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Control of SCOLE

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THIRD ANNUAL SCOLE WORKSHOP

CONTROL OF SCOLE

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MODAL CONTROL

The object is to control the SCOLE using a relatively low order model.

Discretized model: $M\ddot{\underset{\sim}{q}}(t) + K\underset{\sim}{q}(t) = \underset{\sim}{F}(t) + \underset{\sim}{v}(t)$

$\underset{\sim}{q}(t)$ = relatively high-dimensional configuration vector

$\underset{\sim}{v}(t)$ = actuator noise vector

Drastic truncation of the model is proposed by means of a modal expansion.

Open-loop eigenvalue problem: $K\underset{\sim}{u}_i = \omega_i^2 M\underset{\sim}{u}_i, \quad i = 1,2,\ldots,n$

Eigenvalue orthonormality: $\underset{\sim}{u}_j^T M\underset{\sim}{u}_i = \delta_{ij}, \quad \underset{\sim}{u}_j^T K\underset{\sim}{u}_i = \omega_i^2\delta_{ij}$

Modal truncation: $\underset{\sim}{q}(t) = \sum_{i=1}^{c} \underset{\sim}{u}_i \eta_i(t) = U_c\underset{\sim}{\eta}(t)$

$U_c = [\underset{\sim}{u}_1\ \underset{\sim}{u}_2\ \cdots\ \underset{\sim}{u}_c]$ = truncated modal matrix

$\underset{\sim}{\eta}(t)$ = c-dimensional modal vector

MODAL CONTROL (CONT'D)

Truncated modal equations: $\ddot{\eta}_i(t) + \omega_i^2 \eta_i(t) = f_i(t) + v_i(t), \quad i = 1,2,\ldots,c$

$f_i(t) = \underset{\sim}{u}_i^T \underset{\sim}{F}(t)$ = modal control

$v_i(t) = \underset{\sim}{u}_i^T \underset{\sim}{v}(t)$ = modal actuator noise

Modal state equations: $\dot{\underset{\sim}{x}}_i(t) = A_i \underset{\sim}{x}(t) + B_i[f_i(t) + v_i(t)], \quad i = 1,2,\ldots,c$

$$A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & 0 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Actual output vector: $\underset{\sim}{y}(t) = C\underset{\sim}{x}(t) + \underset{\sim}{w}(t)$

C = s×2c matrix with c elements of a given row obtained from U_c and the balance equal to zero

$\underset{\sim}{x}(t)$ = overall modal state

$\underset{\sim}{w}(t)$ = measurement (sensor) noise vector

MODAL CONTROL (CONT'D)

Modal Kalman filter: $\dot{\hat{\underset{\sim}{x}}}(t) = A\hat{\underset{\sim}{x}}(t) + B\underset{\sim}{f}(t) + K(t)[\underset{\sim}{y}(t) - C\hat{\underset{\sim}{x}}(t)]$

A = block-diag A_i, B = block-diag B_i

K = estimator gain matrix

To determine the matrix K, it is necessary to solve first a $2c \times 2c$ matrix Riccati equation for given actuator and sensor noise intensities.

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INDEPENDENT MODAL-SPACE CONTROL (IMSC)

Linear (proportional and rate feedback) control:

$$f_i = -h_i \hat{\dot{\eta}}_i - g_i \hat{\eta}_i$$

h_i, g_i = modal gains

Nonlinear control (on-off):

$$f_i = -k_i,\ \hat{\dot{\eta}}_i \geq d_i; \quad 0,\ |\hat{\dot{\eta}}_i| < d_i; \quad k_i,\ \hat{\dot{\eta}}_i \leq d_i$$

$2d_i$ = width of the deadband region

k_i = magnitude of the modal control force

INDEPENDENT MODAL-SPACE CONTROL (IMSC) (CONT'D)

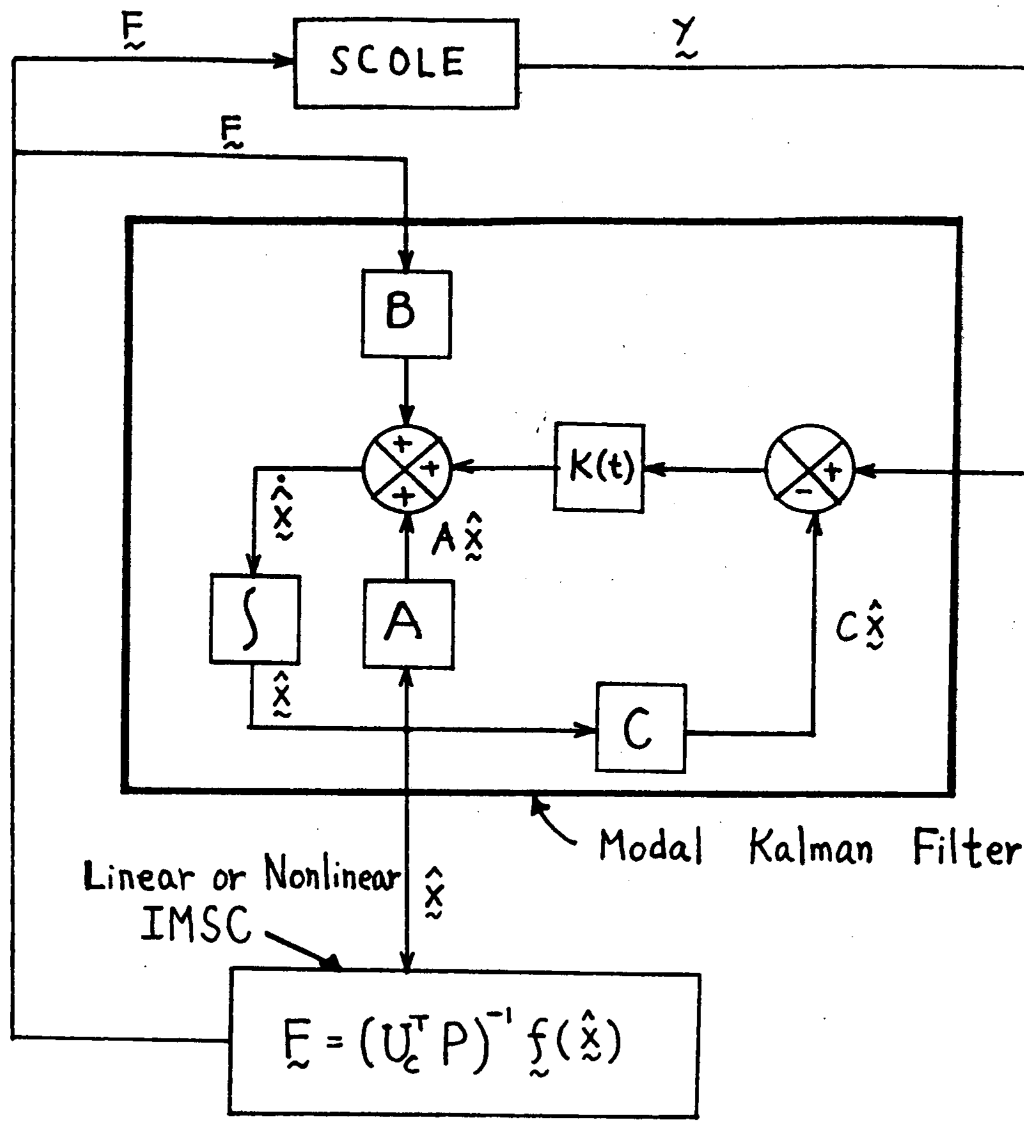
Synthesis of actual controls: let the number of controlled modes coincide with the number of actuators.

Because $\underset{\sim}{F}(t)$ is of smaller dimension than $\underset{\sim}{q}(t)$, let

$$M\ddot{\underset{\sim}{q}}(t) + K\underset{\sim}{q}(t) = P[\underset{\sim}{F}(t) + \underset{\sim}{v}(t)], \quad P = n \times c$$

$$\underset{\sim}{f}(t) = U_C^T P \underset{\sim}{F}(t) \rightarrow \underset{\sim}{F}(t) = (U_C^T P)^{-1} \underset{\sim}{f}(t)$$

$\therefore$ The components of $\underset{\sim}{F}(t)$ are linear combinations of the components of $\underset{\sim}{f}(t)$. When the modal control is nonlinear, the components of $\underset{\sim}{F}(t)$ are quantized and have the appearance of staircase functions


F
SCOLE
Y
F
B
K(t)
A x̂
A
C x̂
x̂
C
Modal Kalman Filter
Linear or Nonlinear
IMSC
x̂
$F = (U_c^T P)^{-1} f(\hat{x})$