## Solar Power Absorption in a Glass Tube

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**APRIL** 1987

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### NASA Technical Memorandum 89063

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#### **Summary**

Equations for the power absorbed by a glass tube have been derived and applied to the particular case of a highly transmissive, fused silica tube irradiated by concentrated sunlight in space near Earth. Calculations were made with the assumption that the focal volume of the solar concentrator entirely envelopes the tube. Tube parameters studied were outside diameter and wall thickness. Absorption data between 155 and 4900 nm were used. Absorption beyond 4900 nm was estimated. Volumetric radiation and the variation of absorptance and emissivity with temperature have been used to show that, with radiative cooling only, large solar concentrations can develop very high temperatures in a fused silica tube.

#### Introduction

NASA has been investigating solar-pumped lasers (refs. 1 and 2) and is now studying space-based concepts for such lasers (refs. 3 and 4). One proposed container for the lasing medium of such a system is a fused silica tube that is to be placed at the focus of a large reflector capable of concentrating sunlight several thousand times at the surface of the tube. Since such concentrations of sunlight may produce high temperatures in the tube, a means is needed to estimate that temperature. This paper provides that means in the form of equations that can be used to calculate the power absorbed in glass tubes. Conversion of the absorbed power to temperature is determined for the particular case of a highly transmissive form of fused silica.

#### **Analysis**

In the derivation of equations which follows, extensive use has been made of the attenuation factor e, which is a function of the absorption coefficient and the path length in glass at a particular wavelength. This absorption coefficient may be calculated from spectral data on the index of refraction and the spectral transmission of the glass (for normally incident light) by use of the following formulas, which are derived from multiple reflection, transmission, and attenuation phenomena in flat glass sheets of thickness d:

$$R = \left(\frac{n-1}{n+1}\right)^2 \tag{1}$$

$$T = \frac{(1-R)^2 e}{1-e^2 R^2} \tag{2}$$

$$e = \exp(-\alpha d) \tag{3}$$

where

R reflectance of interface between indices of refraction 1 and n

 $\alpha$  absorption coefficient

T intensity transmission

(The above equations may be found in other literature (e.g., ref. 5), but are presented here for convenience and continuity in the following analysis.)

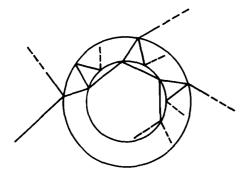
Reflection from, transmission through, and attenuation in a glass tube are not as simple as those of a flat glass plate. If the image of the Sun at the tube is greater than or equal to the outside diameter of the tube, then at any point on the tube surface rays of sunlight enter the tube at all angles of incidence from  $-90^{\circ}$  to  $90^{\circ}$ , as shown in figure 1. (All rays are in a plane perpendicular to the tube axis.) The ray paths inside the tube are in three angular regions: region 1 (fig. 1(a)), where the rays penetrate the inner surface; region 2 (fig. 1(b)), where the rays are totally internally reflected from the inner surface; and region 3 (fig. 1(c)), where the rays never reach the inner surface. The reflectance at each surface is, in general, different because the angle of incidence is different. Absorptance at each entrance angle is different because the path lengths of the corresponding rays differ. An infinite series of reflections is formed between the inner and outer surfaces beginning at any point where a beam enters the glass. In angular region 1, for example, a beam enters the outer surface once and parts of it enter the inner surface an infinite number of times, producing at each entrance an infinite, diminishing series of reflections between inner and outer surfaces. In angular regions 2 and 3, beams do not penetrate the inner surface, but an infinite series of reflections is generated by the beam entering the outer surface.

The key to analyzing this complex model lies in analyzing *one* series of reflections between inner and outer surfaces of the tube. As shown in figure 2, if a beam of unit irradiance enters the tube wall from the inner surface, the total transmitted irradiance can be represented by the following infinite series (see appendix A):

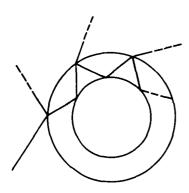
$$T = t_i t_o e \left( 1 + e^2 r_i r_o + e^4 r_i^2 r_o^2 + \dots \right)$$

$$= \frac{t_i t_o e}{1 - e^2 r_i r_o}$$
(4)

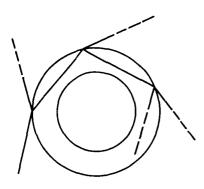
where



(a) Region 1—rays penetrate inner surface.



(b) Region 2—rays totally reflected from inner surface.



(c) Region 3-rays never reach the inner surface.

Figure 1. Light beams incident on a tube. For clarity, some transmitted beams are not shown.

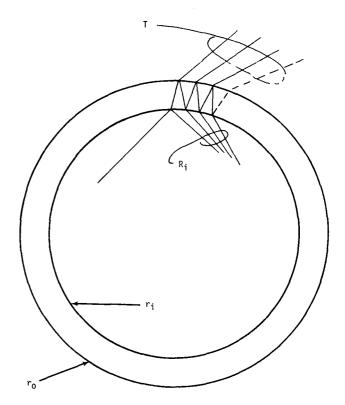


Figure 2. Typical beam path with multiple reflections and transmissions.

 $r_i$  reflectance at inner surface

 $r_o$  reflectance at outer surface

 $t_i$  transmission at inner surface,  $1 - r_i$ 

 $t_o$  transmission at outer surface,  $1 - r_o$ 

(These variables are functions of the polarization of incident radiation. Though not explicitly shown, the contributions of parallel and perpendicular polarizations have been included in the subsequent computations.) Similarly, the total inside reflected irradiance  $R_i$  is given by the series

$$R_{i} = r_{i} + t_{i}^{2} e^{2} r_{o} \left( 1 + e^{2} r_{i} r_{o} + e^{4} r_{i}^{2} r_{o}^{2} + \dots \right)$$

$$= r_{i} + \frac{t_{i}^{2} e^{2} r_{o}}{1 - e^{2} r_{i} r_{o}}$$
(5)

The total absorbed irradiance in the tube wall is thus

$$1 - R_i - T \tag{6}$$

The absorbed irradiance of equation (6) applies to *one* infinite series of reflections between inner and outer tube walls as shown in figure 2. Actually, there are an infinite number of these infinite series. (Each beam from the original series that reenters the tube interior creates another such series with exactly the

same geometry but with less irradiance.) To continue the analysis, let all the beams from the original series that reenter the tube interior be considered as one beam with irradiance  $R_i$ . This collective beam reflects around the tube inner surface, and at each reflection its irradiance is reduced by  $R_i$ . Thus, the absorbed irradiance from the infinite number of infinite series is

$$(1-R_{i}-T)\left(1+R_{i}+R_{i}^{2}+R_{i}^{3}+...\right)$$

$$=\frac{1-R_{i}-T}{1-R_{i}}$$

$$=1-\frac{T}{1-R_{i}}$$
(7)

Recall that the analysis began with a beam of unit irradiance entering the inner surface of the tube. This beam must be related to an external irradiance if the absorption formulas above are to be related to external irradiance. A beam of unit irradiance incident on the exterior of a tube will transmit a fraction T to the interior. (The value T was derived above for a beam coming out of the tube but its value is independent of direction of travel.) Therefore, if the original beam incident on the inner surface has a value of T instead of unity, it may be regarded as the result of unit beam irradiance external to the tube. Accordingly, equation (7) must be multiplied by T. The absorbed irradiance for an infinite number of infinite series of reflections becomes

$$T - \frac{T^2}{1 - R_i} \tag{8}$$

With the addition to equation (8) of the irradiance absorbed from the series of reflections generated by unit irradiance incident from *outside* the wall, the absorbed irradiance  $a(\lambda, \theta)$  becomes

$$a(\lambda, \theta) = (1 - R_o - T) + \left(T - \frac{T^2}{1 - R_i}\right)$$
$$= 1 - R_o - \frac{T^2}{1 - R_i}$$
(9)

where  $\lambda$  refers to the spectral wavelength and the total outside reflectance  $R_0$  is

$$R_o = r_o + \frac{t_o^2 e^2 r_i}{1 - e^2 r_o r_i} \tag{10}$$

Because incident irradiance decreases by the factor  $\cos \theta_1/\cos \theta_2$  inside the glass, where  $\theta_1$  is the angle

of incidence and  $\theta_2$  is the angle of refraction (see appendix B), total absorbed power per unit surface area becomes

$$a(\lambda, \theta) = \left(1 - R_o - \frac{T^2}{1 - R_i}\right) \frac{\cos \theta_1}{\cos \theta_2} \tag{11}$$

Equation (11) applies to region 1 (fig. 1(a)), which results in rays penetrating the inner surface of the tube wall. In region 2 (fig. 1(b)), where there is internal reflection at the tube inner surface, the total absorbed irradiance is

$$a(\lambda, \theta) = (1 - R_o) \frac{\cos \theta_1}{\cos \theta_2}$$
 (12)

In region 3 (fig. 1(c)), the total absorbed irradiance is given by

$$a(\lambda, \theta) = \frac{(1 - e)(1 - r_o)\cos\theta_1}{(1 - er_o)\cos\theta_2}$$
 (13)

The calculation of e above requires a knowledge of absorption length in the glass d. Given  $R_2$ ,  $\tau$ , n, and  $\theta_1$ , as illustrated in figure 3, d can be shown to be

$$d = R_2 \cos \theta_2 - \sqrt{(R_2 \cos \theta_2)^2 - 2R_2\tau + \tau^2}$$
 (14)

where

 $R_2$  outside radius of tube

 $\tau$  wall thickness

 $\theta_2 = \sin^{-1}[\sin(\theta_1/n)]$ 

n index of refraction

For region 3 of figure 1(c),

$$d = 2R_2 \cos \theta_2 \tag{15}$$

#### **Calculations**

The absorbed irradiance in the tube wall is a function of the path length in the tube wall, the index of refraction of glass, the angle of incidence of the irradiance, the absorption coefficient of the glass, and the incident solar intensity. Intensity, index of refraction, and absorption coefficient are spectral variables. Thus, absorbed irradiance must be integrated over the wavelengths at which absorption and solar irradiance are significant (i.e., 155 to 4900 nm). Path length and reflection at tube boundaries are functions of the angle of incidence at the tube surface and the

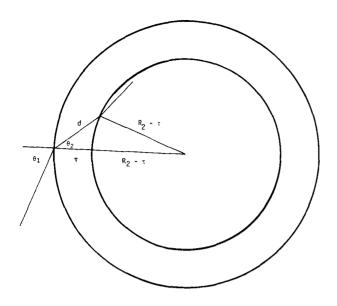


Figure 3. Beam path length in glass wall.

refractive index. Hence, absorbed irradiance must also be integrated over many angles of incidence (between  $0^{\circ}$  and  $90^{\circ}$ ) at each wavelength. Essentially, the following integral was evaluated:

$$I_t = \frac{2}{\pi} \int_0^{\lambda} \int_0^{\frac{\pi}{2}} S(\lambda) a(\lambda, \theta) \ d\theta \ d\lambda \ W/m^2$$
 (16)

where

- $S(\lambda)$  solar spectral intensity distribution, W/m<sup>2</sup>-nm
- $a(\lambda, \theta)$  spectral absorption (eq. (11), (12), or (13), depending on the value of  $\theta_1$ )

Solar spectral intensity data were obtained from reference 6, and refractive index data were obtained from reference 7. Both references present data at wavelengths with nonuniform intervals. Transmission data were obtained from reference 7 by digitizing graphical data presented at irregular intervals. Intermediate values of these three parameters were obtained from a second-order interpolation of data at the next smaller and the next two larger wavelengths. The absorbed irradiance was calculated for a range of tube wall thicknesses for two tube radii of 50 and 5 cm.

#### **Results and Discussion**

Table I shows the refractive index of fused silica, the total reflectance (both surfaces) of fused silica, the absorption coefficient of fused silica, and the solar spectral intensity at various wavelengths. In the spectral region 155 to 220 nm, total reflectance is unusually large because the refractive index is large. The absorption coefficient has a very small value where the solar spectral intensity reaches significant values. Thus, absorbed irradiance in the ultraviolet spectrum is negligible.

In the spectral region 220 to 2780 nm, values of the absorption coefficient are zero (within the accuracy of measurement). Practically no irradiance is absorbed in this spectral region, so no data are presented in table I.

From 2780 to 4900 nm, by far the major portion of irradiance is absorbed. The particular glass chosen for these calculations minimizes absorbed irradiance because a large absorption band at 2700 nm has been eliminated by the removal of OH radicals from the glass.

Beyond 4900 nm,  $7 \text{ W/m}^2$  of irradiance are in the solar spectrum and the absorption coefficient is very large but unknown. It is estimated, therefore, that  $1 \text{ W/m}^2$  is reflected and  $6 \text{ W/m}^2$  are absorbed. Thus,  $6 \text{ W/m}^2$  have been added to each calculated value of absorbed irradiance for the tabulated thickness.

Values of absorbed irradiance for various tube dimensions are listed in table II and plotted in figure 4. The values for large tube wall thicknesses can be used to approximate power absorption in solid rods. The absorbed irradiances expressed by the two curves at the same thickness show the effect of the radius of curvature to be almost negligible.

Since equations (11), (12), and (13) for absorbed irradiance include the independent variables e,  $r_i$ , and  $r_o$ , it is surprising that the absorbed irradiance does not vary with  $r_i$ ! Physically, this means that the tube absorbs as if the interior surface were not there. Only rays that leave the outer surface affect absorbed irradiance. The inner surface serves only to distribute absorption around the tube.

The calculations and these statements are made with the assumption that the tube interior contains no material that absorbs infrared radiation. The presence of infrared-radiation absorbing material in the tube interior would reduce absorbed irradiance in the walls of the tube.

The calculated values of absorbed irradiance in an empty tube are minimum values for two significant reasons: (1) only rays in a plane perpendicular to the tube axis have been considered, and (2) absorption of the glass increases with temperature (ref. 8). In the most general case, incident rays will not be perpendicular to the tube axis, and the absorptive path length in the glass will be increased. This will increase the absorbed irradiance.

Table I. Spectral Properties of Fused Silica and Solar Spectral Intensity

| Wavelength, | Refractive           | Total reflectance | Absorption         | Solar spectral |  |
|-------------|----------------------|-------------------|--------------------|----------------|--|
| nm          | index                | (two surfaces)    | coefficient        | intensity      |  |
| 155         | 1.641361             | 0.111353          | 0.111353 24.042201 |                |  |
| 160         | 1.628813             | .108240           | 10.440855          | .000230        |  |
| 165         | 1.616886             | .105289           | 5.607232           | .000403        |  |
| 170         | 1.605579             | .102498           | 2.634072           | .000630        |  |
| 175         | 1.594894             | .099867           | 1.280840           | .000835        |  |
| 180         | 1.584830             | .097397           | .600185            | .001250        |  |
| 185         | 1.575387             | .095085           | .236414            | .001164        |  |
| 190         | 1.566565             | .092931           | .046188            | .002710        |  |
| 220         | 1.528498             | .083718           | .002016            | .057500        |  |
| 2780        | 1.424320             | .059447           | .003927            |                |  |
| 2785        | 1.424213             | .059423           | .008362            | .039600        |  |
| 2790        | 1.424106             | .059399           | .012766            | .039400        |  |
| 2795        | 1.423999             | .059375           | .017140            | .039200        |  |
| 2800        | 1.423891             | .059351           | .021484            | .039000        |  |
| 2850        | 1.422801             | .059107           | .064847            | .037000        |  |
| 2900        | 1.421684             | .058857           | .104285            | .035000        |  |
|             |                      | 1                 |                    | 1              |  |
| 2950        | 1.420542             | .058601           | .133458            | .033125        |  |
| 3000        | 1.419370             | .058340           | .151974            | .031000        |  |
| 3050        | 1.418169             | .058072           | .158920            | .028300        |  |
| 3100        | 1.416940             | .057798           | .141714            | .026000        |  |
| 3150        | 1.415686             | .057518           | .127101            | .024300        |  |
| 3200        | 1.414400             | .057232           | .115086            | .022600        |  |
| 3250        | 1.413081             | .056939           | .070738            | .020800        |  |
| 3300        | 1.411730             | .056640           | .057293            | .019200        |  |
| 3350        | 1.410345             | .056333           | .076124            | .017825        |  |
| 3400        | 1.408930             | .056020           | .121736            | .016600        |  |
| 3450        | 1.407485             | .055701           | .207387            | .015488        |  |
| 3500        | 1.406010             | .055375           | .298713            | .014600        |  |
| 3550        | 1.404505             | .055044           | .499959            | .014063        |  |
| 3600        | 1.402970             | .054706           | .902126            | .013500        |  |
| 3650        | 1.401405             | .054362           | 1.266029           | .012900        |  |
| 3700        | 1.399810             | .054012           | 1.589068           | .012300        |  |
| 3750        | 1.398185             | .053657           | 1.809609           | .011650        |  |
| 3800        | 1.396530             | .053295           | 1.878325           | .011100        |  |
| 3850        | 1.394845             | .052927           | 1.900677           | .010700        |  |
| 3900        | 1.393130             | .052554           | 1.915146           | .010300        |  |
| 3950        | 1.391385             | .052175           | 2.057062           | .009900        |  |
| 4000        | 1.389610             | .051790           | 2.336959           | .009500        |  |
| 4050        | 1.410607             | .056391           | 2.752873           | .009113        |  |
| 4100        | 1.428891             | .060475           | 3.277175           | .008700        |  |
| 4150        | 1.444462             | .064004           | 4.005107           | .008225        |  |
| 4200        | 1.457321             | .066952           | 4.798062           | .007800        |  |
| 4250        | 1.467468             | .069297           | 5.664125           | .007438        |  |
| 4300        | 1.474901             | .071026           | 6.770718           | .007100        |  |
| 4350        | 1.479623             | .072128           | 7.797080           | .006798        |  |
| 4400        | 1.481632             | .072598           | 8.756014           | .006500        |  |
| 4450        | 1.480928             | .072434           | 8.966105           | .006209        |  |
| 4500        | 1.477512             | .071635           | 8.532039           | .005920        |  |
| 4550        | 1.471383             | .070207           | 7.979737           | .005625        |  |
| 4600        | 1.462541             | .068156           | 7.646193           | .005350        |  |
| 4650        | 1.450988             | .065496           | 8.391921           | .005093        |  |
| 4700        | 1.436721             | .062244           | 9.640547           | .004860        |  |
|             |                      | .058423           | 11.822024          | .004661        |  |
| 4750        | 1.419742             | .054065           | 17.219726          | .004470        |  |
| 4800        | 1.400051<br>1.377647 | .049214           | 21.249766          | .004470        |  |
| 4850        |                      |                   |                    |                |  |

Table II. Absorbed Irradiance for Various Tube Thicknesses

| Tube wall     | Absorbed irradiance,  |  |  |  |  |
|---------------|-----------------------|--|--|--|--|
| thickness, cm | $W/m^2$               |  |  |  |  |
|               | tside radius = 5 cm   |  |  |  |  |
|               |                       |  |  |  |  |
|               | 18.171                |  |  |  |  |
| .4            | 21.767                |  |  |  |  |
| .6            | 23.967                |  |  |  |  |
| 1.0           | 25.614                |  |  |  |  |
| 1.0           | 26.987                |  |  |  |  |
| 1.2           | 28.209                |  |  |  |  |
| 1.4           | 29.351                |  |  |  |  |
| 1.6           | 30.481                |  |  |  |  |
| 2.0           | 31.498                |  |  |  |  |
|               | 32.387                |  |  |  |  |
| 2.2           | 33.177                |  |  |  |  |
| 2.4           | 33.853                |  |  |  |  |
|               | side radius $= 50$ cm |  |  |  |  |
| 0.2           | 18.145                |  |  |  |  |
| .4            | 21.709                |  |  |  |  |
| .6            | 23.869                |  |  |  |  |
| .8            | 25.463                |  |  |  |  |
| 1.0           | 26.766                |  |  |  |  |
| 1.2           | 27.893                |  |  |  |  |
| 1.4           | 28.900                |  |  |  |  |
| 1.6           | 29.819                |  |  |  |  |
| 1.8           | 30.665                |  |  |  |  |
| 2.0           | 31.451                |  |  |  |  |
| 4.0           | 37.104                |  |  |  |  |
| 6.0           | 40.380                |  |  |  |  |
| 8.0           | 42.402                |  |  |  |  |
| 10.0          | 43.708                |  |  |  |  |
| 12.0          | 44.584                |  |  |  |  |
| 14.0          | 45.196                |  |  |  |  |
| 16.0          | 45.643                |  |  |  |  |
| 18.0          | 45.969                |  |  |  |  |
| 20.0          | 46.207                |  |  |  |  |
| 22.0          | 46.387                |  |  |  |  |
| 24.0          | 46.523                |  |  |  |  |

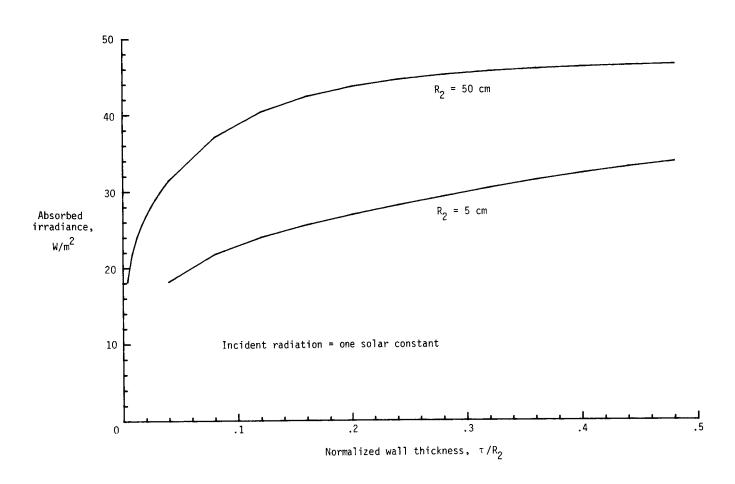


Figure 4. Variation of absorbed irradiance per unit surface area of fused silica (OH free) glass.

The temperature reached by the tube wall will depend on the spectral emission coefficient. That is, in the absence of any other cooling, the tube will reach a temperature at which radiated power will equal absorbed irradiance and, since the absorbed irradiance will be multiplied by the solar concentration factor C at the tube surface,

$$CI_t = C\overline{S}\overline{a} \approx \epsilon \sigma T_t^4$$
 (17)

where

 $\epsilon$  effective emissivity

 $\overline{S}$  effective solar power

σ Stefan-Boltzmann constant, 5.7 × 10<sup>-8</sup> W/m<sup>2</sup>-K<sup>4</sup>

 $\bar{a}$  effective absorptance

 $T_t$  tube temperature

Thus,

$$T_t = \left[ \frac{C\overline{S}}{\sigma} \left( \frac{\overline{a}}{\epsilon} \right) \right]^{\frac{1}{4}} \tag{18}$$

The evaluation of equation (18) is complicated because  $\epsilon$  is also a function of temperature. In contrast to the surface emission of opaque materials, the tube radiates energy throughout its volume. Gardon (ref. 9) has shown that the emissivity of volumetric radiators depends on the product of thickness and absorption coefficient and has evaluated the total hemispherical emissivity of ordinary plate glass for several thicknesses. If the emissivity of fused silica approximates the emissivity of plate glass, the operating temperature of the tube can be estimated.

For purely radial rays, solar concentration C cannot exceed 215 (see appendix C). For this concentration, an iterative solution to equation (18) with the data of Gardon for a 0.2-cm-thick tube shows the temperature to be 519 K. Conversely, if the tube does not exceed the maximum continuous operating temperature of 1225 K, the solar concentration cannot exceed 4450. (In this case rays must have longitudinal as well as radial components.)

The above temperature of 1225 K is conservative and the concentration is liberal because the increase of  $\bar{a}$  above 700 K (ref. 8) has not been included. This

is mainly a surface effect that is absent in the presence of oxygen. In a vacuum, the effect will cause surface heating that will raise the tube temperature by about 25 percent and lower the permissible solar concentration. Conductive cooling may occur to some degree if the lasing medium contacts the tube walls, or it may be introduced intentionally by other means. Regardless, conductive cooling will reduce heating to some degree but will create thermal gradients in the walls of the tube that, if large enough, can damage the tube. Wall temperatures can be reduced by reducing the infrared (IR) irradiation of the tube in several ways: (1) the lasing medium could absorb part of the IR radiation; (2) the tube could be coated with an IR reflector; or (3) the IR radiation could be absorbed or not reflected before it reaches the tube.

#### **Conclusions**

The formulas in this report provide a means for calculating absorbed power (irradiance) in any glass tube and estimating the operating temperature of the tube. The absorbed irradiance varies primarily with tube thickness and is independent of the reflectivity of the inside surface. Variation of absorbed irradiance with tube diameter is insignificant.

In the application of the formulas to tubes made of hydroxyl-free fused silica it was found that, with radiative cooling only, the tube would operate at approximately 519 K with purely radial irradiation and could reach its maximum operating temperature at a solar concentration of approximately 4450. Irradiance is absorbed almost entirely at infrared (IR) wavelengths. Since this form of fused silica absorbs a minimum of IR radiation, other forms of fused silica can be expected to get hotter or tolerate less solar concentration under the same conditions.

Tube heating can be lessened by (1) partial absorption of IR radiation by the lasing medium, (2) coating the tube with an IR reflector, (3) removing IR radiation from the sunlight before incidence, and (4) using a different material that does not absorb significantly. Conductive cooling can lessen the heating but will create thermal gradients in the glass that, if large enough, can damage the tube.

NASA Langley Research Center Hampton, VA 23665-5225 February 25, 1987

#### Appendix A

#### **Transmitted Intensity**

Assume a light ray of unit intensity enters the inner surface of a tube from the interior. At the inner surface a portion  $t_i$  of the ray intensity is transmitted into the tube wall. That portion is attenuated by the factor e before reaching the outer surface. At the outer surface part of the reduced intensity  $t_ie$  is reflected back toward the inner surface and part is transmitted outside the tube. The part transmitted outside the tube is equal to  $t_it_oe$  and is the first part of the transmitted intensity. The portion reflected travels back to the inner surface beginning with intensity  $t_ier_o$  and arrives at the inner surface with intensity  $t_ie^2r_o$ . After reflection at the inner surface the intensity becomes  $t_ie^2r_or_i$  and proceeds, again, to the outer surface. The intensity arriving at the outer surface is reduced again by e to  $t_ie^3r_or_i$ , and  $t_it_oe^3r_or_i$  is transmitted outside the tube. The transmitted intensity becomes

$$T = t_i t_o e + t_i t_o e^3 r_o r_i + \cdots$$

$$= t_i t_o e \left( 1 + e^2 r_o r_i + \cdots \right) \tag{A1}$$

At any point on the outer surface where a transmitted beam emerges there is also a reflection which produces the next transmitted beam. Each succeeding transmitted beam is reduced in intensity by  $e^2r_or_i$ . Hence, the series above for the total transmitted intensity can be continued ad infinitum. Since

$$1 + e^{2}r_{o}r_{i} + e^{4}r_{o}^{2}r_{i}^{2} + e^{6}r_{o}^{3}r_{i}^{3} + \dots = \frac{1}{1 - e^{2}r_{o}r_{i}} \qquad (e^{2}r_{o}r_{i} < 1)$$

then

$$T = \frac{t_i t_o e}{1 - e^2 r_o r_i} \tag{A2}$$

#### Appendix B

## Variation of Radiance With Angle of Incidence

Let  $\theta_1$  be the angle of incidence on a solid surface ( $\theta_1$  is measured from a perpendicular to the surface). At normal incidence ( $\theta_1 = 0$ ), any infinitesimal spot of light on the surface covers a minimum area dA. As  $\theta_1$  increases, the area elongates in the direction of the plane of incidence. This elongation causes the area to

increase by the factor  $1/\cos \theta_1$ , and spot irradiance decreases by  $\cos \theta_1$ .

Inside the solid material, the angle of refraction  $\theta_2$  is smaller than  $\theta_1$  (Snell's law) and decreases the one-dimensional elongation caused by  $\theta_1$ . The area perpendicular to the direction of beam propagation in the material decreases by the factor  $\cos \theta_2$ , so that the beam area inside the solid becomes  $dA \cos \theta_2/\cos \theta_1$ . Since irradiance is inversely proportional to the area of the beam, irradiance decreases by  $\cos \theta_1/\cos \theta_2$ .

#### Appendix C

#### **Maximum Solar Concentration in One Dimension**

If sunlight is incident on the frustum of a reflective right circular cone through the annular area  $\pi\left(r_2^2-r_1^2\right)$  in its base and the apex angle of the cone is 90°, then the rays will be reflected to the smallest possible region, which is the frustum of a much smaller cone centered on the axis of the reflective cone. (See fig. C1.) The radius of the small cone at its base is  $r_2\alpha$  and at the other end is  $r_1\alpha$  ( $\alpha$  is the half-angle of the solar subtense angle,  $4.6543 \times 10^{-3}$  rad). The length of the small cone is  $r_2 - r_1$  from the geometry. The area of the frustum of the small cone is given by

$$\pi(r_1\alpha + r_2\alpha)\sqrt{(r_2 - r_1)^2 + (r_2\alpha - r_1\alpha)^2}$$
 (C1)

Maximum solar concentration C is thus expressed as the ratio of the annular area to the frustum area of the small cone:

$$C = \frac{\pi \left(r_2^2 - r_1^2\right)}{\pi (r_1 \alpha + r_2 \alpha) \sqrt{(r_2 - r_1)^2 + (r_2 \alpha - r_1 \alpha)^2}} = \frac{1}{\alpha \sqrt{1 + \alpha^2}} \approx \frac{1}{\alpha} \approx 215$$
 (C2)

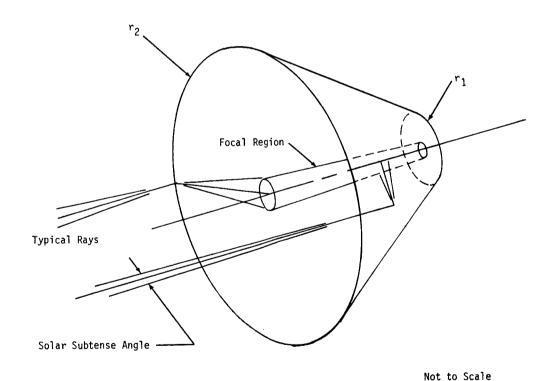


Figure C1. Concentration of sunlight by a conical reflector into a conical focal region.

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#### Standard Bibliographic Page

| 1. Report No.  | 2. Governme   | ent Accession No.   | 3. Recipient's Cat   | alog No.  |  |
|--|---|---|--|---|--|
| NASA TM-89063  |   |   |  |   |  |
| 4. Title and Subtitle  |   |   | 5. Report Date   |   |  |
| Solar Power Absorption in a Glass Tube   |   |   | April 1987   |   |  |
|  |   | 6. Performing Organization Code   |  |   |  |
| 7. Author(s)   |   | 506-41-41-02  |  |   |  |
| Michael D. Williams  |   | 8. Performing Org   | anization Report No.   |   |  |
|  |   |   | L-16055  |   |  |
| 9. Performing Organization Name and Address  |   |   | 10. Work Unit No.  |   |  |
| NASA Langley Research Center   |   |   |  |   |  |
| Hampton, VA 23665-5225   | 11. Contract or C   |   | rant No.   |   |  |
| 12. Sponsoring Agency Name and Address   |   |   | 13. Type of Report and Period Covered                                      |   |  |
| National Aeronautics and Space Administ  |   |   |  | Technical Memorandum  |  |
| Washington, DC 20546-0001  | 14. Sponsoring A  |   | gency Code   |   |  |
| 15. Supplementary Notes  |   |   |  |   |  |
| 16. Abstract   |   |   |  |   |  |
| The optics of a glass tube to be used in examined, and an equation for the power a been developed. The equation has been us of fused silica. The equilibrium temperatu examined, and it shows a significant rise very cylindrical containment vessels for space-batubes. | absorbed fro<br>ed to calcul<br>are reached<br>with large s | om multiple-reflected<br>ate the power absor<br>by the tube with on<br>olar concentrations. | d light beams in<br>bed by a highly<br>lly radiative coo<br>The results ap | the tube wall has<br>transmissive form<br>bling has also been<br>oply specifically to |  |
| 17. Key Words (Suggested by Authors(s)) Solar concentration  | 18. Distribution Statement Unclassified—Unlimited           |   |  |   |  |
| Absorptance  |   |   |  |   |  |
| Fused silica   |   |   |  |   |  |
| Solar-powered lasers   |   |   |  |   |  |
|  |   | Quhian  | Cataman 74   |   |  |
| 19. Security Classif.(of this report)  | Subject Category 74   |   |  |   |  |
| Unclassified  Unclassified   | 20. Security Classif.(of this page) Unclassified            |   | 21. No. of Pages   | 22. Price<br>A02  |  |