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CASCADE MODEL OF CORONAL HEATING

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INTRODUCTION

Coronal heating theories can be classified as either wave-heating or current-heating theories. According to the wave-heating model, convective motions below the solar surface generate MHD waves, which then propagate outward and dissipate their energy in the corona. Observations of line widths in the solar chromosphere seem to rule out the possibility that the corona is heated by acoustic- or slow mode MHD waves (Athay and White 1978), leaving Alfvén- and fast mode waves waves as possible candidates of energy transport. According to the current-heating model, the sub-surface convective motions cause random displacements of the photospheric magnetic footpoints, leading to twisting and braiding of the coronal magnetic field. The field-aligned electric currents associated with these twists are subject to resistive dissipation. The current-heating model applies only to "closed" magnetic structures such as coronal loops, whereas the wave model applies to both open structures (coronal holes) and closed structures (active regions). Another difference between the two models is the time scale τ of the photospheric motions: in wave-heating models, τ is smaller than or equal to the resonance time $2L/v_A$ of the loop, whereas in current-heating models $\tau \gg 2L/v_A$ (L is the loop length, v_A is the Alfvén speed).

The details of the mechanisms responsible for wave- and current dissipation are presently not well understood. The problem is that dissipative processes such as plasma resistivity and viscosity are unimportant on the large spatial scales of observable coronal structures: dissipation can occur only if there are strong gradients in the magnetic- and/or velocity field, with length scales of 1 km or less in the corona. A crucial problem in any theory of coronal heating is, therefore, to explain how such small-scale structures are generated. In the context of the wave-heating theory, phase-mixing of Alfvén waves, due to density inhomogeneities in the solar corona, provides a way to produce small-scale structures (Heyvaerts and Priest 1983, Sakurai and Granik 1984, Steinolfson 1985).

In the current-heating theory, which is the subject of the present paper, the magnetic energy associated with the braided magnetic field must be similarly transferred to smaller scales. The process by which this "cascade" of magnetic energy occurs is not well understood. Parker (1972, 1979, 1983, 1986) suggests that the process is due to an intrinsic nonequilibrium of magnetic fields: the equations of magnetostatic equilibrium seem to allow solutions only for certain restrictive cases, in which the vertical component of the vorticity in the photosphere is essentially a constant of motion (Parker 1986). Since the motions on the Sun do in general not have this nice property, the magnetic field cannot simply adjust to the slow, random motions applied at the photosphere, but is forced to evolve on the Alfvén time scale L/v_A. Parker assumes this dynamical relaxation leads to the formation of discontinuities (current sheets), where magnetic reconnection will occur until the topological constraints are satisfied. Hence, according to the nonequilibrium model the formation of small-scale structures is due to a relaxation process that takes place on the Alfvén time scale.

Recently, I proposed a somewhat different picture of the cascade process (van Ballegooijen 1985, 1986, hereafter papers I and II). According to this model there are no special restrictions on the velocity fields that may be applied at the photospheric boundary: the magnetic field in a coronal loop can evolve slowly through a series of equilibrium states, without the necessity for magnetic reconnection. Hence, in our opinion the dynamical relaxation process proposed by Parker does not occur. The necessary condition for equilibrium found by Parker (1972) was shown to result from an incorrect ordering of terms in Parker's perturbation scheme (cf. appendix of paper I). Therefore, a firm mathematical basis for Parker's concept of nonequilibrium seems to be lacking.

In the absence of nonequilibrium, the magnetic field evolves through a series of equilibrium states. Hence the formation of small-scale structures such as current sheets cannot be due to the relaxation process by which magnetic equilibria are reached; if current sheet formation occurs in closed coronal structures, it must be a result of the quasi-static evolution process. We expect, therefore, that the time scale for current sheet formation is related to the time scale τ of the photospheric motions. To test this hypothesis, we need to understand the properties of braided magnetic fields as they evolve quasi-statically in response to random motions applied at the photospheric boundary. This is a rather difficult 3-dimensional problem, and a general procedure for computing statistical quantities such as the magnetic power spectrum are not known. However, one can gain some insight into the nature of the cascade process from a simplified 2-dimensional problem, in which only a single plane transverse to the mean magnetic field is considered (cf. paper II). In the following this model is briefly discussed.

STATISTICAL MODEL

Consider an initially uniform field $\mathbf{B}_0 = B_0\,\hat{\mathbf{z}}$, extending between two flat boundary plates located at $z = 0$ and $z = L$. We assume that the field is perturbed by a random, incompressible motion in the boundary plates, characterized by a correlation length ℓ and a correlation time τ (we assume $\ell \ll L$). Then the velocity field in the interior of the volume is given by:

$$\mathbf{v} = [v_x(x,y,z,t), v_y(x,y,z,t), 0], \tag{1}$$

and the magnetic field is approximately given by:

$$\mathbf{B} = B_0[b_x(x,y,z,t), b_y(x,y,z,t), 1], \tag{2}$$

where b_x and b_y denote the transverse field components ($b_x, b_y \ll 1$). Assuming ideal MHD, the induction equation can be written as:

$$\frac{db_x}{dt} = \frac{dv_x}{dz} \equiv a_x, \tag{3a}$$

$$\frac{db_y}{dt} = \frac{dv_y}{dz} \equiv a_y, \tag{3b}$$

where d/dz is the spatial derivative along fieldlines, and d/dt is the co-moving time derivative.

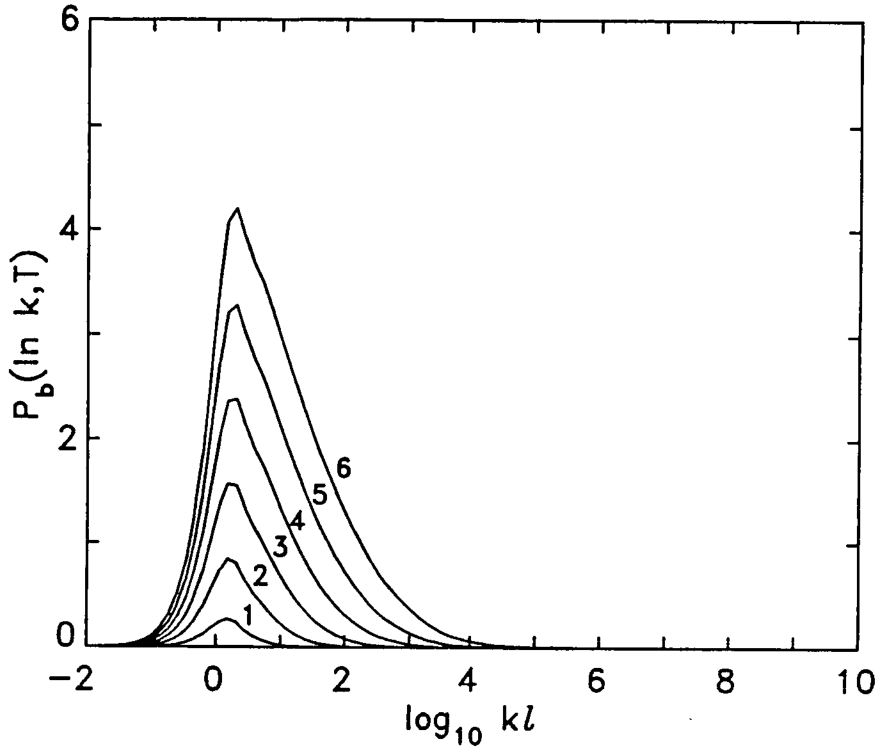
The basic idea of the model is to consider an arbitrary plane $z = z_0$ in the interior of the volume ($0 < z_0 < L$), and to consider the velocity $\mathbf{v}(x,y,z_0,t)$ and the velocity gradient $a_x(x,y,z_0,t)$ at this plane as the *independent* statistical variables. Integration of equation (3a) then yields the transverse field b_x:

$$b_x(\mathbf{R},T) = \int_0^T a_x(\mathbf{r}(t),t)\; dt, \tag{4}$$

where $\mathbf{r}(t) = [x(t), y(t), z_0]$ is the path in the z_0-plane that ends at position $\mathbf{R}$ on time T. The correlation between the b_x-values at two different points $\mathbf{R}_1$ and $\mathbf{R}_2$ in the z_0-plane is:

$$\begin{aligned} C_b(\Delta\mathbf{R},T) &\equiv < b_x(\mathbf{R}_1,T) b_x(\mathbf{R}_2,T) > \\ &= \int_0^T\int_0^T < a_x(\mathbf{r}_1(t'),t') a_x(\mathbf{r}_2(t''),t'') > \; dt'\; dt''. \end{aligned} \tag{5}$$

By making suitable assumptions about the statistical properties of a_x, one can evaluate the right hand side of equation (5), which yields the magnetic correlation function $C_b(\Delta\mathbf{R}, T)$. We omit here the details of the derivation, which is given in paper II. The magnetic power spectrum $P_b(\mathbf{k}, T)$ as function of transverse wavenumber $\mathbf{k}$ is obtained by taking the Fourier transform of C_b with respect to $\Delta\mathbf{R}$.



The results of the calculation are displayed in the above figure, which shows the magnetic power spectrum (per unit $\log k$) as function of dimensionless wavenumber $k\ell$. The different curves correspond to times $T/t_b = 0.4$, 0.8, 1.2, 1.6, 2.0, and 2.4, where t_b is the "braiding" time defined by:

$$t_b \equiv \frac{\ell^2}{u^2 \tau}, \tag{6}$$

and where ℓ, τ and u are the correlation length, correlation time and r.m.s. velocity of the photospheric motions, respectively. Note that magnetic energy, injected into the system at wavenumber $k \sim \ell^{-1}$, is rapidly transferred to larger k: the maximum wavenumber in the spectrum increases exponentially with time, implying a rapid cascade of magnetic energy towards smaller length scales. This cascade takes place on the time scale t_b, which is determined entirely by the statistical properties of the photospheric motions. Note, that t_b is somewhat larger that the correlation time τ, since the fluid displacements over one correlation time are generally smaller than the correlation length ($u\tau \leq \ell$).

The exponential increases of the maximum wavenumber derives from the fact that, in an incompressible random flow, the separation between closely neighboring fluid particles $\mathbf{r}_1(t)$ and $\mathbf{r}_1(t)$ increases exponentially with time; the e-folding time is of order t_b. Hence, the correlation function $< a_x(\mathbf{r}_1(t), t) a_x(\mathbf{r}_2(t), t) >$ appearing on the right hand side of equation (5) vanishes for time differences $(T - t)$ larger than $t_b \ln(\ell/\Delta R)$, when the separation of the particles becomes larger than the correlation length. As a result the correlation function $C_b(\Delta R, T)$ has a logaritmic dependence on ΔR, with a sharp peak at $\Delta R = 0$. The Fourier transform of C_b therefore has significant power at high wavenumbers.

The electric current density, $j_z = (c/4\pi)[\partial B_y/\partial x - \partial B_x/\partial y]$, involves the derivatives of the transverse field, and therefore the power spectrum of current-density fluctuations is given by $P_j(k,T) = k^2 P_b(k,T)$. It can be shown that the integral of $P_j(k,T)$ over wavenumber, which is equal to $< j_z^2(T) > \equiv j_{rms}^2$, increases exponentially with time:

$$j_{rms}(T) \sim \frac{c}{4\pi}\frac{B_0}{L}\exp\left[2\sqrt{2\pi}\frac{T}{t_b}\right]. \tag{7}$$

This should be contrasted with the "free" magnetic energy, which increases only quadratically with time:

$$\frac{< B_\perp^2(T) >}{8\pi} \approx \frac{B_0^2}{8\pi}\frac{2u^4\tau^2T^2}{\ell^2L^2}. \tag{8}$$

Since the cascade of energy towards smaller scales proceeds exponentially in time, magnetic diffusion and reconnection will become important after a time t_1 which depends logaritmically on the magnetic Reynolds number:

$$t_1 \approx t_b\frac{\ln R_m}{6\sqrt{2\pi}}. \tag{9}$$

Here R_m is defined as:

$$R_m \equiv \frac{\ell^2}{\eta t_b}, \tag{10}$$

where η is the magnetic diffusivity based on the classical (Ohmic) resistivity. For the sun, $R_m \sim 10^{10}$, so that $t_1 \approx 1.5t_b$. This implies that only a small number of braids can be introduced into the system before reconnection becomes important.

I suggest that for $t \gg t_1$ a *statistically stationary state* develops, in which there is a continuous transfer of magnetic energy from the scale ℓ where the energy is put in, to the scale $\ell R_m^{-1/2}$ where the energy is dissipated. The dissipation rate E_H in this stationary regime can be estimated as the time derivative of expression (8), evaluated at the time t_1 when reconnection processes first become important; this yields:

$$E_H \sim \frac{B_0^2}{8\pi}\frac{2u^2\tau\ln R_m}{3L^2\sqrt{2\pi}}, \tag{11}$$

i.e., the heating rate depends logaritmically of the Reynolds number. Note that E_H is proportional to the product $u^2\tau$, which is directly related to the effective diffusion constant of the photospheric motions:

$$D = \frac{1}{2}u^2\tau\sqrt{2\pi}. \tag{12}$$

Observations of the spreading of active regions over time scales of months indicate that D is in the range of 150 - 425 km^2/s (DeVore et al. 1985). With $B_0 = 100$ G and $L = 10^5$ km, parameters typical for large active regions, we find $E_H \sim 5 \times 10^{-5}$ $erg/cm^3/s$, which corresponds to an energy flux of 2.5×10^5 $erg/cm^2/s$ at each footpoint. This energy flux is a factor 40 smaller than the observed radiative- and conductive losses in active regions (cf. Withbroe and Noyes 1977). There are a number of possible reasons for this discrepancy:

1) Our assumption that E_H equals the energy input rate at $t = t_1$ probably under-estimates the heating rate, since the input rate may continue to increase for some time after reconnection

first becomes important. To see whether this is the case, it is necessary to include magnetic diffusion in the above analysis.

2) There may be small-scale photospheric motions with $D > 1000\ \mathrm{km}^2/\mathrm{s}$ which have so far escaped detection.

3) It is possible that random photospheric motions are not the primary cause of coronal heating in active regions; periodic motion associated with Alfvén waves may be a more important source of energy.

In summary, we suggest that the quasi-static evolution of coronal magnetic structures is characterized by a cascade of magnetic energy to smaller length scales. This cascade process takes place on a time scale t_b determined entirely by the photospheric motions. The Ohmic heating rate E_H in the statistically stationary state was estimated using observational data on the diffusivity of photospheric motions; E_H turned out to be too small by a factor 40 when compared with observed coronal energy losses. However, given the fact that our theoretical estimate is based on a rather uncertain extrapolation to the diffusive regime, current heating cannot be ruled out as a viable mechanism of coronal heating.

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