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Statistical Aspects of Solar Flares

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and Space Administration

**Scientific and Technical
Information Branch**

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I. INTRODUCTION

Serendipity has often played a major role in science. An example of this is the *chance* observation of a solar flare, observed independently by Carrington and Hodgson in white light on 1 September 1859, just 5 months before the maximum of cycle 10 [1-3]. While this milestone observation revealed something totally new and *unexpected* occurring on the Sun, observation of a second solar flare in white light (by Trouvelot) did not come until much later, on 17 June 1891 during the rise of cycle 13 [2]. Following introduction of the spectroheliograph, which allowed the first Hα photographs of the Sun, and later the spectrohelioscope, which allowed direct *visual* observation of the Sun in Hα and which truly heralded the beginning of routine systematic investigations of solar flares, chances for accidentally catching a solar flare were vastly improved.

Initially, investigations of solar flares often were descriptive of a single event or a small collection of events, and they usually dealt more with associational aspects between solar flares and effects on Earth rather than with statistical or physical properties of flares [4-11]. In the late 1930s and much of the 1940s, more general comments regarding the statistics of flares were described [12-18]. These early statistical studies, based on the flares of cycles 17 and 18, showed that solar flares occur suddenly and are transitory, usually being short-lived and having rapid rise to maximum emission, being localized and not affecting nearby photospheric structures, and being related to changes in the area and magnetic complexity of associated sunspots (i.e., the probability for flare occurrence was recognized as being greater when sunspots are growing and when the magnetic structure of the spots is more complicated). Later studies (in the 1950s and 1960s) have confirmed many of the earlier findings and have added greatly to the understanding of solar flares [19-27], some investigating correlation between flares and radio events on Earth and others investigating distributions of flare duration and rise time and of hemispheric asymmetries, yielding results which sometimes were controversial.

This paper presents results of a study of 850 Hα flares occurring in 1975, just prior to the minimum of cycle 21. Investigated are the statistical properties of these flares (distributions of rise time, decay time, and duration; mean, mode, median, and 90th percentiles; proportions for selected time intervals and for selected groupings of flares), correlations against monthly mean sunspot number and between other parameters (e.g., between decay time and rise time and between duration and rise time), hemispheric asymmetries (north-south, east-west, limb-disk, etc.), the statistical significance of differences found in the distributions and groupings, and a comparison of results reported herein with results reported earlier for different epochs.

II. APPROACH

The 850 flares included in this study have been extracted from the larger compilation of flares contained in issues of *Solar-Geophysical Data* (Part II, Comprehensive Reports, Nos. 371-381; National Geophysical Data Center, Boulder, Colorado, U.S.A.). These study flares represent all events which were well observed throughout their lifetimes (i.e., each was listed as having definite times of start, maximum, and end, thereby allowing unambiguous determination of rise time, decay time, and duration). For events observed by two or more observatories, averages have been used to designate the times of rise, maximum, and end. Also noted for each of the flares have been its Hα importance (i.e., subflares, 1, 2, etc.), location (latitude and central meridian distance), and whether or not it could be associated with large-scale mass motion (denoted by specific letter codes in the *SGD*; i.e., A, H, L, M, R, S, U, and V which will be defined later in Section III). Numbers and proportions of flares by month and by selected groupings have been determined, and mean values for rise time, decay time, and duration have been computed. Also, regression analysis and statistical testing have been accomplished.

III. RESULTS

It is sometimes difficult to compare results from early flare studies with those which are more contemporary partly because a universally accepted and standardized flare area-relative brightness classification was not available until only recently [28]. Consequently, often when an investigator spoke of a *small* flare, what was meant was a flare of importance 1, *not* a subflare. Flares, more or less, were classified by flare area as small, medium, and large, roughly corresponding to Hα importance 1, 2, and 3, respectively. In this study a small flare is considered a subflare. It is also interesting to note that, historically, the very term "solar flare" was not universally adopted until the early 1940s, apparently having been coined by Chapman and Bartels in their book entitled *Geomagnetism* (reprinted in 1962) [21,29-32]. Previously, solar flares were termed "solar chromospheric eruptions" or, more simply, "solar eruptions" or "chromospheric eruptions."

A. Overview

Figure 1 (bottom) illustrates the distributions or numbers of occurrence of flares for selected measures of time (in minutes) for rise time, decay time, and duration, using *all* study flares (i.e., discarding flare importance and/or any other categorization). Bins range in value from 0 (meaning a time <1 min) through 60 min to a catch-all bin for all values >60 min. Figure 1 (top) depicts the cumulative percent as one sums bins from 0 to >60 min. Together, they easily allow a visual determination of the mode(s) and median (50 percent) or any other centile for rise time, decay time, and duration of the Hα study flares. A summary of the more important features of Figure 1 is given in Table 1.

Based on the 850 study flares of 1975, mean rise time, decay time, and duration (using 95 percent confidence intervals) are found to be 5.2 ± 0.4 min, 12.9 ± 0.8 min, and 18.1 ± 1.1 min, respectively. Median values are 3 min, 9 min, and 13 min, respectively. The principal mode for rise time is 2 min, while for decay time and duration the principal modes are 6 and 10 min and 10 and 12 min, respectively. The 90th percentile values for rise time, decay time, and duration are 10.5 min, 25.5 min, and 35.5 min, respectively; thus, 90 percent of the study flares had a value for rise time ≤10.5 min, and so forth. In terms of proportions, nearly 90 percent of the study flares had rise time ≤10 min and >99 percent had rise time ≤30 min; slightly more than half (56 percent) had decay time ≤10 min and about 94 percent ≤30 min; and about one-third (36 percent) had duration ≤10 min and 85 percent ≤30 min. Less than 1 percent of the study flares had rise time >60 min, and only 1.5 percent and 2.5 percent, respectively, had decay time and duration >60 min. While bifurcated subgroupings of flares according to length of rise time (*fast* and *slow* rise time events) or duration (*short-* and *long-lived* events) sometimes have been discussed in the past [20,33,34], Figure 1 suggests that such groupings are purely arbitrary.

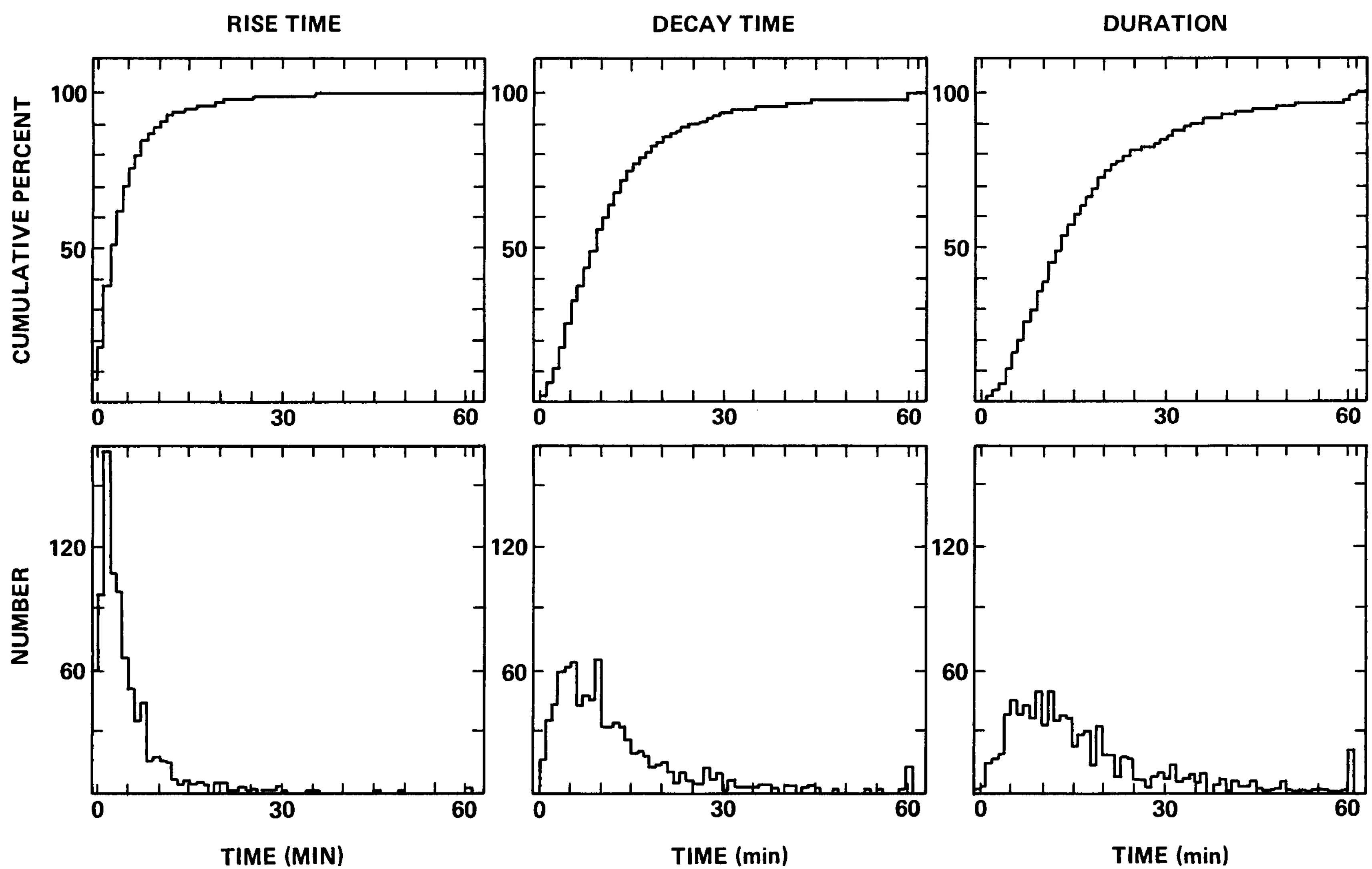

Figure 1. Distributions (bottom) and cumulative percents (top) for rise time, decay time, and duration of Hα solar flares observed in 1975.

TABLE 1. SUMMARY OF STATISTICAL PROPERTIES OF FLARES OCCURRING IN 1975 ($n \doteq 850$)

PARAMETER	RISE TIME	DECAY TIME	DURATION
TIMES (min):			
MEAN	5.2±0.4	12.9±0.8	18.1±1.1
MODE*	2	4 - 6, 10	5 - 15
MEDIAN (Q_{50})	3	9	13
Q_{90}	10.5	25.5	35.5
PROPORTIONS (%):			
≤ 10 min	89.3	56.5	35.5
≤ 30 min	99.2	93.8	85.2
≤ 60 min	99.8	98.5	97.5

*DECAY TIME: 4 min (6.9%), 5 min (7.2%), 6 min (7.5%), 10 min (7.6%)

DURATION: 5 min (4.5%), 6 min (5.3%), 7 min (4.5%), 8 min (5.1%), 9 min (4.2%), 10 min (5.8%), 11 min (3.9%), 12 min (5.9%), 13 min (4.1%), 14 min (4.5%), 15 min (4.2%)

Figure 2 shows the numbers of occurrence and percent occurrence for a number of different subgroupings of the 850 study flares. For example, nearly 90 percent of the study flares are *subflares*, about 4 percent are of importance $\geqslant 1$, and the remainder are of uncertain area (either unknown or the observatories had conflicting reports for its importance). Likewise, nearly 60 percent of the flares are "faint" in terms of the relative brightness, about 20 percent are "normal," 4.5 percent are "bright," and the remainder are of uncertain brightness. Approximately 17 percent of the flares are identified as being associated with large-scale mass motion (to be discussed below). Differences in numbers of occurrence of flares for selected hemispheric groupings (e.g., north-south, east-west, etc.) are found; however, detailed discussion of the asymmetries will be delayed until Section III.D. During 1975, flares are seen to have been more numerous *west* of central meridian and *north* of the equator; also, more flares occur within 45 deg of central meridian (*disk*), as compared to those >45 deg of central meridian (*limb*). No strong east-west asymmetry of limb events is observed. In terms of latitude, 94 percent of the flares are within 15 deg of the equator (*low*), 4 percent at latitudes between 16 and 30 deg (*mid*), and only 1.5 percent at latitudes >30 deg north or south of the equator (*high*). A predominance of low-latitude flares is to be expected for the study flares of 1975, simply because the majority of flares are associated with old or late-cycle spots that occur preferentially near the equator, as compared to new or early-cycle spots that occur at higher latitudes. In the study flares of 1975, no flare occurs at a latitude higher than 39 deg from the equator. In terms of the bifurcated groupings of fast and slow rise time and of short- and long-lived duration (both subgroupings being determined by the mean values of the parameter of interest), flares with rise time $\leqslant 5$ min occur 70 percent of the time and flares of duration $\leqslant 18$ min occur 67 percent of the time.

Tables 2, 3, and 4 give additional information pertaining to the numbers of occurrence and the proportions of mass-motion related flares. Of the 850 study flares, 148 (17 percent) were identified as flares having associated mass motion, based on letter codes given to these events by observers and recorded in *SGD*. These letter codes include A, H, L, M, R, S, U, and V. The letter code A signifies a flare that is associated with an eruptive prominence whose base is <90 deg from central meridian; H is a flare accompanied by a high-speed dark filament; L is an existing filament which shows signs of sudden activity; M is a white-light flare (none were reported in the 850 study flares of 1975, revealing the relative *infrequency* of this type of event occurring, particularly, late in the solar cycle); R is a flare with marked asymmetry in the Hα line, suggesting ejection of high-velocity material; S is a brightness which follows the disappearance of a filament (at nearly the same position); U is a two-ribbon flare (the appearance of two bright branches, either parallel or converging); and V is the occurrence of an explosive phase within the flare (i.e., an important and abrupt expansion in abount 1 min with or without an important intensity increase). These letter codes (and others) are described in the *SGD* (Supplement: Explanation of data reports, any February issue). Letter codes H and U account for one-half and one-fourth, respectively, of the mass-motion related flares, when viewed separately, and account for nearly three-fourths (73 percent) when grouped together (i.e., a flare was designated either H, U, or H/U). Three-fourths of the 148 mass-motion related flares were subflares, 10 percent were flares of Hα importance $\geqslant 1$, and about 15 percent were of indeterminate area. Approximately one-third (30 percent) were associated with "faint" flares, 25 percent with flares of "normal" relative brightness, 10 percent with "bright" flares, and about 34 percent were of indeterminate brightness. For H and U flares, in both cases, about 75 percent were observed to be subflares, while neither were predominated (i.e., greater than half) by any specific single flare brightness group; rather, there was a tendency for H flares to be of faint relative brightness and for U flares to be of normal relative brightness. The following subsections discuss in greater detail other aspects of the statistical properties of the 850 study flares, in total and by subgrouping, and the statistical significance of the results.

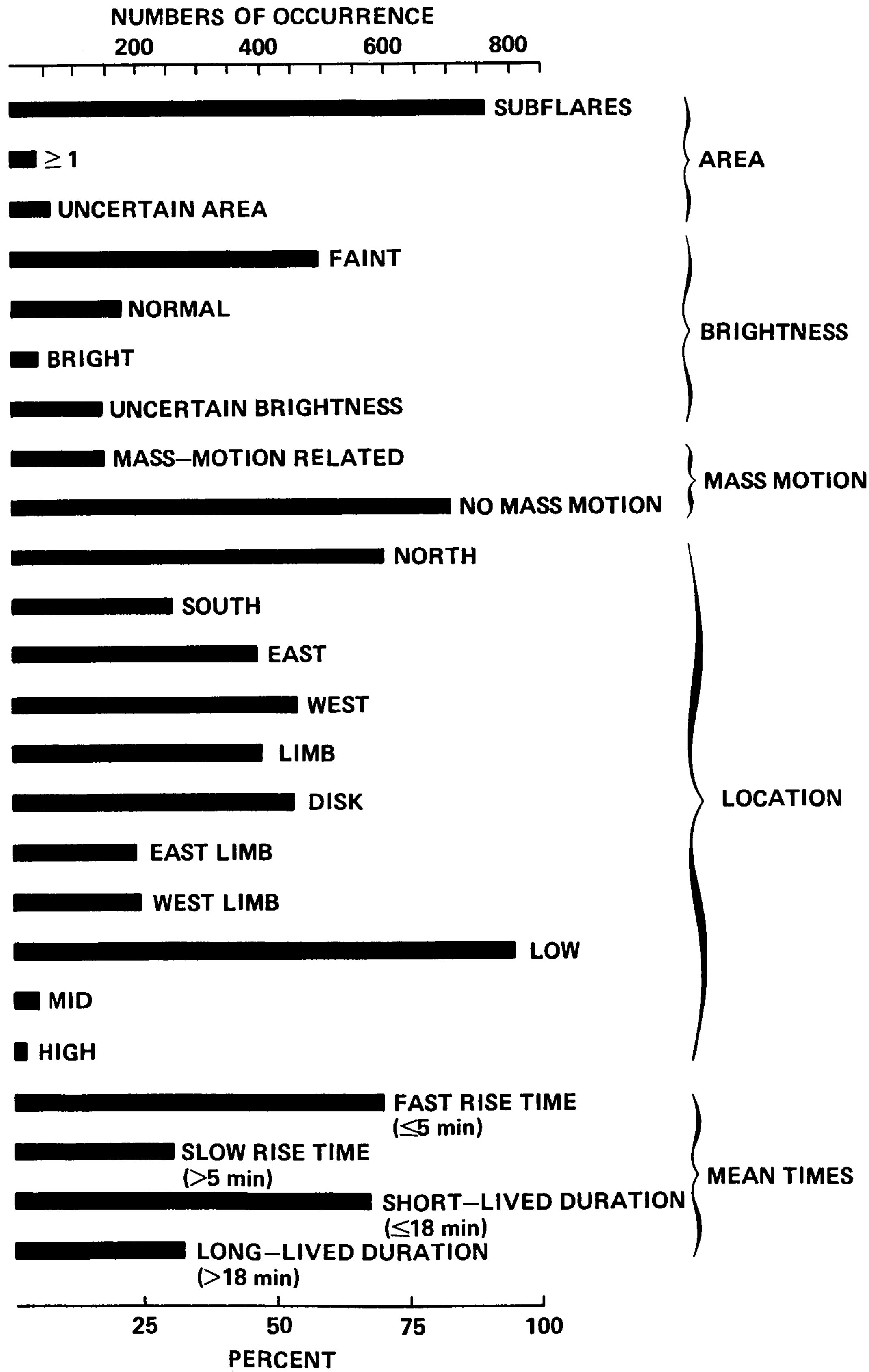

Figure 2. Histograms of numbers of occurrence for selected groupings of flares.

TABLE 2. MASS-MOTION RELATED FLARES DURING 1975 (n = 148) BY CLASS

CLASS	NUMBER	PROPORTIONS ALL (n = 850)	PROPORTIONS TOTAL (n = 148)
A	9	1.1%	6.1%
H	76	8.9%	51.4%
L	24	2.8%	16.2%
M	0	0.0%	0.0%
R	8	0.9%	5.4%
S	5	0.6%	3.4%
U	42	4.9%	28.4%
V	2	0.2%	1.4%

TABLE 3. MASS-MOTION RELATED FLARES DURING 1975 (n = 148) BY GROUPING

GROUP	NUMBER	PROPORTIONS GROUP	PROPORTIONS TOTAL (n = 148)
SUBFLARES (n = 759)	111	14.6%	75.0%
IMPORTANCE ≥ 1 (n = 35)	15	42.9%	10.1%
UNCERTAIN AREA (n = 56)	22	39.3%	14.9%
"FAINT" FLARES (n = 492)	45	9.1%	30.4%
"NORMAL" FLARES (n = 176)	37	21.0%	25.0%
"BRIGHT" FLARES (n = 38)	15	39.5%	10.1%
UNCERTAIN BRIGHTNESS (n = 144)	51	35.4%	34.5%

TABLE 4. SUMMARY OF U AND H FLARES DURING 1975

	U FLARES			H FLARES		
		PROPORTION			PROPORTION	
GROUP	NUMBER	GROUP	TOTAL (n = 42)	NUMBER	GROUP	TOTAL (n = 76)
SUBFLARES (n = 759)	30	4.0%	71.4%	60	7.9%	78.9%
IMPORTANCE ≥ 1 (n = 35)	6	17.1%	14.3%	4	11.4%	5.3%
UNCERTAIN AREA (n = 56)	6	10.7%	14.3%	12	21.4%	15.8%
"FAINT" FLARES (n = 492)	8	1.6%	19.0%	24	4.9%	31.6%
"NORMAL" FLARES (n = 176)	16	9.1%	38.1%	18	10.2%	23.7%
"BRIGHT" FLARES (n = 38)	6	15.8%	14.3%	5	13.2%	6.6%
UNCERTAIN BRIGHTNESS (n = 144)	12	8.3%	28.6%	29	20.1%	38.2%

B. Rise Time, Decay Time, and Duration

Table 5 summarizes (using 95 percent confidence intervals) mean values and standard deviations for a variety of subgroupings of the 850 study flares, in terms of rise time, decay time, and duration, and gives the actual numbers of occurrence for each subgroup, thereby facilitating the assessment of statistical significance of the results. For example, Table 5 shows that, on average, a flare of 1975 had a rise time of 5.2 ± 0.4 min (standard deviation: 6.3 min), a decay time of 12.9 ± 0.8 min (standard deviation: 12.6 min), and a duration of 18.1 ± 1.1 min (standard deviation: 16.1 min). Subflares averaged 4.9 ± 0.4 min, 11.7 ± 0.8 min, and 16.6 ± 1.0 min in terms of rise time, decay time, and duration, respectively, these times being *significantly* (probability P ≤5 percent) different from corresponding times for flares of Hα importance ≥1: 10.4 ± 4.0 min, 30.4 ± 9.0 min, and 40.8 ± 10.9 min, respectively. Thus, with 95 percent level of confidence, one can be assured that, on average, flares of Hα importance ≥1 have *longer* rise time, decay time, and duration than do subflares (based on hypothesis testing of the difference of two means; e.g., Lapin [35]). On average, flares of importance ≥1 are found to be about *twice* as long as subflares, in terms of rise time, decay time, and duration.

Concerning relative brightness, on average, a "bright" flare and a "normal" flare are significantly longer (in terms of decay time and duration) than that of a "faint" flare, while being insignificantly different from each other. Although the mean rise time of a "bright" flare is longer than that for a "faint" or "normal" flare, the difference is insignificant.

Mass-motion related flares, on average, are significantly *longer* (in terms of rise time, decay time, and duration) than those which have not been designated in this way (i.e., non-mass-motion related flares); on average, they are about 1.4 times longer (rise time, decay time, and duration). Two-ribbon flares (U flares), on average, are found to be longer than H flares, but *not* significantly so.

Although no significant differences are noted for any of the hemispheric or latitudinal groupings, flares occurring in the western hemisphere and on the disk (within 45 deg of central meridian), on average, tend to be longer in terms of rise time, decay time, and duration. Southern hemispheric flares tend to be longer in all respects as compared to northern hemispheric flares, and mid-latitude flares tend to be longer than low- or high-latitude flares; high-latitude flares had the shorter times of the latitudinal groups. No significant differences, likewise, are noted between east limb and west limb flares, although, on average, west limb flares are found to be about 1.1 times longer (rise time, decay time, and duration).

Separating flares according to either mean rise time or mean duration yields distributions that are significantly different. Namely, slow rise time flares (rise time RT >5 min), on average, have durations which are significantly longer than fast rise time flares (about 2.5 times longer). Similarly, long-lived duration flares (duration D >18 min), on average, have rise times which are significantly longer than short-lived duration flares (about 3.1 times longer). Thus, an association between rise time and duration is strongly suggested (see below).

TABLE 5. SUMMARY OF RISE TIME, DECAY TIME, AND DURATION FOR THE FLARES OF 1975 BY SELECTED SUBGROUPING

		95% CONFIDENCE INTERVALS FOR MEAN VALUES, STANDARD DEVIATIONS		
GROUP	NUMBER	RISE TIME	DECAY TIME	DURATION
ALL	850	5.2±0.4, 6.3	12.9±0.8, 12.6	18.1±1.1, 16.1
SUBFLARES	759	4.9±0.4, 5.9	11.7±0.8, 10.6	16.6±1.0, 14.1
⩾1	35	10.4±4.0, 12.0	30.4±9.0, 27.3	40.8±10.9, 32.9
UNCERTAIN AREA	56	5.8±1.2, 4.6	18.1±3.7, 13.9	24.0±4.0, 15.3
FAINT	492	5.1±0.6, 6.9	10.8±1.0, 11.1	15.9±1.3, 14.9
NORMAL	176	4.9±0.8, 5.3	15.6±2.1, 14.4	20.5±2.6, 17.6
BRIGHT	38	8.5±3.7, 11.6	21.2±6.4, 20.1	29.7±8.4, 26.3
UNCERTAIN BRIGHTNESS	144	5.0±0.6, 3.9	14.4±1.8, 10.9	19.4±2.1, 13.0
MASS–MOTION RELATED	148	6.7±1.3, 7.9	17.3±2.6, 16.0	24.1±3.4, 21.2
H	76	5.3±1.0, 4.5	14.9±3.1, 13.6	20.2±3.7, 16.3
U	42	8.0±3.1, 10.2	20.6±4.8, 15.9	28.6±6.2, 20.6
NON-MASS-MOTION RELATED	702	4.8±0.4, 5.8	12.0±0.9, 11.6	16.8±1.1, 14.6
NORTH	596	5.0±0.5, 6.0	12.4±0.9, 11.1	17.4±1.2, 14.4
SOUTH	254	5.6±0.8, 6.9	14.2±1.9, 15.6	19.8±2.4, 19.6
EAST	393	4.7±0.5, 5.2	12.9±1.3, 13.2	17.6±1.6, 16.1
WEST	457	5.6±0.6, 7.1	12.9±1.1, 12.1	18.5±1.5, 16.1
LIMB	401	4.8±0.5, 5.4	12.6±1.3, 13.2	17.4±1.6, 15.9
DISK	449	5.5±0.6, 7.0	13.2±1.1, 12.1	18.6±1.5, 16.4
LOW	801	5.2±0.4, 6.3	12.8±0.9, 12.7	18.0±1.1, 16.3
MID	36	5.9±2.2, 6.7	15.1±3.8, 11.5	21.0±4.5, 13.7
HIGH	13	3.8±2.2, 3.6	11.4±6.7, 11.1	15.2±8.2, 13.6
EAST LIMB	198	4.2±0.6, 4.1	12.0±1.8, 12.9	16.3±2.1, 14.8
WEST LIMB	203	5.4±0.9, 6.4	13.2±1.9, 13.5	18.6±2.3, 16.9
FAST RT (RT ⩽5 min)	595	2.5±0.1, 1.4	–	12.6±0.7, 8.8
SLOW RT (RT >5 min)	255	11.5±1.0, 8.4	–	30.9±2.6, 21.3
SHORT–LIVED D (D⩽18 min)	573	3.1±0.2, 2.3	–	10.3±0.4, 4.4
LONG-LIVED D (D>18 min)	277	9.6±1.1, 9.1	–	34.2±2.3, 19.3

Table 6 gives (95 percent confidence intervals) mean values and standard deviations, in terms of rise time, decay time, and duration, for various Hα importance classes. One observes that the bulk (56 percent) of the flares is designated subfaint (SF), the second largest category (18 percent) subnormal (SN), and the third largest (3 percent) subbright (SB). All categories of subflares are insignificantly ($P > 5$ percent) different from each other, in terms of rise time, although faint subflares, on average, are longer than normal subflares which are longer still than bright subflares; bright subflares, on average, are 25 percent shorter in length of rise time than faint subflares. In terms of decay time and duration, insignificant differences are found between SB and SN events and between SB and SF events; however, SN events appear to be significantly longer than SF events (about 1.3 times). For class 1 events, no significant differences in rise time, decay time, and duration are found, although 1B events, on average, are longer than 1F or 1N events (>2 times). A comparison of individual classes of Hα importance reveals no significant differences in rise time between class 1 events and subflares, except for the 1B events; the rise time of 1B flares is significantly longer than the rise time associated with any subflare. The same is true for decay time and duration. Also, the decay time for 1N events is significantly longer than the decay time for SF events.

To assess the statistical significance of associations between decay time and rise time and between duration and rise time for flares in general (discarding Hα importance or other subgroupings), one can place the data in 2x2 contingency tables based on the mean values for the parameters and apply Fisher's exact test [36]. Results of this analysis are depicted in Figure 3. On the lefthand side of Figure 3, the probability P that no preferential association exists between decay time and rise time of flares (i.e., the association is due entirely to chance) is computed to be $P \ll 1$ percent (for the observed table or one more suggestive of a departure from independence). Thus, the analysis indicates that fast decay time ($DT \leq 13$ min) flares tend to be associated with fast rise time flares, while slow decay time flares tend to be related to slow rise time flares. Similarly, on the righthand side of Figure 3, the probability that no preferential association exists between duration and rise time is computed to be $P \ll 1$ percent. Hence, short-lived duration flares tend to be associated with fast rise time flares, and long-lived duration flares tend to be related to slow rise time flares.

Linear regression analysis between decay time and rise time and between duration and rise time are illustrated in Figure 4 (left and right, respectively). For decay time versus rise time, the regression can be approximated as $\widehat{DT} = 8.881 + 0.777\,RT$, where $\widehat{DT}$ is the expected value of decay time reduced from the regression equation and RT is the observed rise time. The correlation coefficient r is only 0.387, implying a coefficient of determination r^2 of only 0.15; thus, the regression equation can only explain about 15 percent of the variation in decay time. For the regression, one finds a standard error of estimate equal to 11.6 min. While Fisher's exact test reveals a rather strong association to exist between decay time and rise time, at least by *quadrants*, regression analysis reveals only a *weak* correlation to exist. Statistical testing [35], however, shows that the difference between the observed slope and the null (= 0) slope is significant, having a t statistic equal to about 12. So, the regression fit provides a much better fit to the data than does a mean fit.

For duration versus rise time, the regression can be approximated as $\widehat{D} = 8.877 + 1.774\,RT$, where $\widehat{D}$ is the expected value of duration and RT is as before. The correlation coefficient r equals 0.693, so the coefficient of determination r^2 is 0.48. Thus, nearly half of the variation in duration can be accounted for by the regression fit. The standard error of estimate is 11.6 min. Both Fisher's exact test and the regression analysis reveal a strong association between duration and rise time for flares. Statistical testing of the slope shows that it is significantly different from the null slope, having a t statistic equal to be about 28.

TABLE 6. SUMMARY OF RISE TIME, DECAY TIME, AND DURATION FOR THE FLARES OF 1975 BY Hα FLARE IMPORTANCE

		95% CONFIDENCE INTERVALS FOR MEAN VALUES, STANDARD DEVIATIONS		
IMPORTANCE	NUMBER	RISE TIME	DECAY TIME	DURATION
SF	475	5.0±0.6, 6.6	10.6±1.0, 10.7	15.7±1.3, 14.7
SN	153	4.8±0.8, 5.1	14.8±1.9, 11.9	19.6±2.3, 14.4
SB	24	3.7±1.8, 4.2	12.2±2.8, 6.6	15.8±4.1, 9.8
1F	6	6.2±8.1, 7.8	21.0±23.1, 22.0	27.2±23.2, 22.1
1N	10	4.7±3.4, 4.7	19.7±7.6, 10.6	24.4±10.4, 14.6
1B	12	17.2±10.2, 16.0	39.9±16.9, 26.5	57.2±18.4, 28.9

$\langle RT \rangle$ = 5.2 min ($\simeq$ 5 min)

DECAY TIME		
	II 156	I 150
	III 439	IV 105
	RISE TIME	

$\langle DT \rangle$ = 12.9 min ($\simeq$ 13 min)

$$P = \frac{306!544!595!255!}{850!156!150!439!105!}$$

$\log P = -18.69171$

$\Rightarrow P \ll 1\%$

$\langle RT \rangle$ = 5.2 min ($\simeq$ 5 min)

DURATION		
	II 98	I 179
	III 497	IV 76
	RISE TIME	

$\langle D \rangle$ = 18.1 min ($\simeq$ 18 min)

$$P = \frac{277!573!595!255!}{850!98!179!497!76!}$$

$\log P = -48.58336$

$\Rightarrow P \ll 1\%$

Figure 3. 2 × 2 contingency tables for decay time versus rise time (left) and duration versus rise time (right). Calculation of the probability that the observed table or one more suggestive of a departure from independence (i.e., due to chance) is shown below. The calculation is facilitated by means of the log N!, values found in Lieberman and Owen [37].

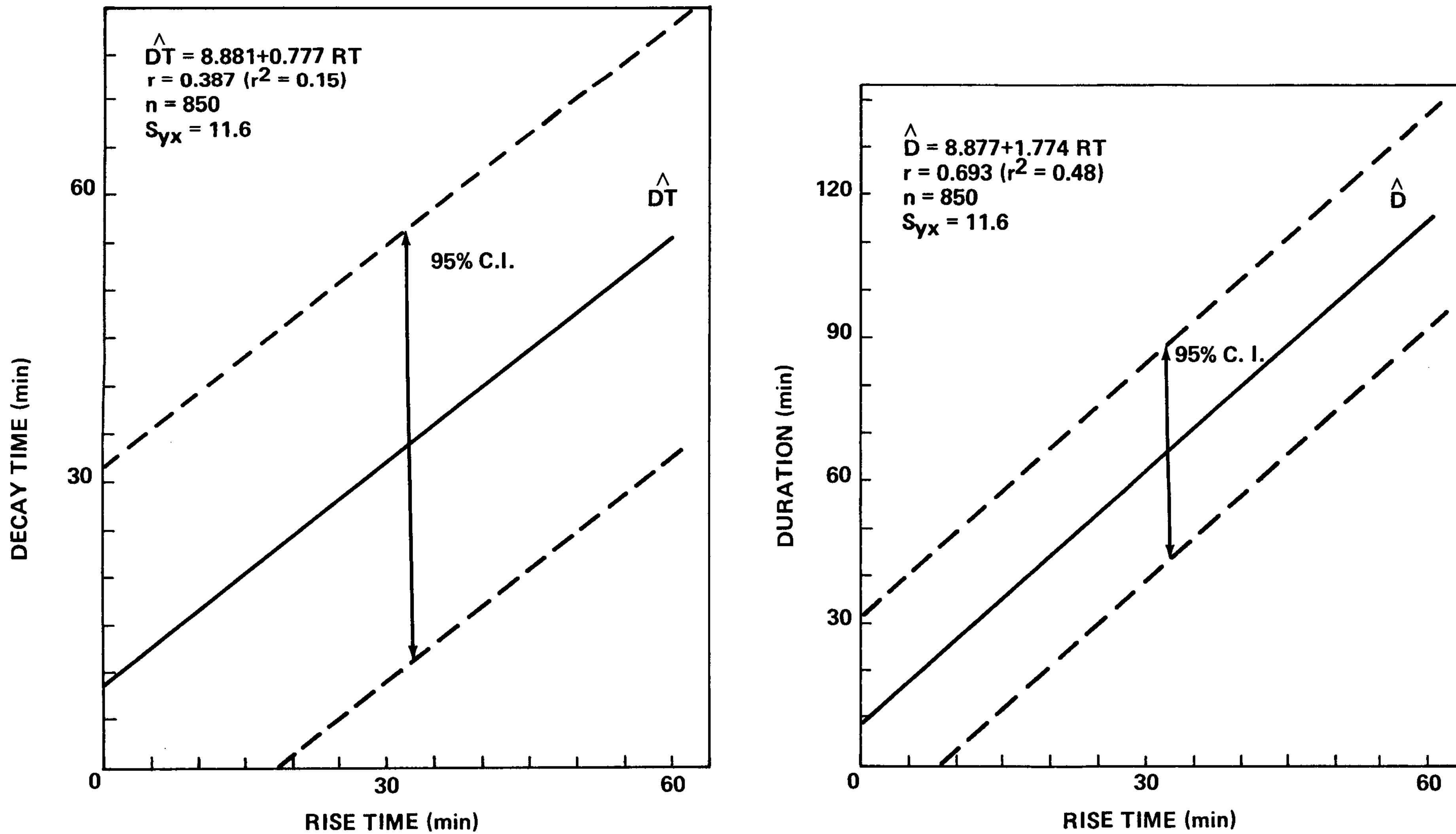

Figure 4. Linear regression fits for decay time versus rise time (left) and duration versus rise time (right). The 95 percent confidence intervals are shown as dashed lines parallel to the regression lines.

C. Correlations Against R

Numbers of occurrence of flares, proportions (by group), mean values (rise time, decay time, and duration), and mean monthly relative sunspot number R are tabulated in Table 7 for January-December 1975. During 1975, flares (in general and for each class) are noted to have been most frequent in August when sunspot number was enhanced, and least frequent in April when sunspot number was depressed. Hence, a strong linear relationship between numbers of flares and sunpot number is anticipated. On average, about 71 study flares occurred each month or about one flare every 10 hr. (Actual rates of occurrence for flares will be higher than these rates, because one must adjust them for no Hα patrol periods and the proportion of study flares to flare entries in the *SGD*.) Sunspot number averaged 15.5 during 1975. On average, a mass-motion related flare occurred about once every 60 hr. In terms of proportions, the greatest proportion (9 percent) of flares with Hα importance $\geq$1 occurred in June (November was nearly as high); the greatest proportion (7 percent) of bright flares occurred in August (April and November were nearly as high); and the greatest proportion (41 percent) of mass-motion related flares occurred in May (December was nearly as high). Mean rise time was shortest (3.7 min) in April and longest (8.7 min) in December. Similarly, mean decay time and duration were shortest in April (8.8 min and 12.5 min, respectively) and longest in June (18.6 min and 25.6 min, respectively). Statistical testing reveals that all monthly averages of rise time, decay time, and duration are insignificantly different from each other, or from the yearly weighted mean values, *except* the mean values of decay time and duration for the month of April, which appear to be significantly low.

In Table 8, a horizontal depiction of a 2x2 contingency table for *each* of the parameters listed in Table 7 (against R) is given, where I refers to the upper righthand quadrant of an XY plot divided into four quadrants by the parameters' mean values (X refers to sunspot number R and Y refers to the parameter being tested); II is the upper lefthand quadrant; III is the lower lefthand quadrant; and IV is the lower righthand quadrant. The entries represent numbers of months and the mean values used in determining the sections (quadrants) are those identified at the bottom of Table 7. As an example, for the case of subflares, its mean value is 63.2. The mean value for R is 15.5. Therefore, quadrant I includes the numbers of months with R $>$15.5 and numbers of subflares $>$63; quadrant II: R $\leq$15.5 and numbers of subflares $>$63; and so on. In this example, I = 3, II = 0, III = 8, and IV = 1. Also listed in Table 8 is the probability that the *observed* distribution is due entirely to chance, based on Fisher's exact test. Continuing the example, then, P = 1.8 percent, a significant result. For the numbers of occurrence group data (the upper portion of Table 8), *all* observed distributions are seen to be either marginally significant (P $\leq$10 percent) or significant (P $\leq$5 percent). For the proportions grouped data (the middle portion of Table 8), only two stand out as possibly being of interest: the distribution of bright flares and the distribution of mass-motion related flares. No significant associations are found between means of rise time, decay time, or duration and sunspot number (the bottom portion of Table 8). Thus, for numbers of occurrence grouped data, in particular, and possibly for selected proportions (bright flares and mass-motion related flares), strong associations are found which may indicate that when sunspot number is above average, numbers (proportions) of events in the tested groups are also above average; conversely, when sunspot number is below average, numbers (proportions) of events in the tested groups are also below average.

TABLE 7. VARIATION OF NUMBERS OF OCCURRENCE, PROPORTIONS, MEAN VALUES, AND R BY MONTH DURING 1975

		NUMBERS OF OCCURRENCE								
MONTH	R	SUBFLARES	≥1	UNCERTAIN AREA	FAINT	NORMAL	BRIGHT	UNCERTAIN BRIGHTNESS	MASS–MOTION RELATED	FLARES
JAN	18.9	46	3	1	33	8	5	4	6	50
FEB	11.5	62		1	43	8	2	10	6	63
MAR	11.5	45	2		35	10	1	1	3	47
APR	5.1	16	1		13	2	1	1	1	17
MAY	9.0	28		1	17	8		4	12	29
JUN	11.4	46	5	4	38	10		7	4	55
JUL	28.2	140	2	8	89	30	1	30	28	150
AUG	39.7	217	11	29	115	58	19	65	54	257
SEP	13.9	40	2	3	26	14	2	3	4	45
OCT	9.1	23			13	9		1	3	23
NOV	19.4	80	8	7	56	16	6	17	20	95
DEC	7.8	16	1	2	14	3	1	1	7	19
SUM		759	35	56	492	176	38	144	148	850
MEAN	15.5	63.2	2.9	4.7	41.0	14.7	3.2	12.0	12.3	70.8

95% Confidence intervals for mean values (all flares, n = 850)

	RISE TIME	DECAY TIME	DURATION
	5.5±1.6	13.9±3.7	19.4±4.7
	5.9±1.6	11.2±2.3	17.0±3.2
	5.7±1.7	16.5±3.9	22.2±4.3
	3.7±1.8	8.8±3.1	12.5±3.9
	4.6±1.2	13.6±3.6	18.2±4.0
	6.9±2.8	18.6±6.2	25.6±7.8
	4.6±0.7	10.8±1.3	15.4±1.7
	4.9±0.7	11.8±1.2	16.7±1.5
	4.1±1.4	13.6±2.5	17.7±3.3
	4.2±2.1	12.1±5.0	16.3±6.7
	5.4±1.2	15.0±3.6	20.4±4.4
	8.7±7.8	12.8±7.4	21.5±13.7
MEAN	5.4±0.9 (5.2±0.4)*	13.2±1.7 (12.9±0.8)*	18.6±2.2 (18.1±1.1)*

*WEIGHT MEANS

TABLE 7. (Concluded)

	PROPORTIONS							
	SUBFLARES	≥1	UNCERTAIN AREA	FAINT	NORMAL	BRIGHT	UNCERTAIN BRIGHTNESS	MASS–MOTION RELATED
	0.92	0.06	0.02	0.66	0.16	0.10	0.08	0.12
	0.98		0.02	0.68	0.13	0.03	0.16	0.10
	0.96	0.04		0.74	0.21	0.02	0.02	0.06
	0.94	0.06		0.76	0.12	0.06	0.06	0.06
	0.97		0.03	0.59	0.28		0.14	0.41
	0.84	0.09	0.07	0.69	0.18		0.13	0.07
	0.93	0.01	0.05	0.59	0.20	0.01	0.20	0.19
	0.84	0.04	0.11	0.45	0.23	0.07	0.25	0.21
	0.89	0.04	0.07	0.58	0.31	0.04	0.07	0.09
	1.00			0.57	0.39		0.04	0.13
	0.84	0.08	0.07	0.59	0.17	0.06	0.18	0.21
	0.84	0.05	0.11	0.74	0.16	0.05	0.05	0.37
MEAN	0.91 (0.89)*	0.04 (0.04)*	0.05 (0.07)*	0.64 (0.58)*	0.21 (0.21)*	0.04 (0.04)*	0.12 (0.17)*	0.17 (0.17)*

*WEIGHTED MEANS

TABLE 8. 2x2 CONTINGENCY TABLES FOR SELECTED GROUPINGS (NUMBERS, PROPORTIONS, AND MEANS) AGAINST R

PARAMETER	I	II	III	IV	PROBABILITY
NUMBER:					
SUBFLARES	3	0	8	1	1.8%
≥ 1	3	1	7	1	6.5%
UNCERTAIN AREA	3	0	8	1	1.8%
FAINT	3	1	7	1	6.5%
NORMAL	3	0	8	1	1.8%
BRIGHT	3	0	8	1	1.8%
UNCERTAIN BRIGHTNESS	3	0	8	1	1.8%
MASS–MOTION RELATED	3	0	8	1	1.8%
FLARES	3	0	8	1	1.8%
PROPORTIONS:					
SUBFLARES	2	5	3	2	42.4%
≥ 1	3	3	5	1	24.2%
UNCERTAIN AREA	2	3	5	2	42.4%
FAINT	1	5	3	3	24.2%
NORMAL	1	2	6	3	50.9%
BRIGHT	3	2	6	1	14.1%
UNCERTAIN BRIGHTNESS	3	3	5	1	24.2%
MASS–MOTION RELATED	4	1	6	1	4.4%
MEANS:					
RISE TIME	2	4	4	2	45.5%
DECAY TIME	2	4	4	2	45.5%
DURATION	2	3	5	2	42.4%

Table 9 summarizes the essential features of regression analyses between the given Y parameters (numbers of occurrence of subflares, flares with importance $\geqslant 1$, etc.) and the X parameter (sunspot number). The essential features include the mean, standard deviation, Y-axis intercept, slope, correlation coefficient r, coefficient of determination r^2, standard error of estimate, and the t statistic for evaluating the statistical significance of the slope (as compared to the null slope; i.e., slope = 0). *All* regressions are found to be significant at >99 percent level of confidence (i.e., $t > 3.169$). Furthermore, the regressions are found to adequately describe the variation of the Y parameters (58 percent $< r^2 <$ 94 percent). Thus, as R increases (decreases), numbers of flares, subflares, etc. increase (decrease). (It is to be remembered that the regressions are valid *only* for $R < 40$, the approximate upper value observed for R in 1975; to extend the regressions to higher values of R will require an expansion of the data base, something which is planned later.)

Although Fisher's exact test (as noted above) revealed a possible association to exist between certain proportions of grouped data and sunspot number, at least by quadrant, regression analyses demonstrate that the associations are *insignificant* (t = 1.0 and 0.3, respectively, for bright flares and mass-motion related flares). Thus, for these and the other proportions, as for monthly mean values of rise time, decay time, and duration, use of yearly mean values is preferred.

D. Asymmetries

Table 10 tabulates the distributions (numbers of occurrence) of flares by selected groups in 15 deg bands of longitude east and west of central meridian. Similarly, Table 11 tabulates distributions in bands of latitude north and south of the equator. Together, these tables reveal an *excess* of flares (in general) in the western and northern hemispheres (Fig. 2). Table 12 summarizes the statistical significance of these and other asymmetries (EAST-WEST, NORTH-SOUTH, LIMB-DISK, and EAST LIMB-WEST LIMB) in the flares of 1975 for the general case and for specific groupings of flares. The significance of the result is recorded in two ways: The first is by means of the *excess*, being a measure of some number times the uncertainty, computed as $d(n/2)^{-1/2}$, where d is the difference in counts between the two hemispheres and n is the total number of flares contained in both groups (see Reid [27]). The second is by means of computing the *actual* probability of obtaining the observed results or one having a larger difference due to chance; i.e., $P[D \geqslant d]$.

As an example, concerning the EAST-WEST asymmetry, one observes 393 flares to have occurred in the eastern hemisphere and 457 in the western hemisphere, yielding a difference d equal to 64. Applying the formula given above, one computes the excess to be about 3.1; thus, a western excess estimated to be about 3.1 times the uncertainty is found. The actual probability of obtaining the observed difference or one larger by chance is computed to be $P[D \geqslant d]$ = 3 percent, a significant result. In general, for the numbers of flares encountered here, when EXCESS < 2, $P[D \geqslant d] >$ 10 percent, implying an insignificant result; when 2 < EXCESS < 3, $P[D \geqslant d]$ is marginally significant, having a value between 5 and 10 percent; when EXCESS > 3, $P[D \geqslant d] <$ 5 percent, a statistically significant result; and when EXCESS > 4, $P[D \geqslant d] <$ 1 percent, a highly significant result.

TABLE 9. LINEAR REGRESSION ANALYSIS FOR SELECTED PARAMETERS AGAINST MONTHLY MEAN SUNSPOT NUMBER (MEAN = 15.458, STANDARD DEVIATION = 9.889)

PARAMETER	MEAN*	STANDARD DEVIATION	INTERCEPT	SLOPE	CORRELATION COEFFICIENT	COEFFICIENT OF DETERMINATION	STANDARD ERROR OF ESTIMATE	t STATISTIC FOR SLOPE
NUMBER:								
SUBFLARES	63.250	59.244	−26.278	5.792	0.967	0.935	15.871	11.969
≥1	2.917	3.450	−1.174	0.265	0.759	0.575	2.351	3.697
UNCERTAIN AREA	4.667	8.117	−6.676	0.734	0.894	0.799	3.809	6.320
FAINT	41.000	31.932	−6.685	3.085	0.955	0.913	9.885	10.236
NORMAL	14.667	15.435	−8.220	1.481	0.949	0.900	5.106	9.513
BRIGHT	3.167	5.340	−3.748	0.447	0.828	0.686	3.143	4.665
UNCERTAIN BRIGHTNESS	12.000	18.781	−15.475	1.777	0.936	0.876	6.952	8.383
MASS–MOTION RELATED	12.333	15.347	−9.893	1.438	0.926	0.858	6.051	7.794
ALL FLARES	70.833	69.488	−34.128	6.790	0.966	0.934	18.758	11.872

*SIMPLE UNWEIGHTED MEAN

TABLE 10. DISTRIBUTIONS OF FLARES (BY GROUP) IN 15 DEG BANDS OF LONGITUDE CENTERED ON CENTRAL MERIDIAN

	ALL	SUB-FLARES	$\geqslant 1$	UN-CERTAIN AREA	FAINT	NORMAL	BRIGHT	UN-CERTAIN BRIGHT-NESS	MASS-MOTION RELATED	RISE TIME FAST ($\leq$5 min)	RISE TIME SLOW ($>$5 min)	DURATION SHORT ($\leq$18 min)	DURATION LONG ($>$18 min)	
90-76E	56	41	5	10	28	14	1	13	6	42	14	44	12	
75-61E	74	66	6	2	44	19	2	9	7	51	23	50	24	EAST
60-46E	68	60	3	5	43	8	2	15	14	56	12	47	21	
45-31E	77	75	1	1	46	22	3	6	17	54	26	50	27	
30-16E	52	49	3	0	33	9	4	6	7	34	15	33	19	
15-0E	66	63	1	2	31	20	5	10	15	48	18	42	24	CM
1-15W	72	66	3	3	50	14	2	6	15	48	24	46	26	
16-30W	75	69	2	4	38	15	4	18	14	48	27	52	23	WEST
31-45W	107	99	0	8	56	25	3	23	18	79	28	72	35	
46-60W	78	75	1	2	50	12	2	14	11	56	22	54	24	
61-75W	70	55	5	10	43	12	2	13	12	47	23	50	20	
76-90W	55	41	5	9	30	6	8	11	12	32	23	33	22	
TOTAL	850	759	35	56	492	176	38	144	148	595	255	573	277	

TABLE 11. DISTRIBUTIONS OF FLARES (BY GROUP) IN BANDS OF LATITUDE, NORTH AND SOUTH OF THE EQUATOR

	ALL	SUB-FLARES	≥1	UN-CERTAIN AREA	FAINT	NORMAL	BRIGHT	UN-CERTAIN BRIGHT-NESS	MASS-MOTION RELATED	RISE TIME FAST (≤5 min)	RISE TIME SLOW (>5 min)	DURATION SHORT (≤18 min)	DURATION LONG (>18 min)
90-31N	12	12	0	0	8	3	0	1	2	8	4	8	4
30-16N	31	23	3	5	12	4	7	8	13	20	11	17	14
15-0N	553	492	23	38	301	121	25	106	97	386	167	383	170
1-15S	248	226	9	13	167	46	6	29	35	177	71	162	86
16-30S	5	5	0	0	3	2	0	0	1	3	2	2	3
31-90S	1	1	0	0	1	0	0	0	0	1	0	1	0
TOTAL	850	759	35	56	492	176	38	144	148	595	255	573	277

TABLE 12. STATISTICAL SIGNIFICANCE OF ASYMMETRIES IN THE FLARES OF 1975

GROUP	EAST–WEST			NORTH–SOUTH			LIMB–DISK			E LIMB–W LIMB		
	d	EXCESS	P[D≥d]	d	EXCESS	P[D≥d]	d	EXCESS	P[D≥d]	d	EXCESS	P[D≥d]
ALL FLARES (n = 850)	64	3.1W	3%	342	16.6N	<1%	48	2.3D	11%	5	0.4WL	84%
SUBFLARES (n = 759)	51	2.6W	7%	295	15.1N	<1%	83	4.3D	<1%	4	0.3WL	87%
IMPORTANCE ≥1 (n = 35)	3	0.7E	74%	17	4.1N	1%	15	3.6L	2%	3	0.8EL	69%
UNCERTAIN AREA (n = 56)	14	3.0W	4%	30	5.7N	<1%	20	3.8L	1%	4	0.9WL	63%
"FAINT" FLARES (n = 492)	42	2.7W	6%	150	9.6N	<1%	16	1.0D	50%	8	0.7WL	65%
"NORMAL" FLARES (n = 176)	8	0.9E	60%	80	8.5N	<1%	34	3.6D	1%	11	1.8EL	24%
"BRIGHT" FLARES (n = 38)	4	0.9W	63%	26	6.0N	<1%	4	0.9D	63%	7	2.4WL	14%
UNCERTAIN BRIGHTNESS (n = 144)	26	3.1W	4%	86	10.1N	<1%	6	0.7L	68%	1	0.2WL	100%
MASS MOTION (ALL) (n = 148)	16	1.9W	22%	76	8.8N	<1%	24	2.8D	6%	8	1.4WL	37%
MASS MOTION (U) (n = 42)	6	1.3W	44%	18	3.9N	1%	20	4.4D	<1%	5	2.1WL	23%
MASS MOTION (H) (n = 76)	20	3.2W	3%	48	7.8N	<1%	12	1.9D	21%	8	2.0WL	22%
FAST RT (RT≤5 min) (n = 595)	25	1.4W	33%	233	13.5N	<1%	27	1.6D	29%	14	1.2EL	44%
SLOW RT (RT>5 min) (n = 255)	39	3.5W	2%	109	9.7N	<1%	21	1.9D	21%	19	2.5WL	10%
SHORT D (D≤18 min) (n = 573)	41	2.4W	9%	243	14.4N	<1%	17	1.0D	50%	4	0.3EL	86%
LONG D (D>18 min) (n = 277)	23	2.0W	19%	99	8.4N	<1%	31	2.6D	7%	9	1.1WL	47%

LEGEND:

For GROUP, n is the number of flares comprising the designated group; U means flares with two bright branches, parallel or converging; *H means flare accompanied by a high-speed dark filament;* RT *means rise time; and* D *means duration.*

For EAST–WEST, NORTH–SOUTH, etc., d is the difference in numbers of flares between the asymmetric groupings; EXCESS is a measure of the significance of d in terms of the uncertainty; P[D ≥d] is the probability of a difference greater than or equal to d occurring by chance (statistically significant differences occur, in general, when EXCESS >3.0, implying P≤5%); E and W mean East and West; N means North; D and L mean Disk and Limb; and EL and WL means East Limb and West Limb. Statistically significant differences are shown within boxes.

Example:

For the group ALL FLARES, 457 occur in the western hemisphere and 393 in the eastern hemisphere, yielding d = 64. The EXCESS is computed following Reid [27]: EXCESS = $d(n/2)^{-2}$ or EXCESS = 3.1 West. The probability of obtaining <u>by chance</u> a difference D≥ d is computed to be 3%.

All asymmetries enclosed in "boxes" in Table 12 are those which have statistically significant or highly significant excesses. Clearly, northern hemisphere flares predominate all categories of flare grouping at >99 percent level of confidence. Other noteworthy asymmetries include a western excess for mass-motion related flares coded H (3.2W, 3 percent); a western excess for slow rise time flares (RT >5 min) (3.5W, 2 percent); a predominance of disk flares for the groups: subflares (4.3D, <1 percent), "normal" flares (3.6D, 1 percent), and mass-motion related flares coded U (4.4D, <1 percent); and a predominance of limb flares for the group Hα importance ≥1 (3.6L, 2 percent). Marginally significant results occur for subflares (2.6W, 7 percent), "faint" flares (2.7W, 6 percent), short-lived duration flares (2.4W, 9 percent), mass-motion related flares (2.8D, 6 percent), long-lived duration flares (2.6D, 7 percent), and slow rise time flares (2.5 WL, 10 percent). Two additional groups are very close to having marginal significance: all flares (2.3D, 11 percent) and "bright" flares (2.4WL, 14 percent).

IV. DISCUSSION

From Table 1, it is recalled that the 850 study flares of 1975 had mean values for rise time, decay time, and duration of 5.2, 12.9, and 18.1 min, respectively. These values include both subflares and larger events. Excluding subflares and flares of indeterminate area, flares of Hα importance ≥1 had mean values of rise time, decay time, and duration of 10.4, 30.4, and 40.8 min. Since early statistical flare studies did not include subflares, comparison of early study results with those found here should be limited to flares of Hα importance ≥1. When appropriate, the comparison will be made for the complete data set. Following is a comparison of early statistical flare results with those found in Section III.

Newton and Barton [12], based on a study of flares occurring in 1935-1936 during the rise of cycle 17, reported that flares appear very suddenly, rise rapidly to maximum emission (in 5 to 10 min), and decay slowly with all traces of the flare having disappeared within 1 hr after it first appears. They further noted that the average duration of a flare spans from about 20 min (for those of lesser intensity) to 40 min (for those of greater intensity). Based on the same timespan, Waldmeier [13] found the mean duration for flares of Hα importance 1 to be about 21 min and for those of importance 2 to be about 38 min.

In the present study, only 3 flares of importance 2 occurred, which is too few to yield a truly meaningful average; however, for the sake of comparison, importance 2 flares are found to have mean values of rise time, decay time, and duration equal to 13.7 min, 49.0 min, and 62.7 min, respectively. Flares of Hα importance 1 have mean values of rise time, decay time, and duration equal to 10.1 min, 28.7 min, and 38.7 min, respectively. A comparison of Waldmeier's results for flares during the rise of cycle 17 with those reported here during the decline of cycle 20 (and within 1 yr of cycle 21 minimum) suggests possibly meaningful differences: importance 1 duration, 21 min versus 38.7 min; and importance 2 duration, 38 min versus 62.7 min. (It is interesting to note that, while cycles 17 and 20 were of comparable amplitude, they were of different cycle lengths; cycle 17 is a short-while-growing cycle and cycle 20 is a long-while-declining cycle [38-40].

Based on a study of 24 flares with known rise times occurring between June 1937 and December 1938, Giovanelli [15] found that flares have a mean rise time of 7 min, with half having a rise time ≤4 min; for flares of Hα importance 1, the mean rise time was 4 min, while being 11 min for flares of importance 2. He further noted that there was a tendency for a flare's rise time to increase with its brightness (i.e., the brighter the flare, the longer its rise time). Flare lifetime was found to average about 30 min [16].

The present study (Fig. 1, Table 1), as reported above, reveals that, in general, flares have a mean rise time equal to about 5.2 min, with about half (median) having a rise time $\leq$3 min; $>$60 percent had a rise time $\leq$4 min. Looking only at events of importance $\geq$1, a mean rise time of 10.4 min, consisting of a mean rise time of 10.1 min for flares of importance 1 and 13.7 min for flares of importance 2, was computed. Based on 35 events of importance $\geq$1, a median (50 percent) of 7 min was deduced, indicating that only about one-third of the events of importance $\geq$1 had a rise time $\leq$4 min. Thus, possibly meaningful differences are noted in mean rise time and duration (30 min versus 40.8 min) and median values of events of importance $\geq$1 between Giovanelli's results from cycle 17 (rise) and here from cycle 20 (decline). The tendency for brighter flares to have longer rise times (cf. Tables 5 and 6) is borne out in the present study, although for subflares the *converse* is true (*brighter* subflares tend to have *shorter* rise times). The present study shows that faint and normal brightness flares are comparable in mean rise time: 5.1 min versus 4.9 min, while bright flares have a longer mean rise time: 8.5 min. (Recall, however, that *none* of these times is significantly different at the 95 percent level of confidence.)

Ellison [17], based on a study of flares spanning 1939 to 1947 (the decline of cycle 17 and the rise of cycle 18), was among the first to introduce the idea of subclasses of flares (see also Richardson [20]). For example, he noted flares to rise quickly to peak intensity, sometimes occurring as a "flash," followed by a slow decay. Some of his other findings include flares that have durations of peak intensity persisting $\leq$5 min, with total lifetimes of an intense flare being many hours; small flares (importance 1) being most frequent during the stages of rapid spot growth or decline, with the most intense flares (importance $\geq$2) seeming to favor the *mature* stage of spot development; number of flares of Hα importance 1 being greater than that for importance 2 which is greater than that for importance 3, with half of 91 flares being classified as flares of importance 1; the mean duration per importance class (based on 109 flares) being 17 min (range 4 to 43 min) for flares of importance 1, 29 min (range 10 to 90 min) for flares of importance 2, 62 min (range 20 to 155 min) for flares of importance 3, and being $\approx$3 hr (range 0.9 to 7.2 hr) for the very largest of flares; and the probability of observing a flare (of any kind) at the limb ($\pm$5 deg) to be about one-tenth that of observing a similar event on the disk.

While flares are often categorized by investigators into particular classes (e.g., fast or slow rise time, short- or long-lived duration, "flash," etc.), distributions of rise time, decay time, and duration deduced for the study flares indicate that such subdivision is purely *arbitrary*. Instead, the distributions (Fig. 1) suggest that each is more appropriately described as a continuum from 0 to $>$60 min, having positive (or rightward) skew. Also, while Ellison found that half of his 91 flares of importance $\geq$1 were flares of Hα importance 1, this study, in contrast, reveals 89 percent of 35 flares to be flares of importance 1. (No importance 3 or larger flares are contained in the list of study flares.) The mean durations (based on 109 flares) that Ellison found for importance 1 and 2 are somewhat low in comparison to that found here: 17 min versus 38.7 min and 29 min versus 62.7 min, respectively. Ellison's observation that the likelihood of observing a flare diminishes as one becomes increasingly close to the limb is borne out for flares in general, and for subflares and faint (possibly even normal) flares (Table 10); however, for flares of Hα importance $\geq$1 or bright flares, *no* dearth of flares in longitudinal bands nearest the limb is seen. In fact, nearly 29 percent of the flares of Hα importance $\geq$1 occur in the bands closest to the limbs, as compared to 17 percent one would have expected, yielding a ratio of observed to expected equal to 1.7. For bright flares, the ratio of observed to expected is 1.4.

Based on 1118 flares recorded at Mt. Wilson from October 1935 through June 1950 (spanning cycle 17 and most of cycle 18), Richardson [20] found, in part, that about 75 percent of all flares are "fast" flares (i.e., a flare which attains maximum area ≤ 0.4 times its lifetime), this percentage being fairly independent of flare importance, and 80 percent of the flares of intensity (importance) 2 and 3 to have reached maximum ≤ 20 min, while flares of intensity 1 or smaller (subflares) reached maximum (same proportion) ≤ 10 min. In the present study, one can *confirm* Richardson's comments regarding the proportion of "fast" flares: 78 percent of the 759 subflares were "fast" flares, having rise times ≤ 0.4 times the mean duration or ≤ 6 min, and 80 percent of the 35 flares of importance ≥ 1 were "fast" flares, having rise times ≤ 0.4 times the mean duration or ≤ 16 min. As aforementioned, only three flares of importance 2 and no flares of importance 3 or larger are contained in the study flare group; consequently, the statistics for *major flares* (those of importance ≥ 2 [34]) is not very reliable. All the study flares of importance ≥ 2 reached maximum ≤ 30 min, having a mean rise time equal to 13.7 min. As already stated, flares of importance 1 had a mean rise time equal to 10.1 min, with 66 percent having a rise time ≤ 10 min; subflares had a mean rise time of 4.9 min, with 89 percent having a rise time ≤ 10 min. For the group Hα importance ≤ 1, 88 percent have a rise time ≤ 10 min (89 percent have a rise time ≤ 10 min for flares in general; see Table 1). Thus, the proportions by importance class for the study flares *do not* agree with that reported by Richardson; namely, 80 percent of the flares (independent of importance class) reach maximum within 10 min.

Based on a sample of flares occurring between February 1951 and April 1953 (the decline of cycle 18), Warwick [22] found, in part, that as either intensity (brightness) or area of a flare increased the mean lifetime increased, while the mean value of the ratio (rise time/duration) decreased; 69 percent of the flares had areas ≤ 100 millionths of the Sun's hemisphere (subflares); the mean rise time of all flares was 12 min and the mean duration was 40 min; for subflares, the mean rise time was about 10 to 11 min and the mean duration was about 31 to 32 min; on the whole, bright flares had a mean duration about *twice* that of faint flares: 65 min versus 32 min, however, faint flares were much more abundant than bright flares: 188 versus 10; the mean rise time for faint and bright flares was about the same: 12 to 13 min; and an *eastern excess* was found which measured 2.17 times the uncertainty (which is of marginal significance, P ≈ 12 percent). In the present study (Table 5), one *confirms* Warwick's finding that mean duration increases as either flare brightness or area increases. For example, the mean duration of subflares was found to be 16.6 min; it was 40.8 min for the larger area flares of Hα importance ≥ 1. The mean duration of faint flares was 15.9 min, while it was 20.5 min for normal flares and 29.7 min for bright flares. Also, the study shows that the ratio of rise time to duration *does* decrease as area increases (0.30 to 0.26), as described by Warwick; however, it does not *strictly* decrease as brightness increases (0.32 to 0.24 to 0.29). While 69 percent of the flares in Warwick's investigation were subflares, here nearly 90 percent were subflares. Also, while she found a mean rise time for flares (independent of importance class) to be 12 min and a mean duration of 40 min, here a mean rise time of 5.2 min and a mean duration of 18.1 min were found, being considerably different (about a factor of 2). The same factor of 2 difference is also found when one compares her mean rise time and duration for subflares (and faint and bright flares) to those reported here. The ratio of numbers of faint to bright flares measured 18.8 to 1 in Warwick's study, but only 12.9 to 1 here. Finally, while she found a nearly marginally significant (2.2E, 12 percent) eastern excess in flares, this study (Table 12) found a statistically significant (3.1W, 3 percent) *western excess*. (East-west asymmetry in flares is really a byproduct of east-west asymmetry in sunspots, since flares are intimately associated with spots. It was Maunder [41] who, studying the spots of 1889-1911 during cycle 13, first reported an east-west asymmetry in the distribution of sunspots, in terms of areas and numbers of spot groups [24,26,27,42-45].

On the basis of photometric light curves of 194 flares between January 1949 and June 1952 (the decline of cycle 18), Dodson et al. [23] found an asymmetry between disk and limb events (139 versus 55), although they did not discuss the significance of this disk excess. They further noted that 54 percent of their study flares were subflares. Dodson et al. could find no convincing relationship between rate of increase of intensity and maximum intensity attained by a flare; however, they reported that the slowest rate of increase appears to diminish the chances of attaining a maximum as bright as the continuous spectrum. Flares with the slowest rates of rise and decline were observed only for flares *within* 55 deg of central meridian; flares with the fastest rates of decline occurred at *greater* than 50 deg from central meridian. For flares of the same Hα importance class, average duration diminished with increasing distance from central meridian. Dodson et al. found that flare area and intensity increase together (i.e., the larger the flare, the brighter the flare), and they found flare duration to be more closely associated with area rather than with maximum intensity. Subflares were found to average about 28 min in duration, importance 1 about 43 min, importance 2 about 66 min, and importance 3 about 84 min. They concluded that there is no evidence to indicate that subflares are not just smaller fainter examples of flare phenomena; also, flares of importance $\geqslant 1$ and subflares were found to have similar disk distributions and rates of increase and decrease in intensity.

The significance of the disk excess, as reported by Dodson et al. [23], can easily be calculated from the numbers they provided. Hence, one deduces this disk excess to be equal to 6.0D or P <1 percent, a highly significant result. This study, however, finds a disk excess that is nearly of marginal significance (2.3D, 11 percent; see Table 12). Also, while their study showed the proportion of subflares to be 54 percent, the reader recalls that subflares in the present investigation accounted for nearly 90 percent of the flares observed in 1975. Dodson et al. could find no convincing relationship between rate of increase of intensity and maximum intensity attained by a flare. If, instead, one compares mean rise time per Hα importance class and Hα importance class (equating SF, SN, SB, etc. with artificially ascribed number values, e.g., 1, 2, 3, etc., respectively), regression analysis (using the values contained in Table 6) yields the following equation: $\widehat{IMP} = 1.810 + 0.244\ RT$. The correlation coefficient r equals 0.663, implying a coefficient of determination r^2 of 0.44, and one computes a standard error of estimate equal to 1.6 and a t statistic equal to 1.779, indicating that the slope is *not* significant even at the 90 percent level of confidence (= 2.132). Thus, while the regression analysis suggests a possible relationship between mean rise time and Hα importance, interpretation of the result is that it is not statistically significant at the 10 percent level of significance, a finding that is supportive of the result found by Dodson et al. A similar analysis comparing Hα importance and mean duration, on the other hand, yields the equation: $\widehat{IMP} = 0.964 + 0.095\ D$. This regression has a coefficient of correlation r equal to 0.796, a coefficient of determination r^2 equal to 0.63, a standard error of estimate equal to 1.3, and a t statistic of 2.608, implying that the result is of marginal significance (P <10 percent). Therefore, while long (slow) rise time does not significantly relate to large area (Hα importance) in flares, long duration, being marginally significant, does suggest that the flare may be of larger area (i.e., of greater Hα importance). While Dodson et al. found the slowest (or longest) duration flares to be within 55 deg of central meridian and the fastest (or shortest duration) flares to be at greater than 50 deg from central meridian, suggesting a dichotomy of flares by duration with short duration flares tending to occur preferentially near the limb and with long duration flares tending to occur preferentially closer to central meridian (disk events), this investigation found a *disk excess* of long duration flares (see Table 12) to be only of marginal significance (2.6D, 7 percent); no limb excess was found for short duration flares. Short duration flares were found to have a marginally significant western excess (2.4W, 9 percent). Both long and short duration flares were observed to have a highly significant northern excess (8.4N and 14.4N, respectively, <1 percent). Supportive of the finding of Dodson et al., the present study found a dimunition of mean duration for flares (of same Hα importance) with increasing distance from central meridian. For example, Table 13 shows that for subflares or flares of Hα importance $\geqslant 1$, mean duration tended to decrease with increasing distance from central meridian. Additionally, limb (>45 deg from central meridian) subflares averaged 15.4 min and limb flares of Hα importance ≥1 averaged 34.0 min, while disk subflares averaged 17.5 min and flares of Hα importance ≥1 averaged 57.9 min. Also corroborated is the finding of Dodson et al. that suggested a positive relationship between area and brightness (i.e., the larger the flare, the brighter the flare). For example, a χ^2 analysis of the numbers of flare by Hα importance contained in Table 6 (expressed as a 2x3 table, comparing flare area–subflare, importance 1–and brightness–faint, normal, bright) shows that the association between area and brightness is *highly significant* ($\chi^2 = 89.57$ versus $\chi^2_{critical} = 13.82$ for $\alpha = 0.001$ or 99.9 percent level of confidence and 2 degrees of freedom). Calculation of the Goodman and Kruskal γ measure of association (Everitt [36]) for the 2x3 table yields $\gamma = 0.808$, indeed, implying a strong positive association between area and brightness, γ being interpreted similarly to that of r in a linear regression analysis. Mean duration found by Dodson et al. for subflares does not agree well with that found in the present study (28 min versus 16.6 min), but for flares of Hα importance 1 and 2 the agreement is very good (43 min versus 38.7 min and 66 min versus 62.7 min, respectively). The similarity of limb-disk distribution for subflares and flares of Hα importance ≥1 found by Dodson et al. was not found there; recall that Table 12 shows that subflares had a highly significant preference to occur on the disk near central meridian rather than near the limb (4.3D, <1 percent), a result in direct *contrast* to that reported for flares of Hα importance ≥1 (3.6L, 2 percent) and reported by Dodson et al.

Bell and Glazer [24] have performed one of the largest statistical studies of flares. Based on 5940 sunspots and 8403 flares occurring during the interval 1937 through 1953 (cycles 17 and 18), that part of their study which can readily be compared to results reported here is that they found no apparent north-south asymmetry in their flare data and that only a slight east-west asymmetry was discerned, both in marked contrast to that reported in this study (see also Reid [27] discussed below). Recall from Table 12 that the present study (for flares of 1975) showed a highly significant north-south asymmetry (16.6N, <1 percent) and a significant east-west asymmetry (3.1W, 3 percent), as well.

Another very large study of the statistical properties of solar flares is one performed by Reid [27] and reported nearly a decade ago in Solar Physics. Based on the analysis of the properties of 2907 flares occurring between March 1958 and December 1965 (the decline of cycle 19), Reid found flare activity to be greatest in 1959, following solar maximum. Also, he found an east-west asymmetry with 56.5 percent of the flares occurring in the western hemisphere, a result in contrast to that reported above by Bell and Glazer [24]. Reid estimated the western excess to be 9.92 times the statistical uncertainty, which is seen to be a highly significant result. Further, he found a preponderance of flares in the northern hemisphere. Subflares accounted for the majority of flares in his study and flare occurrence tended to associate with specific types of spot groups: the majority of flares occurred in spot groups where the number of spots within the group was low, but the largest of flares showed no dependence on number of spots in the group. Reid examined flare distribution and duration of flares with respect to the parameter $(N + 10)/R_F$ for each flare, where N is the number of spots in each flare group and R_F is the relative sunspot number for the day of each flare. He found the majority of flares to occur when the ratio was low, and fewer flares to occur when the ratio was high, and he found a tendency for flares with longer durations to occur when the ratio was small. For subflares, Reid noted that the most frequently observed rise time for his flares was 2 to 3 min and the mean rise time was 3.9 min; flares of Hα importance 1 had rise times fairly evenly divided between 2 and 5 min (peak about 3 min) and had a mean rise time of 6.3 min. For flares of Hα importance >1, he found a wide dispersion in rise time (up to 20 min), with a mean rise time of 12.2 min. For all flares, he found a mean rise time of 5.3 min. Concerning duration, Reid's list of flares contained 11 with durations >2 hr (maximum being 4 hr). The preferred duration for subflares was found to be between 11 and 15 min, with a mean duration of 16.5 min. The preferred duration for flares of Hα importance 1 was found to be between 16 and 25 min, with a mean duration of 28.2 min. For the larger flares, Reid computed a mean duration of 60.5 min (maximum frequency of duration was found to be between 40 and 50 min). Reid also noted a slight tendency for shorter duration flares to occur farther from disk center. He found a slight inverse relationship between the rise time to duration ratio and flare importance. For subflares, he noted the most frequent ratio values to be between 0.1 and 0.3, with a mean value of 0.24; for flares of Hα importance 1, the mean ratio was 0.22; and for larger flares, it was 0.21. For all flares, the mean ratio was 0.23. Lastly, Reid found little variation in the ratio with the phase of the solar cycle.

For the flares of 1975, as shown in Table 12, flares exhibited both a northern excess (16.6N, <1 percent) and a western excess (3.1W, 3 percent), in agreement with that reported by Reid [27] but in contrast to that reported by Bell and Glazer [24] (cf. Warwick [22]; Smith and Smith [26]). Likewise, this study showed that, by far, subflares are the most preponderant form of solar flaring. While Reid examined numbers of flares in comparison to a particular parameter [$(N + 10)/R_F$], in this study monthly numbers of flares were compared directly against monthly mean sunspot number R (Table 9)). For values of R between 5.1 and 39.7, a regression fit was determined; namely, $\hat{N} = -34.128 + 6.790\,R$, where $\hat{N}$ is the expected monthly numbers of flares and R is as defined. The fit was found to have a high correlation coefficient r (= 0.966), a high coefficient of determination r^2 (= 0.934), a small standard error of estimate (= 18.758), and a highly significant t statistic (= 11.872). Thus, more flares occur, on average, when R is high than when R is low. Consequently, peak numbers of flares should occur in the vicinity of solar maximum within a cycle. Regarding mean values for rise time, decay time, and duration, a comparison of results from this study and that due to Reid reveals close agreement. While Reid's list contained 11 flares of duration >2 hr, this study found only two flares with duration >2 hr: one being of importance SF with a duration of 135 min and the other being of importance 2N with a duration of 161 min. Reid's finding of a slight tendency for short duration flares to occur closer to the limb is not borne out in the present investigation (see Table 12). Also, contrary to Reid's finding of a mean value for the rise time to duration ratio being 0.23, this study found the ratio to be 0.29 ± 0.01 (the 95 percent confidence interval); no variation with sunspot number was found ($r = -0.05$; $r^2 = 0.2$ percent), indicating no preferential association with solar cycle, in agreement with Reid.

Finally, on the basis of 1348 flares occurring in 1980, Wilson [46-48] deduced a number of the statistical properties of solar flares. Table 14 summarizes a number of these features. For example, the average flare of 1980 had a rise time of 7.7 ± 0.8 min, a decay time of 22.1 ± 1.7 min, and a duration of 29.8 ± 2.2 min (the 95 percent confidence intervals). When these mean values are compared to those of 1975 (Table 1), one immediately sees that the mean values of rise time, decay time, and duration for flares occurring in 1980 near solar maximum are significantly longer than those found for flares occurring in 1975 near solar minimum. Thus, flares near solar maximum, on average, tend to be longer in rise time, decay time, and duration than flares occurring near solar minimum. A slight reduction in the ratio of rise time to duration is found: 0.26 near maximum versus 0.29 near minimum. In addition to significant differences between the two phases of solar cycle being noticeable in the mean values, differences are discerned in the medians, with the flares of 1980 having the larger median values (only for decay time and duration; median values for rise time are identical). χ^2 analysis of the proportions of flare durations (≤10 min, 11 to 30 min, and 31 to 60 min) for 1975 and 1980 (deduced from Tables 1 and 14) yield a significant result (or association): $\chi^2 = 106.788$ ($\chi^2_{0.001} = 13.82$). Calculation of the Goodman and Kruskal γ measure of association for the 2x3 table yields $\gamma = 0.392$, implying only a weak positive association between phase of solar cycle (year) and duration of flares. Also found in the 1980 flares was a substantial southern excess (3.5S, <5 percent), in contrast to that observed in 1975 near solar minimum. So, changes in the distributions and durations of flares with phase of solar cycle are indicated. To better determine the suggested solar phase dependency, one must examine a larger collection of flare data (spanning an entire solar cycle), using the same consistent set of criteria regarding inclusion of flares in the data base. This remains to be accomplished.

TABLE 13. MEAN DURATION OF FLARE GROUPS BY DISTANCE FROM CENTRAL MERIDIAN

	MEAN DURATION (min)						
CMD*	n	SUBFLARES	n	≥ 1			
90 - 76	82	14.6	10	22.1	LIMB:	SUBFLARES	15.4 min
75 - 61	121	16.9	11	41.2		≥ 1	34.0 min
60 - 46	135	14.6	4	44.2			
45 - 31	174	17.0	1	11.0	DISK:	SUBFLARES	17.5 min
30 - 16	118	17.8	5	82.4		≥ 1	57.9 min
15 - 0	129	18.0	4	39.0			

*CMD: CENTRAL MERIDIAN DISTANCE, IN DEG

TABLE 14. SUMMARY OF STATISTICAL PROPERTIES OF FLARES OCCURRING IN 1980 (n = 1348)

PARAMETER	RISE TIME	DECAY TIME	DURATION
TIMES (min):			
MEAN	7.7±0.8	22.1±1.7	29.8±2.2
MODE*	2	6, 7, 9-13	9, 11, 12, 17
MEDIAN (Q_{50})	3	16	21
Q_{90}	16.5	44	58
PROPORTIONS (%):			
≤ 10 min	81.9	30.5	17.3
≤ 30 min	95.6	78.1	66.2
≤ 60 min	99.0	95.0	90.8

*DECAY TIME:		DURATION	
	6 min (3.9%)		9 min (3.5%)
	7 min (4.3%)		11 min (3.9%)
	9 min (3.6%)		12 min (3.6%)
	10 min (4.4%)		17 min (3.5%)
	11 min (3.6%)		
	12 min (3.5%)		
	13 min (3.9%)		

V. CONCLUSIONS

Examination of the statistical properties of solar flares began in the 1930s (during cycle 17). Even though these properties have now been investigated for about 50 yr, findings have not always been consistent. In fact, two of the most recent and largest statistical studies (one in the 1950s and the other in the 1960s; Bell and Glazer [24] and Reid [27], respectively) have yielded results on asymmetry of flares which are incongruent with each other.

The present study has examined the statistical properties of 850 Hα flares occurring in 1975 (the decline of cycle 20, 1 yr prior to the start of cycle 21). Distributions and cumulative percents for rise time, decay time, and duration of flares were determined. The distributions are characterized as being normal-like with rightward skew. Subdivision of flares into bifurcated groups (e.g., fast or slow rise time flares, short- or long-lived duration flares, etc.) is clearly arbitrary. During 1975, the average flare had a rise time of 5.2 min, a decay time of 12.9 min, and a duration of 18.1 min; on average, it was a subflare (which comprised nearly 90 percent of the total), and was of "faint" relative brightness. Subflares, as a group, had mean values of decay time and duration that were significantly shorter than those determined for flares of Hα importance ≥ 1. Nearly 20 percent of the study flares were associated with mass motion, 73 percent of these being accompanied by high-speed dark filaments or being two-ribbon flares. Mean values for rise time, decay time, and duration for flares with associated mass motion are significantly longer than for those that are not known to be associated with mass motion. Linear regression analysis shows that both decay time and duration of flares can be correlated against rise time, and that the slopes of the regression fits are highly significant. Linear regression analysis also shows that numbers of flares (in general and by group) per month are directly proportional to the monthly mean sunspot number R, all fits being highly significant. A highly significant *northern excess* of flares was apparent in 1975, as was a significant *western excess*. Likewise, a highly significant *disk excess* was observed for subflares, in direct contrast to flares of Hα importance ≥ 1, which had a significant *limb excess*. (Recall that the distinction of disk and limb flares is arbitrary, based on the proximity of central meridian; disk flares occur within 45 deg of central meridian.) Mass-motion related flares were observed to preferentially occur on the disk. No significant excesses were observed comparing east limb events with west limb events. Subflares and flares of Hα importance ≥ 1 have durations which are shorter, on average, for limb events as compared to disk events. Evidence was presented supportive of the hypothesis that the statistical properties of flares change over the solar cycle, with longer duration events occurring near solar maximum as compared to events occurring near solar minimum. Differences between short-while-growing and long-while-declining solar cycles are hinted, as well. To assess these aspects of solar flares, however, will require considerable additional work.

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16. ABSTRACT

A survey of the statistical properties of 850 Hα solar flares during 1975 is presented. Comparison of the results found here with those reported elsewhere for different epochs is accomplished. Distributions of rise time, decay time, and duration are given, as are the mean, mode, median, and 90th percentile values. Proportions by selected groupings are also determined. For flares in general, mean values for rise time, decay time, and duration are 5.2 ± 0.4 min, 12.9 ± 0.8 min, and 18.1 ± 1.1 min, respectively. Subflares, accounting for nearly 90 percent of the flares, had mean values lower than those found for flares of Hα importance ≥ 1, and the differences are statistically significant. Likewise, flares of "bright" and "normal" relative brightness have mean values of decay time and duration that are significantly longer than those computed for "faint" flares, and mass-motion related flares are significantly longer than non-mass-motion related flares. Seventy-three percent of the mass-motion related flares are categorized as being a two-ribbon flare and/or being accompanied by a high-speed dark filament. Slow rise time flares (rise time > 5 min) have a mean value for duration that is significantly longer than that computed for fast rise time flares, and long-lived duration flares (duration > 18 min) have a mean value for rise time that is significantly longer than that computed for short-lived duration flares, suggesting a positive linear relationship between rise time and duration for flares. Indeed, such is the case. Monthly occurrence rates for flares in general and by group are found to be linearly related in a positive sense to monthly sunspot number; as sunspot number increased (decreased), number of flares increased (decreased). Statistical testing reveals the association between sunspot number and numbers of flares to be significant at the 95 percent level of confidence, and the t statistic for slope is significant at >99 percent level of confidence. Dependent upon the specific fit, between 58 percent and 94 percent of the variation can be accounted for with the linear fits. A statistically significant northern hemisphere flare excess ($P < 1$ percent) was found, as was a western hemispheric excess ($P \approx 3$ percent). Subflares were more prolific within 45 deg of central meridian ($P < 1$ percent), while flares of Hα importance ≥ 1 were more prolific near the limbs (>45 deg from central meridian; $P \approx 2$ percent). Two-ribbon flares were more frequent within 45 deg of central meridian ($P < 1$ percent). Slow rise time flares occurred more frequently in the western hemisphere ($P \approx 2$ percent), as did short-lived duration flares ($P \approx 9$ percent), but fast rise tim flares were not preferentially distributed (in terms of east-west or limb-disk). Long-lived duration flares occurred more often within 45 deg o central meridian ($P \approx 7$ percent). Mean durations for subflares and flares of Hα importance ≥ 1, found within 45 deg of central meridian, are 14 percent and 70 percent, respectively, longer than those found for flares closer to the limb. As compared to flares occurring near cycle maximum, the flares of 1975 (near solar minimum) have mean values of rise time, decay time, and duration that are significantly shorter. A flare near solar maximum, on average, is about 1.6 times longer than one occurring near solar minimum.

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