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ROBUST MULTIVARIABLE CONTROLLER DESIGN FOR FLEXIBLE SPACECRAFT

S. M. Joshi and E. S. Armstrong
Spacecraft Control Branch
Guidance and Control Division
NASA Langley Research Center
Hampton, Virginia 23665

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ABSTRACT

Large, flexible spacecraft are typically characterized by a large number of significant elastic modes with very small inherent damping, low, closely spaced natural frequencies, and the lack of accurate knowledge of the structural parameters. This paper summarizes some of our recent research on the design of robust controllers for such spacecraft, which will maintain stability, and possibly performance, despite these problems. Two types of controllers are considered, the first being the linear-quadratic-Gaussian-(LQG)-type. The second type utilizes output feedback using collocated sensors and actuators. The problem of designing robust LQG-type controllers using the frequency domain loop transfer recovery (LTR) method is considered, and the method is applied to a large antenna model. Analytical results regarding the regions of stability for LQG-type controllers in the presence of actuator nonlinearities are also presented. The results obtained for the large antenna indicate that the LQG/LTR method is a promising approach for control systems design for flexible spacecraft. For the second type of controllers ("collocated" controllers), it is proved that the stability is maintained in the presence of certain commonly encountered nonlinearities and first-order actuator dynamics. These results indicate that collocated controllers are good candidates for robust control in situations where model errors are large.

CHARACTERISTICS OF LARGE SPACE STRUCTURES AND RESULTING CONTROL CHALLENGES

- o Large number of significant elastic modes
- o Very small inherent damping
- o Low, closely–spaced natural frequencies
- o Model errors (no. of modes, frequencies, damping ratios, mode–shapes)

▷ These characteristics make even linear design with perfect actuators/sensors difficult!

▷ There is a need for "robust" controllers

ROBUST CONTROLLERS

Robust = Maintain stability and acceptable performance, in spite of

- o Modelling errors
- o Uncertainties
- o Parameter variation
- o Actuator/sensor nonlinearities
- o Failures

ROBUST CONTROLLER DESIGN APPROACHES

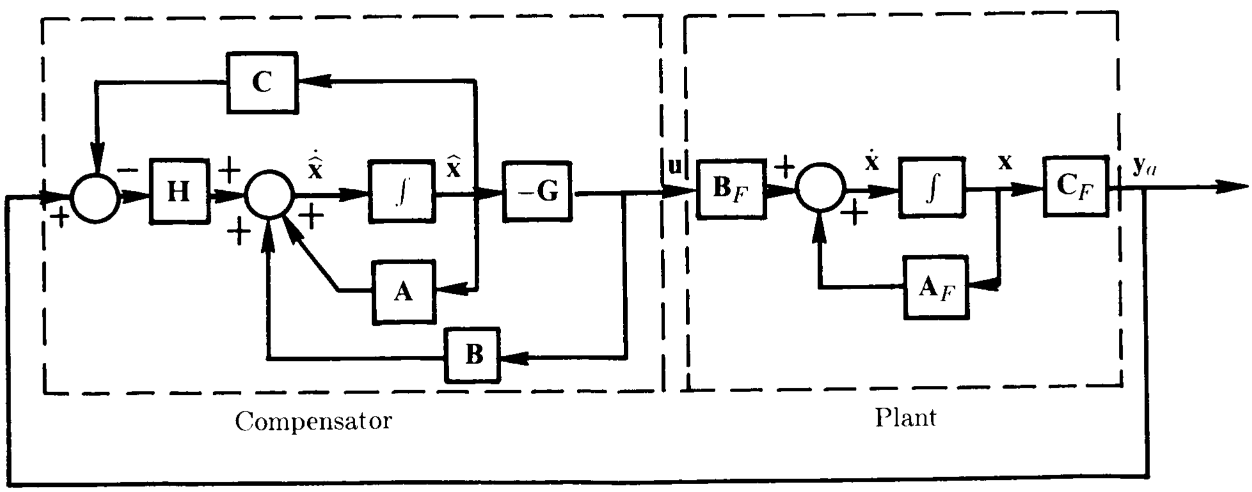
The first approach considered is the LQG-type controller. In order to be practically implementable, it is usually necessary to consider only a reduced-order "design" model for synthesizing the controller. The stability of such reduced-order controllers is not guaranteed because of the control and observation "spillovers" [1,2], and because of errors in the knowledge of the plant parameters. The LQG/LTR method [3,4], which is a frequency-domain method, offers a systematic approach to robust controller design in the presence of modeling uncertainties. In this paper, the LQG/LTR method is briefly described, and the results of its application to a finite element model of the 122-meter hoop-column antenna are presented. Some analytical results on the stability of LQG-type controllers in the presence of realistic actuator nonlinearities are subsequently presented. The second controller design approach consists of "collocated" controllers which utilize actuator/sensor pairs placed at the same (or close) locations on the structure. The stability of such controllers is investigated in the presence of realistic actuator/sensor nonlinearities and also actuator dynamics.

- **I. LQG–TYPE CONTROLLERS**
  - **LQG/LTR method (freq. domain) for robustness to modeling uncertainties**
    - **▷ application to 122 m hoop-column antenna**
  - **Stability in the presence of actuator/sensor nonlinearities**
- **II. "COLLOCATED" CONTROLLERS**
  - **Robustly stable for any number of modes, for any parameter values**
  - **We investigate effect of actuator/sensor nonlinearities and dynamics**

It was proved by Safonov and Athans [5] that the linear quadratic regulator (LQR) which employs state feedback has excellent robustness properties, namely, 60°-phase margin and infinite gain margin. However, when the complete state vector is not available for feedback and an estimator must be used, the resulting LQG-type compensator has no guaranteed robustness properties. The LQG/LTR technique [3,4] offers a method to asymptotically "recover" the robustness properties of the full state feedback controller. The LQG/LTR method basically consists of first defining a desirable "loop gain" in the frequency domain. For obtaining good tracking performance (i.e., loop broken at the output), this is accomplished by using the Kalman-Bucy filter. This loop gain is then "recovered" asymptotically using a model-based (LQG-type) compensator, which simultaneously satisfies certain stability robustness conditions, expressed in terms of frequency-domain singular values.

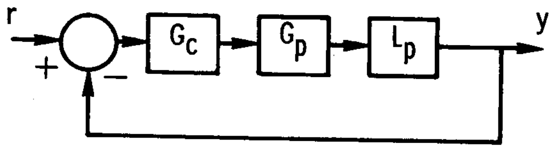
Basic Philosophy

- Define a "desirable" loop gain based on Kalman–Bucy filter (KBF)
- Recover that loop gain using a model–based compensator (i.e., LQ regulator and KBF) while satisfying stability conditions w.r.t. uncertainty.

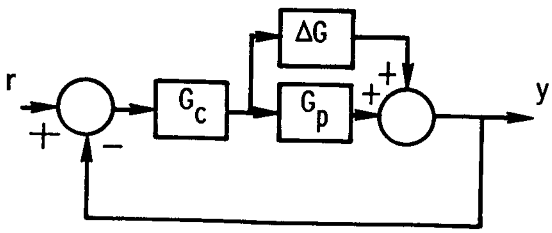


STABILITY ROBUSTNESS CONDITIONS

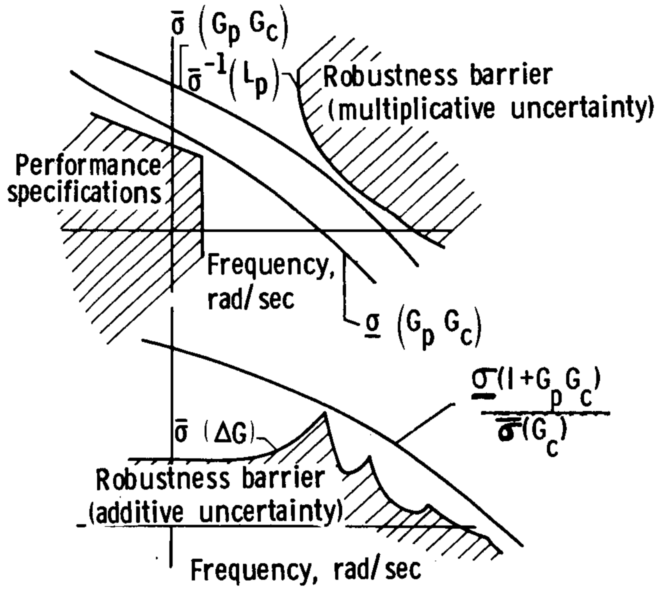
The modeling uncertainty can be expressed either as additive $[\Delta G(s)]$ or multiplicative $[L_p(s)]$. Different sufficient conditions for stability are available for these two formulations. These are expressed in terms of the smallest or the largest singular values of the loop gain, the compensator, and the uncertainty. In the case of flexible spacecraft, all the flexible modes appear in parallel with the rigid-body modes. Therefore, the additive uncertainty model is a natural one for this problem. For satisfying the performance specifications, the $\underline{\sigma}(G_pG_c)$-curve must pass above the "performance barrier" in the low-frequency region. For satisfying the robustness conditions, the $\bar{\sigma}(G_pG_c)$-curve must pass under the high-frequency "robustness barrier" for the multiplicative uncertainty case, while for additive uncertainty, a somewhat more complicated condition has to be satisfied.



Multiplicative uncertainty



Additive uncertainty



LQG/LTR CONTROLLER DESIGN PROCEDURE

The first step in applying the LQG/LTR procedure [4] is to define a reduced-order design model for the large space structure. (In this paper, a sequence of design models with increasingly higher order was considered, starting with a three degree of freedom rigid-body model.) The performance barrier is defined by using the bandwidth specification; e.g., 0.1 rad/sec for the antenna problem. The robustness barrier is defined by the unmodeled structural modes, as well as the parameter uncertainties. The second step is to obtain an "ideal" full state feedback loop gain, using the Kalman-Bucy filter equations (loop is broken at the output for good tracking performance). This loop gain should satisfy the bandwidth specifications. The third step is to design an LQ regulator so that $\sigma(G_p G_c)$ approaches the ideal loop gain in the low-frequency region, and the stability condition is satisfied in the high-frequency region. The final step is to verify the closed-loop stability and performance (eigenvalues, time-responses, etc.) of the entire closed-loop system using the "truth model."

1. Define a design model $G\,(j\omega)$: $\dot{x} = A\,x + B\,u$
   $y = C\,x$

   o Low–freq. performance barrier (bandwidth)

   o High–freq. robustness barrier $L_p(j\omega)$
   (unmodeled dynamics; uncertainties)

2. Design a full state feedback compensator (KBF)–
   Defines "ideal" loop–gain (loop broken at output)

   KBF equations:

   $$A\Sigma + \Sigma A^T + LL^T - \frac{1}{\mu}\Sigma C^T C \Sigma = 0$$

   $$H = \frac{1}{\mu}\Sigma C^T \implies G_{KF} = C\,(sI - A)^{-1} H$$

   Select L and μ to achieve performance specs.

3. Design an LQ regulator to asymptotically recover the freq. response of G_{KF}.

   Compensator: $G_c = G_q\,[sI - A + BG_q + HC]^{-1}\,H$
   where

   $$G_q = B^T P \text{ and } A^T P + PA - PBB^T P + q\,C^T C = 0$$

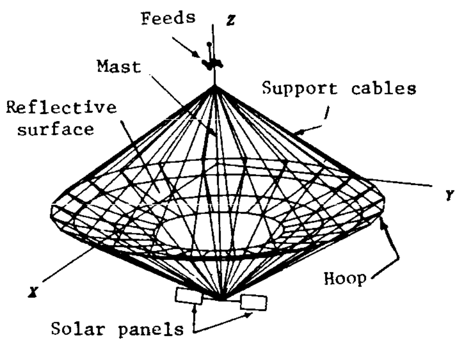
   Recovery is achieved by increasing q : $G_p(s)G_c(s) \rightarrow G_{KF}(s)$

4. Verify closed–loop stability, robustness and performance.

APPLICATION TO THE HOOP-COLUMN ANTENNA

In order to study its applicability, the LQG/LTR method was applied to a finite element model of the 122-meter hoop-column antenna [6]. The three-axis rigid-body attitude angles and the first 10 elastic modes were included in the "truth" model for this investigation. Only one three-axis torque actuator and one three-axis attitude sensor were used. An inherent structural damping of 1 percent was assumed to be present in each elastic mode.

HOOP-COLUMN ANTENNA CONCEPT



ANTENNA PARAMETERS

Mass $\simeq$ 4544.3 kg $\quad I_{xx} = 5.724 \times 10^{6}$ kg-m^2

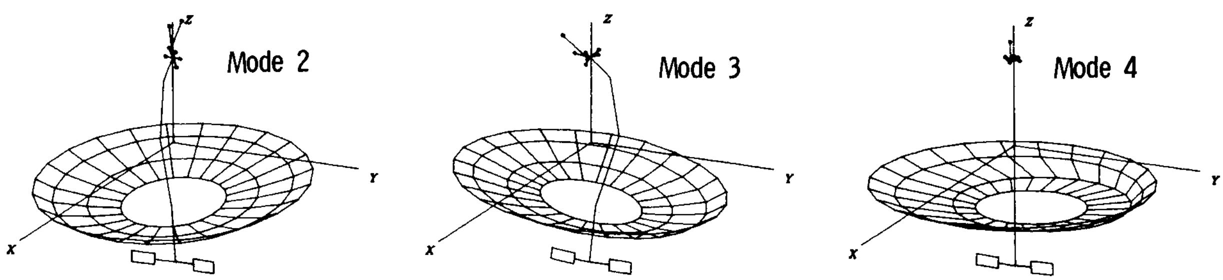
$I_{yy} = 5.747 \times 10^{6}$ kg-m^2 $\quad I_{zz} = 4.383 \times 10^{6}$ kg-m^2

$I_{xz} = 3.906 \times 10^{4}$ kg-m^2 $\quad I_{xy} = I_{yz} = 0$

Structural mode frequencies (rad/sec)

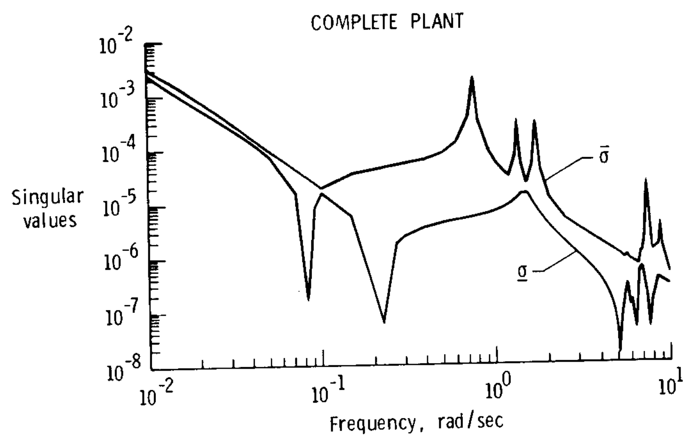
0.75, 1.35, 1.7, 3.18, 4.53, 5.59, 5.78, 6.84, 7.4, 8.78

Typical Antenna Mode-shapes

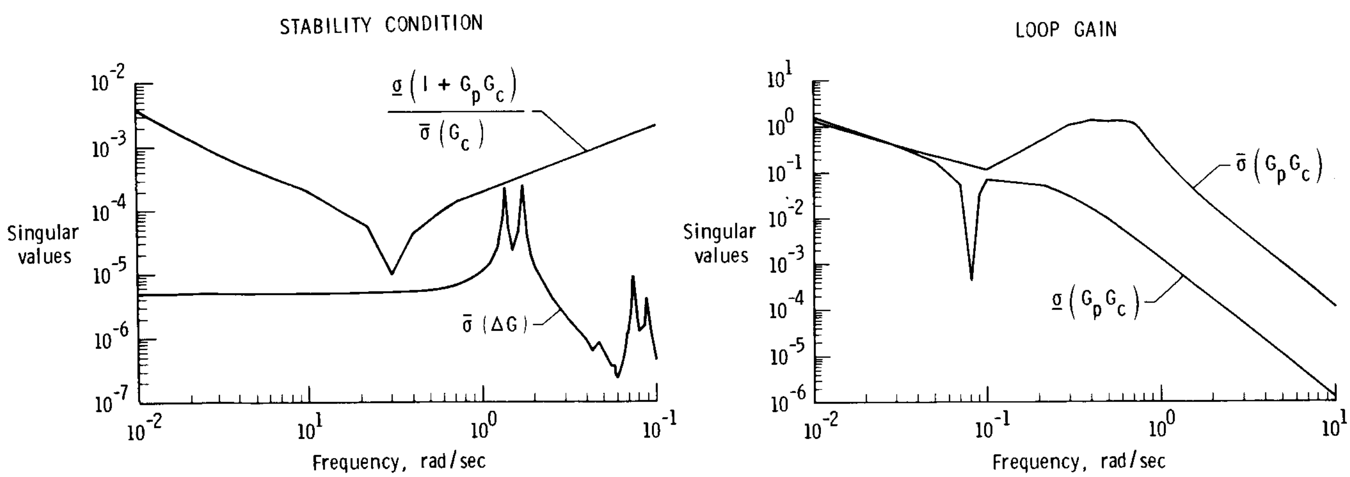


NUMERICAL RESULTS

The first design model used was the sixth order rigid-body model consisting of the three rotational modes, with all the elastic modes being lumped into the additive uncertainty. L and μ were chosen to give good performance characteristics, and the q was increased in the LQR Riccati equation to increase the bandwidth as much as possible without violating the additive uncertainty stability condition. All the computations were performed using ORACLS [7] and a new frequency-domain software package presently under development [8]. It was not possible to obtain the desired bandwidth using the rigid design model. The next step was to use the design model consisting of the rigid-body modes plus the first flexible mode, which is a torsion mode. For this case, somewhat higher bandwidth was obtained, but it still failed to meet the 0.1 rad/sec specification.



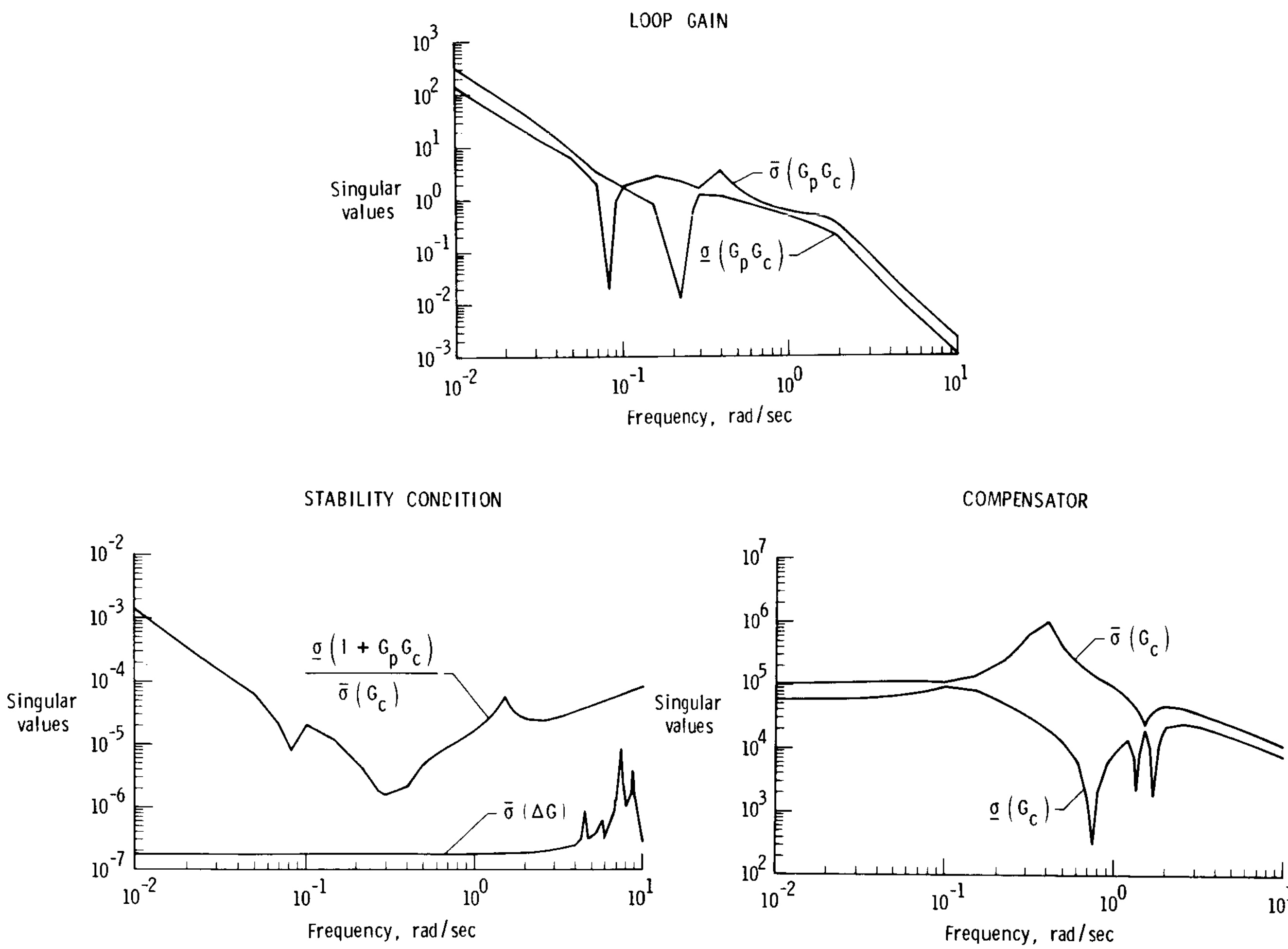
Rigid-body plus one flexible-mode design model



NUMERICAL RESULTS (CONT'D)

The next design model consisted of the rigid body modes and the first three flexible modes (first torsion, pitch bending, and yaw bending modes). For this case, it was possible to obtain the desired 0.1 rad/sec bandwidth while also satisfying the stability condition. However, because of the pair of invariant zeros of G_P near 0.082 rad/sec frequency, the performance is somewhat degraded, as seen by the dip in the $\underline{\sigma}(G_pG_c)$ plot at that frequency. This pair of zeros is close to the $j\omega$ axis and behaves numerically as nonminimum-phase. The frequency of the zero is determined by the sensor/actuator locations.

Rigid-body plus three flexible-modes design model



LQG/LTR METHOD-FINDINGS

The results obtained indicate that it was possible to design a robust controller using the LQG/LTR method. To achieve the required performance and robustness, it was necessary to include at least the first three elastic modes in the design model. Some degradation in performance was caused by the presence of invariant transmission zeros within the desired bandwidth. Since these zeros depend on the actuator and sensor locations, it would be advisible to consider these control aspects in the early design phase of the structure.

- LQG/LTR is a useful method for LSS control
- To meet 0.1 rad/sec bandwidth spec., the design model should include first 3 modes
- Invariant zeros present in the design bandwidth degrade the performance:
  - Zeros depend on the sensor/actuator locations
  - Therefore, control aspects should be considered in early design phase

EFFECT OF ACTUATOR/SENSOR NONLINEARITIES

Ensuring stability in the presence of unmodeled dynamics and parameter uncertainty, which was addressed in the preceding section, is only one aspect of the overall robustness problem. Other considerations include the effect of nonlinearities which are inherently present in components such as the actuators. For example, most real-life actuators have magnitude limits (saturation). Many actuators also have dead-zones, hysteresis, etc. Therefore, it is important to ensure the closed-loop stability in the presence of actuator nonlinearities.

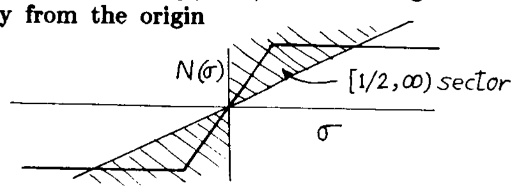
It was proved by Safonov and Athans [5] that the LQ regulator can tolerate nonlinearities in the $[1/2,\infty)$ sector without causing instability. (A nonlinear function $N(\sigma)$ is said to lie in the $[k,\infty)$ sector if $N(0)=0$ and $\sigma[N(\sigma)-k\sigma]>0$.) However, most nonlinearities encountered in practice do not lie in the $[1/2,\infty)$ sector. For example, a saturation nonlinearity lies in that sector in a neighborhood of the origin, but escapes the sector in regions away from the origin. Such (saturation-type) nonlinearities will be termed as "Type-I" nonlinearities. If Type-I nonlinearities are present, it can be proved that [9] there exists a region of attraction such that all trajectories originating in that region will converge to the origin exponentially. The expression obtained for the region of attraction, as well as the accompanying asymptotic properties, provides methods for selecting better performance function weights.

- First consider LQ Regulator (LQR) only:

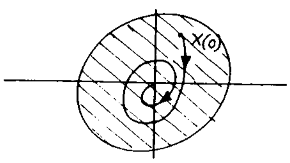
Realistic nonlinearities escape the $[1/2,\infty)$ stability sector:

- Type I Nonlinearities– (saturation–type):

  –These escape the $[1/2,\infty)$ sector in regions away from the origin



  –We prove that there exists a region of attraction (Riccati matrix P provides a natural Lyapunov function)



$$S_a = \{x \mid x^T P x < d\}$$

$$d = \min_{i\in[1,m]} \frac{(\ell_i R_{ii})^2}{b_i^T P b_i}$$

– S_a can be readily determined for a given design

– to make S_a large, increase R or decrease Q.

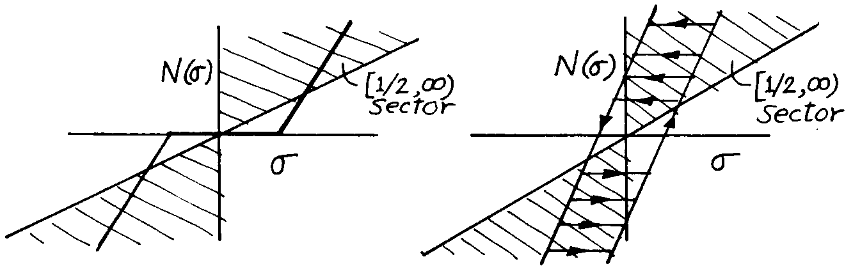
– If $\mathrm{Re}\{\lambda_i(A)\} \le 0$, $S_a \rightarrow E^n$ as $R \rightarrow \infty$.

If not, $S_a \rightarrow$ constant bounded or semi-bounded region in E^n.

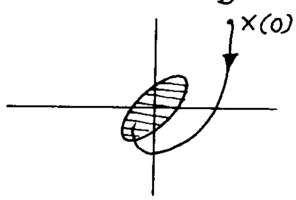
Nonlinearities such as dead-zone or hysteresis lie in the $[1/2,\infty)$ stability sector in regions away from the origin, but escape the region in a neighborhood of the origin. Such nonlinearities will be termed "Type-II" nonlinearities. It can be proved that, in the presence of Type-II actuator nonlinearities, there exists a region of ultimate boundedness such that all the trajectories will enter that region in a finite time, and will remain in that region thereafter [10]. If there are any limit cycles, they will lie inside that region. The expression obtained for the region of ultimate boundedness provides methods for selecting better LQ weighting matrices.

When the full state vector is not available for feedback, a state estimate is used for feedback. Preliminary results for this case have been obtained, and will be included in a paper accepted for publication in the IEEE Transactions on Automatic Control (scheduled for December 1986). Additional work is presently in progress.

- **Type II nonlinearities (dead–zone type)**
  - **Escape $[1/2,\infty)$ sector in a neighborhood of origin**



  - **We prove that there exists a region of ultimate boundedness S_b:**



$$S_b = \{x \mid x^T P x \leq h\}$$

h depends on P, Q, R, α and the nonlinearities

  - **Can readily determine region of ultimate boundedness for a given design**
  - **S_b can be made smaller by increasing α or by reducing R.**

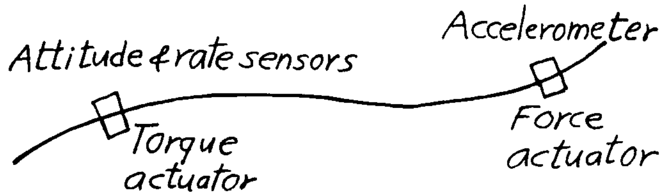
EXTENSION TO LQG CONTROLLERS

- **State estimate is used instead of state vector Work in progress– prelim. results to appear in <u>IEEE Trans. Auto. Contr.</u>**

COLLOCATED CONTROLLERS

This class of controllers consists of pairs of compatible actuators and sensors placed at the same (or close) locations throughout the structure. Thus attitude and rate sensors collocated with torque actuators will constitute a "collocated attitude controller". These controllers use negative definite feedback of the measured attitude and rate. The greatest advantage of such controllers is that, with perfect (linear, instantaneous) actuators, the closed-loop stability is guaranteed for any number of modes and any errors in the knowledge of the parameters. However, the actuators and sensors encountered in practice have nonlinearities and finite bandwidth, thus invalidating these general stability properties.

- Compatible actuators and sensors are placed at same locations



- For control of both rigid-body attitude and elastic motion

- Control input consists of feedback of measured positions and rates (rotational and/or translational)

ADVANTAGES OF COLLOCATED CONTROLLERS

- With perfect (linear, instantaneous) actuators and sensors, stability guaranteed for
  - Any number of modes
  - Any parameter errors

- Simple to implement

PROBLEM: ACTUATORS AND SENSORS HAVE NONLINEARITIES AND PHASE LAGS!

Our Contribution:– We proved that these robustness properties still hold in presence of a wide variety of realistic actuator/sensor imperfections.

COLLOCATED CONTROLLERS (CONT'D)

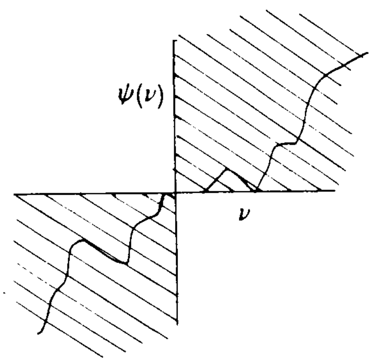
Therefore, we investigated the stability of collocated controllers when imperfect actuators/sensors are present. Using the Lyapunov method and function-space techniques, we proved that the stability properties of such controllers remain intact even in the presence of a variety of actuator/sensor nonlinearities and first-order actuator dynamics [11]. These results substantially increase the applicability of collocated controllers, and also identify them as good candidates for robust control especially when the modeling uncertainty is very large; e.g., during deployment, assembly, or initial operation when the parameters have not yet been identified. Investigation of stability in the presence of higher-order actuator dynamics is being planned.

OUR ROBUSTNESS RESULTS

- o Robust stability for ANY parameter values and ANY no. of modes is still maintained if
  - – actuator NL's are monotonic increasing and sensor NL's belong to 1st and 3rd quadrant
  - – if at least one actuator and sensor per axis is functional
  - – actuators have linear first–order dynamics, and

  Proportional gain < (actuator bandwidth) x (rate gain)
- o With only velocity feedback (for damping enhancement), stable if all NL's belong to 1st and 3rd quadrants, and actuators have 1st order dynamics
- o Research continuing for higher–order actuator dynamics and for obtaining better performance.

Nonlinearity lying in 1st and 3rd quadrants



CONCLUDING REMARKS

The problem of designing robust controllers for flexible spacecraft was addressed using two approaches. The first approach consisted of an LQG-type compensator. It was found that this type of compensator can be robustified against unmodeled dynamics using the loop-transfer-recovery-(LTR) procedure. The presence of transmission zeros can cause performance degradation, and should be considered in the early design phase while selecting actuator/sensor locations. Effects of sensor/actuator nonlinearities were investigated, and expressions were obtained for stability regions. The second design approach considered utilizes "collocated" controllers, which were shown to have excellent robustness properties in the presence of not only modeling errors, but also actuator/sensor nonlinearities and dynamics. Future efforts should attempt to obtain less conservative stability regions in the presence of nonlinearities, and to develop procedures to robustify LQG-type compensators simultaneously against modeling errors and nonlinearities. For collocated controllers, efforts should be directed towards obtaining optimal feedback gains, as well as stability results with higher-order actuator dynamics.

- LQG/LTR is a promising method
  - Consideration must be given to transmission zeros (actuator/sensor location)
- Collocated controllers offer highly robust control in the presence of large modeling uncertainty:
  - Deployment
  - Assembly
  - Initial operation
  - Failure modes

Directions:

- Obtain less conservative results for LQG–type controllers with realistic nonlinearities
  - combine with freq.–domain compensator design methods
  - apply to realistic problem (e.g., SCOLE)
- Study other robust control design methods:
  - H^{∞}– methods
  - SSV Method (Doyle, Wall, Stein, Athans)
  - Stable factorization method (Vidyasagar)
- Develop methods for optimizing collocated controller performance
- Study effect of sampled–data implementation

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