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ON A CRITERION FOR VORTEX BREAKDOWN

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Abstract

A criterion for the onset of vortex breakdown is proposed. Based upon previous experimental, computational, and theoretical studies, an appropriately defined local Rossby number is used to delineate the region where breakdown occurs. In addition, new numerical results are presented which further validate this criterion. A number of previous theoretical studies concentrating on inviscid standing-wave analyses for trailing wing-tip vortices are reviewed and reinterpreted in terms of the Rossby number criterion. Consistent with previous studies, the physical basis for the onset of breakdown is identified as the ability of the flow to sustain such waves. Previous computational results are reviewed and re-evaluated in terms of the proposed breakdown criterion. As a result, the cause of breakdown occurring near the inflow computational boundary, common to several numerical studies, is identified. Finally, previous experimental studies of vortex breakdown for both leading edge and trailing wing-tip vortices are reviewed and quantified in terms of the Rossby number criterion.

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1. *Introduction*

Vortices can be generated in many ways. Of specific interest are vortices generated by a finite plate or sharp-edged body at a non-zero angle of attack. These vortices are often highly stable structures characterized by a strong axial flow. Other examples of vortices with a strong axial velocity component include tornadoes and waterspouts, intake vortices, and swirling flow in pipes and tubes.

Leading-edge vortices shed from a delta wing induce a velocity field that results in increased lift and stability of the wing. However, under certain conditions related to the angle of attack of the wing, these vortices can undergo a sudden and drastic change in structure known as vortex breakdown. This breakdown can adversely alter the aerodynamic characteristics of the wing. A similar vortex bursting phenomena has been observed for trailing wing-tip vortices, which is desirable because these vortices represent a hazard to smaller aircraft in areas of dense air traffic. The fundamental difference between these two classes of vortices lies in their circumferential velocity distributions. Far downstream, as was shown by Batchelor,[1] the circumferential velocity profile of the wing-tip vortex behaves like the two-dimensional Burgers' vortex; whereas Hall[2] has shown that the circumferential velocity distribution of the leading-edge vortex can be approximated using the concept of a viscous subcore very near the axis surrounded by an inviscid rotational conical flow region. Thus, the radial gradients of the circumferential velocity near the axis of the leading-edge vortices are much larger than those of the wing-tip vortices.

The ability to control these vortical structures is an important and active area of research. For example, it is desirable to delay the process over a delta wing and accelerate it for trailing-tip vortices. Unfortunately, a comprehensive scheme to describe the breakdown process and the parameters effecting it is presently lacking, although several theories have been proposed.

Vortex breakdown was first observed experimentally by Peckham and Atkinson.[3] They observed that vortices shed from a delta wing at high angles of attack appeared to "bell out" and dissipate several core diameters downstream from the trailing edge of the wing. Since then, vortex breakdown has been observed in swirling flows in straight pipes, nozzles and diffusers, combustion chambers, and tornadoes. Seven types of breakdown have been identified experimentally,[4] ranging from a mild "spiral" type to a strong "bubble" type breakdown. Observations in the early 1960's spurred considerable effort to develop a theoretical explanation for the vortex breakdown phenomena. Three different classes of phenomena have been suggested as the cause or explanation of breakdown. These are: (1) the concept of a critical state,[5-8] (2) analogy to boundary-layer separation,[2,9] (3) hydrodynamic instability.[10-12]

The critical state theory is based upon the possibility that a columnar vortex can support axisymmetric standing waves. The supercritical state has low-swirl velocities and the flow is unable to support these waves. Subcritical flows have high-swirl velocities and are able to support waves. Vortex breakdown can be thought of as the ability of the flow to sustain standing waves.

In Hall's[2] theory, the breakdown phenomena is taken to correspond to a failure of the quasi-cylindrical approximation. The idea being that when streamwise gradients in the vortex become large the quasi-cylindrical approximation must fail, thus signaling breakdown. This is considered to be analogous to the failure of the boundary-layer equations which signals an impending separation.

Stability theory only allows one to investigate the amplification or decay of infinitesmally small disturbances imposed on the base vortex flow. Breakdown is then assumed to be analogous to laminar-turbulent transition. Of course, as pointed out by Leibovich,[13] breakdown can occur with little sign of instability and a vortex flow may become unstable and not undergo breakdown.

The purpose of this paper is to show that the existence of a critical condition for vortex breakdown can be described in terms of a fundamental local flow parameter. A common idea among several previous theoretical studies will be identified and an analogy to the ideas of Taylor[14] concerning the stability of Couette flow will be noted. The identification of the Rossby number as the key parameter will be discussed and its close relationship with the ideas set forth in previous studies by others is examined. Finally, the Rossby number criterion will be applied to previous computational and experimental results and the numerical results of the present investigators.

2. Previous Theoretical Results

Throughout the remainder of this paper we use a cylindrical polar coordinate system, (r, θ, z), and corresponding velocity components, U in the radial (r) direction, V in the circumferential (θ) direction, and W in the axial (z) direction. In discussing previous work, we adopt the (r, θ, z) convention.

Squire[8] appears to be the first to have performed a theoretical analysis of vortex breakdown. He suggested that if standing waves were able to exist on a vortex core then small disturbances, present downstream, could propagate upstream and cause breakdown. This is analogous to the earlier work of Taylor[14] on the stability of circular Couette flow. There, a linear stability analysis was performed to ascertain the ability of the base flow to support axisymmetric standing-wave disturbances. In all of the cases studied, Squire assumed that the vortex flow was inviscid and axisymmetric. He then sought to determine conditions under which an inviscid, axisymmetric, steady perturbation to the flow could exist. This condition, which was necessary for the existence of a standing wave, was taken to mark the transition between subcritical and supercritical states. Two of the cases studied by Squire are relevant to the present study.

In the first case W was taken to be a constant. V was taken to be that of a solid body rotation inside a core of unit radius and that of a potential vortex outside. That is

$$V = V_o\, r \qquad 0 \leqslant r \leqslant 1$$

$$V = V_o/r \qquad r \geqslant 1 \qquad (1)$$

with V_o a constant. He found that for standing waves to exist a swirl parameter, "k" the ratio of the maximum swirl speed to the axial speed, had to satisfy a criterion

$$k = V_{max}/W \geqslant 1.20 \qquad (2)$$

When k = 1.20 the wave is infinitely long but has a finite wavelength for k > 1.20.

In the second case W was also taken to be a constant, but

$$V = \frac{V_o}{r}\,(1 - e^{-r^2}) \qquad (3)$$

with V_o a nondimensional parameter. Again, Squire found that there was a condition on the swirl parameter "k" for the existence of a standing wave. The condition was

$$k = V_{max}/W \geqslant 1.00, \qquad (4)$$

where we note that

$$V_{max} = 0.638\ V_o \qquad (5)$$

Benjamin[5] examined this phenomena from a different point of view. He considered vortex breakdown to be a finite transition between two dynamically conjugate states of flow. There is subcritical flow, which is defined as the state that is able to support standing waves, and a conjugate supercritical flow which is unable to support standing waves. In this context the work of Squire gives a condition marking the interface between these two states. As in the work of Squire, a universal characteristic parameter was defined which delineates the critical regions of the flow. This parameter, denoted by N, is the ratio of the absolute phase velocities of long wavelength waves which propogate in the axial direction, i.e.,

$$N = \frac{C_+ + C_-}{C_+ - C_-} \tag{6}$$

Here C_+ and C_- are the phase velocities of the waves which propogate with and against the flow, respectively. For $N > 1$ the flow conditions are supercritical and for $N < 1$, subcritical.

Benjamin applied this theory to a specific vortex flow, defined by W a constant and

$$\begin{aligned} V &= V_o\, r && 0 \leqslant r \leqslant 1 \\ V &= V_o/r && 1 \leqslant r \leqslant R \end{aligned} \tag{7}$$

If $R \to \infty$, this is just the combined vortex studied by Squire. Benjamin found that the critical condition was of the same form as Squire's

$$V_{max}/W = \text{constant} \tag{8}$$

The precise value of the constant depends on the value of R but lies between 1.92 when $R = 1$ and 1.20 when $R = \infty$. Thus Benjamin, although starting from a different perspective, arrived at the same critical condition for a combined vortex as did Squire.

As a variation of the phase velocity criterion of Benjamin, Tsai and

Widnall[15] examined a group velocity criterion which follows more directly from the view that the breakdown occurs due to a wave trapping mechanism.[16] Their investigation was of swirling pipe flows where the radial and axial velocity distributions can both be fit to exponential profiles. They used the least squares fit of Garg and Leibovich[17] to calculate the dispersion relation from linear parallel stability theory. The group velocity associated with the various flow profiles was then calculated. The results showed that upstream of breakdown the group velocity of both the symmetric and asymmetric modes was directed downstream. Even though their criticality condition of zero group velocity proved an accurate guide for the various types of breakdown, they were unable to establish a relationship between vortex breakdown and wave trapping.

Finally to complete this brief review of previous theoretical studies, a recent paper by Ito, Suematsu, and Hayase[18] is considered. There, both stationary and unsteady vortex breakdown were examined. They considered the stability of a columnar vortex to small amplitude disturbances. The disturbances can be axisymmetric as well as asymmetric and steady or unsteady. Their analysis yields a criterion for breakdown from the requirement for the existence of solutions to their disturbance equations. A comparison of these results with those of Benjamin for the same case of a finite-radius pipe containing a rigid-body rotation gives the same criterion for breakdown. The important aspect of the Ito et al[18] work lies in their interpretive criterion. Their non-dimensionalization leads to the Rossby number as the relevant parameter. For example, in the case of swirling pipe flow consisting of a solid body rotation, the relevant scales are the axial velocity W, pipe radius r^* and constant angular velocity of the flow, Ω .

It is advantageous to summarize this section by placing these theoretical analyses into perspective. As has been shown there is quantitative agreement among the results of Squire,[8] Benjamin,[5] and Ito et al[18] for the various test problems that have been examined. These analyses have been constrained by either the scope of the analysis (linear, parallel, inviscid) or the narrow class of flows that have been considered. In the study of Tsai and Widnall,[15] the calculation of group velocity is an added task. This is generally not feasible in engineering applications where a criterion based solely on mean quantities may be necessary. Nevertheless, these analyses indicate that a criterion for vortex breakdown is available. In section 4, this Rossby number criterion is applied to a variety of computational and experimental, confined and unconfined flows and its range of applicability is examined. However, before proceeding with this analysis it is instructive to examine the large number of computational studies which have been performed.

3. Previous Numerical Studies

Numerical simulations of vortex breakdown abound.[19-23] In all these cases the flows were restricted to have axial symmetry and a relatively low range of Reynolds numbers. In general, geometries and boundary conditions were chosen to reflect experimentally observed flows. The purpose of the computational experiments was to obtain information concerning the structure of the breakdown region and the various parameters affecting its development. A possible criticism of these numerical experiments is that when breakdown occurs, it invariably does so within a few core diameters of the inflow plane. Due to the existence of an inflow boundary layer inherent in all numerical calculations of this type the results in this region must be suspect. This has been pointed out by Leibovich,[24] but has continued to plague subsequent numerical results.

Grabowski and Berger[21] solved the steady axisymmetric Navier-Stokes equations for a free vortex approximated by a two parameter family of assumed inflow velocity distributions. These were the polynomial profiles given by

Mager[25] in his integral analysis, embedded in an irrotational flow. The equations of motion were written in terms of stretched coordinates in the radial and axial directions. The conditions $\partial(Ur)/\partial r = 0$, $V = const/R$ and $W = 1$ were chosen at the radial boundary, R. At inflow, a parameter, α, allows for wake-like or jet-like axial velocity profiles. The artifical compressibility technique was used to solve the equations of motion. Solutions were obtained which were characteristic of vortex breakdown for Reynolds numbers based on axial velocity and characteristic core radius, up to 200. These solutions were obtained with inflow conditions that were, in many cases, subcritical. The results indicate that breakdown is enhanced by increasing the swirl, and is relatively Reynolds number independent.

Kopecky and Torrance[20] considered axisymmetric swirling flow through a cylindrical tube. This distribution of swirl velocity at inflow behaves as a solid body near the axis and a potential vortex away from the axis, representing a solution to the Navier-Stokes equations for the limiting case of Reynolds number approaching infinity. A parametric study was performed with Reynolds numbers, based on axial velocity at the outer computational boundary and tube radius, ranging from 50 to 500; and swirl ratios from 0.4 to 10. The development of a recirculation zone was demonstrated as the swirl was increased for fixed Reynolds number and viscous core diameter. Similar results were obtained when the core diameter and swirl ratio were fixed while the Reynolds number was increased. In all cases, the breakdown appeared to form very near the inflow boundary.

All of the numerical work completed after that of Kopecky and Torrance,[20] and Grabowski and Berger[21] appears redundant. The authors report breakdown near or at the inflow plane, which appears to be the greatest shortcoming of all these results. The variation of the breakdown behavior to changes in Reynolds number, swirl and axial velocity is similar. In addition, the assumption of axial symmetry limits the usefulness of the results. These discrepancies must be resolved before the relationship between numerically generated breakdown and that observed experimentally can be accurately evaluated.

4. Breakdown Criterion

It is apparent from previous theoretical work that a criticality condition can be established for the onset of breakdown. However, from an examination of the computational studies that have been performed and the experimental studies to be reviewed in this section, it appears that no such criterion has been systematically applied to the various results. In general, the computations are suspect due to the breakdown occurring very close to the inflow boundary. The intent in this section is to show that an appropriately defined local Rossby number can be used to determine the criticality of the flow. This choice is motivated by the fact that both the Squire[8] and Benjamin[5] studies can be reinterpreted in terms of this parameter and the recent work of Ito et al[18] explicitly expresses the result in terms of a Rossby number.

The Rossby number must be defined in a consistent manner with respect to the basic type of vortex flow being considered. It is defined as

$$Ro = \frac{W}{r^{*}\Omega} \qquad (9)$$

where W, r^* and Ω represent a characteristic velocity, length, and rotation rate, respectively. For the velocity profiles consistent with swirling flows, leading edge, and trailing wing-tip vortices we define r^* as the radial distance at which the swirl velocity is a maximum. As pointed out by Leibovich,[13] this is a characteristic viscous length scale appropriate for swirling flows. W represents the axial velocity at r^*. This is justified by the fact that it is a consistent velocity scale for both uniform and radially varying axial velocity profiles, and it is also consistent with the "swirl velocity scale" implied by $r^*\Omega$. A characteristic property of trailing wing-tip vortices is the solid body rotation occurring near the vortex centerline. This rotation rate is considered to be the characteristic rate, Ω, of the vortex. The wing-tip vortices are often described in terms of the two-dimensional Burgers' vortex given as

$$V(r) = \frac{K}{r}\left(1 - \exp\left(-a\, r^2/2\nu\right)\right) \tag{10}$$

Where a is an adjustable constant associated with the core size, ν is the kinematic viscosity, and K is proportional to the circulation. Here, Ω is taken as the limit of V/r as $r \to 0$, i.e.,

$$\Omega = \lim_{r \to 0}\left(\frac{V}{r}\right) = \frac{aK}{2\nu} \tag{11}$$

The characteristic length is taken as

$$r^* = \frac{2\nu}{a} \tag{12}$$

which turns out to be close to the radius of maximum swirl velocity. For the case of the combined vortex considered by Squire[8] and Benjamin[5] the characteristic radius, r*, is 1. Note that the parameter "k" given by Squire for the combined vortex is the inverse of the Rossby number, since the characteristic rate of rotation is given by the solid body rotation of the vortex core, V_0.

Figure 1 is a plot of the Rossby number versus Reynolds number for a variety of computational and experimental studies of swirling flows and trailing wing-tip vortices. Throughout the figure, the open symbols denote no breakdown and the solid symbols denote breakdown. For these computational and confined experimental studies, breakdown is defined as stagnation of the axial velocity on the axis. For the unconfined experimental studies, breakdown is defined as a rapid expansion of the core coupled with a strong deceleration of the axial velocity. The Reynolds number is defined here in terms of the viscous length scale r* and the axial velocity W at the radius r*. A third parameter associated with such flows is the Eckman number, or a "rotational" Reynolds number but in this context it is not an independent parameter. It is, of course, apparent that the Rossby number parameter is expressible in terms of the less fundamental swirl ratio parameter if the chosen characteristic velocities are consistent with the Rossby number definition. For example, in the combined vortex Ωr^* is equal to V_{max}; whereas, for Burger's vortex, and vortices in general, Ωr^* is not equal to V_{max}. The data in the figure show that the computational work to date has been performed at relatively low Reynolds numbers compared to the experimental studies. Since the results are Reynolds number dependent in the range of computational test cases, direct application of inviscid theory in this range is invalid.

The authors have performed numerical calculations using a numerical algorithm for which an axisymmetry condition is not a prerequisite for solution. The algorithm is the three-dimensional extension of the earlier work of Gatski, Grosch, and Rose,[27] using vorticity-velocity variables and a compact discretization of the Navier-Stokes equations. Application of this algorithm to the numerical study of the breakdown phenomena for a variety of flow conditions and parameters forms a part of the doctoral dissertation of R. E. Spall. For a Reynolds number of 200 and a Rossby number of 0.5, breakdown occurred at the inflow plane. For the same Reynolds number and a Rossby number of 0.72, a decrease in axial velocity occurs near inflow, but does not result in breakdown. In addition, for a Reynolds number of 50 and Rossby number of 0.5, breakdown occurred at inflow.

The experimental studies have been conducted for both confined and unconfined flows at higher Reynolds number. Figure 1 shows the results for the confined flows of Garg and Leibovich[17] which are characteristic of the wing-tip class of vortices. Since the data was fit to Burgers' vortex, the Rossby number is easily obtained. Here the Reynolds numbers ranged from 1288 to 2150. The data points representing the bubble form of breakdown are taken approximately 21 cm upstream from the breakdown point and within 6 cm of the beginning of the divergent section of the duct. A single set of data was available from the study of Uchida and Nakamura.[28] This is a confined flow with axisymmetric breakdown occurring at a Rossby number of 0.64. The data point from Singh and Uberoi[26] is for an unconfined trailing wing-tip vortex of a laminar flow wing. In this case the minimum axial velocity rapidly decreases to $0.3\ W_{\infty}$, which suggests vortexbreak down.

The points representing the spiral form are taken near the front of the bubble but occur further downstream (~ 15 cm). Here, the data show a drop in Rossby number as the breakdown point is reached. Note the spiral form of breakdown seems to occur at a higher Rossby number than the bubble form. This is not surprising since the stability mechanisms responsible for this type of breakdown are different than the bubble-type breakdown. Thus, a new higher critical Rossby number is apparently needed as a threshold value for the onset of the spiral form of breakdown.

Figure 2 displays the relationship between Rossby number and Reynolds number for the leading-edge class of vortices. The experimental data was obtained from reports by Owen and Peake,[29] Anders,[30] and Verhaagen and Kruisbrink.[31] Once again open symbols denote no breakdown and closed symbols denote breakdown.

In the study of Owen and Peake,[29] axial core blowing was introduced into vortices shed from delta wings in order to study its effect on breakdown. The symbols in Figure 2 representing this data are variations based on a blowing coefficient, C_μ, at fixed streamwise stations $\frac{z}{c} = 3$ and $\frac{z}{c} = 4$ (c is the chord length of the delta wing). As C_μ increases, the corresponding axial velocity increases and the Rossby number increases past critical. They state that breakdown occurs for the case $C_\mu = 0.0$, while for $C_\mu = 0.05$ and 0.12 the flow is stabilized and no breakdown occurs. For the study of Anders,[30] the variation of the data in Figure 2 is parameterized by the angle of attack of the delta wing. The results for the two angles of attack, $\alpha = 19.3°$ and $\alpha = 28.9°$, at essentially the same downstream location, are shown in the figure. As shown the higher angle of attack causes breakdown to occur closer to the wing leading edge. Verbaagen and Kruisbrink[31] measured the flow properties of the core to support and validate the development of mathematical models. They report that no breakdown occurred. It is important to note that for this class of vortices, as well as for the trailing wing-tip vortices, the Reynolds number range over which the Rossby number criterion holds is significant.

Although the data is sparse and the evaluation of the Rossby number approximate, one may conclude that vortex breakdown for leading-edge vortices occurs at a higher Rossby number than for trailing wing-tip vortices. This may be due to the fact that the swirl velocity profiles are of a different type. Far downstream, the flow outside the core of a trailing wing-tip vortex is nearly irrotational. For a leading-edge vortex, the flow at the edge of the core is rotational and nearly inviscid. In addition, the leading-edge vortex contains a narrow viscous subcore where the radial gradients of the circumferential velocity are extremely large. In contrast, the wing-tip vortex approaches a solid body rotation as the axis is approached. Upstream of breakdown both type vortices can generally be approximated as quasi-cylindrical. The authors can find no analyses that seeks standing-wave solutions to profiles applicable to leading-edge vortices. If these were available, an analytic Rossby number criterion could be obtained. Based on experimental results, it should be near unity.

5. Conclusions

The results shown in Figure 1 make it apparent that experimentally, analytically, and computationally, the critical Rossby number for the symmetric form of trailing wing-tip vortex breakdown for Reynolds numbers greater than 50 is about 0.65. For lower Reynolds numbers, the value of the critical Rossby number is lowered, undoubtedly due to the increased damping effects of viscosity on the wave motions.

Figure 1 sheds light on the proper way to perform computational experiments. The inflow profile should correspond to a Rossby number greater than the critical value. This prevents the possibility of wave-like solutions near the inflow thus precluding breakdown. A mechanism, either inherent in the dynamics of the flow or externally imposed, must then modify the local Rossby number as the flow evolves in the streamwise direction. For example, the decay of a jet-like axial flow due to viscosity or the imposition of an adverse pressure gradient might be sufficient to lower the local Rossby number. Once the critical condition is achieved the possibility of wave-like solutions arises. One would expect a wave propogating upstream to become trapped at this location. If, on the other hand, the Rossby number at inflow is less than the critical value, axisymmetric waves can be expected to propogate to the inflow boundary. Here, the velocity profiles are fixed, thus acting as an "artifical" critical condition. Thus breakdown occurs at this point.

This scenario for numerical computations corresponds to the way in which experiments conducted in tubes have been carried out. A supercritical flow is drawn towards critical as it evolves downstream due to the slight expansion of the tube. At the critical station, breakdown occurs.

The theoretical analysis of Squire, Benjamin and Ito et al reduce to a criterion for the existence of axisymmetric standing waves based on a Rossby number. The exponential profile, (Eq. (10), which most closely models experimental flows, yields a critical Rossby number of 0.64. This value is shown as a dashed line in Figure 1. The experimental data of Garg and Liebovich,[17] interpreted in terms of a Rossby number, shows that the bubble form of breakdown occurs when the local Rossby number falls in the range of 0.63 to 0.67, whereas the spiral form of breakdown occurs when the local Rossby number falls to 0.7. From the available data, the local Rossby number was initially below 0.7 for the cases involving the bubble-type breakdown. Numerical experiments reveal a high Reynolds number limit ($Re > 50$) of about $Ro = 0.6$ for breakdown to occur. For lower Reynolds numbers, a lower Rossby number is required to initiate breakdown.

The situation is less clear for the class of leading-edge vortices of Figure 2. In this case the data is sparse and, when obtainable, it is more difficult to cast in terms of the Rossby number and Reynolds number. However, the data that is shown was obtained from a more diverse parameter base. For example, in the study of Owen and Peake,[29] core blowing was the variable parameter and in the study of Anders,[30] angle of attack was the relevant variable parameter; nevertheless, in both cases, the available data was consistent with the concept of a Rossby number criterion.

It is apparent from the results of this paper that retarding or precluding vortex breakdown is a practical and viable objective. This altering of the vortex characteristics can be accomplished by either reducing the characteristic rotation rate of the vortex or enhancing the streamwise velocity. The rotation rate can be reduced, for example, by imposing transverse pressure gradients, or the streamwise velocity can be enhanced by imposing streamwise pressure gradients. In either approach the effective measure is the Rossby number.

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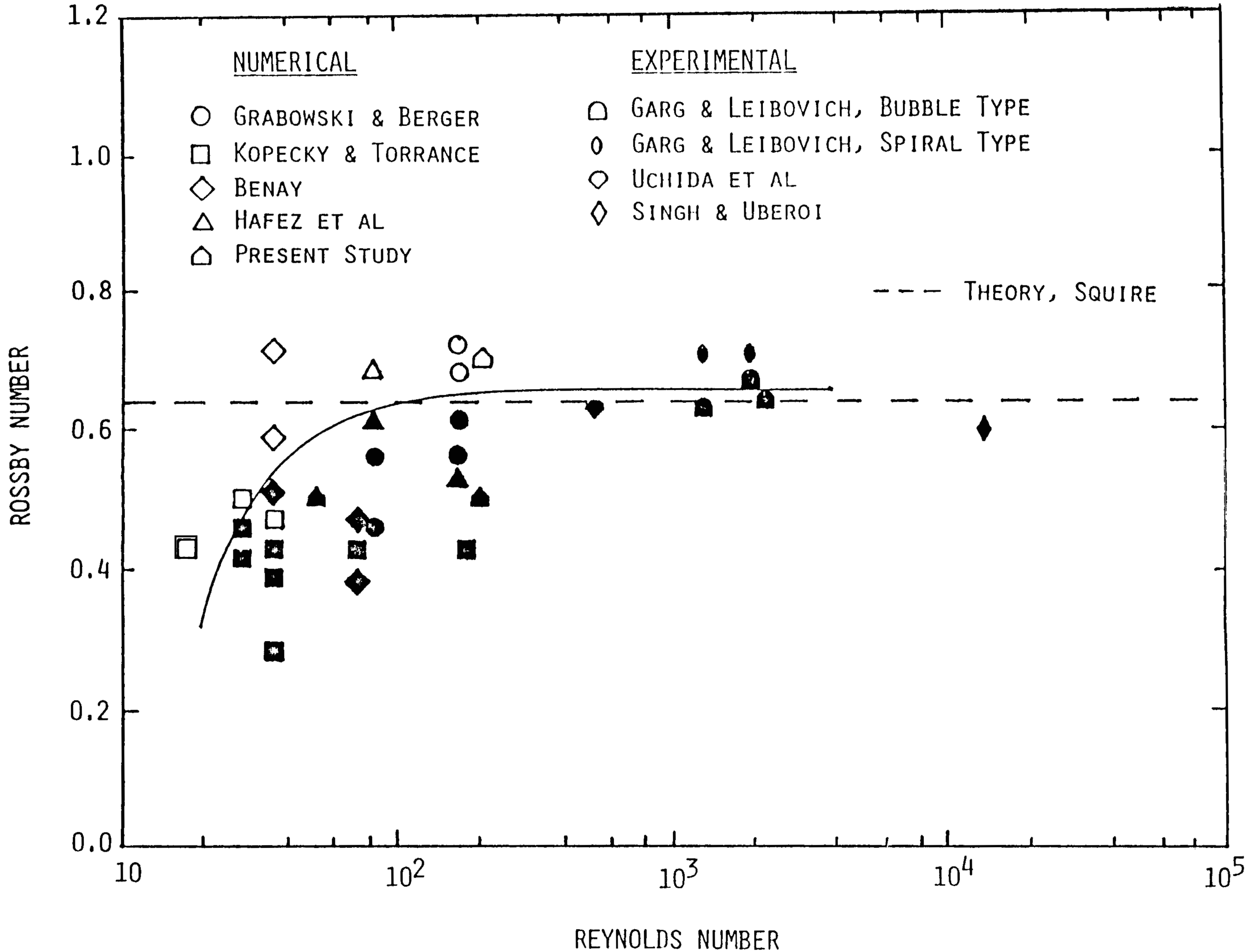
Figure Captions

Figure 1. Rossby Number Dependence of Wing-Tip Vortices.

Figure 2. Rossby Number Dependence of Leading-Edge Vortices.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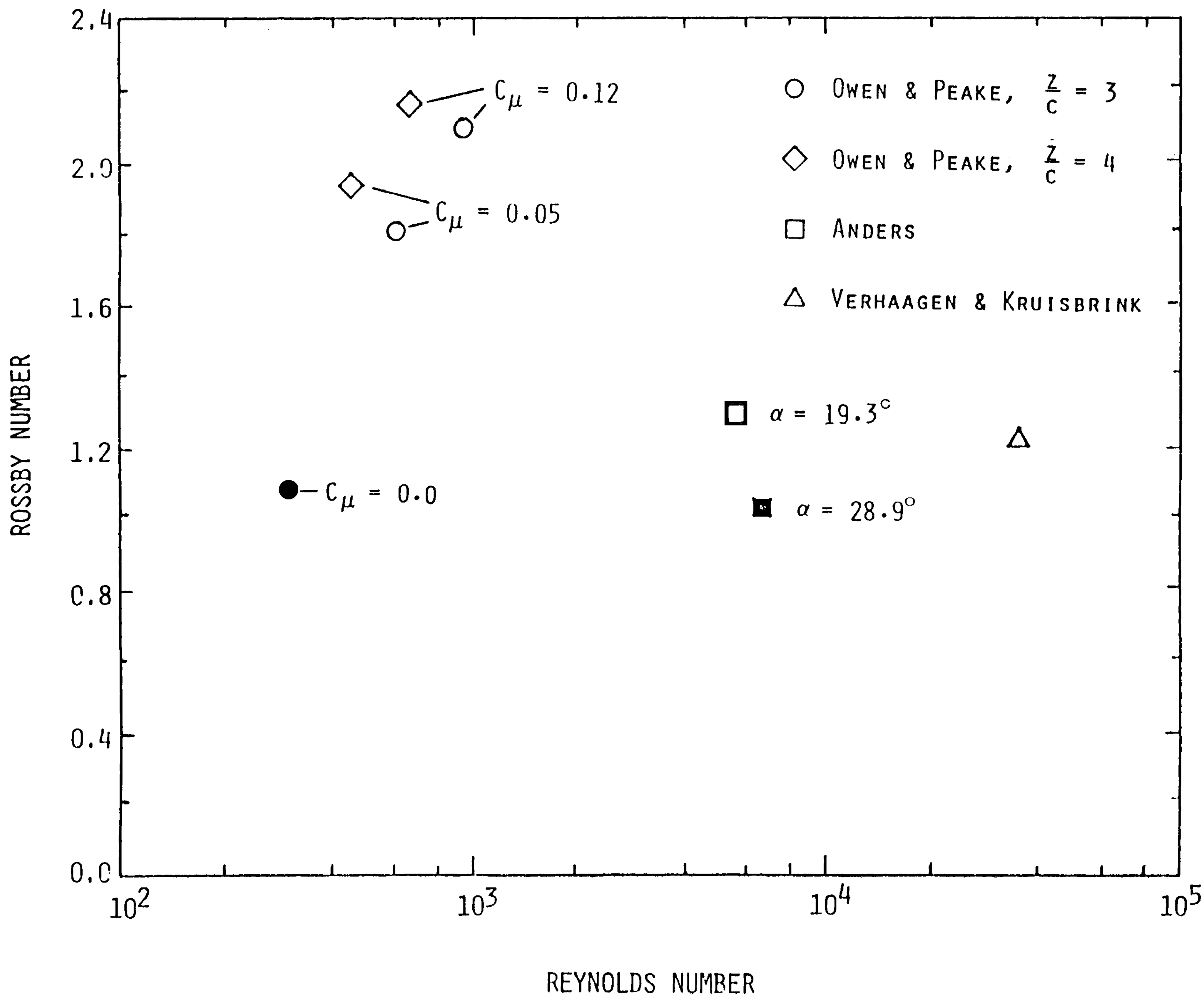
1.0
0.8
0.6
0.4
0.2
0.0
ROSSBY NUMBER
10
10^2
10^3
10^4
10^5
REYNOLDS NUMBER
NUMERICAL
GRABOWSKI & BERGER
KOPECKY & TORRANCE
BENAY
HAFEZ ET AL
PRESENT STUDY
EXPERIMENTAL
GARG & LEIBOVICH, BUBBLE TYPE
GARG & LEIBOVICH, SPIRAL TYPE
UCHIDA ET AL
SINGH & UBEROI
THEORY, SQUIRE

FIGURE 1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2.0
1.6
1.2
0.8
0.4
0.0
ROSSBY NUMBER
10^2
10^3
10^4
10^5
REYNOLDS NUMBER
OWEN & PEAKE, Z/c = 3
OWEN & PEAKE, Z/c = 4
ANDERS
VERHAAGEN & KRUISBRINK
Cμ = 0.12
Cμ = 0.05
Cμ = 0.0
α = 19.3°
α = 28.9°

FIGURE 2

Standard Bibliographic Page

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16. Abstract

A criterion for the onset of vortex breakdown is proposed. Based upon previous experimental, computational, and theoretical studies, an appropriately defined local Rossby number is used to delineate the region where breakdown occurs. In addition, new numerical results are presented which further validate this criterion. A number of previous theoretical studies concentrating on inviscid standing-wave analyses for trailing wing-tip vortices are reviewed and reinterpreted in terms of the Rossby number criterion. Consistent with previous studies, the physical basis for the onset of breakdwon is identified as the ability of the flow to sustain such waves. Previous computational results are reviewed and re-evaluated in terms of the proposed breakdown criterion. As a result, the cause of breakdown occurring near the inflow computational boundary, common to several numerical studies, is identified. Finally, previous experimental studies of vortex-breakdown for both leading edge and trailing wing-tip vortices are reviewed and quantified in terms of the Rossby number criterion.

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