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TRANSPORT OF ABSOLUTE ANGULAR MOMENTUM IN QUASI-AXISYMMETRIC EQUATORIAL JET STREAMS

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It is well known that prograde equatorial jet streams cannot occur in an axisymmetric inviscid fluid, owing to the constraints of local angular momentum conservation ("Hide's theorem"). For a viscous fluid, the constraints of mass conservation prevent the formation of any local maximum of absolute angular momentum m without a means of transferring m against its gradient ∇m *in the meridional plane (e.g., Held and Hou, 1980,* J. Atmos. Sci. *37, 515-533.) The circumstances under which m can be diffused up-gradient by normal molecular viscosity are derived, and illustrated with reference to numerical simulations of axisymmetric flows in a cylindrical annulus. Viscosity is shown to act so as to tend to expel m from the interior outwards from the rotation axis. Such an effect can produce local super-rotation* ($m>\Omega R^2$) *even in a mechanically-isolated fluid (e.g. contained by rigid stress-free boundaries), and is illustrated in a further numerical experiment. The tendency of viscosity to result in the expulsion of m is shown to be analogous in certain respects to a "vorticity-mixing" hypothesis for the effects of non-axisymmetric eddies on the zonally-averaged flow. We show how the advective and 'diffusive' transport of m by non-axisymmetric eddies can be represented by the Transformed Eulerian Mean meridional circulation and the "Eliassen-Palm" (EP) flux of Andrews & McIntyre (1976,* J. Atmos. Sci. *27, 15-30) respectively, in the zonal mean. Constraints on the form and direction of the EP flux in an advective/"diffusive" flow for such eddies are derived, by analogy with similar constraints on the diffusive flux of m due to viscosity. From a consideration of these constraints on* $\underset{\sim}{E}$, $\nabla\cdot\underset{\sim}{E}$, *and* $\underset{\sim}{v}^*$, *and the properties of the super-rotating numerical simulations discussed above, we suggest ways of using observations of the zonal mean flows on Jupiter and Saturn to infer the sources and sinks of m required to maintain the observed flow. The associated form of* $\underset{\sim}{E}$ *can also be used to infer some of the properties of the non-axisymmetric eddies responsible for the transport of m within Jupiter's equatorial jet.*

EQUATORIAL JETS AND ANGULAR MOMENTUM

It is well known from cloud-tracked wind data, obtained from ground-based observations and the Voyager spacecraft that both Jupiter and Saturn exhibit strong westerly (prograde) jet streams near their equators. When measured with respect to the "interior" rotation rate (defined by their radio rotation frequencies), these equatorial jet streams are found to be extremely intense ($u > 100$ m s^{-1} on Jupiter and $u \sim 500$ m s^{-1} on Saturn), and are remarkably persistent features in the large-scale circulations of the major planets. Some recently measured velocity profiles are shown schematically in Fig. 1.

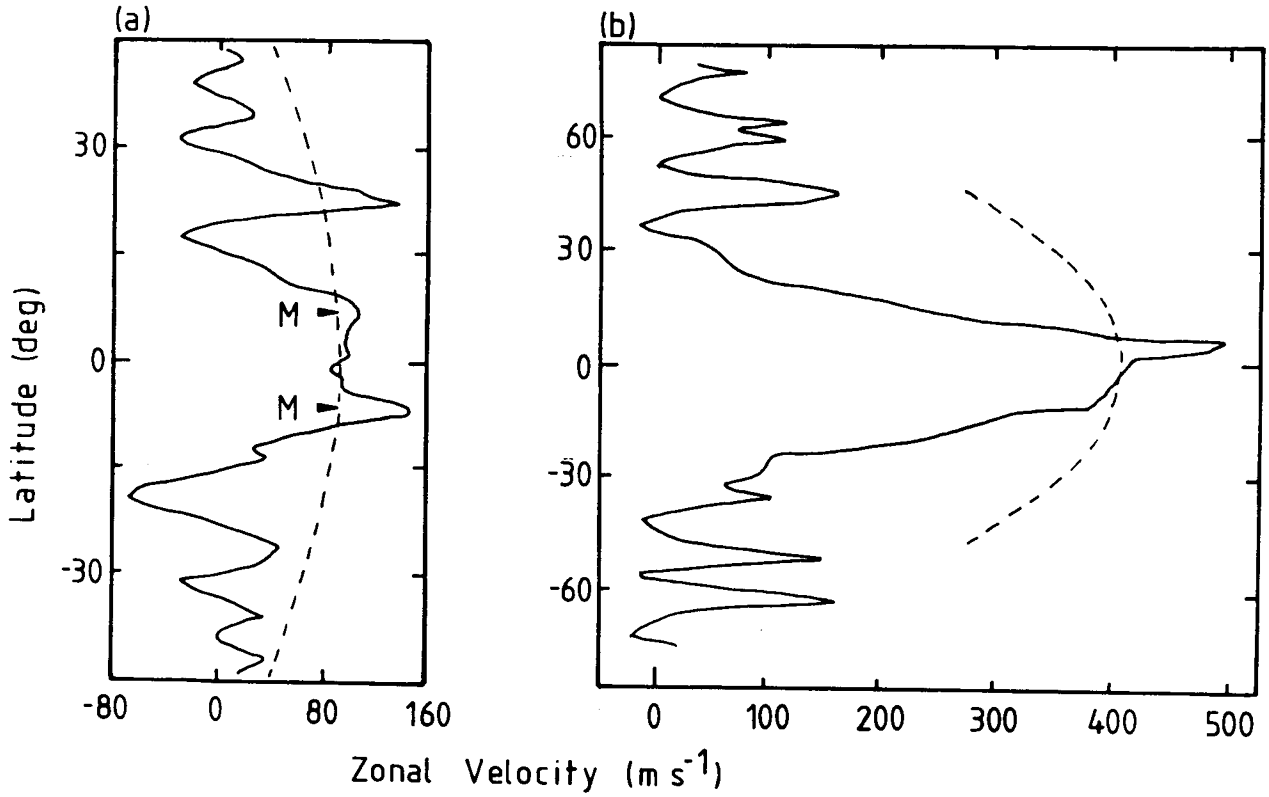

Figure 1. Latitudinal profile of mean zonal velocity (with respect to the measured radio rotation rate) at the cloud tops of (a) Jupiter (adapted from the data of Ingersoll *et al.*, 1981) and (b) Saturn (adapted from the data of Smith *et al.*, 1982). Also shown (dashed) are velocity profiles corresponding to a uniform angular velocity with latitude, equivalent to the approximate rotation period measured at the equator on each planet. The latitudes of the maximum observed angular velocity in Jupiter's equatorial jet are indicated in (a) by M.

The existence of these apparently steady equatorial jet streams poses some intriguing dynamical questions, especially regarding the origin of their angular momentum and the nature of the processes which maintain them. If we consider the simplest possible configuration, in which only axisymmetric motions (e.g., driven by a latitudinal distribution of diabatic heat sources and sinks) are permitted (i.e., we exclude non-axisymmetric pressure gradients, hydromagnetic effects etc.), prograde equatorial jets cannot occur without rather special initial conditions. This is because, in inviscid axisymmetric flow, the angular momentum per unit mass

$$m = (\Omega R \cos \lambda + u) R \cos \lambda \qquad (1)$$

(where Ω is the planetary rotation rate, R the planetocentric radius and λ is latitude) is conserved following the motion in the meridional plane, i.e.,

$$Dm/Dt = 0 \quad (\text{where } D/Dt = \partial/\partial t + \underset{\sim}{v}\cdot\nabla) \qquad (2)$$

Since $u>0$ at the equator requires that $m>\Omega R^2{}_{max}$, (2) requires the initial flow also to contain fluid elements with $m>\Omega R^2{}_{max}$, i.e., with more m than any fluid element initially at rest in the rotating frame (This is sometimes referred to as "Hide's theorem"--see Hide 1969). The volume-integrated angular momentum is similarly constrained to a constant value at all times.

EFFECTS OF INTERNAL VISCOSITY

Since an inviscid, axisymmetric system (initially at rest) cannot produce an equatorial jet, it is natural to consider the effects of Newtonian (molecular) viscosity, while still preserving an axisymmetric flow. Viscosity removes the formal conservation of m but, for a steady flow, some constraints on the flow can still be obtained (using arguments similar to Schneider 1977; Held and Hou 1980) as follows. With viscosity, Eq. (1) becomes

$$Dm/Dt = -\nabla\cdot\underset{\sim}{F} \qquad (3)$$

where $\underset{\sim}{F}$ is the diffusive flux of m due to viscosity which, for an isotropic fluid, is given by

$$\underset{\sim}{F} = -\nu R^2 (\cos^2 \lambda)\ \nabla\gamma \qquad (4)$$

where ν is the kinematic viscosity and

$$\gamma = u/(R \cos \lambda) \qquad (5)$$

the local relative angular velocity.

If we consider any local maximum in m (necessarily, though not exclusively, associated with an equatorial jet), $m = m_0$ (say), we may draw a closed m contour in the meridional plane at $m_0 - \varepsilon$ (where ε may be arbitrarily small; see Fig. 2). Integrating the steady form of (3) over the toroidal volume enclosed by the $m - \varepsilon$ contour, we get

$$
\begin{aligned}
\iiint \nabla \cdot (\underset{\sim}{v} m)\, d\tau &= (m_0 - \varepsilon) \oint \underset{\sim}{v} \cdot \underset{\sim}{n}\, ds \\
&= 0 \text{ (by mass conservation)} \\
&= - \oint \underset{\sim}{F} \cdot \underset{\sim}{n}\, ds \qquad (6)
\end{aligned}
$$

Thus, either $\underset{\sim}{F} \cdot \underset{\sim}{n} = 0$ everywhere, or else $\underset{\sim}{F}$ must possess both inward and outward components in different regions of the m contour (i.e., $\underset{\sim}{F}$ must act up-gradient with respect to ∇m somewhere in the flow). Since $\underset{\sim}{F}$ is (anti-) parallel to $\Delta\gamma$, and not to ∇m (see (4) and (5) above), a necessary condition for the existence of a steady viscous jet is for $\nabla\gamma \cdot \nabla m$ to take either sign in different parts of the flow. It is not necessary to invoke "negative viscosity" phenomena, which are fundamentally associated with non-axisymmetric effects.

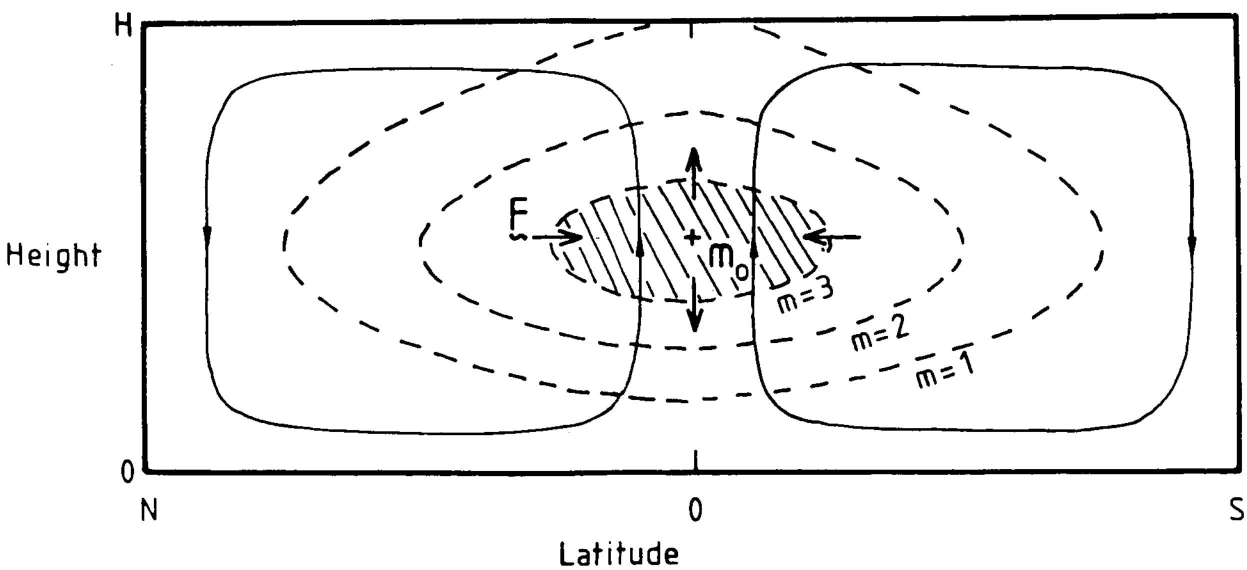

Figure 2. Map of m in a hypothetical flow on a sphere, with a local maximum in m near the equator. Constraints derived for such a flow, associated with a balance between advection (in a meridional circulation, shown by continuous lines) and diffusion, result in a need for $\underset{\sim}{F}$ to act inwards in some places and outwards in others for a closed m contour region (shaded), with $\oint \underset{\sim}{F} \cdot \underset{\sim}{n}\, ds = 0$.

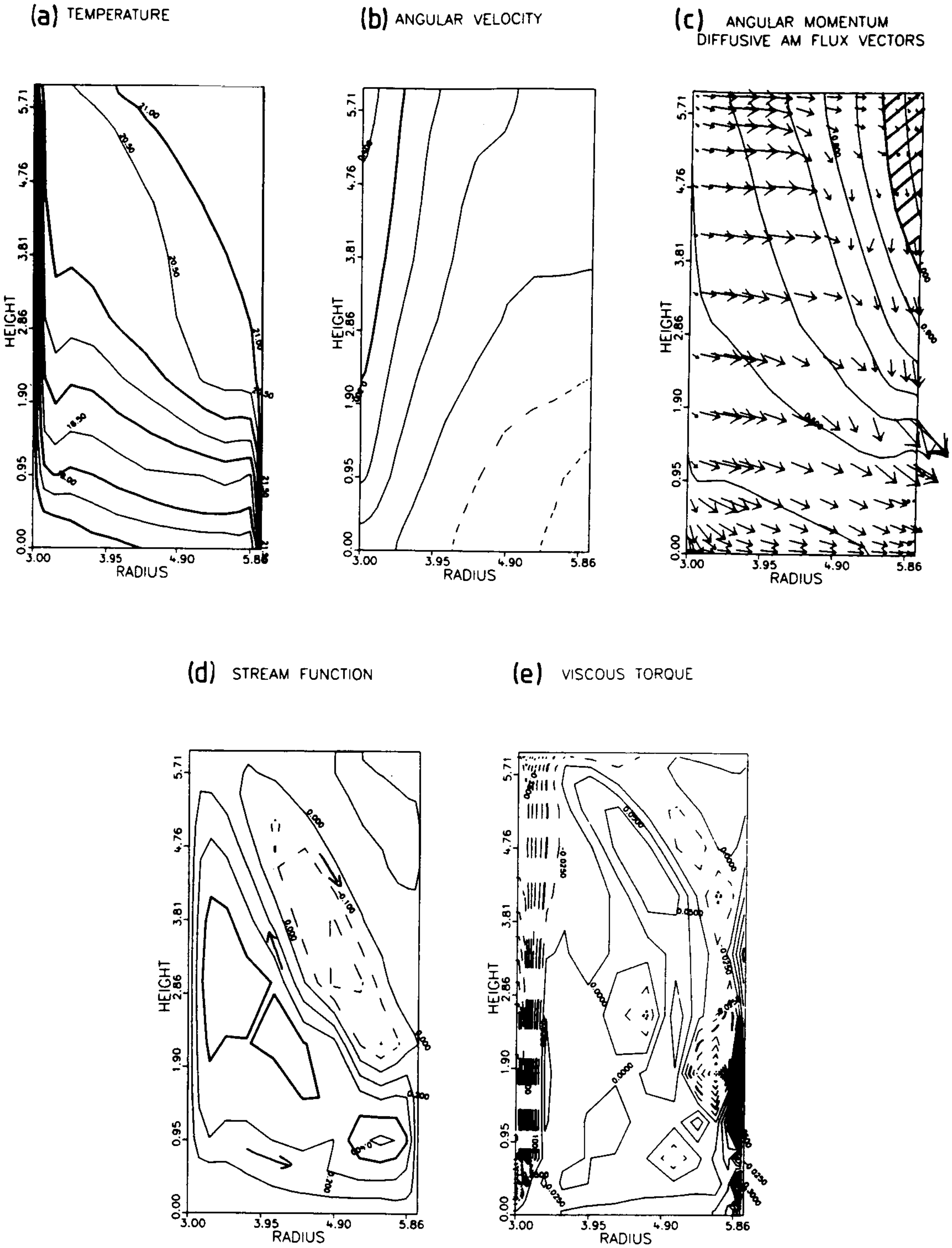

Figure 3. Steady state fields shown in the (r,z) plane for thermally-driven flow (by differential heating at the sidewalls) in a system with stress-free boundaries. (a) Temperature T. (Contour interval [CI] 0.5 deg C); (b) Angular velocity $\gamma = u/r$ (CI = 0.1 rad s^{-1}); (c) m and vectors of F (CI = 0.1 $m/\Omega b^2$); (d) Meridional stream function $\chi*$. (CI = 0.1 $cm^2\ s^{-1}$); (e) Local viscous torque $(-\nabla \cdot \underset{\sim}{F})$. (CI = 0.025 $cm^2\ s^{-2}$).

AN EXAMPLE IN THE CYLINDRICAL ANNULUS

We illustrate the application of these ideas in a numerical simulation of axisymmetric flow in a rotating cylindrical fluid annulus. The fluid is incompressible and Boussinesq (for simplicity), and contained between two coaxial cylindrical surfaces (at $r=a,b$) and horizontal boundaries at $z = 0,H$. Motion is driven by maintaining the two sidewall boundaries at different (constant) temperatures (T_a and T_b), and in the present example, all boundaries are rigid and stress-free (so that $\nabla\gamma\cdot\underset{\sim}{n}=\underset{\sim}{F}\cdot\underset{\sim}{n} = 0$, and the fluid cannot exchange m with its environment). The model used is a grid-point finite-difference formulation of conventional design, with a stretched mesh to resolve boundary layers, and a realistic representation of molecular viscosity and thermal conductivity (being the only sub-gridscale processes, see Hignett *et al.*, 1985 and Read 1986a for further details).

Figure 3 shows a selection of steady state fields in the (r,z) plane. With $T_a = T_b$, a meridional overturning circulation is driven between the two sidewalls which is predominantly thermally-direct (cf. Figs. 3(a) and (d)), though some smaller, indirect cells also occur owing to the effects of a diffusive instability for Prandtl number $\sigma \gg 1$. During the "spin-up" of the flow, m is redistributed by the main meridional circulation (though because of the boundary conditions, the total amount must remain constant). It accumulates near the top of the inner cylinder (top left in Fig. 3), generating a local maximum in γ (see Fig. 3(b)). Since F is related to $\nabla\gamma$ (see Eq. (6)), viscous diffusion transfers m outwards from the γ maximum, tending to transfer m horizontally against its local gradient ∇m. In the steady state, this outward transfer of m exactly balances advection by the meridional circulation, and the resultant distribution of $\underset{\sim}{F}$ and $\underset{\sim}{m}$ is shown in Fig. 3(c). Note that fluid elements near $r=b$, $z=H$ have now acquired $m>\Omega b^2$ in a wedge-shaped region of the flow (shaded in Fig. 3(c)), even though the total angular momentum is unchanged from the initial rest state. Thus, viscosity has acted as a mechanism for <u>angular momentum expulsion</u> towards the outer cylinder.

Since Eq. (6) is satisfied not only for a region bounded by contours of m, but also by impermeable boundaries, it applies to Fig. 3. This can be seen in Fig. 3(c), with $\underset{\sim}{F}$ being largely up-gradient (with respect to ∇m) in the horizontal, and down-gradient in the vertical. The corresponding local torque on the fluid ($-\nabla\cdot\underset{\sim}{F}$) is shown in Fig. 3(e), and consists of a complicated array of sources and sinks which, when integrated over the volume of the fluid, have a resultant of zero.

APPLICATION TO REAL EQUATORIAL JETS

The configuration considered in Fig. 3 is analogous in some respects to an equatorial jet in a thermally-driven flow on an entirely fluid spherical planet (i.e., without a solid underlying surface), if we regard the (r,z) plane as equivalent to the latitude, height) plane and $r=b$ as representing the equator. The resulting advective/diffusive flow is similar in some respects to some simple models of the Jovian atmosphere (e.g. Williams and Robinson 1973; Mayr and Harris 1983), although such models have tended to require a stress-bearing lower boundary to generate a prograde equatorial jet. The experiment in Fig. 3

uses a stress-free lower boundary, yet still supports a local maximum in $m > \Omega_b^2$ in the steady state. The main criterion is that $\nabla\gamma\cdot\nabla m<0$ somewhere in the flow, implying that maxima in γ cannot coincide with maxima in m.

Some evidence for this is apparent in Jupiter's equatorial jet (Fig. 1(a)), which clearly shows a local minimum in $\bar{u}$ (and $\bar{\gamma}$) at the equator itself, and maxima in $\bar{u}$ (and $\bar{\gamma}$) at the poleward extremes of the jet. Transfers of m by molecular viscosity are unlikely to be significant for Jupiter and Saturn's jets, however, and are probably dominated by eddy mixing (which is fundamentally non-axisymmetric). Since $\underset{\sim}{F}$ is related to the gradient of (relative or absolute) vorticity $\bar{\zeta}$, however, viscosity is seen to act in some respects like a vorticity-mixing process (tending to even out gradients in $\bar{\zeta}$), such as has been suggested for a variety of eddy phenomena in fluid mechanics and geophysics (e.g., Taylor, 1915; Rossby, 1947; Green, 1970; Read, 1986b). The experiment in Fig. 3 can be regarded, therefore, as indicating in a general way how an equatorial super-rotation can be driven and maintained by a thermally-driven meridional circulation interacting with eddies which act to mix vorticity.

Are there any useful constraints on non-axisymmetric flows? We present an eddy transfer theorem for closed m contours. If we form the equation for the conservation of zonally-averaged angular momentum m, using the residual Eulerian mean circulation of Andrews & McIntyre (1976), we get (e.g., in spherical geometry using pressure coordinates in the vertical)

$$\partial\bar{m}/\partial t + \underset{\sim}{v}_* \cdot \nabla\bar{m} = - \nabla\cdot\underset{\sim}{E} \; (- \nabla\cdot\underset{\sim}{F}) \qquad (7)$$

where

$$\underset{\sim}{v}_* = [\, \bar{\underset{\sim}{v}} - (\overline{v'\theta'}/\bar{\theta}_p)_p ,\ \bar{\omega} + (\overline{v'\theta'} \cos\lambda/\bar{\theta}_p)_\lambda/(R \cos\lambda] \qquad (8)$$

and represents the (quasi-Langrangian) mean meridional circulation associated with diabatic heating, eddy dissipation and transience. $\underset{\sim}{E}$ is related to the so-called Eliassen-Palm flux, and given by

$$\underset{\sim}{E} = R \cos\lambda[\overline{u'v'} - \bar{u}_p\overline{v'\theta'}/\bar{\theta}_p ,$$

$$\omega'u' + \{(\bar{u} \cos\lambda)_\lambda/(R \cos\lambda) - 2\Omega \sin\lambda\}\ \overline{v'\theta'}/\bar{\theta}_p] \qquad (9)$$

(cf. Andrews & McIntyre 1978). Since $\underset{\sim}{v}*$ obeys a similar continuity equation to $\underset{\sim}{v}$, $\underset{\sim}{E}$ must obey a constraint similar to Eq. (6) for closed, steady m contours, provided we may ignore molecular viscosity. Thus,

$$\oint \underset{\sim}{E} \cdot \underset{\sim}{n} \, ds = 0 \tag{10}$$

around any closed m contour.

Further aspects of the ideas presented in Sections 1-4 above are discussed in greater detail by Read (1986a,b).

DIAGNOSTICS FOR JUPITER'S EQUATORIAL JET?

Given the framework of constraints on equatorial jets associated with a balance between advection of m and its transfer by non-axisymmetric eddies, it may be possible to use observations of Jupiter's equatorial jet to obtain information concerning the processes by which the jet is maintained. We suggest below some strategies for possible diagnostic studies of Jupiter's atmosphere which may bear on this important problem:

a) From detailed observations of the zonal mean flow on Jupiter, we may investigate the properties of candidate eddy processes and their associated sources and sinks of m. The equatorially-localized structure of the jets might suggest the relevance of equatorially-trapped Kelvin and Mixed Rossby-Gravity (MRG) modes, for example, as studied in a simple model of Jupiter's equatorial jet by Maxworthy (1975). Given the thermal and velocity structure of the mean zonal flow, it is possible (subject to certain assumptions) to calculate the properties of steady modes which are compatible with that (steady) mean flow. An example of such a procedure was given by Plumb and Bell (1982), who undertook a similar study in connection with the quasi-biennial oscillation in the terrestrial stratosphere. Given the form of u(y,z) and N^2, the mode structure can be obtained, together with quantities analogous to $\underset{\sim}{E}$ and $\nabla \cdot \underset{\sim}{E}$ for the main Kelvin and MRG modes. The latter could then be tested against the constraints of Eq. (10) for consistency with the distribution of m and a suitable meridional circulation.

b) Since the meridional circulation compatible with the constraints on $\underset{\sim}{E}$ and $\nabla \cdot \underset{\sim}{E}$ is quasi-Lagrangian, it primarily reflects the distribution of diabatic heat sources and sinks in the Jovian atmosphere. This can also be compared, therefore, with the known sources and sinks, e.g., due to radiative forcing obtained from radiation budget studies, to check for consistency of the solution, and to provide further requirements which the eddy processes need to satisfy.

The feasibility of such strategies have yet to be demonstrated for Jupiter, although some progress on a similar scheme for the analysis of data for Venus (from Pioneer Venus project) has been made by Hou (1984), Hou and Goody (1985) and Valdes (1984 and private communication).

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