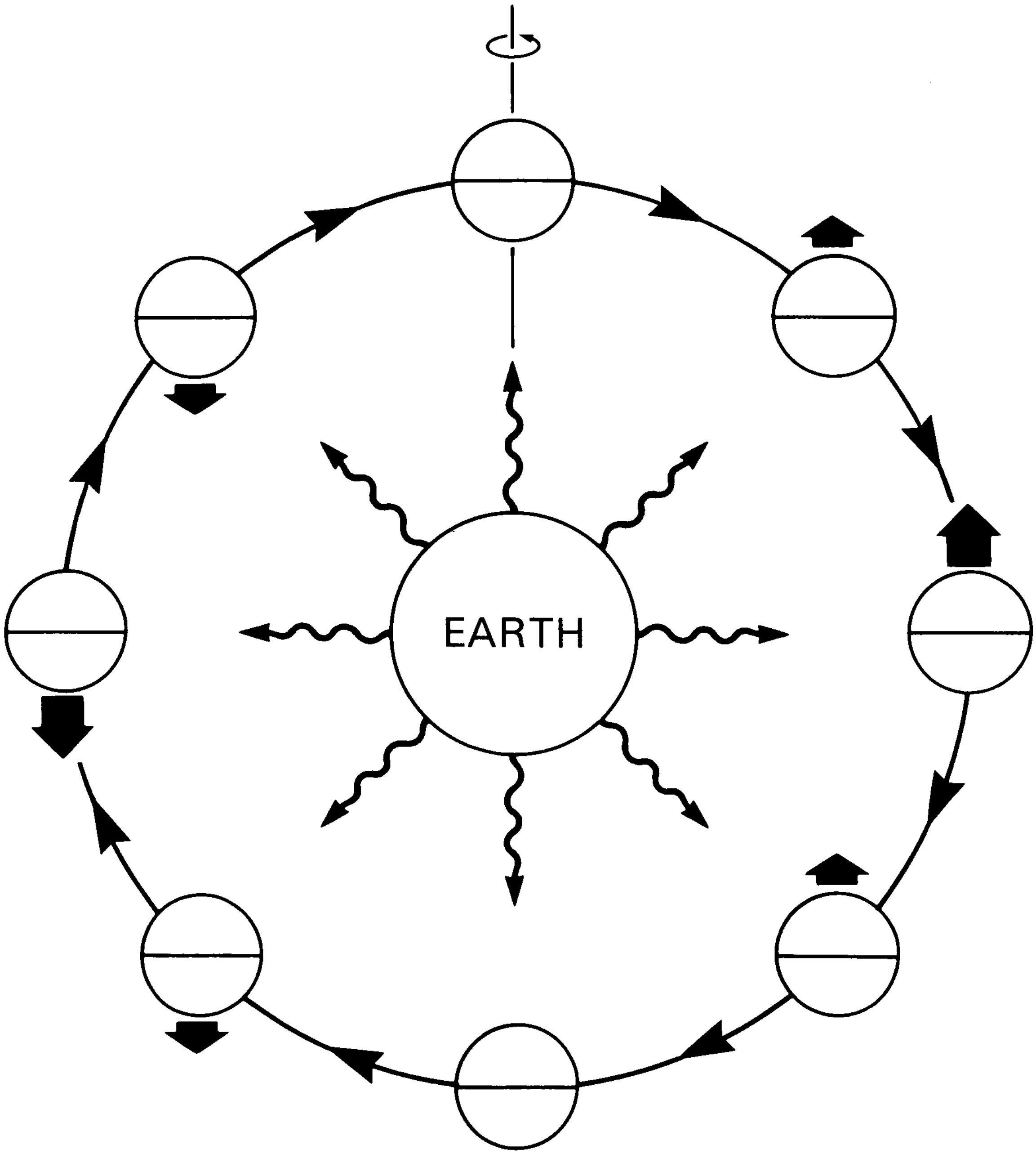

Figure 2. Schematic diagram of the infrared effect when the spin axis is in the plane of the orbit. The earth's infrared radiation (wavy arrows) heat up the retroreflectors (not shown) on Lageos' surface, causing a net force (thick arrows) along the direction of the spin axis. The thermal lag angle is taken to be 90 degrees in this diagram merely to illustrate that there is an along-track deceleration. The actual angle is closer to 37 degrees.

THE RETROREFLECTORS

There are 426 corner-cube laser retroreflectors on Lageos. They cover 42 per cent of Lageos' surface (Johnson et al., 1976). Of these 422 are made of fused silica. The remaining four are made of germanium (Johnson et al., 1976). Only the silica retroreflectors are discussed here. The few made of germanium are not considered at all in this paper.

The relevant properties of a silica retroreflector are given in Table 1. The values for the density ρ, specific heat C_p, thermal conductivity κ, infrared emissivity ϵ, and average temperature T_o come from Bendix (1974, appendices A and K). The diameter of the outer face d comes from Johnson et al. (1976), while the height h from the face to the corner is estimated from their Figure 5, which shows a drawing of a retroreflector. The effective radius R_R of a sphere with about the same volume as as retroreflector comes from approximating the shape as a cone with diameter d and height h: $\pi\, d^2h/12 = 4\pi\, R_R{}^3/3$. The fraction f_o of the face area to the total area comes from measurements made on a cardboard model. The values for R_R and f_o are thus somewhat crude, but should be good enough for the purposes considered here.

RETROREFLECTOR MODEL

The first step in finding the along-track acceleration S from the thermal drag is to thermally model a retroreflector. This is done as follows: the retroreflector is assumed to be a sphere with radius R_R and illuminated by three sources: infrared radiation from the earth; sunlight; and infrared radiation from the aluminum cavity in which the retroreflector sits. The retroreflector is taken to be conductively isolated from the aluminum (Johnson

Table 1
Constants relating to Lageos, the earth, and the universe.

Symbol	Quantity	Numerical value
a	orbital semimajor axis	1.227×10^7 m
C_p	retroreflector specific heat	712 J kg^{-1} K^{-1}
f_o	ratio of face to total area of retroreflector	0.33
I	orbital inclination to earth's equator	109.9 degrees
M_L	mass of Lageos	407 kg
n_L	Lageos mean motion	4.65×10^{-4} s^{-1}
R_L	radius of Lageos	0.3 m
R_R	retroreflector radius	0.01331 m
T_o	retroreflector temperature	276.7 K
ϵ	retroreflector emissivity	0.9
κ	retroreflector thermal conductivity	1.34 W K^{-1} m^{-1}
ρ	retroreflector density	2200 kg m^{-3}
$\Delta F_{IR}{}^o$	sub-earth irradiance at Lageos altitude	62.55 W m^{-2}
R_E	radius of earth	6.371×10^6 m
c	speed of light	2.9979×10^8 m s^{-2}
σ	Stefan-Boltzmann constant	5.67×10^{-8} W m^{-2} K^{-4}

in the above equation gives

$$\Delta\hat{T}(R_R) = \frac{\Delta F_{IR} f_o}{4\sigma T_o^3 + \frac{i\nu\rho C_p R_R}{3\epsilon}}$$

or

$$\Delta\hat{T}(R_R) = \frac{\Delta F_{IR}}{4\sigma T_o^3} \frac{f_o\, e^{-i\delta}}{(1 + \zeta^2)^{1/2}} \tag{10}$$

where

$$\delta = \text{Arc}\tan\zeta \tag{11}$$

and

$$\zeta = \frac{\nu\rho C_p R_R}{12\,\epsilon\sigma T_o^3} \tag{12}$$

The approximations for sin w and cos w will be justified later. Interestingly, κ drops out of (10). Also, note that δ is positive or negative depending on whether ν is positive or negative.

TEMPERATURE DIFFERENCES BETWEEN RETROREFLECTORS

Equation (10) gives the expression for the surface temperature for a given retroreflector. The next step is to find how $\Delta\hat{T}(R_R)$ varies from retroreflector to retroreflector due to differences in their illumination by the earth.

Assume for simplicity that the earth is infinitely far away from Lageos; then the impinging rays come in parallel lines and

$$\Delta F_{IR} = \Delta F_{IR}^{\,o}\, D(\theta, \lambda, \theta_E, \lambda_E)$$

where $\Delta F_{IR}^{\,o}$ is the irradiance at the sub-earth point and

$$D(\theta,\lambda,\theta_E,\lambda_E) = 4\pi \sum_{L=o}^{\infty} \sum_{J=-L}^{+L} \left(\frac{d_L}{2L+1}\right) Y_L^{J*}(\theta_E,\lambda_E)\; Y_L^J(\theta,\lambda)$$

(Rubincam and Weiss, 1986). Here the $Y_L^J(\theta,\lambda)$ are the spherical harmonics used in quantum mechanics; the asterisk (*) means complex conjugate. The coordinates (θ,λ) are the colatitude and longitude, respectively, of a retroreflector on the satellite (see Figure 3). The angle θ is measured from the spin axis, which is the satellite z_L axis. The coordinates of the earth are (θ_E,λ_E) in this system. The d_L are coefficients given in Table IV of Rubincam and Weiss (1986). $\Delta F_{IR}^{\,o}$ is easy to find; due to inverse square attenuation $\Delta F_{IR}^{\,o} = 232\,(R_E/a)^2 = 62.55$ W m^{-2}, where R_E is the earth's radius and a is the orbital semimajor axis.

Consider only the $L = 1$, $J = 0$ term in $D(\theta,\lambda,\theta_E,\lambda_E)$; then $d_1 = ½$ and $D(\theta,\lambda,\theta_E,\lambda_E) = \cos\theta\cos\theta_E/2$. The change in temperature thus depends only on colatitude θ on the satellite. The dependence on longitude λ is

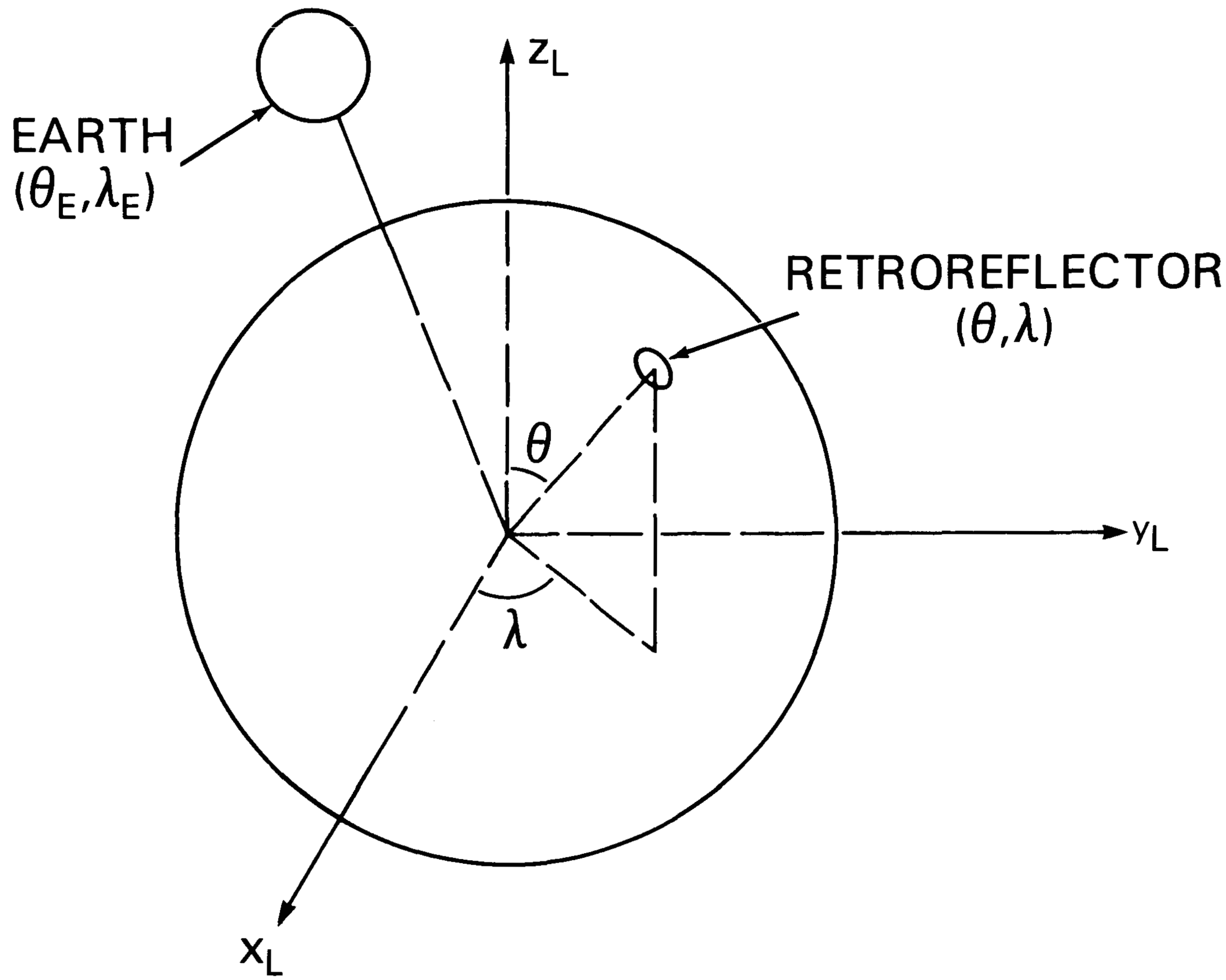

Figure 3. Coordinate system (x_L, y_L, z_L) rigidly attached to Lageos. The z_L axis points along the spin axis of the satellite. The retroreflector is located at (θ,λ), whre θ is colatitude and λ is longitude. The earth is at (θ_E, λ_E).

ignored and assumed to be small due to the rapid rotation of Lageos (a spin period of about 7 s at the end of 1983; E. M. Gaposchkin, private communication, 1985). The temperature also depends on time through θ_E. In fact, the time dependence can be made explicit by writing

$$\cos\theta_E = \frac{e^{in_Lt} + e^{-in_Lt}}{2}, \quad \theta_E = n_Lt$$

without loss of generality. Here n_L is the mean motion of the satellite and its value is given in Table 1.

Using these data in (10) - (12) give

$$\Delta T = B\cos\theta\cos(n_Lt - \delta) \tag{13}$$

where n_L takes the place of ν, and where

$$B = \frac{\Delta F_{IR}{}^{o} f_o}{8\sigma T_o^3 (1+\zeta^2)^{1/2}} = 1.72 \text{ K} \tag{14}$$

and

$$\delta = \text{Arc tan } (0.7465) = 36.7 \text{ degrees} \tag{15}$$

The numerical values come from using Table 1. The $\cos\theta$ factor in (13) shows that there is an axial asymmetry in the retroreflector temperatures across Lageos; one hemisphere is hotter than the other. The $\cos(n_L t - \delta)$ in the same equation shows that the hemispheres alternate in hotness. Equations (13) - (15) thus give the quantitative formulation of the qualitative statements made in the Introduction and are fundamental to this paper. Note that by (8) $(kR_R)^2 = -0.096i$, justifying the approximation for sin w and cos w made earlier. Note also that the lag angle given by (15) corresponds to about 23 minutes of time. This is long compared to the spin period and justifies ignoring the temperature dependence on λ. It is short compared to the orbital period of 3.76 hours.

MAXIMUM DRAG

The last step is to find the along-track acceleration S due to the temperature asymmetry. Consider the flat outer surface of an actual retroreflector, so that the assumption of sphericity is dropped; equations (13) - (15) will, however, still be assumed to be valid. It is easy to show that if the face has area dA and temperature T and emits radiation according to Lambert's law (e.g., Brown, 1965, p. 225), then the photons leaving it carry away momentum at the rate of

$$\frac{dp}{dt} = 2\epsilon\sigma T^4 dA/3c$$

each second normal to the surface; c is the speed of light. Now because $B << T_o$, by (3) and (4) the component of this force in the satellite z_L direction (along the spin axis) is approximately

$$\frac{dp_L}{dt} = -\frac{2\epsilon\sigma}{3c} (T_o^4 + 4T_o^3 \Delta T) \cos\theta \, dA$$

The T_o^4 term drops out when integrating over Lageos' surface, leaving

$$\ddot{z}_L = -(0.41) \frac{4\pi\epsilon R_L^2 \Delta F_{IR}{}^{o} f_o}{9M_L c (1+\zeta^2)^{1/2}} \cos(n_L t - \delta) \tag{16}$$

as the total acceleration due to all of the retroreflectors. The factor 0.41 comes from the fact that the silica retroreflectors cover only 41 percent of Lageos' surface.

The along-track acceleration S is $\ddot{z}_L \sin\theta_E$. Averaging S over one revolution gives

$$\langle S \rangle_{max} = -(0.41) \frac{2\pi\epsilon R_L^2 \Delta F_{IR}{}^{o} f_o}{9M_L c (1 + \zeta^2)^{1/2}} \sin\delta \tag{17}$$

for the case of the spin axis lying in the orbital plane. Substituting numerical values in this equation give

$$\langle S \rangle_{max} = -1.877 \times 10^{-12} \text{ m s}^{-2} \quad (18)$$

This is the extreme value <S> achieves for the thermal model considered here. Any other spin-orbit geometry will give a smaller drag.

The value given by (18) is about 56 per cent the observed average drag. Hence the thermal drag becomes a major contender for explaining the average acceleration as shown in Figure 1 provided the spin-orbit geometry is favorable. The subject of geometry is taken up next.

LAGEOS ALONG-TRACK ACCELERATION

What is desired now is a general expression for <S> when the spin axis is not necessarily in the plane of the orbit. The purpose is to see how <S> varies over long periods of time as the spin axis shifts and the orbit precesses.

Consider the coordinate system shown in Figure 4. The origin is at the center of the earth and the z-axis pierces the North Pole. The x-axis points to the vernal equinox. Assume once again that Lageos' orbit is circular; then Lageos will have a unit position vector (Goldstein, 1950), p. 109.):

$$\begin{aligned} \hat{r} = \; & [\cos\Omega\cos(\omega+f) - \cos I \sin\Omega\sin(\omega+f)] \; \hat{x} \\ & + [\sin\Omega\cos(\omega+f) + \cos I \cos\Omega\sin(\omega+f)] \; \hat{y} \\ & + [\sin I \sin(\omega+f)] \; \hat{z} \end{aligned} \quad (19)$$

in this system. Here I is the inclination of the orbit to the earth's equator (a constant 109.9 degrees), Ω the nodal position in the equatorial plane, ω the argument of perigee, and f the true anomaly. The unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$, lie along their respective axes. The unit along-track vector $\hat{t}$ is

$$\begin{aligned} \hat{t} = \; & [-\cos\Omega\sin(\omega+f) - \cos I \sin\Omega\cos(\omega+f)] \; \hat{x} \\ & + [-\sin\Omega\sin(\omega+f) + \cos I \cos\Omega \cos(\omega+f)] \; \hat{y} \\ & + [\sin I \cos(\omega+f)] \; \hat{z} \end{aligned}$$

The unit vector in the spin direction is

$$\hat{s} = s_x \hat{x} + s_y \hat{y} + s_z \hat{z}$$

where

$$s_x^2 + s_y^2 + s_z^2 = 1$$

The acceleration will lie along the $\hat{s}$-axis and be proportional to $\cos\psi$, i.e. $\ddot{\vec{r}} \propto \cos\psi \, \hat{s}$, where $\cos\psi = \hat{r}_d \bullet \hat{s}$. Here $\hat{r}_d$ is the delayed unit position vector in which $\omega + f - \delta$ replaces $\omega + f$ in (19) to allow for the thermal lag.

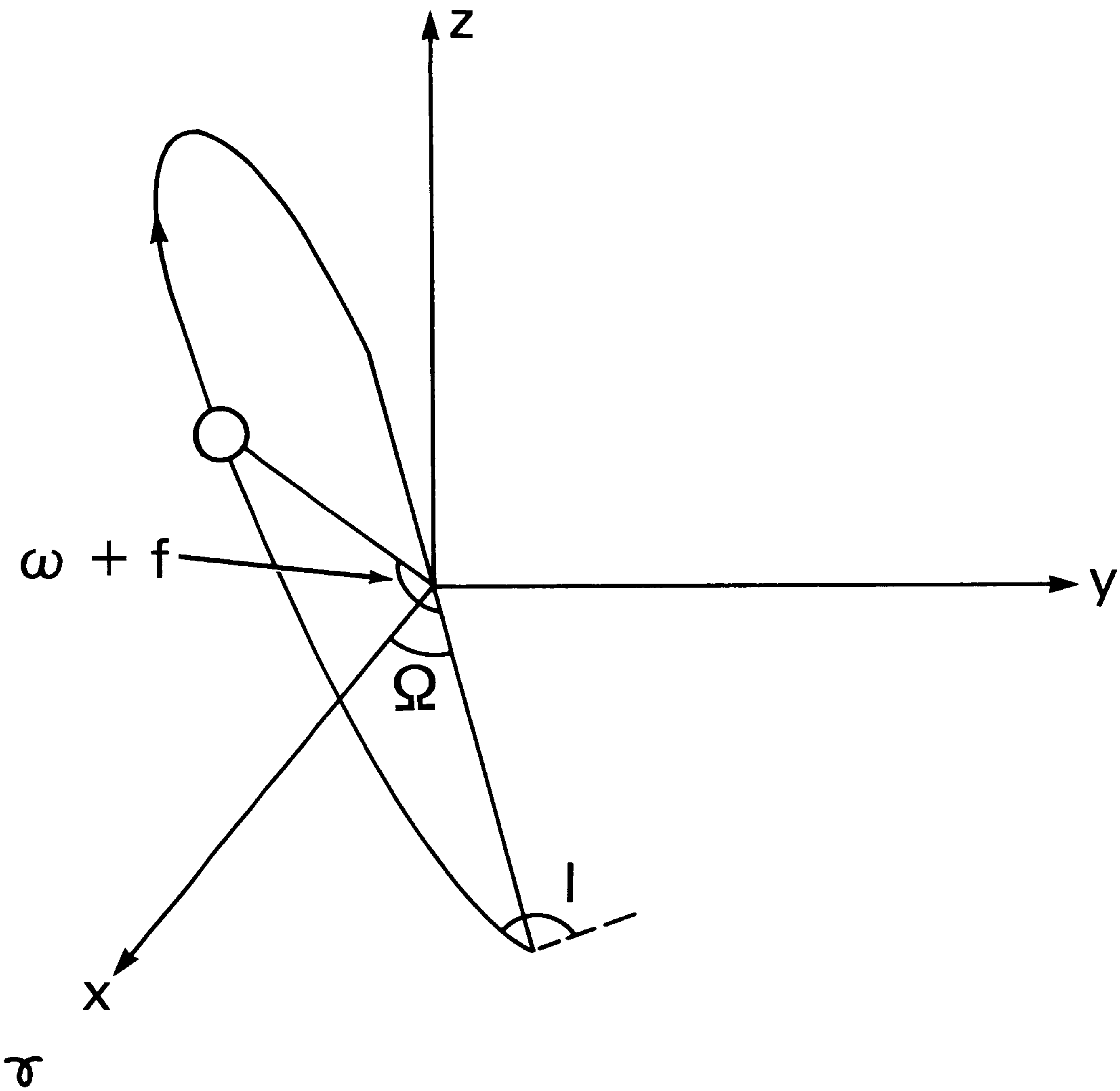

Figure 4. Inertial coordinate system for Lageos' orbit. The x-axis points to the vernal equinox while the z-axis lies along the earth's rotation axis.

The along-track acceleration S is then obtained from $\ddot{\vec{r}}$ via $S = \ddot{\vec{r}} \bullet \hat{t}$. Using (19) and the succeeding equations and averaging $\omega + f$ from 0 to 2π give

$$
\begin{aligned}
\langle S \rangle = \langle S \rangle_{max}[1 - s_z^2 &+ \tfrac{1}{2}(3s_z^2 - 1)\sin^2 I \\
&+ s_z \sin 2I \,(s_y \cos\Omega - s_x \sin\Omega) \\
&+ \tfrac{1}{2}(s_x^2 - s_y^2)\sin^2 I \cos 2\Omega \\
&+ s_x s_y \sin^2 I \sin 2\Omega]
\end{aligned}
\tag{20}
$$

The time dependence of (20) comes in implicitly through s_x s_y, s_z, and Ω. These quantities change only slowly compared to $\omega + f$ and were held fixed in the averaging process. The explicit dependence of Ω on t can be written

$$\Omega = \dot{\Omega}(t - t_o) + \Omega_o$$

where

$$
\begin{aligned}
\dot{\Omega} &= 0.3425 \text{ degrees/day} \\
\Omega_o &= 28.5596 \text{ degrees} \\
t_o &= 42902.5 \text{ (Modified Julian Date)}
\end{aligned}
$$

so that the nodal position moves at a constant rate along the equator. The dependence of s_x, s_y, and s_z on t is a more complicated story. One must find their initial values at launch ($t = t_o$) and then find how the torques on the satellite move the spin axis.

The initial values of s_x, s_y, and s_z can be found from the flight data (J. D. Kraft, oral communication, 1978). Lageos was launched on 4 May 1976 at 4 AM Eastern Daylight Time. Apogee kick motor separation occurred 5457 seconds later at +4.5 degrees geocentric latitude and +20.8 degrees geocentric longitude. The elevation of the spin axis was -11.0 degrees and the azimuth was +158.7 degrees in the topocentric frame. These data yield

$$
\begin{aligned}
s_x &= \mp 0.25433 \\
s_y &= \pm 0.27650 \qquad \text{on 4 May 1976} \\
s_z &= \mp 0.92675
\end{aligned}
$$

for the initial values in the frame of Figure 4. The details of the calculations, which involve the multiplication of rotation matrices, are omitted here. The $\pm$ signs appear because of uncertainty in the sense of rotation of the spin. They create no problems in evaluating (20), since that expression is invariant under change of sign of $\hat{s}$. This merely reaffirms that it is the direction of the spin axis which is important, not the spin vector.

Substituting the above values in (20) gives

$$\langle S\rangle = \langle S\rangle_{max} \bullet [+0.8381 - 0.2229 \cos(\Omega - 222.6 \text{ deg}) - 0.0624 \cos(2\Omega - 85.2 \text{ deg})] \tag{21}$$

This expression shows that the value for $\langle S\rangle$ given by (18) must be reduced by a factor of 0.8381:

$$\langle S\rangle_{sec} = -1.57 \times 10^{-12} \text{ m s}^{-2} \tag{22}$$

to obtain the secular part of S immediately after launch. This is 47 per cent of the observed average of -3.33×10^{-12} m s^{-2}. Also, $\langle S\rangle$ varies sinusoidally with frequencies Ω and 2Ω, with the $\cos\Omega$ term being the principal one. Since the factor in brackets in (21) becomes as small as 0.8381-0.2229-0.0624 = 0.5528, $\langle S\rangle$ varies from (22) by 100 (0.8381-0.5528)/0.8381 = 34 per cent.

The spin axis will change its position in space as Lageos is despun by the earth's magnetic field. The gravitational torque on Lageos is negligible (Barlier et al., 1986). The Lageos spin axis should eventually line up with the earth's spin axis, since the average magnetic field points north-south. In this case $s_x = s_y = 0$ and $s_z = 1$ in (20), giving

$$\langle S\rangle = -1.66 \times 10^{-12} \text{ m s}^{-2}$$

with no periodic terms. This is not much different from (22); hence the average drag will probably not change much as the Lageos spin axis aligns itself with the earth's spin axis. However, no detailed treatment of this problem is given here; both the precession of Lageos' spin axis and the time scale associated with despin have been ignored, as well as the evolution of the orbit when Lageos spins very slowly. These will certainly be of future interest.

DISCUSSION

The thermal model presented here explains a little less than half the observed average drag. The question naturally arises as to whether the thermal drag explains nearly all of it, or whether a better model will give an even smaller contribution to the drag than that computed here. This obviously cannot be answered without studying a more detailed model; so it is worthwhile to discuss in what areas further progress might be made towards a new model.

One of the simplifying assumptions made here was that all of the retroreflectors have the same temperature T_0 in the absence of heating by the earth. But a worst case analysis where the Lageos spin axis points towards the sun gives a temperature difference of 24 K between the antipodal retroreflectors (Bendix 1974, Appendix K). Should this be of concern in a new model?

The answer is probably no, for two reasons. One is that for the initial spin axis orientation the sun never departs from Lageos' equatorial plane by more than 15 degrees (details of the computation are omitted); and never more than 23.5 degrees when the spin axis aligns itself with the earth's axis. Hence both hemispheres are about equally illuminated—far from the worst case where one is in shadow and the other in sunlight. So the differences between retroreflector temperatures are probably only a few degrees. The other reason is that $\langle S\rangle$ as given by (17) is relatively insensitive to changes in T_o when ζ is near 1. Thus it appears that there is not much to be gained by looking at a temporally- and spatially-varying T_o.

In assuming the retroreflectors were spherical only the $\ell = 0$ term in (7) was considered and the higher harmonics were ignored. Carrying through the analysis for $\ell = 1$ shows a negligible effect on $\langle S \rangle$; so the neglect of the $\ell > 0$ terms does not appear to be serious.

Also ignored here is sunlight reflected from the earth; this too will heat up the retroreflectors. But the effect is tiny. Going from no sunlight to full sunlight changes a retroreflector's temperature by only ~ 3 K for a cavity temperature of 303 K (NASA, 1975, Table 3-1). (Lageos' actual cavity temperature is about 328 K; see Bendix, 1974, Appendix K.) The irradiance due to reflected sunlight is about (0.3) (1376)/(4•4) ≈ 26 W m^{-2} at Lageos' altitude. Here 0.3 is the earth's albedo (Stephens et al., 1981) and 1376 W m^{-2} is the irradiance from the sun (Hickey et al., 1980). One factor of 4 comes from spreading the incident sunlight over the earth's surface, while the other comes from the inverse-square law. Therefore the temperature change due to reflected sunlight must be on the order of (26) (3)/1376 = 0.06 K, which is small.

The earth was assumed here to be a point source illuminating Lageos. Actually, the earth subtends a diameter of about 60 degrees as viewed from Lageos, so that more than half its surface is heated by the earth. The extra illumination is offset somewhat by specular reflection (ignored here) off the retroreflectors with large zenith angles, but the matter may be worth further consideration.

Probably the most progress will be made in giving a retroreflector a more realistic shape than a sphere and modeling its interaction (via radiation) with the cavity. NASA (1975) describes a numerical model consisting of 66 nodes where the retroreflector is shaped like a cone and sits in a cylindrical cavity. This perhaps should be the point of departure for a future model, and it will be interesting to see whether the front face temperature changes go up or down; up means higher drag.

There is some evidence that they will go up. Experimental tests show that infrared radiation increases the steady-state front face temperature for the sub-earth retroreflector by about 10 K, for a cavity temperature of 303 K (NASA, 1975, Table 3-1). The time-variable case will divide this amplitude by $2(1 + \zeta^2)^{1/2}$, the factor of 2 coming from $D(\theta, \lambda, \theta_E, \lambda_E)$, and the $(1 + \zeta^2)^{1/2}$ coming from (10). This would give a value of B in (14) of ~4K, assuming the same lag angle, which would about double the drag and explain all of the observed secular decrease in the semimajor axis of the orbit. This clearly warrants further investigation.

Nothing has been said so far about the contribution by the aluminum shell of Lageos to the thermal drag. Its contribution can be shown to be small, as evidenced by the following argument. The worse case analysis by Bendix (1974) gives a maximum temperature difference of 8 K between the two aluminum hemispheres for solar heating. This would make the amplitude 4 K instead of 1.72 K in (13) for sunlight. Since the earth's irradiance (62.55 W m^{-2}) is 22 times smaller than the solar irradiance (1376 W m^{-2}; Hickey et al., 1980), the amplitude for the earth would be approximately 4/22 = 0.18 K, which is about 10 per cent that of the retroreflectors, assuming the same lag angle. The actual case must be even smaller than the worst case. Hence Lageos' aluminum surface explains only about 5 per cent or less of the observed average drag and plays only a small role in the thermal drag.

The model described here cannot explain the observed fluctuations shown in Figure 1. The model gives sinusoidal fluctuations in drag with a period of $360/\dot{\Omega}$ = 1051 days (see equation 21). Its signature is entirely different from the observed one: the biggest observed fluctuations clearly occur when the orbit intersects the earth's shadow. Could diurnal variations in the earth's exitance somehow account for them?

The answer to this question appears to be no. Clouds and ocean maintain a fairly constant exitance throughout the day, according to METEOSAT data (Saunders and Hunt, 1980). The Sahara seems to give the most extreme variation in exitance; but this still only amounts to a change of about 50 Wm^{-2} from day to night (Figure 2 of Saunders and Hunt, 1980). This will be reduced by at least a factor of 4 due to inverse-square or higher attenuation, making it small compared to the uniform 62.55 W m^{-2}. Averaging over many rotations of the earth will make it even smaller, since Lageos is not in an orbit resonant with the earth's rotation. Explanations for the observed fluctuations must be looked for elsewhere. Anselmo et al. (1983) and Barlier et al. (1986) propose radiation pressure from sunlight reflected off of the earth.

The thermal drag depends crucially on the spin-orbit geometry. If the spin axis is normal to the orbital plane, then there is no drag. If the spin axis is in the plane of the orbit as in Figure 2, then there is maximum drag. These facts allow predictions to be made. If the thermal drag is the main cause of the secular decay of Lageos' orbit, then Doppler radar observations should confirm that Lageos' spin axis is never perpendicular to the orbital plane when sizable drag occurs. Also, the drag on Lageos II should be calculable if the spin axis orienation is known. (Lageos II should differ from Lageos only in having an orbital inclination of 59 degrees instead of 110 degreees, assuming a normal launch. Its launch date is now uncertain, due to the Challenger disaster.)

If the thermal drag is the dominant drag mechanism operating on Lageos, then charged particle drag must be smaller than previously believed (Afonso et al., 1980; Mignard, 1980; Rubincam, 1982; Afonso et al., 1985; and Barlier et al., 1986). In any case, the heating of Lageos' retroreflectors by the earth's infrared radiation appears to play a significant role in the secular decrease in the semimajor axis of Lageos' orbit.

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