SENSITIVITY METHOD FOR INTEGRATED STRUCTURE/ACTIVE CONTROL LAW DESIGN

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OUTLINE

This paper describes the development of an integrated structure/active control law design methodology for aeroelastic aircraft applications. The paper gives a short motivating introduction to aeroservoelasticity and the need for integrated structures/controls design algorithims. Three alternative approaches to development of an integrated design method are briefly discussed with regards to complexity, coordination and tradeoff strategies, and the nature of the resulting solutions. This leads to the formulation of the proposed approach which is based on the concepts of sensitivity of optimum solutions and multi-level decompositions. The concept of sensitivity of optimum is explained in more detail and compared with traditional sensitivity concepts of classical control theory. The analytical sensitivity expressions for the solution of the linear, quadratic cost, Gaussian (LQG) control problem are summarized in terms of the linear regulator solution and the Kalman Filter solution. Numerical results for a state-space aeroelastic model of the DAST ARW-II vehicle are given, showing the changes in aircraft responses to variations of a structural parameter, in this case first wing bending natural frequency.

- Introduction

- Design Approach

- Sensitivity of Optimum

- Sensitivity of LQG Solution

- Integrated Design Results

- Concluding Remarks

INTRODUCTION

Aeroservoelasticity is defined as the interaction of unsteady aerodynamics, elastic structure, and automatic control systems of an aircraft. This interaction can be either favorable and unfavorable, that is it can be the source of dynamic responses of the aircraft which force the redesign of the structure or flight control system, or it can actually improve the performance of the aircraft. Examples of aircraft which exhibited aeroservoelastic problems include the F-16, F-18, and the X-29, all of which required flight control system changes before flight. The Lockheed L1011-500 on the other hand employs an active load alleviation system to reduce wing loads and improve range.

The state of the art in aeroservoelastic analysis is now to the point where it is possible in many cases to predict aeroservoelastic interactions before flight test of the vehicle. With this capability in hand, the next logical step is to develop design methodologies which use aeroservoelastic interactions to improve aircraft performance. This paper describes the initial development of one approach to this interdisciplinary design problem, concentrating on integrated design of aircraft structures and control laws.

- Aeroservolasticity Is The Interaction

    Of Aerodynamics, Structures, And Controls

- Favorable And Unfavorable Interactions

- Analysis Methods Maturing

- Integrated Design Methods In Infancy

DESIGN APPROACHES

There are three possible approaches to integrated structure/ control law design, or for that matter, any integrated design. These are the simultaneous or combined approach, the series or sequential approach, and the parallel approach. In the simultaneous approach, the design problem is formulated as a single problem by combining the objectives, requirements, and design variables of the various disciplines into a single set. The design variables are then selected simultaneously to satisfy the design requirements and objectives. The drawbacks to this approach are the resulting size of the design problem and the difficulty of making reasonable tradeoffs when all the design criteria are not satisfied.

In the series approach, the individual disciplinary designs are performed in a logical sequence or series, with each discipline selecting its own design variables to satisfy its own design requirements. System performance is assessed at the end of the sequence, and the process is repeated if necessary in an iterative manner. Again, one of the drawbacks with this approach is difficulty in making tradeoffs between disciplines, although a more serious drawback is that the overall system design is dominated by the discipline which is first in the sequence. For example, if an aircraft structural design is completed first, followed by the flight control design, and unfavorable dynamic iteractions occur, most often the flight control system design is changed extensively to improve the overall dynamics while the structural design is held fixed, even though moderate structural changes may be more effective.

A parallel approach to integrated design has the individual disciplines performing their designs simultaneously but independently. At the completion of the design iteration, the overall system performance is checked and the individual designs undergo iterations. Of course, some form of coordination must occur during the iteration process or no improvement in the system design will be possible. The coordination activity requires that information about the individual designs and the relationships of those designs to the other disciplines must be available. This information is dependent on the actual design methods that are used as well as the type and form of the design requirements, objectives, and design variables. The kinds of information, coordination, and design methods necessary for the sucessful development of a parallel integrated design methodology are still open research questions.

DESIGN APPROACHES
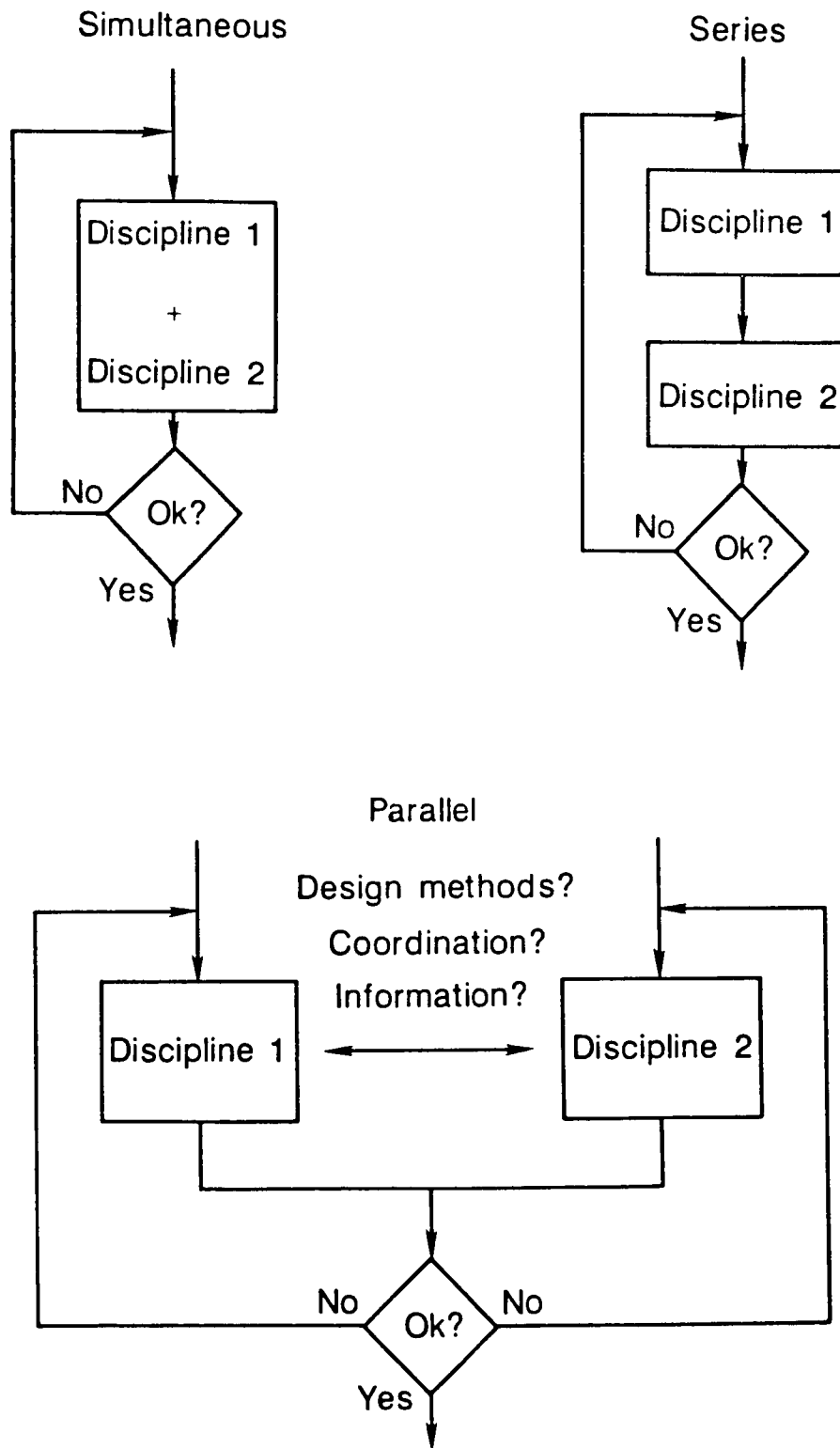
PARALLEL DESIGN

The successful development of a parallel integrated interdisciplinary design methodology requires a coordination strategy, the determination of disciplinary design information requirements, and the selection of design tools for each discipline which are compatible with the coordination strategy and information requirements. Based on research conducted at NASA – Langley Research Center and elsewhere, a multi-level problem decomposition approach [1,2] is used to define a coordination strategy for the integrated structures/control law design method proposed here. This approach breaks the integrated design problem down into a heirarchal structure that naturally reflects the individual disciplinary design requirements as well as the integrated system design requirements and objectives. Selecting optimization techniques for the individual disciplinary design methods allows the use of the concept of the sensitivity of an optimum solution to fixed parameters [3] to define the information requirements of the hierarchical decomposition. That is, for the case of integrated structure/control law design, the sensitivity of the optimum structural design to control law design variables is passed to the coordination level, as is the sensitivity of the optimum control law design to structural design variables. This information is used at the system design level to make design tradeoffs between the disciplines in the interest of improving the system design.

Design Methods: Optimization Techniques

Information: Sensitivity Of Optimum Designs[3]

Coordination: Multilevel Problem Decompositions[1,2]

INTEGRATED STRUCTURE/CONTROL DESIGN

The selection of multi-level hierarchal problem decompositions, optimization techniques, and the sensitivities of optimum solutions leads to the integrated structure/control law design methodology shown below. The control law and structure designs occur simultaneously and in parallel, with the recognition that the two disciplines interact in the actual aircraft. These designs proceed on the basis of the individual discipline design objectives and variables. For example, the structural design might determine structural element sizing to minimize weight while maintaining stress levels, while the control design picks control gains to minimize control energy and maintain adequate stability margins. The sensitivity of the optimum control design to the structural element sizes, and the sensitivity of the optimum structural design to the control law gains are then computed, either by finite differencing of neighboring designs, or by analytical sensitivity of optimum derivative expressions, and passed on to the system level. This sensitivity data are then used as gradient information at the higher level to determine the most effective tradeoffs to achieve desired system performance. A key aspect of the research reported here is the development of analytical sensitivity expressions for the LQG optimal control law problem, eliminating the need for finite difference derivative calculations.
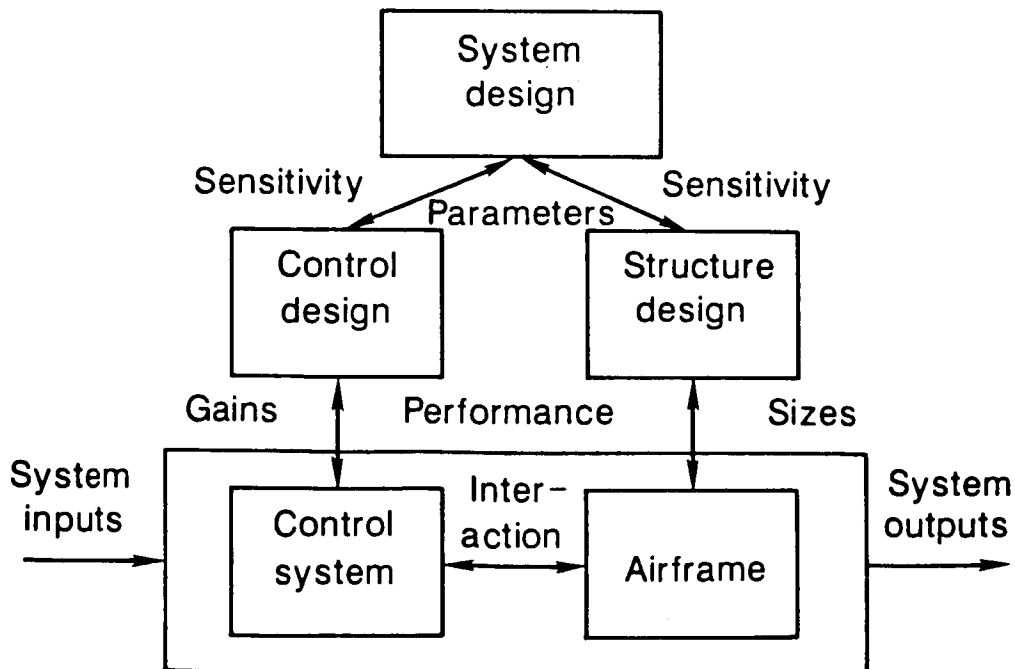
GEOMETRICAL INTERPRETATION OF OPTIMUM SENSITIVITY

The concept of the sensitivity of an optimum solution of an optimization problem to problem parameters which were held fixed during the optimization is illustrated below. Consider a conceptual optimization problem where an objective function $J(u,p)$ is to be minimized by choice of a design variable u, with some design parameter p held fixed at some nominal value p_0. For a different nominal value of the design parameter, say p_1 the optimum solution of the problem will be different, as shown. The sensitivity of the optimum solution with respect to the design parameter p is then the change of the optimum value of the objective function and the change of the design variable at optimum due to changes in the parameter. One approach to calculating these sensitivities is to finite difference solutions of the problem due to perturbations in the parameter. Another approach is to obtain analytical sensitivity expressions by differentiation of the necessary conditions of optimality with respect to the design parameter, and evaluating those expressions at the optimum solution, as advocated in [3]. This approach eliminates the need for numerous perturbed solutions of the problem and the inaccuracies of numerical approximations of the sensitivities.
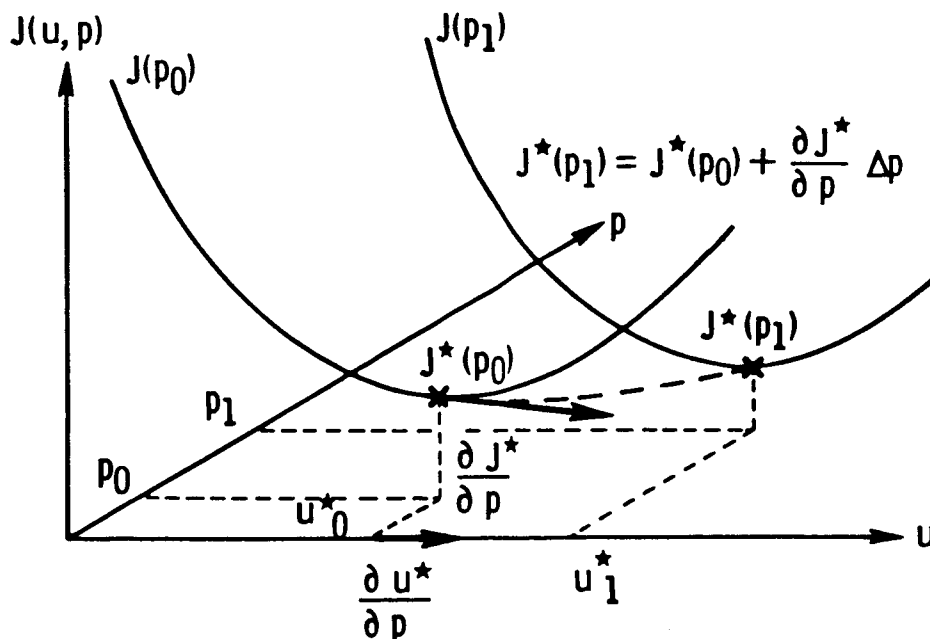
COMPARISON OF SENSITIVITY TYPES

The difference between the sensitivity of optimum analysis and a traditional sensitivity analysis in controls applications can be highlighted as follows. Consider the time response of some output $Y(t,p)$ of an optimally controlled dynamic system due to a specified input. For the nominal value of the design parameter p_0, the optimal control law is computed and the time response is calculated. If the value of the design parameter was to change to p_1, but the control law was to remain the same (that is the control law that is optimal for p_0), then the time response to the same input would change, and a traditional sensitivity analysis could be used to predict that change. On the other hand, if a new control law which is optimal for p_1 is used, the time response would again be different from the original, and from the perturbed control law time response as well. The sensitivity of optimum analysis results can be used to predict this new optimally controlled system time response analytically.
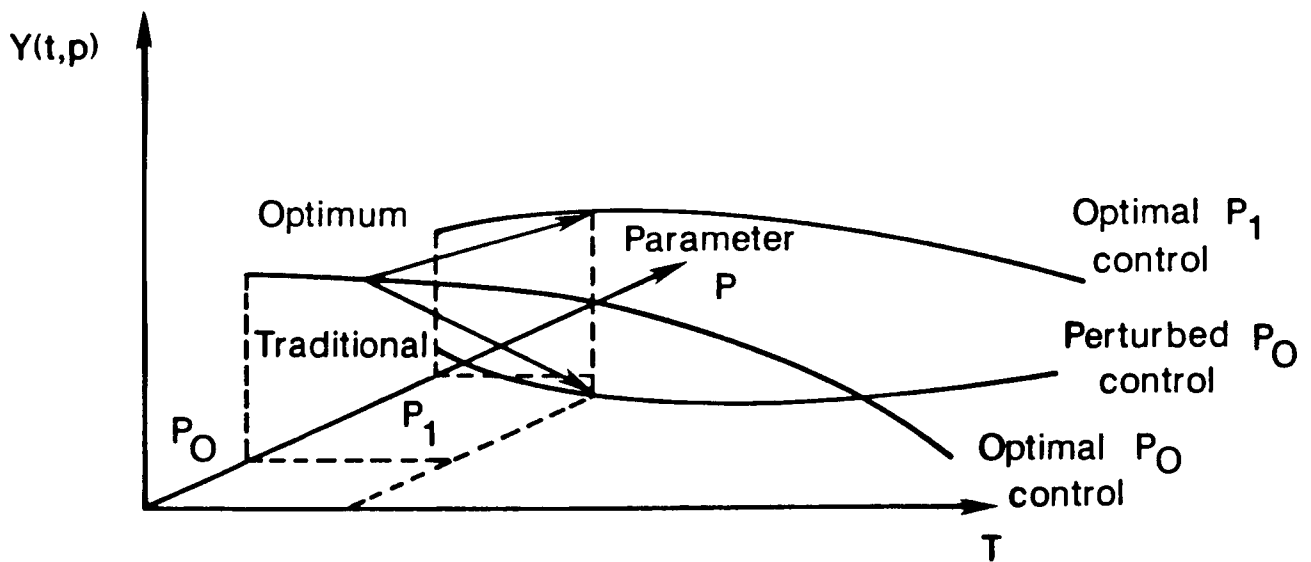
LQG PROBLEM FORMULATION

The optimal control law formulation to be used in the integrated structure/control law design algorithim is the standard linear time invariant system, quadratic cost, Gaussian distributed noise (LQG) optimal control problem. For the purposes of this integrated design methodology, the formulation consists of state space equations of motion, where A is the system state matrix, B is the control input matrix, C is the controlled output matrix, D is disturbance input matrix, and M is the measurement matrix defining the signals to be used for feedback. The vector x is the system state vector, u is the control input, r is a vector of external commands, and w_D, w_U, and v are Gaussian distributed white noise vectors with noise intensity matrices W_D, W_U, and V respectively. The objective function J to be minimized is the expected value of a quadratic function of the controlled outputs y and the control inputs u, where the weighting matrices Q and R are positive semi-definite and positive definite, respectively. It is assumed that the matrices A, B, C, M, Q, R, and W_U are functions of the fixed design parameters p, for which the functional dependence and the derivatives of the matrix elements with respect to the parameters are known. The solution to this optimal control problem is known to be the interconnection of the optimal linear regulator with the optimal Kalman Filter state estimator [4, pg. 390].

$$\dot{x} = A(\bar{p})x + B(\bar{p})(u+r) + Dw_D + B(\bar{p})w_U$$

$$y = C(\bar{p})x$$

$$z = M(\bar{p})x + v$$

$$J = \lim_{T \to \infty} \varepsilon \left\{ \frac{1}{2T} \int_0^T (y^T Q(\bar{p})y + u^T R(\bar{p})u)dt \right\}$$

$$\bar{p} = \left\{ \ldots \ldots p \ldots \right\}^T = \text{vector of fixed parameters}$$

The solution of the LQG optimal control problem is the interconnection of the optimal linear regulator and the optimal Kalman Filter state estimator. The solution of the optimal regulator problem is an optimal feedback gain matrix G determined by the solution for S of a nonlinear matrix Riccati equation, where both equations come directly from the necessary conditions of optimality [5, pg. 148]. The interconnection with the Kalman Filter is defined by feeding back estimates of the system states rather than the actual (unmeasurable) system states. Differentiating the LQG solution equations with respect to the parameter p gives an expression for the sensitivity of the optimal gain matrix G which is in terms of the sensitivity of the Riccati equation solution matrix S. The Riccati sensitivity is obtained from the solution of the linear Lypuanov equation that results from differentiation of the matrix Riccati equation with respect to p. (Note that all the other derivative matrices in the two equations are assumed to be known as part of the problem formulation.) A property of the regulator solution is that the matrix (A-BG) is asymptotically stable, guaranteeing that a unique solution to the Lyapunov equation exists [4, pg. 103].

Necessary conditions

$$u = -R^{-1}B^T S \hat{x} = -G\hat{x}$$

$$0 = A^T S + SA - SBR^{-1}B^T S + C^T QC$$

Differentiate necessary conditions with respect to p

$$\frac{\partial G}{\partial p} = -R^{-1}\frac{\partial R}{\partial p}R^{-1}B^T S + R^{-1}\frac{\partial B^T}{\partial p}S + R^{-1}B^T\frac{\partial S}{\partial p}$$

$$0 = (A-BG)^T\frac{\partial S}{\partial p} + \frac{\partial S}{\partial p}(A-BG) + \left\{\frac{\partial A^T}{\partial p}S + S\frac{\partial A}{\partial p} + \frac{\partial C^T}{\partial p}QC + C^T\frac{\partial Q}{\partial p}C\right.$$

$$\left. + C^T Q\frac{\partial C}{\partial p} - S\left(\frac{\partial B}{\partial p}R^{-1}B^T - BR^{-1}\frac{\partial R}{\partial p}R^{-1}B^T + BR^{-1}\frac{\partial B^T}{\partial p}\right)S\right\}$$

KALMAN FILTER AND OPTIMUM SENSITIVITY

The optimal Kalman Filter solution is similiar to the optimal regulator solution in that the optimal filter gain matrix K is given in terms of the solution T to the filter nonlinear matrix Riccati equation. Differentiation of these two equations with respect to the parameter p gives the filter gain matrix K sensitivity in terms of the sensitivity of the matrix Riccati equation solution T. This sensitivity is calculated from a linear Lyapunov equation obtained by differentiation of the Riccati equation, which again is known to have a unique solution because of the asymptotic stability properties of the coefficient matrix (A–KM).

Necessary conditions :

$$K = TM^T V^{-1}$$

$$0 = AT + TA^T + DW_D D^T + BW_u B^T - TM^T V^{-1} MT$$

Differentiate necessary conditions with respect to p:

$$\frac{\partial K}{\partial p} = \frac{\partial T}{\partial p} M^T V^{-1} + T \frac{\partial M^T}{\partial p} V^{-1}$$

$$0 = (A-KM)\frac{\partial T}{\partial p} + \frac{\partial T}{\partial p}(A-KM)^T + \left\{ \frac{\partial A}{\partial p} T + T\frac{\partial A^T}{\partial p} + \frac{\partial B}{\partial p} W_u B^T \right.$$

$$\left. + B\frac{\partial Wu}{\partial p} B^T + BW_u \frac{\partial B^T}{\partial p} - T\left(\frac{\partial M^T}{\partial p} V^{-1} M + M^T V^{-1}\frac{\partial M}{\partial p}\right) T \right\}$$

The state-space equations of the optimally controlled system can be written in terms of the optimal gain matrices G and K by defining a state estimation error vector e which in turn is used to define a new augmented system state vector. The closed-loop system equations are then as shown, with the new system state matrix shown in partitioned form as a function of K and G. The sensitivity of the new system state matrix with respect to p is calculated in terms of known sensitivity derivative matrices and the optimal gain sensitivities which have already been calculated. These results are used with analytical performance senstivity expressions, such as for eigenvalues and time responses, to find the changes in optimally controlled system performance due to changes in system design parameters p.

Define : $e \triangleq x - \hat{x}$, $\bar{x}^T = \{ x^T \mid e^T \}$, $\bar{w}^T = \{ w^T \mid v^T \}$

Closed Loop

$$\dot{\bar{x}} = \bar{A}\bar{x} + \bar{D}\bar{w}$$
$$y = \bar{C}\bar{x}$$
$$u = \bar{G}\bar{x}$$

$$\bar{A} = \left[\begin{array}{c|c} A - BG & BG \\ \hline 0 & A - KM \end{array} \right] \qquad \frac{\partial \bar{A}}{\partial p} = \left[\begin{array}{c|c} \frac{\partial A}{\partial p} - \frac{\partial B}{\partial p}G - B\frac{\partial G}{\partial p} & \frac{\partial B}{\partial p}G + B\frac{\partial G}{\partial p} \\ \hline 0 & \frac{\partial A}{\partial p} - \frac{\partial K}{\partial p}M - K\frac{\partial M}{\partial p} \end{array} \right]$$

ANALYTICAL PERFORMANCE SENSITIVITIES

Analytical performance senstivity expressions exist for numerous dynamic system performance measures in terms of the sensitivity matrices of state-space systems. These include eigenvalue and eigenvector sensitivities [6], trajectory (time) and frequency response sensitivities [7], sensitivity of covariance responses due to random system inputs and disturbances [8], and singular value sensitivities [9]. These results are used in the integrated structure/control algorithim at the upper level as gradient information to predict performance changes due to changes in the structural design parameters.

- Eigenvalues/Eigenvectors

- Trajectory Responses

- Covariance Responses

- Frequency Responses

- Singular Value Decompositions

COMPARISON OF PREDICTED AND ACTUAL CHANGES

Numerical results have been calculated for an integrated design study of the DAST ARW-II flight test vehicle. This application involved the design of an optimal LQG control law and the prediction of changes in the optimally controlled response of the vehicle due to changes in a structural design parameter, in this case first wing bending natural frequency. For example, changes in mean square wing tip acceleration and mean square aileron deflection rate due to changes in wing first bending frequency for a 12 ft/sec RMS random wind gust environment are shown below. The sensitivities of the mean square responses to the structural parameter are the slopes of the solid and dashed lines, with the lines themselves showing the predicted change in performance if a new optimal control law was implemented for various changes in the parameter. The symbols show the actual change in performance which occurred when the parameter was varied and the resulting new optimal control law was computed and implemented. For + or - 10% variations in the wing first bending frequency the sensitivity based predictions were reasonably accurate. For larger variations, the predictions were not as accurate, although the trends were correct. Note that for reductions in wing first bending frequency, both the acceleration and the control surface deflection rate were reduced, whereas if changes were made only in the control law, the acceleration could only be reduced at the expense of increased aileron deflection rate. This indicates the potential benefit of an integrated structure/control law design approach to improved system performance.
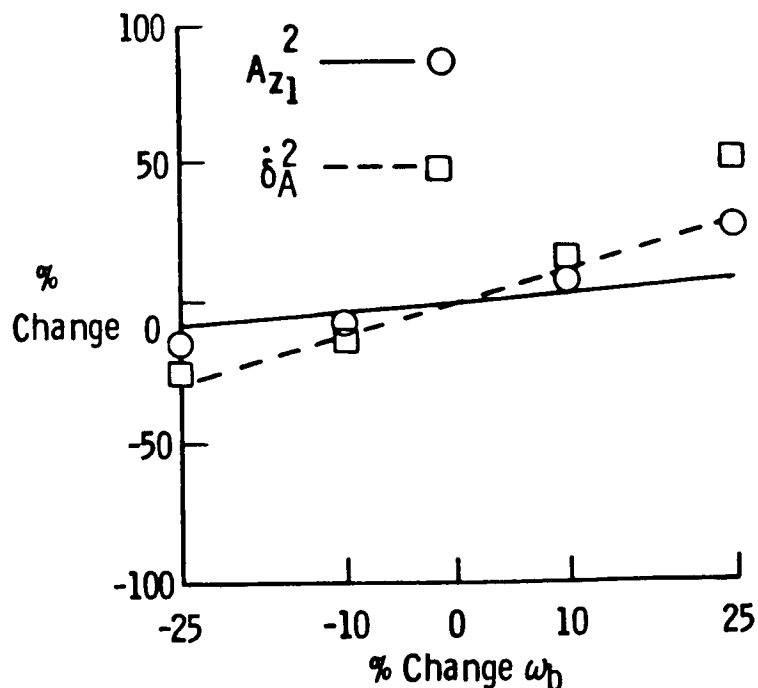
WING BENDING FREQUENCY VARIATIONS

Shown are two controlled system performance measure changes due to changes in wing first bending frequency. The top plot is of changes in the minimum singular value of the loop return difference matrix with the control loops broken at the input to the system. This measure is an indication of the stability robustness of the system with respect to gain and phase variations and unmodelled higher order dynamics, with larger values over the frequency range implying greater robustness. For reductions of 10% and 25% in the wing first bending natural frequency, there is a slight rise in the minimum singular value at the critical low regions between .1 and 1 rad/sec and again near 100 rad/sec. The lower plot shows wing tip acceleration in g's due to a commanded pitch over of the vehicle. For 10% and 25% reductions of nominal wing bending frequency there is a small reduction in transient wing tip acceleration response to the same manuever, although the steady-state acceleration is the same. These results again indicate the possibility for improvements in overall system performance due to integrated structure/control law design, although other structural parameters may provide more significant changes in performance and thus be more useful from a design standpoint.
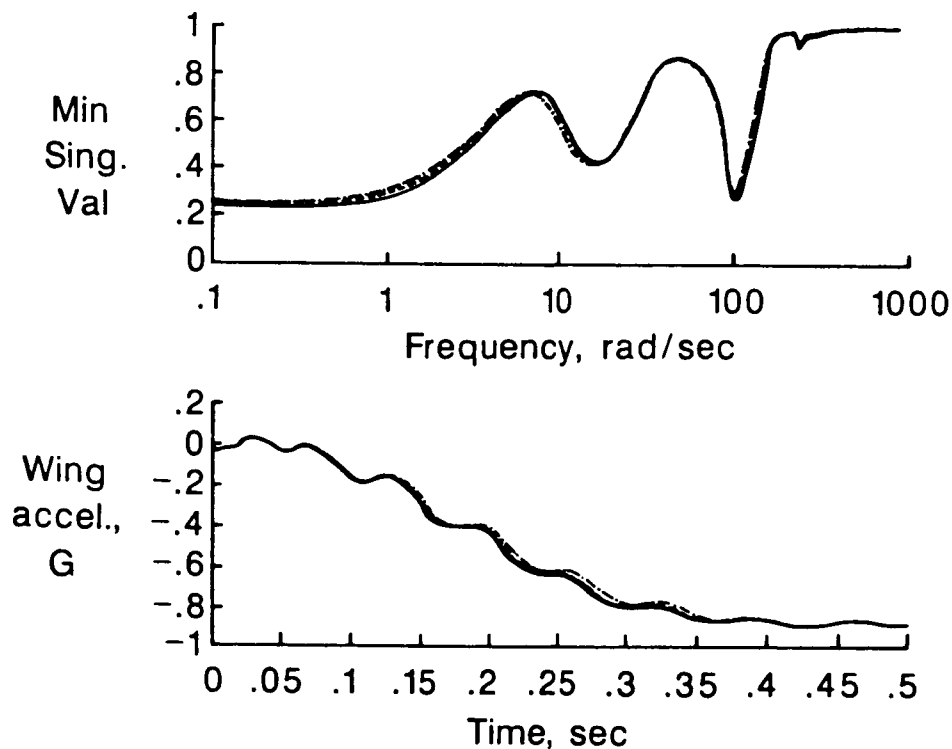
CONCLUSIONS AND FUTURE RESEARCH

An approach to parallel integrated interdisciplinary design using hierarchal decompositions and sensitivity of optimum solution concepts is under development at NASA-Langley Research Center. An implementation of this approach for integrated structure/control law design problems of aeroservoelastic aircraft is currently under way, and numerical results for an example problem indicate that an integrated design could lead to better overall system performance. The development and implementation of the methodology have also shown that senstivity of optimum solutions to problem parameters is required for accurate gradient information at the top (system) level when the parallel disciplinary design approaches are optimization based, and that accurate predictions of performance changes due to reasonable (+ or - 10%) variations in parameters are obtained from the optimum sensitivity results. The continuing research program is working toward the inclusion of more formal structural optimization techniques, and to the development of sensitivity expressions for other, more realistic, optimal control law problem formulations.

- Sensitivity of Optimum Analysis Required When Design Iterations Use Optimization

- Performance Changes Accurately Predicted For Reasonable Parameter Variations

- Overall System Performance Can Be Improved By Parallel Intergrated Design

- Need To Develop Analytical Sensitivity Expressions For More Optimal Control Problems

- Need To Exercise Parallel Design Methodology Fully

REFERENCES

[1] Sobieszczanski-Sobieski, J., "A Linear Decomposition Method for Large Optimization Problems - A Blueprint for Development", NASA TM-83248, Feb. 1982.

[2] Mesarovic, M. D., Macko, D., and Takahara, Y., "Theory of Hierarchical,Multilevel,Systems", Academic Press, 1970.

[3] Sobiesczcanski-Sobieski, J., Barthelemy, J. F., and Riley, K. M., "Sensitivity of Optimum Solutions of Problem Parameters", AIAA Journal, Vol. 20, No. 9, pg. 1291-1299.

[4] Kwakernaak, H., and Sivan, R., "Linear Optimal Control Systems", Wiley-Interscience, 1972.

[5] Bryson, A. E., and Ho, Y. C., "Applied Optimal Control", Halstead Press, 1975.

[6] Nelson, R. B., "Simplified Calculations of Eigenvector Derivatives", AIAA Journal, Vol. 14, No. 9, pg. 1201-1205

[7] Frank, P. M., "Introduction to System Sensitivity Theory", Academic Press, 1978.

[8] Schaecter, D. B., "Closed-Loop Control Performance Sensitivity to Parameter Variations", Journal of Guidance, Control, and Dynamics, Vol. 6, No. 5, pg. 399-402.

[9] Newsom, J. R., and Mukhopadhyay, V., "The Use of Singular Value Gradients and Optimization Techniques to Design Robust Controllers for Multiloop Systems", AIAA 83-2121, AIAA Guidance and Control Conference, August 1983.