N87-18871

APPPLICATION OF A SYSTEM MODIFICATION TECHNIQUE TO DYNAMIC TUNING OF A SPINNING ROTOR BLADE

C. V. Spain
PRC Kentron, Inc.
Aerospace Technologies Division
Hampton, Virginia

INTRODUCTION

An important consideration in the development of modern helicopters is the vibratory response of the main rotor blade. One way to minimize vibration levels is to ensure that natural frequencies of the spinnning main rotor blade are well removed from integer multiples of the rotor speed. This report demonstrates a technique for dynamically tuning a finite-element model of a rotor blade to accomplish that end.

Rotor blades are an ideal subject for this type of analysis because a good structural representation can be achieved with a single string of beam elements and relatively few degrees of freedom. This means that the numerous system stiffness and mass matrices required can be formed with relatively low central processor time. The technique is valid, however, for larger and more complex models.

Because the tuning process involves the independent redistribution of mass and stiffness, it is especially applicable to composite blade designs in which mass and stiffness can be controlled independently by fiber orientation and the use of nonstructural mass.

In the following sections, a brief overview is given of the general purpose finite element system known as Engineering Analysis Language (EAL, ref. 1) which was used in this work. A description of the EAL System Modification (SM) processor is then given along with an explanation of special algorithms developed to be used in conjunction with SM. Finally, this technique is demonstrated by dynamically tuning a model of an advanced composite rotor blade.

This work was accomplished in support of the Interdisciplinary Research Office of NASA Langley Research Center and the objectives were threefold. The first was to establish a technique for tuning the natural frequencies of a spinning rotor blade. The second was to demonstrate the usefulness of the EAL SM processor and to be able to perform sensitivity and modification operations without dependence on additional software. The final objective was to provide guidelines on advanced use of the SM processor, i.e., use beyond the scope of currently available documentation.

ENGINEERING ANALYSIS LANGUAGE (EAL)

EAL is a general purpose finite element system produced by Engineering Information Systems, Inc. It evolved from an earlier finite element program known as SPAR (ref. 2). In its present form, EAL consists of an Executive Control System (ECS) in which the user can execute work flow logic, looping, branching and data storage; and processors (similar to subroutines) which actually perform structural and utility computations. Data input or computations result in data sets which are stored in binary data bases or libraries which can be saved and referred to indefinitely. The user communicates with and uses these features with input known as runstreams.

Reference 1 is the current EAL reference manual, however, the older SPAR reference manual (ref. 2) must be used for the SM processor. EAL version 209 was used in this work.

EAL SYSTEM MODIFICATION (SM) FOR FREQUENCY MODIFICATION

The approach in modifying frequencies is to first specify a set of target (required) eigenvalues corresponding to natural frequencies of the original model. Parameters to be changed must be identified along with limits on acceptable changes. Sensitivities of the eigenvalues to parameter changes must then be calculated. To determine the actual structural changes, the statistical method described in reference 3 is used.

SM operates in 4 phases as described below. The notation used here is generally consistent with the SM description contained in reference 2.

Phase 1: The differences (Δ_X) between the eigenvalue targets (X_T) and current eigenvalues (X) are calculated. That is:

$$\Delta X = X_T - X \tag{1}$$

Phase 2: The purpose of phase 2 is to approximate the sensitivities of eigenvalues (radians^2/sec^2) to specified changes in structural parameters which affect stiffness and/or mass. These specified changes are known as unit parameters. System stiffness change (ΔK) and mass change (ΔM) matrices are formed for each unit parameter.

Because the original model eigenvalue solution is based on equation 2 below, where λ_i is the ith eigenvalue and M, K and Y_i are the system mass, system stiffness and the ith mode shape, respectively, then the modified system can be described by equation 3.

$$\lambda_i M Y_i - K Y_i = 0 \tag{2}$$

$$(\lambda_i + \Delta\lambda_i)(M + \Delta M) - (K + \Delta K)(Y_i + \Delta Y_i) = 0 \tag{3}$$

With some simplifying assumptions (i.e. changes in mode shapes and products of the changes (Δ's) are very small), a reasonable approximation of eigenvalue sensitivity is expressed by equation (4).

$$\Delta\lambda_i = Y_i^T \Delta K Y_i - \lambda_i Y^T \Delta M Y_i \tag{4}$$

The ΔK, ΔM and $\Delta\lambda_i$ are therefore the results of phase 2 which is computationally the most costly phase because the system mass and stiffness matrices must be formed for each unit parameter. Computations in the other 3 phases are trivial in terms of central processor time.

Equation 4 is valid only for a nonspinning structure and must be augmented for a spinning structure as described later.

Phase 3: The actual structural changes needed to realize the targeted eigenvalues are estimated based on equation (5) below which is an adaptation of the work presented in reference 3.

$$\{\Delta P\} = [S_{rr}]\ [NT]^T \left\{ [NT]\ [S_{rr}]\ [NT]^T + [S_{ee}] \right\}^{-1} [N]\ \{\Delta x\} \tag{5}$$

where:

ΔP is a set of multipliers which reflects the total estimated structural modifications needed in terms of corresponding unit parameters.

S_{rr} is the covariance or weighting matrix. The diagonal terms, each corresponding to the unit parameters in sequence, allow for the relative weighting of those parameters. In this application, values are set at unity and reset in later iterations if the parameter change limits are being exceeded.

N is a matrix containing reciprocals of the current eigenvalues ($1/\lambda_i$).

T is the sensitivity matrix consisting of ($\Delta\lambda_i$'s) with the rows corresponding to the number of targets and the columns to the number of unit parameters.

S_{ee} is the target tolerance matrix associated with acceptable variances of the resulting eigenvalues from the targets.

ΔX is as described in equation (1).

The purpose of using this method is to achieve the targeted eigenvalues with minimum change to the structure. S_{rr} can be used to influence how much a particular unit parameter is changed. For example, a unit parameter which can be changed with small penalty or is not likely to exceed the prescribed change limits may be assigned an S_{rr} value of 1.0, whereas, a unit parameter which should be changed as little as possible may be assigned a value of 0.1. S_{ee} values normally range from 0.0 (when a more exact attainment of the targeted eigenvalue is being sought) to 0.1 (when only an approximate result is needed). As described in reference 4, S_{ee} values of 0.001 when most S_{rr} values are 1.0 normally provide satisfactory results.

Phase 4: Each term of the ΔP matrix is compared to the parameter change limits data set (described below). If any of the limits are exceeded, a ΔPX matrix is formed where the smaller terms (from ΔP or limits) are used. ΔPX (ΔP if no limits were exceeded) is then used to actually change the structural parameter data sets of the finite-element model.

To test the results after the completion of phase 4, new mass and stiffness matrices must be formed, and the original process of computing mode shapes and frequencies is repeated. Normally, two or three iterations are sufficient to achieve the desired results if reasonable targets, unit parameters and change limits were selected. A complete iteration is the execution of phases 1-4 and testing of the results by calculating frequencies of the modified structure.

Prior to executing SM, EAL data sets must be established defining the targets, parameters, change limits, weighting and target tolerances. The EAL data set names for these inputs are given below followed by brief descriptions.

TVAL - Target (desired) eigenvalues (radian^2/sec^2) preceded by mode sequence numbers.

PARA - Each PARA data set is a group of changes (incremental element parameter or rigid mass) expressed as a fraction of the existing value. Each data set is then considered a unit parameter in SM computations.

SEE - Target tolerance matrix (S_{ee}).

SRR - Covariance or weighting matrix (S_{rr}).

DPLI - Parameter change limits (minimums and maximums) expressed as multiples (+ or -) of unit parameters defined in the PARA data sets.

AUGMENTATION TO THE SM PROCESSOR

In this application, it was necessary to develop three algorithms to augment the SM processor. These were implemented in the EAL Arithmetic Utility System (AUS) processor. The first was to add the centrifugal stiffening effect of the mass change (ΔM) matrices to the sensitivity matrix. The second was to revise the weighting matrix (S_{rr}) when the original values resulted in too many values of the change limits data set (DPLI) being violated by the ΔP matrix, thus causing structural changes which were inadequate in achieving targeted results. The third was to update the change limits after a complete iteration so that in the next iteration, the change limits data set (DPLI), which is based on a fraction of the current structural data set values, expresses the same engineering limits in terms of mass or stiffness originally intended.

To correct the sensitivity matrix, an additional system stiffness matrix must be formed for each nonzero ΔM matrix formed in phase 2. This matrix [ΔKC] reflects the centrifugal effect of the spinning ΔM and is formed using the AUS SPIN command to calculate a centrifugal force matrix and the elastic and centripetal contributions to stiffness. The Static Solution (SSOL) processor is used to calculate deflections due to the centrifugal force. The resulting stresses are embedded in the element state data sets by the GSF processor. Geometric stiffness changes are then calculted using the KG processor. The elastic, centripetal and geometric stiffness contributions are then summed to form [ΔKC] which is used to finalize the sensitivities as follows:

$$\Delta\lambda_{iTOTAL}=\lambda_i+Y_i\ [\Delta KC]\ Y_i^T \qquad (6)$$

where λ_i is given by equation 4.

The weighting matrix (S_{rr}) is revised when limits (DPLI) are violated by (ΔP). This is accomplished simply by multiplying each term of the (S_{rr}) matrix by the ratios of corresponding terms of the ΔPX and ΔP matrices. That is,

$$\left[S_{rr}\right]_{NEW} = \left[S_{rr}\right]_{OLD} \begin{bmatrix} \frac{\delta PX_1}{\delta P_1} & & \\ & \frac{\delta PX_2}{\delta P_2} & \\ & & \ddots \end{bmatrix} \quad (7)$$

which has the effect of reducing those S_{rr} terms corresponding to unit parameters which are tending to be changed beyond their allowable limits in phase 3. This process is repeated until the resulting ΔPX matrix resulting from phase 4 does not, in the judgement of the user, differ too greatly from the ΔP matrix. If this cannot be achieved, the targets may be unachievable based on the selected parameters and change limits.

The updating of the change limits (DPLI) for the subsequent iteration is achieved by the following process which updates each term of the DPLI matrix to retain the original engineering value.

$$[\Delta PLI]_{NEW} = \begin{bmatrix} \frac{L_{1OLD} - \Delta PX_1}{1 + f_1 \Delta PX_1} & \frac{L_{2OLD} - \Delta PX_2}{1 + f_2 \Delta PX_2} \cdots \\ \frac{U_{1OLD} - \Delta PX_1}{1 + f_1 \Delta PX_1} \cdots & \text{(Etc for each parameter)} \end{bmatrix} \quad (8)$$

where:

ΔPLI_{NEW} = New parameter change limits data set.

L_{1OLD}, L_{2OLD} = Old lower limits for parameters 1 and 2.

U_{1OLD} = Old upper limit for parameter 1.

ΔPX_1, ΔPX_2 = The final changes for parameters 1 and 2 produced in SM phases 3 and 4.

f_1, f_2 = The fraction used in defining a unit change for parameters 1 and 2 in the PARA data sets. For this process to work, the fraction must be uniform within a given PARA data set.

DEMONSTRATION

The finite element model (see figure 1) used in this report is based on a preliminary design of an advanced composite main rotor blade developed by Mark W. Nixon of the U.S. Army Aerostructures Research Group at Langley Research Center. Table I gives the mass and stiffness properties of the baseline model which resulted from a composite analysis program also developed by Mr. Nixon. Table II provides the constraints or parameter changes which cannot be exceeded during the tuning process. These constraints are based on the designer's estimate of what changes can be reasonably made without sacrificing the structural integrity or performance of the rotor blade.

Additional constraints on the problem were that bending stiffness, if modified, must be changed uniformly over large segments of the blade. The minimum allowable mass moment of inertia about the hub was 19000 lb-in-sec^2 for autorotation capability.

The objective of the tuning process was to minimize resonances caused when flexible mode frequencies were too close to integer multiples of the rotor speed up to eight per revolution (8P). The main rotor speed was 263 RPM (4.3833 HZ) and a criterion of at least .2P separation was used. Table III lists the unacceptable frequency ranges along with the natural frequencies of the original model and those of the modified model following the first and second tuning iterations.

The overall process which was conducted interactively is depicted in figure 2. Figure 3 contains the actual EAL runstreams used in the process. The runstreams in combination with this paper and the references should provide adequate guidelines for a new SM user.

Modes 1 and 2 are the flatwise and edgewise rigid body modes, and due to the physics of a spinning rotor blade, cannot be significantly altered. Modes 3 through 7 were therefore targeted for modification. Due to blade twist, modes 3, 5, 6 and 7 are combined flatwise/edgewise bending modes whereas mode 4 is predominantly torsion. It appeared reasonable to drive all of the bending mode frequencies to approximately .25P below the nearest P multiple while allowing the torsion mode to remain close to its original frequency. A study of the sensitivities indicated that to drive frequencies in opposite directions would have required unacceptably large changes in certain parameters. The selected target frequencies are listed in Tables III and IV. Table IV also lists all of the SM inputs.

Results of two complete iterations are summarized in Table III and figure 4. Figure 4 shows the ratio of calculated to target frequencies plotted against the iteration number ("0" iteration being the original model). A ratio of 1.0 would indicate complete convergence with the target value. The first iteration did not move all of the frequencies to acceptable ranges (Table III) but did move all of them towards the targets as shown in figure 4. The second iteration produced frequencies out of the unacceptable ranges and very close to the targeted frequencies. The total weight of the blade increased from 250.54 lb to 265.30 lb and the mass moment of inertia about the hub increased from 19007.1 to 19780.7 lb-in-sec^2. Table V summarizes the final structural properties of the modified rotor blade model.

CONCLUDING REMARKS

A sensitivity technique useful in minimizing vibrations associated with helicopter rotor blades has been demonstrated. This and similar techniques can be effective in achieving desired performance with minimum change to the basic structure. This is especially true for spinning structures because centrifugal stiffening complicates the intuitive process of changing mass and stiffness to tune natural frequencies.

An advantage of the process described in this report is that the modification capability is built into the structural analysis program. This eliminates the need for data transfer and development or use of external software.

The EAL System Modification processor has applications beyond that for which it was originally produced and documented, as demonstrated here for a spinning structure. As long as the equations for calculating appropriate sensitivities are known, structural modification can be computed to achieve any targeted response such as mode shapes, static deflections, stress and bending moments and loads due to dynamic loads.

REFERENCES

1. Whetstone, W. D.: EISI-EAL Engineering Analysis Language Reference Manual. Engineering Information Systems, Inc., San Jose, CA, July, 1983.

2. Whetstone, W. D.: EISI/SPAR Reference Manual, Volume 1 Program Execution. Engineering Information Systems, Inc., Saratoga, CA, January, 1979.

3. Collins, Jon D.; Hart, Gary C.; Hassleman, T. K.; and Kennedy, Bruce: Statistical Identification of Structures. AIAA Journal, Vol. 12, No. 2, Feb. 1974, pp. 185-190.

4. Robinson, James C.: Application of a Systematic Finite-Element Model Modification Technique to Dynamic Analysis of Structures, AIAA Paper No. 82-0730. Presented at 23rd AIAA/ASME/ASCE/AHS SDM Conference, May 10-12, 1982, New Orleans, LA.

TABLE I. - MODEL PROPERTIES

Joint No.	Joint Location (z,in)	Lumped Mass (lb)	Lumped inertia about z-axis (lb-in-sec^2)
1	0	0	.2415
2	16.1	0	.2700
3	18.0	0	.0585
4	20.0	0	.1230
5	26.2	0	.1860
6	32.4	0	.1860
7	38.6	0	.1905
8	45.1	0.32	.1551
9	51.5	0.645	.1161
10	58.0	0.645	.1161
11	64.4	1.93	.3474
12	96.6	3.22	.5796
13	128.8	3.22	.5796
14	161.0	3.22	.5796
15	193.2	2.415	.4347
16	209.3	1.61	.2898
17	225.4	1.61	.2898
18	241.5	1.61	.31395
19	257.6	1.61	.36225
20	273.7	1.61	.3864
21	289.8	1.125	.2700
22	296.2	0.645	.1548
23	302.7	0.645	.1548
24	309.1	0.32	.2068
25	315.6	0.35	.2580
26	322.0	0.35	.1280

Beam Section	Joints Spanned	Edgewise Stiffness EI_{11}* (LBF-in^2)	Flatwise Stiffness EI_{22}* (LBF-in^2)	Twist Angle LE Down (DEGR)	Distributed Weight (Lbs/in)	Cross Sectional Area In^2	Torsional Stiffness GJ (LBF-in^2)
1	1 - 2	900.0	900.0	26.0	2.29	44.44	100.00
2	2 - 3	.0001	.0001	25.34	2.29	44.44	87.50
3	3 - 4	.0001	.0001	25.26	2.29	44.44	87.50
4	4 - 5	580.0	360.0	25.17	2.20	44.44	75.50
5	5 - 6	580.0	360.0	24.92	2.20	44.44	75.50
6	6 - 7	580.0	360.0	24.66	2.20	44.44	75.50
7	7 - 8	580.0	298.0	24.41	2.60	44.44	60.00
8	8 - 9	1260.0	25.89	24.14	0.35	78.84	17.125
9	9 - 10	1260.0	25.89	23.87	0.35	78.84	17.125
10	10 - 11	1260.0	25.89	23.60	0.35	78.84	17.125
11	11 - 12	1260.0	25.89	23.34	0.35	78.84	17.125
12	12 - 13	1260.0	25.89	22.01	0.35	78.84	17.125
13	13 - 14	1260.0	25.89	20.68	0.35	78.84	17.125
14	14 - 15	1260.0	25.89	19.35	0.35	78.84	17.125
15	15 - 16	1260.0	25.89	18.02	0.35	78.84	17.125
16	16 - 17	1260.0	25.89	17.34	0.35	78.84	17.125
17	17 - 18	1260.0	25.89	16.69	0.35	78.84	17.125
18	18 - 19	1260.0	25.89	16.03	0.35	78.84	17.125
19	19 - 20	1260.0	25.89	15.67	0.35	78.84	17.125
20	20 - 21	1260.0	25.89	14.70	0.35	78.84	17.125
21	21 - 22	1260.0	25.89	14.03	0.35	78.84	17.125
22	22 - 23	1260.0	25.89	13.77	0.35	78.84	17.125
23	23 - 24	1260.0	25.89	13.51	0.35	78.84	17.125
24	24 - 25	580.0	24.0	13.23	0.90	44.44	60.00
25	25 - 26	580.0	24.0	12.77	3.3706	44.44	60.00

Total mass moment of inertia about x axis (hub): 19007.1 lb in sec^2. Total weight: 250.54 lb.

* Stiffness paramaters are with respect to a local reference frame which is rotated the amount of the twist angle from the global frame shown on figure 1.

TABLE II. - CHANGE LIMITS

ORIGINAL PAGE IS OF POOR QUALITY

Joint No.	Lumped Mass %	Section No.	Edgewise Stiffness %	Flatwise Stiffness
1	0	1	0	0
2	0	2	0	0
3	0	3	0	0
4	0	4	0	0
5	0	5	0	0
6	0	6	0	0
7	0	7	0	0
8	0	8	±20	±10
9	-50 +100	9	±20	±10
10	-50 +100	10	±20	±10
11	-50 +100	11	±20	±10 *
12	-50 +100	12	±20	±10
13	-50 +100	13	±20	±10
14	-50 +100	14	±20	±10
15	-50 +100	15	±20 *	±10
16	-50 +100	16	±20	±10
17	-50 +100	17	±20	±10
18	-50 +100	18	±20	±10
19	-50 +100	19	±20	±10 *
20	-50 +100	20	±20	±10
21	-50 +100	21	±20	±10
22	-50 +100	22	±20	±10
23	-50 +100	23	±20	±10
24	-50 +100	24	0	0
25	±100	25	0	0
26	±100			

Minimum Allowable Mass Moment of Inertia About X Axis: 19000 lb in sec^2
* Items in brackets must be changed uniformly as a group.

TABLE III. - MODEL NATURAL FREQUENCIES COMPARED TO UNACCEPTABLE RANGES

MULTIPLE, M	UNACCEPTABLE RANGES, HZ (MP±.2P)
1	3.507 - 5.260
2	7.890 - 9.643
3	12.273 - 14.027
4	16.657 - 18.410
5	21.040 - 22.793
6	25.423 - 27.177
7	29.807 - 31.560
8	34.190 - 35.943

WHERE P=263rpm OR 4.3833HZ

	FREQUENCIES			
MODE	TARGET	ORIGINAL	ITER 1	ITER 2
3	12.054	12.488*	12.067	12.030
4	16.0896	16.090	16.010	15.988
5	20.8208	22.460*	21.062*	20.928
6	23.0125	25.056	23.158	22.949
7	33.9708	36.368	34.326*	34.114

*IN UNACCEPTABLE RANGE

TABLE IV. - SYSTEM MODIFICATION INPUT DATA

TARGET FREQUENCIES (TVAL SM)[1]

SEQUENCE NO.	MODE NO.	EIGENVALUE
1	3	5736.3299 (12.05 HZ)
2	4	10219.9920 (16.09 HZ)
3	5	17114.1744 (20.82 HZ)
4	6	20906.7892 (23.01 HZ)
5	7	45558.7855 (33.97 HZ)

UNIT PARAMETERS (PARA SM n)

n	DATA TYPE	LINE[2] NO.	FRACTION	COLUMN[3] NO.
1	RIGID MASS	25	.1	1 TO 3
	RIGID MASS	26	.1	1 TO 3
2	RIGID MASS	9	.1	1 TO 3
	RIGID MASS	10	.1	1 TO 3
3	RIGID MASS	11	.1	1 TO 3
4	RIGID MASS	12	.1	1 TO 3
5	RIGID MASS	13	.1	1 TO 3
6	RIGID MASS	14	.1	1 TO 3
7	RIGID MASS	15	.1	1 TO 3
8	RIGID MASS	16	.1	1 TO 3
9	RIGID MASS	17	.1	1 TO 3
10	RIGID MASS	18	.1	1 TO 3
11	RIGID MASS	19	.1	1 TO 3
12	RIGID MASS	29	.1	1 TO 3
13	RIGID MASS	21	.1	1 TO 3
14	RIGID MASS	22	.1	1 TO 3
	RIGID MASS	23	.1	1 TO 3
15	RIGID MASS	24	.1	1 TO 3
16	EDGEWISE STIFFNESS (EI_{11})	8 TO 23	.1	4
17	FLATWISE STIFFNESS (EI_{22})	16 TO 23	.1	6
18	FLATWISE STIFFNESS (EI_{22})	8 TO 15	.1	6

TARGET TOLERANCES (SEE SM)

TARGET NO.	MODE NO.	TOLERANCE
1	3	.001
2	4	.1[4]
3	5	.001
4	6	.001
5	7	.001

INITIAL COVARIANCE[5] AND CHANGE LIMITS (SRR SM AND DPLI SM)

UNIT PARAMETER NO.	COVARIANCE	LIMITS[6]
1	1	-10,+10
2-15	1	-5,+10
16	1	-2,+2
17	1	-1,+1
18	1	-1,+1

NOTES: These data correspond to the input in the EAL runstream in figure 3b.

1. Names in parentheses are EAL data set names.
2. Line number of structural data set corresponds to joint for rigid masses and beam segment number for stiffnesses.
3. The unit parameter is a set of numbers computed from multiplying the fraction times the structural values in the indicated lines and columns.
4. A tolerance value of 0.1 rather than 0.001 indicates that it is less critical for the final frequency to be very close to the target value.
5. These values were modified in the iteration process.
6. Limits of -5 to plus 10 means that the structural parameter cannot be reduced by more than 5 x (FRACTION) x (EXISTING VALUE) nor increased more than 10 x (FRACTION) x (EXISTING VALUE).

TABLE V. - FINAL MODIFIED STRUCTURAL PROPERTIES

JOINT NO.	LUMPED MASS (lb)	BEAM SECTION	EDGEWISE STIFFNESS EI_{11} (LBF in^2)	FLATWISE STIFFNESS EI_{22} LBF in^2)
9	1.284	8	1046.96	23.311
10	1.284	9	1046.96	23.311
11	3.785	10	1046.96	23.311
12	6.440	11	1046.96	23.311
13	6.286	12	1046.96	23.311
14	6.401	13	1046.96	23.311
15	1.883	14	1046.96	23.311
16	1.025	15	1046.96	23.32
17	1.881	16	1046.96	23.32
18	3.190	17	1046.96	23.32
19	3.220	18	1046.96	23.32
20	3.201	19	1046.96	23.32
21	0.5713	20	1046.96	23.32
21	0.3225	21	1046.96	23.32
23	0.3225	22	1046.96	23.32
24	0.1815	23	1046.96	23.32
25	0.1275			
26	0.1275			

TOTAL MASS MOMENT OF INERTIA ABOUT X AXIS (HUB): 19780 lb-in sec^2.

TOTAL WEIGHT: 265.30 lb

NOTE: All other properties unchanged from Table I.

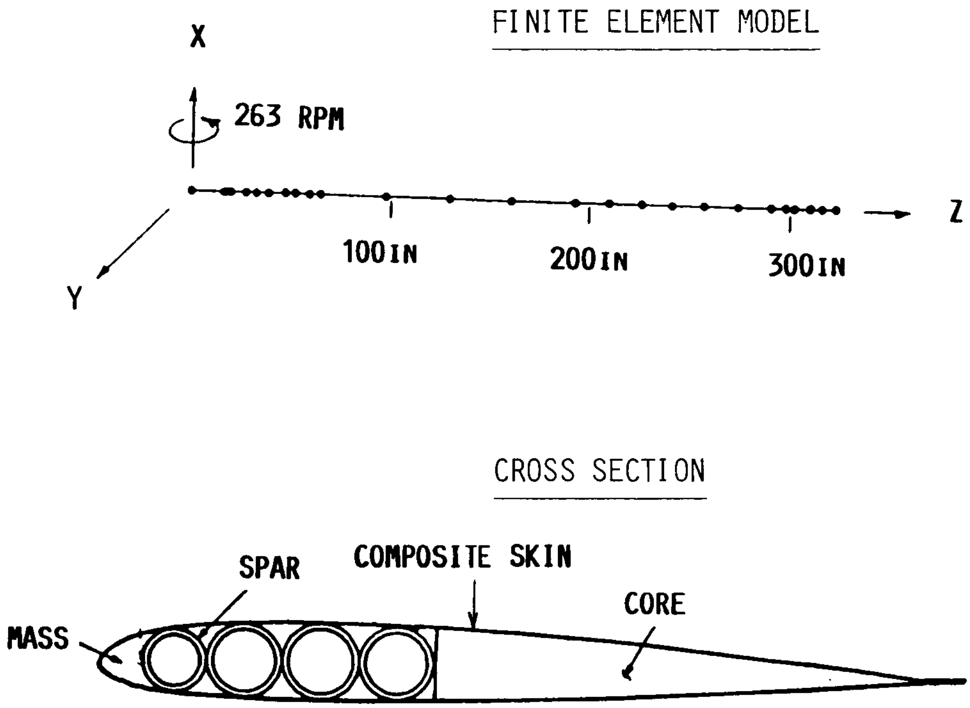

Figure 1.- Rotor blade model.

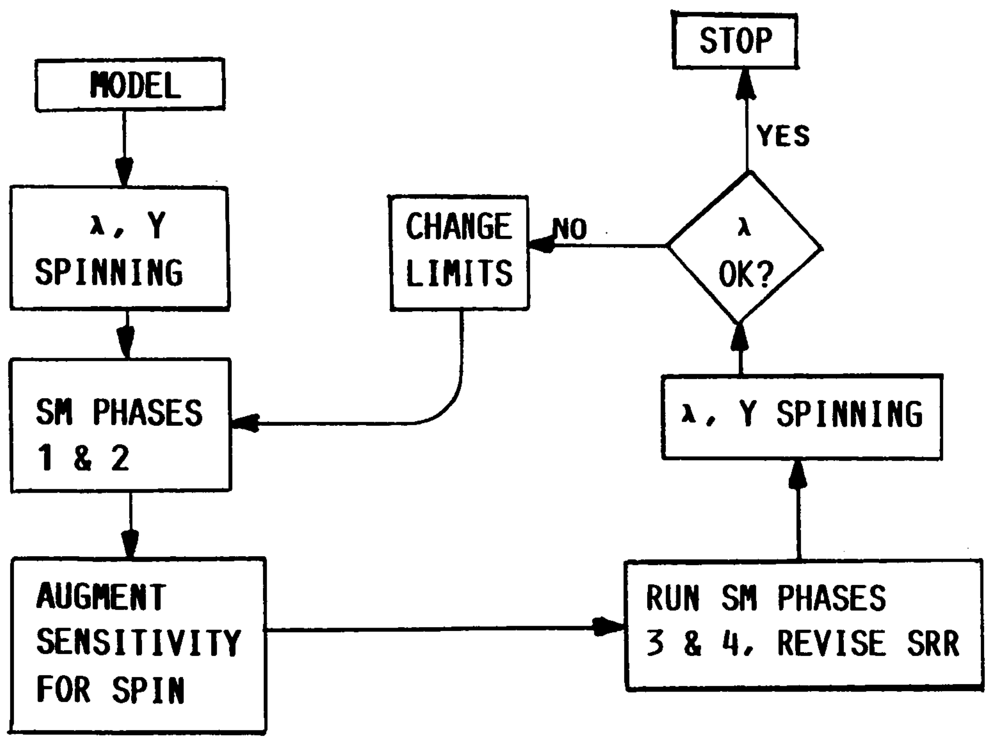

Figure 2.- Procedure for tuning frequencies of a spinning structure.

```
*XQT TAB
START 26
JLOC
1  0.  0.   00.
2  0.  0.  16.1
3  0.  0.  18.0
4  0.  0.  20.0
5  0.  0.  26.20
6  0.  0.  32.40
7  0.  0.  38.60
8  0.  0.  45.10
9  0.  0.  51.50
10 0.  0.   58.00
11 0.  0.   64.40
12 0.  0.   96.60
13 0.  0.   128.80
14 0.  0.   161.00
15 0.  0.   193.20
16 0.  0.   209.30
17 0.  0.   225.40
18 0.  0.   241.50
19 0.  0.   257.60
20 0.  0.   273.70
21 0.  0.   289.80
22 0.  0.   296.20
23 0.  0.   302.70
24 0.  0.   309.10
25 0.  0.   315.60
26 0.  0.   322.00
CON=1
ZERO 1 2 3 4 5 6:1
MATC
1  1.+6  .25  0. $G=.4+6
BA
$GIVN I1 A1 I2 A2 A F
GIVN 1  900.0 0. 900.0 0. 44.44 100.00
GIVN 2  .0001 0. .0001 0. 44.44 87.500
GIVN 3  .0001 0. .0001 0. 44.44 87.500
GIVN 4  580.0 0. 360.0 0. 44.44 75.000
GIVN 5  580.0 0. 360.0 0. 44.44 75.000
GIVN 6  580.0 0. 360.0 0. 44.44 75.000
GIVN 7  580.0 0. 298.0 0. 44.44 60.000
GIVN 8  1260. 0. 25.89 0. 78.84 17.125
GIVN 9  1260. 0. 25.89 0. 78.84 17.125
GIVN 10 1260. 0. 25.89 0. 78.84 17.125
GIVN 11 1260. 0. 25.89 0. 78.84 17.125
GIVN 12 1260. 0. 25.89 0. 78.84 17.125
GIVN 13 1260. 0. 25.89 0. 78.84 17.125
GIVN 14 1260. 0. 25.89 0. 78.84 17.125
GIVN 15 1260. 0. 25.89 0. 78.84 17.125
GIVN 16 1260. 0. 25.89 0. 78.84 17.125
GIVN 17 1260. 0. 25.89 0. 78.84 17.125
GIVN 18 1260. 0. 25.89 0. 78.84 17.125
GIVN 19 1260. 0. 25.89 0. 78.84 17.125
GIVN 20 1260. 0. 25.89 0. 78.84 17.125
GIVN 21 1260. 0. 25.89 0. 78.84 17.125
GIVN 22 1260. 0. 25.89 0. 78.84 17.125
GIVN 23 1260. 0. 25.89 0. 78.84 17.125
GIVN 24 580.0 0. 24.00 0. 44.44 60.000
GIVN 25 580.0 0. 24.00 0. 44.44 60.000
MREF
 1   1 1 -1 .89879
 2   1 1 -1 .90378
 3   1 1 -1 .90438
 4   1 1 -1 .90505
 5   1 1 -1 .90690
 6   1 1 -1 .90880
 7   1 1 -1 .91061
 8   1 1 -1 .91255
 9   1 1 -1 .91447
10   1 1 -1 .91636
11   1 1 -1 .91817
12   1 1 -1 .92712
13   1 1 -1 .93557
14   1 1 -1 .94351
15   1 1 -1 .95095
16   1 1 -1 .95455
17   1 1 -1 .95787
18   1 1 -1 .96112
19   1 1 -1 .96281
20   1 1 -1 .96727
21   1 1 -1 .97017
22   1 1 -1 .97126
23   1 1 -1 .97234
24   1 1 -1 .97346
25   1 1 -1 .97527
RMASS
CM .00259067 .00259067
   8    0.32
   9    0.645
   10   0.645
   11   1.93
   12   3.22
   13   3.22
   14   3.22
   15   2.415
   16   1.61
   17   1.61
   18   1.61
   19   1.61
   20   1.61
   21   1.125
   22   0.645
   23   0.645
   24   0.32
25   .35
26   .35
NSWS NON STRUCT WT DISTR
   1    2.29
   2    2.29
   3    2.29
   4    2.20
   5    2.20
   6    2.20
   7    2.60
   8    0.35
   9    0.35
   10   0.35
   11   0.35
   12   0.35
   13   0.35
   14   0.35
   15   0.35
   16   0.35
   17   0.35
   18   0.35
   19   0.35
   20   0.35
   21   0.35
   22   0.35
   23   0.35
   24   0.90
   25   3.3706
*XQT ELD
E21
GROUP 1' ROTOR BLADE
NSECT=1:NMAT=1:NNSW=1
INC NSECT=1
INC NNSW=1
INC NREF=1
1 2 1 25
*XQT E
RESET G=386.
*XQT AUS
TABL(NI=6 NJ=26):IN6
I=6: J=    1:  .241500E+00
I=6: J=    2:  .270000E+00
I=6: J=    3:  .585000E-01
I=6: J=    4:  .123000E+00
I=6: J=    5:  .186000E+00
I=6: J=    6:  .186000E+00
I=6: J=    7:  .190500E+00
I=6: J=    8:  .155100E+00
I=6: J=    9:  .116100E+00
I=6: J=   10:  .116100E+00
I=6: J=   11:  .347400E+00
I=6: J=   12:  .579600E+00
I=6: J=   13:  .579600E+00
I=6: J=   14:  .579600E+00
I=6: J=   15:  .434700E+00
I=6: J=   16:  .289800E+00
I=6: J=   17:  .289800E+00
I=6: J=   18:  .313950E+00
I=6: J=   19:  .362250E+00
I=6: J=   20:  .386400E+00
I=6: J=   21:  .270000E+00
I=6: J=   22:  .154800E+00
I=6: J=   23:  .154800E+00
I=6: J=   24:  .206800E+00
I=6: J=   25:  .258000E+00
I=6: J=   26:  .128000E+00
DEM2=SUM(DEM IN6)
M+RM = SUM(RMAS DEM2 )
*XQT DCU
DISA 1 EQNF
*XQT EKS
*XQT TAN
*XQT K
SPDP=2
*XQT AUS
SPIN: M+RM K 27.5413 0.0 0. 0. 0. 0. $ 27.5413 RAD/SEC
*XQT RSI
RESET K=KSPN
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
SPDP=2
*XQT AUS
KECG=SUM(KSPN KG)
*XQT RSI
RESET K=KECG
*XQT EIG
RESET INIT=11 NREQ=7 M=M+RM K=KECG OUTL=1
*XQT AUS
RIG=RIGID(1)
MR=PROD(M+RM RIG)
GM=XTYD(RIG MR)
GMWT=UNION(386. GM)
*XQT DCU
PRINT 1 GM
PRINT 1 GMWT
*XQT EXIT
```


Figure 3a.- EAL runstream for calculation of natural frequencies of the spinning structure.

ORIGINAL PAGE IS
OF POOR QUALITY

```
*XQT AUS
TABL(NI=2 NJ=5):TVAL SM
J=1:3. 5736.3299 $12.054 HZ
J=2:4. 10219.9920 $ 16.0896
J=3:5. 17114.1744 $ 20.8208
J=4:6. 20906.7892$ 23.0125
J=5:7. 45558.7855$ 33.9708Z
TABL(NI=5 NJ=2):PARA SM 1:J=1:18. 25. .1 1. 3.
                   J=2:18. 26. .1 1. 3.
TABL(NI=5 NJ=2):PARA SM 2:J=1:18. 09. .1 1. 3.
                          J=2:18. 10. .1 1. 3.
TABL(NI=5 NJ=1):PARA SM 3:J=1:18. 11. .1 1. 3.
TABL(NI=5 NJ=1):PARA SM 4:J=1:18. 12. .1 1. 3.
TABL(NI=5 NJ=1):PARA SM 5:J=1:18. 13. .1 1. 3.
TABL(NI=5 NJ=1):PARA SM 6:J=1:18. 14. .1 1. 3.
TABL(NI=5 NJ=1):PARA SM 7:J=1:18. 15. .1 1. 3.
TABL(NI=5 NJ=1):PARA SM 8:J=1:18. 16. .1 1. 3.
TABL(NI=5 NJ=1):PARA SM 09:J=1:18. 17. .1 1. 3.
TABL(NI=5 NJ=1):PARA SM 10:J=1:18. 18. .1 1. 3.
TABL(NI=5 NJ=1):PARA SM 11:J=1:18. 19. .1 1. 3.
TABL(NI=5 NJ=1):PARA SM 12:J=1:18. 20. .1 1. 3.
TABL(NI=5 NJ=1):PARA SM 13:J=1:18. 21. .1 1. 3.
TABL(NI=5 NJ=2):PARA SM 14:J=1:18. 22. .1 1. 3.
                           J=2:18. 23. .1 1. 3.
TABL(NI=5 NJ=1):PARA SM 15:J=1:18. 24. .1 1. 3.
TABL(NI=5 NJ=16):PARA SM 16:J=1 16
9. 8.  .1 4. 4.
9. 9.  .1 4. 4.
9. 10.  .1 4. 4.
9. 11.  .1 4. 4.
9. 12.  .1 4. 4.
9. 13.  .1 4. 4.
9. 14.   .1 4. 4.
9. 15.   .1 4. 4.
9. 16.  .1 4. 4.
9. 17.  .1 4. 4.
9. 18.   .1 4. 4.
9. 19.  .1 4. 4.
9. 20.  .1 4. 4.
9. 21.  .1 4. 4.
9. 22.  .1 4. 4.
9. 23.   .1 4. 4.
TABL(NI=5 NJ=8):PARA SM 17:J=1 8
9. 16.  .1 6. 6.
9. 17.  .1 6. 6.
9. 18.   .1 6. 6.
9. 19.  .1 6. 6.
9. 20.  .1 6. 6.
9. 21.  .1 6. 6.
9. 22.  .1 6. 6.
9. 23.   .1 6. 6.
TABL(NI=5 NJ=8):PARA SM 18:J=1 8
9. 8.  .1 6. 6.
9. 9.  .1 6. 6.
9. 10.  .1 6. 6.
9. 11.  .1 6. 6.
9. 12.  .1 6. 6.
9. 13.  .1 6. 6.
9. 14.   .1 6. 6.
9. 15.   .1 6. 6.
TABL(NI=1 NJ=5):SEE SM:J=1 5:0.001 .1 .001 .001 .001
TABL(NI=1 NJ=18):SRR SM:J=1 18:1.
TABL(NI=2 NJ=18):DPLI SM:J=1:-10. 10.
                     J=2 15:-5.0 10.
                  J=16:-2. 2.
                  J=17:-1. 1.
                  J=18:-1. 1.
*XQT EXIT
```

Figure 3b.- EAL runstream for input to the SM processor.

```
*XQT SM $DEVELOP DM FOR EACH PARAM
RESET NPARA=18 G=386. OUTL=1 NUUX=2
OPER 1 1 0 0
*XQT RSI
RESET K=K$LAST KECG
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 1
DEFINE DMT=DM SM 1 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK1 SPAR 36 1
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 2
DEFINE DMT=DM SM 2 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK2 SPAR 36 2
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 3
DEFINE DMT=DM SM 3 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK3 SPAR 36 3
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 4
DEFINE DMT=DM SM 4 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK4 SPAR 36 4
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 5
DEFINE DMT=DM SM 5 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK5 SPAR 36 5
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 6
DEFINE DMT=DM SM 6 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK6 SPAR 36 6
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 7
DEFINE DMT=DM SM 7 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK7 SPAR 36 7
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 8
DEFINE DMT=DM SM 8 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK8 SPAR 36 8
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 9
DEFINE DMT=DM SM 9 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK9 SPAR 36 9
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 10
DEFINE DMT=DM SM 10 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK10 SPAR 36 10
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 11
DEFINE DMT=DM SM 11 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK11 SPAR 36 11
```

Figure 3c.- EAL runstream for developing the sensitivity matrix.

```
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 12
DEFINE DMT=DM SM 12 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK12 SPAR 36 12
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 13
DEFINE DMT=DM SM 13 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK13 SPAR 36 13
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 14
DEFINE DMT=DM SM 14 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK14 SPAR 36 14
*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 15
DEFINE DMT=DM SM 15 1:DMA=UNION(DMT)
SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT DCU
CHAN 1 KG SPAR 36 0 DK15 SPAR 36 15
$*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 16
$DEFINE DMT=DM SM 16 1:DMA=UNION(DMT)
$SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
$*XQT SSOL
$RESET K=KSPN
$*XQT GSF
$RESET EMBED=1
$*XQT KG
$*XQT DCU
$CHAN 1 KG SPAR 36 0 DK16 SPAR 36 16
$*XQT AUS$ DEVEL INERTIAL STIFFNESS FOR PARAM 17
$DEFINE DMT=DM SM 17 1:DMA=UNION(DMT)
$SPIN:DMA K 27.5413 0. 0. 0. 0. 0.
$*XQT SSOL
$RESET K=KSPN
$*XQT GSF
$RESET EMBED=1
$*XQT KG
$*XQT DCU
$CHAN 1 KG SPAR 36 0 DK17 SPAR 36 17
*XQT AUS
DEFI VM5=VIBR MODE 1 1 3 7
YK1=PROD(DK1 VM5)
YK2=PROD(DK2 VM5)
YK3=PROD(DK3 VM5)
YK4=PROD(DK4 VM5)
YK5=PROD(DK5 VM5)
YK6=PROD(DK6 VM5)
YK7=PROD(DK7 VM5)
YK8=PROD(DK8 VM5)
YK9=PROD(DK9 VM5)
YK10=PROD(DK10 VM5)
YK11=PROD(DK11 VM5)
YK12=PROD(DK12 VM5)
YK13=PROD(DK13 VM5)
YK14=PROD(DK14 VM5)
YK15=PROD(DK15 VM5)
$YK16=PROD(DK16 VM5)
$YK17=PROD(DK17 VM5)
SE01=XTYD(VM5 YK1 )
SE02=XTYD(VM5 YK2 )
SE03=XTYD(VM5 YK3 )
SE04=XTYD(VM5 YK4 )
SE05=XTYD(VM5 YK5 )
SE06=XTYD(VM5 YK6 )
SE07=XTYD(VM5 YK7 )
SE08=XTYD(VM5 YK8 )
SE09=XTYD(VM5 YK9 )
SE10=XTYD(VM5 YK10)
SE11=XTYD(VM5 YK11)
SE12=XTYD(VM5 YK12)
SE13=XTYD(VM5 YK13)
SE14=XTYD(VM5 YK14)
SE15=XTYD(VM5 YK15)
$SE16=XTYD(VM5 YK16)
$SE17=XTYD(VM5 YK17)
TABL(NI=5 NJ=1):SE16
TABL(NI=5 NJ=1):SE17
TABL(NI=5 NJ=1):SE18
SENK=UNION(SE01 SE02 SE03 SE04 SE05 SE06 SE07>
SE08 SE09 SE10 SE11 SE12 SE13>
SE14 SE15 SE16 SE17 SE18)
SENB=SUM(SENK SENS)
*XQT DCU
CHAN 1 SENB AUS 1 1 SENS MATR 0 1
*XQT AUS
DEFI BA9=BA BTAB 2 9:B9=UNION(BA9)
DEFI RM18=RMAS BTAB 2 18:R18=UNION(RM18)
*XQT SM
RESET OUTL=2 NUVX=2 NUDP=2 NPARA=18 G=386.
OPER 0 0 1 1
*XQT AUS
BA BTAB 2 9=UNION(B9)
RMAS BTAB 2 18=UNION(R18)
*XQT EXIT
```

Figure 3c.- EAL runstream for developing the sensitivy matrix (concluded).


```
*XQT AUS
DEFI DPA=2 DP SM 1 1
DEFI DPXA=2 DPX REV 1 1
RA=RECIP(DPA)
RAX=PROD(RA DPXA)
RAXT=RTRAN(RAX)
RAX2=PROD(SRR RAXT)
SRR SM 1 1=UNION(RAX2)
DEFI BA9=BA BTAB 2 9:B9=UNION(BA9)
DEFI RM18=RMAS BTAB 2 18:R18=UNION(RM18)
*XQT SM
RESET OUTL=2 NUVX=2 NUDP=2 NPARA=18 G=386.
OPER 0 0 1 1
*XQT AUS
BA BTAB 2 9=UNION(B9)
RMAS BTAB 2 18=UNION(R18)
*XQT DCU
PRINT 1 SRR
PRINT 2 DP SM
PRINT 2 DPX
*XQT EXIT
```

Figure 3d.- EAL runstream for revision of the covariance (weighting) matrix.

```
*XQT SM
RESET OUTL=2 NUUX=2 NUDP=2 NPARA=18 G=386.
OPER 0 0 0 1
*XQT AUS
TABL(NI=1 NJ=18):SRR SM:J=1 18:1.
DEFI DPSM 2 DPX REV
DPST=RTRAN(DPSM)
TABL(NI=2 NJ=18):DPCH
TRAN(SOUR=DPST ILIM=1 JLIM=18 DSKIP=1 SBASE=0 DBASE=0)
TRAN(SOUR=DPST ILIM=1 JLIM=18 DSKIP=1 SBASE=0 DBASE=1)
DEFI DPO=DPLI
DPNE=SUM(DPO -1. DPCH)
DPFR=SUM(SRR .1 DPSM)
DPRE=RECIP(DPFR)
TABL(NI=2 NJ=18):RADP
TRAN(SOUR=DPRE ILIM=1 JLIM=18 DSKIP=1 SBASE=0 DBASE=0)
TRAN(SOUR=DPRE ILIM=1 JLIM=18 DSKIP=1 SBASE=0 DBASE=1)
DPN2=PROD(RADP DPNE)
DPLI SM=UNION(DPN2)
*XQT E
RESET G=386.
*XQT EKS
*XQT K
*XQT AUS
MNEW=SUM(DEM2 RMAS)
MDIF=SUM(MNEW -1. M+RM)
M+RM=SUM(DEM2 RMAS)
SPIN:M+RM K 27.5413 0. 0. 0. 0. 0.
*XQT RSI
RESET K=KSPN
*XQT SSOL
RESET K=KSPN
*XQT GSF
RESET EMBED=1
*XQT KG
*XQT AUS
KECG=SUM(KSPN KG)
*XQT RSI
RESET K=KECG
*XQT EIG
RESET INLIB=1 M=M+RM K=KECG
*XQT AUS
RIG=RIGID(1)
MR=PROD(M+RM RIG)
GM=XTYD(RIG MR)
GMWT=UNION(386. GM)
RMWT=UNION(386. RMAS)
*XQT DCU
PRINT 1 GM
PRINT 1 GMWT
PRINT 1 RMWT
PRINT 1 BA BTAB 2 9
PRINT 2 SENS:PRINT 2 DP
*XQT EXIT
```

Figure 3e.- EAL runstream for calculation of frequencies of the modified structure and recomputation of the change limits for the next iteration.

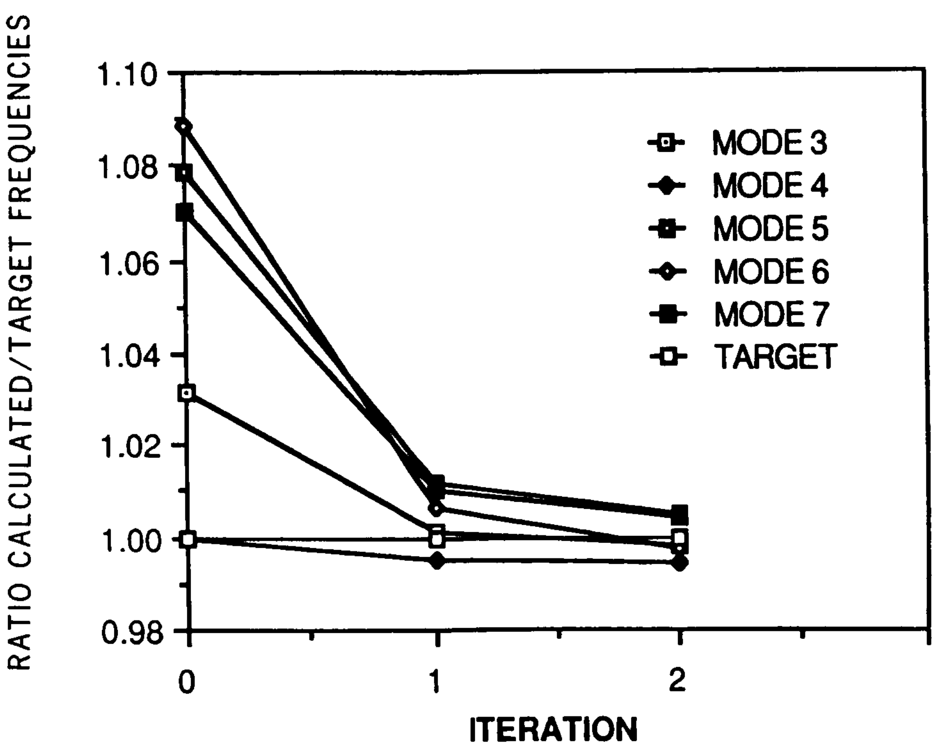

Figure 4.- Results of modification.