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A High Resolution Shock Capturing Scheme for High Mach Number Internal Flow

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A HIGH RESOLUTION SHOCK CAPTURING SCHEME FOR HIGH MACH NUMBER INTERNAL FLOW

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SUMMARY

An accurate shock capturing scheme is developed for high speed flow. The flux limited dissipation scheme can be converted to a symmetric TVD scheme in the case of a scalar conservation law. Application to an oblique shock wave problem shows the improved resolution and accuracy when compared to the results obtained by the adaptive dissipation method, while exhibiting comparable computational efficiency.

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INTRODUCTION

It has been a difficult task for computational aerodynamicists to capture the nonoscillatory shock waves. Unbounded growth of spurious oscillations often resulted in numerical instability. Central difference schemes produce good solutions in smooth regions of the flow, but are prone to oscillations in the neighborhood of shock waves. It is well-known that these wiggles can be suppressed by the introduction of a carefully controlled blend of first and third order numerical dissipation terms. Recently the authors showed that the strong oblique shock waves in high Mach number flows can be captured by the method of adaptive dissipation (ref. 1).

It has long been recognized that upwind differencing can eliminate undesirable oscillations near shock waves. Stemming from the mathematical theory of scalar conservation laws, Harten proposed the concept of total variation diminishing (TVD) schemes (ref. 2). TVD schemes preserve the monotonicity of an initially monotone profile, because the total variation would increase if the profile ceased to be monotone. Second order schemes can be constructed by using multipoint extrapolation formulas to estimate the numerical flux, or by adding higher order dissipative terms. In either case flux limiters are then needed to control the signs of the coefficients of a semi-discrete approximation to the hyperbolic system (refs. 2 to 5). However, TVD schemes in two or more space dimensions are at most first order accurate (ref. 6). Nevertheless, two-dimensional results using one-dimensional second order TVD schemes and dimensional splitting show sharp resolution of discontinuities without oscillations (ref. 7). Jameson constructed an efficient scheme which is converted to a TVD scheme in the case of a scalar conservation law using flux limited dissipation (ref. 5). In this paper, we introduce a modified form of the scheme and apply it to high Mach number flow calculations. Computed solutions are compared with the results obtained by the adaptive dissipation. The LU implicit scheme (ref. 1) is used as a baseline algorithm.

GOVERNING EQUATIONS

The Euler equations are obtained from the Navier-Stokes equations by neglecting viscous terms. Let ρ, u, v, E, H, and p be the density, Cartesian velocity components, total energy, total enthalpy, and pressure, and let x and y be Cartesian coordinates. Then for a two-dimensional flow these equations can be written as

$$\frac{\partial W}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0 \tag{1}$$

where W is the vector of dependent variables, and F and G are convective flux vectors

$$\begin{aligned} W &= (\rho,\rho u,\rho v,\rho E)^T \\ F &= (\rho u,\rho u^2 + p,\rho vu,u(\rho E + p))^T \\ G &= (\rho v,\rho uv,\rho v^2 + p,v(\rho E + p))^T \end{aligned} \tag{2}$$

The pressure is obtained from the equation of state

$$p = \rho(\gamma - 1)\left\{E - \frac{1}{2}(u^2 + v^2)\right\} \tag{3}$$

These equations are to be solved for a steady state $\partial W/\partial t = 0$ where t denotes time.

SEMI-DISCRETE FINITE VOLUME METHOD

A convenient way to assure a steady state solution independent of the time step is to separate the space and time discretization procedures. In semi-discrete finite volume method one begins by applying a semi-discretization in which only the spatial derivatives are approximated. The use of a finite volume method for space discretization allows one to handle arbitrary geometries and helps one to avoid problems with metric singularities that are usually associated with finite difference methods. The scheme reduces to a central difference scheme on a Cartesian grid, and is second order accurate in space provided that the mesh is smooth enough. It also has the property that uniform flow is an exact solution of the difference equations.

LU IMPLICIT SCHEME

Let the Jacobian matrices be

$$A = \frac{\partial F}{\partial W} , \qquad B = \frac{\partial G}{\partial W}$$

and let the correction be

$$\delta W = W^{n+1} - W^{n}$$

where n denotes the time level.

The linearized implicit scheme for a system of nonlinear hyperbolic equations such as the Euler equations can be formulated as

$$\left\{ I + \beta \, \Delta t \, (D_x A + D_y B) \right\} \delta W + \Delta t \, R = 0 \tag{4}$$

where R is the residual

$$R = D_x F(W^n) + D_y G(W^n)$$

Here D_x and D_y are central difference operators that approximate $\partial/\partial x$ and $\partial/\partial y$.

If $\beta = 1/2$ the scheme remains second order accurate in time, for other values of β, the time accuracy drops to first order. The unfactored implicit scheme (eq. (4)) produces a large block banded matrix which is very costly to invert and requires huge storage. An unconditionally stable implicit scheme which has error terms at most of order $(\Delta t)^2$ in any number of space dimensions can be derived by the LU factorization.

$$\left\{ I + \beta \, \Delta t \, (D_x^- A^+ + D_y^- B^+) \right\} \left\{ I + \beta \, \Delta t \, (D_x^+ A^- + D_y^+ B^-) \right\} \delta W + \Delta t \, R = 0 \tag{5}$$

where D_x^- and D_y^- are backward difference operators and D_x^+ and D_y^+ are forward difference operators. The reason for splitting is to ensure the diagonal dominance of lower and upper factors as well as to make use of the built-in implicit dissipation.

Here, A^+, A^-, B^+, and B^- are constructed so that the eigenvalues of "+" matrices are nonnegative and those of "−" matrices are nonpositive.

$$\begin{aligned} A^+ &= \frac{1}{2}(A + r_A I), \qquad & A^- &= \frac{1}{2}(A - r_A I) \\ B^+ &= \frac{1}{2}(B + r_B I), \qquad & B^- &= \frac{1}{2}(B - r_B I) \end{aligned} \tag{6}$$

where

$$r_A \geq \max(|\lambda_A|), \qquad r_B \geq \max(|\lambda_B|) \tag{7}$$

Here, λ_A and λ_B represent eigenvalues of Jacobian matrices. Equation (5) can be inverted in two steps. The LU implicit scheme needs the inversion of sparse triangular matrices which can be done efficiently without using large storage. This scheme has only two factors in three dimensions.

FLUX LIMITED DISSIPATION SCHEME

Consider the scalar conservation law

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x} f(u) = 0 \tag{8}$$

It is well known that the total variation

$$TV = \int_{-\infty}^{\infty} \left|\frac{\partial u}{\partial x}\right| dx$$

cannot increase. Suppose now that a multi-point semi-discrete approximation to equation (8) is expressed in the form

$$\frac{du_i}{dt} = \sum_{q=-Q}^{Q-1} c_q(i)(u_{i-q} - u_{i-q-1}) \tag{9}$$

The discrete total variation is

$$TV = \sum_{i=-\infty}^{\infty} |u_i - u_{i-1}|$$

It can be shown (ref. 8) that this will not increase if and only if

$$c_{-1}(i-1) \geq c_{-2}(i-2). \; . \; . \geq c_{-Q}(i-Q) \geq 0$$

and (10)

$$-c_0(i) \geq -c_1(i+1). \; . \; . \geq -c_{Q-1}(i+Q-1) \geq 0$$

Now consider the scheme

$$\frac{du_i}{dt} + \frac{1}{\Delta x}(h_{i+1/2} - h_{i-1/2}) = 0 \tag{11}$$

where $h_{i+1/2}$ is an approximation to the flux across the boundary between the $(i+1)^{st}$ and i^{th} cells. Denoting $f(u_i)$ by f_i, define the numerical flux as

$$h_{i+1/2} = \frac{1}{2}(f_{i+1} + f_i) + d_{i+1/2} \tag{12}$$

where $d_{i+1/2}$ is a dissipative flux.

$$d_{i+1/2} = \alpha_{i+1/2}\left\{B(e_{i+3/2}, e_{i+1/2}) - 2\, e_{i+1/2} + B(e_{i+1/2}, e_{i-1/2})\right\} \tag{13}$$

where $e_{i+1/2} = \rho_{i+1} - \rho_i$ for example.

B (p,q) is known as Roe's min mod function (ref. 4) and can be defined as

$$B(p,q) = \left\{s(p) + s(q)\right\} \min(|P|, |q|) \tag{14}$$

where

$$s(p) = \begin{cases} \frac{1}{2} \text{ if } P \geq 0 \\ -\frac{1}{2} \text{ if } P < 0 \end{cases}$$

Here

$$\alpha_{i+1/2} = \min\left(1/2,\ k_o + k_1 \bar{\nu}_{i+1/2}\right)(R_{i+1} - R_i) \qquad (15)$$

where k_o and k_1 are the constants and

$$\bar{\nu}_{i+1/2} = \max(\nu_{i+1/2}, \nu_{i+1}, \nu_i, \nu_{i-1})$$

$$\nu_i = \left| \frac{P_{i+1} - 2P_i + P_{i-1}}{P_{i+1} + 2P_i + P_{i-1}} \right|$$

The spectral radius R can be estimated as the value of

$$R = | \Delta y u - \Delta x v | + c\sqrt{\Delta x^2 + \Delta y^2}$$

on the edge separating cells (i+1,j) and (i,j) in two-dimensions. c is the speed of sound.

RESULTS

A two-dimensional schematic high speed inlet is selected as a model problem to test the performance of the new shock capturing scheme. A 54 by 32 H-mesh is shown in figure 1. Figure 2 shows the plot of Mach number contours for the Mach 5 flow using the adaptive dissipation method. Improved result by the TVD scheme is shown in figure 3. The plots of Mach number along the centerline are compared in figure 4 for the adaptive dissipation method and in figure 5 for the TVD scheme. While both methods capture nonoscillatory shock waves, the results of the TVD scheme show sharper resolution and improved accuracy. Considering that the computational mesh lines are not aligned with the oblique shock wave, the result seems to be satisfactory. The convergence rate for the TVD scheme (fig. 7) is found to be comparable to that for the adaptive dissipation method (fig. 6). Moreover, the computational work per cycle of the present symmetric TVD scheme is comparable to that of the adaptive dissipation scheme. In general, symmetric TVD schemes offer a significant reduction of computational complexity in comparism with upwind TVD schemes, while exhibiting comparable shock capturing capabilities.

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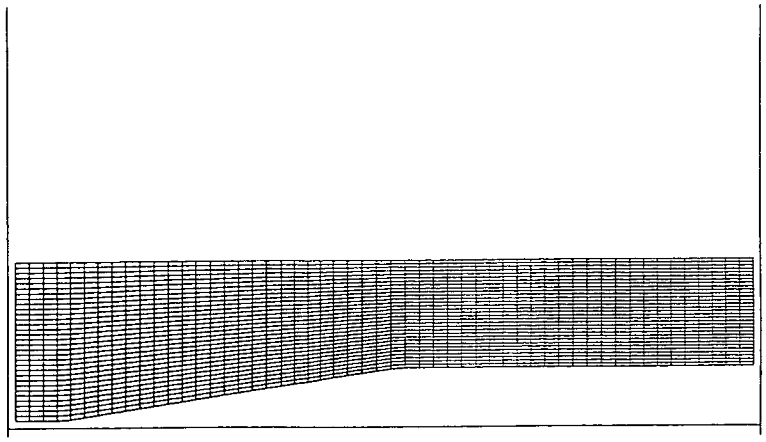

FIGURE 1. - COMPUTATIONAL GRID FOR A SCHEMATIC HIGH SPEED INLET.

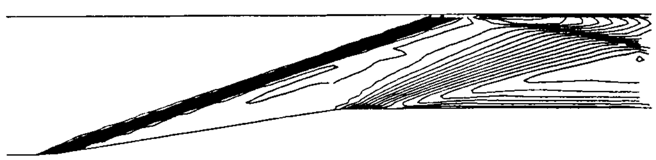

FIGURE 2. - MACH NUMBER CONTOURS FOR MACH 5 FLOW (ADAPTIVE DISSIPATION SCHEME).

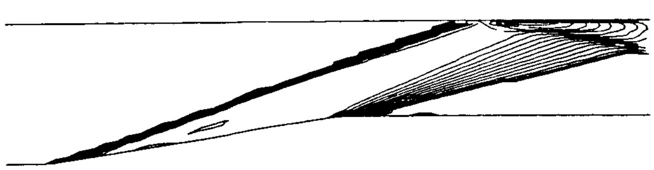

FIGURE 3. - MACH NUMBER CONTOURS FOR MACH 5 FLOW (PRESENT METHOD).

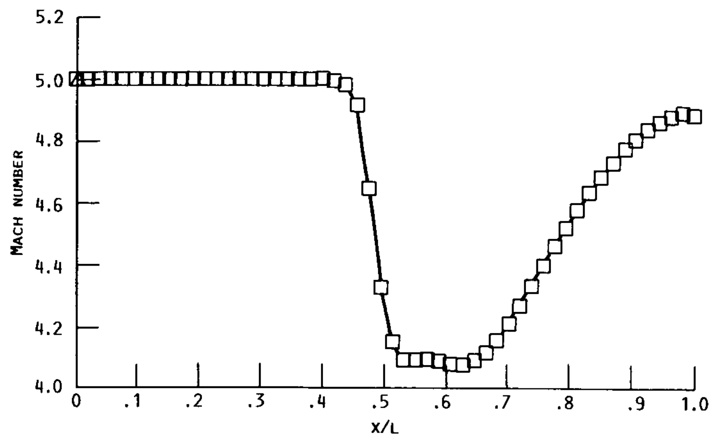

FIGURE 4. - MACH NUMBER ALONG THE CENTERLINE (ADAPTIVE DISSIPATION SCHEME).

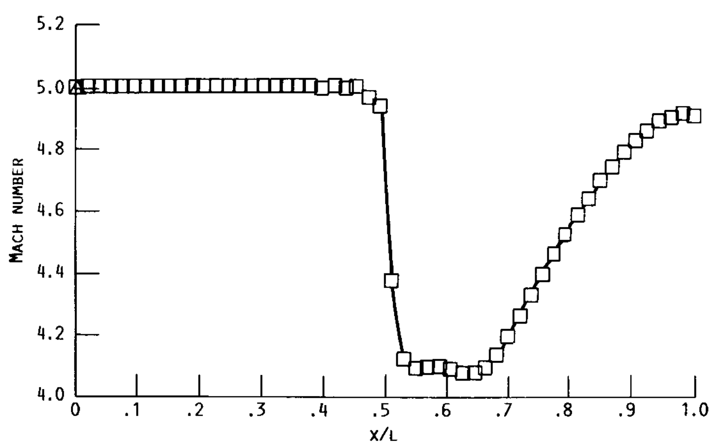

FIGURE 5. - MACH NUMBER ALONG THE CENTERLINE (PRESENT METHOD).

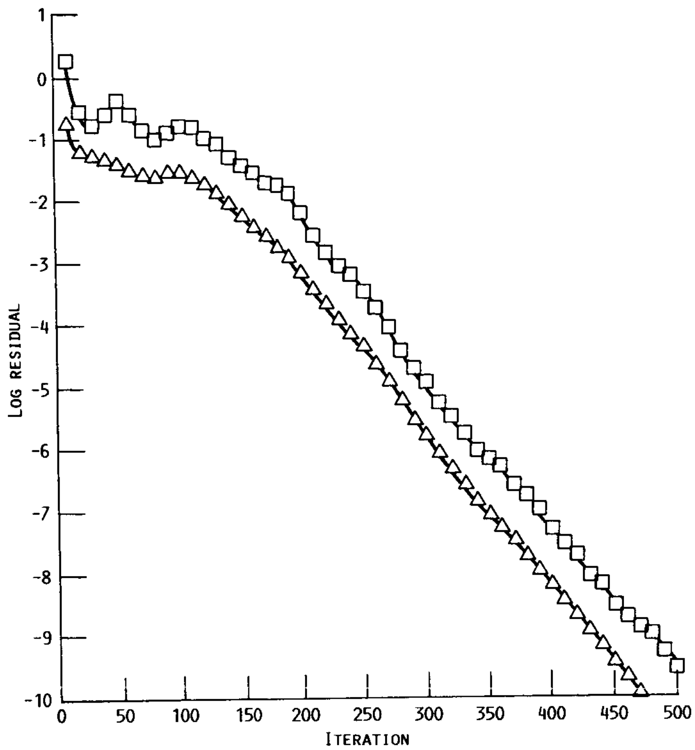

FIGURE 6. - CONVERGENCE HISTORY (ADAPTIVE DISSIPATION SCHEME).

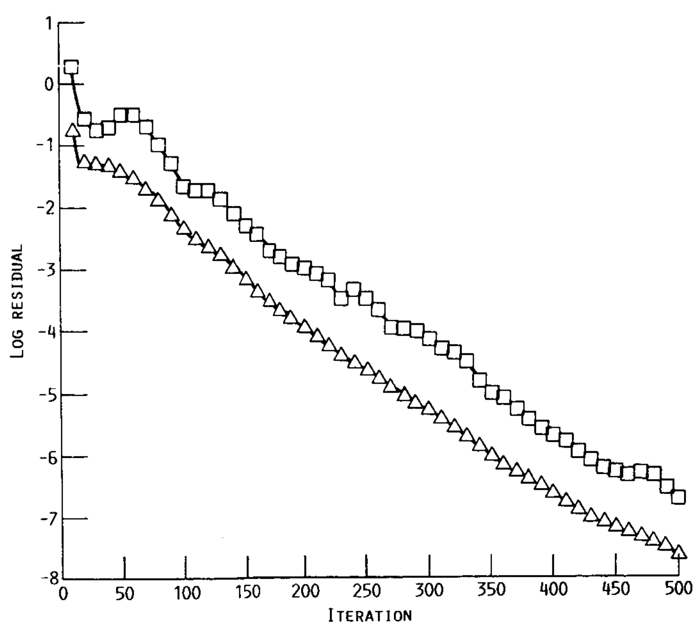

FIGURE 7. - CONVERGENCE HISTORY (PRESENT METHOD).

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