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CONVECTIVE FORCING OF GLOBAL CIRCULATIONS ON THE JOVIAN PLANETS

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Examples of convection in rotating layers are presented to illustrate how convection can drive global circulations on the Jovian planets. One example is derived from an analytical solution for the onset of convection in a rotating, ideal fluid layer (Hathaway, 1984, Ap. J. *276, 316-324.) For rapid rotation the convective motions become largely two-dimensional and produce Reynold stresses which drive large scale flows. The initial tendency is to produce a prograde equatorial jet and a meridional circulation which is directed toward the poles in the surface layers. Fully nonlinear numerical simulations for the slowly rotating solar convection zone show that the meridional circulation does not reach the poles (Glatzmaier, 1984,* J. Comp. Phys *55, 461-484). Instead a multicellular meridional circulation is produced which has a downward flowing branch in the mid-latitudes. For more rapidly rotating objects such as Jupiter and Saturn this meridional circulation may consist of a larger number of cells. Axisymmetric convective models then show that prograde jets form at the downflow latitudes. A nonlinear numerical simulation of convection in a prograde jet is presented to illustrate the interactions which occur between convection and these jets. Without rotation the convection removes energy and momentum from the jet. With rotation the convection feeds energy and momentum into the jet. The conversion rates are similar to those found for motions in the vicinity of Jupiter's North Temperate Belt. A movie of this simulation will be shown.*

Those of us who would like to construct realistic models for the dynamics of Jupiter's atmosphere and interior are faced with a bit of a dilemma...actually it's a major problem. There is a very wide range of length scales that are dynamically important for forcing flows on the giant planets. The planetary rotation is important for a broad range of lengths from the global scale of tens of thousands of kilometers down to a scale of 100 kilometers or less. What I would like to present today are two examples of these dynamically important flows. The first is from an analytical model that I have developed with the hope of getting a handle on how to parameterize small scale convection in a global circulation model. The second example is a numerical simulation of convection in a zonal flow which illustrates the interactions between the convection, rotation and a zonal shear flow.

The first slide shows an example of the type of flow that is produced from the analytical solution of the equations of motion for convection in a rotating layer. The solution is for an ideal fluid, that is, a fluid without dissipation, in a rotating layer in which the rotation vector is tilted from the vertical to represent various latitudes (Hathaway, 1984). This example represents a fairly rapidly rotating case in which the ratio of the buoyancy time to the rotation period is about three. What is found is that the fluid tends to flow upward and toward the pole in a direction nearly parallel to the rotation axis but with much spiraling about this trajectory. Much of the kinetic energy in the flow is in the horizontal spiraling motions rather than the vertical flow.

This solution is used to produce a stress tensor that can be employed in a global circulation model for the large scale flows. The stress tensor is formed by taking the product of the different components of the fluid velocity and then averaging over the volume of the convective eddy. The second slide (Fig. 1) shows the latitude dependence of the stress tensor components for cells like those shown in the first slide. The curves are labeled with the tensor components in spherical polar coordinates. Comparing the diagonal components shows that there is about a factor of 10 more kinetic energy in the horizontal motions than in the vertical motions for this case where rotation is three times faster than buoyancy. More important for forcing the global circulations are the off diagonal components, the products of eastward and northward velocities or northward and radial velocities. The stress tensor shown here gives an equatorward flux of zonal momentum, given by the $\theta\phi$ component, which converges at the equator and would tend to produce a rapidly rotating equator. There is also an upward flux of latitudinal momentum that would tend to produce a meridional circulation that is directed toward the poles in the surface layers.

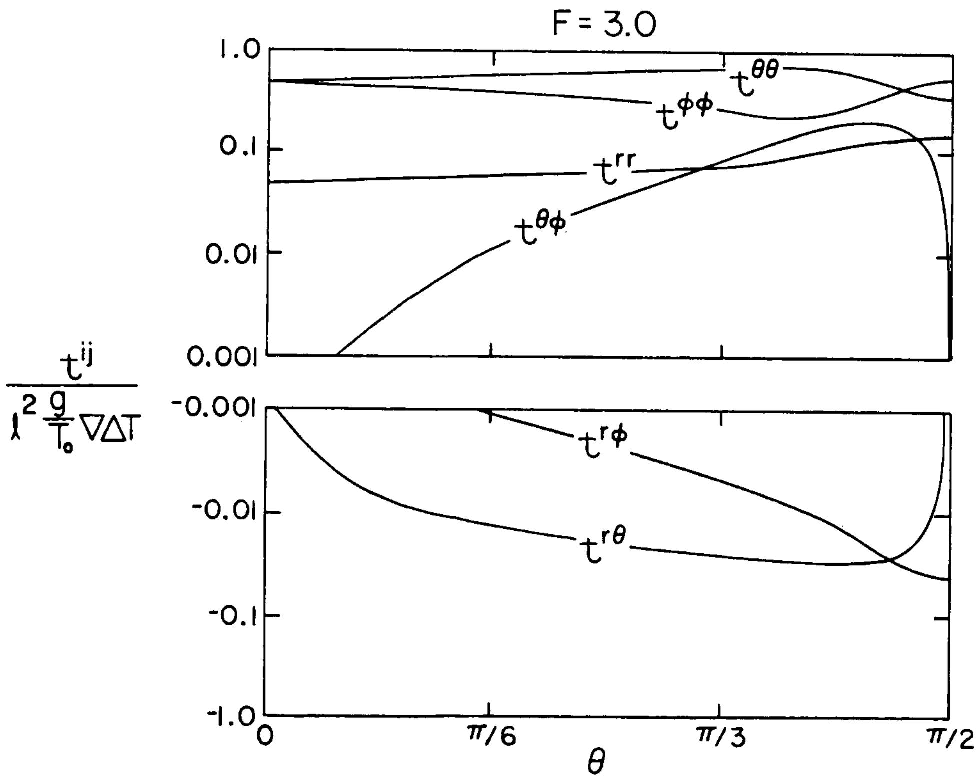

Figure 1. Latitude dependence of the stress tensor components.

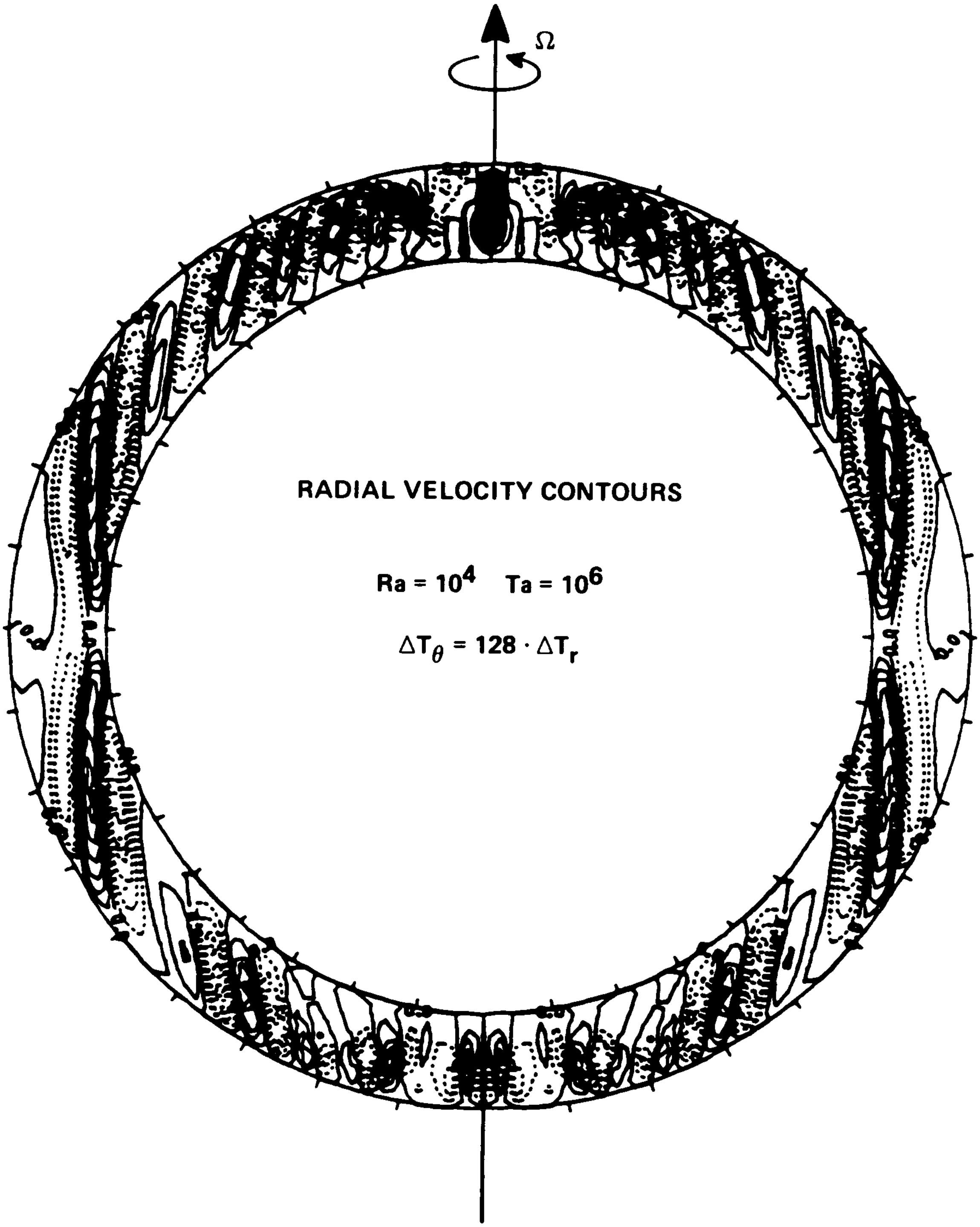

Figure 2. Contour plot of the radial velocity for an axisymmetric flow forced by a thermal gradient.

This linear convection model only gives the initial tendencies for the global circulation. The actual flows produced by this stress tensor can only be determined from a global circulation model which includes both the stress tensor and the nonlinear interactions between the various flow components. A variety of effects and feedbacks can alter the form of the global circulation from what might be expected given the form of the stress tensor. Fully non-linear models which explicity calculate the convection as well as the global circulation have been produced by Gilman (1977) and Glatzmaier (1984) for the Sun. They find that for the Sun, which is a slow rotator, the stresses produced by the convection do, in fact, produce a rapidly rotating equator and a meridional circulation which is directed toward the poles in the surface layers. However, the meridional circulation does not travel all the way to the poles but instead turns inward at midlatitudes. I propose that on rapidly rotating objects like Jupiter and Saturn this process may go even further and break up the meridional circulation into a series of cells. If I cheat a bit (actually I have to cheat a lot) I can produce a circulation that does just that.

The third slide (Fig. 2) shows the result of a calculation for an axisymmetric flow which is forced by a thermal gradient rather than by a stress tensor. Here the equator is hot and the poles are cold. This tends to drive a meridional circulation in a manner similar to that of the stress tensor. With rapid rotation, here rotation is ten times faster than buoyancy, the meridional circulation breaks up into a series of cells whose sides are nearly parallel to the rotation axis. This slide shows a contour plot of the radial velocity. Associated with this radial flow is a zonal flow in the form of a series of prograde and retrograde jets. Prograde jets are formed in the downdrafts and retrograde jets are formed in the updrafts essentially by conservation of angular momentum in the flow. In this calculation I don't get a rapidly rotating equator because I only include the axisymmetric motions and don't have the stress tensor forcing by the small scale convection.

The second part of this talk is concerned with what happens to small scale convection which is imbedded in zonal shear flows like those produced in the meridional circulation shown in this last slide. The fourth slide (Fig. 3) shows the basic idea behind this study. Consider a zonal flow with a jet-like profile in latitude together with convection in the form of a chevron pattern. (Cloud patterns of this type can be seen on Jupiter, particularly around the 23 deg N latitude jet.) Without rotation the fluid rises along the axes of these convective rolls, spreads outward at the top and then sinks downward in the downdrafts. We find that there is a correlation between the velocity components such that westward flows are associated with flow into the jet maxima and eastward flows are associated with flows into the jet minima. This process extracts momentum and energy from the zonal jet. We get the opposite effect if we look at what happens with rotation. If we take all these horizontal vectors and turn them by the Coriolis force then the velocity correlations are reversed. The fluid moving into the jet maxima is moving to the east and the fluid moving into the jet minima is moving to the west. These velocity correlations feed momentum and energy into the jet.

To see what type of convection pattern is formed, and how it interacts with the jet, Richard Somerville and I have run some numerical calculations for

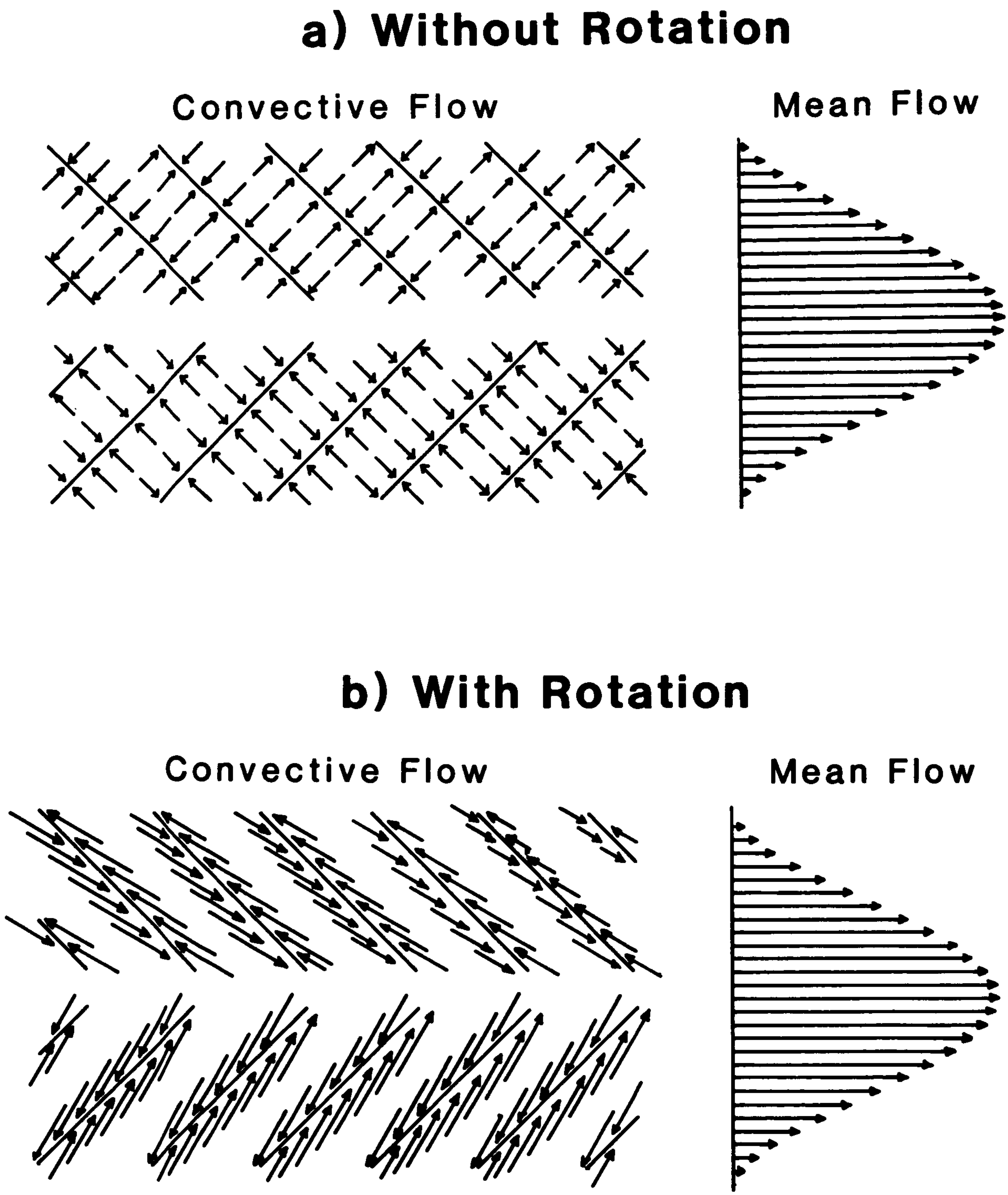

Figure 3. Schematic depiction of correlation between convective velocity components and the zonal mean flow.

three-dimensional and time-dependent convection in a jet-like zonal flow. This prescribed zonal flow has a sine-squared profile in latitude and is constant with depth. The computational domain is periodic in both horizontal directions so the jet profile repeats to the north and south. The jet maximum (eastward) is located halfway between the northern and southern boundaries.

The next slide shows the convective motions for the nonrotating case. On either side of the jet maximum, where the shear is strongest, convection cells are produced which are aligned with the shear flow in an east/west direction. A chevron pattern is not produced in this nonrotating case.

The next slide shows the convective motions for a moderately rotating case in which the rotation period and the buoyancy time are about equal. Here the convection starts to form a chevron pattern and the fluid motion is turned by the Coriolis force so that the velocity components are correlated in the same sense as suggested by the slide shown earlier (Fig. 3). The next slide shows a simulation in which the rotation rate has been increased slightly. Here the chevron pattern is more striking and there is a larger correlation between the velocity components that would feed momentum into the jet. The next slide shows a rapidly rotating case in which the convection is actually quenched where the shear is large. The tilted rotation vector stabilizes convective rolls oriented east/west while the shear flow stabilizes convective rolls oriented north/south. Where these two processes compete the convection is inhibited leaving convection only in the maximum of the jet where the shear vanishes.

I now have 3000 transparencies to show you in the form of a movie. The first part of the movie shows the basic state--hot on the bottom, cool on the top and with the imposed zonal flow. Again, the domain is periodic in both horizontal directions so that fluid that flows off the east end reappears at the west end and flow off the north face returns at the south face.

The second part of the movie shows the nonrotating case. This was started from the basic state with small random perturbations in the temperature field. It takes a while for the convection to get started but a pattern starts to form after a while. This pattern has convective rolls aligned with the shear where the shear is strong and then ridges or cells that move along with the flow in the flow maximum. On the two vertical faces of the domain you can see the overturning motions.

DR. STONE: Does the convection interact with the mean flow here or not?

DR. HATHAWAY: It does. I have the imposed mean flow and an additional mean flow is produced by the convection. In the following slides I'll show what the mean flow actually looks like.

The third part of the movie shows a rotating case which was also started from the basic state with small random perturbations in the temperature field. The convection takes a bit longer to get started here because of the stabilizing effects of rotation. We first see the wave-like features in the jet maximum and somewhat later the convection on either side where the shear is stronger.

But it does end up forming a chevron pattern by cells that are tilted into the flow where the shear is strong. It's a bit hard to see exactly how the velocities are correlated in these cells, but the slides I'll show afterwards indicate that there is a flux of momentum into the jet rather than out of it.

DR. LEOVY: Is that flux both in the horizontal and the vertical or just in the horizontal?

DR. HATHAWAY: Primarily in the horizontal, but there is an added twist to the story at the end.

That's the end of the movie. We should now go on to the next slide.

This slide (Fig. 4) shows the mean flow for the nonrotating case. Latitude increases upward and eastward flows are plotted toward the right. The dotted line is the imposed zonal flow. The solid line is the zonal flow after the convection has had a chance to interact with the flow. For this nonrotating case eastward momentum is taken out of the peak of the jet and deposited in the troughs so that there is less momemtum and kinetic energy in the mean flow. In the right hand panel of this slide are plotted the quantities that others have used in analyzing the Voyager images of Jupiter and Saturn. The latitudinal flux of zonal momentum is shown with a dashed line and the gradient of the mean zonal flow is shown with a solid line. These two quantities are anti-correlated indicating that the flux is down the gradient. Momentum is removed from the jet and added to the convection.

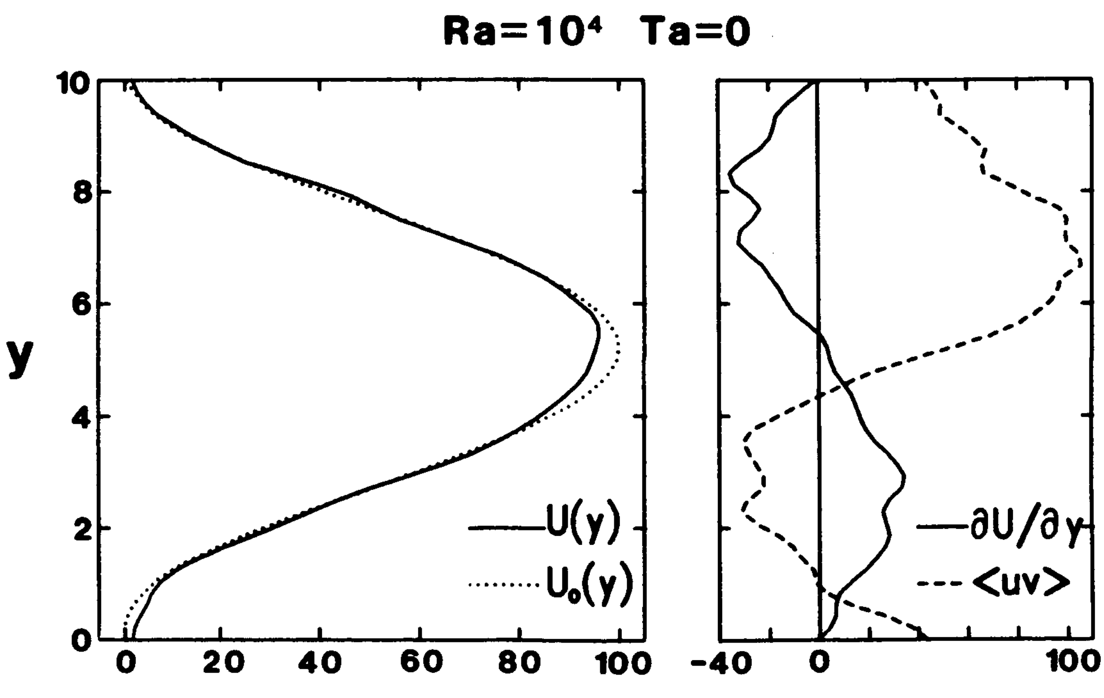

Figure 4. Zonal mean velocity, its latitudinal gradient, and the eddy momentum flux as computed for a nonrotating model.

The next slide (Fig. 5) shows the same quantities for the rotating case shown in the movie. The left hand panel shows that the mean flow is slightly enhanced although the effect is not very large. However, the right hand panel shows a positive correlation between the momentum flux and the momentum gradient which indicates a flux of momentum up the gradient that feeds momentum into the jet. If I apply this model to Jupiter, taking the depth of the layer as 1000 km and the distance between jet maxima as 10,000 km, then the jet has a maximum velocity of about 100 m s^{-1} and the momentum flux is as high as 35 m^2s^{-2}. This corresponds well with the magnitude of the momentum flux found by Ingersoll *et al.* (1981). Judging from the last talk there seems to be a question as to what the magnitude of this effect really is but there does seem to be agreement on the sign of this momentum flux for Jupiter.

In spite of the magnitude of the momentum flux we find in these simulations, we don't produce a very strong jet. The next slide (Fig. 6) shows the final twist to this story. It turns out that there are several other terms that act on the mean flow. One that hasn't been mentioned is the Coriolis force on the vertical flow. Downflows produce eastward flows and upflows produce westward flows. We find that there is a meridional circulation induced by the convection in this last simulation with rising motion on the equatorward side of the jet and falling motion on the poleward side. The Coriolis force acting on these vertical flows is nearly anticorrelated with the divergence of the momentum flux as shown in this slide. These two forcing terms for the mean zonal flow counteract each other so that a stronger jet is not produced. When we look at the other forcing terms we find that they are unimportant for the depth-averaged zonal flow. The Coriolis force acting on the induced latitudinal flow gives no net contribution, the force is in one direction on the top and in the opposite direction on the bottom.

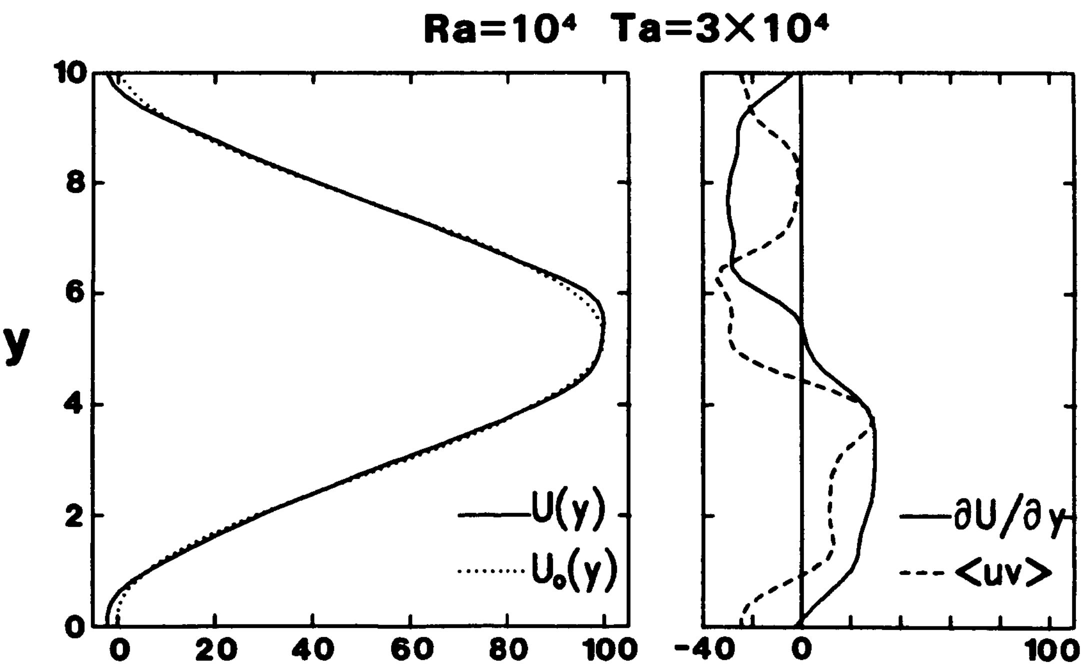

Figure 5. Zonal mean velocity, its latitudinal gradient, and the eddy momentum flux as computed for a model with rotation.

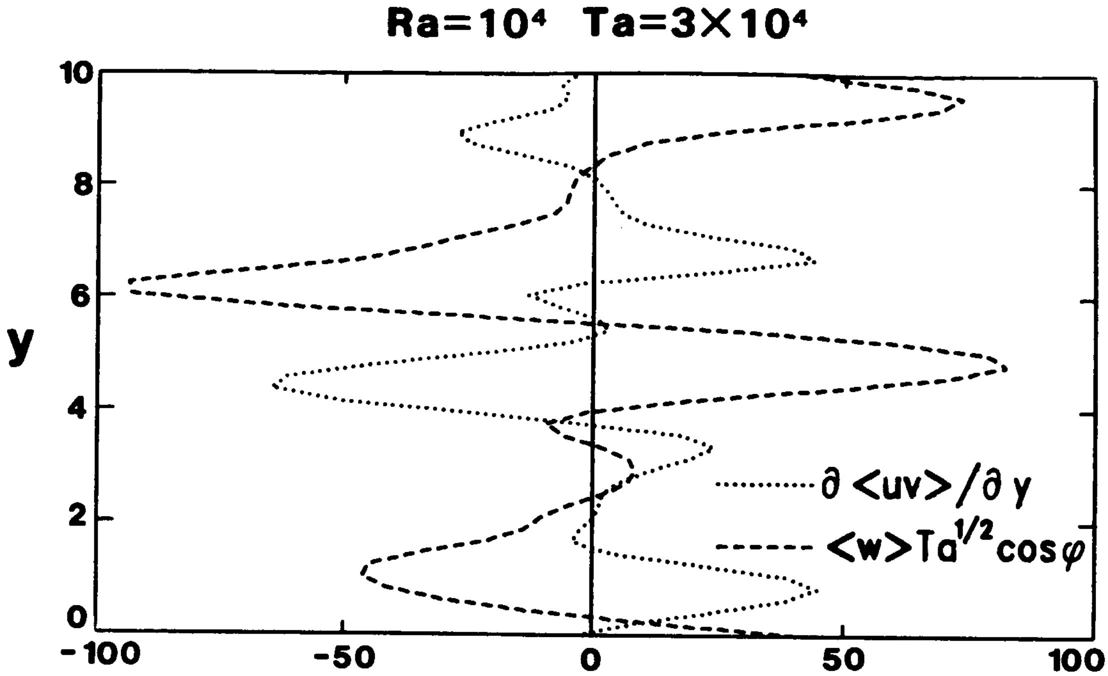

Figure 6. The Coriolis torque on vertical motion and the divergence of eddy momentum flux as computed for the rotating model.

This induced meridional circulation may be of interest to those concerned with the cloud structure and aerosols. I think it's also the same type of circulation that Michael Flasar presented in his model of overturning flow around the jets at stratospheric levels. I hope that this drives home the point that there are additional forcing terms that we need to look at. Although the Coriolis force on the mean vertical flow may be one of the hardest things to measure, in this model, at least, it was probably the dominant term. With that I'll end my talk.

REFERENCES

Gilman, P. A. (1977). Nonlinear dynamics of Boussinesq convection in a deep rotating spherical shell. I. *Geophys. Astrophys. Fluid Dynam.* 8, 93-135.

Galtzmaier, G. A. (1984). Numerical simulations of stellar convective dynamos. I. The model and the method. *J. Comput. Phys.* 55, 461-484.

Hathaway, D. H. (1984). A convective model for turbulent mixing in rotating convection zones. *Astrophys. J.* 276, 316-324.

Ingersoll, A. P., R. F. Beebe, J. L. Mitchell, G. W. Garneau, G. M. Yagi, and J. P. Mueller (1981). Interactions of eddies and mean zonal flow on Jupiter as inferred from Voyager 1 and 2 images. *J. Geophys. Res.* 86, 8733-8743.

DR. STONE: I'd like to ask a question myself. Were those equilibrium solutions that you were showing? They had fully evolved as much as they could?

DR. HATHAWAY: I think you can see from the movie, I ran it until they were statistically steady. As far as I could tell it looked like there weren't going to be any more changes. They certainly do produce small fluctuations but I think the statistics were fairly steady at the time I stopped calculations.

DR. HUBBARD: Can I ask you to defend your results in terms of what I would consider to be a more realistic interior model. I mean, you're taking a layer as I understand it, which is 1000 km thick or so and you have some sort of rigid lower boundary to it whereas on the real Jupiter the equation of state gradually goes over to a non-ideal form over something like that depth. But there is certainly no abrupt transition to some other domain.

DR. HATHAWAY: Yes. My choice of depth was motivated by one thing and that was to get the size of the cells somewhat similar to what we observe in the north temperate belt and there are a lot of things that can change that. Within this model I have rigid tops and bottoms, the size of the convection tends to be on the order of the depth of the layer. What you get for more realistic situations is a little hard to say. I mean, there are problems even in the Earth's atmosphere in producing very broad meso-scale eddies where the width of the eddies are much bigger than the depth of the layer. It was a bit of an arbitrary decision, but the choice shouldn't change the character of the interaction much. The previous calculation I showed for the spherical shells chose the depth to be where the fluid changes from molecular to metallic hydrogen. This seems to be a more reasonable place to put an interface.

DR. EMANUEL: What do you imagine might change if you were to repeat the experiment on the equatorial beta plane? I guess what I'm driving at is, is it possible that this mechanism might have anything to do with the equatorial jet?

DR. HATHAWAY: It could. Basically all you need to get the equatorial jet with the convection are cells that are elongated in the north/south direction. That's basically the thing you need. You can go through the same exercise drawing vectors and seeing which way the Coriolis force turns them. I think that the key for producing equatorial acceleration is cells that are elongating north/south. That is also the sense you get out of convective models in spherical shells or those I've done on inclined planes where the rotation vector is tilted. The rotation vector tends to elongate cells in that direction. However, I can't do these calculations on a beta plane. A beta plane has a rotation vector which tilts more as you go toward the north. I've got periodic boundary conditions, so I can't do that.

DR. FELS: I'm sorry. I'm not quite clear on just what's in your model. It's a fully non-linear invisid...?

DR. HATHAWAY: Which one, the latter model?

DR. FELS: The one that generated the movie, yeah.

DR. HATHAWAY: The one that generated the movie is wholly viscous. I've solved the Navier-Stokes equations in a plane parallel layer with a rigid top and bottom and rotation about an axis that's tilted from the vertical. I then impose the jet, and then solve the equations for the flow, and see how it evolves. I still have to include something that's essentially forcing the basic jet. The convection here doesn't maintain it afterward. It does feed momentum into it, but it doesn't really maintain it.

DR. INGERSOLL: Sort of a body force?

DR. HATHAWAY: Yes, I need a pressure gradient, or some other forcing. That's why I had two parts to my talk. In the first part of my talk I wanted to suggest that you can produce jets by these meridional circulations and it may be that these maintain the jets. Once you have convection, you then get these momentum fluxes.

DR. STONE: Don't you have an externally imposed meridional pressure gradient?

DR. HATHAWAY: That's right. That's what I have within the model.

DR. STONE: There's no feedback on that pressure gradient?

DR. HATHAWAY: That's right.

DR. FELS: So would the jet grind down?

DR. HATHAWAY: It depends on what you have for forcing, whether it's strictly by the meridional flow or by a thermal wind or what. That something isn't specified. I just have a force within my equations that maintains that basic shear profile and then the convection interacts with the shear and produces its own flow which in one case was opposed to the imposed flow and in the other case was to increase the imposed flow.

DR. CANUTO: Can I ask a question about these Reynolds stresses you have computed? You show a computed graph of $\tau^{r\phi}$ versus θ and presuming the geometry was right, do you really believe that the absolute values are also correct? You use a linear model for those.

DR. HATHAWAY: What I have to do there is go to mixing length theory to get an amplitude for motions.

DR. CANUTO: That's what you do to get the absolute value?

DR. HATHAWAY: That's right. With my linear model all I can do is describe the geometry of the flow, for the amplitude I have to go to mixing length theory.

DR. STOKER: You talked about vertical motion flowing along the parallel to the rotation vector. Does that happen for more shallow flows? I mean, if your flow is 100 km rather than 1000 km, will it still...

DR. HATHAWAY: It depends upon the ratio of the time scales. If you have shallow flows that are very rapid with respect to the rotation period, I wouldn't expect that. If the flows are slow so that a turnover time is very long compared to rotation period I would expect the same thing, that you get a sloping flow nearly in the direction of the rotation axis.