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The Stability of Coronal and Prominence Magnetic Fields

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INTRODUCTION

The magnetic fields in Prominences and Active Regions (Coronal Arcades) may be susceptible to a variety of instabilities. Ideal MHD instabilities are the fastest growing and criteria for checking stability are complicated by the line tying effect of the dense photosphere. In general, the line tying introduces a coupling of modes and obtaining stability cirteria for a given prominence or arcade field involves either solving partial differential equations (Hood and Priest 1981, Hood 1983, Cargill *et al* 1986) or coupled O.D.E's (Einaudi & Van Hoven 1983) (from a truncated Fourier series). This can be a very time consuming exercise. What is needed is a simpler test applicable to any field.

LOCALISED MODES

Progress can be made by studying localised instabilities or Ballooning modes (Conner *et al* 1979; Dewar and Glasser 1983). By using a WKB approach, (Dewar and Glasser 1983), the idea is to study instabilities localised about a given magnetic flux, with a fast variation perpendicular to the equilibrium field and a slow variation along the field lines. This filters out the stable Alfven and magnetoacoustic waves. Thus, all displacements to coronal arcades are of the form

$$\underset{\sim}{\xi}(r,\theta,z,t) = \underset{\sim}{\xi}(r,\theta)e^{i(S(r,\theta,z)/\epsilon + \omega t)} \tag{1}$$

where ϵ, the instability length scale, $\ll R_0$ the equilibrium length scale and $S(r,\theta,z)$, ω and the amplitudes $\underset{\sim}{\xi}(r,\theta)$ are all $O(1)$ quantities. (For coronal loops see Hood 1986a). The slow variation along the field occurs only when

$$\underset{\sim}{k}.\underset{\sim}{B} = \nabla S.\underset{\sim}{B} = 0. \tag{2}$$

For cylindrically symmetric fields, with the photosphere situated at $\theta = \pm\pi/2$, a solution to Eq (2) is

$$S = S(r) + z - q\theta, \tag{3}$$

where $q = rB_z/B_\theta$. Substituting (1) and (3) into the linearised equations of motion (Hood 1986a) gives

$$B\frac{d}{ds}\left[\frac{k^2}{B}\frac{d\xi}{ds}\right] - 2K_s\left[\frac{\gamma\mu pB^2}{\gamma\mu p + B^2}\left[B\frac{d\eta}{ds} - 2K_s\xi\right] + \frac{\mu dp}{d\psi}\xi\right] + \frac{\rho\mu\omega^2k^2}{B^2}\xi = 0 \tag{4}$$

$$B \frac{d}{ds} \left[\frac{\gamma\mu p B^2}{\gamma\mu p + B^2} \left[B \frac{d\eta}{ds} - 2K_s \xi \right] \right] + \mu\rho\omega^2 B^2 \eta = 0, \tag{5}$$

where, for cylindrical arcades, $Bd/ds = B_\theta d/rd\theta$, $K_s = -B_\theta/rB^2$, $dp/d\psi = -(dp/dr)/B_\theta$, $k^2 = |\nabla S|^2$. Strictly speaking the solutions to Eqs (4) and (5) with the line tying boundary conditions define a dispersion relation for $k^2 = k^2(r;\omega^2)$. The radial integration must satisfy a Bohr Sommerfeld condition (Dewar and Glasser 1983, Hood 1986a) and this defines the physical growth rate. However, if the least stable mode is of interest, the procedure is simpler. Now set $S'(r) \equiv 0$ and solve Eqns (4) and (5) to obtain $\omega^2(r)$. The *minimum* value of $\omega^2(r)$ defines the physical value of ω^2 for the least stable mode. If $\omega^2 < 0$, then there is an instability but, if $\omega^2 > 0$, the localised modes are stable.

For example, Hood (1986a) considered the field

$$B_\theta = B_o(r/b)/(1 + r^2/b^2) \qquad B_z = B_o\,/(1 + r^2/b^2)$$

$$\mu p = B_o^2(1 - \lambda^2)/2(1 + r^2/b^2)$$

and solved equations (4) and (5) to obtain $\omega^2(r)$ as shown in Fig. 1.

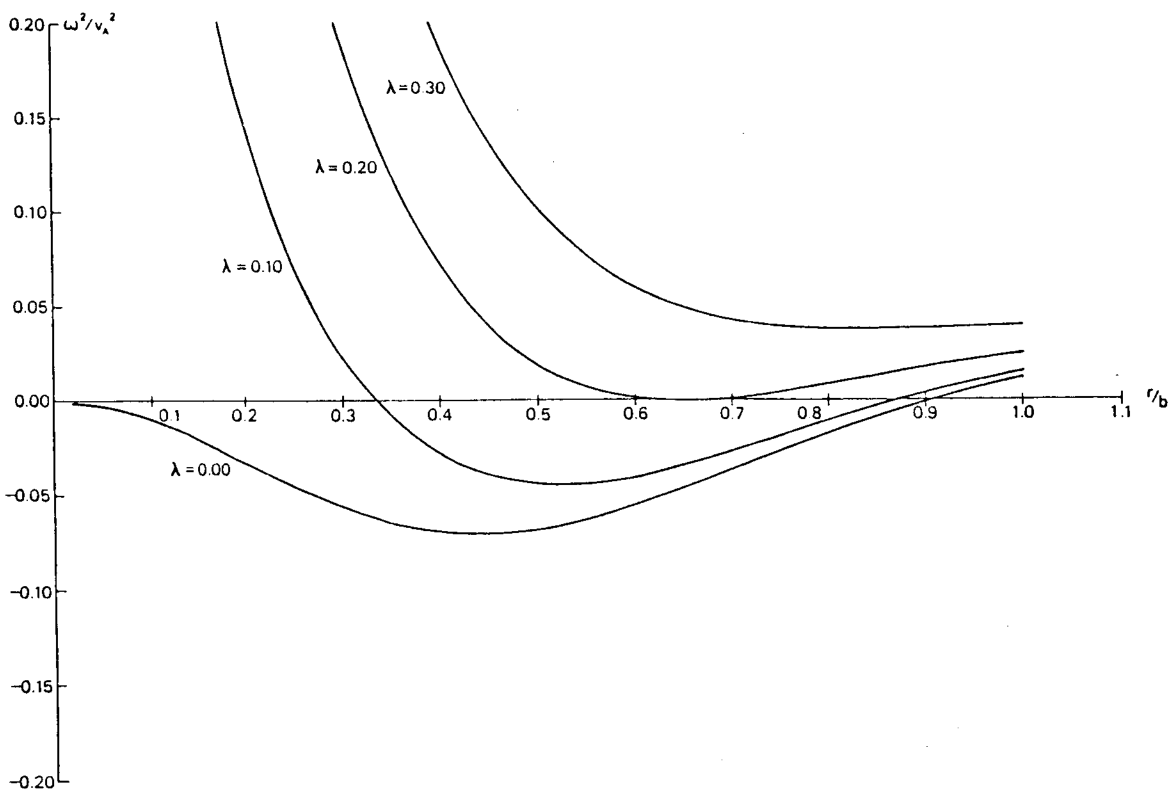

Fig. 1. The eigenvalue ω^2 of equations (4) and (5) is shown as a function of the radial coordinate for various values of λ. The physical value of ω^2, for the least stable mode, is given by the minimum value. ω^2 is measured in units of $v_A^2 = B_o^2/\mu\rho b^2$ and $\gamma = 1$.

Equations (4) and (5) can be converted into quadratic form by multiplying by ξ and η, respectively and integrating. Then the sign of ω^2 can be determined by trial functions. Thus, it can be shown, (Hood 1986a), that the field is *definitely unstable* if

$$\frac{\pi^2}{4} B_z^2 \left[\frac{q'}{q}\right]^2 + \frac{2\mu p'}{r} + \frac{B^2}{r^2} + \frac{\gamma\mu p B_\theta^2}{r^2(\gamma\mu p + B^2)} < 0 \qquad (6)$$

(see Spicer 1976 for a heuristic derivation with $\gamma = 0$). Equation (6) has been derived using the rigid boundary conditions, $\xi = 0$ on the photosphere but $\gamma = 0$ simulates the flow through boundary conditions of $\xi_\perp = 0$ and $\xi_\parallel \neq 0$ (Einaudi and Van Hoven 1983). The first term is the shear stabilisation, the second term is the driving term due to adverse pressure gradients. The last two terms provide line tying stabilisation due to Alfven waves and compressional slow waves.

A CHECK ON LINE TYING CONDITIONS

Using the ballooning approximation, the photospheric line tying conditions can be investigated (Hood 1986b). Equations (4) and (5) were solved including a density variation along the field lines and, when the density difference between the photosphere and corona was increased to realistic values, the value of ω^2 rapidly approached the value predicted by the rigid boundary conditions. In addition, the values of ξ and η at the photosphere tend to zero. This suggests that the rigid boundary conditions are the correct boundary conditions (at least for localised modes).

THE EFFECT OF GRAVITY

The effect of gravity has been included by, for example, Zweibel 1981, Melville *et al* 1986a. Melville *et al* considered the linear equilibria of Zweibel and Hundhausen (1982) and showed that as soon as a magnetic island appeared the field became unstable. Interestingly enough, each field line (except the O point itself) was unstable *before* it formed an island. The analysis has been extended to other fields (Melville *et al* 1986b), and preliminary results suggest that, as the effect of gravity is increased, the fields become more susceptible to the Rayleigh-Taylor instability. However, a simple test, similar to Equation (6), has yet to be developed.

CONCLUSIONS

The significance of the localised instabilities is not yet fully understood. The nonlinear coupling of these modes may give rise to an explosive instability, with the modes coupling to longer wavelengths, (Mondt and Weiland, 1985). On the other hand, if the modes saturate early, then the main effect of the instability may be an enhancement of transport coefficients. Nonetheless, Equation (6) provides a simple test for the stability of cylindrical magnetic fields.

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