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STATISTICAL ANALYSIS OF FAILURE DATA ON CONTROLLERS AND SSME TURBINE BLADE FAILURES

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STATISTICAL ANALYSIS OF FAILURE DATA ON

CONTROLLERS AND SSME TURBINE BLADE FAILURES

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ABSTRACT

The expressions for the maximum likelihood functions are given when the failure data is censored at a given point or at multiple points, or when the data comes in groups. Different models applicable to failure data are presented with their characteristics. A graphical method of distinguishing different models by using cumulative hazard function is discussed.

For the failure data on controllers the model is determined by cumulative hazard function and chi-square goodness of fit. Using the Weibull Model the maximum likelihood estimators of the shape parameter and the failure rate parameter are obtained. The confidence intervals, meantime between failures, and B1 are determined. Similarly, for the data on SSME blade failures the maximum likelihood estimators are obtained for the Weibull parameters. The variances, confidence intervals, meantime between failures and reliability are determined. The analysis is carried under assumption of grouped data as well as randomly placed data.

1. Introduction and Objectives

In Patil (1985) report I introduced the stochastic process, failure times, and reliability of a complex system. Also, I discussed the constant and variable failure rate models. The models developed were applicable to independent as well as nested components. The methodology was applied to the Space Telescope Solar Array System. Abernethy (1985) discussed some aspects of failures on SSME turbine blade failures for simulated data. Mario Rheinfurth (1985) made a study of reliability analysis of HPOTP first stage blade failures assuming the Weibull Model for the grouped data. He obtained estimates for the failure rate and shape parameter for the Weibull Model.

In the present study we shall consider general methodology for obtaining Maximum Likelihood Estimators (MLE) for the complete data, censored or multicensored data or grouped and multicensored data. We shall discuss five different models for failure data. The models we shall discuss are: The Exponential Model, Gamma Model, Weibull Model, Lognormal Model, and Extreme Value Model. For these models, we shall discuss briefly the hazard function and graphically represent the hazard functions.

The methodology discussed will be applied for the suitability of the models for the controller failure data. For the assumed model, the MLE's of the failure rate and shape parameter in the model will be obtained. The variances of the estimators and confidence intervals for the parameters will be obtained. Also, the estimated reliability curves and estimated MTBF will be found.

For the HPOTP turbine blade grouped data, the MLE's of the parameters and their variances will be obtained. The analysis will also be done assuming observations are equally spaced or randomly spaced. From the estimators, the Reliability Curve and MTBF will be presented.

2. The Maximum Likelihood Functions

Let $t_1, t_2, \ldots, t_n$ be the failure times and the probability density function (p.d.f.) be $f(t,\theta)$, then the likelihood function of $t_1, t_2, \ldots t_n, \theta$ is

$$L(t_1,t_2 \ldots t_n,\theta) = f(t_1,\theta) \ldots\ldots f(t_n,\theta), \quad 0<t_i<\infty, \; i = 1,\ldots n \tag{1}$$

If the observations are censored at time t_o i.e. the experiment is terminated at time t_o and r failures ($r<n$) occurred before t_o. Since the probability of a failure after time t_o is $1-F(t_o)$, where $F(t)$ is the distribution function of T, and $t_1, t_2 .. t_r$ are independent the likelihood function can be written as

$$L(t_1, t_2, ... t_r, t_o, \theta) = f(t_1, \theta) ... f(t_r, \theta)\ (1-F(t_o))^{n-r},$$

$$0<t_i< \infty \quad i = 1,2..r \qquad (2)$$

If r = n then (2) reduces to (1).

If each of the units are censored at different times, $t_1, ... t_r$ are failures times and $t_{r+1}, ... t_n$ denote the censoring times of the unfailed units. Then the likelihood function is given by

$$L(t_1, t_2 .. t_r, t_{r+1}, ... t_n, \theta) =$$

$$f(t_1, \theta) ..... f(t_r, \theta) \prod_{i=r+1}^{n} (1-F(t_i)) \qquad (3)$$

If $t_i = t_1 = ... = t_n = t_o$ (3) reduced to (2).

If the failures are counted at the end of certain intervals, then the observations occur in groups. Suppose k observations occur in (t_{11}, t_{12}), k_2 in $(t_{21}, t_{22}), ... k_r$ observations in the interval (t_{r1}, t_{r2}) and remaining $n - (k_1 + k_2 + .. + k_r)$ observations multicensored as above. Since probability of k_i observations occurring in (t_{i1}, t_{i2}) is

$$(F(t_{i1} - F(t_{i1}))^{k_i}$$

the likelihood function can be written as

$$L(t_{11}, t_{12}, t_{21}, t_{22}, ... t_{r1}, t_{r2}, t_{r+i}, ... t_n)$$

$$= \prod_{i=1}^{r} (F(t_{i2}) - F(t_{i1}))^{k_i} \prod_{i=r+1}^{n} (1-F(t_i))$$

$$0<t_{i1}<t_{i2}, \quad i = 1,2.....,r \qquad (4)$$

These likelihood functions can be used to find the MLE's and their variances.

3. Different Models

In this section we shall discuss different models which could be applied to failure rate data. We shall give the density function, distribution function, the reliability function, MTBF, the hazard function, and the cumulative hazard function for each model. We consider five models: Exponential Model, Gamma Model, Weibull Model, Lognormal Model, and Extreme Value Model.

3.1 The Exponential Model

The p.d.f:

$$f(t) = \lambda e^{-\lambda t}, \; t > 0 \tag{5}$$

The constant $\lambda > 0$, is the failure rate. The distribution function

$$F(t) = 1-e^{-\lambda t}, \; t > 0 \tag{6}$$

The reliability

$$R(t) = 1-F(t) = e^{-\lambda t} \tag{7}$$

The MTBF

$$MT = \int_0^\infty R(t)dt = \int_0^\infty e^{-\lambda t} \, dt = \frac{1}{\lambda} \tag{8}$$

The hazard function

$$h(t) = \frac{f(t)}{1-F(t)} = \lambda \tag{9}$$

The cumulative hazard function is

$$H(t) = \int_0^t h(u)du = \int_0^t \lambda du = \lambda t \tag{10}$$

which is linear in t.

3.2 The Gamma Model

The p.d.f. of t is

$$f(t) = \lambda^k t^{k-1} e^{-\lambda t} / \Gamma(k) \tag{11}$$

The distribution function

$$F(t) = \frac{1}{\Gamma(k)} \int_0^t \lambda^k k^{k-1} e^{-\lambda x} dx = \tag{12}$$

$$\frac{1}{\Gamma(k)} \int_0^{\lambda t} u^{k-1} e^{-u} du = \Gamma_k (\lambda t) / \Gamma(k)$$

where $\Gamma_k(.)$ is the incomplete gamma function. The reliability R(t) is

$$R(t) = 1 - \Gamma_k(\lambda t)/\Gamma(k) \tag{13}$$

The MTBF is

$$M_T = \int_0^\infty R(t) dt = \frac{1}{\Gamma(k)} \int_0^\infty t \lambda^{k-1} e^{-\lambda t} dt = \frac{k}{\lambda} \tag{14}$$

The hazard function

$$h(t) = \frac{\lambda^k e^{-\lambda t} t^{k-1}}{1 - \Gamma_k(\lambda t)} \tag{15}$$

$$H(t) = - \ln (1 - \Gamma_k (\lambda t)/\Gamma(k)) \tag{16}$$

For k = 1, the model reduces to exponential model.

3.3 The Weibull Model

The p.d.f. is

$$f(t) = \lambda \beta t^{\beta - 1} e^{-\lambda t^\beta} \tag{17}$$

The distribution function and reliability are

$$F(t) = 1-e^{-\lambda t^{\beta}}, \qquad (18)$$

$$R(t) = e^{-\lambda t^{\beta}} \qquad (19)$$

MTFB is

$$M_T = \int_0^{\infty} e^{-\lambda t^{\beta}} dt = \frac{\Gamma(1+1/\beta)}{\lambda^{1/\beta}} \qquad (20)$$

The hazard function and cumulative hazard function are

$$h(t) = \lambda \beta t^{\beta-1} \qquad (21)$$

$$H(t) = \lambda t^{\beta} \qquad (22)$$

For $\beta = 1$ the model becomes exponential model.

3.4 The Lognormal Model

The failure rate t has lognormal model when lnt has normal distribution hence the p.d.f. of t is

$$f(t) = \frac{1}{t\sigma\ (2\pi)^{1/2}}\ e^{-1/2(\frac{\ln t}{\sigma})^2},\ 0<t<\infty . \qquad (23)$$

The distribution function and reliability are given by

$$F(t) = \Phi\ (\frac{\ln t}{\sigma}), \qquad (24)$$

$$R(t) = 1-\Phi(\frac{\ln t}{\sigma}) = \Phi(-\frac{\ln t}{\sigma}) \qquad (25)$$

where Φ (.) is the distribution function of the standard normal random variable. The MTBF is

$$M_T = E(T) = \int_0^{\infty} t \frac{1}{t\sigma(2\pi)^{1/2}} e^{-\frac{1}{2}\left(\frac{\ln t}{\sigma}\right)^2} dt \qquad (26)$$

$$= e^{\sigma^2/2}$$

The hazard function and cumulative hazard are

$$h(t) = \frac{1}{t\sigma(2\pi)^{1/2}} \frac{e^{-\frac{1}{2}\left(\frac{\ln t}{\sigma}\right)^2}}{\Phi\left(-\frac{\ln t}{\sigma}\right)}, \qquad (27)$$

$$H(t) = -\ln \Phi\left(-\frac{\ln t}{\sigma}\right). \qquad (28)$$

3.5 The Extreme Value Model

The p.d.f. of t is

$$f(t) = \frac{1}{a} e^{t/a} \exp(-e^{t/a}), \ -\infty < t < \infty \qquad (29)$$

The distribution function and reliability are given by

$$F(t) = 1 - \mathrm{exb}(-e^{+t/a}) \qquad (30)$$

$$R(t) = \exp(-e^{t/a}) \qquad (31)$$

The MTBF is

$$M_T = \int_{-\infty}^{\infty} t \ e^{t/a} \exp(-e^{t/a}) dt = -a\gamma \qquad (32)$$

where r is the Eular's constant. The hazard function and cumulative hazard are

$$h(t) = \frac{1}{a} e^{t/a}, \; -\infty < t < \infty \tag{33}$$

$$H(t) = e^{t/a}, \; -\infty < t < \infty \tag{34}$$

If we take $t_1 = e^t$ then t_1 has Weibull distribution. The models have been given in Lawless (1982).

4. Comparison of Different Models

In this section we discuss the graphs of hazard functions and their use in differentiating different models. Hazard function given the probability that individual will last a time $t + \Delta t$ given that the individual has lasted up to time t. There are usually three types of hazard function: (i) Decreasing hazard function; for this type there is larger failure rate at the beginning we call infant failure model. (ii) Increasing hazard function. The failure rate goes up as age of the unit goes up. These are fatigue models. (iii) Bath tub model where both types of failure can occur. There are few instances these models we now consider hazard functions of the models.

(i) Exponential Model - For this model hazard rate remains constant. The graph of the hazard function is parallel to X axis.

(ii) Gamma Model - For this model for $k < 1$ the failure rate is decreasing for $k = 1$ hazard is constant and for $k > 1$ the failure rate goes up. The graph of h(t) is shown in figure (i). This model is fairly flexible.

(iii) Weibull Model - Like Gamma Model for $\beta < 1$ the failure rate is decreasing, for $k = 1$ hazard function is constant and for $\beta > 1$ the hazard function is increasing. This model is flexible and useful in many situations. The graph of the hazard function for different values of β is given in figure (ii).

(iv) Lognormal Model - The hazard function is sharply decreasing if there is a high mortality rate then this model may be used.

(v) Extreme Value Model - For this model the hazard function of the graph increases exponentially. Also,

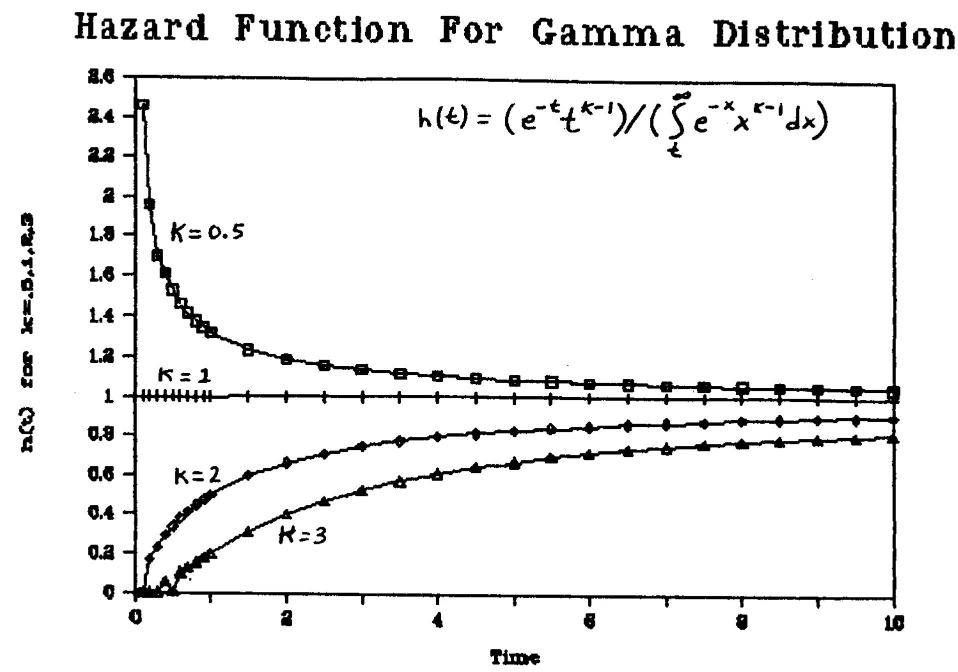


FIGURE i

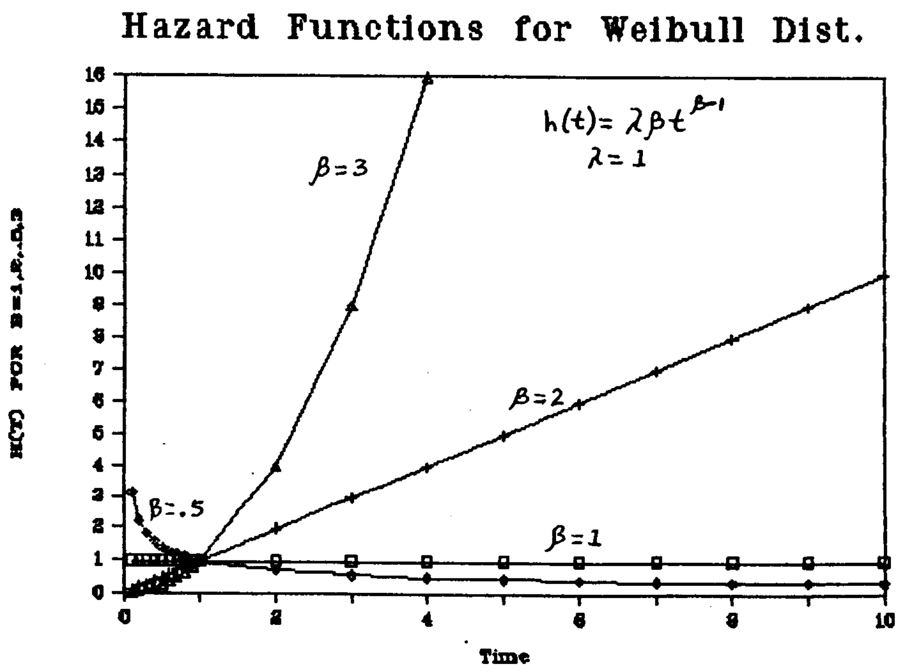


FIGURE ii

the failure distribution varies from $-\infty$ to ∞. This model is not very useful. The model further can be determined by plotting the estimated cumulative hazard function which we shall discuss in application.

5. Statistical Analysis of Controller Failures

In this section we shall apply mathematical model to fit the controller failure data.

5.1 Controller Failure Data

Data on controller failures is given in two different sets. The data is given in the appendix. First set of data is collected by Honeywell and provided by Jack Matheny and Russ Mattox. The data consists of 26 failures and 24 different censoring times in hours. The failure times include factory testing plus field testing. The failure definition was determined in consultation with Russ Mattox and Oliver Burnett of Rockwell International. The failure times are estimated failure times. The total time to the failures, and the total factory failure times were observed. The estimated failure times were estimated by subtracting factory failure times from all the failure times.

5.2 Estimated Cumulative Hazard Function

Different models could be distinguished by the estimated cumulative hazard function. The method is discussed in Nelson (1969). We use Nelson's graphical method to distinguish among Exponential, Weibull, and Lognormal Models.

We shall describe the method of distinguishing the models.

An estimator of a cumulative hazard function H(t) for the multicensored model is denoted by

$$\hat{H}(t)$$

and is obtained as follows:

The failure times and censoring times are ranked together from largest to smallest, then the estimated hazard function is 100 times reciprocal of the rank of the failure time and

$$\hat{H}(t) = \hat{H}$$

is the running total of the estimated hazard functions. Different models distinguished from the graph of some function of $\hat{H}$ versus time.

Exponential Model: The relation between t and $\hat{H}$ is

$$t = (1/\lambda)\hat{H}. \tag{35}$$

Hence, if the scatter plot of t against $\hat{H}$ fits a line through origin the data indicates exponential model.

Weibull Model: The relation between t and $\hat{H}$ is

$$\ln t = \frac{1}{\beta}\ln \hat{H} + \ln\lambda \tag{36}$$

Hence, if the scatter diagram on log scale is linear then the data indicates Weibull fit. If the slope is 1 then the model reduces to exponential model.

Lognormal Model: The relation between t and $\hat{H}$ is

$$t = \mu + \sigma\Phi^{-1}(1-e^{-\hat{H}}), \tag{37}$$

For this case, if the scatter diagrams of t against

$$\Phi^{-1}(1-e^{-\hat{H}})$$

fits a straight line then the data indicates a possible fit of Lognormal Model. The graphs of scatter diagrams of t against functions of $\hat{H}$ for Weibull Model is given in figure (iii). The data indicates fairly good linear fit for the Weibull graph. Also, the histograms of 26 failures is shown in figure (iv). If the observed frequency is O and theoretical frequency is e, then for large n>20,

$$\Sigma(O-e)^2/e$$

has a chi-square distribution with degrees of freedom as the number of classes less the number of parameters. If

Weibull Distribution (Censored)

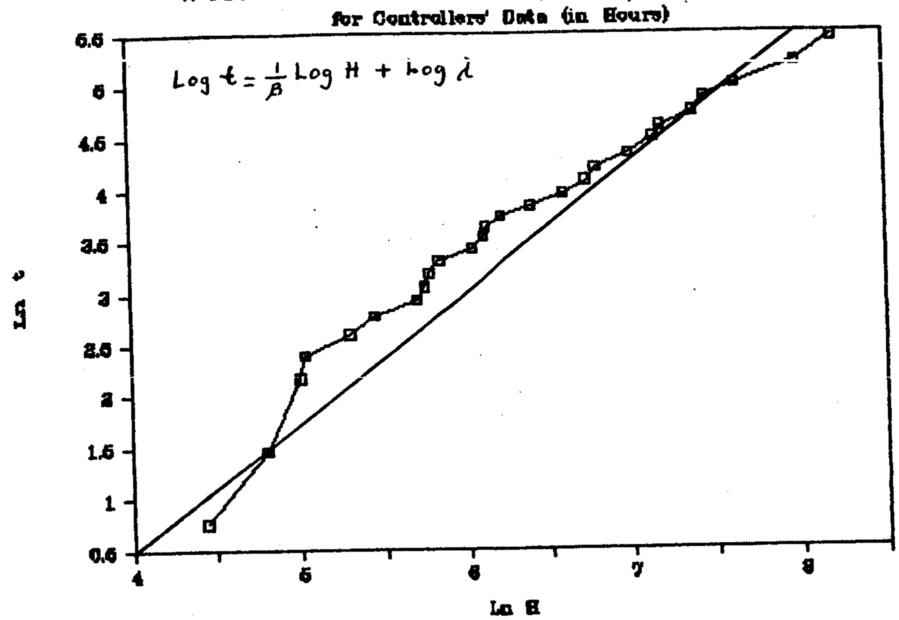


FIGURE iii

Histogram for Failures against Time

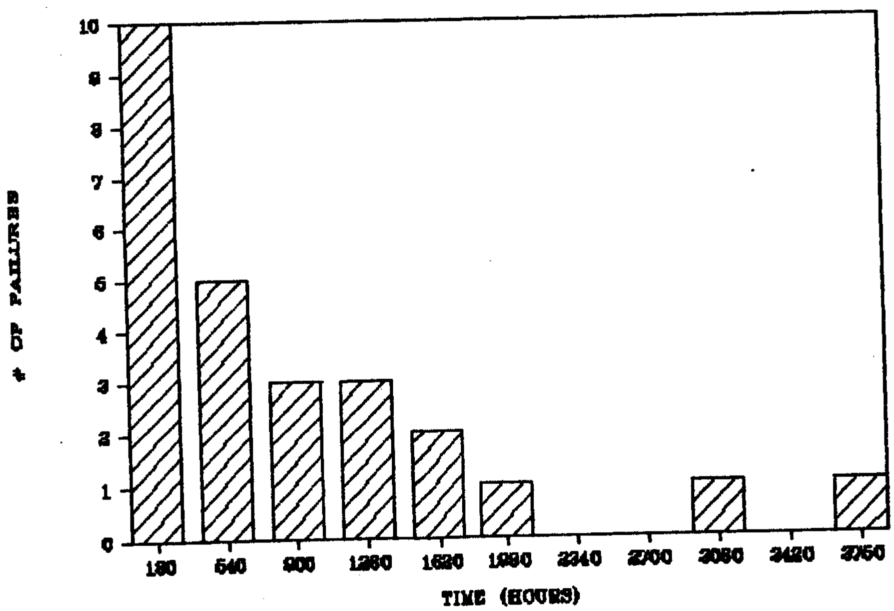


FIGURE iv

$$\Sigma(O-e)^2/e < 95\text{th percentile}$$

of the chi-square then observed distribution represents theoretical distribution. For the data

$$\Sigma(O-e)^2/e \text{ is } 10.8 <$$

the critical point 16.1, hence, the Weibull Model is acceptable.

5.3 MLE of the Weibull Parameters

The maximum likelihood function for the multicensored Weibull Model for r failures $t_1, t_2, .. t_r$ and $t_{r+1}, ... t_n$ censoring times is given by

$$L(t_i,..,t_n) = \prod_{i=1}^{r} \lambda\beta\, t_i^{\beta-1} \exp(-\lambda t_i^{\beta}) \prod_{i=r+1}^{n} \exp(-\lambda t_i^{\beta}) \qquad (38)$$

The log likelihood can be written as

$$\ln L = r\ln\lambda + r\ln\beta + (\beta-1)\sum_{i=1}^{r} \ln t_i - \lambda\sum_{i=1}^{n} t_i^{\beta} \qquad (39)$$

Taking partial derivatives with respect to λ and β and equating to zero, the MLE for λ is given by

$$\hat{\lambda} = r/\sum_{i=1}^{n} t_i^{\beta} \qquad (40)$$

and $\hat{\beta}$ is given by

$$\frac{1}{\beta} + \frac{1}{r}\sum_{i=1} \ln t_i - \left(\sum_{i=1}^{n} t_i^{\beta} \ln t_i\right) / \left(\sum_{i=1}^{n} t_i^{\beta}\right) = 0 \qquad (41)$$

For the controller data r = 26, n = 50, using $t_1, ... t_{50}$ the observations in the appendix 1 and solving (41) iteratively we find

$$\hat{\beta} = 1.108 \qquad (42)$$

and substituting $\hat{\beta}$ in (40) we find

$$\hat{\lambda} = 3.014 \times 10^{-4} \tag{43}$$

since β is greater than one there is little aging effect.

5.4 The Variances, MTBF, and Reliability

To find the estimated variance we find second derivatives of likelihood function as

$$\frac{\partial^2 \ln L}{\partial \lambda^2} = -r/\lambda^2 \tag{44}$$

$$\frac{\partial^2 \ln L}{\partial \lambda \partial \beta} = -\sum_{i=1}^{n} t_i^{\beta} \ln t_i = \frac{r}{\lambda \beta} + \frac{1}{\lambda} \sum_{i=1}^{r} \ln t_i \tag{45}$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = -r/\beta^2 - \lambda \sum_{i=1}^{n} t_i^{\beta} (\ln t_i)^2 \tag{46}$$

Using (44)-(46) and estimates in (42) and (43), the estimated information matrix can be obtained. The inverse of this matrix gives the variance covariance matrix. The variances of the estimators λ and β are

$$\hat{\text{Var}}\ (\hat{\lambda}) = 1.12 \times 10^{-7}$$

$$\hat{\text{Var}}\ (\hat{\beta}) = 2.45 \times 10^{-2}$$

The 95% confidence interval for λ and β are given by

$(-3.76 \times 10^{-4}, 9.79 \times 10^{-4})$

$(.800, 1.42)$

The estimated meantime for failures is

$$\hat{M}_T = \Gamma(1+1/\hat{\beta})/(\hat{\lambda})^{1/\hat{\beta}} = 1448.4 \text{ hrs.}$$

Since there are two channels for the controllers, the reliability for the two channels is given by

$$\hat{R}(t) = 2\,\hat{R}_1(t) - ((\hat{R}_1(t))^2 \tag{47}$$

The graph of $\hat{R}(t)$ against time t is shown in the figure (v). Also, the B1- life represents the operating time for which the reliability of the system is 99%. For the given data with Weibull Model B1 life is 197.5 hours.

6. Analysis of Failures of SSME Turbine Blades

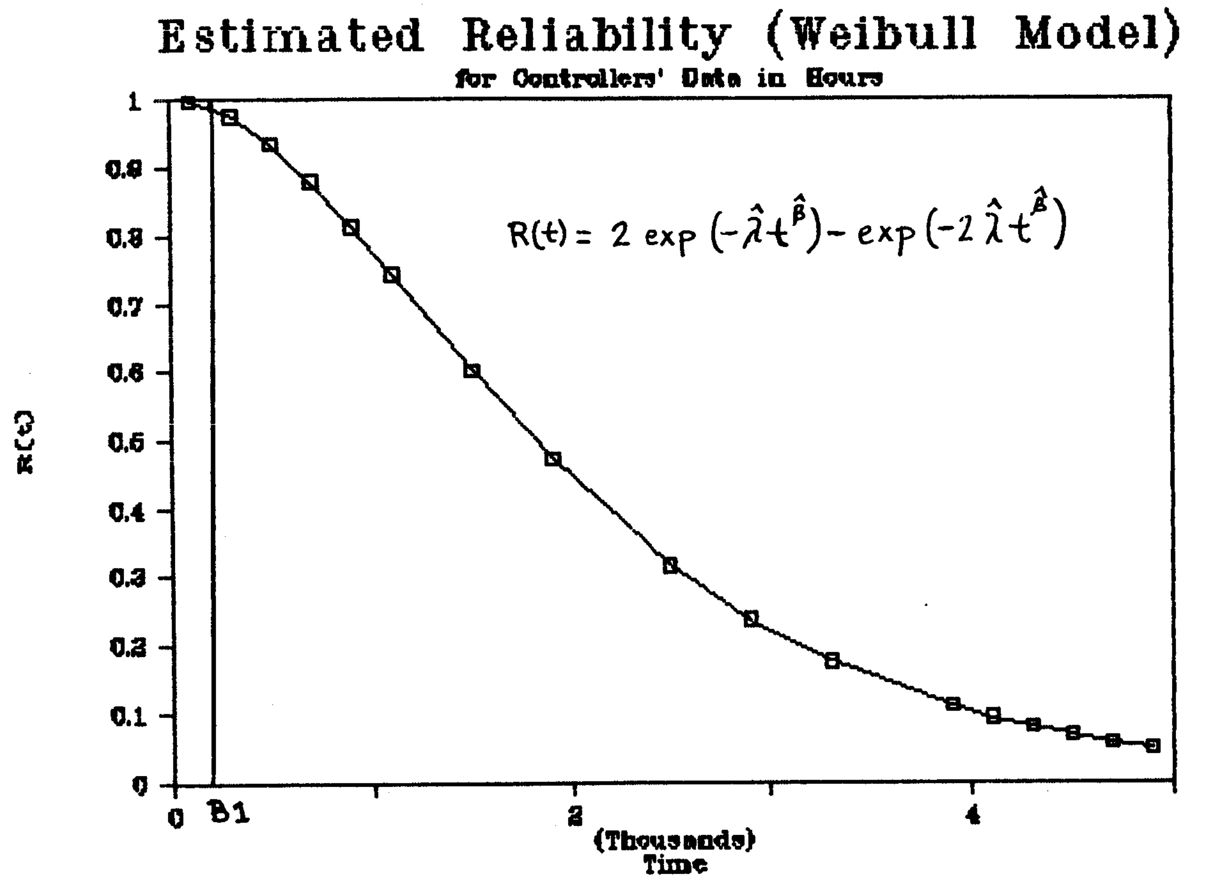
In this section we analyze the 1st stage turbine blade downstream shank cracks data for chambered blades.

6.1 Turbine Blade Failure Data

Data on HPOTP turbine blade cracks is provided by Billy Gonterman of Rockwell International. The data is given in the appendix 2. There were a total of 67 test runs of 78 blades in each run and cracks were observed at 100% power level of the engine. The exact failure times of cracks are not available, however, the interval in which cracks occur are observed. There were 10 runs in which a total of 15 cracks occurred, in 57 runs there were no cracks. These 57 times can be taken as censoring times. The data is analyzed as grouped data and also as randomized data.

6.2 Grouped Model

Following Rheinfurth (1986) we shall use the Weibull Model. Let K_i cracks be observed in the interval (t_{i1}, t_{i2}) i=1,2,..,10. Also, the censoring times for first 10 runs are $t_{i2} = t_i$, i=1,2,..10, t_i, i=11,...,67 are the censoring times for zero cracks. Using equation (4) the log likelihood under Weibull Model can be written as



Estimated Reliability (Weibull Model)
for Controllers' Data in Hours
$R(t) = 2\exp(-\hat{\lambda}t^{\hat{\beta}}) - \exp(-2\hat{\lambda}t^{\hat{\beta}})$
R(t)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
B1
2
4
(Thousands)
Time

FIGURE v

$$\ln L = \sum_{i=1}^{10} k_i \ln(\exp(-\lambda t_{i2}^{\beta}) - \exp(-\lambda t_{i1}^{\beta}))$$

$$- \lambda [78 \sum_{i=1}^{67} t_i^{\beta} - \sum_{i=1}^{10} k_i \; t_i^{\beta}] \tag{48}$$

Taking the partial derivatives of lnL and letting

$$V_{i1} = \exp(-\lambda t_{i1}^{\beta}), V_{i2} = \exp(-\lambda t_{i1}^{\beta}), \; V_i = V_{i1} - V_{i2}$$

we find the estimating equations for λ and β are

$$\frac{\partial \ln L}{\partial \lambda} = \sum_{i=1}^{10} k_i (V_i)^{-1} (-t_{i1}^{\beta} \; V_{i1} + t_{i2}^{\beta} \; V_{i2}) + \sum_{i=1}^{10} k_i \; t_i^{\beta}$$

$$-78 \sum_{i=1}^{67} t_i^{\beta} = 0, \tag{49}$$

$$\frac{\partial \ln L}{\partial \beta} = \sum_{i=1}^{10} k_i (V_i)^{-1} (-\lambda t_{i1}^{\beta} \; \ln t_{i1} \; V_{i1} +$$

$$\lambda t_{i2}^{\beta} \ln t_{i2} \; V_{i2})$$

$$-\lambda [- \sum_{i=1}^{10} k_i \; t_i^{\beta} \ln t_i + 78 \sum_{i=1}^{67} t_i^{\beta} \ln t_i] = 0 \tag{50}$$

Putting the values of k_i, t_{i1}, t_{i2} $i=1,\ldots,10$; t_i, $i=1,2..67$ from the appendix 2 equations (49) and (50) are solved simultaneously and iteratively for λ and β . First equation (50) is brought close to zero for various β and λ values then (49) was solved so that two successive values of λ trap the value of the equation to zero. The estimates of β and λ are

$$\hat{\beta} = 1.78 \tag{51}$$

$$\hat{\lambda} = 1.704439 \times 10^{-9} \tag{52}$$

For determining the variance we find,

$$-\frac{\partial^2 \ln L}{\partial \lambda^2} = \sum_{i=1}^{10} k_i (V_i)^{-2} (-t_{i1}^{\beta} V_{i1} + t_{i2}^{\beta} V_{i2})^2$$

$$- \sum_{i=1}^{10} k_i (V_i)^{-1} (t_{i1}^{2\beta} V_{i1} - t_{i2}^{2\beta} V_{i2}), \tag{53}$$

$$\frac{\partial^2 \ln L}{\partial \beta \partial \lambda} = -\sum_{i=1}^{10} k_i V_i^{-1} (-\lambda t_{i1}^{\beta} V_{i1} + t_{i2}^{\beta} V_{i2})$$

$$+ \lambda [78 \sum_{i=1}^{67} t_i^{\beta} \ln t_i - \sum_{i=1}^{10} t_i^{\beta} \ln t_i]$$

$$-\frac{\partial^2 \ln L}{\partial \beta^2} = \sum_{i=1}^{10} k_i (V_i)^2 (-\lambda t_i^{\beta} \ln t_{i1} V_{i1}$$

$$+ \lambda t_i^{\beta} \ln t_{i2} V_{i2})^2, \tag{54}$$

$$- \sum_{i=1}^{10} k_i (V_i)^{-1} (-\lambda t_{i1}^{\beta} (\ln t_{i1})^2 V_{i1} (1 - \lambda t_{i1}^{\beta}) +$$

$$\lambda t_{i2}^{\beta} (\ln t_{i2})^2 (1 - \lambda t_{i2}^{\beta}))$$

$$+(78 \sum_{i=1}^{67} t_i^{\beta} (\ln t_i)^2 - \sum_{i=1}^{10} k_i t_i^{\beta} (\ln t_i)^2) \tag{55}$$

Using equations (53) through (55), $\hat{\beta}$, $\hat{\lambda}$ given in (51) and (52) and data in appendix 2, we find the estimates of variances as

$$\text{Var } \hat{\lambda} = 1.647528 \times 10^{-12}$$

$$\text{Var } \hat{\beta} = 1.936758 \times 10^{-19}$$

Using the estimators of β and λ the MTBF is obtained as

$$\text{MTBF} = \frac{\Gamma(1 + 1/\hat{\beta})}{(\hat{\lambda})^{1/\hat{\beta}}} = 99667 \text{ secs.} = 27.69 \text{ hrs.}$$

The estimated reliability is given by

$$R(t) = \exp(-\hat{\lambda} t^{\hat{\beta}})$$

From the reliability we find B1 life as B1 = 7780 secs. = 2.16 hrs.

6.3 Randomized Model

In the grouped model to find the estimators two nonlinear simultaneous equations are needed to solve. The equations do not converge easily to the required degree. Also, the variances of estimators are large. An alternative approach is to assume that the individual failures occur randomly in the given interval and take these random values as the failure times and analyze the modified data.

The interval (t_{i1}, t_{i2}) contains k_i failures. Let $t_{i1}, t_{i2}, \ldots t_{ik_i}$ i=1,2..10 be the random failure times in the ith interval. Using these values the log likelihood for the Weibull Model can be written as:

$$\ln L = 10 \ln\lambda + 10 \ln\beta - (\beta-1) \sum_{i=1}^{10} \sum_{j=1}^{k_i} \ln t_{ij} - \lambda\left(\sum_{i=1}^{10} \sum_{j=1}^{k_i} t_{ij}^{\beta} + 78 \sum_{i=1}^{67} t_i^{\beta} - \sum_{i=1}^{10} k_i t_i^{\beta}\right) \tag{56}$$

Using (56) the estimator for λ is given by

$$\hat{\lambda} = 10/\left(78 \sum_{i=1}^{67} t_i^{\hat{\beta}} + \sum_{i=1}^{10} \sum_{j=1}^{k_i} t_{ij}^{\hat{\beta}} - \sum_{i=1}^{10} k_i t_i^{\hat{\beta}}\right) \tag{57}$$

and the estimating equation for β is

$$10/\hat{\beta} - \sum_{i=1}^{10} \sum_{j=1}^{k_i} \ln t_{ij}$$

$$- \left(78 \sum_{i=1}^{67} t_i^{\hat{\beta}} \ln t_i - \sum k_i t_i^{\beta} \ln t_i + \sum_{i=1}^{10} \sum_{j=1}^{k_i} t_{ij}^{\beta} \ln t_{ij}\right) /$$

$$\left(78 \sum_{i=1}^{67} t_i^{\beta} - \sum_{i=1}^{10} k_i t_i^{\beta} + \sum_{i=1}^{10} \sum_{j=1}^{k_i} t_{ij}^{\beta}\right) = 0 \tag{58}$$

For the information matrix we find

$$- \frac{\partial^2 \ln L}{\partial \lambda^2} = k/\lambda^2 \tag{59}$$

$$- \frac{\partial^2 \ln L}{\partial \beta \partial \lambda} = \left((k/\beta) - \sum_{i=1}^{10} \sum_{j=1}^{k_i} \ln t_{ij}\right)/\lambda \tag{60}$$

$$- \frac{\partial^2 \ln L}{\partial \beta^2} = k/\beta^2 - \lambda \left(78 \sum_{i}^{67} t_i^{\beta} (\ln t_i)^2 - \sum_{i=1}^{10} k_i t_i^{\beta} (\ln t_i)^2\right.$$

$$\left. + \sum_{i=1}^{10} \sum_{j=1}^{k_i} t_{ij}^{\beta} (\ln t_{ij})^2 \right) \tag{61}$$

Using (57), (58), and the data in the appendix 2 and the random numbers from random number generator program, the estimators for β and λ are obtained as

$\hat{\beta} = 1.8002$
$\hat{\lambda} = 1.8869 \times 10^{-9}$
$\hat{\eta} = 70187$ secs.

The estimates of variances and related quantities are

Var $\hat{\beta} = .120$
Var $\hat{\lambda} = 2.901 \times 10^{-17}$
MTBF = 62376.81 secs. = 17.32 and B1 life is 1.513 hrs.

The variances in this case are smaller indicating the observations have smaller spread. Also, the convergence for the equation in β is fast.

7. Conclusions

For the controller failure data Weibull Models fits well. The MTBF assuming censored Weibull Model is 1,448 hours. If one uses simple Exponential Model, MTBF is 881. It is advisable to use censored models which take into account the time for the units which did not fail. The B1 life using Weibull Model is 197.5 hours.

For SSME blade failures using grouped Weibull Model MTBF obtained is 27.69 hours. The variances of the estimators are also obtained for the parameters in MTBF. The B1 life is 2.16. The drawback of the method is that to find the estimators one needs to solve two simultaneous nonlinear equations. Alternatively the randomly placed model can be used. For this method MTBF is 17.32 hours and B1 life is 1.5 hours. This method depends on seed numbers used in the random number generators so it is better to make number of runs with different seed points and average the results.

Other models like Gamma Model may give the better fit for controller failure data. The maximum likelihood estimating equations involve incomplete gamma functions solving these equations need sophisticated programming techniques. These problems need further investigation.

REFERENCES

1. Abernethy, Ken, "A Monte Carlo Study of Weibull Reliability Analyses for Space Shuttle Main Engine Components," (1985) Summer Faculty Report.

2. Lawless, J.F., "Statistical Models and Methods For Life Time Data," (1982) Wiley Series in Probability and Mathematical Statistics.

3. Nelson, Wayne, "Hazard Plotting For Incomplete Failure Data," (1969) Journal of Quality Technology.

4. Patil, S.A., "Reliability Models Applicable to Space Telescope Solar Array Assembly System," (1985) Summer Faculty Report.

5. Rheinfurth, Mario, "Weibull Distribution Based on Maximum Likelihood with Interval Inspection Data," (1985) NASA Technical Memorandum.

APPENDIX 1

Controllers' Failure Data

CONTROLLER	# FAILED	FAILURE TIME (HOURS)	CENSORING TIME (HOURS)
1 P7	1	3683	485
2 P8	2	608	878
		1324	
3 F5	2	740	318
		2977	
4 F6	1	150	4171
5 F4	5	232	142
		300	
		344	
		842	
		1609	
6 F9	1	1727	0
7 F10	1	200	2098
8 F11	1	2053	23
9 F12	1	1270	287
10 F13	1	1121	729
11 F15	2	153	80
		900	
12 F16	1	505	807
13 F17	1	313	354
14 F18	2	323	7
		456	
15 F19	1	452	720
16 F20	0	-	799
17 F21	1	419	140
18 F22	0	-	1192
19 F23	0	-	612
20 F24	0	-	577
21 F25	0	-	499
22 F26	0	-	497
23 F27	0	-	828
24 F28	2	85	365
		121	
25 F29	0	-	227
TOTALS=	26	22907	16835

APPENDIX 2

Chamfered Blades' Failure Data

RUN #	# OF CRACKS	INSP. TIME 1 (SECS)	INSP. TIME 2 (SECS)	CENSOR. TIME (SECS)
1	4	4000	5996	5996
2	1	2100	4252	4252
3	1	1100	3441	3441
4	1	1200	3360	3360
5	1	1100	3281	3281
6	3	0	3112	3112
7	1	0	1767	1767
8	1	0	1859	1859
9	1	0	1500	1500
10	1	0	1260	1260
11	0	0	-	7840
12	0	0	-	6332
13	0	0	-	5982
14	0	0	-	4652
15	0	0	-	4563
16	0	0	-	4135
17	0	0	-	3550
18	0	0	-	3500
19	0	0	-	3491
20	0	0	-	3310
21	0	0	-	3241
22	0	0	-	3211
23	0	0	-	3155
24	0	0	-	3138
25	0	0	-	3101
26	0	0	-	3077
27	0	0	-	2946
28	0	0	-	2888
29	0	0	-	2868
30	0	0	-	2827

RUN #	# OF CRACKS	INSP. TIME 1 (SECS)	INSP. TIME 2 (SECS)	CENSOR. TIME (SECS)
31	0	0	-	2810
32	0	0	-	2771
33	0	0	-	2584
34	0	0	-	2448
35	0	0	-	2349
36	0	0	-	2085
37	0	0	-	1859
38	0	0	-	1857
39	0	0	-	1825
40	0	0	-	1792
41	0	0	-	1767
42	0	0	-	1609
43	0	0	-	1555
44	0	0	-	1509
45	0	0	-	1491
46	0	0	-	1427
47	0	0	-	1332
48	0	0	-	1307
49	0	0	-	1305
50	0	0	-	1273
51	0	0	-	1260
52	0	0	-	1231
53	0	0	-	1016
54	0	0	-	1010
55	0	0	-	932
56	0	0	-	895
57	0	0	-	750
58	0	0	-	721
59	0	0	-	666
60	0	0	-	525
61	0	0	-	392
62	0	0	-	372
63	0	0	-	300
64	0	0	-	252
65	0	0	-	250
66	0	0	-	250
67	0	0	-	250