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THE GROWTH OF RADIATIVE FILAMENTATION MODES IN SHEARED MAGNETIC FIELDS

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ABSTRACT

Observations of prominences show them to require well-developed magnetic shear and to have complex small-scale structure. We show here that these features are reflected in the results of the theory of radiative condensation. We have studied, in particular, the influence of the nominally negligible contributions of perpendicular (to $\underset{\sim}{B}$) thermal conduction. We find a large number of unstable modes, with closely spaced growth rates. Their scale widths across $\underset{\sim}{B}$ show a wide range of longitudinal and transverse sizes, ranging from much larger than to much smaller than the magnetic shear scale, the latter characterization applying particularly in the direction of shear variation.

INTRODUCTION

Coronal prominences owe their existence to a condensation process which occurs as a consequence of an instability in the thermal equilibrium of a diffuse medium (Parker, 1953; Field, 1965; Hildner, 1974). The condensation mechanism relies on optically thin radiation whose dependence on thermodynamic variables (e.g., density and temperature) is such that a cool, dense perturbation loses more energy through radiation than it gains through adiabatic and non-adiabatic heating processes and thermal conduction.

Prominences and filaments (as seen on the disk) in the solar atmosphere are often observed to form above a magnetic neutral (polarity-inversion) line in regions of increasing magnetic field shear as indicated by photospheric magnetograms (Martin, 1973; Leroy, 1978). Local heat conduction, which is strongly attenuated in directions perpendicular to the magnetic field, would dominate radiation and other forms of energy transport if no field were present, thereby suppressing the thermal instability. Thus, one can expect the equilibrium field structure to exert a strong influence over the formation of prominences.

These empirical and physical considerations have motivated previous computational studies of the dynamics of the thermal instability in a sheared magnetic field (Chiuderi and Van Hoven, 1979; Van Hoven and Mok, 1984; Van Hoven et al., 1984; Sparks and Van Hoven, 1985). One result of these theoretical studies is that a sheared background field is necessary for the existence of a true localized condensation. Secondly, it is surprising that the width of the condensation in a direction perpendicular to the shear layer does not correspond to those points at which the magnetic field has tilted sufficiently that the radiation loss is roughly balanced by parallel thermal conduction, nor is it affected by realistic values of the perpendicular conductivity.

The present study is devoted to the delineation of some additional consequences of the presence of anisotropic thermal conduction in a sheared-field filament. One finds that new unstable excitations appear, with complicated transverse variations (multiple nodes), and that perpendicular conduction provides modes with shorter wavelengths and faster growth.

FORMULATION

To model a sheared, active-region, magnetic field, we use the planar force-free form $\underset{\sim}{B}/B_o = F(y/a)\underset{\sim}{e}_z + G(y/a)\underset{\sim}{e}_x$ where $F(0) = 0$ and $F^2 + G^2 = 1$, which is consistent with uniform temperature (T) and density (ρ). [The example we use is $F(y/a) = \tanh y/a$.]

We describe the plasma dynamics by the compressible ideal MHD limit, and the energetics by the heat-transport equation (Chiuderi and Van Hoven, 1979)

$$\frac{dp}{dt} - \gamma p \underset{\sim}{\nabla}\cdot\underset{\sim}{v} = (\gamma-1)\left[H_o - \rho^2\Phi(T) + \underset{\sim}{\nabla}\cdot\underset{\approx}{\kappa}\cdot\underset{\sim}{\nabla}T\right] \tag{1}$$

which includes an unspecified (and unknown) constant heat input for thermal-equilibrium balance, optically thin radiation losses and anisotropic heat flow. If one considers $T(y,z,t) = T_0 + T_1(y)\exp(\nu t + ikz)$, one can linearize (1) as

$$\frac{\partial T_1}{\partial t} + (\gamma-1)\frac{\partial \rho_1}{\partial t} = \Omega_\rho T_1 - \Omega_T \rho_1 - \left[k^2a^2F^2\Omega_\parallel + k^2a^2G^2\Omega_\perp - \Omega_\perp(T_1''/T_1)\right] \tag{2}$$

where T_1 and ρ_1 are fractional perturbations and $T_1' = a\partial T_1/\partial y$. The heat-flow rates for the non-adiabatic terms on the right side of (2) include the radiation rate Ω_ρ at constant density and the generalized parallel-plus-perpendicular thermal conduction rate $\Omega_\kappa \propto \kappa T_o/a^2 p_o$ in the square bracket on the right.

When the two-dimensional dynamic equations are simplified, they reduce to the set of coupled equations (Van Hoven and Mok, 1984; Sparks and Van Hoven, 1985)

$$\begin{aligned} q'' - \alpha^2F^{2\prime}(\nu^2+\alpha^2)(\nu^2+\alpha^2F^2)^{-1}q' - (\alpha^2+\alpha_\nu^2)q &= 0 \\ T_1 = \left[(\gamma-1)\nu - \Omega_T\right]\left[\nu-\Omega_\rho+\Omega_\kappa\right]^{-1}(\alpha_\nu^2/\nu^2)q & \end{aligned} \tag{3}$$

where $q = p_1 + \underset{\sim}{B}_o\cdot\underset{\sim}{B}_1/\mu_o$ is the total pressure perturbation. The normalized wave numbers are $\alpha \equiv ka$ and

$$\alpha_\nu^2 \equiv \nu^4(\nu - \Omega_\rho + \Omega_\kappa)$$

$$\times[\nu^2(\nu-\Omega_\rho+\Omega_\kappa) + \tfrac{1}{2}\beta\gamma(\nu^2+\alpha^2F^2)(\nu-\Omega_p+\Omega_\kappa/\gamma)]^{-1}$$

which depends implicitly on T_1''/T_1 (through Ω_κ) when $\Omega_\perp \neq 0$ and provides the only energy-transport contribution to the $\underline{q}$ equation.

The solutions of equations (3), with boundary conditions requiring localization of the excitation near $y = 0$ and exponential decay as $y \to \pm\infty$, provide eigenfrequencies (growth rates) ν which depend on ka, and eigenfunctions $T_1(y)$. Significant information about the allowable values of ν(ka), and about the structure of the eigenfunctions, can be obtained from the poles and zeros of α_ν and from considering local solutions in a uniform field (Chiuderi and Van Hoven, 1979; Sparks and Van Hoven, 1985).

RESULTS

The simplest radiative-instability case to consider in a sheared field has $\Omega_\kappa = 0$ so that radiation merely competes against adiabatic compression. The solutions of (3a) then provide a series of modes which have the essential characteristics of the solutions to more complete formulations. The modes exhibit increasing numbers of y-direction nodes, and growth rates approaching $\Omega_c = (\Omega_\rho + \tfrac{1}{2}\beta\gamma\Omega_p)/(1+\tfrac{1}{2}\beta\gamma)$, the radiation rate for which perpendicular (to $\underset{\sim}{B}$) plasma motions occur at constant total pressure. In fact, these solutions have the typical property that the total-pressure perturbation $\underline{q}$ is much smaller than the thermal pressure p_1, especially at shorter wavelengths (Sparks and Van Hoven, 1985).

The addition of the nominally dominant effects of parallel thermal conduction $\Omega_\parallel$ does not change the situation very much. The principal modification is the introduction of a pole in α_ν^2, which prevents the existence of solutions at wavelengths shorter than the $s^2 = \infty$ curve of Fig. 1 (Chiuderi and Van Hoven, 1979).

It is necessary, finally, to add the effects of perpendicular thermal conduction $\Omega_\perp$, which should not (*a priori*) be important, to be able to obtain a reasonably complete treatment of the structure and growth of these sheared-field radiative modes (Van Hoven and Mok, 1984; Van Hoven et al., 1986).

In order to see the important effects of $\Omega_\perp$, I have shown a qualitative growth-rate curve in Fig. 1, which also displays the various instability rates identified by Field (1965). The behavior of the eigensolutions is different on the two sides of the curve $s^2 \equiv \alpha^2+\alpha_\nu^2(F=1,\Omega_\perp=0) \to \infty$ [or $\alpha = \alpha_p(\nu)$]. The growing modes on the lower left are variants of the original sheared-field modes found by Chiuderi and Van Hoven (1979). The addition of $\kappa_\perp$ to the energy transport resolves the steep gradients of these solutions (Van Hoven and Mok, 1984) and allows them to have multiple radial nodes, within a width given by Eq. (18) of this earlier paper. It is somewhat unusual that the "fundamental" (no nodes) transverse-variation mode has the lowest growth rate. The practical consequence of this fact is unclear, however, since the growth-rate curves effectively lie on top of each other for typical solar coronal parameters.

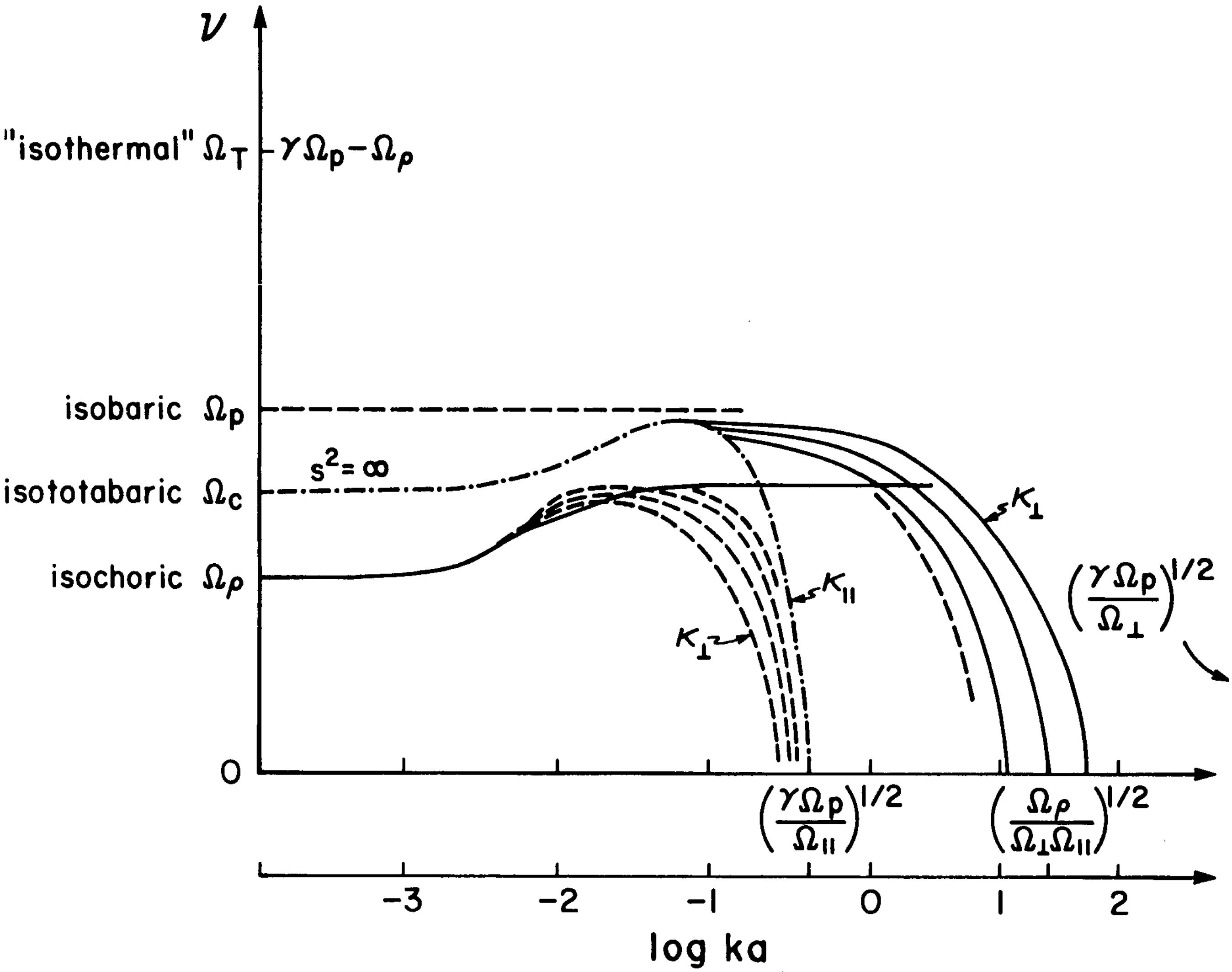

Figure 1. Radiative filamentation growth rates vs wavenumber.

The faster-growing, shorter-wavelength modes on the upper right in Fig. 1 only appear as a result of the presence of $\kappa_\perp$ (Van Hoven, Sparks, and Tachi, 1986). [Small as this coefficient is ($\kappa_\perp/\kappa_\parallel \sim 10^{-10}$ in the corona), it resolves certain singularities which appear when $\kappa_\perp \to 0$.] For these modes, the fundamental is the fastest growing, although the inter-mode spacing near the peak is (again) negligible in practice. The transverse structure of these eigensolutions is unusual, for $\nu > \Omega_c$, in that the (negative) peak of the temperature perturbation is located away from the axis on each side (Van Hoven, Sparks, and Tachi, 1986). The inner and outer edges of the temperature peak(s) can be scaled from the equivalent of Fig. 1. One takes the α positions $[\alpha_p(\nu)]$ of the $s^2 = \infty$ curve at the relevant eigenvalue of $\nu(\alpha)$, divides it by α = ka and calculates $y/a = \tanh^{-1}(\alpha_p/\alpha)$. The resulting widths for reasonable parameters are $\sim 10^{-3}$a (located at $\sim \pm 10^{-3}$a) for $\lambda/2 \sim 0.3$a, but one must remember that a number of modes grow at nearly the same rate. The outer edges of these modes, for which $\alpha_p^2 \approx \gamma(\Omega_p - \nu)/\Omega_\parallel$, exhibit the only direct dependence on the parallel conductivity.

DISCUSSION

The principal conclusion of our study of the linear eigensolutions of the radiative filament-condensation instability in a sheared magnetic field is that the theoretical results are nearly as complex as the observational results. The range of growth rates for the radiative instability is small, with $\Omega_\rho \sim 10^{-3.9} n_9 T_6^{-2} \lesssim \nu < \Omega_p = 1.8\Omega_\rho$ for the usual estimate $\Phi(T) \propto T^{-1}$ (Hildner, 1974) at coronal temperatures $T \sim 10^6 T_6$ K and number densities $n \sim 10^9 n_9$ cm^{-3}.

Not only is the range of growth rates narrow, but there are a large number of distinct eigenmodes within this range. There are two groups, one with wavelengths (the vertical width of the characteristic knife-blade form) greater than the magnetic shear scale, and the second with mainly shorter wavelengths. In most cases, there is an approximate balance between magnetic pressure increases and thermal pressure decreases.

These eigenmodes also exhibit a complex structure in the transverse direction, equivalent to the horizontal thickness of the knife-blade filament. The temperature (and density) profiles oscillate in this direction, with a number of nodes. The long wavelength modes are concentrated in the center of the shear layer, but the shorter modes often show a hollow profile with the coolest layers separated from the shear center. These latter excitations extend to the point where radiation is overtaken by parallel thermal conduction (which is relatively strong for short wavelengths).

We will not know which of these many excitations is (are) the dominant one(s) until we complete a series of nonlinear computations which are now in progress.

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REFERENCES

Chiuderi, C. and Van Hoven, G.: 1979, Astrophys. J. Letts. 232, L69.
Field, G. B.: 1965, Astrophys. J. 142, 531.
Hildner, E.: 1974, Solar Phys. 35, 123.
Leroy, J.-L.: 1978, Astron. Astrophys. 64, 247.
Martin, S. F.: 1973, Solar Phys. 31, 3.
Parker, E. N.: 1953, Astrophys. J. 117, 431.
Sparks, L. and Van Hoven, G.: 1985, Solar Phys. 97, 283.
Van Hoven, G. and Mok, Y.: 1984, Astrophys. J. 282, 267.
Van Hoven, G., Tachi, T., and Steinolfson, R. S.: 1984, Astrophys. J. 280, 391.
Van Hoven, G., Sparks, L., and Tachi, T.: 1986, Astrophys. J. 300, 249.