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Issues in Modeling and Controlling the SCOLE Configuration

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I. MODELLING OF THE SCOLE CONFIGURATION

- PARAMETRIC STUDY OF THE IN-PLANE SCOLE SYSTEM - FLOQUET STABILITY ANALYSIS
- THREE DIMENSIONAL FORMULATION OF THE SCOLE SYSTEM DYNAMICS
  - Rotational Equations of Motion
  - Structural Analysis - Boundary Conditions
  - Generic Modal Equations
- WHAT WE CAN LEARN ABOUT THE OPEN LOOP SYSTEM?
  - Consider SCOLE configuration without offset of the mast attachment to the reflector and without flexibility
  - Consider SCOLE configuration without mast flexibility but with offset in the direction of orbit (strawman)
  - Consider SCOLE configuration with offsets in two directions but neglecting mast flexibility
  - Consider general SCOLE system dynamics
- IMPLICATIONS FOR CONTROL STRATEGIES

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II. CONTROL ISSUES:

- CONTROL OF LARGE STRUCTURES WITH DELAYED INPUT IN THE CONTINUOUS TIME DOMAIN
- CONTROL WITH DELAYED INPUT IN THE DISCRETE TIME DOMAIN
- CONTROL LAW DESIGN FOR SCOLE USING LQG/LTR TECHNIQUE
- OPTIMAL TORQUE CONTROL FOR SCOLE SLEWING MANUEVERS
    - Kinematical and Dynamical Equations
    - Optimal Control - Two Point Boundary Value Problem
    - Estimation of Unknown Boundary Conditions
    - Numerical Results
    - Discussion and Further Recommendations

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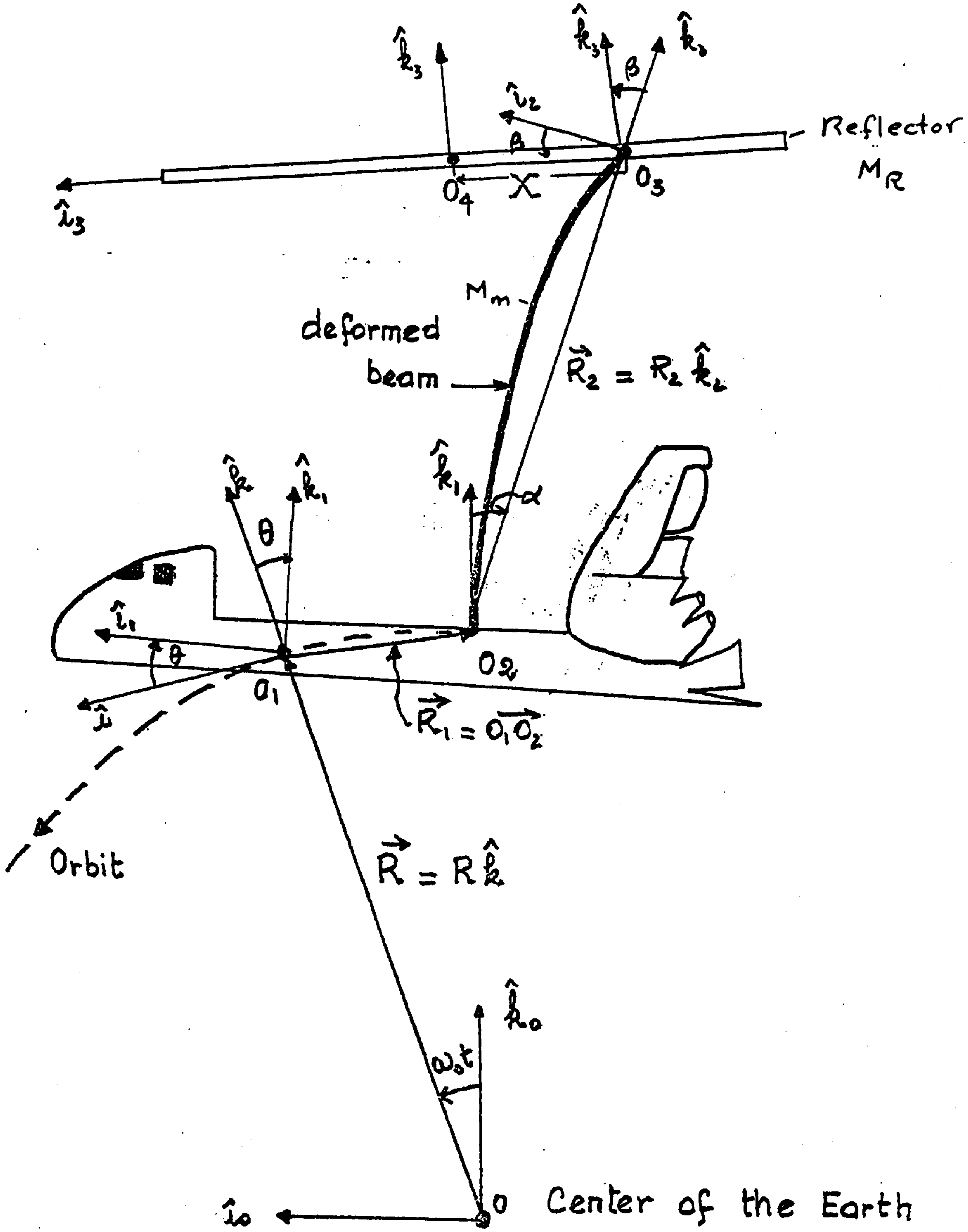

Fig. 2.1. SCOLE System Geometry in the Deformed State (2-D)

Parametric Study of the System

Let us assume that the interface point between the reflector and the mast is at the center of mass of the reflector

$\rightarrow X = 0 \rightarrow \lambda = 0 = C_5 = C_6$

Under this assumption, the equation becomes

$$-\theta'' + C_2/C_1 \, d/d\tau\left[\theta\cos(\Omega\tau+\phi)\right] + \Omega^2 C_3/C_1 \cos(\Omega\tau+\phi)$$
$$- C_4/2C_1 \, \Omega \sin\left[2(\Omega\tau+\phi)\right] - 3/C_1 (I_{11} - I_{33})\,\theta = 0 \qquad (2.5)$$

which in the absence of gravity gradient, yields the following first integral of the motion:

$$-\theta' + C_2/C_1 \left[\theta\cos(\Omega\tau+\phi) + \Omega C_3/C_1 \sin(\Omega\tau+\phi)\right.$$
$$\left.+ C_4/4C_1 \cos\left[2(\Omega\tau+\phi)\right]\right] = K \qquad (2.6)$$

This equation is plotted in the phase plane (θ', θ) for different values of μ and Ω. (Figs. 2.2)

Floquet Analysis

The angular motion about an axis perpendicular to the orbit plane in the absence of gravity gradient is described by:

$$\theta'' = \left[-C_5/C_1 + C_2/C_1 \cos\Omega\tau\right]\theta' - \left[\frac{C_2\Omega}{C_1}\sin\Omega\tau\right]\theta \qquad (2.7)$$

Case 2. No gravity gradient, but offset.

$$p(\tau) = \begin{bmatrix} \frac{c_2}{c_1}\cos\Omega\tau - \frac{c_5}{c_1} & -\frac{c_2}{c_1}\Omega\sin\Omega\tau \\ 1 & 0 \end{bmatrix}$$

$[\dot{Z}(\tau)] = [P(\tau)]\,[Z(\tau)]$

$$\dot{Z}_{11} = P_{11}Z_{11} + P_{12}Z_{21} \quad (1)$$

$$\dot{Z}_{12} = P_{11}Z_{12} + P_{12}Z_{22} \quad (2)$$

$$\dot{Z}_{21} = P_{21}Z_{11} + P_{22}Z_{21} \quad (3) \text{ which becomes}$$

$$\dot{Z}_{21} = Z_{11} \text{ since } P_{21} = 1 \text{ and } P_{22} = 0$$

$$\dot{Z}_{22} = P_{21}Z_{12} + P_{22}Z_{22} \quad (4) \text{ which becomes}$$

$$\dot{Z}_{22} = Z_{12}$$

from (3) $\ddot{Z}_{21} = \dot{Z}_{11}$, substituted into (1) yields

$$\ddot{Z}_{21} = P_{11}\dot{Z}_{21} + P_{12}Z_{21}$$

similarly from (4) $\ddot{Z}_{22} = \dot{Z}_{12}$, substituted into (2) yields

$$\ddot{Z}_{22} = P_{11}\dot{Z}_{22} + P_{12}Z_{22}$$

since $\frac{c_5}{c_1}$ = constant $\quad \frac{d}{dt}P_{11} = P_{22}$

Then

$$\ddot{Z}_{21} = P_{11}\dot{Z}_{21} + \dot{P}_{11}Z_{21} = \frac{d}{dt}(P_{11}Z_{21})$$

and $$\ddot{Z}_{22} = P_{11}\dot{Z}_{22} + \dot{P}_{11}Z_{22} = \frac{d}{d\tau}(P_{11}Z_{22})$$

These two last equations are integrated and the following results for Z_{21} and Z_{22} obtained.

$$\dot{Z}_{21} = P_{11}Z_{21} + K_1$$

$$\dot{Z}_{22} = P_{11}Z_{22} + K_2$$

but from (3), $\dot{Z}_{21}(\tau) = Z_{11}(\tau)$ and from (4)

$$\dot{Z}_{22}(\tau) = Z_{12}(\tau)$$

Therefore, $\quad \dot{Z}_{21}(0) = Z_{11}(0) = P_{11}(0)Z_{21}(0) + K_1 \rightarrow K_1 = 1$

since $Z_{11}(0) = 1$ and $Z_{21}(0) = 0$

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$$\dot{Z}_{22}(0) = Z_{12}(0) = 0 = P_{11}(0)Z_{22}(0) + K_2$$

$$\rightarrow -\frac{c_5}{c_1} + \frac{c_2}{c_1} + K_2 = 0 \text{ or } K_2 = -\frac{c_2}{c_1} + \frac{c_5}{c_1}$$

The two last equations integrated once, yield

$$\dot{Z}_{21} = P_{11}Z_{21} + 1$$

$$\dot{Z}_{22} = P_{11}Z_{22} - \left(\frac{c_2-c_5}{c_1}\right)$$

Solution of the first order equations

$$\frac{dZ_{22}}{d\tau} - P_{11}Z_{22} = -\left(\frac{c_2-c_5}{c_1}\right) \quad (1)$$

The presence of $\frac{dZ_{22}}{d\tau}$ and $P_{11}Z_{22}$ in the equation suggests a product of the type $\phi(\tau)Z_{22}(\tau)$

$$\text{but } \frac{d}{d\tau}(\phi Z_{22}) = \frac{d\phi}{d\tau} Z_{22} + \phi \frac{d}{d\tau} Z_{22} \quad (2)$$

Multiplying (1) by $\phi(\tau)$ yields

$$\phi \frac{dZ_{22}}{d\tau} - \phi P_{11}Z_{22} = -\phi \left(\frac{c_2-c_5}{c_1}\right)$$

which can become

$$\frac{d}{d\tau}(\phi Z_{22}) = -\phi\left(\frac{c_2-c_5}{c_1}\right)$$

if one can find $\phi(\tau)$ (integrating factor) such that

$$\frac{d\phi}{d\tau} = -\phi P_{11}$$

$$\rightarrow \ln\phi(\tau) = \int -P_{11}\, d\tau = \int -\frac{c_2}{c_1}\cos\Omega\tau\, d\tau + \int \frac{c_5}{c_1} d\tau$$

$$\ln\phi(\tau) = -\frac{c_2}{c_1\Omega}\sin\Omega\tau + \frac{c_5}{c_1}\tau + K$$

$$\text{or } \phi(\tau) = \exp\left[-\frac{c_2}{c_1\Omega}\sin\Omega\tau\right] . e^{\left(\frac{c_5}{c_1}\tau + K\right)}$$

$$\text{from } \frac{d(Z_{22}\phi)}{d\tau} = -\phi\,\frac{(c_2-c_5)}{c_1}$$

$$Z_{22} = \frac{1}{\phi}\int \phi \frac{(c_5-c_2)}{c_1} d\tau$$

$$Z_{22} = \exp\left[\frac{c_2}{c_1\Omega}\sin\Omega\tau - \frac{c_5}{c_1}\tau - K\right]\left(\frac{c_5-c_2}{c_1}\right)\int \exp\left[-\frac{c_2}{c_1\Omega}\sin\Omega\tau + \frac{c_5}{c_1}\tau + K\right] d\tau$$

$$\exp\left[-\frac{c_2}{\Omega c_1}\sin\Omega\tau\right] \simeq 1 - \frac{c_2}{c_1}\tau + \frac{c_2^2}{c_1^2}\tau^2/2 - \left\{\left(\frac{c_2}{c_1}\right)^2 - \Omega^2\frac{c_2}{c_1}\right\}\tau^3/6 + \cdots$$

$$\exp\left[c_5\tau/c_1\right] \simeq 1 + c_5\tau/c_1 + (c_5\tau/c_1)^2 \cdot \frac{1}{2} + \cdots$$

$$\text{Therefore, } \exp\left[-\frac{c_2}{\Omega c_1}\sin\Omega\tau + c_5\tau/c_1\right] \simeq 1 + \left(\frac{c_5-c_2}{c_1}\right)\tau + \left(\frac{c_5-c_2}{c_1}\right)^2\tau^2/2 + \cdots$$

$$Z_{22} = \exp\left[\frac{c_2}{c_1\Omega}\sin\Omega\tau - \frac{c_5}{c_1}\tau - K\right]\left(\frac{c_5-c_2}{c_1}\right)e^K\left(\tau + \frac{c_5-c_2}{c_1}\tau^2/2 + \left(\frac{c_5-c_2}{c_1}\right)^2\tau^3/6 + \cdots + K\right)$$

$$Z_{22}(0) = 1 \rightarrow \frac{(c_5-c_2)}{c_1}K_1 = 1 \rightarrow K_1 = \frac{c_1}{c_5-c_2}$$

$$Z_{22} = \exp\left[\frac{C_2}{\Omega C_1}\sin\Omega\tau - \frac{C_5}{C_1}\tau\right]\left\{1 + \left(\frac{C_5 - C_2}{C_1}\right)\tau + \left(\frac{C_5 - C_2}{C_1}\right)^2\frac{\tau^2}{2} + \left(\frac{C_5 - C_2}{C_1}\right)^3\frac{\tau^3}{6} + \cdots\right\}$$

since $\dot{Z}_{22} = Z_{12}(\tau) = \exp\left[\frac{C_2}{C_1\Omega}\sin\Omega\tau - \frac{C_5}{C_1}\tau\right]\left\{\left(\frac{C_5 - C_2}{C_1}\right) + \left(\frac{C_5 - C_2}{C_1}\right)^2\tau + \cdots\right\}$

$$+\left(\frac{C_2}{C_1}\cos\Omega\tau - \frac{C_5}{C_1}\right)\exp\left[\frac{C_2}{\Omega C_1}\sin\Omega\tau - \frac{C_5}{C_1}\tau\right]\left\{1 + \left(\frac{C_5 - C_2}{C_1}\right)\tau + \left(\frac{C_5 - C_2}{C_1}\right)^2\frac{\tau^2}{2} + \cdots\right\}$$

$$Z_{12} = \exp\left[\frac{C_2}{C_1\Omega}\sin\Omega\tau - \frac{C_5}{C_1}\tau\right]\left\{-\frac{C_2 + C_2\cos\Omega\tau}{C_1} + \left[\left(\frac{C_2\cos\Omega\tau - C_5}{C_1}\right)\left(\frac{C_5 - C_2}{C_1}\right)\right.\right.$$

$$\left.\left. + \left(\frac{C_5 - C_2}{C_1}\right)^2\right]\tau + \left[\left(\frac{C_2\cos\Omega\tau - C_5}{C_1}\right)\left(\frac{C_5 - C_2}{C_1}\right)^2 + \left(\frac{C_5 - C_2}{C_1}\right)^3\right]\frac{\tau^2}{2} + \cdots\right\}$$

$$Z_{12}(\tau) = \exp\left[\frac{C_2}{C_1\Omega}\sin\Omega\tau - \frac{C_5}{C_1}\tau\right]\left\{\frac{C_2}{C_1}(1 + \cos\Omega\tau) + \left(\frac{C_5 - C_2}{C_1}\right)\left(\frac{C_2\cos\Omega\tau - C_2}{C_1}\right) + \cdots\right.$$

$$Z_{12}(\tau) = \exp\left[\frac{C_2}{\Omega C_1}\sin\Omega\tau - \frac{C_5}{C_1}\tau\right]\left[\frac{C_2}{C_1}(\cos\Omega\tau - 1)\right]\left[1 + \frac{C_5 - C_2}{C_1}\tau + \left(\frac{C_5 - C_2}{C_1}\right)^2\frac{\tau^2}{2} + \cdots\right.$$

$$\frac{dZ_{21}}{d\tau} = P_{11}Z_{21} + 1 \quad \text{where } P_{11} = \frac{c_2}{c_1}\cos\Omega\tau - \frac{c_5}{c_1}$$

Integrating factor ϕ ; $\frac{d\phi}{d\tau} = -\phi' P_{11}$

$\rightarrow \phi = \exp\left[-\frac{C_2}{\Omega C_1}\sin\Omega\tau + \frac{C_5}{C_1}\tau + K\right]$ and

$$Z_{21} = \frac{1}{\phi}\int\phi\,d\tau = \mathrm{Exp}\left[\frac{C_2}{\Omega C_1}\sin\Omega\tau - \frac{C_5}{C_1}\tau - K\right]\int\phi\,d\tau$$

$$\exp\left[-\frac{C_2}{\Omega C_1}\sin\Omega\tau + \frac{C_5}{C_1}\tau\right] \simeq 1 + \left(\frac{C_5 - C_2}{C_1}\right)\tau + \left(\frac{C_5 - C_2}{C_1}\right)^2\frac{\tau^2}{2} + \cdots$$

Integrating term by term yields,

$$Z_{21} = \exp\left[\frac{C_2}{C_1}\sin\Omega\tau - \frac{C_5}{C_1}\tau\right]\left[\tau + \left(\frac{C_5 - C_2}{C_1}\right)\frac{\tau^2}{2} + \left(\frac{C_5 - C_2}{C_1}\right)^2\frac{\tau^3}{6} + \cdots + K'\right]$$

since $Z_{21}(0) = K' = 0 \Rightarrow$

$$Z_{21}(\tau) = \exp\left[\frac{C_2}{\Omega C_1}\sin\Omega\tau - \frac{C_5}{C_1}\tau\right]\left[\tau + \left(\frac{C_5 - C_2}{C_1}\right)\frac{\tau^2}{2} + \left(\frac{C_5 - C_2}{C_1}\right)^2\frac{\tau^3}{6} + \cdots\right]$$

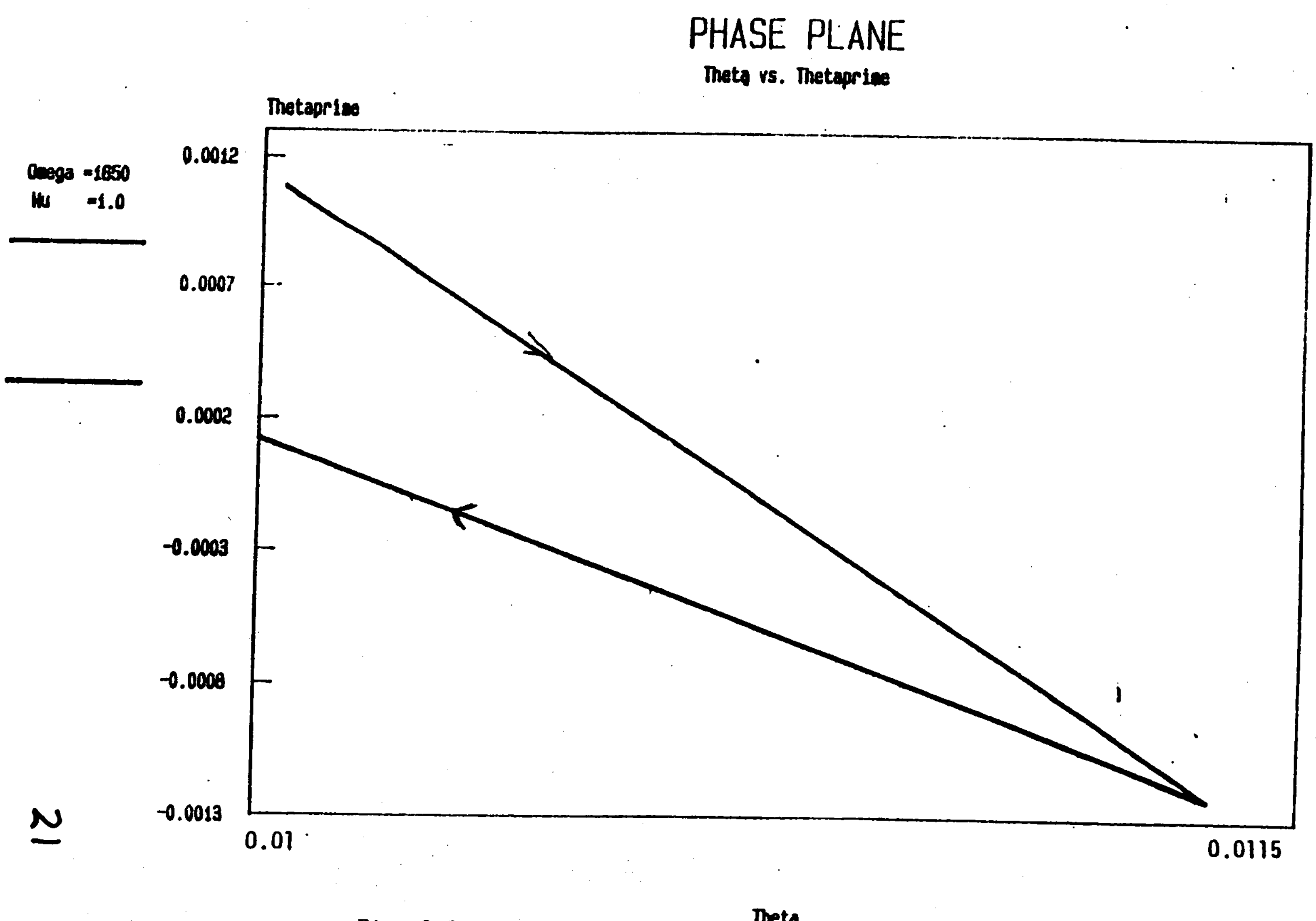
$$\dot{Z}_{21} = Z_{11}(\tau) = \exp\left[\frac{C_2}{\Omega C_1}\sin\Omega\tau - \frac{C_5}{C_1}\tau\right]\left\{1 + \left(\frac{C_5 - C_2}{C_1}\right)\tau + \left(\frac{C_5 - C_2}{C_1}\right)^2\frac{\tau^2}{2} + \cdots\right\}$$

$$+\left[\frac{C_2}{C_1}\cos\Omega\tau - \frac{C_5}{C_1}\right]\exp\left[\frac{C_2}{\Omega C_1}\sin\Omega\tau - \frac{C_5}{C_1}\tau\right]\left[\tau + \left(\frac{C_5 - C_2}{C_1}\right)\frac{\tau^2}{2} + \left(\frac{C_5 - C_2}{C_1}\right)^2\frac{\tau^3}{6} + \cdots\right]$$

$$Z_{11}(\tau) = \exp\left[C_2/\Omega C_1 \sin\Omega\tau - C_5/C_1\,\tau\right]\left[1 + (C_5 - C_2 + C_2\cos\Omega\tau - C_5)\frac{\tau}{C_1} + \cdots\right.$$

$$Z_{11}(\tau) = \exp\left[\frac{C_2}{\Omega C_1}\sin\Omega\tau - \frac{C_5}{C_1}\tau\right]\left[1 + \frac{C_2}{C_1}(\cos\Omega\tau - 1)\tau + \frac{C_2}{C_1}(C_5 - C_2)(\cos\Omega\tau - 1)\tau\right.$$
$$\left. + \cdots\right]$$

It is seen that $Z_{11}(0) = 1$


PHASE PLANE
Theta vs. Thetaprime
Thetaprime
Omega =1650
Nu =1.0
0.0012
0.0007
0.0002
-0.0003
-0.0008
-0.0013
0.01
0.0115
Theta

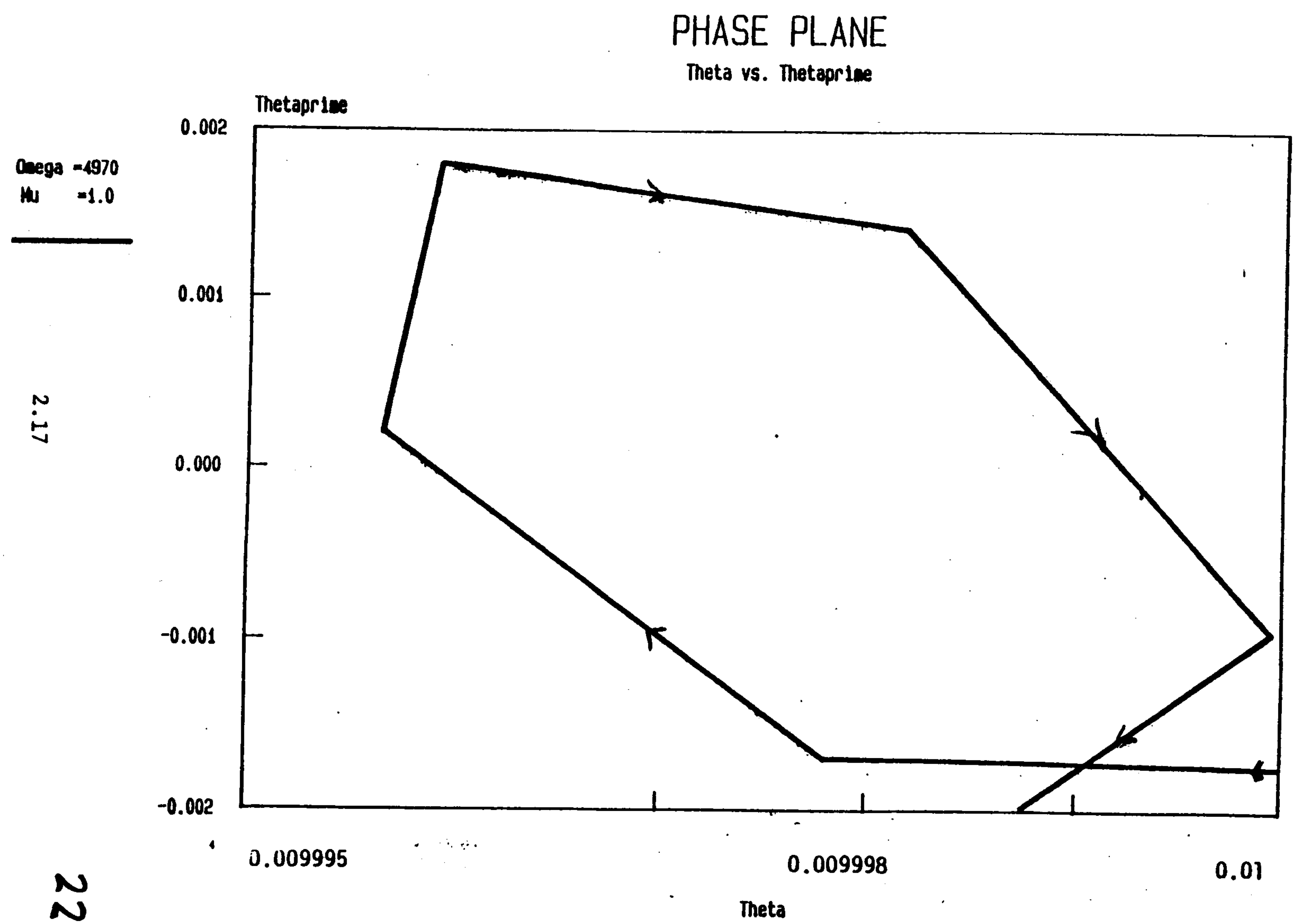


Fig. 2.2b SCOLE System-Phase Plane Trajectories - No Offset, No Gravity Gradient

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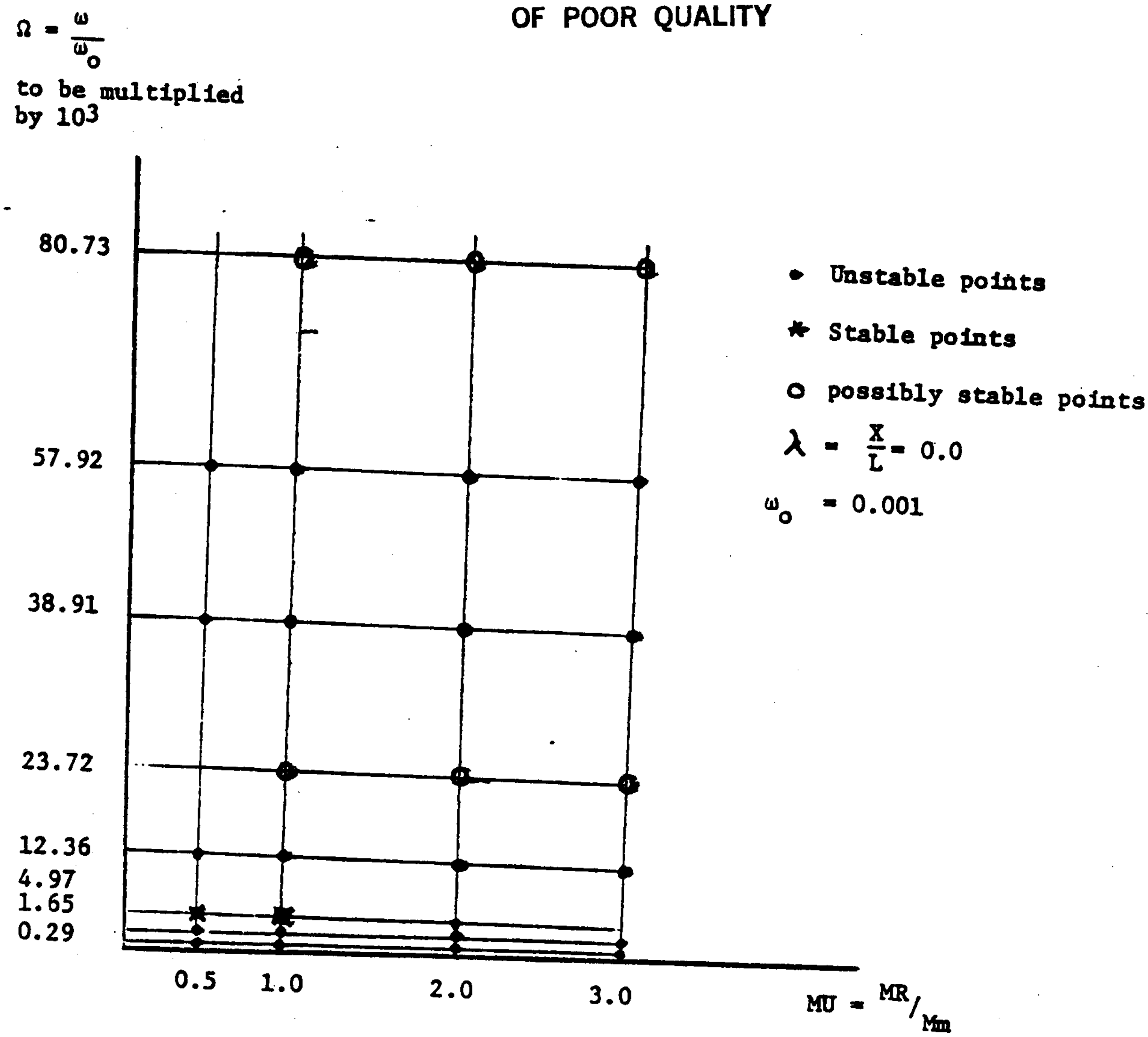

Fig. 2.3 Floquet Stability Diagram - SCOLE Configuration-No Offset No Gravity Gradient.

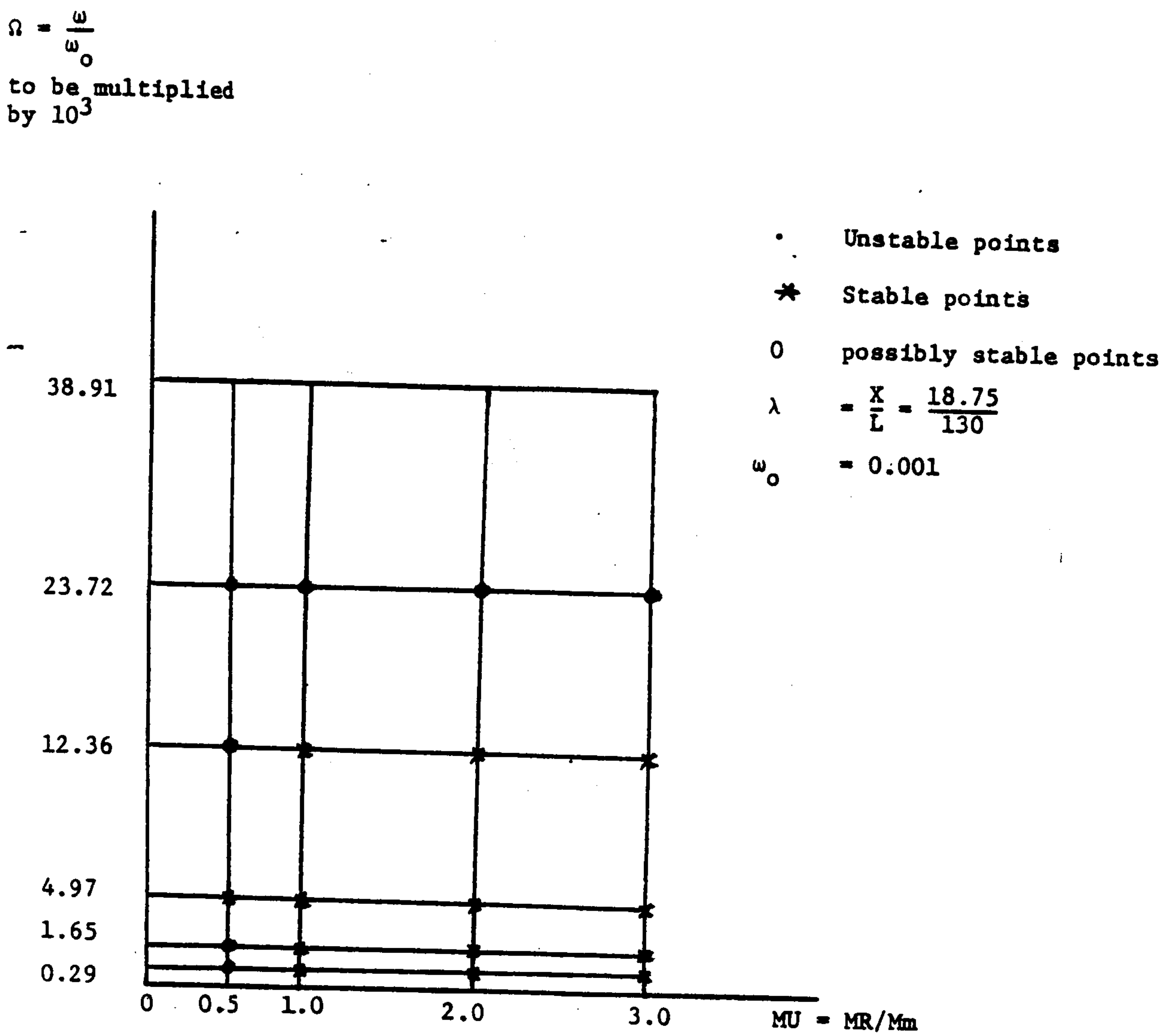

Fig. 2.5 Floquet Stability Diagram - SCOLE Configuration
No Gravity Gradient

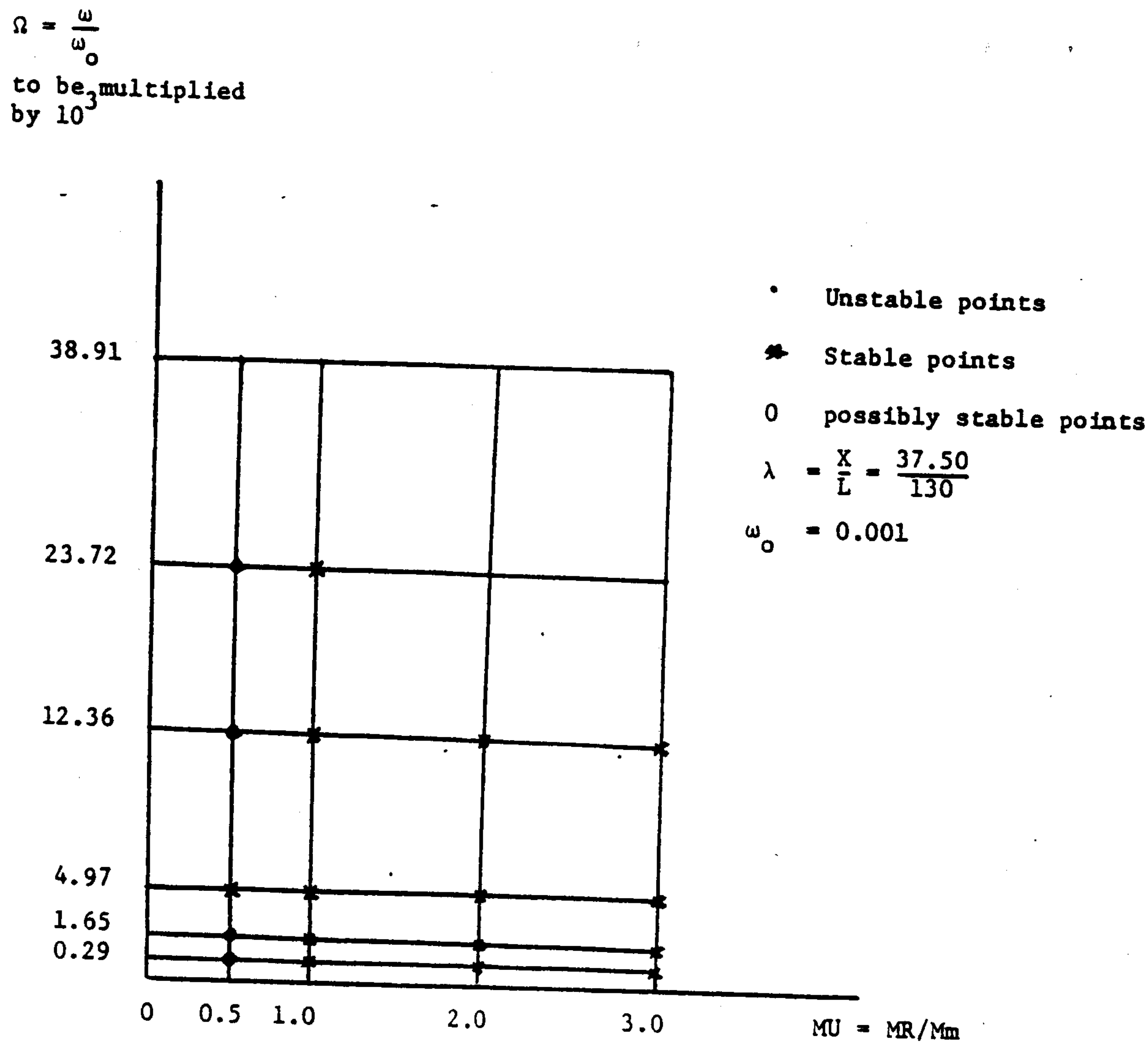

Fig. 2.4 Floquet Stability Diagram - SCOLE Configuration No Gravity Gradient.

FLOQUET STABILITY ANALYSIS
2D SCOLE OPEN-LOOP SYSTEM

- Offset of the mast attachment point on the reflector results in an increase in the number of stable points for the lower frequencies

- Number of stable points increases for $M_R/M_m > 1.0$

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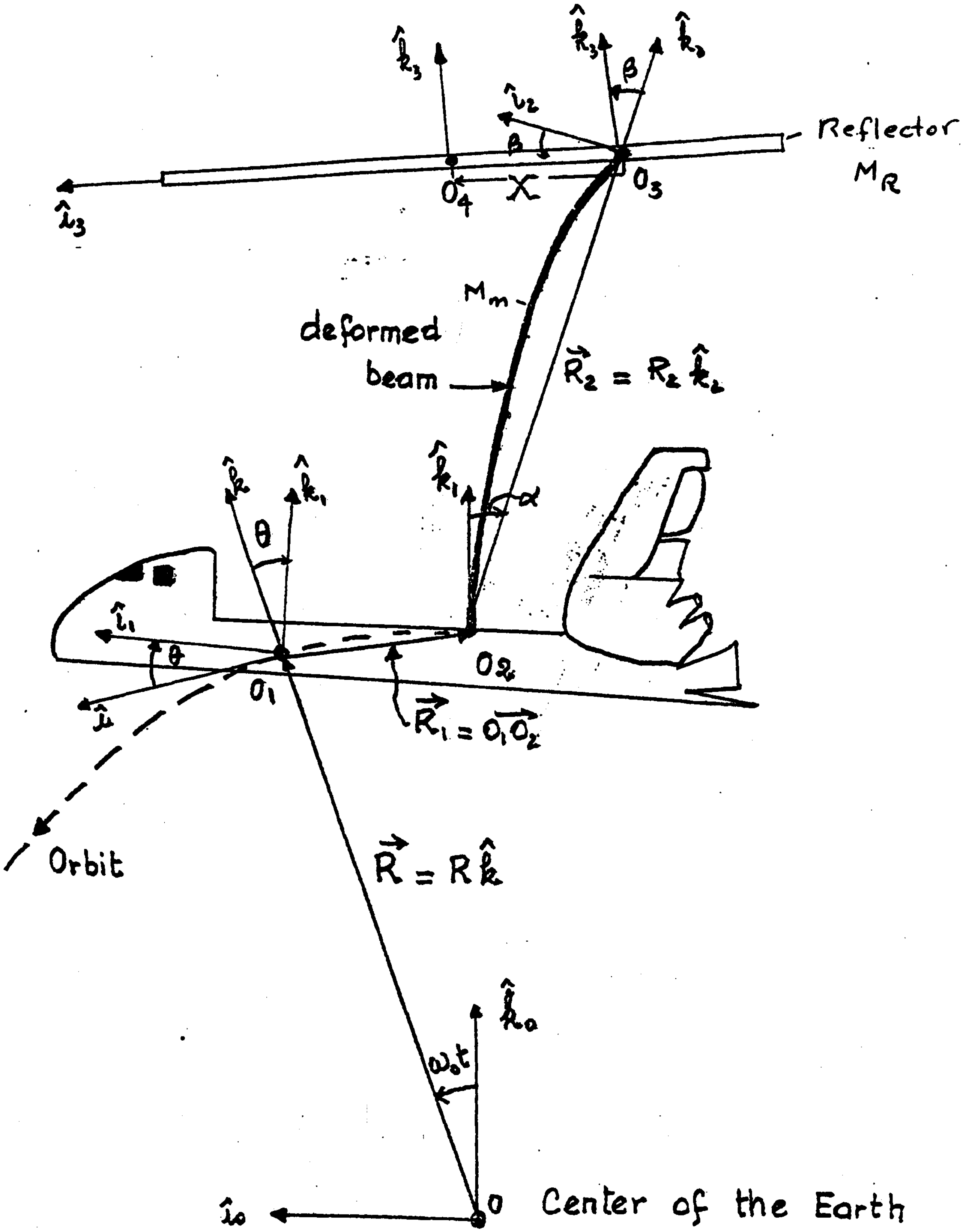

Fig. 2.1. SCOLE System Geometry in the Deformed State (2-D)

A. Angular Momentum of the Shuttle About its Mass Center, G

The angular momentum of the Shuttle, taken as a rigid body in a circular orbit, consists of contributions due to rotation about its center of mass plus the translation along the orbit.

$$\vec{Hs}/_G = \bar{\bar{I}}_G \; \vec{\omega s}/_{Ro} \qquad (1.9)$$

where

$$\bar{\bar{I}}_G = \begin{bmatrix} 905,443 & 0 & 145,393 \\ 0 & 6,784,100 & 0 \\ 145,393 & 0 & 7,086,601 \end{bmatrix} = \left[Iij\right]$$

in R(x,y,z) (1.10)

$$\vec{\omega s}/_{Ro} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k} \qquad (1.11)$$

B. Angular Momentum of the Beam about G

Consider an element of mass, dm, of the beam located at some point, p, such that $\vec{GP} = \vec{r}_o + \vec{q} = \vec{r}$

where:

(1.12) $\vec{r}_o = -z\hat{k}$ is the position vector of p in the undeformed state

(1.13) $\vec{q}(z,t) = u\hat{i} + v\hat{j}$ in which, u and v are the x and y components of the mode shape vector.

The angular momentum of dm about G, $d\,\vec{Hm}/_G$ is given by:

$$d\,\vec{Hm}/_G = \vec{r} \times \frac{d}{dt}(-R\hat{k}+\vec{r})\Big|_{Ro} dm \qquad (1.14)$$

$$d\,\vec{Hm}/_G = (\vec{r} \times \omega_o R \hat{i} + \vec{r} \times \frac{d}{dt}\vec{r}\Big|_{Ro})\, dm$$

which is expressed explicitly as:

$$d\vec{H}_{m/G} = \left\{(-z\hat{k} + u\hat{i} + v\hat{j}) \times \omega_0 R\hat{i} + (-z\hat{k} + u\hat{i} + v\hat{j}) \times \frac{d}{dt}(-z\hat{k} + u\hat{i} + v\hat{j})/_{R_0}\right\} dm$$

where

$$\frac{d\vec{r}}{dt}/_{R_0} = \frac{d}{dt}(-z\hat{k} + u\hat{i} + v\hat{j})_{R_0} = (\dot{u} - \omega_z v - z\omega_y)\hat{i} + (\dot{v} + \omega_z u + z\omega_x)\hat{j} + (\omega_x v - u\omega_y)\hat{k}$$

$$\vec{r} \times \frac{d\vec{r}}{dt}/_{R_0} = -z(\dot{u} - \omega_z v)\hat{j} + z(\dot{v} + \omega_z u)\hat{i} + u(\dot{v} + \omega_z u)\hat{k} - u(\omega_x v - u\omega_y)\hat{j}$$

$$- v(\omega_y u - \omega_x v)\hat{i} - v(\dot{u} - \omega_z v)\hat{k} + z^2(\omega_y\hat{j} + \omega_x\hat{i}) + z(u\omega_x + v\omega_y)\hat{k}$$

After substituting the different terms into equation (1.14), the following expression results:

$$d\vec{H}_{m/G} = \Big\{\left[z(\dot{v} + \omega_z u) + v(\omega_x v - \omega_y u) + z^2\omega_x\right]\hat{i}$$

$$+ \left[-z\omega_0 R - z(\dot{u} - \omega_z v) + u(\omega_y u - \omega_x v) + z^2\omega_y\right]\hat{j}$$

$$+ \left[-v\omega_0 R + u(\dot{v} + \omega_z u) - v(\dot{u} - \omega_z v) + z(u\omega_x + v\omega_y)\right]\hat{k}\Big\} dm \quad (1.15)$$

Since $u(z,t) = \Sigma\, p_x^n(t)s_x^n(z)$ and $v(z,t) = \Sigma\, p_y^n(t)s_y^n(z)$, (1.16)
we consider for one mode in the open-loop situation,

$$\dot{u} = -\omega_x' \sin(\omega_x' t + \alpha)s_x(z) \text{ and}$$
$$\dot{v} = -\omega_y' \sin(w_y' t + \gamma)s_y(z) \quad (1.17)$$

Assuming small elastic displacements such that, $\frac{s_i s_j}{\ell^2} << 1$ and $s_i^{\,2}(z)/\ell^2 << 1$, and dividing $\overrightarrow{dHm}/_G$ by $\Omega\ell^2$, where Ω is an assigned frequency and ℓ a reference length, then,

$$\frac{d\overrightarrow{H}_{m/G}}{\Omega\ell^2} \simeq \frac{1}{\Omega\ell^2}\Big\{(z\dot{v}+z\omega_z u+\omega_x z^2)\hat{i}+(-z\omega_0 R-z\dot{u}+\omega_z z v+z^2\omega_y)\hat{j}+(-\omega_0 R v+\omega_x u z+\omega_y z v)\hat{k}\Big\}\rho dz \tag{1.18}$$

where ρ is the mass per unit length of the beam. After multiplying both sides of this equation by $\Omega\ell^2$, there results:

$$d\overrightarrow{H}_{m/G} \simeq \Big\{(z\dot{v}+zu\omega_z+z^2\omega_x)\hat{i}+(-zR\omega_0-z\dot{u}+zv\omega_z+z^2\omega_y)\hat{j}+(-vR\omega_0+zu\omega_x+zv\omega_y)\hat{k}\Big\}\rho dz \tag{1.19}$$

The total angular momentum of the mast about G is obtained by integrating (1.19) over the total length of the mast,

$$\overrightarrow{H}_{m/G} = \int_0^{-\ell} d\overrightarrow{H}_{m/G} \tag{1.20}$$

The ten terms appearing in $\overrightarrow{dHm}/_G$ are integrated using integral tables- e.g.

$$\int_0^{-\ell} z\dot{v}\rho dz = -\rho\omega_y' \sin(\omega_y' t+\gamma)\int_0^{-\ell} z(A_2\sin\beta_2 z+B_2\cos\beta_2 z+C_2\sinh\beta_2 z+D_2\cosh\beta_2 z)\,dz =$$

$$-\rho\omega_y' \sin(\omega_y' t+\gamma)\Big[A_2\Big(-\frac{\sin\beta_2\ell}{\beta_2^2}+\frac{\ell\cos\beta_2\ell}{\beta_2}\Big)+B_2\Big(\frac{\ell\sin\beta_2\ell}{\beta_2}+\frac{\cos\beta_2\ell}{\beta_2^2}-\frac{1}{\beta_2^2}\Big)$$
$$+C_2\Big(-\frac{\ell\cosh\beta_2\ell}{\beta_2}+\frac{\sinh\beta_2\ell}{\beta_2^2}\Big)+D_2\Big(\frac{\ell\sinh\beta_2\ell}{\beta_2}-\frac{\cosh\beta_2\ell}{\beta_2^2}+\frac{1}{\beta_2^2}\Big)\Big]$$

To simplify the notation, let

$$f_i(\beta_i) = \left\{ A_i \left(\frac{l \cos\beta_i l}{\beta_i} - \frac{\sin\beta_i l}{\beta_i^2} \right) + B_i \left(\frac{\cos\beta_i l}{\beta_i^2} + \frac{l \sin\beta_i l}{\beta_i} - \frac{1}{\beta_i^2} \right) \right.$$

$$\left. + C_i \left(\frac{\sinh\beta_i l}{\beta_i^2} - \frac{l \cosh\beta_i l}{\beta_i} \right) + D_i \left(\frac{l \sinh\beta_i l}{\beta_i} - \frac{\cosh\beta_i l}{\beta_i^2} + \frac{1}{\beta_i^2} \right) \right\} \quad (1.21)$$

$$g_i(\beta_i) = A_i \left(\frac{1}{\beta_i} - \frac{\cos\beta_i l}{\beta_i} \right) - B_i \frac{\sin\beta_i l}{\beta_i} - D_i \frac{\sinh\beta_i l}{\beta_i}$$

$$+ C_i \left(\frac{\cosh\beta_i l}{\beta_i} - \frac{1}{\beta_i} \right) \quad (1.22)$$

After substitution of the f_i, g_i and $\frac{Mm}{\ell}$ for $p_{x,y}$ in the expression of $\vec{H}m/_G$, one arrives at:

$$\vec{H}_{m/G} = \frac{Mm}{l} \left\{ \left[\omega_z \cos(\omega_x' t + \alpha) f_1 - \omega_y' \sin(\omega_y' t + \gamma) f_2 - \omega_x \frac{l^3}{3} \right] \hat{i} \right.$$

$$+ \left[\omega_0 R \frac{l^2}{2} + \omega_x' \sin(\omega_x' t + \alpha) f_1 + \omega_z \cos(\omega_y' t + \gamma) f_2 - \omega_y \frac{l^3}{3} \right] \hat{j}$$

$$\left. + \left[\omega_x \cos(\omega_x' t + \alpha) f_1 + \omega_y \cos(\omega_y' t + \gamma) f_2 + \omega_0 R \cos(\omega_y' t + \gamma) g_2 \right] \hat{k} \right\} \quad (1.23)$$

C. Angular Momentum of the Reflector about G.

Since small deflections for the beam are assumed, the reflector can be assumed to be located at a constant distance from G, the Shuttle mass center.

Using the transfer theorem for the angular momentum, (See Appendix IIIA)

$$\vec{H}_{r/G} = \bar{\bar{I}}_{r/O_1}\,\vec{\omega}_{r/R_o} + M_r \vec{GO_1} \times \frac{d}{dt}(\vec{OO_1})_{/R_o} \quad (1.24)$$

where $\bar{\bar{I}}_{r/O_1}$ and $\vec{\omega}_{r/R_o} = \vec{\omega}_{r/s} + \vec{\omega}_{r/s}$ (1.25) are both expressed in the same coordinate system, $R_2(x_2, y_2 z_2)$, moving with the reflector. In R_2 (principal axes of inertia of the reflector),

$$\bar{\bar{I}}_{r/O_1} = \begin{bmatrix} Ir_1 & 0 & 0 \\ 0 & Ir_2 & 0 \\ 0 & 0 & Ir_3 \end{bmatrix} = \begin{bmatrix} 4{,}969 & & 0 \\ 0 & 4{,}969 & 0 \\ 0 & & 9{,}938 \end{bmatrix} \quad (1.26)$$

$$\vec{\omega}_{r/R_o} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k} + \dot{\psi}_r \hat{i} + \dot{\theta}_r \hat{j}' + \dot{\Phi}_r \hat{k}_2 \quad (1.27)$$

with $\hat{j}' = \sin \Phi_r\, \hat{i}_2 + \cos \Phi_r \hat{j}_2$

therefore,

$$\vec{\omega}_{r/R_o} = (\omega_x + \dot{\psi}_r)\hat{i} + \omega_y \hat{j} + \omega_z \hat{k} + \dot{\theta}_r \sin \Phi_r \hat{i}_2 + \dot{\theta} \cos \Phi_r \hat{j}_2 + \dot{\Phi}_r \hat{k}_2 \quad (1.28)$$

$$\overrightarrow{H}_{r/G} = \omega_1 I_{r1} \hat{i}_2 + \omega_2 I_{r2} \hat{j}_2 + \omega_3 I_{r3} \hat{k}_2$$

$$+ M_r \left\{ (bl + c(v+Y))\hat{i} - (al + c(u+X))\hat{j} + (b(u+X) - a(v+Y))\hat{k} \right\} \quad (1.34)$$

Where

$$a = \left\{ (\omega_0 R + \dot{u} - \omega_y l - \omega_3 v - \omega_3 Y T_{11} + X \omega_3 T_{21}) + (\omega_1 Y - \omega_2 X) T_{31} \right\}.$$

$$b = \left\{ \dot{v} - \omega_x l + \omega_3 u - \omega_3 Y T_{12} + \omega_3 X T_{22} + (\omega_1 Y - \omega_2 X) T_{32} \right\}$$

$$c = \left\{ \omega_x v - \omega_y u - \omega_3 Y T_{13} + \omega_3 X T_{23} + (\omega_1 Y - \omega_2 X) T_{33} \right\}$$

D. Angular Momentum of the System about G

The angular momentum of the system = the sum of the angular momentum of each component evaluated at the same point

$$\vec{H}_{system/G} = \vec{H}_{S/G} + \vec{H}_{m/G} + \vec{H}_{r/G}$$

$$= (\omega_x I_{11} + \omega_z I_{13})\hat{i} + (\omega_y I_{22} \qquad)\hat{j} + (\omega_x I_{31} + \omega_z I_{33})\hat{k}$$

$$+ \frac{M_m}{\ell}\Big\{\Big[\omega_z \cos(\omega_x' t+\alpha) f_1 - \omega_y' \sin(\omega_y' t+\gamma) f_2 - \omega_x \frac{\ell^3}{3}\Big]\hat{i}$$

$$+\Big[\omega_0 R \frac{\ell^2}{2} + \omega_x' \sin(\omega_x' t+\alpha) f_1 + \omega_z \cos(\omega_y' t+\gamma) f_2 - \omega_y \frac{\ell^3}{3}\Big]\hat{j}$$

$$+\Big[\omega_x \cos(\omega_x' t+\alpha) f_1 + \omega_y \cos(\omega_y' t+\gamma) f_2 - \omega_0 R \cos(\omega_y' t+\gamma) g_2\Big]\hat{k}\Big\}$$

$$+ M_r\Big\{(b\ell + c(v+Y))\hat{i} - (a\ell + c(u+X))\hat{j} + (b(u+X) - a(v+Y))\hat{k}\Big\}$$

$$+ \omega_1 I_{r1}\hat{i}_2 + \omega_2 I_{r2}\hat{j}_2 + \omega_3 I_{r3}\hat{k}_2 \qquad (1.35)$$

In the expression for the total angular momentum, the last term will now be expressed in R(x,y,z) by simply transforming $\hat{i}_2$, $\hat{j}_2$, and $\hat{k}_2$ into functions of $\hat{i}$, $\hat{j}$, and $\hat{k}$ as follows:

$$\left.\begin{aligned}
\hat{i}_2 &= \cos\Phi_r \cos\theta_r\,\hat{i} + \sin\Phi_r \cos\psi_r\,\hat{j} + \sin\Phi_r \sin\psi_r\,\hat{k} \\
\hat{j}_2 &= -\sin\Phi_r \cos\theta_r\,\hat{i} + (\cos\Phi_r \cos\psi_r - \sin\Phi_r \sin\theta_r \sin\psi_r)\,\hat{j} \\
&\quad + (\sin\Phi_r \sin\theta_r \cos\psi_r + \cos\Phi_r \sin\psi_r)\,\hat{k} \\
\hat{k}_2 &= \sin\theta_r\,\hat{i} - \cos\theta_r \sin\psi_r\,\hat{j} + \cos\theta_r \cos\psi_r\,\hat{k}
\end{aligned}\right\} \qquad (1.36)$$

Rotational Equations of Motion. (Torque Free)

In the linear range,

$$\dot{H}_x + \omega_y H_z - \omega_z H_y = 0 \qquad \text{(A)}$$

$$\Rightarrow \Big\{ (\ddot{\psi} - \omega_0 \dot{\phi}) I_{11} + (\ddot{\phi} + \omega_0 \dot{\psi}) I_{13} + \frac{M_m}{\ell} \Big[(\ddot{\phi} + \omega_0 \dot{\psi}) \cos(\omega_x' t + \alpha) f_1$$

$$- \omega_x' (\dot{\phi} + \omega_0 \psi) \sin(\omega_x' t + \alpha) f_1 - \omega_y'^2 \cos(\omega_y' t + \gamma) f_2 - (\ddot{\psi} - \omega_0 \dot{\phi}) \frac{\ell^3}{3} \Big]$$

$$+ (\ddot{\psi} - \omega_0 \dot{\phi} + \ddot{\psi}_r - \omega_0 \dot{\Phi}_r) I_{r1} + \omega_0 \dot{\Phi}_r I_{2r} \Big\} + (\omega_0^2 \phi - \omega_0 \dot{\psi}) I_{31}$$

$$+ (\omega_0 \dot{\phi} + \omega_0^2 \psi)(I_{22} - I_{33} \qquad) + \frac{M_m}{\ell} \Big[(\omega_0^2 \phi - \omega_0 \dot{\psi}) \cos(\omega_x' t + \alpha) f_1$$

$$+ (\omega_0^2 - 2\omega_0 \dot{\theta}) \cos(\omega_y' t + \gamma) f_2 - (\omega_0 \dot{\theta} - \omega_0^2) R \cos(\omega_y' t + \gamma) g_2 \Big]$$

$$+ \omega_0^2 \psi_r I_{2r} - (\psi_r \omega_0^2 + \omega_0 \dot{\phi} + \omega_0^2 \psi + \omega_0 \dot{\Phi}_r) I_{r3} + M_r (u + X) \Big[(\dot{\theta} - \omega_0) \dot{v}$$

$$+ \ell (\omega_0^2 \phi - \omega_0 \dot{\psi}) - (\omega_0 \dot{\phi} + \omega_0^2 \psi) u - X (\omega_0^2 \psi_r + \omega_0 \dot{\Phi}_r + \omega_0^2 \psi + \omega_0 \dot{\phi})$$

$$+ \omega_0^2 \psi_r X \Big] - M_r (v + Y) \Big[(\omega_0 \dot{\theta} - \omega_0^2) R + (\dot{\theta} - \omega_0) \dot{u} + (\omega_0 \dot{\theta} - \omega_0^2) \ell$$

$$+ (\omega_0 \dot{\phi} + \omega_0^2 \psi) v + Y (\omega_0^2 \psi_r + \omega_0 \dot{\Phi}_r + \omega_0 \dot{\phi} + \omega_0^2 \psi) - \omega_0^2 \theta_r X \Big]$$

$$+ M_r \ell \Big[\ddot{v} + (\ddot{\psi} - \omega_0 \dot{\phi}) \ell + X (\ddot{\phi} + \omega_0 \dot{\psi} + \ddot{\Phi}_r) \Big] + M_r X \omega_0 \dot{v}$$

$$+ M_r Y \Big[Y (\ddot{\psi} + \ddot{\psi}_r - \omega_0 (\dot{\phi} - \dot{\Phi}_r) - X (\ddot{\theta} + \ddot{\theta}_r) \Big] = 0 \qquad (1.45)$$

$$\dot{H}_y + \omega_z H_x - \omega_x H_z = 0 \qquad (8)$$

$$\Leftrightarrow \quad \ddot{\theta} I_{22} + \frac{M_m}{\ell}\Big[\omega_x'^2 \cos(\omega_x' t+\alpha) f_1 + (\ddot{\phi} + \omega_0 \dot{\psi}) \cos(\omega_y' t+\gamma) f_2$$

$$- 2\omega_y' (\dot{\phi} + \omega_0 \psi) \sin(\omega_y' t+\gamma) f_2 - \ddot{\theta}\frac{\ell^3}{3} + \omega_0 (\dot{\psi} - \omega_0 \phi) \cos(\omega_y' t+\gamma)(f_2 + g_2 R$$

$$+ I_{2r}(\ddot{\theta} - \ddot{\theta}_r) - M_r \ell \Big[\ddot{u} - \ddot{\theta}\ell - Y(\ddot{\phi} + \omega_0 \dot{\psi} + \ddot{\Phi}_r + \omega_0 \dot{\psi}_r)$$

$$+ \omega_0 \dot{\theta}_r X\Big] - 2M_r X \omega_0 \dot{u} - M_r X Y(\ddot{\psi} - \omega_0 \dot{\phi} + \ddot{\psi}_r - \omega_0 \dot{\Phi}_r)$$

$$- M_r X^2 (\ddot{\theta} + \ddot{\theta}_r) - M_r Y(\dot{\psi} - \omega_0 \phi)(\omega_0 R + \omega_0 \ell) = 0 \quad (1.47)$$

$$\dot{H}_z + \omega_x H_y - \omega_y H_x = 0 \qquad (c)$$

$$\Rightarrow \Big\{ (\ddot{\psi} - \omega_0 \dot{\phi}) I_{31} + (\ddot{\phi} + \omega_0 \dot{\psi}) I_{33} + \frac{M_m}{\ell}\Big[(\dot{\psi} - \omega_0 \phi) \cos(\omega_x' t + \alpha) f_1$$

$$+ \omega_x' (\dot{\psi} - \omega_0 \phi) \sin(\omega_x' t + \alpha) f_1 + \ddot{\theta} \cos(\omega_y' t + \gamma) f_2 - \omega_y' (\dot{\theta} - \omega_0) \sin(\omega_y' t + \gamma) f_2$$

$$+ \omega_0 \omega_y' R \sin(\omega_y' t + \gamma) g_2 \Big] + M_r (u + X) \big[\ddot{v} + \ddot{\psi} \ell - \omega_0 \dot{\phi} \ell + \ddot{\phi} u + \dot{\phi} \dot{u} + \omega_0 \dot{\psi} u$$

$$+ \omega_0 \psi \dot{u} + X (\dot{\psi}_r \omega_0 + \ddot{\phi} + \omega_0 \dot{\psi} + \ddot{\Phi}_r - \omega_0 \dot{\psi}_r) \big] + M_r \dot{u} \big[\dot{v} + (\dot{\psi} - \omega_0 \phi) \ell + (\dot{\phi} + \omega_0 \psi) u$$

$$+ X (\dot{\phi} + \omega_0 \psi + \dot{\Phi}_r) \big] - M_r (v + Y) \big[\ddot{u} - \ddot{\theta} \ell - \ddot{\phi} v - \dot{\phi} \dot{v} - \omega_0 \dot{\psi} v - \omega_0 \psi \dot{v}$$

$$- Y (\dot{\psi}_r \omega_0 + \ddot{\Phi}_r + \omega_0 \dot{\psi} + \ddot{\phi}) + \omega_0 X \dot{\theta}_r \big] - M_r \dot{v} \big[\omega_0 R + \dot{u} - (\dot{\theta} - \omega_0) \ell$$

$$- (\dot{\phi} + \omega_0 \psi) v + \omega_0 \theta_r X - Y (\psi_r \omega_0 + \dot{\phi} + \omega_0 \psi + \dot{\Phi}_r) \big] - (\dot{\psi} - \omega_0 \phi) \omega_0 I_{22}$$

$$+ \frac{M_m}{\ell} (\dot{\psi} - \omega_0 \phi) \left(\omega_0 R \frac{\ell^2}{2} + \omega_x' \sin(\omega_x' t + \alpha) f_1 + \omega_0 \frac{\ell^3}{3} \right) - \omega_0 (\dot{\psi} - \omega_0 \Phi) I_{r2}$$

$$- M_r \ell (\dot{\psi} - \omega_0 \phi)(\omega_0 R + \dot{u} + \omega_0 \ell) - M_r (u + X)^2 (\dot{\psi} - \omega_0 \phi) + \omega_0 (\dot{\psi} - \omega_0 \phi) I_{11}$$

$$+ \omega_0 (\dot{\phi} + \omega_0 \psi) I_{13} + \omega_0 \frac{M_m}{\ell} (\dot{\phi} + \omega_0 \psi) \cos(\omega_x' t + \alpha) f_1 + (\dot{\theta} - \omega_0) \omega_y' \sin(\omega_y' t + \gamma) f$$

$$- M_m \ell^2 (\dot{\psi} - \omega_0 \phi) + \omega_0 (\dot{\psi} - \omega_0 \phi + \dot{\psi}_r - \omega_0 \Phi_r) I_{r1} + \omega_0^2 \Phi_r I_{2r}$$

$$+ M_r \omega_0 \big\{ \dot{v} \ell + (\dot{\psi} + \omega_0 \phi) \ell^2 + X \ell (\psi_r \omega_0 + \dot{\Phi}_r$$

$$+ \dot{\phi} + \omega_0 \psi) - \omega_0 \psi_r X \ell + Y \omega_0 u + Y^2 (\dot{\psi} + \dot{\psi}_r - \omega_0 \phi - \omega_0 \Phi_r)$$

$$- X (\dot{\theta} - \omega_0 + \dot{\theta}_r) = 0 \qquad (1.49)$$

2. STRUCTURAL ANALYSIS

A. Governing Differential Equations

The governing partial differential equations for the system (beam) are comprised of two one-plane-bending equations (2.1) and (2.2) and one axial torsion equation, (2.9).

All these equations assume small displacements and slopes, uniform density and distribution of stiffness, and the torsional equation is derived for a circular shaft.

for the x-z plane bending: $$-\frac{\partial^2(u(z,t))}{\partial t^2} = \frac{(EI)_x}{\rho A}\frac{\partial^4 u(z,t)}{\partial z^4} \qquad (2.1)$$

for the y-z plane bending: $$-\frac{\partial^2(v(z,t))}{\partial t^2} = \frac{(EI)_y}{\rho A}\frac{\partial^4 v(z,t)}{\partial z^4} \qquad (2.2)$$

where ρ is the density of the beam, A its cross sectional area, and $(EI)_x$, $(EI)_y$ its (x-z) and (y-z) plane bending stiffnesses, respectively.

Assuming separation of variables for u(z,t), one may write $u(z,t) = r_x(z)p_x(t)$, and equation (2.1) can then be rewritten as:

$$\ddot{P}_x/P_x = -\frac{(EI)_x}{\rho A}\frac{r_x^{(4)}}{r_x} \qquad (2.3)$$

This equation is valid if and only if both sides are equal to a constant: $-\omega_x'^2$

Therefore $\ddot{P}_x + \omega_x'^2 P_x = 0$ (2.4), which integrates into

$P_x(t) = \cos(\omega_x' t + \alpha)$, (2.5); where α is a phase angle.

$$r_x^{(4)} - \omega_x'^2\frac{\rho A}{(EI)_x} r_x = 0 \Rightarrow r^{(4)} - \beta_x^4 r_x = 0 \quad \text{where } \beta_x^4 = \frac{\rho A}{(EI)_x}\omega_x'^2 \qquad (2.6)$$

this equation yields.

$$r_x = A_1 \sin\beta_x z + B_1 \cos\beta_x z + C_1 \sinh\beta_x z + D_1 \cosh\beta_x z$$

$$\Rightarrow \quad U(z,t) = \cos(\omega_x' t + \alpha)\{A_1 \sin\beta_x z + B_1 \cos\beta_x z + C_1 \sinh\beta_x z + D_1 \cosh\beta_x z\} \quad (2.7)$$

A similar reasoning enables us to find the solution of equation (2.2) in the following form:

$$v(z,t) = \cos(\omega_y' t + \gamma)\{A_2 \sin\beta_y z + B_2 \cos\beta_y z + C_2 \sinh\beta_y z + D_2 \cosh\beta_y z\} \quad (2.8)$$

Finally the z axis torsional bending is described by:

$$(2.9) \quad \frac{\partial^2 \Phi(z,t)}{\partial t^2} = \frac{G}{\rho} \frac{\partial^2 \Phi(z,t)}{\partial z^2}$$

where G is the modulus of rigidity of the beam.

Assuming $\Phi(z,t) = \tilde{\theta}(z) P_z(t)$ and substituting it into equation (2.9) yields:

$$\ddot{P}_z / P_z = -\frac{G}{\rho} \frac{\tilde{\theta}^{(2)}}{\tilde{\theta}} = -\omega_z'^2 \quad (2.10)$$

$$\ddot{P}_z / P_z = -\omega_z'^2 \Rightarrow P(t) = \cos(\omega_z' t + \delta) \quad \text{and} \quad (2.11)$$

$$G/\rho \; \frac{\tilde{\theta}^{(2)}}{\tilde{\theta}} = \omega_z'^2 \Rightarrow \quad \tilde{\theta}(z) = A_3 \sin\beta_z z + B_3 \cos\beta_z z \quad (2.12)$$

Therefore,

$$\Phi_1(z,t) = \cos(\omega_z' t + \delta)\{A_3 \sin\beta_z z + B_3 \cos\beta_z z\} \quad (2.13)$$

B. Boundary Conditions (I-X) and Natural Frequencies of Vibration

The following relationships between shear, moment, and beam displacement are used in the boundary conditions

$$V_x = -\frac{EI}{L^3} u^{(3)} \qquad V_y = -\frac{EI}{L^3} v^{(3)}$$

$$M_x = -\frac{EI}{L^2} v^{(2)} \qquad M_y = -\frac{EI}{L^2} u^{(2)} \quad \text{and}$$

$$M_z = \frac{GI_p}{L} \frac{\partial \Phi}{\partial \varepsilon} \tag{2.17}$$

where, V_x = shear force in the x direction

V_y = " " " " y direction

M_x M_y and M_z the moment x,y, and z components, respectively.

I_p is the beam polar moment of inertia. Let M_s be the mass of the Shuttle while M_r is the mass of the reflector. The displacement in the x direction of a point located at z = 0 is given by $u(0,t)-\Delta y_o\Phi(0,t)$ and that in the y direction by $v(0,t) + \Delta x_o\Phi(0,t)$ where Δx_o, Δy_o are the coordinates of the c.m. of the end body (Shuttle).

Now, an attempt will be made to cast the 10 equations describing the boundary conditions into the following matrix form:

[M] {A} = 0 which has a non-trivial solution only when det [M] = 0.

Since there is no offset at the Shuttle end, $\Delta Xo = \Delta Yo = 0$. Therefore B.C. (I) becomes

$$\frac{EI}{\ell^3} r_x^{(3)}\Big/_{\varepsilon=0} = + M_s \omega^2 r_x\Big/_{\varepsilon=0} .$$

Explicitly

$$(EI/\ell^3)\beta_1^3\{-A_1 + C_1\} = M_s\omega^2\{B_1 + D_1\} \qquad \text{(I')}$$

B.C. (II) becomes

$$\frac{EI}{\ell^3} r_y^{(3)}\Big/_{\varepsilon=0} = M_s \omega^2 r_y\Big/_{\varepsilon=0}$$

$$(EI/\ell^3)\beta_2^3\{-A_2 + C_2\} = M_s\omega^2\{B_2 + D_2\} \qquad \text{(II')}$$

<u>Equation (III')</u>

$$\frac{EI}{\ell^3}\beta_1^3\{-A_1\cos\beta_1 + B_1\sin\beta_1 + C_1\cosh\beta_1 + D_1\sinh\beta_1\}$$

$$= -\omega^2 M_r\{-A_1\sin\beta_1 - B_1\cos\beta_1 - C_1\sinh\beta_1 - D_1\cosh\beta_1$$

$$+ \Delta Y_L A_3\sin\beta_3 + \Delta Y_L B_3\cos\beta_3\}$$

Equation IV'

$$\frac{EI}{\ell^3}\beta_2^3\{-A_2\cos\beta_2+B_2\sin\beta_2+C_2\cosh\beta_2+D_2\sinh\beta_2\}$$

$$=-\omega^2 Mr\{-A_2\sin\beta_2-B_2\cos\beta_2-C_2\sinh\beta_2-D_2\cosh\beta_2$$

$$-\Delta X_L A_3\sin\beta_3-\Delta X_L B_3\cos\beta_3\}$$

Equation V'

$$\frac{EI}{\ell^2}\beta_2^2\{-B_2+D_2\}=-\omega^2\{\frac{I_{xxs}}{\ell}\beta_2(A_2+C_2)-\frac{I_{xys}}{\ell}\beta_1(A_1+C_1)\}$$

Equation VI'

$$\frac{EI}{\ell^2}\beta_1^2\{-B_1+D_1\}=-\omega^2\{\frac{I_{xys}}{\ell}\beta_2(A_2+C_2)+\frac{I_{yys}}{\ell}\beta_1(A_1+C_1)\}$$

Equation VII'

$$\frac{EI}{\ell^2}\beta_2^2\{-A_2\sin\beta_2-B_2\cos\beta_2+C_2\sinh\beta_2+D_2\cosh\beta_2\}=$$

$$-\omega^2\{\frac{I_{xxr}}{\ell}\beta_2(A_2\cos\beta_2-B_2\sin\beta_2+C_2\cosh\beta_2+D_2\sinh\beta_2)$$

$$-\frac{I_{xyr}}{\ell}\beta_1(A_1\cos\beta_1-B_1\sin\beta+C_1\cosh\beta_1+D_1\sinh\beta_1)\}$$

Equation VIII'

$$\frac{EI}{\ell^2}\beta_1^2\{-A_1\sin\beta_1-B_1\cos\beta_1+C_1\sinh\beta_1+D_1\cosh\beta_1\}=$$

$$-\omega^2\{\frac{I_{xyr}}{\ell}\beta_2(A_2\cos\beta_2-B_2\sin\beta_2+C_2\cosh\beta_2+D_2\sinh\beta_2)$$

$$-\frac{I_{yyr}}{\ell}\beta_1(A_1\cos\beta_1-B_1\sin\beta_1+C_1\cosh\beta_1+D_1\sinh\beta_1)\}$$

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Equation IX'

$$\frac{GI_P}{\ell}\beta_3 A_3 = -\omega^2 I_{33s} B_3$$

Equation X '

$$\frac{GI_P}{\ell}\beta_3 (A_3 \cos\beta_3 - B_3 \sin\beta_3) = -\omega^2 \{ - I_{33r} (A_3 \sin\beta_3 + B_3 \cos\beta_3)$$

$$+ M_r (- \Delta X_L [A_2 \sin\beta_2 + B_2 \cos\beta_2 + C_2 \sinh\beta_2 + D \cosh\beta_2]$$

$$+ \Delta Y_L [A_1 \sin\beta_1 + B_1 \cos\beta_1 + C_1 \sinh\beta_1 + D_1 \cosh\beta_1])$$

3. GENERIC MODE EQUATIONS

Consider an elemental mass, dm, of the body whose instantaneous position from the center of mass of the Shuttle is $\vec{r}$. The equations of motion of dm can be written as

$$\vec{a}\,dm = L(\vec{q}) + \vec{f}dm + \vec{e}dm \tag{3.1}$$

where $\vec{a}$ is the inertial acceleration of dm; $\vec{f}$, the gravitational force per unit mass; $\vec{e}$, the external force per unit mass; $\vec{q}$, the elastic displacement of dm; and L, a linear operator which when applied to the small elastic displacement, $\vec{q}$, yields the elastic forces acting on dm.

The gravitational force per unit mass $\vec{f}$, can be expressed as

$$\vec{f} = \vec{f}o + M\vec{r} \tag{3.2}$$

where $\vec{f}o$ is the gravitational force per unit mass as the center of mass of the body considered and M_o = matrix operator.

In what follows, the generic mode equations will be derived based on a Newton-Euler formulation. The principal assumptions made in this development are: 1) within each component of the system, the mass and structural properties are uniformly distributed; 2) the material of each component is isotropic; 3) the system is deformed in such a manner that it experiences only small strains (within the linear range); 4) elastic displacements are small as compared with the characteristic linear dimensions of the system; 5) the natural mode shapes of free vibrations of the system are known à priori; 6) the system is nominally earth pointing; 7) the system is considered to be closed: no mass transfer across its boundaries.

The vector equation (3.1) can be written in the frame moving with each body as:

$$[\vec{a}_{cm} + \ddot{\vec{r}} + 2\vec{\omega}\times\dot{\vec{r}} + \dot{\vec{\omega}}\times\vec{r} + \vec{\omega}\times(\vec{\omega}\times\vec{r})]\,dm = L(\vec{q}) + (\vec{f}+\vec{e})\,dm \qquad (3.3)$$

Note that $\dot{\bar{r}}$ and $\ddot{\bar{r}}$ are the velocity and acceleration of dm as seen from the body fixed frame. The symbol $\vec{\omega}$ refers to the inertial angular velocity of the body. The instantaneous position vector, $\vec{r}$, of dm can be written as

$$\vec{r} = \vec{r}_o + \vec{q} \qquad (3.4)$$

where $\vec{r}_o$ is the position vector of dm with respect to G, center of mass of the Shuttle, in the undeformed state; $\vec{q}$ is the elastic displacement of dm. Hence

$$\dot{\vec{r}} = \dot{\vec{q}} \quad \text{and} \quad \ddot{\vec{r}} = \ddot{\vec{q}} \qquad (3.5)$$

For small amplitude elastic displacements, one can write the elastic displacement, $\vec{q}$, as a superposition of the various modal contributions according to

$$\vec{q} = \sum_{n=1}^{\infty} A_n(t)\, \vec{\Phi}^{(n)}(\vec{r}_o) \qquad (3.6)$$

where $A_n(t) = P^{(n)}(t)\,(r_x^2 + r_y^2 + \tilde{\theta}^2)^{1/2}$ = modal amplitude

and

$$\vec{\Phi}^{(n)}(\vec{r}_o) = \frac{r_x\hat{i} + r_y\hat{j} + \tilde{\theta}\hat{k}}{\sqrt{r_x^2 + r_y^2 + \tilde{\theta}^2}} \qquad (3.7)$$

The mode shape $\bar{\phi}^{(n)}(z)$ is associated with the natural frequency, ω_n, and satisfies the following conditions

$$\int_M \vec{\Phi}^{(m)} \cdot \vec{\Phi}^{(n)}\, dm = \delta_{mn} M_n \qquad (3.8)$$

where M_n is the generalized mass in the n^{th} mode.

$$L(\vec{\Phi}^{(n)}) = -\rho \omega_n^2 \vec{\Phi}^{(n)} \tag{3.9}$$

$$\int_M \vec{\Phi}^{(n)} dm = \vec{0} \tag{3.10}$$

and

$$\int_M \vec{r}_0 \times \vec{\Phi}^{(n)} dm = \vec{0} \tag{3.11}$$

This here assumes that the structural frequencies are much greater than the 1.745 hour/orbit $\Rightarrow \omega_0 = 0.001$ rad/s orbital angular velocity. This enables one to use, with a high degree of accuracy, the mode shape functions corresponding to non-rotating structures. The generic mode equation is obtained by taking the modal components of all internal, external and inertial forces acting on the system, i.e.,

$$\int_M \vec{\Phi}^{(n)} \cdot \left[\vec{a}_{cm} + \ddot{\vec{r}} + 2\vec{\omega} \times \dot{\vec{r}} + \dot{\vec{\omega}} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r}) \right] dm$$
$$= \int_M \vec{\Phi}^{(n)} \cdot \left[L(\vec{q})/dm + \vec{f} + \vec{e} \right] dm \tag{3.12}$$

The various terms appearing in equation (3.12) can now be expanded as follows:

$$\int_M \vec{\Phi}^{(n)} \cdot \vec{a}_{cm} = \vec{a}_{cm} \cdot \int_M \vec{\Phi}^{(n)} dm = \vec{0} \tag{3.13}$$

$$\int_M \vec{\Phi}^{(n)} \cdot \ddot{\vec{r}}\, dm = \int_M \vec{\Phi}^{(n)} \cdot \ddot{\vec{q}}\, dm \tag{3.14}$$

$$\int_M \vec{\Phi}^{(n)} \cdot (2\vec{\omega} \times \dot{\vec{r}})\, dm = 2 \int_M \vec{\Phi}^{(n)} \cdot (\vec{\omega} \times \dot{\vec{q}})\, dm \qquad (3.15)$$

$$\int_M \vec{\Phi}^{(n)} \cdot (\dot{\vec{\omega}} \times \vec{r})dm = \int_M \vec{\Phi}^{(n)} \cdot (\dot{\vec{\omega}} \times \vec{r}_o)dm + \int_M \vec{\Phi}^{(n)} \cdot (\dot{\vec{\omega}} \times \vec{q})dm \qquad (3.16)$$

$$\int_M \vec{\Phi}^{(n)} \cdot (\vec{\omega} \times (\vec{\omega} \times \vec{r}))\, dm = \int_M \vec{\Phi}^{(n)} \cdot \vec{\omega} \times (\vec{\omega} \times \vec{r}_o) dm + \int_M \vec{\Phi}^{(n)} \cdot \vec{\omega} \times (\vec{\omega} \times \vec{q}) dm \qquad (3.17)$$

$$\int_M \vec{\Phi}^{(n)} \cdot \frac{L(\vec{q})}{dm}\, dm = -\omega_n^2 A_n M_n \qquad (3.18)$$

$$\int_M \vec{\Phi}^{(n)} \cdot \vec{f} dm = \int_M \vec{\Phi}^{(n)} dm \cdot \vec{f}_o + \int_M \vec{\Phi}^{(n)} \cdot M \ddot{\vec{r}}_o\, dm + \int_M \vec{\Phi}^{(n)} \cdot M \ddot{\vec{q}}\, dm \qquad (3.19)$$

$$\int_M \vec{\Phi}^{(n)} \cdot \vec{e} dm = E_n \qquad (3.20)$$

where E_n is the modal contribution of the external forces in the n^{th} mode.

Gravity Gradient Torque, $\vec{N}$.

Assumed that CG of entire system coincides with CG of Shuttle.

($x_G = 0.036$ ft; $y_G = -0.063$ ft; and $z_G = -0.379$ ft)

$$\boxed{\vec{N} = 3\,\omega_0^2\,\hat{a}_1 \times \mathbb{I}_{syst/G}\,\hat{a}_1}$$

$$\mathbb{I}_{syst/G} = \begin{bmatrix} I_{11} + I_{r1} + M_m l^2/3 & 0 & I_{13} \\ 0 & I_{22} + I_{r2} + M_m l^2/3 & 0 \\ I_{13} & 0 & I_{33} + I_{r3} \end{bmatrix} =$$

$$\mathbb{I}_{syst/G} = \begin{bmatrix} I_1 & 0 & I_4 \\ 0 & I_2 & 0 \\ I_4 & 0 & I_3 \end{bmatrix}$$

$$\hat{a}_1 = \sin\theta\cos\phi\,\hat{\imath} - (\cos\theta\sin\psi + \sin\theta\sin\phi\cos\psi)\hat{\jmath} + (\sin\theta\sin\phi\sin\psi - c\theta$$

$$\boxed{\vec{N} = 3\omega_0^2\left\{[\psi(I_3 - I_2)]\hat{\imath} + [I_4 - \theta(I_1 - I_3)]\hat{\jmath} - I_4\psi\hat{k}\right\}}$$

System with offset.

$$\mathbb{I}_{syst/G} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

$$\vec{N} = 3\omega_o^2 \Big\{ [I_{yz} + \psi(I_{zz} - I_{yy}) - \theta I_{xy}]\hat{\imath} + [I_{xz} - \psi I_{xy} - \theta(I_{xx} - I_{zz})]\hat{\jmath} + (\theta I_{yz} + \psi I_{xz})\hat{k} \Big\}$$

I. MODELLING OF THE SCOLE CONFIGURATION

- PARAMETRIC STUDY OF THE IN-PLANE SCOLE SYSTEM - FLOQUET STABILITY ANALYSIS

- THREE DIMENSIONAL FORMULATION OF THE SCOLE SYSTEM DYNAMICS
  - Rotational Equations of Motion
  - Structural Analysis - Boundary Conditions
  - Generic Modal Equations

✓ • WHAT WE CAN LEARN ABOUT THE OPEN LOOP SYSTEM?
  - Consider SCOLE configuration without offset of the mast attachment to the reflector and without flexibility
  - Consider SCOLE configuration without mast flexibility but with offset in the direction of orbit (strawman)
  - Consider SCOLE configuration with offsets in two directions but neglecting mast flexibility
  - Consider general SCOLE system dynamics

- IMPLICATIONS FOR CONTROL STRATEGIES

SCOLE (No flexibility, No offset)

$$\ddot{\psi}[I_{11} + M_m l^2/3 + M_r l^2 + I_{r1}] + \ddot{\phi} I_{13} - \omega_0 \dot{\phi}[I_{11} - I_{22} + I_{33} + I_{r1} - I_{r2} + I_{r3}] + \omega_0^2 \phi I_{31}$$

$$- \omega_0^2 \psi [I_{33} - I_{22} + I_{r3} - I_{r2} + 3(I_3 - I_2) - M_m l^2/3] = 0$$

$$\ddot{\psi} I_{31} + \ddot{\phi}(I_{33} + I_{r3}) + \omega_0 \dot{\psi}[I_{11} + I_{33} - I_{22} + I_{r1} + M_m \frac{Rl}{2} - M_r Rl + I_{r3} - I_{r2}] - \omega_0^2 \phi [I_{11} - I_{22} + I_{r1} - I_{r2} + M_m Rl/2 - M_r Rl]$$

$$+ \omega_0^2 \psi (I_{13} + 3I_4) = 0$$

$$\ddot{\theta}[I_{22} + I_{r2} + M_r l^2 + M_m l^2/3] + 3\omega_0^2 \theta (I_1 - I_3) - 3\omega_0^2 I_4 = 0$$

The "θ, pitch" equation decouples from the two others.

and since $I_1 - I_3 < 0$ and $I_{22} + I_{r2} + M_r l^2 - M_m l^2/3 > 0$

$\Rightarrow$ Instability in that d° of freedom.

Furthermore, the last equation if set as:

$$d\ddot{\theta} + e\,\theta + f = 0$$

yields

$$\underline{\underline{\theta(t) = A' e^{\sqrt{e/d}\,t} + B' e^{-\sqrt{e/d}\,t} - f/e}}$$

The two other equations can be recast in the following state matrix format:

$$\begin{bmatrix} \dot{\phi} \\ \ddot{\phi} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \eta_3 & -\eta_4 & -\eta_1 & -\eta_2 \\ 0 & 0 & 0 & 1 \\ \eta_7 & \eta_8 & \eta_6 & \eta_5 \end{bmatrix} \begin{bmatrix} \phi \\ \dot{\phi} \\ \psi \\ \dot{\psi} \end{bmatrix}$$

SCOLE (No flexibility but "X" offset)

$$\ddot{\psi}\left[I_{11} + M_m \ell^2/3 + M_r \ell^2 + I_{r1}\right] + \ddot{\phi}\left(I_{13} + M_r X \ell\right) - \omega_0 \dot{\phi}\left[I_{11} - I_{22} + I_{33} + I_{r1} + I_{r3} - I_{r2}\right] - \omega_0^2 \phi\left(I_{13} + M_r X \ell\right) - \omega_0^2 \psi\left[I_{33} - I_{22} - M_m \ell^2/3 + I_{r3} - I_{r2} - M_r \ell^2 + 3\left(I'_{33} - I'_{yy}\right)\right] = 0$$

where $I'_{33} = I_{33} + I_{r3} + M_r X^2$ and

$$I'_{yy} = I_{22} + I_{r2} + M_r\left(X^2 + \ell^2\right) + M_m \ell^2/3$$

$$\ddot{\theta}\left[I_{22} + I_{2r} + M_r\left(X^2 + \ell^2\right) + M_m \ell^2/3\right] + 3\omega_0^2 \theta\left(I'_{xx} - I'_{33}\right) - 3\omega_0^2\left(I_{13} + M_r X \ell\right) = 0$$

where $I'_{xx} = I_{11} + I_{r1} + M_m \ell^2/3 + M_r \ell^2$

again, the pitch equation decouples from the yaw and roll equations.

Since $I'_{xx} - I'_{33} < 0 \Rightarrow$ Instability in that d° of freedom.

$$\ddot{\phi}\left[I_{33}+I_{r3}+M_r X^2\right]+\ddot{\psi}\left[I_{31}+M_r X \ell\right]+\omega_0\dot{\psi}\left[I_{11}-I_{22}\right.$$

$$\left.+I_{33}+I_{r1}-I_{r2}+I_{r3}+M_m R\ell/2-M_r R\ell\right]-2\omega_0\dot{\phi}M_r X\ell$$

$$-\omega_0^2\phi\left[I_{11}-I_{22}+I_{r1}-I_{r2}-M_r(X^2+R\ell)+M_m R\ell/2\right]$$

$$+\omega_0^2\psi\left\{4(I_{13}+M_r X\ell)\right\}=0$$

SCOLE with rigid mast and offset in both the "X" and "Y" directions

$$\ddot{\psi}\left[I_{11} + M_m \ell^2/3 + M_r(\ell^2 + Y^2) + I_{r1}\right] + \ddot{\phi} I_{xz} - \ddot{\theta} M_r X Y$$

$$- \omega_o \dot{\phi}\left[I_{11} - I_{22} + I_{33} + I_{r1} - I_{r2} + I_{r3}\right] - \omega_o \dot{\theta} M_r Y(\ell + R)$$

$$- \omega_o^2 \psi\left[I_{33} - I_{22} + I_{r3} - I_{r2} + M_r(Y^2 - \ell^2) - M_m \ell^2/3 + 3(I_{33} - I_{xx})\right]$$

$$+ \omega_o^2 \phi I_{xz} + 3\omega_o^2 \theta I_{xy} + \omega_o^2\left[M_r Y(\ell + R) - 3 I_{yz}\right] = 0$$

$$\ddot{\theta}\left[I_{22} + M_m \ell^2/3 + I_{r2} + M_r(\ell^2 + X^2)\right] - \ddot{\phi} M_r Y \ell + \ddot{\psi} M_r X Y$$

$$+ \omega_o \dot{\phi} M_r X Y + \omega_o^2 \phi M_r Y(\ell + R) + 3\omega_o^2 \psi I_{xy} + 3\omega_o^2 \theta (I_{xx} - I_{zz})$$

$$- 3\omega_o^2 I_{xz} - \omega_o \dot{\psi} M_m Y R = 0$$

$$\ddot{\phi}\left[I_{33} + I_{r3} + M_r(X^2 + Y^2)\right] + \ddot{\psi} I_{zx} + \omega_o \dot{\psi}\left[I_{11} - I_{22} + I_{33} + I_{r1} + M_m R\ell/2\right.$$

$$\left. M_r R \ell - I_{r2} + I_{r3}\right] - \omega_o^2 M_r X Y + \ddot{\theta} M_r Y \ell + M_r X Y \dot{\theta} \omega_o - 3\omega_o^2 \theta I_{yz}$$

$$\omega_o^2 \phi\left[-M_m R\ell/2 + I_{11} - I_{22} + I_{r1} - I_{r2} + M_r(R\ell - Y^2 + X^2)\right] + \omega_o^2 \psi (4 I_{xz}) = 0$$

IMPLICATIONS FOR LINEAR CONTROL STRATEGIES

After suppression of mast vibrations, linear system eqns. have constant coefficients, control laws can be synthesized based on LQR techniques.

(A) For the special cases where the in-plane rotational dynamics separate from the out-of-plane dynamics, separate control laws can be generated for pitch and the roll-yaw systems.

(B) When reflector offset results in coupling between the in-plane and out-of-plane systems, a bias momentum scheme could be considered so that the controllers serve to decouple the system via removal of the relevant coupling terms. Care should be taken so that saturation will not occur.

(C) Since the vibration frequencies of the mast are much greater than those of the gravity-gradient forced rigid rotational modes, actuators placed at strategic points on the mast could be used for quick removal of the vibrations without inducing substantial disturbances on the rigid modes. Once the mast deformations have been reduced to a specified level, the techniques described in (A) and/or (B) could than be utilized.

II. CONTROL ISSUES:

✓ • CONTROL OF LARGE STRUCTURES WITH DELAYED INPUT IN THE CONTINUOUS TIME DOMAIN

✓ • CONTROL WITH DELAYED INPUT IN THE DISCRETE TIME DOMAIN

✓ • CONTROL LAW DESIGN FOR SCOLE USING LQG/LTR TECHNIQUE

• OPTIMAL TORQUE CONTROL FOR SCOLE SLEWING MANUEVERS

  • Kinematical and Dynamical Equations

  • Optimal Control - Two Point Boundary Value Problem

  • Estimation of Unknown Boundary Conditions

  • Numerical Results

  • Discussion and Further Recommendations

IV.B STABILITY ANALYSIS OF A SECOND ORDER SYSTEM WITH DELAYED INPUT

The vibration analysis of large space structures is performed using modal analysis and modal coordinates, transforming n coupled second order differential equations or partial differential equations into n decoupled second order differential equations of the form

$$x_i + \omega_i^2 x_i = f_i \qquad i=1,2,\ldots,n \tag{1}$$

where x_i=i th modal coordinate

ω_i=i th natural frequency

f_i= influence of the actuators on the i th mode, and

the control law of the form

$$f_i = 2\zeta_i \omega_i x_i \tag{2}$$

controls and stabilizes the system (1). The effect of delay in the control force was investigated with numerical simulation for the following numerical example.[1]

$$\ddot{x}_i + 6\dot{x}_i(t-h) + 36x_i=0 \tag{3}$$

It was observed that for delay, $h > 0.15$, instability results.

The analytical verification of the above observation is obtained as follows[2]:

The roots of the characteristic equation

$$G(s,h) = \sum_{i=0}^{n} P_i(s)\, e^{-shi} = 0 \tag{4}$$

can be evaluated from the auxilary equation

$$\sum_{i=0}^{n} P_i(s)(1-Ts)^{2i}\,(1+Ts)^{2n-2i} = 0 \tag{5}$$

where $$e^{-j\omega h} = \left[\frac{1-j\omega T}{1+j\omega T}\right]^2 \tag{6}$$

Applying the above result to equation (3), the corresponding characteristic equation is given by:

$$G(s,h) = \sum_{i=0}^{1} P_i(s)\, e^{-shi} \qquad (7)$$

where $P_0(s) = s^2 + 36$

$$P_1(s) = 6S \qquad (8)$$

The auxilary equation is written as

$$T^2S^4 + (2T + 6T^2)\, S^3 + (1 + 36T^2 - 12T)S^2 + (72T + 6)\, S + 36 = 0 \qquad (9)$$

Using the Routh-Hurwitz criterion, Equation (9) has imaginary roots for $T=0.0426$ at $\omega=9.7$. Using relation (6), h can be evaluated as:

$$\omega h = \pi/2 \qquad (10)$$

or $h \cong 0.16$ (11)

It is also brought to our attention[3] that the above result can be arrived at without the approximation (6) for a second order system as follows:

The characteristic equation for system (1) with the control law of the form

$$f_i = -2\zeta_i\omega_i x_i(t-h) \qquad (12)$$

is written as

$$S^2 + 2\zeta_i\omega_i e^{-hS}\, S + \omega_i^2 = 0 \qquad (13)$$

To evaluate the minimum h for which equation (13) has unstable roots replace S by $j\omega$ as:

$$-\omega^2 + j2\zeta_i\omega_i e^{-j\omega h}\,\omega + \omega_i^2 = 0 \qquad (14)$$

Using $e^{-j\omega h} = \cos\omega h - j\sin\omega h$, (15)

Equation (14) can be written as:

$$(-\omega^2 + 2\zeta_i\omega_i\omega\sin\omega h + \omega_i^2) + j\,(2\zeta_i\omega_i\cos\omega h) = 0 \qquad (16)$$

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Thus for equation (16) to be valid

$$\cos\omega h = 0$$

$$\text{or} \quad \omega h = \frac{\pi}{2}(2P + 1) \qquad P = 0,1,2,\ldots \tag{17}$$

and

$$\omega^2 - 2\zeta_i\omega_i\omega\sin\omega h - \omega_i^2 = 0 \tag{18}$$

the roots of Equation (18) are

$$\omega = \omega_i \{\zeta_i \sin\omega h \pm \sqrt{1 + \zeta_i^2}\} \tag{19}$$

Taking the positive ω and substituting into (17)

$$h = \frac{\pi(1+2P)}{2\omega_i\{\zeta_i \sin\omega h + \sqrt{1+\zeta_i^2}\}} \tag{20}$$

Thus giving

$$h_{min} = 0.1618 \tag{21}$$

for the numerical example (3).

Thus the example second order system considered with the natural period of oscillation of 1 second can not tolerate more than 0.16 seconds of delay without becoming unstable. Thus the general problem of delay in control input must be carefully considered in the control system implementation of large space structures.

the beginning. However, the delay in input in the discrete time domain can be relatively easily solved as shown below.[10]

The dynamic system described as:

$$X(i+1) = \sum_{j=0}^{m} A_j X(i-j) + \sum_{j=1}^{\ell} B_j U(i-j) \tag{53}$$

can be written as

$$\underbrace{\begin{bmatrix} x(i+1) \\ x(i) \\ x(i-m+1) \\ u(i-1) \\ u(i) \\ u(i-2) \\ \vdots \\ u(i-\ell+1) \end{bmatrix}}_{Z(i+1)} = \underbrace{\begin{bmatrix} A_0 & A_1 \ldots A_m & B_1 & B_2 \ldots B_1 \\ I & 0\ldots0 \; 0 & 0 & 0\ldots0 \\ 0 & 0\ldots I \; 0 & 0 & 0\ldots0 \\ 0 & 0\ldots0 \; 0 & 0 & 0..00 \\ 0 & 0\ldots0 \; 0 & I & 0..00 \\ 0 & 0 & 0 & I..00 \\ & & 0 & 0 \;\; 00 \\ 0 & 0\ldots0 \; 0 & 0 & 0..I0 \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} X(i) \\ x(i-1) \\ x(i-m) \\ U(i-1) \\ U(i-2) \\ \\ \\ U(i-\ell) \end{bmatrix}}_{Z(i)} + \underbrace{\begin{bmatrix} B_0 \\ 0 \\ 0 \\ I \\ 0 \\ \\ \\ 0 \end{bmatrix}}_{\tilde{B}} U(i) \tag{54}$$

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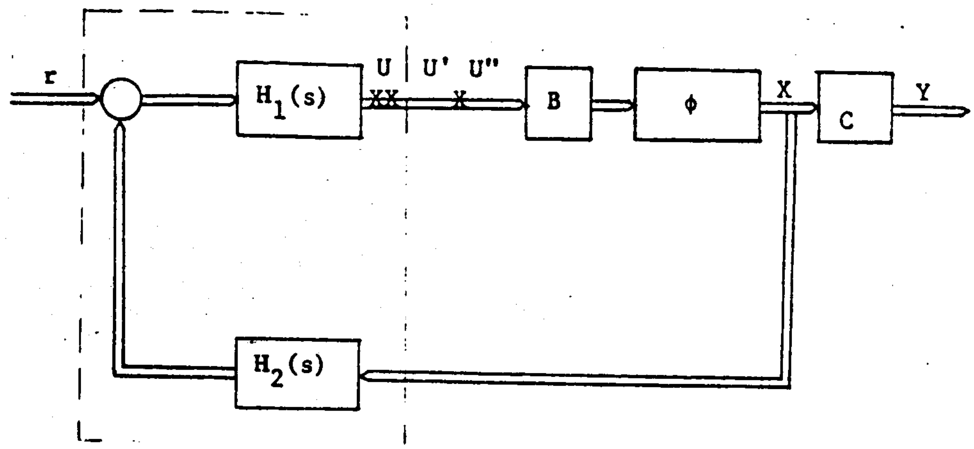
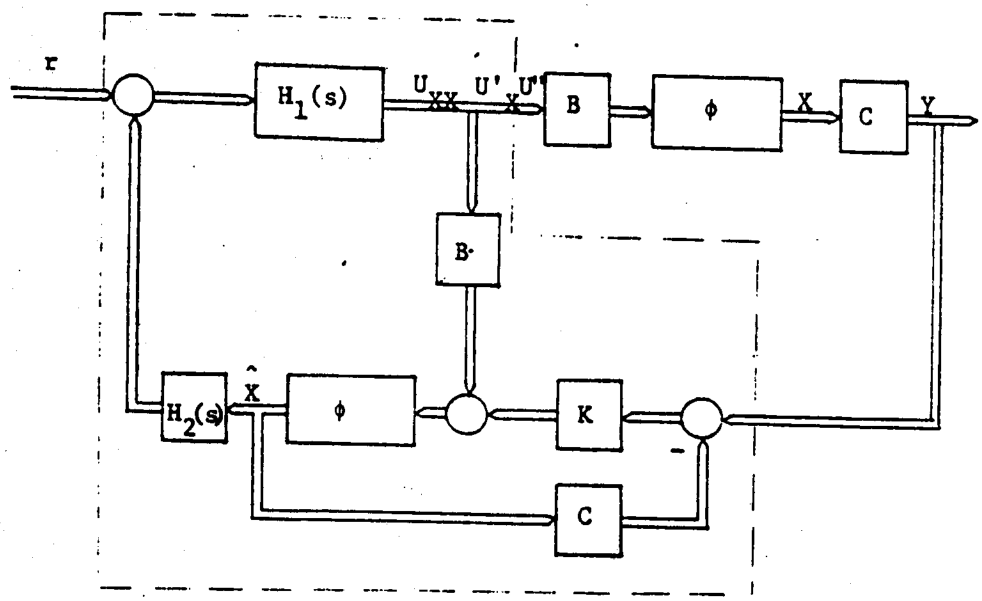
which can be written as:

$$Z(i+1) = \tilde{A}\, Z(i) + \tilde{B}\, U(i) \tag{55}$$

Thus the augumented dynamic system (52) can be solved as a standard control problem. The only disadvantage is the increase in dimensionality of an already large dimensional problem.



a) Full State Feedback



b) Observer Based Implementation

$$\phi = (sI-A)^{-1}$$

IMPORTANT PROPERTIES OF THE TWO TYPES OF IMPLEMENTATIONS:

1. THE CLOSED LOOP TRANSFER FUNCTION MATRICES FROM COMMAND r TO STATE X ARE IDENTICAL IN BOTH IMPLEMENTATION

2. THE LOOP TRANSFER FUNCTION MATRICES FROM CONTROL SIGNAL U′ TO CONTROL SIGNED U (LOOP BROKEN AT XX) ARE IDENTICAL IN BOTH IMPLEMENTATIONS

3. THE LOOP TRANSFER FUNCTION FROM CONTROL SIGNAL U″ TO CONTROL U (LOOPS BROKEN AT POINT X) ARE GENERALLY DIFFERENT. THEY ARE IDENTICAL IF THE OBSERVER DYNAMICS SATISFY:

   $K\,[I + C\,(SI-A)]^{-1} = B[C(SI-A)^{-1}B]^{-1}$ FOR ALL S.

For Full State Feedback

$$X = \phi B U''$$

For observer Based Implementation

$$(I + \phi KC)\hat{X} = \phi BU' + \phi KC\phi BU''$$

$$\hat{X} = (\phi^{-1} + KC)^{-1} (BU' + KC\phi BU'')$$

$$\hat{X} = (I + \underline{\phi K}C)^{-1}\phi (BU' + KC\phi BU'')$$

$$= (I - \phi K(I + C\phi K)^{-1}C) \phi (BU' + KC\phi BU'')$$

$$= \phi[B(C\phi B)^{-1} - K (I + C\phi K)^{-1}] C\phi BU'$$

$$+ \phi[K - K (I + C\phi K)^{-1} C\phi K] C\phi BU''$$

$$= \phi[B(C\phi B)^{-1} - K (I + C\phi K)^{-1}] C\phi BU'$$

$$+ \phi K [I - (I + C\phi K)^{-1} C\phi K] C\phi BU''$$

$$= \phi[B(C\phi B)^{-1} - K (I + C\phi K)^{-1}] C\phi BU'$$

$$+ \phi [K (I + C\phi K)^{-1}] C\phi BU''$$

use $(I + AB)^{-1} = [I - A(I+BA)^{-1}B]$

An observer Adjustment Procedure:

$$k(q) = \Sigma\ (q)\ C^T\ R^{-1}$$

$$A\Sigma + \Sigma\ A^T + Q(q) - \Sigma\ C^T R^{-1} C\ \Sigma = 0$$

Q and R are treated as design Parameters

[For Kalman Filters, these are noise intensity matrices]

$$Q\ (q) = Q_0 + q^2\ BVB^T$$

$$R = R_0$$

For $q=0$ $K(q)$ is the nominal Kalman gain

For $q \to \infty$

$$\frac{KRK^T}{q^2} \to BVB^T$$

or

$$\frac{K}{q} \to BV^{\frac{1}{2}}\ (R^{\frac{1}{2}})^{-1}$$

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