N87-18873

APPLICATION OF DESIGN SENSITIVITY ANALYSIS FOR GREATER IMPROVEMENT ON MACHINE STRUCTURAL DYNAMICS

Masataka YOSHIMURA
Department of Precision Engineering
School of Mechanical Engineering
Kyoto University
Kyoto, JAPAN

SUMMARY

This paper presents methodologies for greatly improving machine structural dynamics by using design sensitivity analyses and evaluative parameters. First, design sensitivity coefficients and evaluative parameters of structural dynamics are described. Next, the relations between the design sensitivity coefficients and the evaluative parameters are clarified. Then, design improvement procedures of structural dynamics are proposed for the following three cases: (1) addition of elastic structural members, (2) addition of mass elements, and (3) substantial changes of joint design variables. Cases (1) and (2) correspond to the changes of the initial framework or configuration, and (3) corresponds to the alteration of poor initial design variables. Finally, numerical examples are given for demonstrating the availability of the methods proposed in this paper.

1. INTRODUCTION

In usual design optimization of machine structures, a framework pattern for the complete structure is definite and initial design variables which are usually tentatively given are modified so that the objective function is improved. In such design optimization, design sensitivity coefficients of evaluative parameters can be used for finding the most preferable design change directions. However, improvement of the product performance or characteristics, which is attained under the condition of a constant framework and using poor initial design variables, often is not satisfactory. Furthermore, machine structural dynamics depend on characteristics at many natural modes, and on damping characteristics which are yet unclear. Hence, the relationships between the machine structural dynamics and design variables are very complicated. Application of design sensitivity analyses to optimization of structural dynamics is not simple.

This paper proposes design decision making methods of structural dynamics which intend to greatly increase product performance of machine structures. First, evaluative parameters of structural dynamics are listed, and design sensitivity coefficients of the parameters are derived. Next, the relations between the design sensitivity coefficients and the parameters of displacement, internal vibratory force, and energy distributions are analyzed. Based on the analyses, priorities among the evaluative parameters are clarified. Then, using the design sensitivity analyses and the relations between parameters, design improvement procedures of structural dynamics are constructed for each of the three cases: (1) addition of elastic structural members, (2) addition of mass elements, and (3) substantial changes of joint design variables. Addition of elastic structural members and mass elements on the original design is utilized for decreasing the static compliance and for balancing the frequency response over the frequency range, respectively. Substantial changes of joint design variables are made for balancing the frequency response and for increasing damping ratios. Finally, the effectiveness of the procedures is demonstrated by applying them to a structural model.

2. EVALUATIVE PARAMETERS FOR STRUCTURAL DYNAMICS AND INFORMATION FOR DESIGN CHANGES

A machine structure has point E where vibrational (excitational) force or static force generates, and point G where vibrational or static displacement produced by that force causes reduction of the machine performance. The transfer function of a vibrational system defining the relation between the input force at point E and the displacement output at point G is expressed as the "frequency response".

Fig. 1 shows an example of the receptance frequency response R(=D/F) which is obtained from the displacement D at point G caused by the harmonic force F at point E.

According to the requirements for the product performance, the following changes of the characteristics are required:

(1) decrease the static compliance f_s,
(2) increase/decrease a natural frequency ω_n,
(3) increase the damping ratio ζ_n at a natural mode,
(4) decrease the receptance value R_n at a natural mode.

In the case of machine tools, the maximum receptance value $R_{n \cdot max}$ at the cutting point is evaluated for increasing the stability against regenerative chatter (refs. 1 and 2), and natural frequencies are evaluated for diminishing the forced vibrational troubles. Even in other machines' cases concerned with transient dynamic response, some treatment among (1) through (4) can be applied. Hence, the "frequency response" is the most fundamental characteristic of structural dynamics.

In the following nomenclature, "direct" means that the point and direction of the exciting (or static) force are the same as the pick-up point and direction of displacement, while "cross" means that those points and directions are not the same.

2.1 Evaluative Parameters of Frequency Response

The equation of motion in a linear vibrational system having multiple-degrees of freedom is expressed by the following equation:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + i[H]\{X\} + [K]\{X\} = \{F\} \tag{1}$$

where [M], [K], [C], and [H] are the mass, stiffness, viscous damping, and hysteretic damping matrices, respectively; where {X} and {F} are the column vectors representing the displacements and the forces; and where i designates the imaginary unit.

The angular natural frequency at an arbitrary nth natural mode is denoted as ω_n. For easy expansion of equations, a displacement eigenvector $\{X_n\}$ at each of the natural modes is normalized as follows:

$$\{X_n\}^T [M] \{X_n\} = 1 \quad (\text{then, } \{X_n\}^T [K] \{X_n\} = \omega_n^2)$$

The equation showing the relation between {X} and {F} at a given angular frequency ω is expressed using receptance matrix $[R(\omega)]$ as follows:

$$\{X\} = [R(\omega)] \{F\} \tag{2}$$

The receptance matrix under the assumption of the proportional damping vibrational system is obtained using the orthogonality relations of displacement eigenvectors:

$$[R(\omega)] = \sum_{m=1}^{\infty} \left[\frac{[f_m]}{1 - (\frac{\omega}{\omega_m})^2 + 2i \frac{\omega}{\omega_m} \zeta_m} \right] \tag{3}$$

where $[f_m]$, ω_m, and ζ_m are respectively the modal flexibility matrix, angular natural frequency, and damping ratio at the mth natural mode. The modal flexibility matrix (ref. 3) is obtained using the displacement eigenvector $\{X_m\}$ and stiffness matrix [K] as follows:

$$[f_m] = \frac{\{X_m\}\{X_m\}^T}{\{X_m\}^T[K]\{X_m\}} \tag{4}$$

Damping ratio ζ_m at the mth natural mode is obtained for a viscous damping vibrational system as follows:

$$\zeta_m = \frac{\omega_m \{X_m\}^T[C]\{X_m\}}{2\{X_m\}^T[K]\{X_m\}} \tag{5}$$

When no other natural frequencies having large modal flexibility exist near the nth natural frequency, the receptance value at the nth angular natural frequency, ω_n, is approximated from equation (3) using the following equation:

$$[R(\omega_n)] \doteqdot -\frac{i[f_n]}{2\zeta_n} + \sum_{\substack{m=1\\m\neq n}}^{\infty} \left[\frac{[f_m]}{1-(\frac{\omega_n}{\omega_m})^2} \right] \tag{6}$$

Since $[R(o)]$ is equivalent to static compliance $[f_s]$ by substituting zero for ω in eq. (3), the following relation is established between the modal flexibility matrix, $[f_m]$, and the static compliance matrix, $[f_s]$.

$$[f_s] = \sum_{m=1}^{\infty} [f_m] \tag{7}$$

By selecting diagonal elements at the j-row and j-column of matrices $[f_s]$ and $[f_m]$ in eq. (7), the following relation is obtained (ref. 3).

$$f_{s(j,j)} = \sum_{m=1}^{\infty} f_{m(j,j)} \tag{8}$$

Since the values of $f_{m(j,j)}$ are always positive, the relation in eq. (8) means that the summation of $f_{m(j,j)}$ at all natural modes is equal to the value of static compliance $f_{s(j,j)}$.

2.2 Design Sensitivity Coefficients of Evaluative Parameters (ref. 3)

The design variables are denoted by a vector $\mathbf{b} = \{b_1, b_2, \cdots, b_N\}^T$, where N is the number of design variables. Design sensitivity coefficients, $\partial\omega_n/\partial\mathbf{b}$, and $\partial\{X_n\}/\partial\mathbf{b}$ of an angular natural frequency, ω_n, and a displacement eigenvector, $\{X_n\}$, with respect to a design variable vector, $\mathbf{b}$, are obtained by applying the orthogonality relations of displacement eigenvectors to the eigenvalue equation of motion partially differentiated with respect to $\mathbf{b}$, as follows:

$$\frac{\partial\omega_n}{\partial\mathbf{b}} = \frac{1}{2\omega_n}\frac{\partial\omega_n^2}{\partial\mathbf{b}} = \frac{1}{2\omega_n}\{X_n\}^T\left[\frac{\partial[K]}{\partial\mathbf{b}} - \omega_n^2\frac{\partial[M]}{\partial\mathbf{b}}\right]\{X_n\} \tag{9}$$

$$\frac{\partial\{X_n\}}{\partial\mathbf{b}} = -\frac{1}{2}\{X_n\}^T\frac{\partial[M]}{\partial\mathbf{b}}\{X_n\}\{X_n\} + \sum_{\substack{m=1\\m\neq n}}^{\infty}\left\{\frac{\{X_m\}^T\left[\frac{\partial[K]}{\partial\mathbf{b}} - \omega_n^2\frac{\partial[M]}{\partial\mathbf{b}}\right]\{X_n\}\{X_m\}}{\omega_n^2 - \omega_m^2}\right\} \tag{10}$$

Using equations (9) and (10), design sensitivity coefficients of modal flexibilities are derived from eq. (4):

$$\frac{\partial[f_n]}{\partial\mathbf{b}} = -\frac{1}{\omega_n^4}\{X_n\}\{X_n\}^T\{X_n\}^T\frac{\partial[K]}{\partial\mathbf{b}}\{X_n\}$$

$$+\frac{1}{\omega_n^2}\sum_{\substack{m=1\\m\neq n}}^{\infty}\left[\frac{\{X_m\}^T\left[\frac{\partial[K]}{\partial\mathbf{b}} - \omega_n^2\frac{\partial[M]}{\partial\mathbf{b}}\right]\{X_n\}\left[\{X_m\}\{X_n\}^T + \{X_n\}\{X_m\}^T\right]}{\omega_n^2 - \omega_m^2}\right] \tag{11}$$

Similarly, design sensitivity coefficients of damping ratios ζ_n for a viscous damping vibrational system are derived from eq. (5) as follows:

$$\frac{\partial\zeta_n}{\partial\mathbf{b}} = \frac{1}{\{X_n\}^T[K]\{X_n\}}\left\{\frac{1}{4\omega_n}\frac{\partial\omega_n^2}{\partial\mathbf{b}}\{X_n\}^T[C]\{X_n\}\right.$$

$$+\omega_n \frac{\partial \{X_n\}^T}{\partial b} [C] \{X_n\} + \frac{1}{2} \omega_n \{X_n\}^T \frac{\partial [C]}{\partial b} \{X_n\}$$

$$\left. - 2\zeta_n \frac{\partial \{X_n\}^T}{\partial b} [K] \{X_n\} - \zeta_n \{X_n\}^T \frac{\partial [K]}{\partial b} \{X_n\} \right\} \tag{12}$$

Design sensitivity coefficients with respect to fundamental structural elements of spring elements, concentrated mass elements, and damping elements are obtained from eqs. (11) and (12).

(i) Spring element

Spring stiffness k of a spring element at point J (for example, a joint) in the machine structural model is considered as a design variable. The design sensitivity coefficient of the direct modal flexibility $f_{n(C,\,C)}$ at the nth natural mode at point C is:

$$\frac{\partial f_{n(C,C)}}{\partial k} = - f^2_{n(C,J)} + 2 \sum_{\substack{m=1 \\ m \neq n}}^{\infty} \left(\frac{f_{m(C,J)} \cdot f_{n(C,J)}}{\frac{\omega_n^2}{\omega_m^2} - 1} \right) \tag{13}$$

where $f_{n(C,\,J)}$ and $f_{m(C,\,J)}$ are the cross modal flexibilities at the nth and the mth natural modes, respectively. The design sensitivity coefficient of the damping ratio at the nth natural mode is:

$$\frac{\partial \zeta_n}{\partial k} = - \frac{\zeta_n}{2} f_{n(J,J)} \tag{14}$$

where $f_{n(J,\,J)}$ is the direct modal flexibility at point J.

(ii) Concentrated mass element

The mass, M_I, of a concentrated mass element at point I in a machine structural model is considered as a design variable. The design sensitivity coefficient of the direct modal flexibility $f_{n(C,\,C)}$ at point C is:

$$\frac{\partial f_{n(C,C)}}{\partial M_I} = 2\omega_n^2 f_{n(C,I)} \sum_{\substack{m=1 \\ m \neq n}}^{\infty} \left(\frac{f_{m(C,I)}}{1 - \frac{\omega_n^2}{\omega_m^2}} \right) \tag{15}$$

(iii) Damping element

In a viscous damping vibrational system, the design sensitivity coefficient of the damping ratio with respect to viscous damping coefficient c of a damping element at point J is:

$$\frac{\partial \zeta_n}{\partial c} = \frac{\omega_n}{2} f_{n(J,J)} \tag{16}$$

2. 3 Information of Energy Distributions

2. 3. 1 Relationships between changes of natural frequencies and energy distribution rates (ref. 4)

It is assumed that the stiffness matrices at subsystems s and r of the machine structure are $[K_s]$ and $[K_r]$, the mass matrices at subsystems s and r are $[M_s]$ and $[M_r]$, and the displacement eigenvectors corresponding to subsystems s and r are $\{X_n\}_s$ and $\{X_n\}_r$. Now, the values of the stiffness matrix $[K_s]$ at subsystem s increase (or decrease) α times to become $[K'_s]$, and the values of the mass matrix $[M_r]$ at subsystem r increase (or decrease) β times to become $[M'_r]$ as shown in eqs. (17) and (18):

$$[K'_s] = [K_s] + \alpha[K_s] \tag{17}$$

$$[M_r'] = [M_r] + \beta[M_r] \tag{18}$$

α and β being small values. The variable component $d\omega_n^2$ of the square of an angular natural frequency ω_n is obtained as follows:

$$d\omega_n^2 = \frac{\alpha \{X_n\}_s^T [K_s] \{X_n\}_s - \beta\omega_n^2 \{X_n\}_r^T [M_r] \{X_n\}_r}{\{X_n\}^T [M] \{X_n\}} \tag{19}$$

The following equation is obtained multiplying both sides of eq. (19) by $1/\omega_n^2$:

$$\frac{d\omega_n^2}{\omega_n^2} = \frac{\alpha \{X_n\}_s^T [K_s] \{X_n\}_s - \beta\omega_n^2 \{X_n\}_r^T [M_r] \{X_n\}_r}{\omega_n^2 \{X_n\}^T [M] \{X_n\}} \tag{20}$$

In eq. (20), $\{X_n\}_s^T[K_s]\{X_n\}_s$ and $\omega_n^2\{X_n\}_r^T[M_r]\{X_n\}_r$ are respectively twice the potential energy (strain energy) at subsystem s and the kinetic energy at subsystem r in the initial structural design. Hence, those have positive values. $\omega_n^2\{X_n\}^T[M]\{X_n\}$ is twice the maximum kinetic energy in the complete structure which also has a positive value.

The following rule is established from eq. (20): when the design change is conducted so that the rigidity is increased (that is, α has a positive value) / decreased (that is, α has a negative value) at the member or the element which has the larger potential energy distribution, or the mass is decreased (that is, β has a negative value) / increased (that is, β has a positive value) at the member or the element which has the larger kinetic energy distribution, the natural frequency increases / decreases more effectively.

2. 3. 2 Relationships between design sensitivity coefficients and energy distribution rates

The maximum potential energies stored in the whole machine structure and in the spring element with spring stiffness k at point J at the nth natural mode are denoted as V_{Tn} and V_{Jn}, respectively. The design sensitivity coefficients of the natural frequency ω_n and the damping ratio ζ_n at the natural mode with respect to spring stiffness k have the relation with the potential energy distribution rate, V_{Jn}/V_{Tn}, as shown in the following equations.

$$\frac{\partial\omega_n^2}{\partial k} = \frac{\omega_n^2}{k}\left(\frac{V_{Jn}}{V_{Tn}}\right) \tag{21}$$

$$\frac{\partial\zeta_n}{\partial k} = -\frac{\zeta_n}{2k}\left(\frac{V_{Jn}}{V_{Tn}}\right) \tag{22}$$

A similar relation for the modal flexibility at point C is derived when the modal flexibility at a natural mode is far greater than that at any other natural mode (ref. 3):

$$\frac{\partial f_{n(C,C)}}{\partial k} \cong -\frac{f_{n(C,C)}}{k}\left(\frac{V_{Jn}}{V_{Tn}}\right) \tag{23}$$

In this case, the modal flexibility at the natural mode can be decreased by increasing the spring stiffness of the spring element having the high potential energy distribution.

2. 4 Information of Static Displacement and Internal Vibratory Force

(i) Static displacement

It is assumed that a machine structural model is installed in a hypothetical system T which is filled with a substance having a sufficiently small rigidity, as shown in Fig. 2. Now, two points, P_1 and P_2, are chosen on the machine structural model, and between the two points a thin circular tube (or a thin square bar) is conceived. Then, it can be considered that a circular tube (or a square bar) member exists between points P_1 and P_2.

When the evaluative parameter is the direct static compliance $f_{s(C,C)}$ at point C, the design sensitivity coefficient of $f_{s(C,C)}$ with respect to the spring stiffness k_P in the axial direction of the member between points P_1 and P_2 is obtained as follows:

$$\frac{\partial f_{s(C,C)}}{\partial k_P} = -\left(\frac{X_C \cdot (X_{P_1} - X_{P_2})}{2V_s}\right)^2 = -f^2_{s(C,P)} \tag{24}$$

where X_C is the relative displacement between points A and B caused by the static force at point C, X_{P_1} and X_{P_2} are the displacements at points P_1 and P_2 in the axial direction of the member between points P_1 and P_2, and V_s is the total strain energy of the structural model at the displacement state; $f_{s(C,P)}$ is the cross static compliance between points C and P. As understood from eq. (24), the design sensitivity of the direct static compliance $f_{s(C,C)}$ with respect to a hypothetical spring between two points having the largest relative displacement is greatest. Hence, the displacement distribution on the machine structural model can be used as the information for adding an elastic member when the static compliance is required to be decreased.

(ii) Internal vibratory force

When the internal vibratory force at a structural member or a joint is small, it can be understood that the member or the joint has a small effect on the vibrational characteristics. If the force is negligibly small, removal of the member or the joint may have negligible influence on the dynamic characteristics.

On the other hand, when the internal vibratory force F_J is great at a joint, the following two cases exist:

(1) the potential energy distribution rate at the joint is great,

(2) the potential energy distribution rate at the joint is small.

In case (1), the joint has a great effect on the vibrational characteristics, and even small changes of the joint design variables bring about a great change of the characteristics. In case (2), such small changes of the joint design variables cause little change of the vibrational characteristics. Great changes of the joint design variables are necessary for a great change of the characteristics.

The relation on the frequency domain between the excitational input force F_E and the internal vibratory force F_J at a joint similar to the relation between the excitational input force and the displacement shown in eq. (3), is obtained as follows:

$$\frac{F_J}{F_E}(\omega) = \sum_{m=1}^{\infty} \frac{h_{EJm}}{1 - \left(\frac{\omega}{\omega_m}\right)^2 + 2i\frac{\omega}{\omega_m}\zeta_m} \tag{25}$$

where h_{EJm} is the modal internal force coefficient at the mth natural mode. The value of h_{EJm} is subject to very little change due to variations in damping. Hence, values of h_{EJm} can be used for relatively evaluating magnitudes of internal vibratory forces.

Spring stiffness k_J of a spring element at a joint of a machine structural model generally has the relation with the angular natural frequency ω_n at the nth natural mode and the modal internal force coefficient h_{EJn} of the spring element as shown in Fig. 3. Case (1) corresponds to the design at (hypothetical) point Q within the region S, while case (2) corresponds to the design at (hypothetical) point H within the region T.

It can be understood from Fig. 3 that a joint spring element having a great internal vibratory force but a small potential energy distribution rate has a great latent effect on the vibrational characteristics, although the value of the design sensitivity coefficient at that design point is small.

2. 5 Considerations on the Evaluative Parameters and Information for Design Improvement

The following concluding remarks are obtained for the evaluative parameters:

(1) As can be understood from eq. (8), the static compliance f_s has a direct influence on the modal flexibility values at natural modes.

(2) As can be understood from eq. (11), the design sensitivity coefficient of modal flexibility is influenced by the characteristics at many other natural modes. This fact means that the modal flexibility f_n is determined by the systematic balance over the complete structure. Hence, the modal flexibility needs systematic analyses.

(3) As can be understood from eq. (14), the design sensitivity coefficient of the damping ratio at a natural mode does not include the influence of characteristics at the other natural modes. In an approximate sense, the damping ratio at a natural

mode can be changed by adjusting only the characteristics at the natural mode.

(4) As can be understood from eq. (9), the design sensitivity coefficient of natural frequency ω_n does not include the influence of characteristics at the other natural modes.

Higher priority of evaluation must be given to the evaluative parameters which need systematic analyses. If evaluative parameters which can be determined by the local effect are fixed before the systematic evaluation, a great improvement of the product performance cannot be expected. From the above consideration, priority for evaluation of the frequency response should be given in the order of (1) f_s, (2) f_n, and (3) ζ_n and ω_n.

Features of other information for design improvement such as energy distributions, static deformation distributions, and internal vibratory forces are as follows:

(a) In design changes based on energy distributions, it is not necessary to define a specific design variables. Parts of the structure which need increased rigidity or decreased weight can be macroscopically grasped. In usual design practice, first of all, it is required to know where the weak points (regions) in the structure are. In this case, evaluations based on energy distributions (refs. 1 and 2) are effective.

(b) The static displacement distribution can be used as the information for adding elastic structural members.

(c) The magnitude of the internal vibratory force at a natural mode indicates the degree of influence of the structural member or the joint on the vibrational characteristics. That can be used as a sort of sensitivity information.

3. STRATEGIES FOR GREATER IMPROVEMENT OF STRUCTURAL DYNAMICS

In usual design problems, many characteristic and evaluative factors often interact mutually. The relationships between design variables and evaluative factors are very complicated. When the optimum design is required for such design problems, many local optimum solutions often exist in the feasible design space. Therefore, it is very difficult to obtain a design solution which brings about great improvement of the product performance. Table 1 shows the procedures which have been developed for solving those problems. Based on the clarification of competitive and cooperative relationships between characteristics, the procedures are divided into three phases as shown in Table 1 (ref. 5).

In the following, some technical strategies for greater improvement of structural dynamics will be described. Addition of elastic members in Section 3. 1 can be used in the procedures of phases 1 and 2 in Table 1; addition of mass elements in Section 3. 2 can be used in the procedures of phase 2 in Table 1; and substantial changes of joint design variables in Section 3. 3 can be used in the procedures of phase 3 in Table 1.

The improvement or modification of receptance values is most difficult in structural dynamics. Hence, characteristics related with the receptance frequency response will be mainly discussed.

3. 1 Addition of Elastic Structural Members

In the procedures shown in Table 1, first of all, the static compliance is minimized. When sufficient reduction of the static compliance cannot be attained by changes of design variables (such as cross-sectional dimensions of the structural members), addition of new structural members are useful only if change in the framework is possible.

The procedures for decreasing the static compliance $f_{s(C,C)}$ by addition of an elastic structural member are as follows:

Step 1. Detect points P_1 and P_2 having a negative value of the right side part of eq. (24) of which the absolute value is maximum in the feasible region of the machine structural model.

Step 2. Define a thin member region between points P_1 and P_2, and equalize the Young modulus of the member element with that of the other structural members.

Step 3. Repeat the search for optimum cross-sectional design variables until the objective function converges. At each iteration of the search, the locations of points P_1 and P_2 are slightly moved so that the right side part of eq. (24) has the greatest negative value.

When the objective is to minimize the direct modal flexibility $f_{n(C,C)}$ at the nth natural mode at point C, eq. (13) can be used as the design sensitivity coefficient with respect to the spring stiffness k_P in the axial direction of the hypothetical member between points P_1 and P_2. The forementioned procedures for the static compliance can also be applied for minimizing the modal flexibility by transforming eq. (24) into eq. (13).

3. 2 Addition of Mass Elements

The design sensitivity coefficient of the modal flexibility $f_{n(C,\,C)}$ at the nth natural mode at point C of a machine structural system with respect to the small hypothetical mass M_I at point I(such as shown in Fig. 2) is given in eq. (15).

The procedures for reducing the modal flexibility $f_{n(C,\,C)}$ by means of the addition of a mass element are as follows:

Step 1. In order to detect a point where a mass element should be added, search for a point I having a regative value on the right side part of eq. (15) of which the absolute value is maximum in the feasible region of the machine structural system, and add a small mass element at point I.

Step 2. If the modal flexibility $f_{n(C,\,C)}$ is sufficiently small or has reached the convergence point, the added mass element is adopted for the final design. Otherwise, go to Step 3.

Step 3. Modify the point I so that the right side part of eq. (15) has a negative maximum absolute value, and increase the magnitude of the mass at point I and return to Step 2.

3.3 Substantial Changes of Joint Design Variables

In a usual searching process for an optimum design solution, initial design variables are slightly changed so that the objective function is most effectively minimized (or maximized). Hence, if an initial design variable has a low sensitivity for changing the objective function and is widely different from the optimum solution, it takes a very long time to reach the optimum solution, and the design variable often converges into some local optimum point without reaching the optimum solution.

From the standpoint of static rigidity (that is, reciprocal of the static compliance), the rigidity of a joint is required to be as great as possible. However, from the standpoint of dynamic characteristics, the rigidity of a point is required to have a specific value or a value within a specific region in the following cases:

(i) when a change in the ratio f_n/f_s of the modal flexibility f_n to the static compliance f_s is required,

(ii) when an increase of the damping ratio at the natural mode is required.

When the spring stiffness of a joint in the initial design of a structural model has the value at point H as shown in Fig. 3, the design sensitivity coefficient at the point is very small, and potential energy is scarcely stored at the spring. Hence, the spring stiffness may not be changed largely to the region S, and a sufficient change of the vibrational characteristics cannot be generated. In order to attain objective (i) or (ii), the spring stiffness at the joint should be reduced to the region S. If the internal vibratory force at the spring element is large at point H, there is a high possibility to realize objective (i) or (ii) effectively with this spring element.

The procedures for realizing objective (i) or (ii) are as follows:

Step 1. Detect a spring element having the great internal vibratory force but a small potential energy distribution rate among spring elements of all the joints (the spring stiffness k_J at the spring element has a value within the region T as shown in Fig. 3).

Step 2. Decrease the spring stiffness k_J to a value within the region S as shown in Fig. 3.

Step 3. Start the search for the optimum value after having reduced the spring stiffness k_J to get a new initial value.

4. NUMERICAL EXAMPLE

The procedures described in Section 3 are demonstrated on the machine structural model shown in Fig. 4. Fig. 5 shows the simulation model for structural analysis. At the initial design shown in Fig. 6(a), the spring stiffness values at joints J_1, J_2, J_3 and J_4(see Fig. 5) were large enough for avoiding degradation of the static rigidity. The relative receptance frequency response between points A and B in Y-direction for this initial model is shown in Fig. 7(a). The receptance value at the 1st natural mode is very large, and the ratio f_1/f_s of the modal flexibility f_1 at the 1st natural mode to the static compliance f_s is 0.96. The three kinds of procedures proposed in Section 3 were successively added on the same structural model.

4.1 Addition of an Elastic Member

The objective in this step is to decrease the static compliance f_s by adding a circular tube within the shaded region in Fig. 5. Fig. 6(b) shows the final design obtained according to the procedures described in Section 3.1. The receptance frequency response for the design is shown in Fig. 7(b). The static compliance f_s decreases from 2.36×10^{-6} m/N at the initial design to 1.33×10^{-6} m/N. The incremental percentage of the total weight of the structural model by addition of the elastic member is only 0.0134%.

4.2 Addition of a Mass Element

The objective in this step is to decrease the maximum modal flexibility value at the 1st natural mode. A mass element was added at point I of the model as shown in Fig. 6(c) according to the procedures described in Section 3.2. Fig. 7(c) shows the receptance frequency response after the design change. The maximum modal flexibility value decreased by 6%.

4.3 Substantial Changes of Joint Design Variables

The objective in this step is to minimize the maximum receptance value over the whole frequency range. The modal internal force coefficient h_{EJ1} of the 1st natural mode was large at the spring element in Y-direction of joint J_4. The spring stiffness k_J of the spring element was 1.0×10^8 N/m. Since the potential energy distribution rate at the spring element was very small (that means the design sensitivity coefficient is also very small), the spring stiffness k_J was greatly reduced to the value of 2.0×10^5 N/m. After this spring stiffness value was set as an initial design variable of k_J, the spring stiffness k_J and the damping coefficients of all joints (J_1 through J_4) were determined so that the maximum receptance value was minimized. Fig. 7(d) shows the receptance frequency response after the proposed procedures. By these procedures, two requirements (i) great reduction of the ratio f_n/f_s of the modal flexibility f_n at the natural mode having the greatest receptance value to the static comprance f_s and (ii) great increase of the damping ratio at the natural mode (decribed at Section 3.3) were simultaneously accomplished.

It can be understood from comparison of the receptance frequency response in Fig. 7(d) with that in Fig. 7(a) that the proposed procedures are effective for greater improvement of the vibrational characteristics (the maximum receptance value).

5. CONCLUDING REMARKS

Methodologies for greatly improving machine structural dynamics by using design sensitivity analyses and evaluative parameters were proposed. The features are as follows:

(1) Addition of elastic members and mass elements is carried out using information of displacement distributions and design sensitivity analyses for altering the initial framework of a structural model.

(2) Substartial changes of joint design variables are conducted using information of internal vibratory forces and potential energy distributions for the improvement of the poor initial design variables.

REFERENCES

1. Yoshimura, M.: Analysis and Optimization of Structural Dynamics of Machine Tools by a Synthesis of Dynamic Rigidity Program System, Proc. 16th International Machine Tool Design and Research Conference, Sept. 1976, pp. 209–215.

2. Yoshimura, M.: Computer-Aided Design Improvement of Machine Tool Structure Incorporating Joint Dynamics Data, Annals of the CIRP, Vol. 28, No. 1, 1979, pp. 241–246.

3. Yoshimura, M.: Design Sensitivity Analysis of Frequency Response in Machine Structures, ASME Journal of Mechanisms, Transmissions, and Automation in Design, Vol. 106, No. 1, 1984, pp. 119–125.

4. Yoshimura, M.: Optimum Design of Machine Structures with Respect to an Arbitrary Degree of Natural Frequency and a Frequency Interval between Adjacent Natural Frequencies, Bulletin of Japan Society of Precision Engineering, Vol. 14, No. 4, 1980, pp. 236–242.

5. Yoshimura, M.: Design Optimization of Machine-Tool Dynamics Based on Clarification of Competitive-Cooperative Relationships Between Characteristics, ASME Paper No. 86-DET-24, 1986.

Table 1. Procedures for the design optimization method based on clarification of competitive-cooperative relationships between characteristics (ref. 5)

	Design variables	Range of modeling, type of modeling and analytical method
Phase 1	o Design variables of structural members and elements having an influence on the static rigidity	o Modeling for a structure on the static force loop o Static rigidity analysis
Phase 2	o Design variables of structural members and elements and joint stiffnesses having no influence on the static rigidity	o Modeling for a complete structure o Vibrational analysis for an undamped vibrational system
Phase 3	o Damping coefficients of all joints	o Modeling for a complete structure o Vibrational analysis for a non-proportionally damped vibrational system

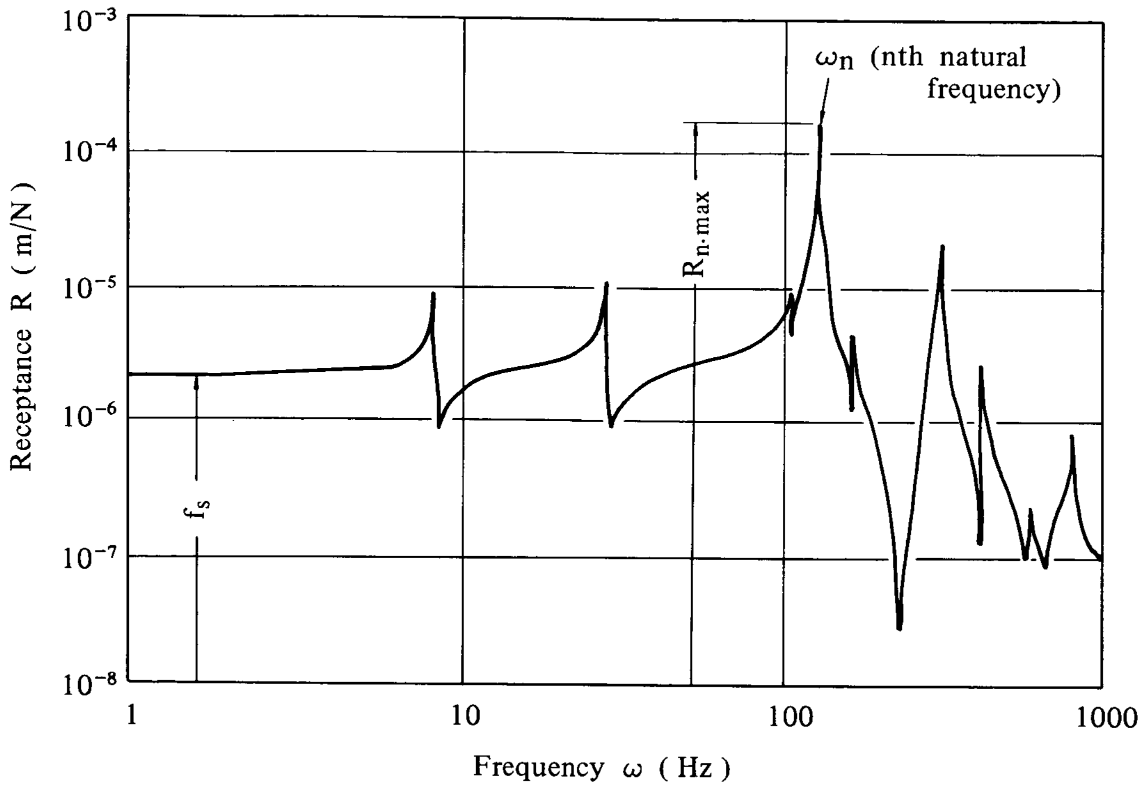

Fig. 1 An example of receptance frequency response

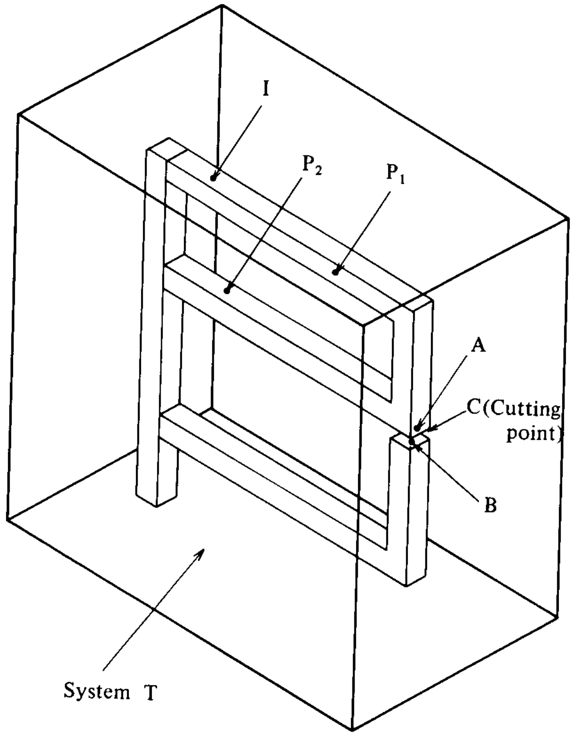

Fig. 2 A structural model in a hypothetical system T

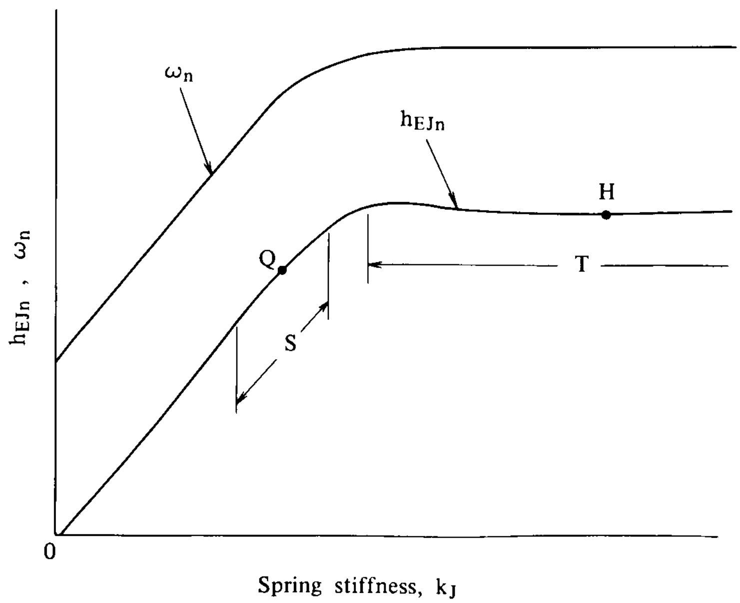

Fig. 3 Relation between the spring stiffness k_J at a joint, and the modal internal force coefficient h_{EJn} and the natural frequency ω_n at the nth natural mode

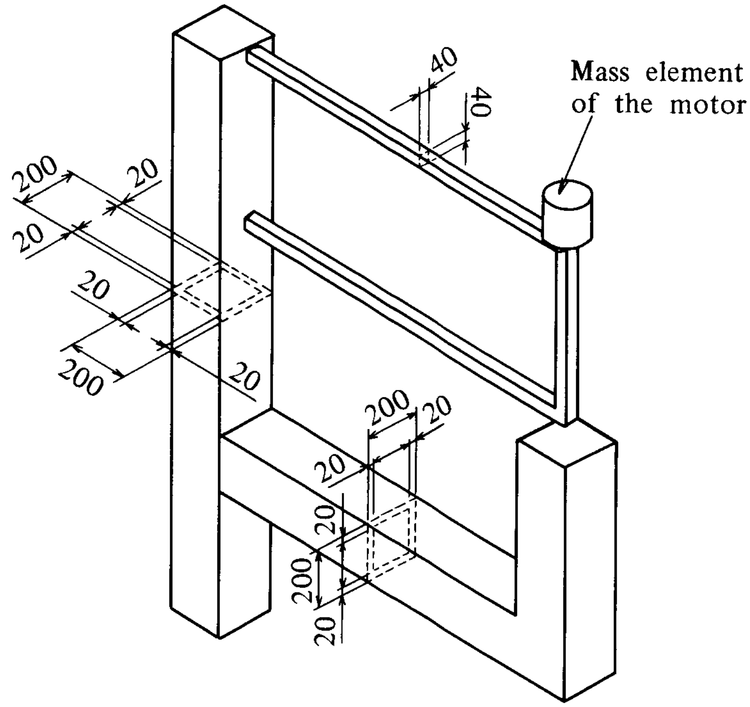

Fig. 4 Structural model of a machine tool
(unit: mm)

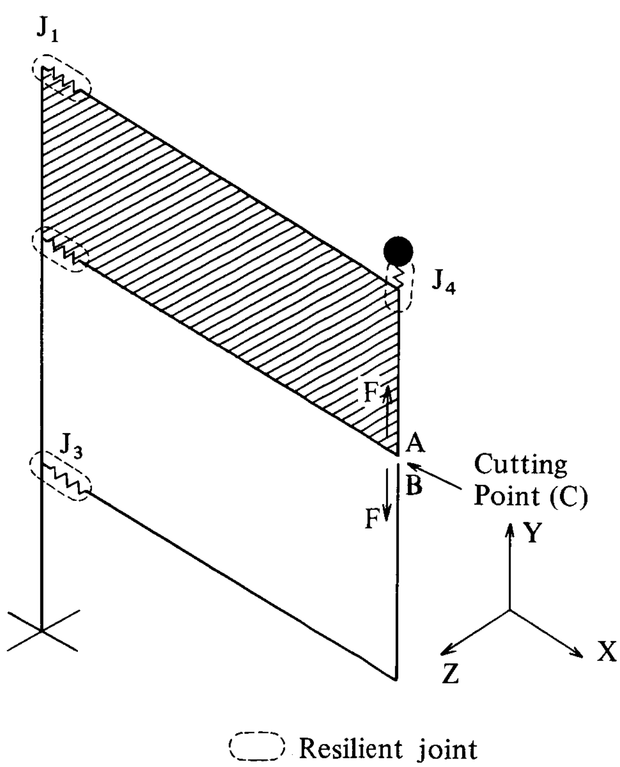

Fig. 5 Simulation model for structural analysis

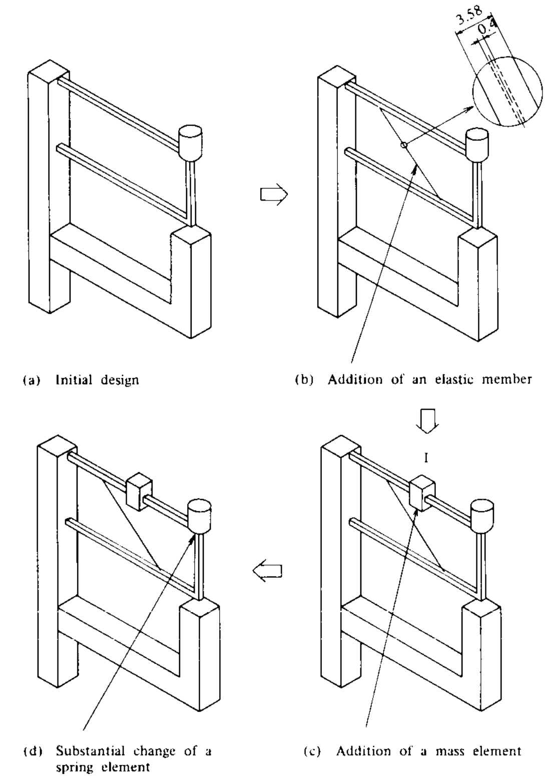

Fig. 6 Alteration of structural configuration by a series of design changes (unit: mm)

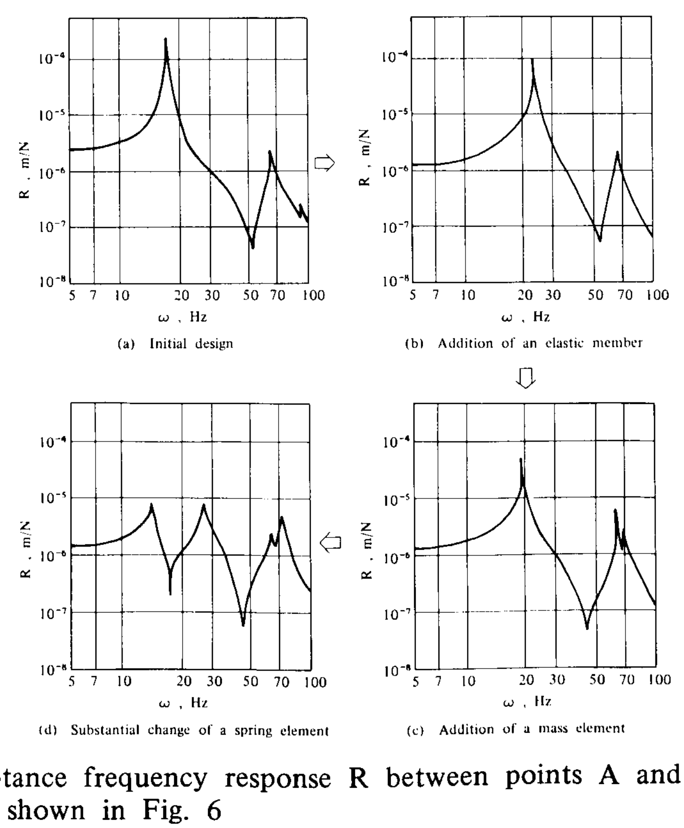

Fig. 7 Relative receptance frequency response R between points A and B corresponding to the design changes shown in Fig. 6