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STOCHASTIC MODELING AND CONTROL SYSTEM
DESIGNS OF THE NASA/MSFC GROUND FACILITY
FOR LARGE SPACE STRUCTURES - THE MAXIMUM
ENTROPY/OPTIMAL PROJECTION APPROACH

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ABSTRACT

In the Control Systems Division of the Systems Dynamics Laboratory of the NASA/MSFC, a Ground Facility (GF), in which the dynamics and control system concepts being considered for Large Space Structures (LSS) applications can be verified, has been designed and built under Dr. Henry Waites' supervision. One of the important aspects of the GF is to design an analytical model which will be as close to experimental data as possible so that a feasible control law can be generated.

In this study, using Hyland's Maximum Entropy/Optimal Projection Approach, we developed a procedure in which the maximum entropy principle is used for stochastic modeling and optimal projection technique is used for a reduced-order dynamic compensator design for a given high-order plant.

ACKNOWLEDGEMENTS

I am extremely honored by being chosen by Dr. Henry B. Waites and Dr. Mike Freeman to participate a second time in the Summer Faculty Research Program. I acknowledge with sincere gratitude the "reboost" that working with Dr. Waites has given to my career as a professor and researcher. Dr. Waites not only provided interesting and challenge research topics to me, he also made himself available to me whenever I needed him. I know that I took a lot of valuable time from him this summer; I thank him for his kindness and patience.

INTRODUCTION

In the Control Systems Division of the Systems Dynamics Laboratory of the NASA/MSFC, a Ground Facility (GF), in which the dynamics and control system concepts being considered for Large Space Structure (LSS) applications can be verified, has been designed and built under Dr. Henry B. Waites [9] supervision. One of the important aspects of the GF is to design an analytical model which will be as close to experimental data as possible so that feasible control laws can be generated.

There are several approaches to design an analytical model and generate control laws for GF/LSS. One of them, Hyland's [1,2,3,4,5] Maximum Entropy/Optimal Projection (MEOP) approach, particularly draws our attention and interest.

One of the major problems in designing high-performance control systems is that of robustness. Maximum Entropy modeling directly addresses this problem by incorporating into the dynamic model a representation of ignorance regarding physical parameters.

Optimal projection technique is used to design quadratically optimal, reduced-order dynamic controllers for high-order systems.

Hyland combined MEOP design approaches and applied to a structural system having uncertainties in the stiffness matrix.

The purpose of this report is to study the feasibility of applying Hyland's MEOP approach to GF/LSS.

OBJECTIVES

The objectives of this work were to:

      (1)   Study Hyland's MEOP approach,

      (2)   Draw a flow chart for MEOP approach,

      (3)   Apply MEOP approach to GF/LSS, and

      (4)   Make comments and recommendations.

LSS DETERMINISTIC ANALYTICAL MODEL

The deterministic analytical model for a Large Space Structure can be described as

$$\dot{X} = AX + BU$$

$$Y = CX \tag{1}$$

where

$$X = [\eta_1, \eta_1, \ldots, \eta_N, \eta_N]^t, \qquad U = [F, M]^t,$$

$$A = \begin{bmatrix} 0 & 1 & & & 0 \\ -\omega_1^2 & -2\zeta_1\omega_1 & & & \\ & & & & \\ & 0 & & 0 & 1 \\ & & & -\omega_N^2 & -2\zeta_N\omega_N \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ \phi_1(X_F) & \phi_1'(X_M) \\ \vdots & \\ 0 & 0 \\ \phi_N(X_F) & \phi_N'(X_M) \end{bmatrix} \qquad C = \begin{bmatrix} 0, & \phi_1(X_S), & \ldots, & 0, & \phi_N(X_S) \\ 0, & \phi_1'(X_S), & \ldots, & 0, & \phi_N'(X_S) \end{bmatrix}$$

η = generalized displacement

ω = modal radians frequency

ζ = modal damping

ϕ = eigenvector ϕ' = eigenvector slope

F = force input M = torque input

X_S = sensor X_F = force displacement

X_M = moment displacement Y = sensor outputs.

This is a finite-element model of a large flexible space struc-
ture which is, generally, an extremely high-order system. The
size of the model and the coupling between sensors and actuators
render classical control-design methods useless and but confound
attempts to use LQG to obtain a controller of manageable order.
These difficulties motivated MEOP approach.

HYLAND'S STOCHASTICAL MODEL

The high-order, uncertain model associated with (1) can be stated
as:

$$\dot{X} = (\bar{A} + \sum_{i=1}^{p} \alpha_i A_i)X + (\bar{B} + \sum_{i=1}^{p} \alpha_i B_i)U + W_1$$

$$\text{(2)}$$

$$Y = (\bar{C} + \sum_{i-1}^{p} \alpha_i C_i)X + W_2$$

where

$\bar{A}, \bar{B}, \bar{C}$ = nominal dynamic matrices

α_i = zero-mean, unit intensity, uncorrelated white
  noise

A_i, B_i, C_i = uncertain patterns

W_1 = disturbance noise, a Wiener process

W_2 = observation noise, a Wiener process.

Uncertainties in the dynamics matrix, A, the control input matrix, B, and the sensor output matrix, $\bar{C}$, are all modeled via the maximum entropy approach [6,8].

The object is to design a lower order dynamic controller with state X_C (dim X_C < dim X) by choosing the controller matrices A_C, B_C, and C_C so as to minimize the indicated quadratic performance criterion described as follows:

$$\dot{X}_C = A_C X_C + B_C Y$$

$$U = C_C X_C$$

(3)

with performance criterion:

$$J(A_C, B_C, C_C) = \lim_{t \to \infty} E[X^T R_1 X + 2X^T R_{12} U + U^T R_2 U]$$

where R_1, R_2, R_{12} are penalty matrices.

OPTIMAL COMPENSATOR GAINS

Determination of A_C, B_C, and C_C requires that we first solve the basic design equations (4) for the quantities Q, P, $\hat{Q}$, $\hat{P}$, and τ.

$$0 = A_X Q + Q A_S^T + \tilde{A}Q\tilde{A}^T + V_1 + (\tilde{A} - \tilde{B}R_{2S}^{-1}\tilde{P}_S)\hat{Q}(\tilde{A} - \tilde{B}R_{2S}^{-1}\tilde{P}_S)^T$$

$$-\tilde{Q}_S V_{2S}^{-1}\tilde{Q}_S^T + \tau_1 \tilde{Q}_S V_{2S}^{-1}\tilde{Q}_S^T \tau_1^T$$

$$0 = A_S^T P + P A_S + \tilde{A}^T P \tilde{A} + R_1 + (\tilde{A} - \tilde{Q}_S V_{2S}^{-1}\tilde{C})^T \hat{P}(\tilde{A} - \tilde{Q}_S V_{2S}^{-1}\tilde{C})$$

$$-\tilde{P}_S^T R_{2S}^{-1}\tilde{P}_S + \tau_1^T \tilde{P}_S^T R_{2S}^{-1}\tilde{P}_S \tau_1$$

(4)

$$0 = (A_S - B_S R_{2S}^{-1}\tilde{P}_S)\hat{Q} + \hat{Q}(A_S - B_S R_{2S}^{-1}\tilde{P}_S)^T + \tilde{Q}_S V_{2S}^{-1}\tilde{Q}_S^T$$

$$- \tau_1 \tilde{Q}_S V_{2S}^{-1}\tilde{Q}_S^T \tau_1^T$$

$$0 = (A_S - \tilde{Q}_S V_{2S}^{-1} C_S)^T \hat{P} + \hat{P}(A_S - \tilde{Q}_S V_{2S}^{-1} C_S) + \tilde{P}_S^T R_{2S}^{-1} \tilde{P}_S$$

$$- \tau_1^T P_S^T R_{2S}^{-1} \tilde{P}_S \tau_1$$

where

$$\text{Rank } \hat{Q} = \text{Rank } \hat{P} = \text{Rank } \hat{Q}\hat{P} = N_C$$

$$\tau = \hat{Q}\hat{P}(\hat{Q}\hat{P})^{\#} \quad , \quad \tau_1 = I_N - \tau$$

\# means a group generalized inverse.

$$\hat{Q}\hat{P} = G^T M \Gamma, \quad \Gamma G^T = I_{N_C}$$

$$\tilde{A}Q\tilde{A}^T = \sum_{i=1}^{p} A_i Q A_i^T, \quad \tilde{A}Q\tilde{B} = \sum_{i=1}^{p} A_i Q B_i, \text{ etc.,}$$

$$A_S = A + \tfrac{1}{2}\tilde{A}^2, \quad B_S = B + \tfrac{1}{2}\tilde{A}\tilde{B}, \quad C_S = C + \tfrac{1}{2}\tilde{C}\tilde{A},$$

$$R_{2S} = R_2 + \tilde{B}^T(P + \hat{P})\tilde{B}, \quad V_{2S} = V_2 + \tilde{C}(Q + \hat{Q})\tilde{C}^T$$

$$Q_S = QC_S^T + V_{12} + \tilde{A}(Q + \hat{Q})\tilde{C}^T$$

$$\tilde{P}_S = B_S^T P + R_{12}^T + \tilde{B}^T(P + \hat{P})\tilde{A}$$

V_1 = intensity matrix of W_1.

V_2 = intensity matrix of W_2

V_{12} = $E[W_1 W^T]$.

Notice that the first two equations in (4) are Riccati equations and the last two are modified Lyapunov equations.

After (4) is solved, the controller gains can be found by

$$A_C = \Gamma(A_S - B_S R_{2S}^{-1}\tilde{P}_S - \tilde{Q}_S V_{2S}^{-1}C_S)G^T$$

$$B_C = \Gamma\tilde{Q}_S V_{2S}^{-1}$$

$$C_C = -R_{2S}^{-1}\tilde{P}_S G^T.$$

PROCEDURE OF APPLYING HYLAND'S MEOP APPROACH

There are two phases in applying Hyland's MEOP approach to GF/LSS. The first phase is modeling and the second phase is solving matrix equation (4). The following chart will describe the procedures applied on those two phases.

MODEL BUILDING PHASE

┌───┐
│ DEFINE UNCERTAIN PATTERNS, A_I, B_I, C_I │
└───┘
                    ↓

```
┌─────────────────────────────────────────────────┐
│   USE MAXIMUM ENTROPY PRINCIPLE TO DETERMINE     │
│         THE PROBABILITY LAW OF α_I               │
└─────────────────────────────────────────────────┘
                        │
                        ▼
        ┌─────────────────────────────────────┐
        │   TEST IF α_I IS WHITE NOISE         │
        └─────────────────────────────────────┘
                        │
                        ▼
    ┌─────────────────────────────────────────────┐
    │  ESTIMATE INTENSITY MATRICES V_1, V_2, V_12  │
    └─────────────────────────────────────────────┘
                        │
                        ▼
        ┌─────────────────────────────────────┐
        │    SET UP STOCHASTIC MODEL (2)       │
        └─────────────────────────────────────┘
```
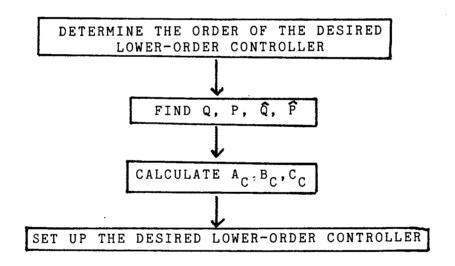
SOLVE MATRIX EQUATIONS PHASE

```
        ┌─────────────────────────────────────┐
        │  DETERMINE THE ORDER OF THE DESIRED  │
        │       LOWER-ORDER CONTROLLER         │
        └─────────────────────────────────────┘
                        │
                        ▼
            ┌─────────────────────────────┐
            │   FIND Q, P, Q̂, P̂           │
            └─────────────────────────────┘
                        │
                        ▼
            ┌─────────────────────────────┐
            │   CALCULATE A_C, B_C, C_C    │
            └─────────────────────────────┘
                        │
                        ▼
   ┌────────────────────────────────────────────────┐
   │  SET UP THE DESIRED LOWER-ORDER CONTROLLER      │
   └────────────────────────────────────────────────┘
```

CONCLUSION AND RECOMMENDATION

MEOP approach has many advantages including:

(1) By using the maximum entropy principle, the probability distribution which maximizes a priori ignorance must be the least presumptive (i.e., least likely to invent data).

(2) Hyland proved that the stochastic model induced by the maximum entropy principle is a Stratonovich multiplicative white noise model.

(3) Optimal projection equations (4) are in terms of covariance, cost matrices, and provide a generalization of standard LQG theory.

(4) MEOP imbeds stochastic effects in the model to begin with. Therefore, the system keeps its linear property during the whole process.

There are some obstacles in applying MEOP approach too. They are:

(1) No definite rule to determine uncertain patterns.

(2) Need to verify α_I is white noise to insure the resulted stochastic model is feasible.

(3) The criterion of determining the desired lower-order controller is not specified.

(4) Feasibility of existing algorithm for solving matrix equations (4) needs to be confirmed.

In all, MEOP is an advanced approach which combines maximum entropy principle and optimal projection technique to generate control laws for Large Space Structures. It is a highly sophisticated but theoretical proved method. The authors think the future of MEOP is very bright and strongly recommending having an in-house package for MEOP developed.

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