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MAGNETIC HELICITY AS A CONSTRAINT ON CORONAL DISSIPATION

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Introduction

An attractive scenario for the heating of coronal active regions has been developed in a series of papers by Parker (1983 and the references therein). The main idea is that the slow footpoint motions in the photosphere distort the overlying coronal field structures in a random fashion, and since there may not be equilibrium configurations without current sheets available for the coronal magnetic fields for arbitrary footpoint motions, we have a non-equilibrium situation giving rise to current sheet formations and energy dissipation. However, it seems virtually impossible to study the detailed dynamics of this non-equilibrium process with the present techniques of MHD. The only way of estimating the heating due to magnetic dissipation seems to be to invoke more global considerations which avoid the problem of describing the detailed dynamics. Parker (1983) suggested that one can calculate the heating by estimating the work done by the footpoints on the coronal magnetic fields. Sturrock and Uchida (1981) estimated the energy in the twists produced by the footpoint motions and assumed that the whole of that energy is available for dissipation. However, recently Heyvaerts and Priest (1984; see also Browning and Priest 1986) pointed out that because of the constraint imposed by the magnetic helicity conservation, all the energy that is fed into the corona by footpoint motions may not be available for dissipation. It has been rigorously demonstrated by Berger (1984) that the time-scale for magnetic helicity decay in coronal magnetic structures is indeed orders of magnitude larger than any relevant time-scale for coronal heating. Heyvaerts and Priest (1984) are certainly correct in pointing out that when footpoints put some energy in the corona, only that much of it can dissipate which is consistent with helicity conservation. However, we want to show that when one extends the Heyvaerts-Priest model to a statistics of completely random footpoint motions and takes sufficiently long time averages, the conservation of helicity *introduces no effective constraint* on energy dissipation, and all the energy in the twists is, in principle, available for dissipation, as proposed by Sturrock and Uchida (1981).

Taylor's Hypothesis and Magnetic Dissipation

The magnetic helicity for a magnetic configuration, which is the volume integral $K = \int \vec{A} . \vec{B} \, dV$, is a measure of the linkage of flux lines and can be easily shown to be independent of gauge if the magnetic configuration is bounded by a closed surface on which $\vec{B} . \hat{n} = 0$ everywhere. For a perfectly conducting plasma bounded by such a surface, Woltjer (1958) showed that the magnetic helicity is a constant of motion. But what happens if the plasma is not perfectly conducting and has a small but finite resistivity ? Taylor (1974) advanced the provocative hypothesis that the total magnetic helicity of an isolated volume of plasma can still be considered to be approximately conserved over the time scale of energy decay. In other

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words, when a plasma relaxes to the final static equilibrium state through non-equilibrium dynamical processes, we can think of those processes as a way of minimizing energy while keeping helicity (and any other conserved quantity) constant. For a plasma with negligible gas pressure compared to magnetic pressure, if one minimizes the energy with the helicity conservation as a constraint, then one obtains the force-free equation

$$\vec{\nabla} \times \vec{B} = \mu \vec{B}, \tag{1}$$

where μ is the Lagrange multiplier in the variational procedure. Taylor's hypothesis was remarkably successful in describing the final field configurations in the well-known "reversed pinch" experiments involving laboratory plasma relaxation (Taylor 1974).

Most astrophysical systems are not bounded by $\vec{B}.\vec{\hat{n}} = 0$ surfaces. So, in order to apply the above ideas to astrophysical plasmas, one has to generalize to bounding surfaces with field lines threading through them. For such open systems, Berger and Field (1984) showed that it is still possible to define a relative magnetic helicity with respect to a ground state potential configuration and make sure that all the physically relevant quantities turn out to be gauge-invariant. In such an open system, there can also be a helicity flow across the bounding surface. For a plane surface like the photosphere with footpoint motions lying in the plane, the rate of flow is given by

$$(\text{helicity flow}) = -2\int (\vec{A}_P . \vec{v})(\vec{B} . \vec{ds}), \tag{2}$$

where $\vec{A}_P$ is the divergence-free vector potential for a potential field with the appropriate flux boundary conditions (for details see Berger 1984). One can generalize Woltjer's theorem (and probably Taylor's hypothesis) for such systems if one keeps track of the changes in the value of helicity within the volume as a result of flows across the boundary.

With this rather terse background, let us briefly summarize the main ideas presented by Heyvaerts and Priest (1984). Let us consider a magnetic region in the corona, which is initially in the Taylor state with helicity K_i and corresponding minimized energy W_i. The individual flux tubes in this region can conceivably interact with each other through neutral point reconnections at the boundaries, but, if the region is sufficiently separated from other magnetic regions, the whole region can be thought of as a candidate for the application of Taylor's hypothesis. As a result of footpoint motions, there will be a flow of both helicity and energy into that coronal magnetic region, with the energy flow across the boundary given by

$$(\text{energy flow}) = \frac{1}{4\pi} \int (\vec{B} . \vec{v})(\vec{B} . \vec{ds}). \tag{3}$$

Let ΔK and ΔW be the additional amounts of helicity and energy put into the coronal region in some interval of time. Then the final magnetic configuration should have a helicity $K_i + \Delta K$, and suppose the energy of the corresponding Taylor-relaxed state is $W_i + \Delta W_f$. We do not expect *a priori* ΔW and ΔW_f to be equal, and the difference $\Delta W_{diss} = \Delta W - \Delta W_f$ is dissipated away. In other words, the whole of the energy ΔW put in the corona by the footpoint motions is not available for dissipation, but only a fraction ΔW_{diss} of it. One remarkable result of the detailed calculations was that when one considered the plasma to relax instantaneously to the Taylor state in response to footpoint motions, one found $\Delta W_{diss} = 0$. Only when the finite relaxation time (which should be small compared to deformation time scales for the model to work) is taken into account, ΔW_{diss} comes out to be non-zero.

Another interesting application of Taylor's hypothesis in astrophysics was made in the study of extragalactic jets, which provided an explanation of the non-axisymmetric oscillations observed in some jets and the associated magnetic structures inferred from synchrotron radiation (Königl and Choudhuri 1985). In this problem also, it was found out that magnetic energy is available for dissipation only when the finite time for relaxation to Taylor state is taken into account (Turner 1986; Choudhuri and Königl 1986).

It has recently been pointed out that quite generally no magnetic energy can be available for dissipation, if a magnetic system always relaxes to the Taylor state instantaneously in response to changes at the boundary (Browning and Priest 1986; Berger 1985). This result helps us to understand the conclusions arrived at both by Heyvaerts and Priest (1984) and by Choudhuri and Königl (1986). A physical explanation of this result is also not difficult to give. Since the magnetic field in the Taylor state is well-behaved and without neutral points, we do not expect dissipation to take place if the system always remains in the Taylor state because of instantaneous relaxation. However, when the relaxation time is finite (but small compared to the deformation time), if we start deforming a magnetic system away from a Taylor state, initially it will tend to depart from the Taylor state until current sheets form, and afterwards dissipation may prevent further departures from Taylor state. Taylor's hypothesis is a particularly powerful tool in deriving the magnetic configuration of the final relaxed state in terms of conserved quantities and boundary conditions. However, when we apply this hypothesis to study the energy dissipation problem, we find that dissipation arises only due to departures from the Taylor state, and consequently any final expression for energy dissipation necessarily involves some arbitrary parameter describing the measure of departure from the Taylor state (or equivalently describing the relaxation time scale). It is found that the relaxation time has to be a few tens of Alfvén time in order to give the right sort of dissipation (Browning and Priest 1986; Choudhuri and Königl 1986).

It is interesting to note that Parker (1983) arrived at a similar conclusion from completely different considerations. He pointed out that the footpoints are able to do sufficient work on the coronal magnetic structures only if we allow stresses to build up for some time. In order to get the right value of the heating rate, the relaxation time for these stresses has to be of the order of a day (i.e. about 100 Alfvén transit times for a coronal loop, depending on the values chosen for different quantities). Parker (1983) also estimated that this relaxation time corresponds to a reconnection rate which is the geometric mean between the Sweet-Parker and the Petschek rates.

Extension of Heyvaerts-Priest Model for a Statistics of Completely Random Footpoint Motions

In a magnetic region in the corona, two processes go on side by side. One is the process of the growth of complexities in the field structures as a result of footpoint motions. The other process is dissipation, which attempts to burn away the increasing complexities. Except for a runaway situation where complexities build up more quickly than they can be dissipated away, we expect these two processes eventually to reach some sort of balance. In other words, we expect a "steady state" in the statistical sense such that the complexity of magnetic structures would statistically be maintained at the same level, provided the timescale of evolution of the coronal structure as a whole is much larger than all the time scales involved. In such a "steady state", whatever energy the footpoints are putting in the corona has to be dissipated away in order to preserve the balance. If helicity conservation prevents a part of this energy from being dissipated, then we have to figure out what eventually happens to this undissipated energy.

We can resolve this puzzle by extending the Heyvaerts-Priest model to a statistics of completely random footpoint motions. In order to understand the basic physics of the process, let us start with a magnetic region which is initially in the potential configuration, and then let the footpoint motions distort it. Following Berger and Field (1984), we measure the magnetic helicity relative to potential fields with the same flux boundary conditions. So, by definition, the initial helicity of our system is zero. Let us now consider a flux tube in the region that is being twisted. Neglecting curvature (which is not expected to change the basic physics), we imagine a cylindrically symmetric flux tube to be anchored between two parallel planes (initially without twist because its field is potential) and to be twisted by a rotationally symmetric velocity field at the bottom plane. Since the magnetic field can be upward or downward, and the velocity field can be clockwise or anti-clockwise, we can have four cases as shown in Fig. 1. Initially $(\vec{B}.\vec{v})$ for all four of them is zero so that we see from (3) that there is no energy flow at first. However, the twist due to footpoint motion gives rise to a ϕ-component of the magnetic field, and when that component is taken into account, it is easy to see that $(\vec{B}.\vec{v})(\vec{B}.\vec{ds})$ is positive for all the four cases, i.e. energy is put into the corona in every case. Now let us look at the helicity flow. The divergence-free vector potential $\vec{A}_P$ for the initial field is in the ϕ-direction, one way for the upward field and the other way for the downward field. The footpoint motions we are considering do not change the flux boundary condition and hence do not change $\vec{A}_P$. We thus find that $(\vec{A}_P.\vec{v})(\vec{B}.\vec{ds})$ is of one sign for the cases (a) and (c), and is of the opposite sign for the cases (b) and (d). In other words, when we sum over the four cases, there is no net helicity flow though there is net energy flow. If we use the gauge chosen by Heyvaerts and Priest (1984) and just use the fact that $\vec{A} \rightarrow -\vec{A}$ when $\vec{B} \rightarrow -\vec{B}$, then also we end up with the same conclusion.

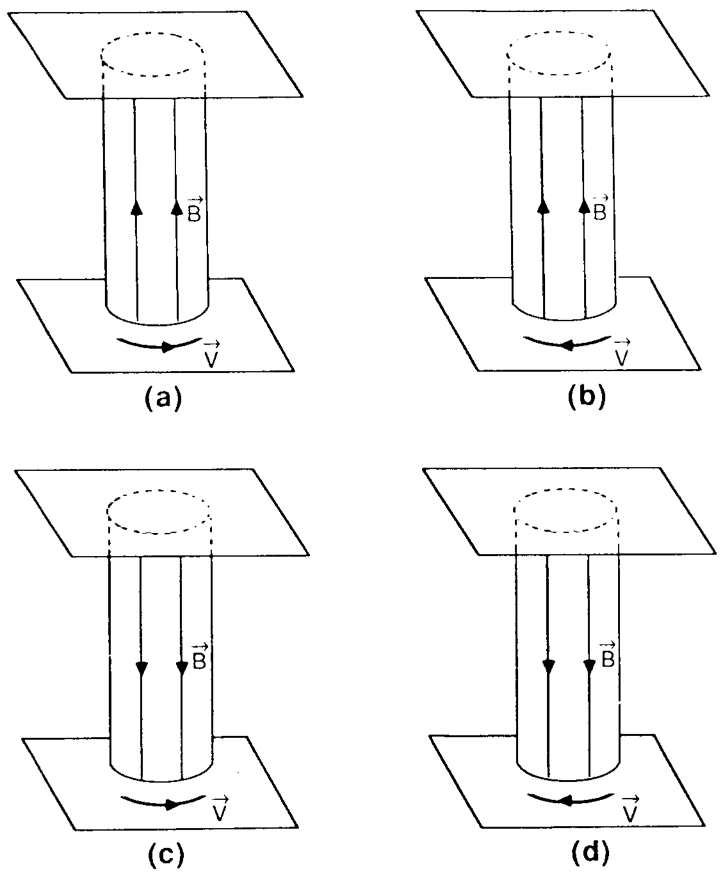

Figure 1

It is straightforward to generalize these conclusions for the case of flux tubes braiding around each other. So long as the footpoint motions are as likely to twist the tubes one way as the other way, the average statistical result will be the addition of energy to the coronal structures without addition of magnetic helicity. The footpoint motions will merely add positive helicity in some regions and negative helicity in others. There are bound to be statistical fluctuations in the value of helicity. However, if one starts from a zero-helicity state and takes sufficiently long time averages, the average value of helicity would remain zero. A Taylor state with zero magnetic helicity is, by virtue of our definition of helicity, a potential field in which all the twist has been dissipated away. We thus conclude that Taylor's hypothesis imposes no constraint on energy dissipation for completely random footpoint motions. We have also seen that a magnetic system has to depart from the Taylor state to some extent if dissipation is to take place. Consequently, we expect that the footpoint motions and the dissipation together would maintain a coronal magnetic region at a steady level of complexity sufficiently removed from a potential configuration such that whatever energy goes into the work done by the footpoints ultimately comes out in the dissipation of the twists.

It is perhaps interesting to ask at this point if footpoint motions in the sun are really completely random so that coronal magnetic structures always have statistically zero helicity. One occasionally sees structures in the corona which seem to contain magnetic fields twisted in one way. However, this can arise only if these magnetic structures erupt through the photosphere with the twist already present, or else if there is a systematic component in footpoint motions. It is conceivable that solar rotation may have some subtle effect on the convection cells so that they preferably tend to put helicity of one sign in the corona, or there may be shearing motions in the photosphere due to dynamical reasons we are still ignorant of. Apart from such minor effects, we expect the footpoints to put mainly energy in the corona with very little net helicity.

Conclusion

Taylor's hypothesis has provided us a model for the relaxed magnetic configurations of not only laboratory plasmas, but also of astrophysical plasmas (Königl and Choudhuri 1985). However, energy dissipation is possible only for systems which depart from a strict Taylor state, and hence one has to introduce a parameter describing that departure, when one uses Taylor's hypothesis to estimate the dissipation (Heyvaerts and Priest 1984; Choudhuri and Königl 1986). An application of Taylor's hypothesis to the problem of coronal heating provides us with new insight into this difficult problem. When particular sorts of footpoint motions put energy and helicity in the corona, the conservation of helicity puts a constraint on how much of the energy can be dissipated. However, on considering a random distribution of footpoint motions, this constraint gets washed away, and Taylor's hypothesis is probably not going to play any significant role in the actual calculation of relevant physical quantities in the coronal heating problem.

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