N87-20877

CONDENSATION MODES IN MAGNETIZED PLASMAS

Chang-Hyuk An
NASA/Marshall Space Flight Center

ABSTRACT

I have studied condensation modes in magnetized cylindrical plasmas, concentrating on how magnetic field affects the stability. It is found that the effects of magnetic field (shear, twist, and strength) on the condensation modes are different depending on the wave vector. For modes whose wave vector is not perpendicular to magnetic field lines the plasma motion is mainly along the field lines; the effects of magnetic field on the modes are negligible except on the heat flow parallel to the field line. For a mode which is localized near a surface where the wave vector is perpendicular to the field line, the plasma moves perpendicular to the line carrying the field line into the condensed region; magnetic field affects the mode by building up magnetic pressure in the condensed region.

The stability of condensation modes strongly depends on how density and temperature vary with field twist. The stable nature of global quiescent prominence magnetic configurations implies that prominences form for low field twist for which ideal MHD modes are stable; plasma temperature should increase with field twist for stable prominence formation.

I. INTRODUCTION

Solar prominences are a very intriguing phenomenon. They are cool dense material imbedded in hot and tenuous coronal plasmas. After Field(1965) various authors(Nakagawa 1970; Hildner 1971; Heyvaerts 1974; Chiuderi and Van Hoven 1979) have studied condensation modes in a plane slab geometry to understand the prominence formation. I have studied the condensation modes(An 1984a, 1985) assuming that prominences are formed in a cylindrical magnetic geometry(Anzer and Tandberg-Hanssen 1970; Pneuman 1983). This magnetized cylindrical plasma is subject to ideal MHD as well as radiative(condensation) instabilities. For coronal conditions, the radiative time scale($t_r \sim 10^3$ sec.) is much longer than the MHD time scale($t_m \sim 1$ sec.), and condensation modes appear as a first order of ε $(=(t_m/t_r)^2 \sim 10^{-6})$ (An 1985). The possible coexistance of ideal MHD and condensation modes with very different time scales requires a careful analytic manipulation for the study of condensation modes. I developed a mathematical techique for the study(An 1985).

Questions about condensation modes in magnetized plasmas are how magnetic field (twist, shear, and strength) affects the modes and how the stability depends on different choice of temperature and density profiles. Here, magnetic shear arises from different field twist on each flux surface. Magnetic shear is an important stabilizing mechanism for ideal MHD instabilities and has been considered to be important for condensation modes(Chiuderi and Van Hoven 1979). I will show that magnetic structure (twist and shear) does not have important effects on condensation modes and will prove mathematically the insignificance of MHD effects on the modes for general magnetic field configurations. Since condensation modes are hydrodynamic in nature it is necessary to study the stability of an equilibrium with different temperature and density profiles for better understanding the prominence formation and stability.

II. DERIVATION OF CONDENSATION MODE EQUATION

I assume that the plasma is governed by ideal MHD equation with radiation and heat conduction. The ambient heating rate is assumed constant in time and the optically thin radiative energy loss rate is used. Due to the two components of magnetic field (longitudinal B_z and poloidal B_θ), the field lines are twisted. Since MHD and radiative time scales are very different we make the governing equations dimensionless. Physical quantities, P, B, ρ , T are normalized with their standard values and time t is normalized by the radiative time scale t_r . Here radiative and MHD time scales are defined as $t_r = P_o/R$ and $t_m = \rho_o a^2/B_o^2$ respectively. The quantity ε is defined as $\varepsilon = (t_m/t_r)^2$ which is much smaller than 1 for coronal plasmas. The quantities P_o , ρ_o, and B_o are standard ambient coronal plasma pressure, density, and magnetic field and a is the radius of the cylinder cross section. H is the ambient heating and R is the radiative energy loss rate. β is defined as P_o/B_o^2 . I derive the linear stability equation by linearizing the governing equations and assuming that the perturbed quantities have the form $f(\vec{r},t) = f(r)\exp[i(m\theta + kz) + \omega t]$. The second order differential equation for linear stability has the following form,

$$LX=0 \tag{1}$$

Here X is an eigenfunction and L is a second order differential operator and depends on ω , ε , and equilibrium quantities. The condensation mode equation can be derived, noting $\varepsilon \ll 1$, by expanding L and X in power of ε and by taking zero and first order equations. The explicit expression of zero and first order equations are as follows.

$$\frac{d}{dr}\left[\frac{(k\cdot B_0)^2}{rK}\frac{dX_0}{dr}\right] - \left\{\left[\frac{2mB_\theta(k\cdot B_0)}{r^3K}\right]' + \frac{1}{r}\left[(k\cdot B_0)^2 - 2B_\theta\left(\frac{B_\theta}{r}\right)' - \frac{4k^2B_\theta^2}{r^2K}\right]\right\}X_0 = 0\,. \tag{2}$$

$$L_0X_1 + \left(\frac{X_0'\rho_0}{rK}\right)' - \left[\frac{X_0\rho_0(\gamma-1)dR/dr}{rK(\phi\rho_0+\gamma P_0\omega)}\right]'$$
$$+ X_0\left\{\frac{2kB_\theta(kB_\theta - mB_z/r)\rho_0[(\gamma-1)/r]dR/dr}{rK(k\cdot B_0)^2(\phi\rho_0+\gamma P_0\omega)} - \frac{\rho_0(2kB_\theta)^2}{r^3K(k\cdot B_0)^2}\right\} - \frac{\rho_0}{r}X_0 = 0\,. \tag{3}$$

Here $K = k^2 + m^2/r^2$ and $\phi = \frac{(\gamma-1)T_0}{\rho_0}\left[\frac{3}{2}\left(\frac{\partial R}{\partial T}\right)_P + \frac{T_0^{5/2}}{B_0^2}\frac{t_r}{t_c}(k\cdot B_0)^2\right]$
Prime(') denotes a derivative with r.

For the derivation of eq.(2) and (3) we assume that $\vec{k}\cdot\vec{B} \neq 0$ and $\phi\rho_o + \gamma P_o\omega = 0$ in the plasma. The condition $\vec{k}\cdot\vec{B} \neq 0$ means that the plasma should be stable to the ideal MHD mode. In order to study condensation modes, we have to solve equation (2) and (3) for a given boundary condition of X_o and X_1 .

III. STABILITY

The effects of magnetic fields on condensation modes vary depending on whether $\vec{k}\cdot\vec{B}$ is zero or not in the plasma. Let us discuss how the magnetic field affects the mode of $\vec{k}\cdot\vec{B}=0$. We cannot use Equ.(3) because it is derived assuming $\vec{k}\cdot\vec{B}\neq 0$ in the plasma. According to An(1984b,c) the eigenfunction of Equ(1) becomes localized near $r=r_s$ where $\vec{k}\cdot\vec{B}=0$ as m and k go to infinity. In this limit the growth rate of the condensation mode is

$$\omega = -\left(\frac{\bar{Q}}{\bar{P}} + \frac{W}{Y\bar{P}}\right), \tag{4}$$

Here,

$$\bar{P} = \rho_0(C_m^2 + C_s^2) ;$$

$$\bar{Q} = \frac{(\gamma - 1)}{\gamma} C_s^2\left[\left(\frac{C_m^2\gamma}{C_s^2} + 1\right)\left(\frac{\partial R}{\partial T}\right)_\rho - \frac{\rho}{T}\left(\frac{\partial R}{\partial \rho}\right)_T\right] ;$$

$$W = -\frac{2B_\theta^2}{r^2}(\gamma - 1)\,\beta\,\frac{dR}{dr} - \frac{C_m^2 C_s^2(\gamma - 1)^2}{\gamma}\,\frac{4\rho_0 B_\theta^4}{\bar{P}r^3B_0^2}\left(\frac{\partial R}{\partial T}\right)_s ;$$

$$C_s^2 = \beta\gamma P_0/\rho_0 , \qquad C_m^2 = B_0^2/\rho_0 ;$$

$$\left(\frac{\partial R}{\partial T}\right)_s = \left(\frac{\partial R}{\partial T}\right)_\rho + \frac{\rho_0}{(\gamma - 1)T_0}\left(\frac{\partial R}{\partial \rho}\right)_T .$$

$$Y = \frac{B_{0\theta}^2 B_{0z}^2(q'/q)^2}{4rB_0^2} + \frac{2B_{0\theta}^2\beta P_0'}{r^2B_0^2} + \frac{4B_{0\theta}^4\beta\gamma P_0}{r^3B_0^2(B_0^2 + \beta\gamma P_0)} .$$

The growth rate, Equ.(4), does not have a heat conduction term because the term appears with $\vec{k}\cdot\vec{B}$, which is zero at $r=r_s$. The local mode will stay unstable while other global modes are stabilized by heat conduction. Equ.(4) shows that magnetic field directly affects the stability. If W<0, magnetic shear plays a stabilizing effect through Y which determines ideal MHD stability. As the magnetic field goes to infinity with plasma pressure fixed, ω becomes the isochoric mode growth rate; magnetic pressure does not allow plasma condensation.

For $\vec{k}\cdot\vec{B}\neq 0$, Equ.(3) can be used for condensation modes. I have studied the stability of two different equilibria with a longitudinal current density profile defined as $J_z(r) = J_0(1-r^2)^\alpha$.
The magnitude of α determines magnetic shear; a higher value of α produces higher shear. Profile(A) has the density and temperature profiles defined as $\rho(r)=1+\delta_1(1-r^2)$ and $T(r)=P(r)/\rho(r)$ respectively while profile (B) has $T(r)=1-\delta_2(1-r^2)$ and $\rho(r)=P(r)/T(r)$. Since plasma pressure increases with field twist, T(r) of (A) and $\rho(r)$ of (B) increase with the twist while $\rho(r)$ of (A) and T(r) of (B) do not change. By solving Equs (2) and (3) for different q we study the stability of condensation modes for profile (A) and (B). Here q_0 is $q(=2\pi B_z r/AB_\theta)$ at r=0 and A is an aspect ratio. q stands for the degree of field twist; higher q implies lower field twist.

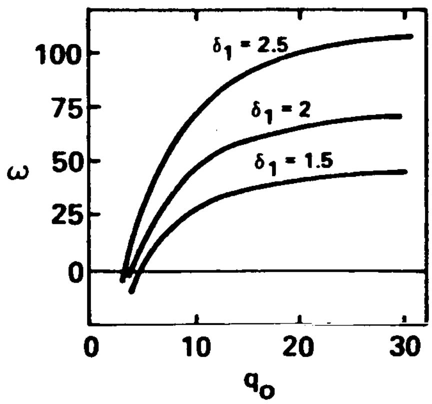

FIG. 1

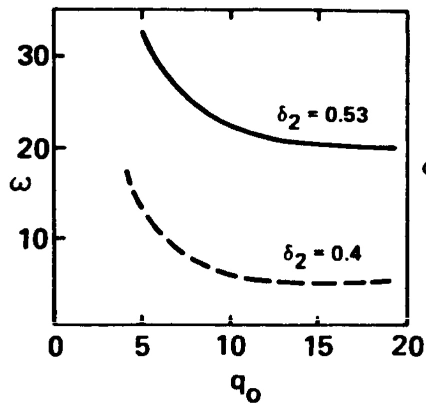

FIG. 2

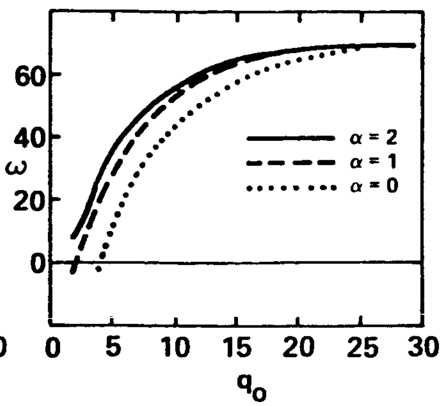

FIG. 3

Fig. 1 shows growth rate ω v.s. q_o for profile(A). Note that the temperature increases but density is constant as twist increases(i.e. lower q_o values). Higher field twist stabilizes the condensation mode due to the increase of temperature (higher heat conduction and lower radiation loss). Growth rate increases with δ_1 for given q_o because density at r=0 increases as $1+\delta_1$ resulting in higher radiation rate. Fig.(2) shows the stability of profile(B). It shows that the mode becomes more unstable as twist increases because density of (B) increases with field twist. The growth rate of δ_2=0.53 is higher than that of δ_2=0.4 for a given q_o because temperature at r=0 decreases with δ_2 as $1-\delta_2$ resulting in lower heat conduction and higher radiation. Fig.1 and 2 have poloidal and longitudinal mode number m and n equal to 1, aspect ratio A=10, and longitudinal magnetic field 10 Gauss. Because we use α =0 the equilibrium does not have shear, in other words, every flux surface has same field twist. Fig. 3 shows the stability of equilibria with different α values. The result seems to show that magnetic shear affects the condensation mode. However, a careful study shows that the different stability is not due to magnetic shear but due to different pressure profiles caused by different choice of α.

It is found from the results that the effect of magnetic field on the global condensation modes with $\vec{k}\cdot\vec{B}\neq 0$ is negligible while the local mode with $\vec{k}\cdot\vec{B}=0$ is affected by magnetic field. Why are the effects of the field on the two modes different? We find the answer by deriving a relation between parallel and perpendicular components of plasma displacements.

$$\left[i\,\frac{2kB_\theta B_0^2}{r^2}\,X + \frac{iB_0^2}{r}\left(\frac{mB_z}{r} - kB_\theta\right)X' - B_0^2 K(\xi \times B_0)_r\right]\frac{(k \cdot B_0)}{kB_\theta - mB_z/r} = \epsilon\rho_0\,\omega^2(\xi \cdot B_0)\,.$$

If $\vec{k}\cdot\vec{B}\neq 0$ the perpendicular displacement is $\epsilon\rho_0\omega^2$ times the parallel component. Since $\epsilon \sim 10^{-6}$, the equation implies that plasma moves mainly parallel to the field line when it condenses. For $\vec{k}\cdot\vec{B}=0$ parallel motion is zero, in other words, plasma moves perpendicular to the field lines carrying the field lines into the condensed region; magnetic field affects the modes.

IV. CONCLUSION

The stability of condensation modes strongly depends on how density and temperature vary with field twist. If plasma temperature increases with twist but density does not (e.g. profile(A)), then condensation modes are unstable for low field twist-when ideal MHD modes are stable. This result implies that prominences form in a globally stable magnetic configuration, which may explain the stable nature of prominences. On the other hand, if plasma density increases with twist but temperature does not (e.g. profile(B)), condensation modes becomes more unstable. If plasmas obey profile (B), we may not observe stable quiescent prominences.

If the effect of magnetic field on condensation modes is insignificant, what is the role of the field on the prominence formation? We may say that the magnetic field has active and passive roles in the formation. The passive role is to insulate the prominence material from hot corona and to guide the plasma motion along the field lines when condensing. The active roles are to trap the outflowing plasmas (solar wind, spicules, and etc.) which then accumulate on the field lines, and to hold and support them against gravity. Depending on the strength and configuration, the magnetic field can trap the plasma effectively, enhancing the density sufficiently to initiate condensation, and can support the condensed plasmas to form a prominence(An, et.al 1986). Without the passive role, however, no prominence can form in a hot coronal plasma.

This research is supported by NAS/NRC and NASA Office of Solar and Heliospheric Physics and Office of Space Plasma Physics.

REFERENCES

An, C.-H., 1984a, Astrophys. J., 276, 755
____ , 1984b, Astrophys. J., 276, 352
____ , 1984c, Astrophys. J., 284, 422
____ , 1985, Astrophys. J., 298, 409
An, C.-H., Suess, S.T., Tandberg-Hanssen, E., and Steinolfson, R.S., 1986, Solar Phys., (in press)
Anzer, U. and Tandberg-Hanssen, E., 1970, Solar Phys., 11, 61
Chiuderi, C., and Van Hoven, G., 1979, Astrophys. J. (Letters), 232,L69
Field, G.B., 1965, Astrophys. J. 142, 531
Heyvaerts, J., 1974, Astron. Astrophys. 37, 65
Hildner, E., 1971, Ph.D. thesis, Univ. of Colorado
Nakagawa, Y., 1970, Solar Phys., 12, 419
Pneuman, G.W., 1983, Solar Phys. 88, 219