N87-17739

MANNED MARS MISSION
SUNLIGHT AND COMMUNICATION OCCULTATIONS

Jack Mulqueen
Marshall Space Flight Center, AL

ABSTRACT

Calculations are presented for the 1999 opposition class mission and a procedure for obtaining similar occultation data for any other given Mars mission is given. Occultation data for a Mars orbiter in a 24.5 hour parking orbit and a Mars base have been calculated for: sunlight occultation - the time in darkness; and radio communication occultation - the communication losses between the lander and the orbiter, the lander and Earth, and orbiter and Earth.

CALCULATIONS

Mars Orbiter Sunlight Occultation

To find the time in darkness for a Mars Orbiting Spacecraft it is necessary to determine the orientation of the parking orbit with respect to the Sun. This is done by finding the angle between the semi-major axis of the orbit and the Mars to Sun line. This angle, α , (see Figure 1), is found using the following equation:

$$\alpha x = RAP - VE - P - L$$

The values of RAP, VE, P and L are found using the trajectory data (1) and the Planetary Handbook (2).

Once α is known it is possible to find the points on the orbit corresponding to the beginning and end of occultation, defined as ψ_1, and ψ_2 . This is done using the equation of the parking orbit and a transformation between a reference frame centered on the orbit (x, y) and one that lies along the Mars-Sun line (x' , y'). These reference frames are shown in Figure 2. The second frame, (x' , y') is defined as a rotation of the orbit centered frame (x, y) through an angle α followed by a translation d, where d is defined as:

$$d = ae \sin \alpha$$

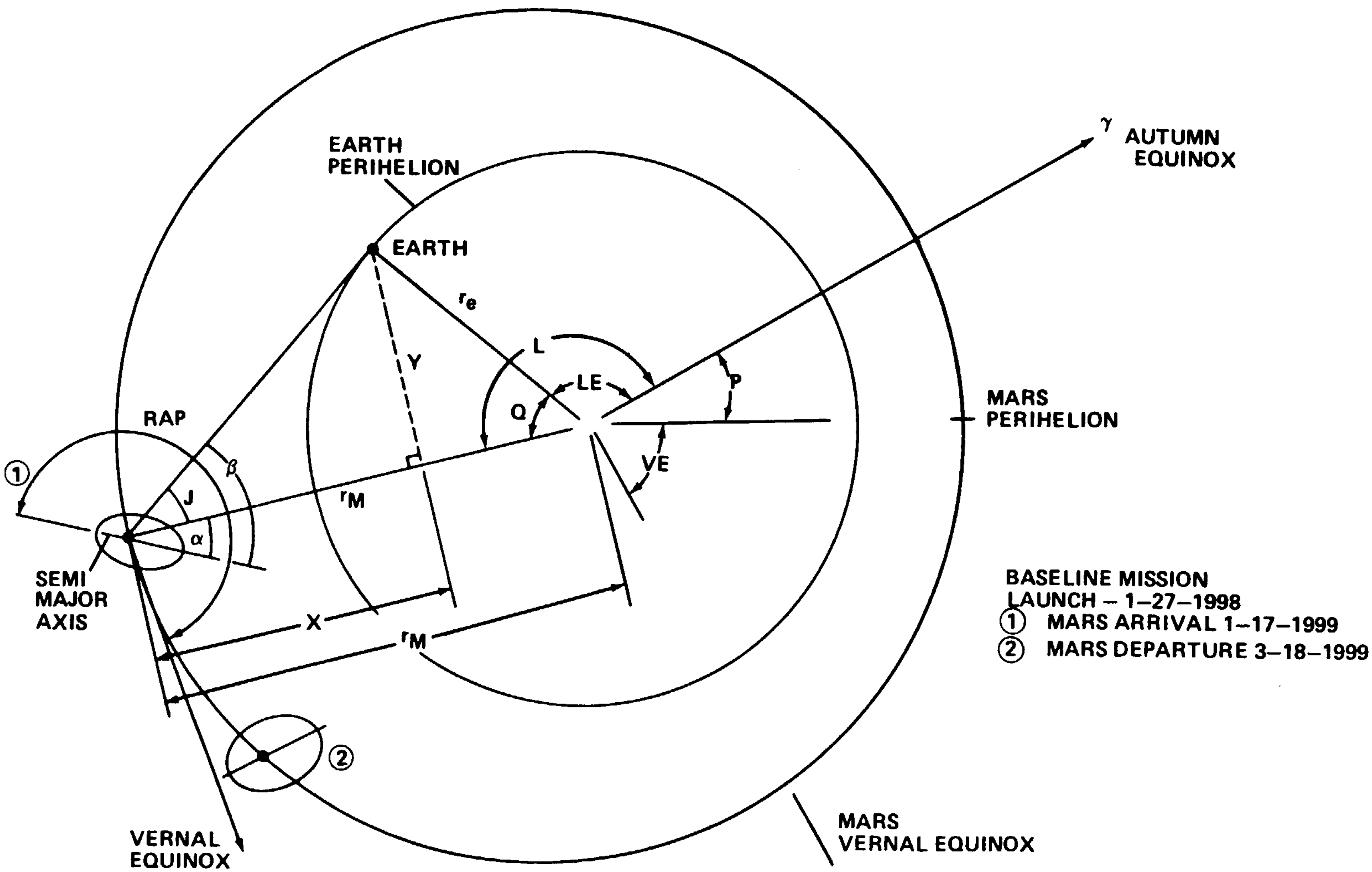
The quantity ae represents the distance from the center of the orbit to the focus of the orbit which is the center of Mars.

The values of ψ_1 and ψ_2 are found by finding the values of x' that correspond to y' equal to $\pm$ the radius of Mars (RM), then con-

3216–85

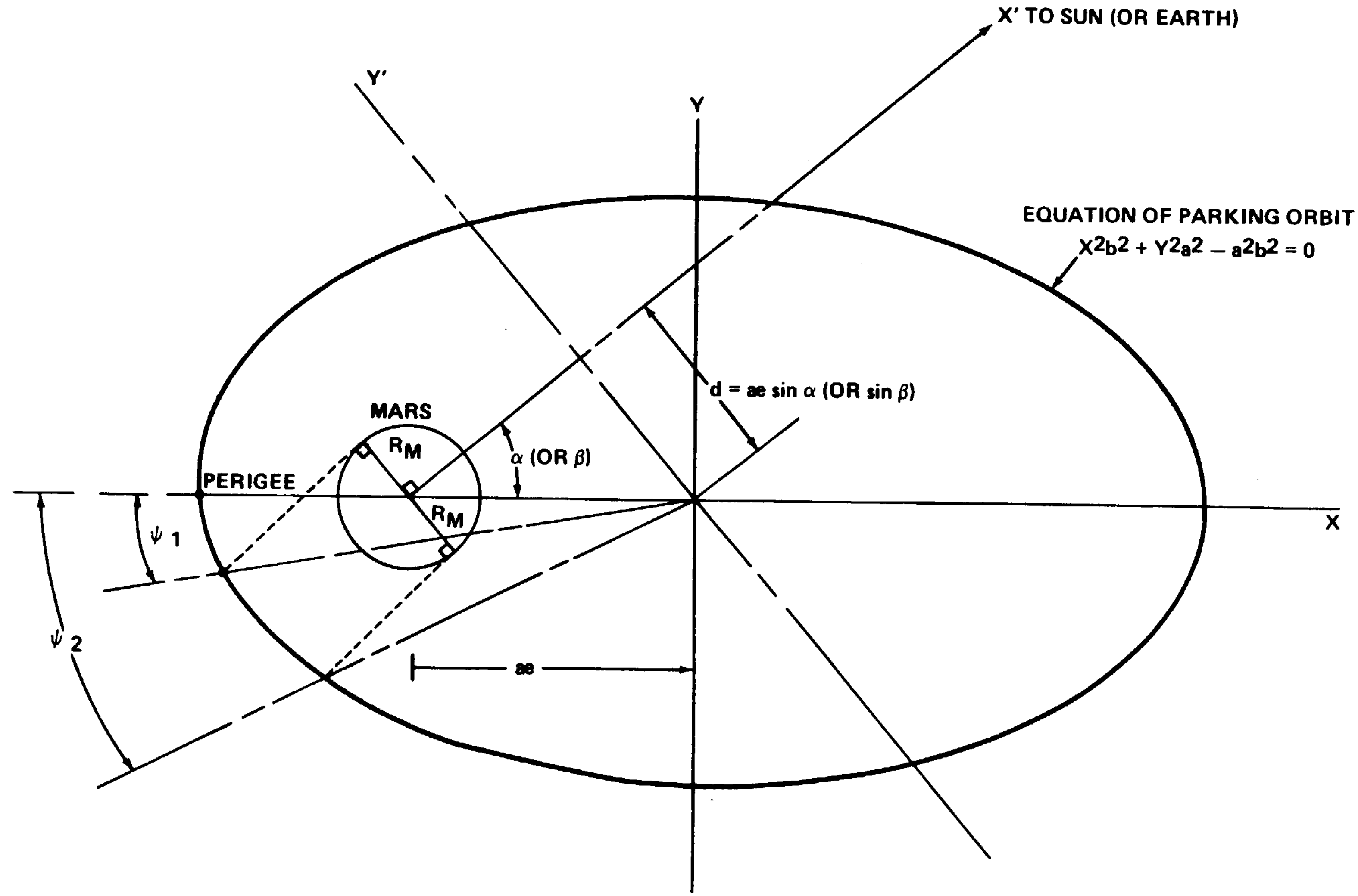
FIGURE 1

GEOMETRY FOR DETERMINING MARS ORBITER OCCULTIONS



3157–85

FIGURE 2
GEOMETRY FOR CALCULATING MARS ORBITER OCCULTATIONS



verting to the x,y frame. [Once ψ_1 and ψ_2 are known,] Kepler's time equation then can be used to find the time from perigee to ψ_1 and ψ_2. The difference in the two times represents the duration of the occultation.

The transformation between the two coordinate frames are found in the following manner:

First, a rotation of the x,y frame through an angle α to yield the x_1, y_1 axes (see Figure 1).

$$x_1 = x \cos\alpha + y \sin\alpha$$
$$y_1 = -x \sin\alpha + y \cos\alpha$$

Next, a translation along the y axis a distance d to yield the x', y' axes.

$$x' = x_1 = x \cos\alpha + y \sin\alpha$$
$$y' = y_1 - d = -x \sin\alpha + y \cos\alpha - d$$

In matrix notation the transformation between the x,y frame and the x', y' frame can be written as:

$$\begin{bmatrix} x' \\ y' + d \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

or:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} x' \\ y' + d \end{bmatrix}$$

To find the values of x' corresponding to $y' = \pm$ RM the equation of the orbit must be found in terms of x' and y'. The equation of the orbit in the x,y frame is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

or:

$$x^2b^2 + y^2a^2 - a^2b^2 = 0$$

using the transformation:

$$x = x' \cos\alpha - (y' + d) \sin\alpha$$
$$y = x' \sin\alpha + (y' + d) \cos\alpha$$

The equation of the orbit in terms of x' and y' is:

$$x'^2 [b^2\cos^2\alpha + a^2\sin^2\alpha] + x' [2(y' + d)(a^2 - b^2)\sin\alpha\cos\alpha] + [(y' - d)^2 (b^2\sin^2\alpha + a^2\cos^2\alpha) - a^2b^2] = 0$$

Given $y' = \pm RM$ the corresponding values of x' can be found using the quadratic formula, then these values of x' and y' can be converted to the x,y frame using the transformation matrices. The values of ψ_1 and ψ_2 are found using the x and y coordinates of the orbit at the beginning & end of occultation.

$$\psi_1 = \tan^{-1}\left(\frac{y}{x}\right)_{\text{beginning}} \qquad \psi_2 = \tan^{-1}\left(\frac{y}{x}\right)_{\text{end}}$$

The duration of the occultation is found by using Kepler's time equation to calculate the time from perigee at ψ_1 and ψ_2.

$$t_\rho = \sqrt{\frac{a^3}{\mu}}\,(\psi - e\sin\psi)$$

The duration of the occultation is:

$$\Delta t = t_{\rho_2} - t_{\rho_1}$$

The orbiting spacecraft is occulted for a period of Δt once during each orbit. The value of Δt changes during the staytime as the longitude of Mars changes and the orbit precesses. To obtain the minimum and maximum occultations, Δt should be calculated on Mars arrival and departure.

Mars Lander Sunlight Occultation

The time in darkness that a Mars lander would be subjected to is highly dependant on the latitude of the landing site and the heliocentric longitude of Mars. The amount of daylight varies on Mars just as it does on Earth, since its equator is inclined to its orbit by 23.984 degrees. To calculate the time in darkness, the following calculations are required.

The geometry shown in Figures 3a and 3b represents the orientation of the axis of rotation of Mars with respect to the Sun. η represents the angle between the polar axis and the vertical as viewed perpendicular to the Mars-Sun line. Using Figure 3a, η can be found:

3218–85

FIGURE 3
GEOMETRY FOR FINDING η

FIG. 3a.

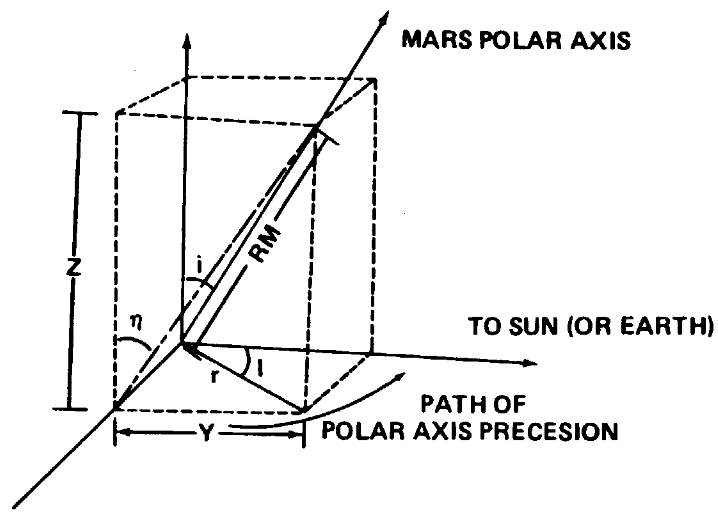
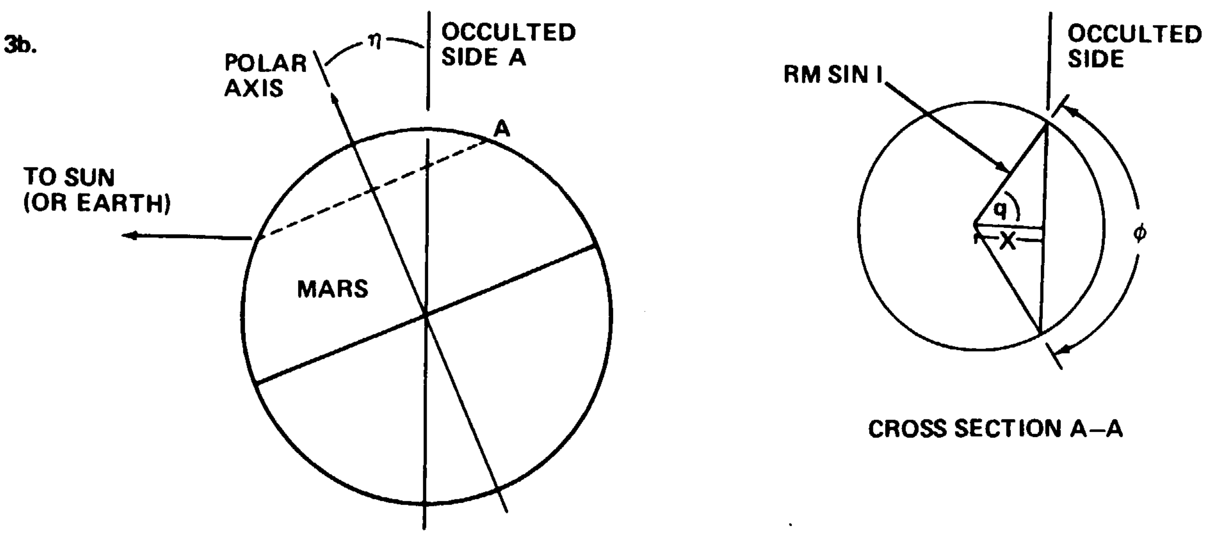


FIG. 3b.



$$i = 23.984 \text{ DEG}$$

$$r = RM \sin i$$

$$y = M \cos L = RM \sin i \cos L$$

$$z = RM \cos i$$

$$\tan \eta = \left(\frac{\cos i}{\cos L} \right)$$

Figure 3b shows the orientation of Mars with respect to the Sun and a cross section of a particular latitude shows how much of that latitude is in the sunlight. The time in darkness is found using the following calculations and Figure 3b.

$$\tan \eta = \left(\frac{RM \sin \ell}{x} \right)$$

$$x = \left(\frac{RM \sin \ell}{\tan \eta} \right)$$

$$q = \cos^{-1} \left(\frac{x}{RM} \right) = \cos^{-1} \left(\frac{\sin \ell}{\cos \eta} \right)$$

$$\emptyset = 2q = 2 \cos^{-1} \left[\left(\frac{\sin \ell}{\cos L} \right) \tan i \right]$$

$$\omega = \frac{2 \pi}{24.5 \text{ hrs}}$$

$$\Delta T = \frac{\emptyset}{\omega} = \frac{1}{\pi} \cos^{-1} \left[\frac{\sin \ell \tan i}{\cos L} \right]$$

<u>Communication</u> <u>Occultation</u> <u>Between</u> <u>The</u> <u>Mars</u> <u>Orbiter</u> <u>And</u> <u>Earth</u>

This calculation is performed using the same procedure as the Mars orbiter sunlight occultation except instead of α being used in the calculations, a different angle, β is used. β is defined as the angle between the semi-major axis of the parking orbit and the Mars to Earth line. The following calculations are required (see Fig. 1).

$$Q = \left| L - LE \right|$$

$$r_e = \left[\frac{a (1-e^2)}{1 + e \cos\theta} \right] \text{ Earth}$$

$$r_m = \left[\frac{a\ (1-e^2)}{1 + e\ \cos\theta}\right] \text{ Mars}$$

$$X = r_m - r_e \cos Q$$

$$Y = r_e \sin Q$$

$$J = \tan\left(\frac{Y}{X}\right)$$

$$\beta = \alpha - J$$

Once β is found the procedures for finding the communication occultations are identical to the sunlight occulations. Starting with equation 1, β is substituted for α .

Communication Occultation Between The Mars Lander and Earth

The communications occultation between the Mars lander and the Earth is dependent on the same parameters that influence the sunlight occultation. Since the plane of the Earth's orbit is inclined only 1.849 degrees to the Mars orbit plane, the value of the duration of the communications occultation would be essentially equal to the duration of the sunlight occultation for a given latitude.

Communication Occultation Between The Mars Lander and The Mars Orbiter

The communication occultation of the Mars lander and the Mars orbiter is obtained by finding when the angle between the local vertical at the landing site and the position of the orbiter is greater than 90 degrees. The geometry for this calculation is shown in Figure 4. It was assumed that the orbiter is directly above the lander when it is at perigee. The angle c represents the angle between the vertical and the orbiter. Communication is occulted as long as c is greater than 90 degrees. Using Figure 4, c was obtained with the following calculations. Given a, e, μ and ψ.

$$\Theta = 2 \tan^{-1}\left[\sqrt{\frac{1 + e}{1 - e}} \tan\left(\frac{\psi}{2}\right)\right]$$

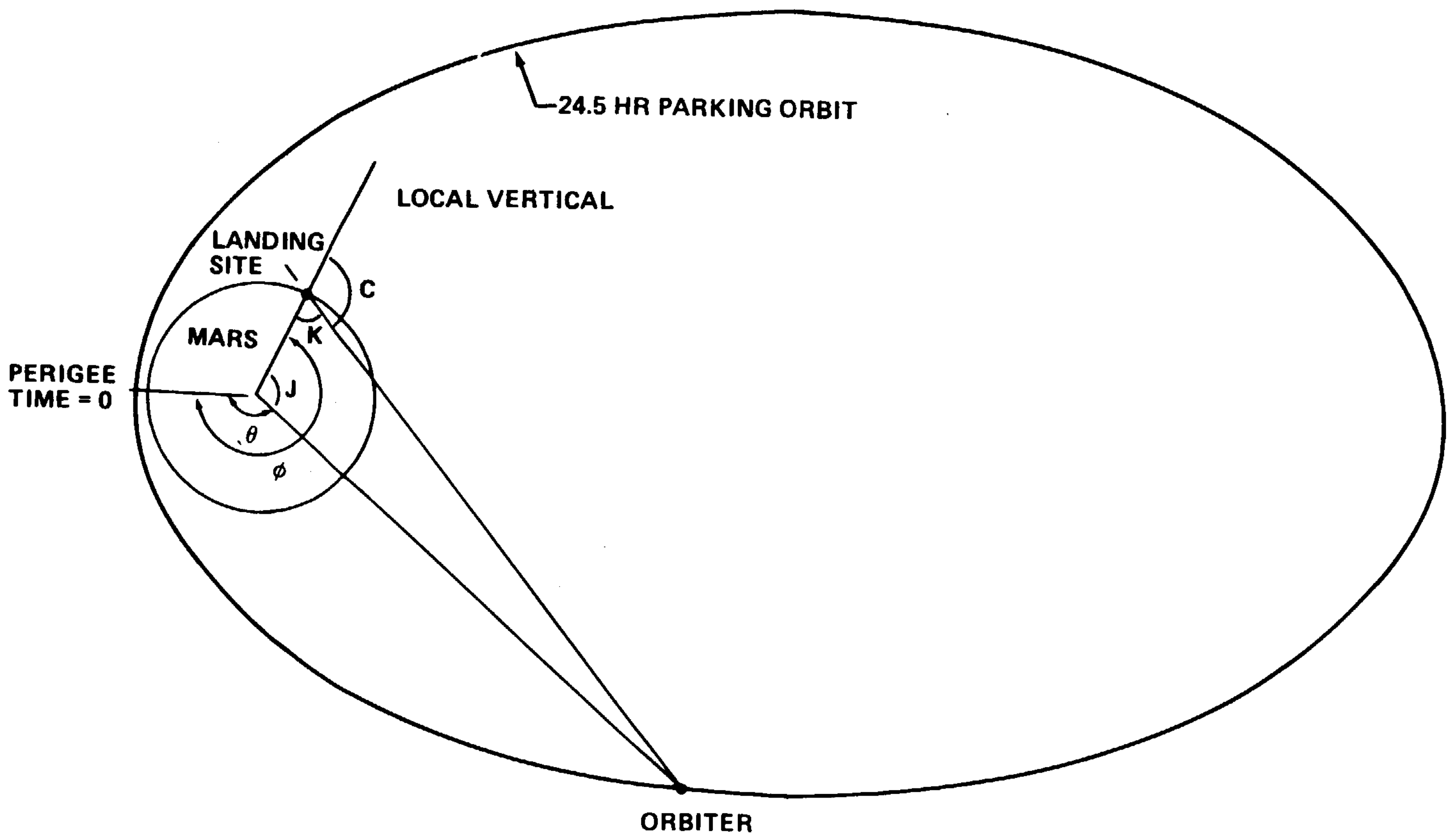
$$t = tp = \sqrt{\frac{a_3}{\mu}}\ (\psi - e \sin\psi)$$

$$\omega = \frac{2\pi}{24.5} \ \frac{\text{RAD}}{\text{HRS}}$$

3217–85

FIGURE 4

MARS LANDER – ORBITER COMMUNICATION GEOMETRY



$\Phi = 2\pi - \omega t$ NOTE: Φ is equal to this only when the parking orbit is retrograde, i.e., direction is opposite Mars's rotation.

$$J = \left| \Theta - \Phi \right|$$

$$h = \sqrt{RM^2 + r^2 - 2\,RM\,r \cos j}$$

$$k = \sin^{-1}\left[\frac{r \sin J}{h}\right]$$

$$c = \pi - K;\ h \geq r$$

$$c = K\ ;\ h < r$$

DATA FOR BASELINE, 1999 MISSION

24.5 hour parking orbit

$r_p = 3900$ km $r_a = 36829.2$ km

$a = 20364.63$ km

$e = .8084915$

MARS ORBITER SUNLIGHT OCCULTATION

	MARS ARRIVAL	MARS DEPARTURE
RAP =	273.816 deg	275.242 deg
VE =	-67.01 deg	-67.01 deg
P =	335.323 deg	335.323 deg
L =	168.915 deg	195.73 deg
α =	16.588 deg	8.801 deg
ΔT =	.267 hr (16.06 min.)	.264 hr (15.85 min.)

MARS LANDER SUNLIGHT OCCULTATION

No landing site has been baselined so, assume occultation of 1/2 Mars day,

$$\Delta t = 12.25 \text{ hrs}$$

COMMUNICATION OCCULTATIONS

ORBITER TO EARTH

	MARS ARRIVAL	MARS DEPARTURE
L =	168.91	195.73
LE =	115.	176.
Q =	53.91	19.73
Θ_E =	12.747	73.477

Θ_M =	193.588	220.41
r_e =	.98367 AU	.99498 AU
r_m =	1.6604 AU	1.62526 AU
X =	1.0809 AU	.68449 AU
Y =	1.315 AU	.33589 AU
J =	50.59 deg	26.138 deg
β =	34.002 deg	17.337 deg
ΔT =	.2823 hr (16.93 min.)	.2677 hr (16.06 min.)

LANDER TO EARTH

Since no landing site has been chosen, assume communication occultation occurs for half of the Mars day.

$$\Delta T = 12.25 \text{ hr}$$

LANDER TO ORBITER

These calculations were made by calculating c for ψ = 0 to 360 degrees, The results are shown below:

tp (hr)	ψ(deg)	c (deg)
0	0	0
.3	18	79.2
.4	24	108.7 occultation begins
7.5	140	89.2 occultation ends
12.25	180	0
17.0	220	89.2
17.2	222	93.2 occultation begins
24.1	336	108.7 occultation ends

ΔT = 7.1 hrs occuring twice every orbit.

REFERENCES

1. Young, A., "Mars Mission Concepts and Opportunities"; MSFC paper in Section II.
2. Planetary Flight Handbook, Volume 3, NASA SP 35 Part 1, 1963.

LIST OF SYMBOLS

a = semi-major axis of an orbit

b = semi-minor axis or an orbit

c = angle between local vertical at landing sight and the Mars orbiter

d = distance from origin of x,y reference frame to the origin of the x_1 ,y_1 frame

e = orbit eccentricity

i = inclination of Mars' equator to the ecliptic

l = latitude of landing site

L = heliocentric longitude of Mars

LE = heliocentric longitude of Earth

P = heliocentric longitude of Mars perihelion

RM = radius of Mars

RAP = right ascension of perigee of Mars parking orbit

r = distance from center of Mars to orbiter

r_e = distance from center of sun to center of Earth

r_m = distance from center of sun to center of Mars

tp_1 = time from perigee that occultation begins

tp_2 = time from perigee that occultation ends

Δt = duration of occultation

VE = heliocentric longitude of Mars vernal equinox

x,y = coordinates in the x,y reference frame (Mars parking orbit)

x' , y' = coordinates in the x' , y' reference frame

α = angle between semi-major axis of parking orbit and Mars-Sun line

β = angle between semi-major axis of parking orbit and Mars-Earth line

ψ = eccentric anomaly

η = angle between vertical and the Mars polar axis as seen perpendicular to the Mars-Sun line

μ = gravitational constant km_3/sec_2

Θ = true anomaly