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Optimal Torque Control for SCOLE Slewing Maneuvers

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OPTIMAL TORQUE CONTROL FOR SCOLE SLEWING MANUEVERS

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Optimal Torque Control
for SCOLE Slewing Maneuvers

PURPOSE:

TO SLEW THE SCOLE FROM ONE ATTITUDE TO THE REQUIRED ATTITUDE, AND MINIMIZE AN INTEGRAL PERFORMANCE INDEX WHICH INVOLVES THE CONTROL TORQUES.

CONTENTS:

1. KINEMATICAL AND DYNAMICAL EQUATIONS
2. OPTIMAL CONTROL ___ TWO-POINT BOUNDARY-VALUE PROBLEM (TPBVP)
3. ESTIMATION OF UNKNOWN BOUNDARY CONDITIONS
4. NUMERICAL RESULTS
5. DISCUSSION AND FURTHER RECOMMENDATIONS

71

1. Kinematical and Dynamical Equations

(Rigid SCOLE Configuration)

$$\dot{q} = (1/2)\, \underset{\sim}{\tilde{W}}\, q \qquad (1)$$

$$I\, \dot{w} = -\tilde{w}\, I\, w + u \qquad (2)$$

where q ___ Euler Parameter Vector $q = [q_0 \; q_1 \; q_2 \; q_3]^T$

w ___ Angular Velocity Vector $w = [w_1 \; w_2 \; w_3]^T$

u ___ Control Torque Vector $u = [u_1 \; u_2 \; u_3]^T$

$$\underset{\sim}{\tilde{W}} = \begin{bmatrix} 0 & -w_1 & -w_2 & -w_3 \\ w_1 & 0 & w_3 & -w_2 \\ w_2 & -w_3 & 0 & w_1 \\ w_3 & w_2 & -w_1 & 0 \end{bmatrix} \qquad \tilde{w} = \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix}$$

$$I = \begin{bmatrix} I_{11} & -I_{12} & -I_{13} \\ -I_{12} & I_{22} & -I_{23} \\ -I_{13} & -I_{23} & -I_{33} \end{bmatrix}$$

where (Ref.1)

$I_{11} = 1132503$, $I_{22} = 7037447$, $I_{33} = 7113962$,

$I_{12} = -7555$, $I_{13} = 115202$, $I_{23} = 52293$ (Slug-ft^2)

or

$I_{11} = 1535474$, $I_{22} = 9500821$, $I_{33} = 9645235$,

$I_{12} = -10243$, $I_{13} = 156193$, $I_{23} = 70933$ (Kg-m^2)

Transfer I to a diagonal form by an orthogonal matrix $C^{-1} = C^T$,

$$C = \begin{bmatrix} 0.9993143 & -0.0311151 & 0.0192393 \\ -0.0361684 & 0.9273053 & 0.3742533 \\ -0.0132577 & -0.3743042 & 0.9271262 \end{bmatrix}$$

$$C^T I C = \begin{bmatrix} I_1 & & \\ & I_2 & \\ & & I_3 \end{bmatrix} = I_m$$

where subindex, m, represents the principal axes system.

I = 1130233 , I = 6936292 , I = 7137342 (Slug-ft^2)

From (2), the dynamical equation becomes

$$C^T I CC^T w = -C^T \tilde{w} CC^T I CC^T w + C^T u$$

or

$$I_m \dot{w}_m = - \tilde{w}_m I_m w_m + u_m \tag{3}$$

where

$$u = C u_m, \qquad w = C w_m$$

Similarily, we have

$$\dot{q}_m = (1/2) \tilde{\underline{w}}_m q_m \tag{4}$$

Eq.(3) can be written as

$$\dot{w}_m = - I_m^{-1} \tilde{w}_m I_m w_m + I_m u_m \tag{5}$$

For simplicity, we drop subindex m in the following derivation.

2. Optimal Control ___ Two-Point Boundary-Value Problem (TPBVP)

Cost Function

$$J = (1/2)\int_{t_0}^{t_f} u^T u \; dt = (1/2)\int_{t_0}^{t_f} u^T u \; dt$$

The Hamiltonian, H, for the system (4),(5) is

$$H = (1/2)\; u^T u + p^T \dot{q} + r^T \dot{w}$$

By means of Pontryagin's Principle, the necessary conditions for minimizining J, are

$$\dot{p} = -\{\partial H/\partial q\} \quad ===> \quad \dot{p} = (1/2)\; \underline{\tilde{W}}\, p \qquad (6)$$

$$\dot{r} = -\{\partial H/\partial w\} \quad ===> \quad \dot{r} = [Jw]\, r + (1/2)[q]\, p \qquad (7)$$

$$0 = \{\partial H/\partial u\} \quad ===> \quad u = -I^{-1} r \qquad (8)$$

plus (4) and (5), where $p=[p_0 \; p_1 \; p_2 \; p_3]^T$, $r = [r_1 \; r_2 \; r_3]^T$ are the costates corresponding to q and w, respectively.

$$[Jw] = \begin{bmatrix} 0 & J_2 w_3 & J_3 w_2 \\ J_1 w_3 & 0 & J_3 w_1 \\ J_1 w_2 & J_2 w_1 & 0 \end{bmatrix} \qquad \begin{aligned} J_1 &= (I_3 - I_2)/I_1 \\ J_2 &= (I_1 - I_3)/I_2 \\ J_3 &= (I_2 - I_1)/I_3 \end{aligned}$$

$$[q] = \begin{bmatrix} q_1 & -q_0 & -q_3 & q_2 \\ q_2 & q_3 & -q_0 & q_1 \\ q_3 & -q_2 & q_1 & -q_0 \end{bmatrix}$$

After substitution of u from (8) into (5), we get

$$\dot{w} = -J_{ww} - I^{-2} r \qquad (9)$$

where

$$J_{ww} = [\; J_1 w_2 w_3 \quad J_2 w_3 w_1 \quad J_3 w_1 w_2 \;]^T$$

74

Let $z = [q_0 \; q_1 \; q_2 \; q_3 \; w_1 \; w_2 \; w_3 \; p_0 \; p_1 \; p_2 \; p_3 \; r_1 \; r_2 \; r_3]^T = [z_1 \; z_2]^T$

$z_1 = [q_0 \; q_1 \; q_2 \; q_3 \; w_1 \; w_2 \; w_3]^T$, $z_2 = [p_0 \; p_1 \; p_2 \; p_3 \; r_1 \; r_2 \; r_3]^T$

Eqs.(4),(5),(7),(9) can be written as

$$\dot{z} = F(z) \tag{10}$$

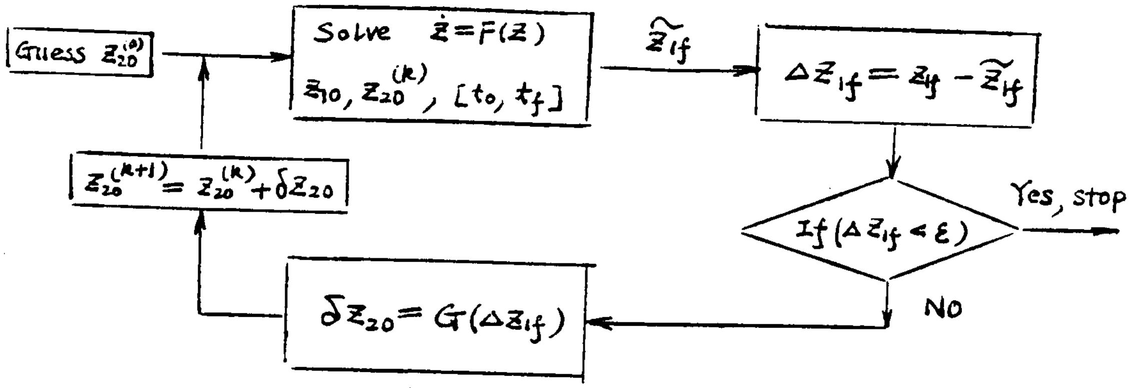
The boundary conditions

$$\begin{aligned} &z_1(t_0) \;,\; z_1(t_f) \quad \text{are known,} \\ &z_2(t_0) \;,\; z_2(t_f) \quad \text{are unknown.} \end{aligned} \tag{11}$$

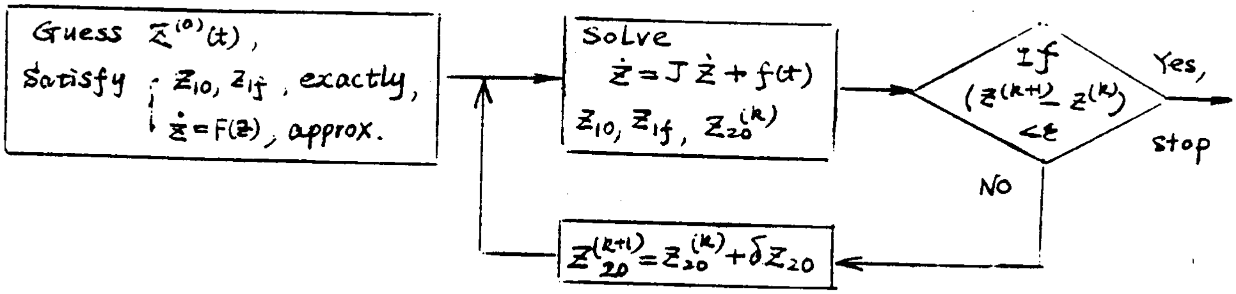
This is the TPBVP. If we find the unknown boundary values, $z_2(t_0)$, then we can integrate (10) to get r, and from (3) we obtain the control torque vector, u.

Brief Review of Methods

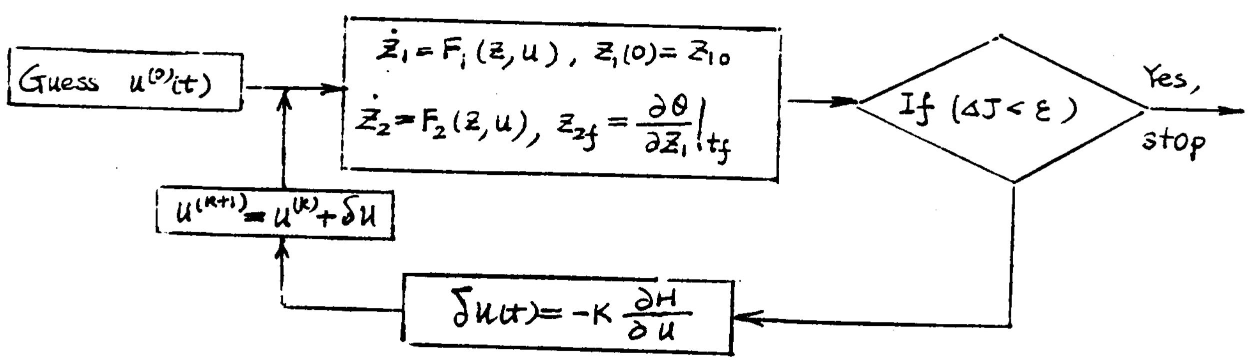
(1) Shooting Methods(Ref.3)



(2) Quasilinerization Methods(Ref.3,4)



(3) Gradient Methods(Ref.4)



(4) Other Methods(Ref.2)

Minimize $B^2 = p^T p$

subject to the terminal contraints $z_1(t_f) = z_{1f}$

3. Estimation of Unknown Boundary Conditions

3.1 Special Case of Slewing Motion

The SCOLE rotates about an arbitrary axis $\vec{\varepsilon}$ fixed in both body axes system and inertial space coordinate system, i.e., the Euler rotation. From the physical point of view, the rotation is very simple, its rotation angle is small, and therefore may consumes less energy (torque). In view of our cost function, it is reasonable to think that the optimal slewing is near the Euler rotation. Considering the analytical solution about single principal axis maneuver in Ref.2, we define a rotation angle $\theta(t)$, about an arbitrary axis $\vec{\varepsilon}$,

$$\theta(t) = \theta_0 + \dot{\theta}_0 t + (1/2)\ddot{\theta}_0 t^2 + (1/6)\dddot{\theta}_0 t^3 \tag{12}$$

For the given boundary conditions

$$\theta(0) = 0, \quad \dot{\theta}(0) = \dot{\theta}_0\,(=0), \quad \theta(t_f) = \theta_f\,,(=20^\circ), \quad \dot{\theta}(t_f) = 0, \tag{13}$$

we have

$$\begin{aligned} \ddot{\theta}_0 &= (6\,\theta_f/t_f^2) - (4\,\dot{\theta}_0/t_f) \\ \dddot{\theta}_0 &= -(12\,\theta_f/t_f^3) + (6\,\dot{\theta}_0/t_f^2) \end{aligned} \tag{14}$$

After substitution of θ and $\vec{\varepsilon}$ into (10), we can get $z_2^{(0)}(0)$, the initial guess of the costates at initial time $t=t_0$.

3.2 Some Porperties of the Costates, p_i

Since $\qquad q^T q = 1$

we have $\qquad p^T p = \beta^2 =$ constant , but $\beta^2 \neq 1$

β is an unknown which is usually determined by iteration, thus

$[\ q_f \quad w_f\ ]^T$ ===> 6 independent conditions

$[\ p_i \quad r_i\ ]^T$ ===> 7 unknowns to be determined

Fortunately, for the problem discussed in this paper, we can prove that 1 of the 4 unknowns p_i can be arbitrarily selected.

4. Numerical Results

Without loss of generality, we choose

$$q = [\,1\ 0\ 0\ 0\,]^T, \quad q = [\,q_{0f}\ \ q_{1f}\ \ q_{2f}\ \ q_{3f}\,]^T$$

so $\quad \theta_f = 2\arccos(q_{0f})\,, \qquad \varepsilon_j = q_{jf}\,\mathrm{sign}(q_{0f})/\sqrt{1-q_{0f}^2}\,, \quad j=1,2,3$

or $\quad q_{0f} = \cos(\theta_f/2)\,, \qquad q_{jf} = \varepsilon_j\,\sin(\theta_f/2)\,, \qquad j=1,2,3$

where θ_f, ε_j, can be chosen according to the practical problem. For example, $\varepsilon_{M_1}=0.87463125$, $\varepsilon_{M_1}=0.159326134$, $\varepsilon_{M_3}=0.454357417$

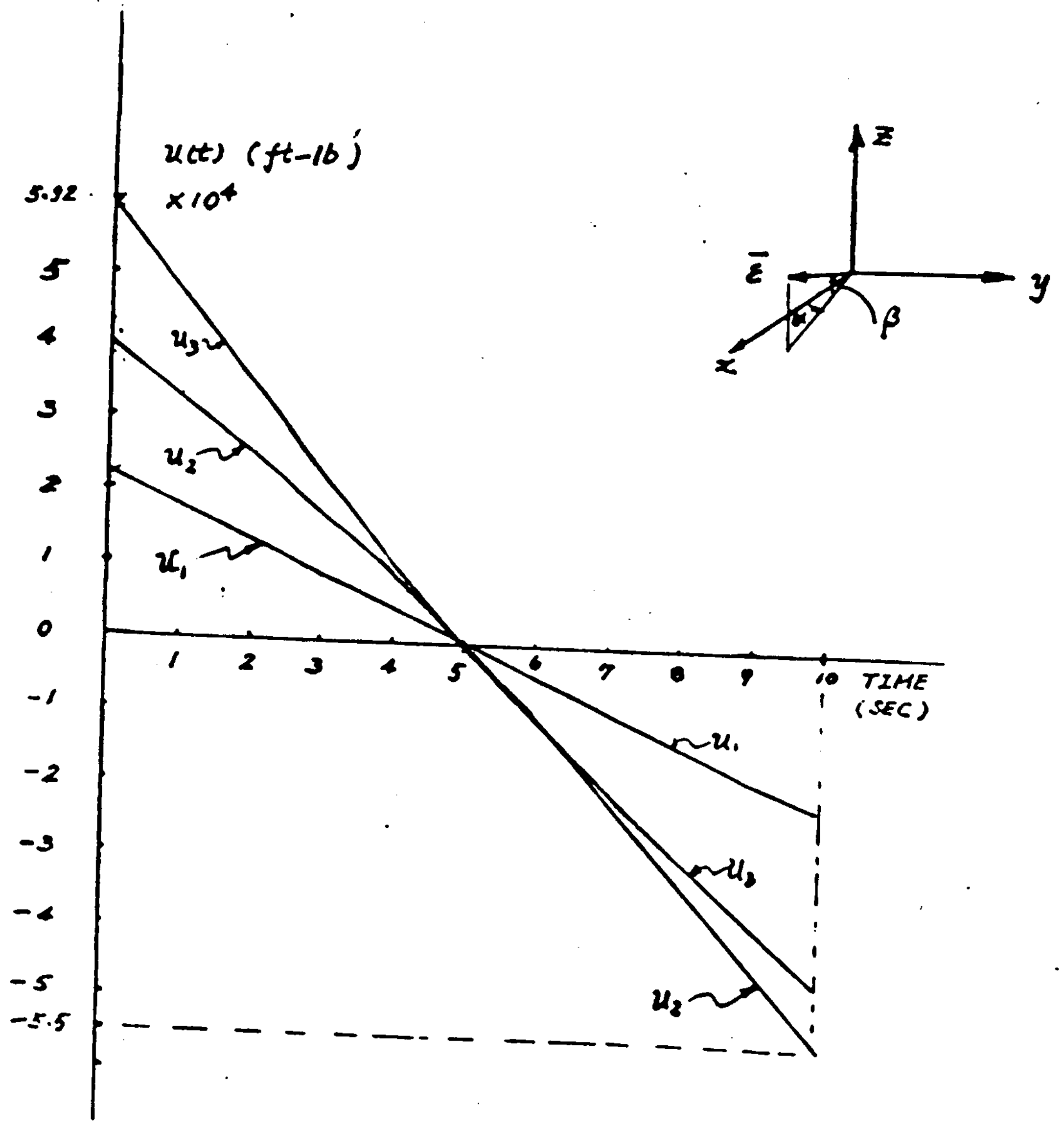


Fig. 2. CONTROL TORQUE

Table I Slewing Data and Boundary Values

$I_1 = 1130283$	$I_2 = 6035292$	$I_3 = 7137342$	(slug-ft^2)
	States		
	Initial	Final	
q_0	1	0.93483775	
q_1	0	0.15187820	
q_2	0	0.02935107	
q_3	0	0.07890037	
w_1	0	0	
w_2	0	0	
w_3	0	0	
	Costates $(p_0 = 0)$ $\times 10^{12}$		
No.of Iter.	p_1	p_2	p_3
0	-0.009360907	-0.069113961	-0.193909345
1	-0.009526038	-0.039331742	-0.201133079
2	-0.009602009	-0.039403392	-0.201193294
3	-0.009602806	-0.039408936	-0.201193267
4	-0.009602806	-0.039408936	-0.201193267
	r_1	r_2	r_3
0	-0.023402267	-0.172734901	-0.484773363
1	-0.023757945	-0.105295499	-0.501347027
2	-0.023705125	-0.105472443	-0.501903771
3	-0.023705005	-0.105472654	-0.501903709
4	-0.023705005	-0.105472654	-0.501903709

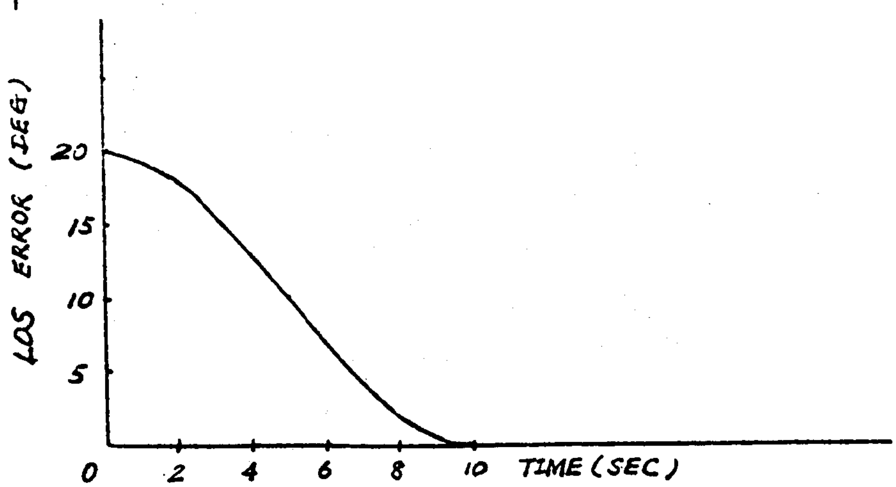


FIG. 1. LINE-OF-SIGHT ERROR

5. Discussion and Further Recommendations

(1) Consider the Distribution of u on the Shuttle and the Reflector.

(2) Time-Optimal Slewing, (Rigid Configuration),

- Cost Function

$$J = \int_{t_0}^{t_f} dt$$

Solve the TPBVP by Shooting Methods

(3) Include the Flexibility in the Problems.

$$z = [q_0\ q_1\ q_2\ q_3\ w_1\ w_2\ w_3\ A_1\ A_2\ \ldots\ A_n\ p_0\ p_1\ p_2\ \ldots\ ]^T$$

$$[\ 1 \times 14 + 2n\ ]$$

n = No. of flexible appendage modes included

80

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