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# Slew Maneuver Dynamics of the Spacecraft Control Laboratory Experiment

by

Y. P. Kakad

Univ. of No. Carolina at Charlotte

# SLEW MANEUVER DYNAMICS OF THE SPACECRAFT CONTROL LABORATORY EXPERIMENT (SCOLE)

Y. P. KAKAD

DEPT. OF ELECTRICAL ENGINEERING
UNIVERSITY OF NORTH CAROLINA AT CHARLOTTE
CHARLOTTE, N.C. 28223

$$C=\begin{bmatrix} \cos\theta_2\cos\theta_3 & -\cos\theta_2\sin\theta_3 & \sin\theta_2 \\ (\sin\theta_1\sin\theta_2\cos\theta_3 + \sin\theta\cos\theta) & (-\sin\theta_1\sin\theta_2\sin\theta_3 + \cos\theta\cos\theta) & -\sin\theta_1\cos\theta_2 \\ (-\cos\theta_1\sin\theta_2\cos\theta_3 + \sin\theta_3\sin\theta_1) & (\cos\theta_1\sin\theta_2\sin\theta_3 + \cos\theta_3\sin\theta_1) & \cos\theta_1\cos\theta_2 \end{bmatrix} \quad (1)$$

where if $\vec{i}, \vec{j}, \vec{k}$ represent the dextral set of orthogonal unit vectors fixed in the body-fixed frame, then $\theta_1$ is the rotation of $\vec{i}$, $\theta_2$ is the rotation of $\vec{j}$ and $\theta_3$ is the rotation of $\vec{k}$.

The angular velocity of the orbiter can be transformed from the inertial frame to the body-fixed frame for the body-three angles as

$$\underline{\omega} = M^T \dot{\underline{\theta}} \quad (2)$$

The total kinetic energy expression of the system can be given as [4]

$$T = T_0 + T_1 + T_2 \quad (3)$$

where $T_0$ is the kinetic energy of the shuttle and is given as

$$T_0 = 1/2\, m_1 \underline{V}^T \underline{V} + 1/2\, \underline{\omega}^T I_1 \underline{\omega} \quad (4)$$

The kinetic energy of the flexible beam is $T_1$ and it

$$T_1 = 1/2 m\underline{V}_0^T \underline{V}_0 + 1/2\, \underline{\omega}^T J\, \underline{\omega} - m\underline{V}_0^T\, \underline{\tilde{c}}\, \underline{\omega} + 1/2 \int \underline{\dot{d}}^T \underline{\dot{d}}\, dm + \underline{V}_0^T \int \underline{\dot{d}}\, dm + \underline{\omega}^T \int \underline{\tilde{a}}\underline{\dot{d}}\, dm + 1/2 \int [\dot{u}_x' \; \dot{u}_y' \; \dot{u}_\psi'] \begin{bmatrix} \dot{u}_x' \\ \dot{u}_y' \\ \dot{u}_\psi' \end{bmatrix} dI \qquad (5)$$

$$T_1 = 1/2m\underline{V}_0^T\,\underline{V} + 1/2\underline{\omega}^T J\underline{\omega} - m\underline{V}_0^T\,\underline{\tilde{c}}\,\underline{\omega} + m\sum_{i=1}^{n}\dot{q}_i^{\,2} + \underline{V}_0^T\,\underline{\dot{\alpha}}$$

$$+\underline{\omega}^T\underline{\dot{\beta}} + 1/4\rho\left[\sum_{i=1}^{n} p_{5i}\dot{q}_i^{\,2} + \sum_{i=1}^{n} p_{6i}\dot{q}_i^{\,2}\right] \qquad (6)$$

where

$$u_x = \sum_{i=1}^{n}\phi_{xi}(s)q_i(t)$$

$$u_y = \sum_{i=1}^{n}\phi_{yi}(s)q_i(t) \qquad (7)$$

$$u_\psi = \sum_{i=1}^{n}\phi_{\psi i}(s)q_i(t)$$

and

$$p_{1i} = \int_0^L \phi_{xi}(s)\;ds$$

$$p_{2i} = \int_0^L \phi_{yi}(s)\;ds$$

$$p_{3i} = \int_0^L s\phi_{xi}(s)\;ds$$

$$p_{4i} = \int_0^L s\phi_{yi}(s)\;ds \qquad (8)$$

$$p_{5i} = \int_0^L (s\phi'_{xi})^2 ds$$

$$p_{6i} = \int_0^L (s\phi'_{yi})^2 ds$$

and

$$\underline{\dot{\alpha}}(t) = \begin{bmatrix} \sum_{i=1}^{n} P_{1i}\ \dot{q}_i \\ \sum_{i=1}^{n} P_{2i}\ \dot{q}_i \\ 0 \end{bmatrix} \tag{9}$$

$$\underline{\dot{\beta}}(t) = \rho \begin{bmatrix} \sum_{i=1}^{n} P_{4i}\ \dot{q}_i \\ \sum_{i=1}^{n} P_{3i}\ \dot{q}_i \\ 0 \end{bmatrix} \tag{10}$$

The kinetic energy $T_2$, of the tip mass (the reflector) is

$$\begin{aligned} T_2 = {} & 1/2\ m_2 \underline{V}_0^T \underline{V}_0 - m_2 \underline{V}_0^T \underline{\tilde{a}}(L)\underline{\omega} + m_2 \underline{V}_0^T \underline{\dot{d}}(L) \\ & - 1/2\ m_2 \underline{\omega}^T \underline{\tilde{a}}(L) \underline{\tilde{a}}(L) \underline{\omega} + m_2\ \underline{\omega}^T \underline{\tilde{a}}(L) \underline{\dot{d}}(L) \\ & + 1/2\ m_2\ \underline{\dot{d}}^T(L) \underline{\dot{d}}(L) + 1/2\ \underline{\Omega}^T I_2 \underline{\Omega} \end{aligned} \tag{11}$$

where

$$\underline{\Omega} = \underline{\omega} + \begin{bmatrix} \dot{u}_x{}'(L) \\ \dot{u}_y{}'(L) \\ \dot{u}_\psi(L) \end{bmatrix} \tag{12}$$

$$T = 1/2\, m_2\, \underline{V}_0^T \underline{V}_0 - m_2 \underline{V}_0^T \underline{\tilde{a}}(L)\, \underline{\omega} + m_2 \underline{V}_0^T \underline{\dot{d}}(L)$$
$$- 1/2 m_2 \underline{\omega}^T \underline{\tilde{a}}(L) \underline{\tilde{a}}(L)\, \underline{\omega} + m_2 \underline{\omega}^T \underline{\tilde{a}}(L) \underline{\dot{d}}(L)$$
$$+ 1/2\, m_2 \left[ \sum_{i=1}^{n} \sum_{j=1}^{n} \phi_{xi}(L)\, \phi_{xj}(L)\, \dot{q}_i \dot{q}_j + \sum_{i=1}^{n} \sum_{j=1}^{n} \phi_{yi}(L)\, \phi_{yj}(L)\, \dot{q}_i \dot{q}_j \right]$$
$$+ 1/2\, \underline{\dot{P}}^T I_2 \underline{\dot{P}} + 1/2\, \underline{\omega}^T I_2 \underline{\omega} \qquad (13)$$

where

$$\underline{\dot{P}} = \left[ \sum_{i=1}^{n} \phi_{xi}'(L)\, \dot{q}_i(t) \quad \sum_{i=1}^{n} \phi_{yi}'(L)\, \dot{q}_i(t) \quad \sum_{i=1}^{n} \phi_{\psi i}(L)\, \dot{q}_i\ (t) \right] \qquad (14)$$

Substituting $T_0$, $T_1$ and $T_2$ from the foregoing equations into equation (3), the total kinetic energy expression can be written as

$$T = 1/2\, m_0\, \underline{V}^T \underline{V} + \underline{\omega}^T H \underline{V} + 1/2\, \underline{\omega}^T I_0\, \underline{\omega} + \underline{V}^T A_1 \underline{\dot{q}}$$
$$+ \underline{\omega}^T A_2 \underline{\dot{q}} + 1/2\, \underline{\dot{q}}\ A_3\, \underline{\dot{q}} \qquad (15)$$

where

$$m_0 = m_1 + \rho L + m_2 \tag{16}$$

$$H = (\rho L + m_0)\underline{\tilde{r}} + m_2\underline{\tilde{a}}(L) + \rho L\underline{\tilde{c}} \tag{17}$$

$$I_0 = I_1 + J + I_2 \tag{18}$$

and also

$$A_1\,\underline{\dot{q}} = \underline{\dot{\alpha}} + m_2\,\underline{\dot{d}}(L) \tag{19}$$

$$A_2\,\underline{\dot{q}} = \underline{\tilde{r}}\,\underline{\dot{\alpha}} + \underline{\dot{\beta}} + m_2\underline{\tilde{r}}\underline{\dot{d}}(L) + m_2\underline{\tilde{a}}(L)\underline{\dot{d}}(L) \tag{20}$$

$$A_3 = \begin{bmatrix} \ddots & & 0 \\ & \rho L + m_2 + p_{5i} + p_{6i} & \\ 0 & & \ddots \end{bmatrix} + \phi^T(L)\,I_2\,\phi(L) \tag{21}$$

The matrix $\phi'^T(L)$ is given as

$$\phi'^T(L) = \begin{bmatrix} \phi'_{1x}(L) & 0 & 0 \\ 0 & \phi'_{1y}(L) & 0 \\ 0 & 0 & \phi'_{1\psi}(L) \\ \cdots\cdots & \cdots\cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots & \cdots\cdots \\ \cdots\cdots & \cdots\cdots & \cdots\cdots \\ \phi'_{ix}(L) & 0 & 0 \\ 0 & \phi'_{iy}(L) & 0 \\ 0 & 0 & \phi_{i\psi}(L) \end{bmatrix} \tag{22}$$

$$m_0\dot{\underline{V}} - H\dot{\underline{\omega}} + A_1\ddot{\underline{q}} = \underline{N}_1 + \underline{F}(t) \tag{26}$$

where the nonlinear term $N_1$ is given as

$$\begin{aligned} \underline{N}_1 &= -C^T\dot{C}\,(m_0\underline{V} - H\underline{\omega} + A_1\dot{\underline{q}}) \\ &= \tilde{\underline{\omega}}\,(m_0\underline{V} - H\underline{\omega} + A_1\dot{\underline{q}}) \end{aligned} \tag{27}$$

Similarly, using equation (2) and the chain rule in the Lagrange's equations, the rotational equations are obtained as

$$H\dot{\underline{V}} + I_0\dot{\underline{\omega}} + A_2\ddot{\underline{q}} = \underline{G}(t) + \underline{N}_2 \tag{28}$$

where $\underline{G}(t)$ is the net moment about the mass center of the orbiter and is given as

$$\underline{G} = \underline{G}_0 + (\underline{r} + \underline{a}) \times \underline{F}_2 \tag{29}$$

and the nonlinear term $\underline{N}_2$ is given in terms of transformations M and C, and $\underline{\omega}$, $\underline{V}$ and $\underline{\theta}$. The vibration equations of the beam can be obtained by again using Lagrange's equations and the potential energy function

$$U = 1/2\ \underline{q}^T K\, \underline{q} \tag{30}$$

where the stiffness matrix K is given as

$$K = \begin{bmatrix} \ddots & & \\ & \dfrac{EI(\beta_i)^4}{L^3} & \\ & & \ddots \end{bmatrix} \tag{31}$$

The vibration equations are

$$A_1^T\dot{\underline{V}} + A_2^T\dot{\underline{\omega}} + A_3\ddot{\underline{q}} = -K\underline{q} \tag{32}$$

$$I_0 \, \dot{\underline{\omega}} + A_2 \ddot{\underline{q}} = \underline{G}(t) + \underline{N}_2(\underline{\omega}) \tag{33}$$

$$A_2^T \dot{\underline{\omega}} + A_3 \ddot{\underline{q}} = -K\underline{q} \tag{34}$$

Equation (33) can be rewritten as

$$\dot{\underline{\omega}} = I_0^{-1} [ \underline{G} + \underline{N}_2(\underline{\omega}) - A_2 \ddot{\underline{q}} ] \tag{35}$$

The first three Euler parameters are defined as

$$\underline{\varepsilon} \triangleq \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \underline{\lambda} \sin \psi/2 \tag{36}$$

$$\varepsilon_4 \triangleq \cos \psi/2 \tag{37}$$

$$\frac{d\underline{\varepsilon}}{dt} \triangleq 1/2 \, ( \varepsilon_4 \underline{\omega} + \underline{\varepsilon} \times \underline{\omega}) \tag{38}$$

$$\frac{d\varepsilon_4}{dt} = -1/2 \, \underline{\omega} \cdot \underline{\varepsilon} \tag{39}$$

$$\underline{\omega} = 2 \left( \varepsilon_4 \frac{d\underline{\varepsilon}}{dt} - \dot{\varepsilon}_4 \underline{\varepsilon} - \underline{\varepsilon} \times \frac{d\underline{\varepsilon}}{dt} \right) \tag{40}$$

$$\dot{\underline{\varepsilon}} = \frac{d\underline{\varepsilon}}{dt} = \underline{h} \, ( \underline{\varepsilon}, \underline{\omega}) \tag{41}$$

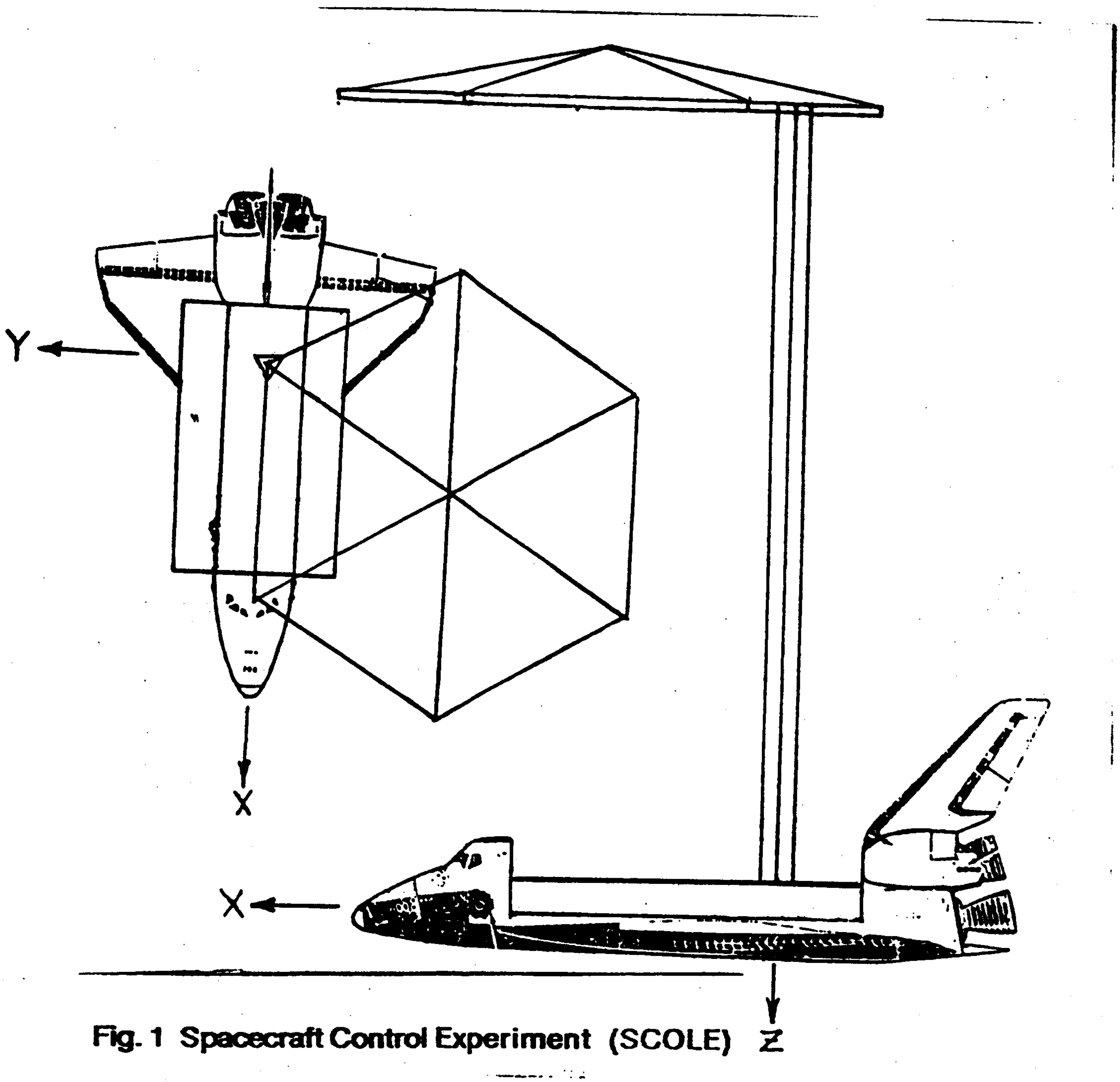


Fig. 1 Spacecraft Control Experiment (SCOLE)

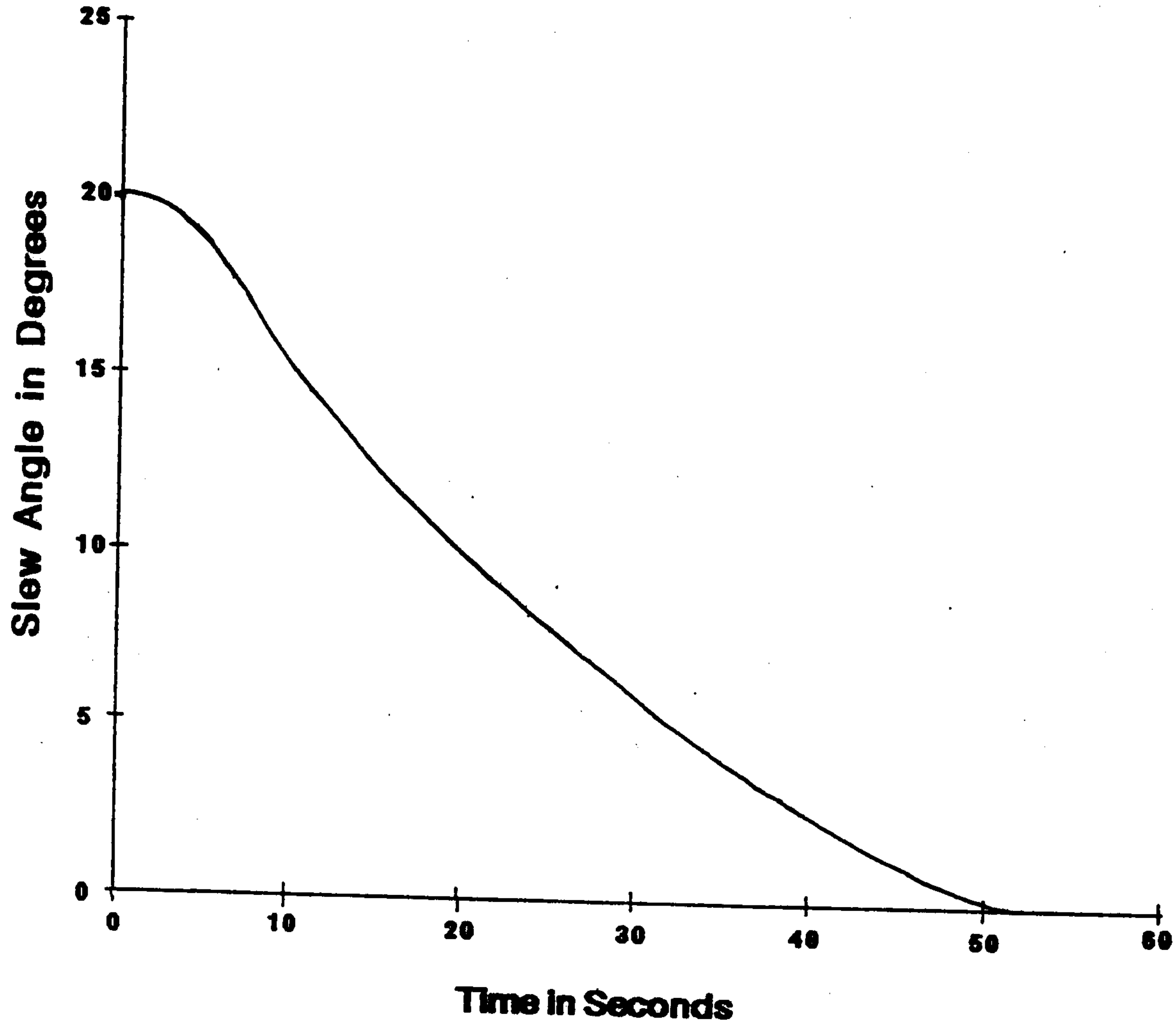


Fig. 6 **Slew Angle vs. Time (Axis of Rotation)**

3i + j + 5k

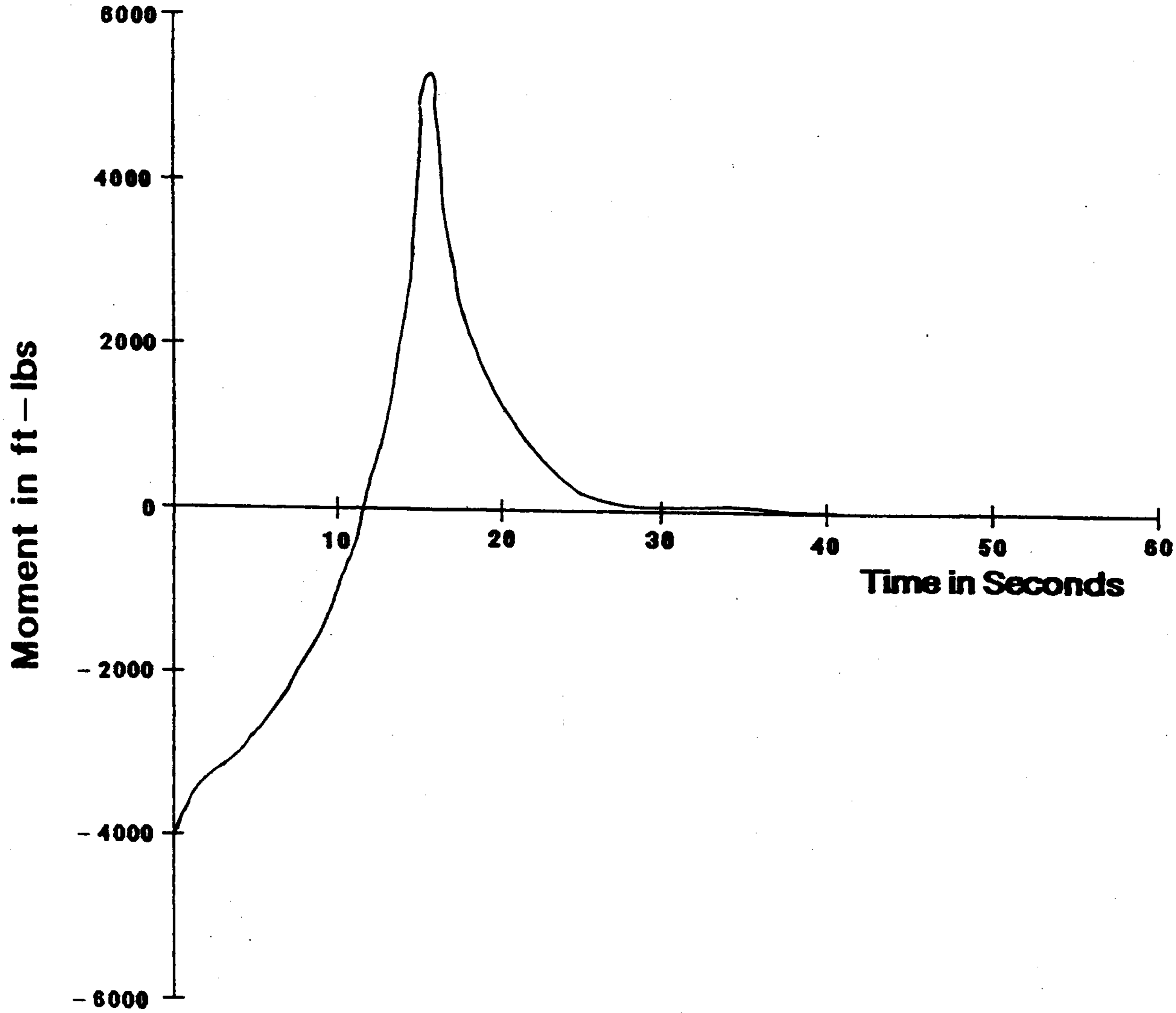


Fig. 7 Moment Component $G_1$ vs. Time
(Axis of Rotation)
$3i + j + 5k$

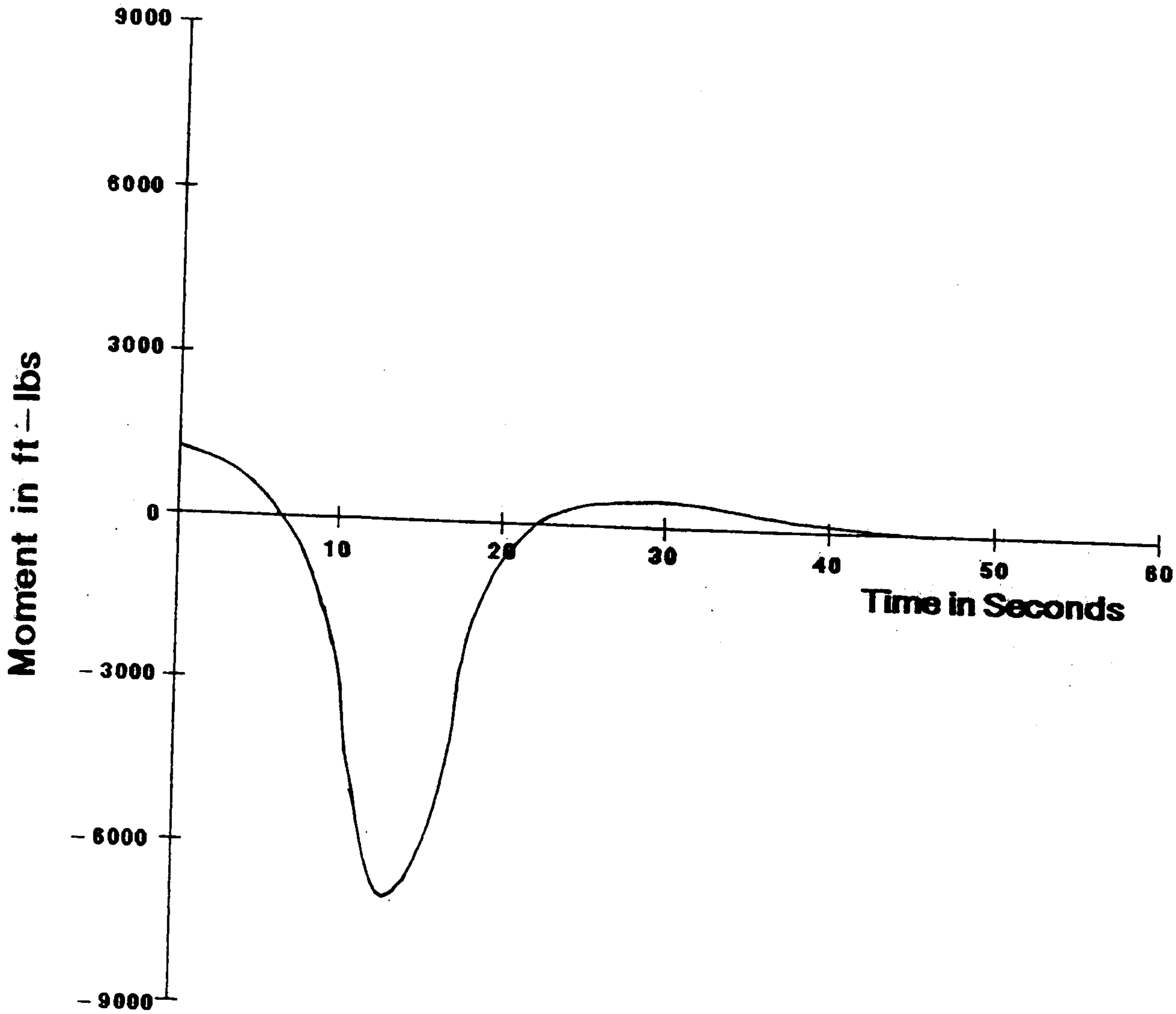


Fig. 8 Moment Component $G_2$ vs. Time
(Axis of Rotation)
$3i + j + 5k$

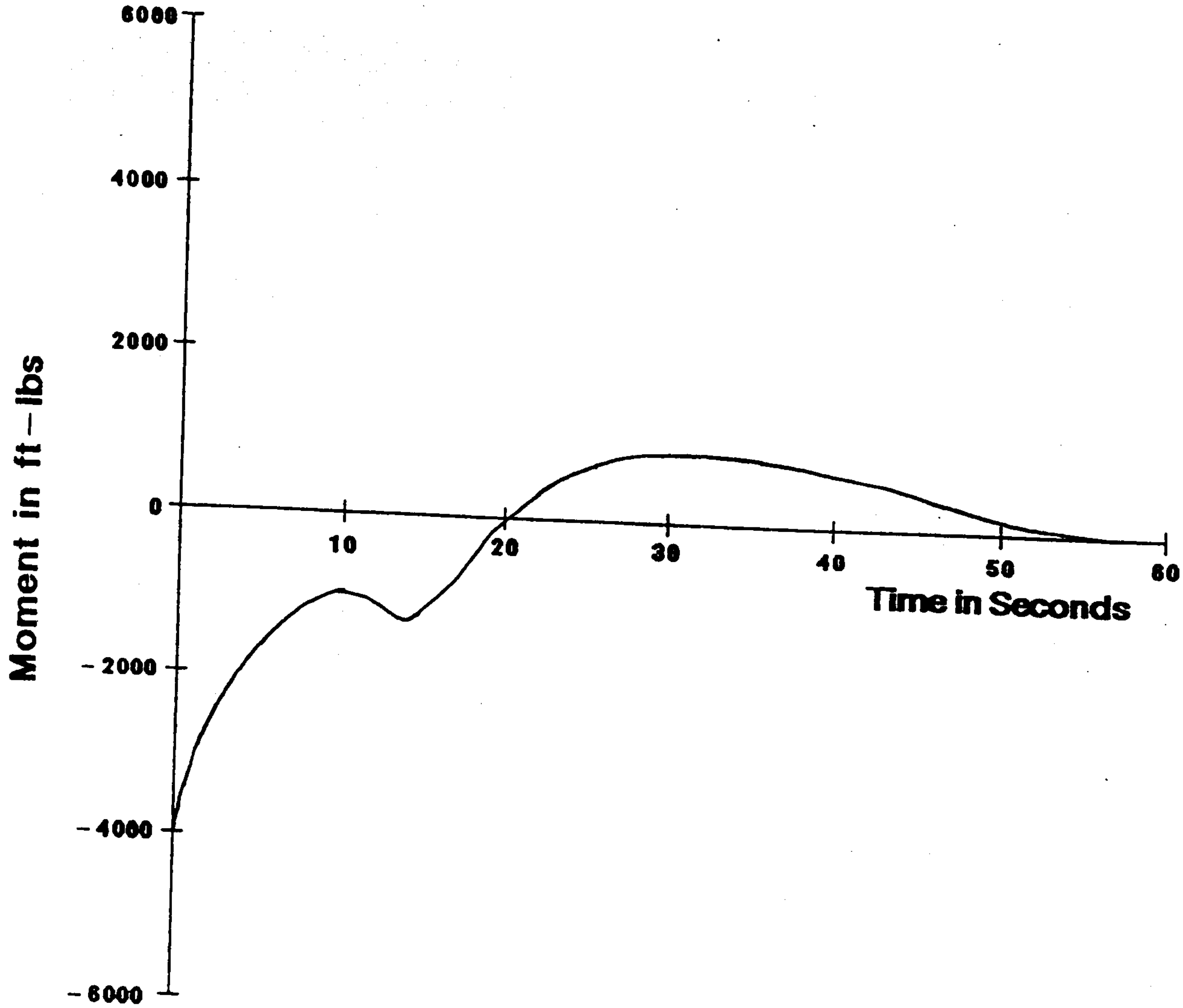


Fig. 9 **Moment Component $G_3$ vs. Time**
**(Axis of Rotation)**
**$3i + j + 5k$**