THE RESULTS OF A LIMITED STUDY OF APPROACHES TO THE DESIGN, FABRICATION, AND TESTING OF A DYNAMIC MODEL OF THE NASA IOC SPACE STATION

EXECUTIVE SUMMARY

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Due to the breadth of the study, it was necessary to discuss numerous topics with many individuals within NASA, the aerospace community, the advanced composites industry, and the rubber specialists industry. All of them gave freely of their time, and their advice is deeply appreciated. The writer would particularly like to note the contributions of members of NASA-Langleys Structures and Dynamics Division and its Fabrication Division. Information they provided was particularly helpful in defining planned model test facilities and state-of-the-art techniques in model construction as well as insights into developmental trends for future full scale space station hardware.

INTRODUCTION

For many decades, structural dynamicists have sought simple, expedient, and cost effective means to better understand the dynamic response of complex structures. This search has frequently led to dynamic models for reasons including the following:

1. The forces and the manner in which they interact to produce dynamic phenomena, including mechanical, friction, or fluid driven instabilities, are not adequately understood.
2. The ability to analytically formulate and solve the governing equations is limited or uncertain.
3. The gap between the analyst and physical reality is often difficult to bridge without some experience with representative hardware.

Despite a high pace of progress with computer oriented analysis, experiments will continue to be necessary in the foreseeable future to check the adequacy of theoretical derivations, interpretations, and applications. Because the space station will be designed for fractional g operations, the dynamic model provides the only realistic option for assembling and testing it as an integrated system. Such systems studies would appear propitious for the worlds largest flimsy structure which must be oriented and stabilized to accuracies of the order of 0.1 degree arc.

The dynamic model also provides a convenient and effective means to evaluate the dynamic response of major subassemblies which represent the station during the various phases of on-orbit construction and is also a valuable tool for assessing the impact of changes in the basic configuration, due to growth or redirection, on systems responses.

This report covers the results of limited studies which explore various options relative to the design, fabrication, and testing of a dynamic model of the IOC space station. An attempt was made to review as many aspects of the task as feasible and to evolve practical approaches which will aid in the model design, fabrication, and testing phases, and broaden the base of organizations capable of providing an effective model to NASA.

SCOPE OF STUDY

A limited study was made to evaluate options for the design, construction, and testing of a dynamic model of the space station. Since the definition of the space station structure is still evolving, the IOC reference configuration was used as the general guideline.

The results of the studies, as given in the main report, treat: general considerations of the need for and use of a dynamic model; factors which deal with the model design and construction; and a proposed system for supporting the dynamic model in the planned Large Spacecraft Laboratory.

Consideration was given to various topics under these three general headings as follows:

1. **GENERAL CONSIDERATIONS**

   1.1 The role of a dynamic model in the prediction of the structural dynamics of the space station.

   1.2 Approach to model design, construction, and testing.

   1.3 Selection of model scale and scale factors.

2. **DESIGN AND FABRICATION OF MODEL**

   2.1 Primary Truss Structure

      2.1.1 Approximation of allowance for joint free play for pointing accuracy.

      2.1.2 Considerations for a tube connector device to vary and control joint stiffness.

      2.1.3 Determination of effective stiffness of a structure and a joint in series.

      2.1.4 Review of scaling of extensions within a joint and an approximation of relative motions.

      2.1.5 Considerations for supporting the model for testing by attachment of tangs from the truss joints.

2.1.6 Feasibility of fabrication and testing of graphite epoxy tubes.

2.1.7 Use of air or water pressure to remove thin walled composite tubes from cylindrical mandrels.

2.1.8 Technique for model tube selection / grading.

2.2 Modules and other masses.

2.3 Solar arrays and large antenna dishes.

3. DESIGN AND FABRICATION OF MODEL SUPPORT SYSTEM

3.1 Minimization of gravitation effects.

3.2 Convenience, simplicity, and minimum costs of model tests.

3.3 Safety of model structures and personnel during model assembly and testing.

3.4 Discussion of factors relating to influence of gravitational effects on model support system.

3.5 Determination of model support frequencies on cable mounting system.

3.5.1 Determination of pendular natural frequencies.

3.5.2 Determination of plunging and rotational natural frequencies.

3.6 Determination of composition of cable below support platform.

3.7 Summary of frequency separation for a 1/4 inch scale model with a suspension length of 120 feet.

3.8 Nature of cables and their properties.

3.9 Considerations on selection of rubber for use in space station model supports.

3.10 Calculation of amount of rubber cord for model support.

3.11 Approximation of weight of rubber cord for model support.

3.12 Approximation of lateral natural frequencies of model support cables.

3.13 Summary of experimental data obtained from static and dynamic tests of a rubber sample.

3.14 Considerations relative to the number of elastic cables employed for model support.

3.15 Investigation of use of coil and reversed loop springs for model support.

Appendix I - Results of Experimental Tests of Additional Rubber Samples

Appendix II - Analysis of a Beam Suspended by Cables and Undergoing Combined Bending and Pendular Motions

A. THE ROLE OF A DYNAMIC MODEL IN THE PREDICTION OF THE STRUCTURAL DYNAMICS OF A SPACE STATION

As currently conceived, the space station will consist of an assembly of special purpose structures. These include the shuttle orbiter (when attached); pressurized vessels for personnel habitat, laboratories and supplies; solar panels for energy collection and radiators for thermal control; antennae for communications; and truss structures for interconnection and support of all of these components. When these components, all designed for minimum weight, are assembled in orbit, they will cover an area approximately the size of a baseball field. Because of its size, configuration, and the need for high structural efficiency, the integrated structure will be characterized by slow body movements and low frequency structural responses.

The space station will be continually subjected to unsteady (time dependent) forces during its assembly and operational use in space. A major concern is the reaction of the space station to external forces used to reposition, reorient, or stabilize it. If these forces are coupled to the structure in such a way that they are dependent on the displacement, velocity, or acceleration of the deformations of the structure, proper phasing of the control forces with respect to the structural deformations is necessary to avoid feeding energy into the structural deformations and driving the structure to unacceptable amplitudes or failure. The analyses necessary to design the integrated structural / propulsive systems to avoid unstable coupling requires a means for expressing the spatial relationships for the motions of the structure. Any of several closed sets of functions can be used for this purpose but the most convenient set is the set of natural mode shapes for the undamped structure. This closed set of functions, the infinity of specific shapes wherein the inertial forces generated by the vibrations of the structure at the corresponding natural frequency exactly balance the elastic forces, offers the advantages that they are orthogonal and characteristic. Orthogonality reduces the mathematical coupling by the vanishing of all integrals which involve products of deformations of more than one mode - a substantial simplification for the analyst. The characteristics property is advantageous because the natural mode shapes are readily excited and "stand out" when the structure is shaken at or near the natural frequency corresponding to the mode of interest.

What is the impact of the foregoing statements? First, prediction of the response of the space station structure to external applied forces is critically dependent on a correct definition of the structural properties of the integrated station in each and all of its operational configurations. The correctness of the structural definition is reflected in the ability of the analyst to predict the natural frequencies and mode shapes of the integrated space station structure as determined by comparison of experimental and analytical results. Second, upon achievement of agreement between the calculated and measured natural mode shapes and their corresponding natural frequencies, the motions of the structure can be represented by linear superposition of a "limited" number of these natural modes. As a guideline to determining what constitutes a limited number, a reasonable approach is to include all modes whose natural frequencies range between 0.2 and 5 times the frequency of the exciting or driving force. However, it should be noted that finite element representations of the structure which adequately predict its characteristics will also adequately predict its dynamic response since the structural characteristics are the principal unknowns in the response problem. The dynamic model provides the best and perhaps the only tool available to the designer to verify the equations, and the values of the physical parameters in them, used to analytically define the space station in its actual flight condition. It can also be used to study any subcase such as those associated with partial construction during assembly, changes in configuration such as those associated with movements of the shuttle orbiter, or changes in payloads.

B. APPROACH TO MODEL DESIGN, CONSTRUCTION AND TESTING

The actual configuration of the space station which will ultimately fly is not yet known but the general concensus seems to be that it will be quite similar to the IOC configuration, Figure 1. A desirable dynamic model would be one which provides opportunities for study of the overall dynamic characteristics of the "current" configuration at the time the model is built plus the flexibility to be easily modified to reflect changes in configuration as the program progresses. In many cases, model test results highlight the need for, and guide development of, changes in full scale structures. The modular concept proposed in the main report for the model provides such options.

Because of the large size of the model and the high flexibility of its structure, it appears impractical to obtain model support frequencies low enough to eliminate interference between the model support system and the model natural modes. Interference implies coupling in cases where motions of the model are partially restrained by the support system. In other cases, proximity of frequencies make it difficult to establish motions of the model which do not involve the superposition of elastic and rigid body modes. Two approaches to alleviation of this problem are recommended. First, minimize the interference by making the support system cables as long as possible and by attachement of model excitation equipment in such ways as to minimize the excitation of rigid body motions of the model on the support cables; and, second, include the gravitational restraint forces in the differential equations of motion used to predict the model (and full scale) characteristics and forced responses. All of the gravitationally induced terms in the equations will contain g, which, when it exists enables prediction of the model responses, and when it vanishes, enables prediction of the scaled full scale station responses.

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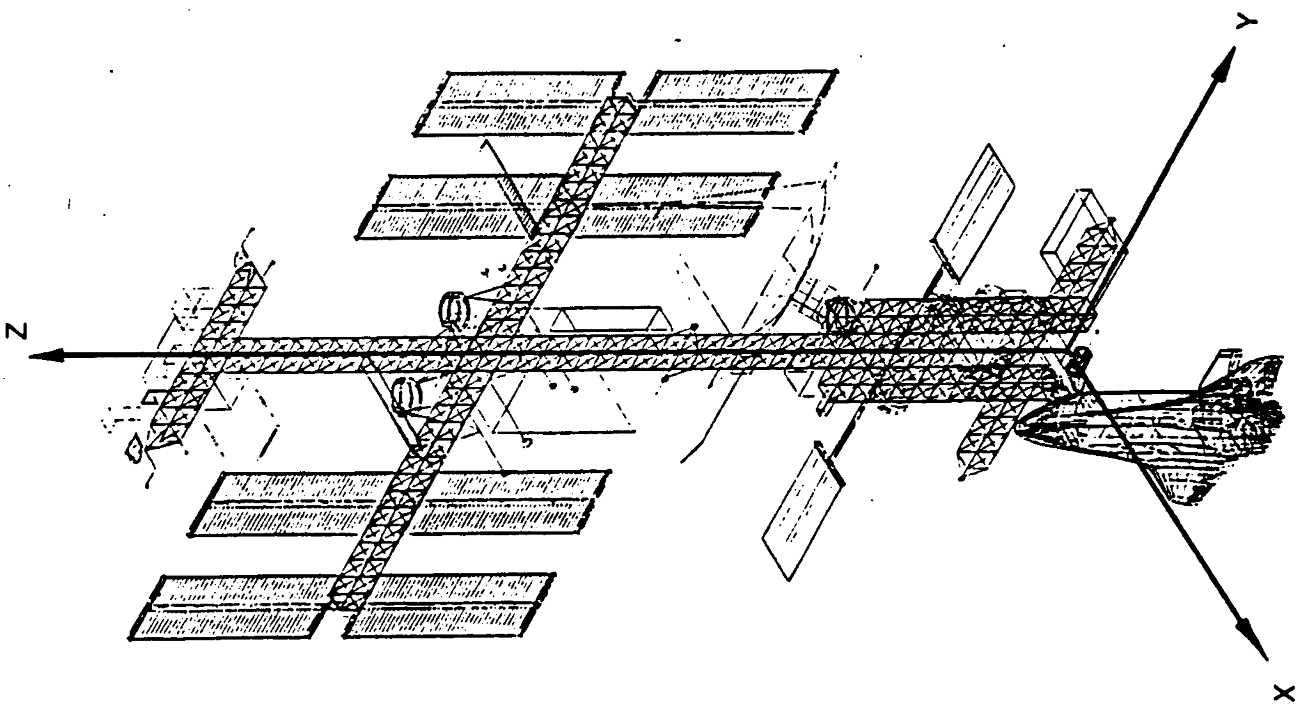

Figure 1. - IOC Reference Space Station - Isometric

The testing of the space station model will be a unique experience because of its large size, its slow response, and its fragility. The approach outlined for the design and fabrication of the model support system appears to offer the only practical means for housing, supporting and testing the model as an integrated system. It will be a difficult but feasible task, the difficulty primarily arising from the need to minimize the effects of the support system on the dynamic characteristics of the model.

C. SELECTION OF MODEL SCALE AND SCALE FACTORS

Theoretically, the limitations on the scaling of a dynamic model reduce to the fact that both the model and the full scale structure must satisfy the same dimensionless equations of motion for the phenomena under study. Stated another way, the ratios of corresponding pairs of forces (and moments) on the model must equal those for the full scale vehicle. From the mathematical viewpoint, this is a straightforward task achievable with any dynamically similar model, replica or distorted, large or small, capable of generating all significant forces and moments in the correct ratios. But the model which staisfies these necessary conditions must satisfy some tough physical conditions to provide data which will identify, or improve the understanding of the dynamic reponse of the full scale space station in orbit. The two more important physical considerations are brought about by the fact that the space station will fly principally under zero gravity conditions and outside the atmosphere, whereas the model tests must be conducted at 1 g and in air at atmospheric pressure.

The fact that the model must be tested at 1 g means that is must be supported in some manner which imposes restraints on its dynamic response. The effects of these restraints on the response can be measured in terms of the ratios of the model's natural frequencies (assuming $g = 0$) to the models support frequencies. It is desirable to make these ratios as high as possible to minimize model restraint interference.

High ratios mean small models and long, soft support systems, i.e.:

$$(\omega_{structure} / \omega_{support}) \propto (\sqrt{\text{support length}} / \text{model length}).$$

The practical need to test the model in air at atmospheric pressure leads to the imposition of aerodynamic damping forces and apparent air mass forces on the model which have no counterpart for the full scale space station flying in orbit. But, for replica scaling where the natural frequency of the model is inversely proportioned to its size, the ratios of the unwanted aerodynamic forces (apparent mass and damping) to the model inertial forces associated with vibrations are independent of model size, or scale. Hence, the aerodynamic forces do not impact the selection of the model size - their minimization requires the model designer to select structures such as screens, rods, cables, etc., to properly simulate the mass and stiffness distributions of structures such as solar panels, and radiators which have high area to mass ratios.

Thus, the selection of model scale reduces to trade-offs between the ability to build the model and the ability to test it. The ability to build the model is a function only of the model; the ability to test it is also contingent upon the provision of a facility to provide an adequate test volume. Also, because of the lack of experience in dynamic analysis of large, flimsy, joint dominated structures, it is desirable to make the model as large as test capabilities will permit. The combination of these and other factors as discussed in the main report and elsewhere leads the writer to recommend a 1/4 scale model. A summary of key factors in this recommendation includes the following:

1. The model can be supported in the planned Large Spacecraft Laboratory with a minimum of interference between model characteristics and model restraints.
2. The principal model structural elements are expected to be graphite epoxy tubes. The 1/4 scale tubes will be about 1/2 inch diameter with wall thickness of about 0.010 inch. On the basis of his recent review of the technology for the manufacture of graphite epoxy tubes, the writer believes the technology exists to make suitable tubes for the 1/4 scale dynamic model.

3. The proposed joint structure for the model truss is feasible at 1/4 scale and offers the opportunity to "tailor" the model mass and stiffness, attach modular and payload masses to the truss structure, and attach the elastic cables for supporting the model.

4. The 1/4 scale space station model will be compatible with the existing 1/4 scale model of the shuttle orbiter. This could represent considerable cost savings.

5. The 1/4 scale model will span approximately 100 feet by 75 feet in planform and weigh about 10,000 lbs. under maximium loading conditions. Its lowest natural frequency will be about 0.5 Hz. The writer believes that if the model is carefully built and tested it should be possible to extrapolate the results and experience from a 1/4 scale model to the prediction and understanding of the dynamic reponse of the full scale space station. It is noted in passing that 1/10 scale dynamic models of numerous smaller aerospace structures ranging from helicopters to launch vehicles have been eminently successful.

The scale factors for the model are based on replica scaling, i.e., those properties of each model element which is necessarily scaled should be scaled as though the element were a replica. For example, the model elements which would represent the habitability modules for a complete replica model would be so stiff that treating them as rigid elements would have negligible impact on the overall dynamic reponse of the model. But their masses, mass moments of inertia, and stiffness of the attachments of the masses to the keel are significant and must be scaled as though they were replica elements. Using these design guidelines, the model scale factors are as given in Figure 2.

SCALE FACTORS FOR PROPOSED MODEL OF IOC SPACE STATION

Primary Factors - Replically Scaled Elements

Length (L_M/L_F)	λ
Mass $(\rho_M/\ \rho_F)\ (L_M/L_F)^3$ $\quad \rho_M = \rho_F$	λ^3
Time (T_M/T_F)	λ

Derived Factors

Area $(L_M/L_F)^2$	λ^2
Volume $(L_M/L_F)^3$	λ^3
Area Moment of Inertia $(L_M/L_F)^4$	λ^4
Displacement (L_M/L_F)	λ
Velocity $(L_M/L_F)\ (T_F/T_M)$	1
Linear Acceleration $(L_M/L_F)\ (T_F/T_M)^2$	λ^{-1}
Angular Acceleration $(T_F/T_M)^2$	λ^{-2}
Structural Frequency (T_F/T_M)	λ^{-1}
Pendular Frequency $(g_M/g_F)\ (L_F/L_M)^{1/2}$ $\quad g_M = g_F$	$\lambda^{-1/2}$
Force $(M_M/M_F)\ (L_M/L_F)\ (T_F/T_M)^2$	λ^2
Torque $(M_M/M_F)\ (L_M/L_F)^2\ (T_F/T_M)^2$	λ^3
Stress $(M_M/M_F)\ (L_M/L_F)\ (T_F/T_M)^2\ (L_F/L_M)^2$	1
Mass Movement of Inertia $(M_M/M_F)\ (L_M/L_F)^2$	λ^5
Gravity Beam Column Effect $(M_M/M_F)\ (g_M/g_F)\ (L_F/L_M)^2$	λ

Figure 2. -- Scale Factors for Replically Scaled Model.

D. DESIGN AND FABRICATION OF MODEL SUPPORT SYSTEM

Current plans for the design of the large spacecraft structures laboratory permit the installation of the model in the orientation shown in Figure 3. This is the recommended orientation for several reasons including: minimization of orientational effects, convenience, simplicity, minimum costs of model tests, and safety of model structures and personnel during model assembly and testing.

As shown in the derivations given in the main report, all natural frequencies of the model support system are proportioned to $1/\sqrt{\ell}$. Since the frequencies of the elastic modes of the model will be higher than the support frequencies, large values of ℓ produce wider separations between natural frequencies for elastic modes and rigid body support modes. The recommended model test configuration shown in Figure 3 provides the highest values for ℓ and thus minimizes coupling.

The recommended model test configuration offers the advantage that nearly all of the model assembly is accomplished with personnel positioned on the floor and working at levels between the floor and shoulder height. In a few instances, it will be necessary to work from a low mobile platform, but no situations are envisioned where model technicians or research personnel are required to work at heights above about 20 feet. This is primarily accomplished by suspending the model from the overhead platform which can be moved as needed from floor to ceiling.

The assembly and testing of the space station model will be a unique experience because of its large size and fragility. These factors impact the safety of test personnel and the utility of an expensive piece of test hardware.

The full scale space station will be designed to function under accelerations of the order of 0.04 g, and as a consequence of the need to minimize the weight to orbit, little structural "fat" is expected. Hence the model, scaled to the same stress level as the prototype, will not be able to support itself under 1 g loads except in small sections. The proposed, essentially continuous support system, effectively eliminates that problem.

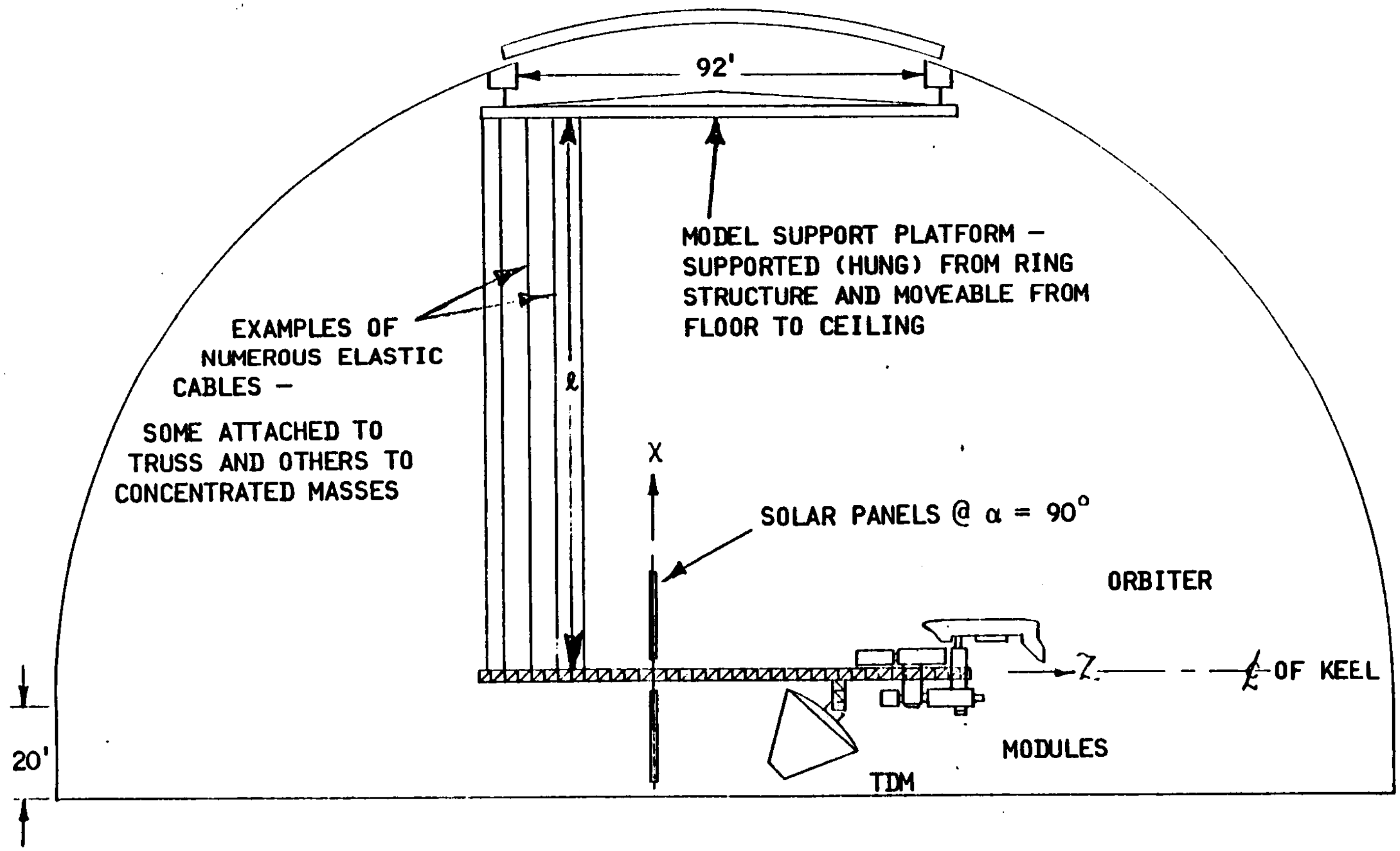

Figure 3. - Outline of Recommended Model Support Arrangement in the Large Spacecraft Laboratory.

Also, because of the fragility of the joints and the tubular members of the truss structure, model test technicians must work with extreme caution to avoid application of damaging model loads. Ground based access to most parts of the model will permit the exercise of reasonable precautions while expediting execution of the model assembly and testing tasks.

E. CONCLUSIONS AND RECOMMENDATIONS

The results of the studies lead to the following conclusions and recommendations:

1. It is proposed that the model be 1/4 scale and that replica scaling be used, i.e., that the natural frequencies of the model be four times the corresponding values for the full scale vehicle.

2. It is proposed that the tubular truss elements (keel, extended keel, transverse boom, etc.) be made as nearly replica as technology and available resources will permit. An alternative to replica joints is proposed which will enable parametric investigation of joint stiffness, free-play, non-linearity, and damping as desired.

3. It is recommended that all modules and other lumped masses which have characteristic natural frequencies substantially higher than the fundamental frequencies of the integrated space station be represented on the model by rigid bodies which have appropriately scaled masses, inertias, and attachment stiffnesses.

4. Because of the high apparent mass ratio of the air surrounding model solar array and antenna components during tests, it is recommended that these components be simulated by open grid structures having appropriate mass and stiffness distributions.

5. The combination of many factors associated with supporting the model for testing suggests that the best, and only necessary, model support configuration is the one which places the plane of the keel and transverse boom near and parallel to the floor. In this orientation, the model will be supported by approximately 100 elastic cables which will maintain the rigid body model frequencies substantially below the frequencies of the lower elastic modes.

6. Apparent air mass, support system masses and gravitational force restraints will all impact the model test results to some degree. It is believed that the proposed model design and test procedures will minimize these effects to the extent that full scale hardware responses in their absence will be highly predictable from model test results.

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DUE TO THE BREADTH OF THE STUDY, IT WAS NECESSARY TO DISCUSS NUMEROUS TOPICS WITH MANY INDIVIDUALS WITHIN NASA, THE AEROSPACE COMMUNITY, THE ADVANCED COMPOSITES INDUSTRY AND THE RUBBER SPECIALISTS INDUSTRY. ALL OF THEM GAVE FREELY OF THEIR TIME AND THEIR ADVICE IS DEEPLY APPRECIATED. THE WRITER WOULD PARTICULARLY LIKE TO NOTE THE CONTRIBUTIONS OF MEMBERS OF NASA-LANGLEYS STRUCTURES AND DYNAMICS DIVISION AND ITS FABRICATION DIVISION. INFORMATION THEY PROVIDED WAS PARTICULARLY HELPFUL IN DEFINING PLANNED MODEL TEST FACILITIES AND STATE-OF-THE-ART TECHNIQUES IN MODEL CONSTRUCTION AS WELL AS INSIGHTS INTO DEVELOPMENTAL TRENDS FOR FUTURE FULL SCALE SPACE STATION HARDWARE.

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SUMMARY

A LIMITED STUDY WAS MADE TO EVALUATE OPTIONS FOR THE DESIGN, CONSTRUCTION AND TESTING OF A DYNAMIC MODEL OF THE SPACE STATION. SINCE THE DEFINITION OF THE SPACE STATION STRUCTURE IS STILL EVOLVING, THE IOC REFERENCE CONFIGURATION WAS USED AS THE GENERAL GUIDELINE.

THE RESULTS OF THE STUDIES, AS GIVEN IN THE REPORT, TREAT GENERAL CONSIDERATIONS OF THE NEED FOR AND USE OF A DYNAMIC MODEL, FACTORS WHICH DEAL WITH THE MODEL DESIGN AND CONSTRUCTION, AND A PROPOSED SYSTEM FOR SUPPORTING THE DYNAMIC MODEL IN THE PLANNED LARGE SPACECRAFT LABORATORY.

THE RESULTS OF THE STUDIES LEAD TO THE FOLLOWING RECOMMENDATIONS:

1. IT IS PROPOSED THAT THE MODEL BE 1/4 SCALE AND THAT REPLICA SCALING BE USED, I.E. THAT THE NATURAL FREQUENCIES OF THE MODEL BE 4 TIMES THE CORRESPONDING VALUES FOR THE FULL SCALE VEHICLE.
2. IT IS PROPOSED THAT THE TUBULAR TRUSS ELEMENTS (KEEL, EXTENDED KEEL, TRANSVERSE BOOM, ETC.) BE MADE AS NEARLY REPLICA AS TECHNOLOGY AND AVAILABLE RESOURCES WILL PERMIT. AN ALTERNATIVE TO REPLICA JOINTS IS PROPOSED WHICH WILL ENABLE PARAMETRIC INVESTIGATION OF JOINT STIFFNES, FREE-PLAY, NON-LINEARITY AND DAMPING AS DESIRED.
3. IT IS RECOMMENDED THAT ALL MODULES AND OTHER LUMPED MASSES WHICH HAVE CHARACTERISTIC NATURAL FREQUENCIES SUBSTANTIALLY HIGHER THAN THE FUNDAMENTAL FREQUENCIES OF THE INTEGRATED SPACE STATION BE REPRESENTED ON THE MODEL BY RIGID BODIES WHICH HAVE APPROPRIATELY SCALED MASSES, INERTIAS AND ATTACHMENTS STIFFNESSES.

4. BECAUSE OF THE HIGH APPARENT MASS RATIO OF THE AIR SURROUNDING MODEL SOLAR ARRAY AND ANTENNA COMPONENTS DURING TESTS, IT IS RECOMMENDED THAT THESE COMPONENTS BE SIMULATED BY AN OPEN GRID HAVING APPROPRIATE MASS AND STIFFNESS DISTRIBUTIONS.

5. THE COMBINATION OF MANY FACTORS ASSOCIATED WITH SUPPORTING THE MODEL FOR TESTING SUGGEST THAT THE BEST, AND ONLY NECESSARY, MODEL SUPPORT CONFIGURATION IS THE ONE WHICH PLACES THE PLANE OF THE KEEL AND TRANSVERSE BOOM NEAR AND PARALLEL TO THE FLOOR. IN THIS ORIENTATION, THE MODEL WILL BE SUPPORTED BY APPROXIMATELY 100 ELASTIC CABLES WHICH WILL MAINTAIN THE RIGID BODY MODEL FREQUENCIES SUBSTANTIALLY BELOW THE FREQUENCIES OF THE LOWER ELASTIC MODES.

6. APPARENT AIR MASS, SUPPORT SYSTEM MASSES AND GRAVITATIONAL FORCE RESTRAINTS WILL ALL IMPACT THE MODEL TEST RESULTS TO SOME DEGREE. IT IS BELIEVED THAT THE PROPOSED MODEL DESIGN AND TEST PROCEDURES WILL MINIMIZE THESE EFFECTS TO THE EXTENT THAT FULL SCALE HARDWARE RESPONSES IN THEIR ABSENCE WILL BE HIGHLY PREDICTABLE FROM MODEL TEST RESULTS.

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INTRODUCTION

FOR MANY DECADES, STRUCTURAL DYNAMICISTS HAVE SOUGHT SIMPLE, EXPEDIENT AND COST EFFECTIVE MEANS TO BETTER UNDERSTAND THE DYNAMIC RESPONSE OF COMPLEX STRUCTURES. THIS SEARCH HAS FREQUENTLY LED TO DYNAMIC MODELS FOR REASONS INCLUDING THE FOLLOWING:

1. THE FORCES AND THE MANNER IN WHICH THEY INTERACT TO PRODUCE DYNAMIC PHENOMENA, INCLUDING MECHANICAL, FRICTION, OR FLUID DRIVEN INSTABILITIES, ARE NOT ADEQUATELY UNDERSTOOD.
2. THE ABILITY TO ANALYTICALLY FORMULATE AND SOLVE THE GOVERNING EQUATIONS IS LIMITED OR UNCERTAIN.
3. THE GAP BETWEEN THE ANALYST AND PHYSICAL REALITY IS OFTEN DIFFICULT TO BRIDGE WITHOUT SOME EXPERIENCE WITH REPRESENTATIVE HARDWARE.

DESPITE ALL OF THE AVAILABLE COMPUTERS, EXPERIMENTS WILL CONTINUE TO BE NECESSARY IN THE FORESEEABLE FUTURE TO CHECK THE ADEQUACY OF THEORETICAL DERIVATIONS, INTERPRETATIONS AND APPLICATIONS. BECAUSE THE SPACE STATION WILL BE DESIGNED FOR FRACTIONAL g OPERATIONS, THE DYNAMIC MODEL PROVIDES THE ONLY REALISTIC OPTION FOR ASSEMBLING AND TESTING IT AS AN INTEGRATED SYSTEM. SUCH SYSTEMS STUDIES WOULD APPEAR PROPITIOUS FOR THE WORLDS LARGEST FLIMSY STRUCTURE WHICH MUST BE ORIENTED AND STABILIZED TO ACCURACIES OF LESS THAN 0.1 DEGREE ARC.

THE DYNAMIC MODEL ALSO PROVIDES A CONVENIENT AND EFFECTIVE MEANS TO EVALUATE THE DYNAMIC RESPONSE OF MAJOR SUBASSEMBLIES WHICH REPRESENT THE STATION DURING THE VARIOUS PHASES OF ON-ORBIT CONSTRUCTION AND IS IS ALSO A VALUABLE TOOL FOR ASSESSING THE IMPACT OF CHANGES IN THE BASIC CONFIGURATION, DUE TO GROWTH OR REDIRECTION, ON SYSTEMS RESPONSES.

THIS REPORT COVERS THE RESULTS OF LIMITED STUDIES WHICH EXPLORE VARIOUS OPTIONS RELATIVE TO THE DESIGN,

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FABRICATION AND TESTING OF A DYNAMIC MODEL OF THE IOC SPACE STATION. AN ATTEMPT WAS MADE TO REVIEW AS MANY ASPECTS OF THE TASK AS FEASIBLE AND TO EVOLVE PRACTICAL APPROACHES WHICH WILL AID IN THE MODEL DESIGN, FABRICATION AND TESTING PHASES, AND BROADEN THE BASE OF ORGANIZATIONS CAPABLE OF PROVIDING AN EFFECTIVE MODEL TO NASA.

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1. GENERAL CONSIDERATIONS

THE PURPOSE OF THIS PART OF THE REPORT IS TO REVIEW THE BASIC NATURE OF THE SPACE STATION, THE ROLE A MODEL CAN PLAY IN THE DYNAMIC ANALYSIS OF THE PARTIAL OR INTEGRATED STATION STRUCTURE AND SYSTEMS, THE SUGGESTED APPROACH TO THE MODEL DESIGN, CONSTRUCTION AND TESTING, AND RECOMMENDATIONS ON MODEL SCALING. THE MATERIAL PRESENTED IS HOPEFULLY HELPFUL TO PROJECT ENGINEERS AND ANALYSTS AS WELL AS STRUCTURAL DYNAMICS SPECIALISTS.

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1.1 THE ROLE OF A DYNAMIC MODEL IN THE PREDICTION OF THE STRUCTURAL DYNAMICS OF A SPACE STATION

THE PURPOSE OF THIS NOTE IS TO DISCUSS THE ROLE OF A DYNAMICALLY SIMILAR MODEL IN THE PREDICTION OF THE DYNAMIC RESPONSE OF A SPACE STATION. THE DYNAMICALLY SIMILAR MODEL MAY OR MAY NOT BE A STRUCTURAL REPLICA (WHERE DIFFERENCES BETWEEN THE MODEL AND FULL SCALE STRUCTURES ARE ESSENTIALLY A MATTER OF SCALE OR SIZE) BUT IT MUST FAITHFULLY REPRESENT THE MASS AND STIFFNESS DISTRIBUTIONS OF THE SPACE STATION IN INSTANCES WHERE THESE DISTRIBUTIONS RESULT IN NATURAL FREQUENCIES AND MODE SHAPES IN THE FREQUENCY DOMAIN WHICH ENCOMPASSES PERTINENT FULL SCALE STRUCTURAL DEFORMATIONS. IT IS ALSO DESIRABLE THAT THE MODEL REFLECT THE DAMPING DISTRIBUTION OF THE FULL SCALE VEHICLE BUT THIS IS PROBABLY NOT ACHIEVABLE AND NOT REALLY ESSENTIAL FOR APPLICATION OF MODEL TEST RESULTS FOR PREDICTION OF FULL SCALE RESPONSES.

AS CURRENTLY CONCEIVED, THE SPACE STATION WILL CONSIST OF AN ASSEMBLY OF SPECIAL PURPOSE STRUCTURES. THESE INCLUDE THE SHUTTLE ORBITER (WHEN ATTACHED); PRESSURIZED VESSELS FOR PERSONAL HABITAT, LABORATORIES AND SUPPLIES; SOLAR PANELS FOR ENERGY COLLECTION AND RADIATORS FOR THERMAL CONTROL; ANTENNAE FOR COMMUNICATIONS; AND TRUSS STRUCTURES FOR INTERCONNECTION AND SUPPORT OF ALL OF THESE COMPONENTS. WHEN THESE COMPONENTS, ALL DESIGNED FOR MINIMUM WEIGHT, ARE ASSEMBLED IN ORBIT, THEY WILL COVER AN AREA APPROXIMATELY THE SIZE OF A BASEBALL FIELD. BECAUSE OF ITS SIZE, CONFIGURATION, AND THE NEED FOR HIGH STRUCTURAL EFFICIENCY, THE INTEGRATED STRUCTURE WILL BE CHARACTERIZED BY SLOW BODY MOVEMENTS AND LOW FREQUENCY STRUCTURAL RESPONSES.

THE SPACE STATION WILL BE CONTINUALLY SUBJECTED TO

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UNSTEADY (TIME DEPENDENT) FORCES DURING ITS ASSEMBLY AND OPERATIONAL USE IN SPACE. THESE FORCES WILL BE OF THREE BASIC TYPES: GRAVITY GRADIENT BODY FORCES WHICH TEND TO KEEP THE MAJOR AXIS OF THE STATION ORIENTED ALONG THE EARTHS RADII; FORCES DUE TO CHANGES IN THE INTERNAL MOMENTUM OF THE SYSTEM; AND EXTERNALLY APPLIED IMPULSIVE FORCES PROVIDED BY DOCKING OR BY PROPULSIVE SYSTEMS AS MAY BE NECESSARY TO REORIENT OR REPOSITION THE STATION. FROM THE STRUCTURAL DYNAMICS VIEWPOINT, THE LATTER TWO ARE OF PRIMARY INTEREST.

FORCES REPRESENTING CHANGES IN THE INTERNAL MOMENTUM OF THE STATION ARE GENERALLY IMPULSIVE AND THE MAJOR CONCERN IS THAT THE DISTURBANCES THEY CAUSE BE SMALL RELATIVE TO ALLOWABLE ON BOARD LIMITS FOR RESEARCH AND HABITABILITY, AND THAT THE DAMPING OF THE STRUCTURES CAUSE THEM TO DECAY QUICKLY.

THE MAJOR CONCERN IS THE REACTION OF THE SPACE STATION TO EXTERNAL FORCES USED TO REPOSITION, REORIENT, OR STABILIZE IT. IF THESE FORCES ARE COUPLED TO THE STRUCTURE IN SUCH A WAY THAT THEY ARE DEPENDENT ON THE DISPLACEMENT, VELOCITY, OR ACCELERATION OF THE DEFORMATIONS OF THE STRUCTURE, PROPER PHASING OF THE CONTROL FORCES WITH RESPECT TO THE STRUCTURAL DEFORMATIONS IS NECESSARY TO AVOID FEEDING ENERGY INTO THE STRUCTURAL DEFORMATIONS AND DRIVING THE STRUCTURE TO UNACCEPTABLE AMPLITUDES OR FAILURE. THE ANALYSES NECESSARY TO DESIGN THE INTEGRATED STRUCTURAL/PROPULSIVE SYSTEMS TO AVOID UNSTABLE COUPLING REQUIRES A MEANS FOR EXPRESSING THE SPATIAL RELATIONSHIPS FOR THE MOTIONS OF THE STRUCTURE. ANY OF SEVERAL CLOSED SETS OF FUNCTIONS CAN BE USED FOR THIS PURPOSE BUT THE MOST CONVENIENT SET IS THE SET OF <u>NATURAL MODE SHAPES</u> FOR THE UNDAMPED STRUCTURE. THIS CLOSED SET OF FUNCTIONS, THE INFINITY OF SPECIFIC SHAPES WHEREIN THE INERTIAL FORCES GENERATED BY THE VIBRATIONS OF THE STRUCTURE AT THE CORRESPONDING NATURAL FREQUENCY

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EXACTLY BALANCE THE ELASTIC FORCES, OFFERS THE ADVANTAGES THAT THEY ARE ORTHOGONAL AND CHARACTERISTIC. ORTHOGONALITY REDUCES THE MATHEMATICAL COUPLING BY THE VANISHING OF ALL INTEGRALS WHICH INVOLVE PRODUCTS OF DEFORMATIONS OF MORE THAN ONE MODE – A SUBSTANTIAL SIMPLIFICATION FOR THE ANALYST. THE CHARACTERISTICS PROPERTY IS ADVANTAGEOUS BECAUSE THE NATURAL MODE SHAPES ARE READILY EXCITED AND "STAND OUT" WHEN THE STRUCTURE IS SHAKEN AT OR NEAR THE NATURAL FREQUENCY CORRESPONDING TO THE MODE OF INTEREST.

WHAT IS THE IMPACT OF THE FOREGOING STATEMENTS? FIRST, PREDICTION OF THE RESPONSE OF THE SPACE STATION STRUCTURE TO EXTERNALL APPLIED FORCES IS CRITICALLY DEPENDENT ON A CORRECT DEFINITION OF THE STRUCTURAL PROPERTIES OF THE INTEGRATED STATION IN EACH AND ALL OF ITS OPERATIONAL CONFIGURATIONS. THE CORRECTNESS OF THE STRUCTURAL DEFINITION IS REFLECTED IN THE ABILITY OF THE ANALYST TO PREDICT THE NATURAL FREQUENCIES AND MODE SHAPES OF THE INTEGRATED SPACE STATION STRUCTURE AS DETERMINED BY COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS. SECOND, UPON ACHIEVEMENT OF AGREEMENT BETWEEN THE CALCULATED AND MEASURED NATURAL MODE SHAPES AND THEIR CORRESPONDING NATURAL FREQUENCIES, THE MOTIONS OF THE STRUCTURE CAN BE REPRESENTED BY LINEAR SUPERPOSITION OF A "LIMITED" NUMBER OF THESE NATURAL MODES. AS A GUIDELINE TO DETERMINING WHAT CONSTITUTES A LIMITED NUMBER, A REASONABLE APPROACH IS TO INCLUDE ALL MODES WHOSE NATURAL FREQUENCIES RANGE BETWEEN 0.2 AND 5 TIMES THE FREQUENCY OF THE EXCITING OR DRIVING FORCE. HOWEVER, IT SHOULD BE NOTED THAT FINITE ELEMENT REPRESENTATIONS OF THE STRUCTURE WHICH ADEQUATELY PREDICT ITS CHARACTERISTICS WILL ALSO ADEQUATELY PREDICT ITS DYNAMIC RESPONSE SINCE THE STRUCTURAL CHARACTERISTICS ARE THE PRINCIPAL UNKNOWNS IN THE RESPONSE PROBLEM.

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THUS THE DYNAMIC MODEL PROVIDES THE BEST AND PERHAPS THE ONLY TOOL AVAILABLE TO THE DESIGNER TO VERIFY THE EQUATIONS, AND THE VALUES OF THE PHYSICAL PARAMETERS IN THEM, USED TO ANALYTICALLY DEFINE THE SPACE STATION IN ITS ACTUAL FLIGHT CONDITION. IT CAN ALSO BE USED TO STUDY ANY SUBCASE SUCH AS THOSE ASSOCIATED WITH PARTIAL CONSTRUCTION DURING ASSEMBLY, CHANGES IN CONFIGURATION SUCH AS THOSE ASSOCIATED WITH MOVEMENTS OF THE SHUTTLE ORBITER, OR CHANGES IN PAYLOADS.

1.2 APPROACH TO MODEL DESIGN, CONSTRUCTION AND TESTING

THE ACTUAL CONFIGURATION OF THE SPACE STATION WHICH WILL ULTIMATELY FLY IS NOT YET KNOWN BUT THE GENERAL CONCENSUS SEEMS TO BE THAT IT WILL BE QUITE SIMILAR TO THE IOC CONFIGURATION OUTLINED IN FIGURES 1 TO 3, EXTRACTED FROM REFERENCE 1. THE SPACECRAFT CONSISTS OF A BACKBONE TRUSS SYSTEM (KEEL, KEEL EXTENSIONS AND BOOMS) FOR INTERCONNECTION OF MAJOR AND MINOR SUBSTRUCTURES INCLUDING HABITABILITY, LABORATORY AND LOGISTIC MODULES, SOLAR ARRAYS AND ANTENNAE, AND RADIATOR SYSTEMS. SINCE THE PHYSICAL PROPERTIES OF THE STATION ARE NOT YET DEFINED, AND WHEN DEFINED ARE SUBJECT TO CHANGE BY GROWTH AND REDIRECTION, A DESIRABLE DYNAMIC MODEL WOULD BE ONE WHICH PROVIDES OPPORTUNITIES FOR STUDY OF THE OVERALL DYNAMIC CHARACTERISTICS OF THE "CURRENT" CONFIGURATION AT THE TIME THE MODEL IS BUILT PLUS THE FLEXIBILITY TO BE EASILY MODIFIED TO REFLECT CHANGES IN CONFIGURATION AS THE PROGRAM PROGRESSES. IN MANY CASES, MODEL TEST RESULTS HIGHLIGHT THE NEED FOR AND GUIDE DEVELOPMENTAL CHANGES IN FULL SCALE STRUCTURES. THE MODULAR CONCEPT PROPOSED FOR THE MODEL, AS DISCUSSED IN PART 2 - DESIGN AND FABRICATION OF THE MODEL, PROVIDES SUCH OPTIONS.

BECAUSE OF THE LARGE SIZE OF THE MODEL AND THE HIGH FLEXIBILITY OF ITS STRUCTURE, IT APPEARS IMPRACTICAL TO OBTAIN MODEL SUPPORT FREQUENCIES LOW ENOUGH TO ELIMINATE INTERFERENCE BETWEEN THE MODEL SUPPORT SYSTEM AND THE MODEL NATURAL MODES. INTERFERENCE IMPLIES COUPLING IN CASES WHERE MOTIONS OF THE MODEL ARE PARTIALLY RESTRAINED BY THE SUPPORT SYSTEM. IN OTHER CASES, PROXIMITY OF FREQUENCIES MAKE IT

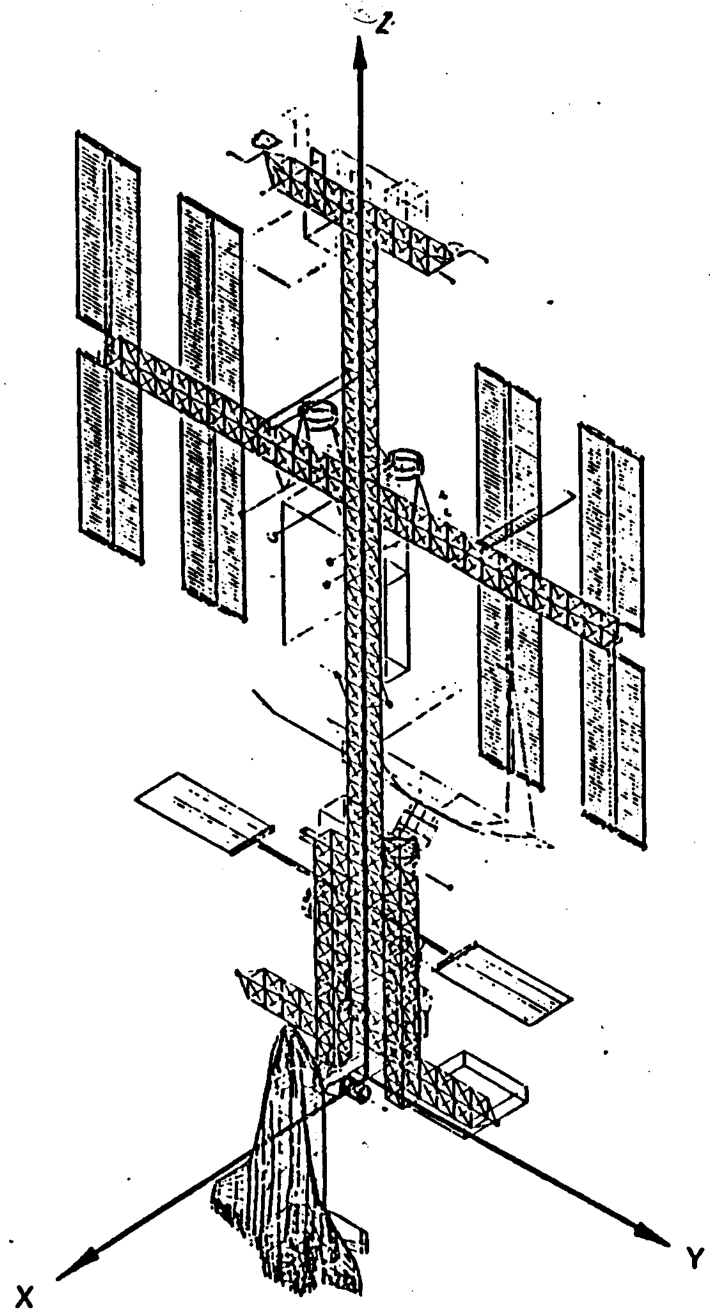


FIGURE 1.- IOC REFERENCE SPACE STATION - ISOMETRIC

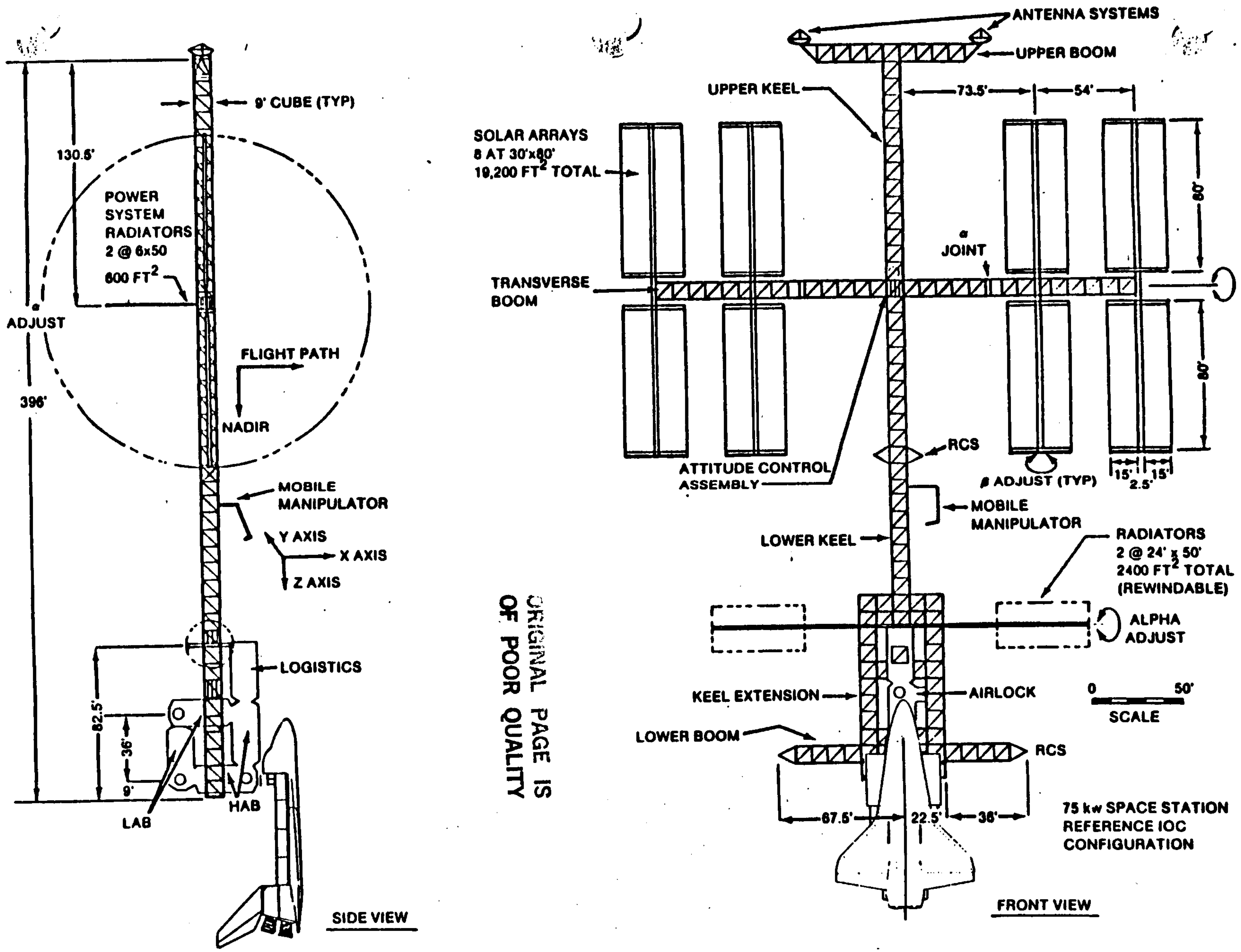
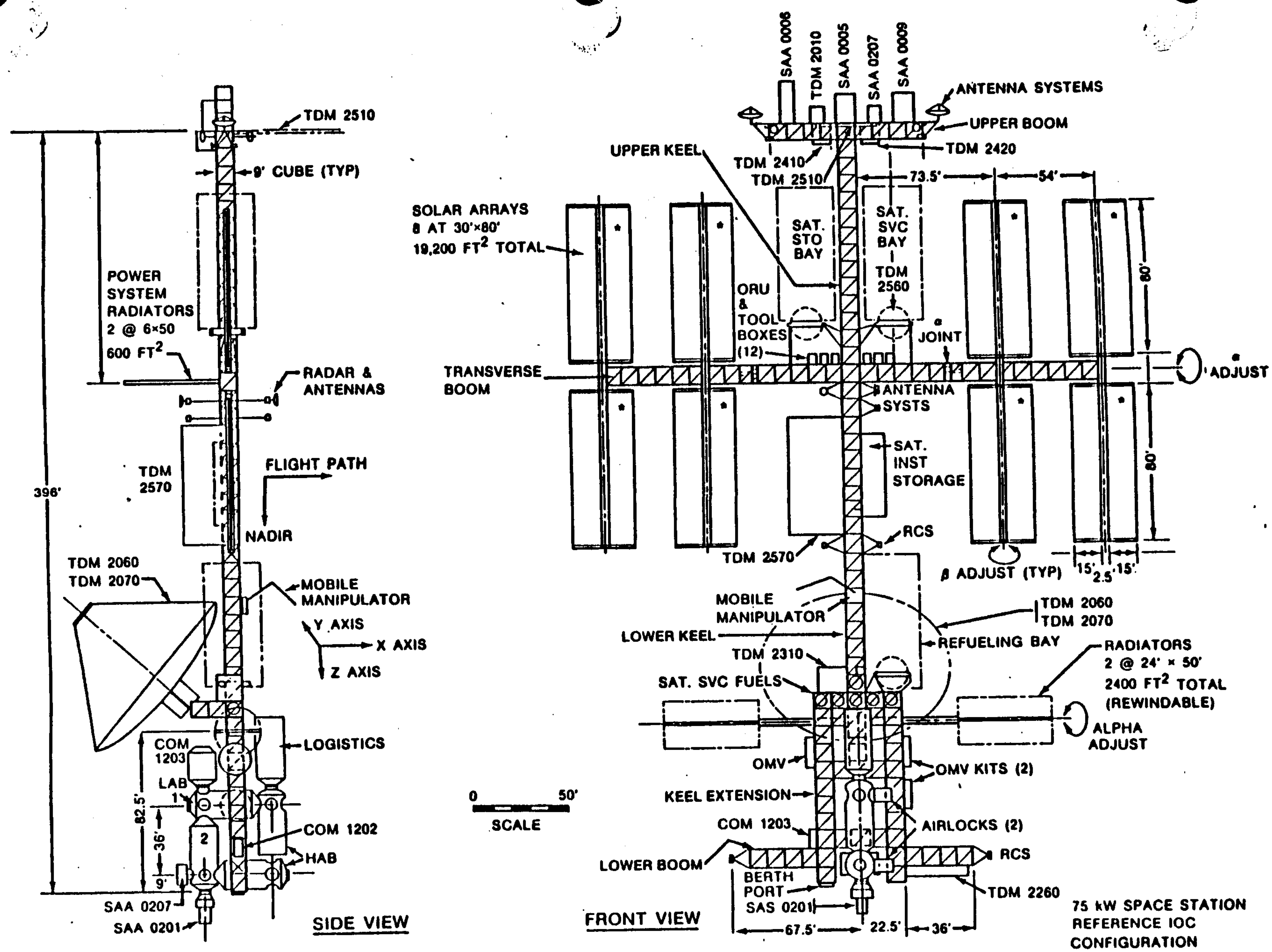


FIGURE 2.- IOC REFERENCE SPACE STATION - OVERVIEW



*NOTE: SOLAR DYNAMICS SYSTEM A PRIME OPTION (NOT SHOWN)

FIGURE 3.- IOC REFERENCE SPACE STATION - COMPONENT DEFINITION

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DIFFICULT TO ESTABLISH MOTIONS OF THE MODEL WHICH DO NOT INVOLVE THE SUPERPOSITION OF ELASTIC AND RIGID BODY MODES. TWO APPROACHES TO ALLEVIATION OF THIS PROBLEM ARE RECOMMENDED. FIRST, MINIMIZE THE INTERFERENCE BY MAKING THE SUPPORT SYSTEM CABLES (SEE SECTION 3) AS LONG AS POSSIBLE AND BY ATTACHMENT OF MODEL EXCITATION EQUIPMENT IN SUCH WAYS AS TO MINIMIZE THE EXCITATION OF RIGID BODY MOTIONS OF THE MODEL ON THE SUPPORT CABLES. AND SECOND, INCLUDE THE GRAVITATIONAL RESTRAINT FORCES IN THE DIFFERENTIAL EQUATIONS OF MOTION USED TO PREDICT THE MODEL (AND FULL SCALE) CHARACTERISTICS AND FORCED RESPONSES. ALL OF THE GRAVITATIONALLY INDUCED TERMS IN THE EQUATIONS WILL CONTAIN g, WHICH, WHEN IT EXISTS ENABLES PREDICTION OF THE MODEL RESPONSES, AND WHEN IT VANISHES ENABLES PREDICTION OF THE SCALED FULL SCALE STATION RESPONSES.

THE TESTING OF THE SPACE STATION MODEL WILL BE A UNIQUE EXPERIENCE BECAUSE OF ITS LARGE SIZE, ITS SLOW RESPONSE, AND ITS FRAGILITY. THE APPROACH OUTLINED IN SECTION 3 - DESIGN AND FABRICATION OF MODEL SUPPORT SYSTEM APPEARS TO OFFER THE ONLY PRACTICAL MEANS FOR HOUSING, SUPPORTING AND TESTING THE MODEL AS AN INTEGRATED SYSTEM. IT WILL BE A DIFFICULT BUT FEASIBLE TASK, THE DIFFICULTY PRIMARILY ARISING FROM THE NEED TO MINIMIZE THE EFFECTS OF THE SUPPORT SYSTEM ON THE DYNAMIC CHARACTERISTICS OF THE MODEL.

1.3 SELECTION OF MODEL SCALE AND SCALE FACTORS

THEORETICALLY, THE LIMITATIONS ON THE SCALING OF A DYNAMIC MODEL REDUCE TO THE FACT THAT BOTH THE MODEL AND THE FULL SCALE STRUCTURE MUST SATISFY THE SAME DIMENSIONLESS EQUATIONS OF MOTIONS FOR THE PHENOMENA UNDER STUDY. STATED ANOTHER WAY, THE RATIOS OF CORRESPONDING PAIRS OF FORCES (AND MOMENTS) ON THE MODEL MUST EQUAL THOSE FOR THE FULL SCALE VEHICLE. FROM THE MATHEMATICAL VIEWPOINT, THIS IS A STRAIGHTFORWARD TASK ACHIEVABLE WITH ANY DYNAMICALLY SIMILAR MODEL, REPLICA OR DISTORTED, LARGE OR SMALL, CAPABLE OF GENERATING ALL SIGNIFICANT FORCES AND MOMENTS IN THE CORRECT RATIOS. BUT THE MODEL WHICH SATISFIES THESE NECESSARY CONDITIONS MUST SATISFY SOME TOUGH PHYSICAL CONDITIONS TO PROVIDE DATA WHICH WILL IDENTIFY, OR IMPROVE THE UNDERSTANDING OF, THE DYNAMIC RESPONSE OF THE FULL SCALE SPACE STATION IN ORBIT. THE TWO MORE IMPORTANT PHYSICAL CONSIDERATIONS ARE BROUGHT ABOUT BY THE FACT THAT THE SPACE STATION WILL FLY PRINCIPALLY UNDER ZERO GRAVITY CONDITIONS AND OUTSIDE THE ATMOSPHERE WHEREAS THE MODEL TESTS MUST BE CONDUCTED AT 1 g AND IN AIR AT ATMOSPHERIC PRESSURE.

THE FACT THAT THE MODEL MUST BE TESTED AT 1 g MEANS THAT IT MUST BE SUPPORTED IN SOME MANNER WHICH IMPOSES RESTRAINTS ON ITS DYNAMIC RESPONSE. THE EFFECTS OF THESE RESPONSES CAN BE MEASURED IN TERMS OF THE RATIOS OF THE MODEL'S NATURAL FREQUENCIES (ASSUMING $g = 0$) TO THE MODELS SUPPORT FREQUENCIES. IT IS DESIRABLE TO MAKE THESE RATIOS AS HIGH AS POSSIBLE TO MINIMIZE MODEL RESTRAINT INTERFERENCE. HIGH RATIOS MEAN SMALL MODELS AND LONG, SOFT SUPPORT SYSTEMS.

THE PRACTICAL NEED TO TEST THE MODEL IN AIR AT ATMOSPHERIC PRESSURE LEADS TO THE IMPOSITION OF AERODYNAMIC DAMPING FORCES AND APPARENT AIR MASS FORCES ON THE MODEL WHICH HAVE NO COUNTERPART FOR THE FULL SCALE SPACE STATION FLYING IN ORBIT. BUT, FOR REPLICA SCALING ($\omega \propto \frac{1}{L}$), THE RATIO OF THE UNWANTED AERODYNAMIC FORCES (APPARENT MASS AND DAMPING) TO THE MODEL INERTIA FORCES ASSOCIATED WITH VIBRATIONS IS INDEPENDENT OF MODEL SIZE OR SCALE. HENCE THE AERODYNAMIC FORCES DO NOT IMPACT THE SELECTION OF THE MODEL SIZE - THEIR MINIMIZATION FORCES THE MODEL DESIGNER TO SELECT STRUCTURES SUCH AS SCREENS, RODS, CABLES, ETC. TO PROPERLY SIMULATE THE MASS AND STIFFNESS DISTRIBUTIONS OF STRUCTURES SUCH AS SOLAR PANELS, AND RADIATORS WHICH HAVE HIGH AREA TO MASS RATIOS.

THUS THE SELECTION OF MODEL SCALE REDUCES TO TRADEOFFS BETWEEN THE ABILITY TO BUILD THE MODEL AND THE ABILITY TO TEST IT. THE ABILITY TO BUILD THE MODEL IS A FUNCTION ONLY OF THE MODEL; THE ABILITY TO TEST IT IS ALSO CONTINGENT ON THE PROVISION OF A FACILITY TO PROVIDE AN ADEQUATE TEST VOLUME. ALSO, BECAUSE OF THE LACK OF EXPERIENCE IN DYNAMIC ANALYSIS OF LARGE, FLIMSY, JOINT DOMINATED STRUCTURES, IT IS DESIRABLE TO MAKE THE MODEL AS LARGE AS TEST CAPABILITIES WILL PERMIT. THE COMBINATION OF THESE AND OTHER FACTORS AS DISCUSSED IN THIS REPORT AND ELSEWHERE LEADS THE WRITER TO RECOMMEND A 1/4 SCALE MODEL. A SUMMARY OF KEY FACTORS IN THIS RECOMMENDATION INCLUDES THE FOLLOWING:

1. THE MODEL CAN BE SUPPORTED IN THE PLANNED LARGE SPACECRAFT LABORATORY WITH A MINIMUM OF INTERFERENCE BETWEEN MODEL CHARACTERISTICS AND MODEL RESTRAINTS.
2. THE PRINCIPAL MODEL STRUCTURAL ELEMENTS ARE EXPECTED TO BE GRAPHITE EPOXY TUBES. THE 1/4 SCALE TUBES WILL BE ABOUT 1/2 INCH DIAMETER WITH WALL THICKNESS OF ABOUT 0.010 INCH. ON THE BASIS

OF HIS RECENT REVIEW OF THE TECHNOLOGY FOR THE MANUFACTURE OF GRAPHITE EPOXY TUBES, THE WRITER BELIEVES THE TECHNOLOGY EXISTS TO MAKE SUITABLE TUBES FOR THE 1/4 SCALE DYNAMIC MODEL.

3. THE PROPOSED JOINT STRUCTURE FOR THE MODEL TRUSS IS FEASIBLE AT 1/4 SCALE AND OFFERS THE OPPORTUNITY TO "TAILOR" THE MODEL MASS AND STIFFNESS, ATTACH MODULAR AND PAYLOAD MASSES TO THE TRUSS STRUCTURE, AND ATTACH THE ELASTIC CABLES FOR SUPPORTING THE MODEL.
4. THE 1/4 SCALE SPACE STATION MODEL WILL BE COMPATIBLE WITH THE EXISTING 1/4 SCALE MODEL OF THE SHUTTLE ORBITER. THIS COULD REPRESENT CONSIDERABLE COST SAVINGS.
5. THE 1/4 SCALE MODEL WILL SPAN APPROXIMATELY 100 FT. BY 75 FT. IN PLANFORM AND WEIGH ABOUT 10,000 LB. UNDER MAXIMUM LOADING CONDITIONS. ITS LOWEST NATURAL FREQUENCY WILL BE ABOUT 0.5 HZ. THE WRITER BELIEVES THAT IF THE MODEL IS CAREFULLY BUILT AND TESTED IT SHOULD BE POSSIBLE TO EXTRAPOLATE THE RESULTS AND EXPERIENCE FROM A 1/4 SCALE MODEL TO THE PREDICTION AND UNDERSTANDING OF THE DYNAMIC RESPONSE OF THE FULL SCALE SPACE STATION. IT IS NOTED IN PASSING THAT 1/10 SCALE MODELS OF NUMEROUS SMALLER AEROSPACE STRUCTURES RANGING FROM HELICOPTERS TO LAUNCH VEHICLES HAVE BEEN EMINENTLY SUCCESSFUL.

THE SCALE FACTORS FOR THE MODEL ARE BASED ON REPLICA SCALING - i.e., THOSE PROPERTIES OF EACH MODEL ELEMENT WHICH IS NECESSARILY SCALED SHOULD BE SCALED AS THOUGH THE ELEMENT WERE REPLICA. FOR EXAMPLE, THE MODEL ELEMENTS WHICH WOULD REPRESENT THE HABITABILITY MODULES FOR A COMPLETE REPLICA MODEL WOULD BE SO STIFF THAT TREATING THEM AS RIGID ELEMENTS WOULD

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HAVE NEGLIGIBLE IMPACT ON THE OVERALL DYNAMIC RESPONSE OF THE MODEL. BUT THEIR MASSES, MASS MOMENTS OF INERTIA, AND STIFFNESS OF THE ATTACHMENTS OF THE MASSES TO THE KEEL ARE SIGNIFICANT AND MUST BE SCALED AS THOUGH THEY WERE REPLICA ELEMENTS. USING THESE DESIGN GUIDELINES, THE MODEL SCALE FACTORS ARE AS GIVEN IN ~~FIGURE 4.~~ THE FOLLOWING TABLE.

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SCALE FACTORS FOR PROPOSED MODEL OF IOC SPACE STATION

PRIMARY FACTORS - REPLICALLY SCALED ELEMENTS

LENGTH (L_M/L_F)		λ
MASS $(\rho_M/\rho_F)(L_M/L_F)^3$	$\rho_M = \rho_F$	λ^3
TIME (T_M/T_F)		λ

DERIVED FACTORS

AREA $(L_M/L_F)^2$		λ^2
VOLUME $(L_M/L_F)^3$		λ^3
AREA MOMENT OF INERTIA $(L_M/L_F)^4$		λ^4
DISPLACEMENT (L_M/L_F)		λ
VELOCITY $(L_M/L_F)(T_F/T_M)$		1
LINEAR ACCELERATION $(L_M/L_F)(T_F/T_M)^2$		λ^{-1}
ANGULAR ACCELERATION $(T_F/T_M)^2$		λ^{-2}
STRUCTURAL FREQUENCY (T_F/T_M)		λ^{-1}
PENDULAR FREQUENCY $(g_M/g_F)(L_F/L_M)^{1/2}$	$g_M = g_F$	$\lambda^{-1/2}$
FORCE $(M_M/M_F)(L_M/L_F)(T_F/T_M)^2$		λ^2
TORQUE $(M_M/M_F)(L_M/L_F)^2(T_F/T_M)^2$		λ^3
STRESS $(M_M/M_F)(L_M/L_F)(T_F/T_M)^2(L_F/L_M)^2$		1
MASS MOMENT OF INERTIA $(M_M/M_F)(L_M/L_F)^2$		λ^5
GRAVITY BEAM COLUMN EFFECT $(M_M/M_F)(g_M/g_F)(L_F/L_M)^2$		λ

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~~FIGURE 4 - SCALE FACTORS FOR REPLICALLY SCALED MODEL~~

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2 DESIGN AND FABRICATION OF THE MODEL

AS NOTED IN SECTION 1.2 AND FIGURES 1 TO 3, THE IOC REFERENCE SPACE STATION CONSISTS OF A PRIMARY TRUSS STRUCTURE TO WHICH SPECIAL PURPOSE COMPONENTS OR ELEMENTS WILL BE ATTACHED. THESE INCLUDE THE LABS, THE SHUTTLE ORBITER, THE SOLAR POWER SYSTEMS, RADIATORS, ANTENNAES, ETC. IT SEEMS HIGHLY PROBABLE THAT THE ACTUAL IOC SPACE STATION WILL BE CONSTRUCTED IN SIMILAR FASHION BECAUSE THIS GENERAL CONFIGURATION OFFERS THE POTENTIAL OF HIGH STRENGTH-TO-WEIGHT RATIO STRUCTURES, EASE OF ACCESS FOR ON-ORBIT ASSEMBLY BY DEPLOYMENT OR ERECTION, AND A VERSATILE BASE FOR PAYLOAD ATTACHMENT AND SERVICING. THUS, IT IS ASSUMED THAT THE DYNAMIC MODEL OF THE SPACE STATION WILL SIMULATE, AND REPLICATE WHERE APPROPRIATE, STRUCTURES VERY SIMILAR TO THE IOC REFERENCE CONFIGURATION.

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2.1 PRIMARY TRUSS STRUCTURE

THE PRIMARY TRUSS STRUCTURE OF THE SPACE STATION WILL PROBABLY CONSIST OF TUBULAR ELEMENTS MADE OF GRAPHITE EPOXY COMPOSITE MATERIALS JOINED IN A TETRAHEDRAL ARRANGEMENT SIMILAR TO THAT SHOWN IN FIGURE 4. THE STRUCTURE IS A REPETITIVE ARRANGEMENT OF BAYS WHERE EACH BAY MAY BE CONSIDERED AS CONSISTING OF 4 LONGERONS, 4 BATTENS AND 4 DIAGONALS. WHETHER THE STRUCTURE IS DEPLOYED OR ERECTED IN ORBIT WILL ONLY IMPACT THE MODEL TO THE EXTENT THAT THE METHOD OF ASSEMBLY MAY INFLUENCE THE MASS AND STIFFNESS OF SOME FLIGHT STRUCTURES, & HENCE, MODEL STRUCTURES.

ON THE BASIS OF THE PREVIOUS ASSUMPTIONS, THE MODEL DESIGNER IS PRINCIPALLY INTERESTED IN THE FOLLOWING QUESTIONS RELATIVE TO THE TRUSS:

1. HOW DO YOU BUILD THE MODEL TRUSS STRUCTURE SO IT DYNAMICALLY SIMULATES THE PROPERTIES OF THE FULL SCALE TRUSS STRUCTURE?
2. HOW DO YOU PROVIDE FOR THE IMPOSITION OF THE LOADS IMPOSED BY THE VARIOUS MASSES WHICH CONSTITUTE THE REMAINDER OF THE STATION?
3. HOW DO YOU PROVIDE FOR ATTACHMENT OF THE MODEL SUPPORT SYSTEM REQUIRED FOR MODEL TESTS

IMPLICIT IN THE PHRASE "DYNAMICALLY SIMULATES" IS THE PROVISION FOR APPROPRIATE DISTRIBUTIONS OF MASS, STIFFNESS, AND DAMPING, WITH EMPHASIS ON THE POSSIBILITY OF FREE PLAY AND FLEXIBILITY IN THE JOINTS IF ASSEMBLED IN A MANNER SIMILAR TO THAT SHOWN IN FIGURE 4.

AS NOTED IN SECTION 2.1.1, THE SPACE STATION POINTING REQUIREMENTS DICTATE THAT THE FREE PLAY IN THE JOINTS OF THE TRUSS MUST BE EXTREMELY SMALL, i.e., AVERAGEING LESS THAN 0.001 INCHES PER JOINT. IN ALL PROBABILITY, THE JOINTS WILL BE DESIGNED WITH A FREE PLAY

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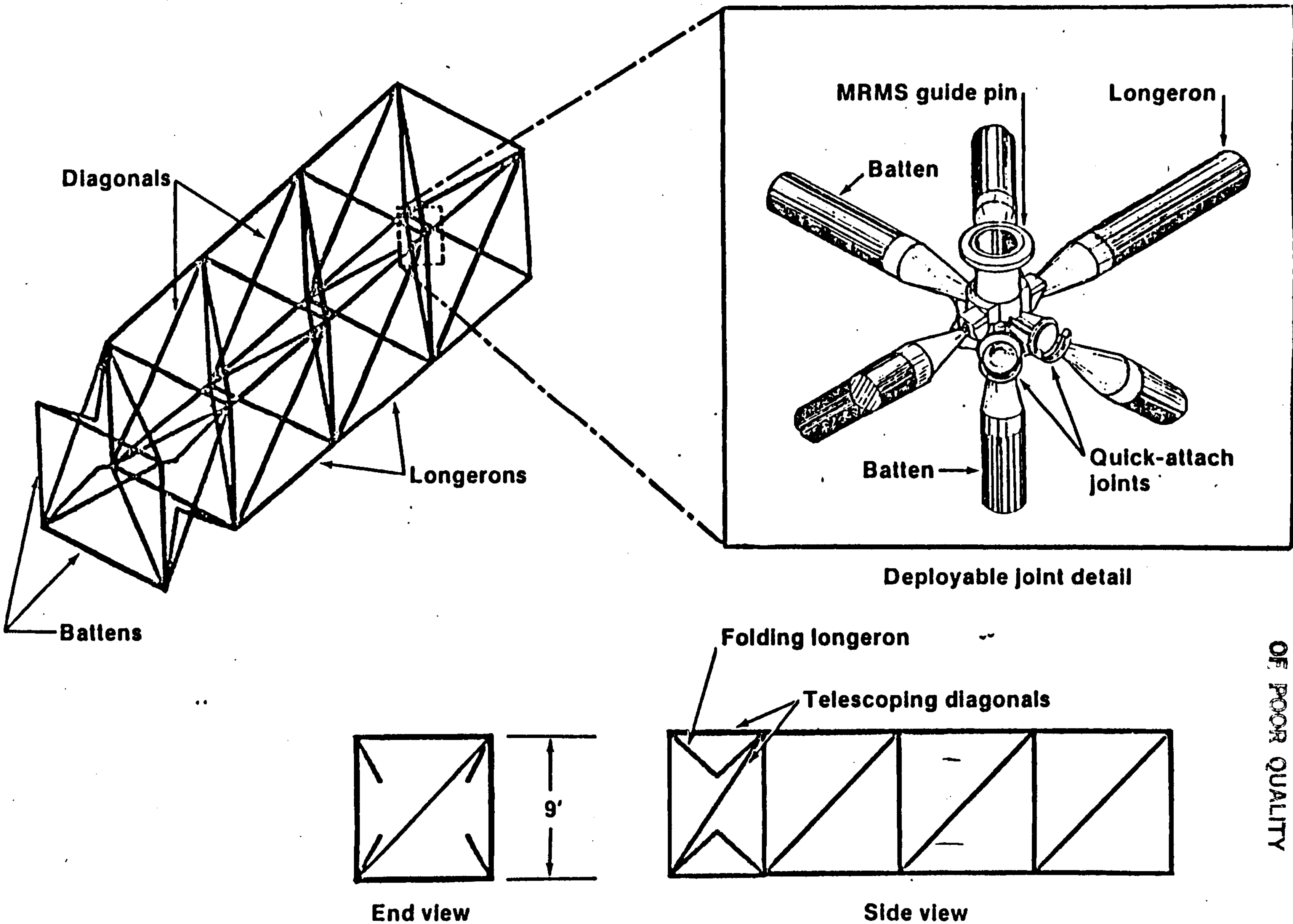

FIGURE 4.- SCHEMATIC OF DEPLOYABLE BEAM AND DETAIL OF JOINT

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LOCKOUT SYSTEM. THE PROBLEM OF FREE PLAY FOR THE MODEL IS EVEN MORE DIFFICULT FOR TWO REASONS — ONE, THE TOLERANCES MUST BE REDUCED BY THE MODEL SCALE FACTOR, AND TWO, IT WOULD BE MUCH EASIER TO HOLD CLOSE TOLERANCES ON JOINT ELEMENTS THE SIZE OF THE FULL SCALE STRUCTURE THAN IT WOULD BE ON ELEMENTS THE SIZE OF THE MODEL

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2.1.1 APPROXIMATION OF ALLOWABLE FOR JOINT FREE PLAY FOR POINTING ACCURACY

AS NOTED ON PAGE 1-22 OF REF. 2, THE ROCKWELL INTERNATIONAL REPORT WHICH COVERS STUDIES MADE FOR NASA/MSFC FOR DEVELOPMENT OF DEPLOYABLE STRUCTURES FOR LARGE SPACE PLATFORMS, A POINTING ACCURACY OF 0.05 to 0.10 DEG IS REPRESENTATIVE. ASSUMING THE KEEL HAS 44 SECTIONS (43 JOINTS) AS SHOWN IN THE IOC REFERENCE CONFIGURATION DESCRIPTION (FIG. 2), THAT THE POINTING ACCURACY IS MEASURED RELATIVE TO THE BASE (LABORATORIES), AND THAT THE DEFLECTIONS PRODUCED IN EACH JOINT ARE ADDITIVE AS EXPECTED FOR LOW FREQUENCY MOTIONS, THE ALLOWABLE ANGLE PER JOINT FOR THE AVERAGE POINTING ACCURACY IS:

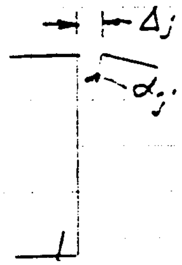
$$\alpha_j = \frac{0.075}{43}\,\frac{1}{57.3} = 3.04 \times 10^{-5} \text{ RAD}$$

ASSUME THAT HALF OF THIS JOINT ROTATION IS A RESULT OF FREE PLAY & HALF IS A RESULT OF ELASTIC DEFORMATION. THEN FOR A NINE-FOOT DEEP TRUSS, THE FREE-PLAY MUST BE LIMITED TO:

$$\Delta_j = \frac{1}{2} \text{ TRUSS DEPTH} \times \alpha_j$$

$$= \frac{1}{2}\left(\frac{9 \times 12}{2}\right) \times 3.04 \times 10^{-5} \text{ in}$$

$$= 0.00082 \text{ in}$$



SINCE THE MODEL SCALES LINEARLY

$$\frac{(\Delta_j)_M}{(\Delta_j)_F} = \lambda = \frac{1}{4}$$

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HENCE, $(\Delta_j)_M \approx 0.000205$ — PROBABLY IMPOSSIBLE TO ACHIEVE IN A REASONABLE MODEL CONSTRUCTION ENVIRONMENT.

2.1.2 CONSIDERATIONS FOR A TUBE CONNECTOR DEVICE TO VARY AND CONTROL JOINT STIFFNESS

SINCE FULL SCALE TRUSS AND JOINT DETAILS ARE NOT YET KNOWN, AND SINCE IT IS HIGHLY DESIRABLE TO BUILD RESEARCH VERSATILITY INTO THE MODEL, IT IS RECOMMENDED THAT A TECHNIQUE FOR MODEL CONSTRUCTION BE EMPLOYED WHICH ENABLES CHANGES IN THE DYNAMIC CHARACTERISTICS OF THE MODEL STRUCTURE. THIS TECHNIQUE IS ILLUSTRATED IN THIS SECTION. THE IDEA IS TO BUILD THE STRUCTURAL ELEMENTS AS LIGHT, STRONG AND STIFF AS POSSIBLE AND INCORPORATE FACTORS SUCH AS JOINT FREE-PLAY, RESILIENCE AND DAMPING IN A CONTROL ELEMENT AS APPROPRIATE. THUS THE BASIC ELEMENTS OF THE TRUSS STRUCTURE WOULD BE THE TUBES, THE JOINTS AND THE CONNECTOR. THE TUBES WOULD BE AS CLOSE TO REPLICA CONSTRUCTION AS TECHNICALLY FEASIBLE; THE JOINTS WOULD BE INVESTMENT CASTINGS MADE WITH MULTIPLE PRONGS OR CONNECTORS TO INTERCONNECT THE VARIOUS TRUSS MEMBERS, TO ATTACH OTHER SPACE STATION COMPONENTS, AND TO ATTACH THE SUPPORT CABLES; AND THE CONNECTORS WOULD PROVIDE FOR TUBE INSTALLATION AND REPLACEMENT AS WELL AS FOR "TUNING" THE DYNAMIC CHARACTERISTICS OF THE TRUSS ELEMENTS.

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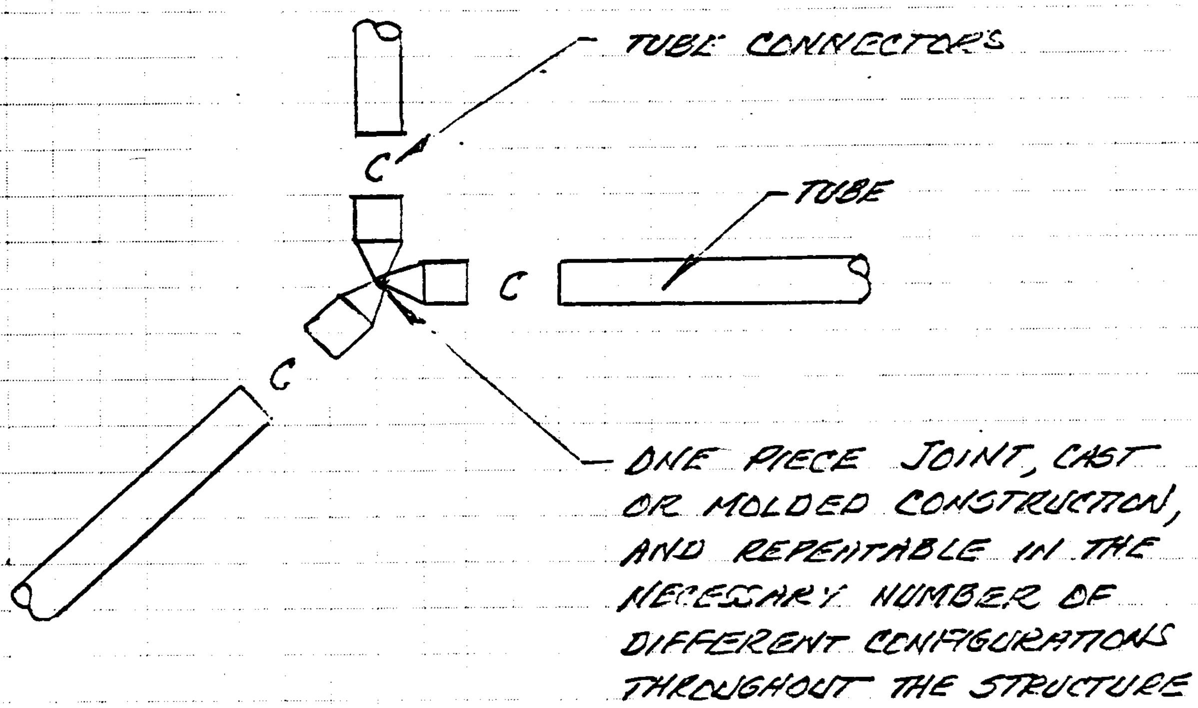
THE FOLLOWING PARAGRAPHS PROVIDE FURTHER DETAILS ON THE PROPOSED JOINT SYSTEM.

ASSUME THAT EACH JOINT IS A ONE PIECE CAST OR MOLDED SYSTEM WHICH HAS NO ARTICULATION BUT HAS TUBE CONNECTION PRONGS COMMENSURATE WITH CONFIGURATION OF FULL SCALE JOINTS



ASSUME THAT EACH END OF EACH TUBE IS FITTED WITH A TUBE CONNECTOR WHICH, WHEN COMBINED WITH THE JOINT AND TUBE PROVIDES THE DESIRED TUBE STIFFNESS, NON-LINEARITY, DAMPING, ETC. EACH CONNECTOR MUST HAVE THE FOLLOWING PROPERTIES:

1. ALLOW ANY TUBE TO BE REMOVED AND REPLACED OR ADJUSTED IN LENGTH WITH EASE

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2. INEXPENSIVE, HIGHLY REPRODUCIBLE

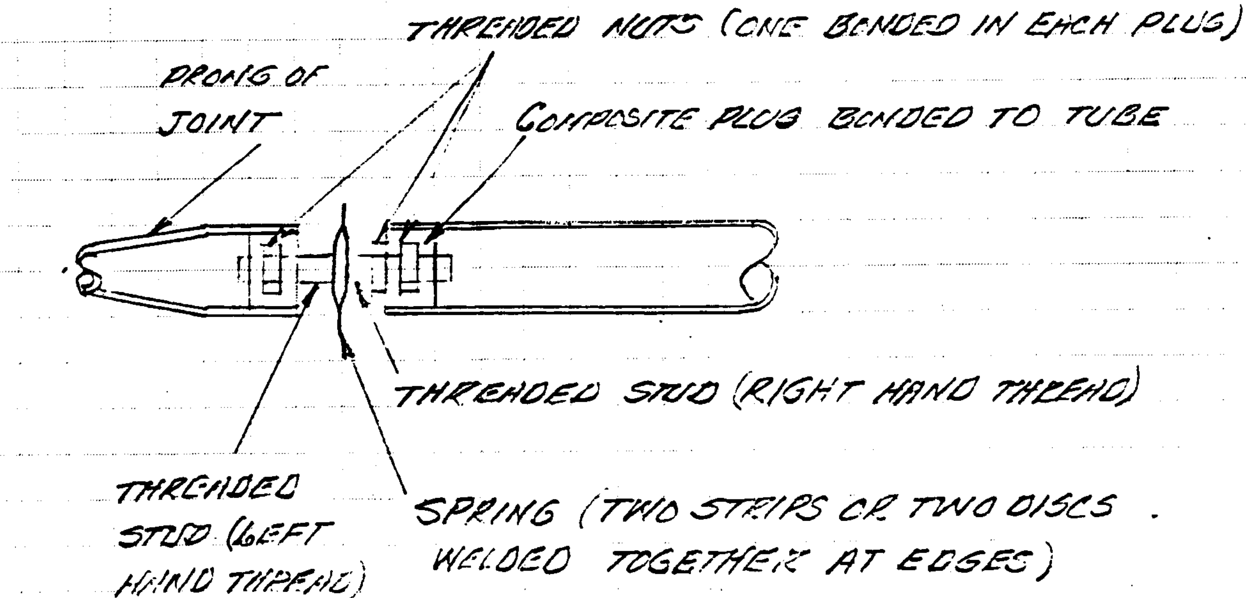
3. STIFFNESS CHARACTERISTS UNIFORM & PREDICTABLE

4. LIGHTWEIGHT

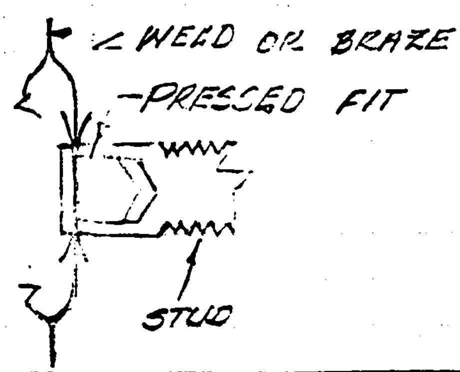
5. LOW INHERENT DAMPING

6. OPTIONS FOR BUILDING IN JOINT NON-LINEARITY IF DESIRED

IT IS BELIEVED THAT THESE PROPERTIES ARE PROVIDED TO A HIGH DEGREE BY THE CONNECTOR SYSTEM SKETCHED BELOW



SPRING DETAIL



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THE CONNECTORS SHOWN PROVIDE SEVERAL OPTIONS FOR VARIATIONS IN JOINT PROPERTIES INCLUDING THE FOLLOWING:

1. STIFFNESS

VARIATIONS IN SPRING DIAMETER, MATERIAL, MATERIAL THICKNESS, ETC.

2. MASS

MAKE SYSTEM AS LIGHT AS POSSIBLE. ADD TAPE TO TUBES TO INCREASE MASS AS DESIRED

3. DAMPING

INHERENTLY LOW. FILL AND COAT SPRINGS WITH VISCOUS ELASTOMERS TO INCREASE DAMPING

4. FREE PLAY

CONTROL BY THREAD CLEARANCE BETWEEN STUD AND NUTS. PLATE STUDS AS NECESSARY. USE STUDS AND NUTS FROM SAME LOT FOR UNIFORMITY

5. NON-LINEARITY

PUT LEAF WASHERS ON OUTSIDE OF SPRINGS

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FIGURE 5 SHOWS A SKETCH OF A PROPOSED MODEL JOINT WHICH WAS SIZED ON THE BASIS OF THE FOLLOWING CALCULATIONS

THE ASSUMPTION IS MADE THAT A FORCE ACTING AT THE CENTER OF MASS OF THE SPACE STATION WILL ACCELERATE THE STATION AT A RATE OF 0.04 g. THEN, FOR AN ALL-UP STATION WEIGHT OF 600000 lb AND CONSIDERING THAT THE FORCES ARE TRANSMITTED THROUGH 4 LONGERONS, THE AREA REQUIRED TO CARRY THE COMPRESSION AND TENSION LOADS IN EACH LONGERON IS

$$A = \frac{F}{\sigma_A} = \frac{600{,}000}{2} \times .04 \, \frac{1}{\sigma_A} \times \frac{1}{4}$$

ASSUMING THAT THE MATERIAL IS ALUMINUM AND THAT $\sigma_A = 12{,}500$ psi,

$$A = \frac{600{,}000 \times 0.04}{2 \times 12{,}500 \times 4} = 0.24 \text{ in}^2/\text{STRUT}$$

$$= \frac{\pi}{4} d_F^{\,2}$$

THEREFORE $d_F = \sqrt{\frac{.24 \times 4}{\pi}} = 0.55$ in

FOR A 1/4 SCALE REPLICA MODEL,

$$d_M = \frac{d_F}{4} = \frac{0.55}{4} = 0.1375 \cdots \text{SAY } 1/8''$$

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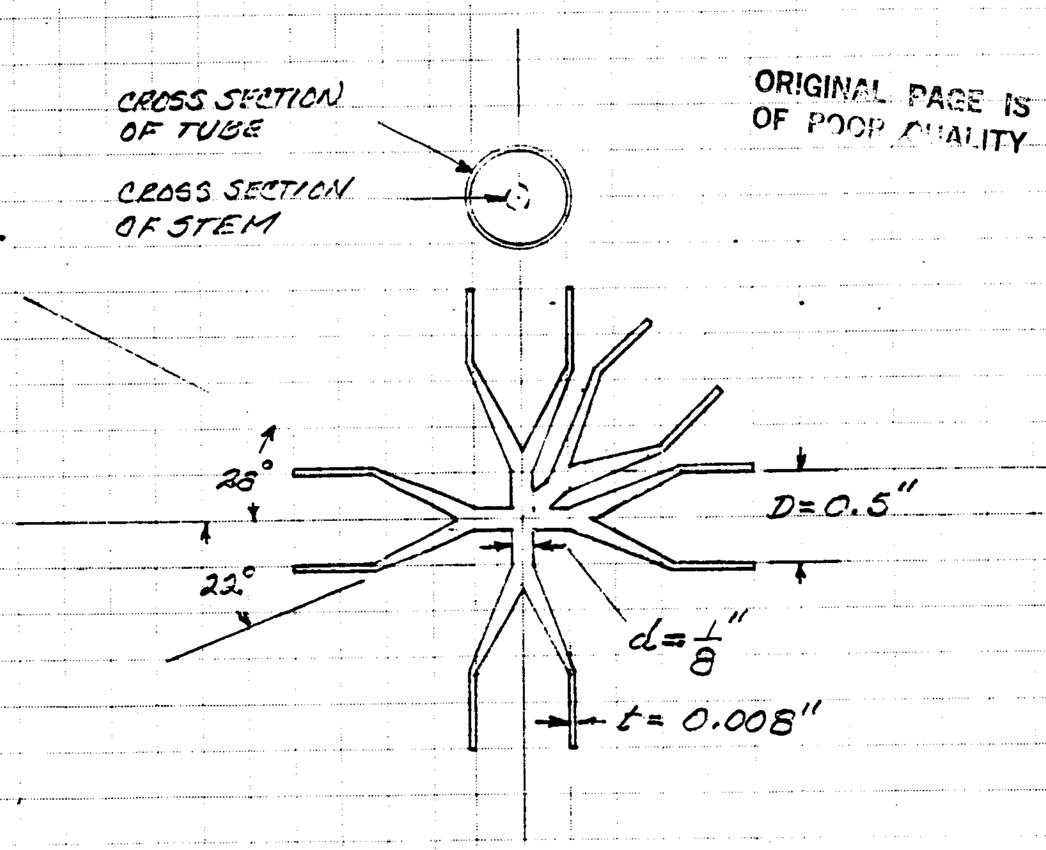
SKETCH OF SPACE STATION MODEL JOINT

MATERIAL – CASTING ALUMINUM



OTHER PRONGS WILL ALSO EXIST
PERPENDICULAR TO THE PLANE
OF THE PAPER BUT MAY BE ADDED
BY WELDING THE WAX MOLDS DURING
THE CASTING PROCESS

$$\pi D t = \frac{\pi}{4} d^2 \quad \text{or} \quad t = \frac{d^2}{4D} = 0.0078''$$

FIGURE 5.- SKETCH OF SPACE STATION MODEL JOINT

FIGURE 6 SHOWS AN INVESTMENT CASTING OF CASTING ALUMINUM WHICH WAS MADE BY THE NASA-LANGLEY FABRICATION DIVISION TO ASSESS THE CAPABILITY OF BUILDING JOINTS SIMILAR TO THAT SKETCHED IN FIGURE 5. THE MODELMAKERS FOUND THAT THE WALL THICKNESS OF 0.008" AS CALLED FOR IN FIGURE 5 WAS NOT ACHIEVABLE BY THE LOST WAX PROCESS AND HAD TO INCREASE THE THICKNESS TO ABOUT 0.013" TO ACHIEVE A GOOD REPEATABLE PRODUCT. EVEN SO, THE WEIGHT OF THE JOINT, WHICH ACCOMODATES ALL THE TUBES EXPECTED FOR A TYPICAL JOINT, IS APPROXIMATELY EQUIVALENT TO THAT OF AN ALUMINUM ROD 5/32"D AND 7.5"L, OR ABOUT 1/4 OZ.

JOINTS OF THE TYPE SHOWN IN FIGURE 6 SATISFY SEVERAL OTHER REQUIREMENTS. FIRST, BY WELDING ANY DESIRABLE ELEMENTS TO THE JOINT DURING THE WAX STAGE, HIGH STRENGTH COMPONENTS CAN BE PROVIDED FOR ATTACHING THE OTHER MASSES (PAYLOADS, ANTENNAS, ETC) TO THE KEEL STRUCTURE AND FOR ATTACHING THE MODEL SUPPORT CABLES.

A SECOND FACTOR TO BE DEALT WITH IS MODEL COST. THE QUESTION WHICH SHOULD RIGHTLY BE ASKED IS, WHY CAN'T YOU BUILD REPLICA JOINTS OF THE FULL SCALE HARDWARE? THE ANSWER IS, YOU CAN. FINE SWISS WATCHES ARE MORE DIFFICULT TO BUILD AND THEY ARE BUILT EVERY DAY. BUT THE HIGH COSTS OF TOOLING IS SPREAD OVER TENS OF THOUSANDS OF WATCHES AND ONLY ONE SPACE STATION MODEL IS ANTICIPATED. THE COST OF REPLICA MODEL JOINTS WOULD EXCEED THE COSTS OF FULL SCALE JOINTS. FURTHERMORE, COMPROMISES MUST BE MADE THROUGH MODEL TESTS IN THE EARTHS ATMOSPHERE AND GRAVITY FIELD WHICH IMPOSE HIGHER DYNAMIC CONSTRAINTS. MODEL JOINTS SUCH AS THESE APPEAR TECHNICALLY RESPONSIVE AND COST EFFECTIVE.

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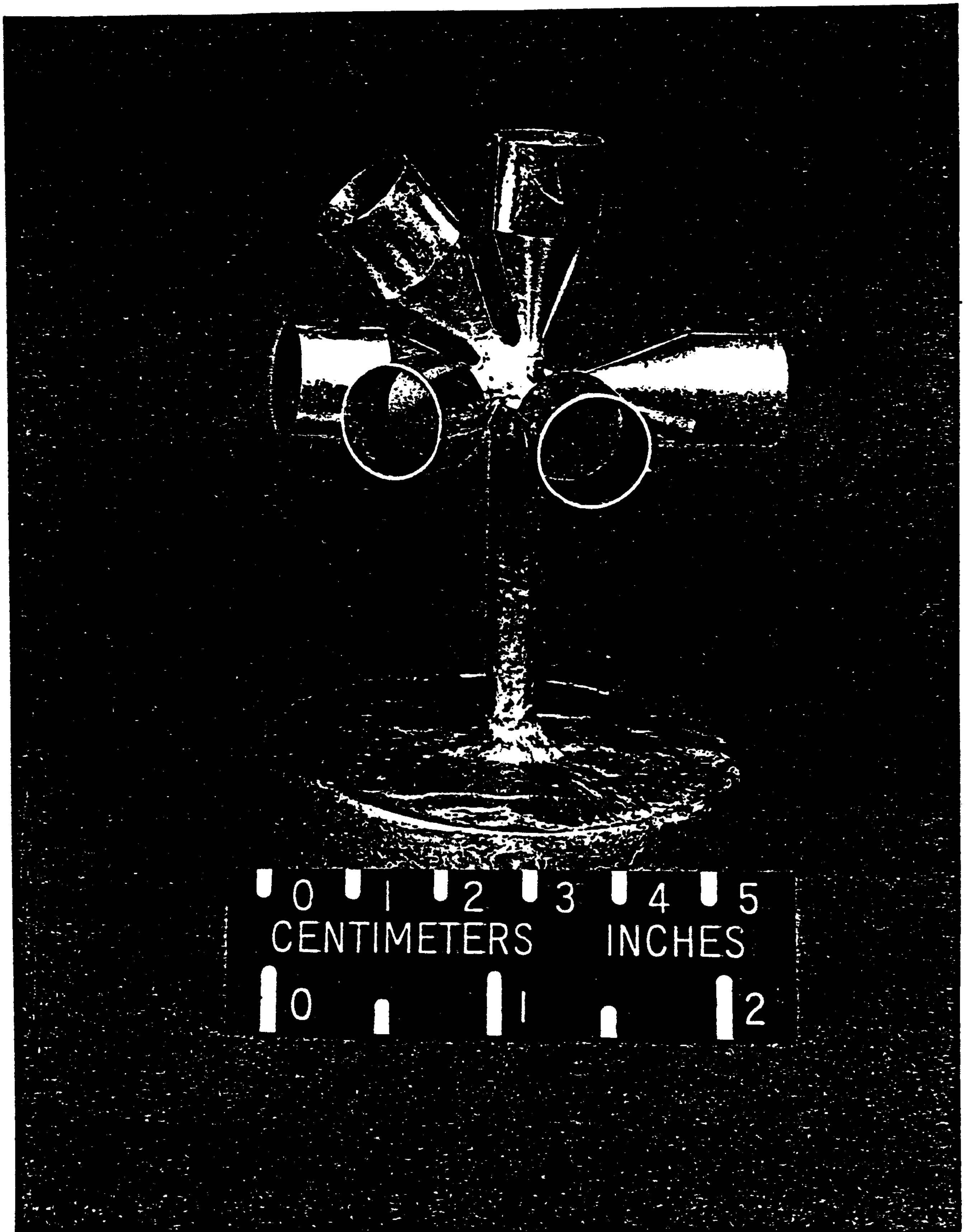

FIGURE 6.- INVESTMENT CASTING OF ALUMINUM JOINT OF TYPE PROPOSED FOR SPACE STATION MODEL

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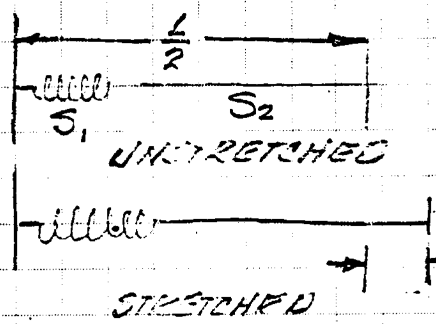
AN APPROXIMATION OF VARIABLE SPRING CHARACTERISTICS REQUIRED FOR THE MODEL TUBE CONNECTOR TO SIMULATE THE FULL SCALE JOINT PROPERTIES IS GIVEN AS FOLLOWS.



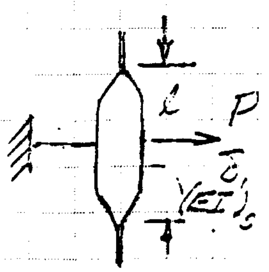
$$\delta = \delta_1 + \delta_2$$

$$= \delta)_{S_1} + (\delta)_{S_2}$$

ASSUME THAT 80% OF THE STRETCH TAKES PLACE IN THE JOINT, i.e., $\delta_1 = 4\delta_2$

ASSUME THAT δ_2 IS THE DEFLECTION FOR A 1/2" D GRAPHITE TUBE WITH WALL THICKNESS OF 0.015" AND A LENGTH OF 13.5" (A TUBE CONNECTOR ASSUMED AT EACH END OF THE TUBE)

THE SPRING S_1 IS AS SHOWN. IT CAN BE REPRESENTED BY TWO FIXED-FIXED BEAMS IN SERIES.



$$\delta_1 = 2\,\frac{P l^3}{192 (EI)_s}$$

THE DEFLECTION OF THE TUBE IS DERIVED BY

$$\sigma_2 = \frac{P}{A} = \epsilon E_T = \frac{\delta_2}{L/2} E_T \quad \text{AND}$$

$$\delta_2 = \frac{L}{2}\,\frac{P}{AE_T}$$

NOW BY EQUATING δ_1 to $4\delta_2$

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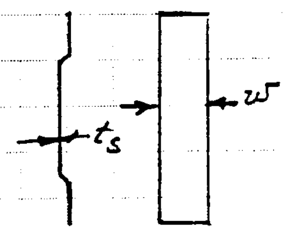
$$\frac{2Pl^3}{192E_sI_s} = 4\,\frac{LP}{2A_tE_T}$$

WHENCE

$$I_s = \frac{1}{192}\left(\frac{E_T}{E_s}\right)\left(\frac{l^3}{L}\right)A_t \qquad A_t = \pi(1/2)\,t_T$$

$$= \frac{1}{192}\left(\frac{21\times10^6}{30\times10^6}\right)\left(\frac{(0.5)^3}{27}\right)\pi\frac{1}{2}(.015)$$

$$= 3.977\times10^{-7} = \frac{1}{12}w\,t_s^3$$



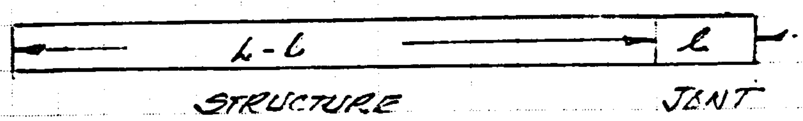
w	t
1/4	0.0267
3/8	0.0233
1/2	0.0212

$$t_s^3 = \frac{4.772\times10^{-6}}{w}$$

THE CONCLUSION FROM THESE CALCULATIONS IS THAT THE SPRINGS COULD BE MADE FROM STEEL SPRING STOCK OF APPROX. 0.025 THICKNESS

OTHER FACTORS REVIEWED RELATIVE TO JOINT DETAILS INCLUDE THE EFFECTIVE STIFFNESS OF A STRUCTURE AND A JOINT IN SERIES, THE RELATIVE MOTIONS DUE TO JOINT AND TUBE DEFORMATIONS, AND THE SCALING OF EXTENSIONS WITHIN A HINGED JOINT. DETAILS OF THESE ANALYSES ARE GIVEN IN SECTIONS 3.1.3 - 3.1.5 WHICH FOLLOW.

2.1.3 DETERMINATION OF EFFECTIVE STIFFNESS OF A STRUCTURE AND A JOINT IN SERIES



THE EFFECTIVE STIFFNESS IS DETERMINED AS FOLLOWS

$$\delta_{eff} = \delta_s + \delta_j = \epsilon_s l_s + \epsilon_j l_j = \frac{\sigma_s}{E_s} l_s + \frac{\sigma_j}{E_j} l_j$$

$$= P\left(\frac{L-l}{(EA)_s} + \frac{l}{(EA)_j}\right) = \frac{PL}{(EA)_{eff}}$$

$$\therefore \frac{(EA)_{eff}}{(EA)_s} = \frac{1}{1 + \frac{l}{L}\left(\frac{(EA)_s}{(EA)_j} - 1\right)}$$

WHAT ARE REASONABLE VALUES ?

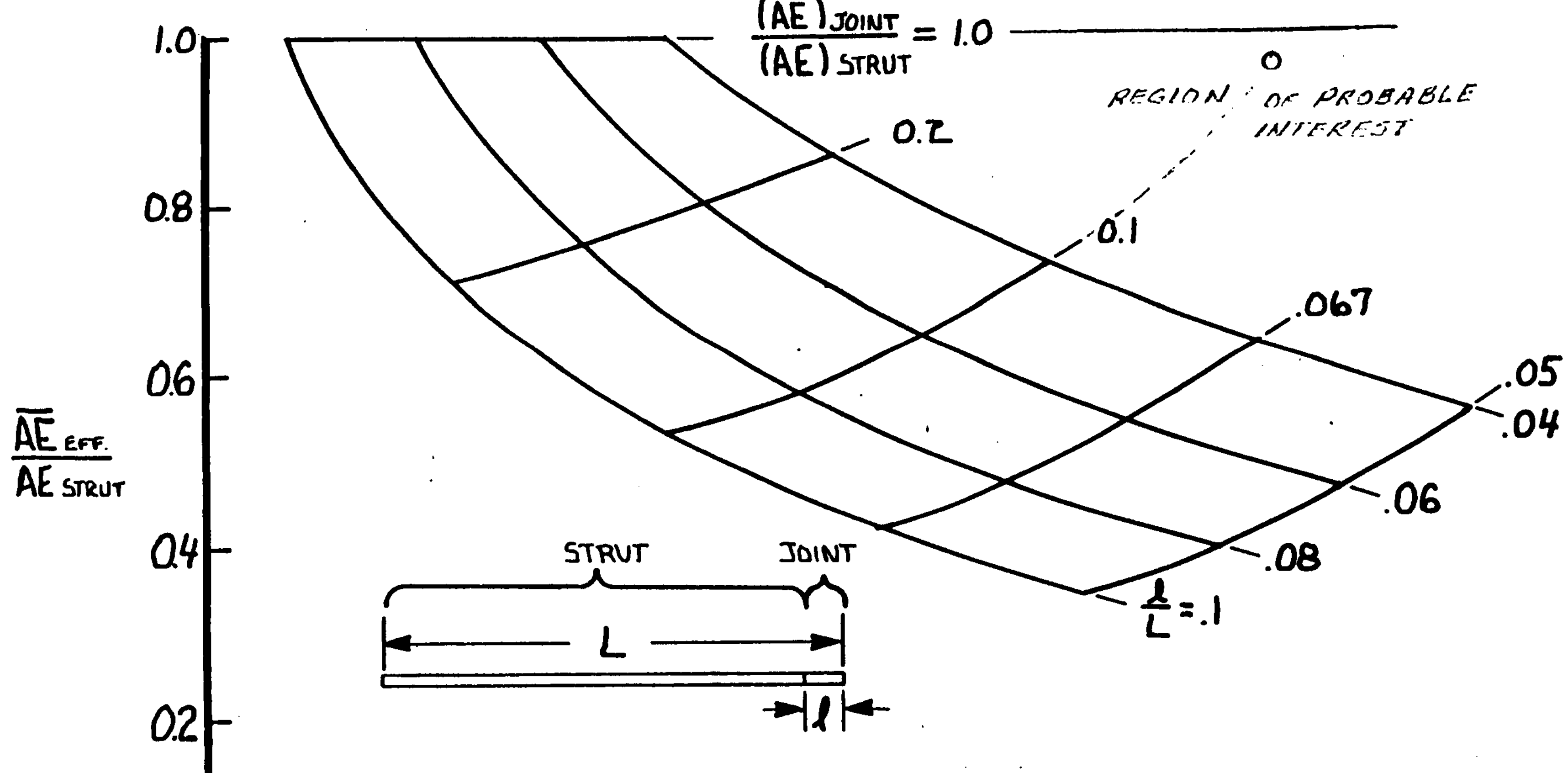
LET $l = 1''$, $L = 9' = 108''$ $\frac{l}{L} = 0.0093$

$(EA)_s = 10\,(EA)_j$

$$\frac{(EA)_{eff}}{(EA)_s} = \frac{1}{1 + .0093\,(10-1)} = \frac{1}{1 + .084} = 0.92$$

A NASA DERIVED CARPET PLOT IS SHOWN IN FIGURE 7.

EFFECTIVE STRUT STIFFNESS CONSIDERING JOINT EFFECTS



$$\frac{\overline{AE}_{\text{EFFECTIVE}}}{AE_{\text{STRUT}}} = \frac{1}{1 + \frac{\ell}{L}\left(\frac{AE_S}{AE_J} - 1\right)}$$

FIGURE 7.- EFFECT OF JOINT STIFFNESS ON THE EFFECTIVE STIFFNESS OF A STRUT.

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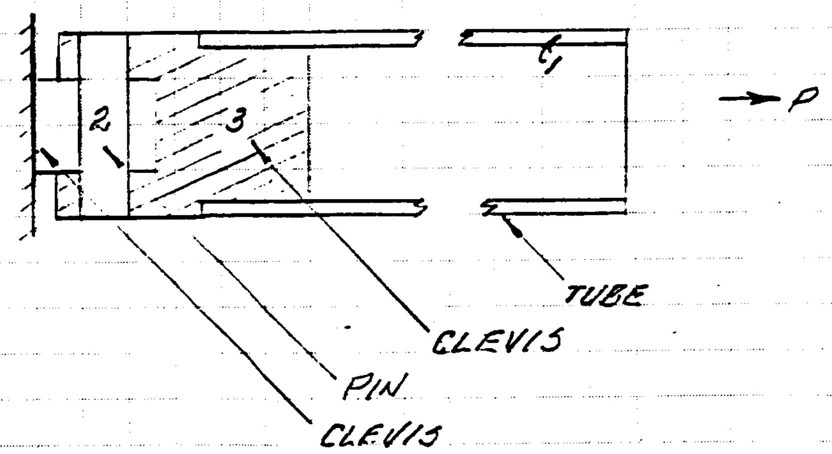
2.1.4 REVIEW OF SCALING OF EXTENSIONS WITHIN A JOINT (HINGE) AND AN APPROXIMATION OF RELATIVE MOTIONS



$$\delta = \sum_{i=1}^{n} \delta_i \quad \text{AND}$$

δ_1 = EXTENSION OF TUBE $\propto \dfrac{K_1 P L_1}{d_1 t_1 E_1}$

δ_2 = EXTENSION DUE TO PIN BENDING

$$\propto k_2 \frac{P\, l_2^3}{E_2\, d_2^4}$$

δ_3 = EXTENSION DUE TO PIN SHEAR

$$\propto k_3 \frac{P}{d_2 G_2}$$

δ_4 = EXTENSION DUE TO CLEVIS STRETCHING AT PIN

$$\propto k_4 \frac{d_2 P}{(w-d)\, t\, E_3}$$

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δ_5 = EXTENSION DUE TO PIN SHEARING

$$\propto k_5 \frac{d_2 P}{E_2 d_2 t}$$

δ_6 = EXTENSION DUE TO CLEVIS BEARING

$$\propto k_6 \frac{d_2 P}{E_3 d_2 t}$$

THE FORCE $P \propto Ma \propto M\omega^2 L$, FOR REPLICA SCALING

$$\frac{M_M}{M_F} = \lambda^3 = \left(\frac{L_M}{L_F}\right)^3; \; \& \; \frac{\omega_M^2}{\omega_F^2} = \frac{1}{\lambda^2}$$

$$\therefore \frac{P_M}{P_F} = \left(\frac{M_M}{M_F}\right)\left(\frac{\omega_M}{\omega_F}\right)^2\left(\frac{L_M}{L_F}\right) = \left(\lambda^3\right)\left(\frac{1}{\lambda^2}\right)\left(\lambda\right) = \lambda^2 \text{ AND}$$

IT FOLLOWS THAT

$$\frac{\delta_{i,M} / L_{i,M}}{\delta_{i,F} / L_{i,F}} = \left(\frac{P_M}{P_F}\right)\left(\frac{\mathcal{L}_F}{\mathcal{L}_M}\right)^2 = 1 \text{ WHEN } E_M = E_F \; \& \; G_M = G_F$$

AND $\mathcal{L}$ IS ONE OF THE CHARACTERISTIC LENGTHS. THUS ALL EXTENSIONS OF THE JOINT OF A REPLICA MODEL SCALE DIRECTLY AS THE SIZE OF THE JOINT.

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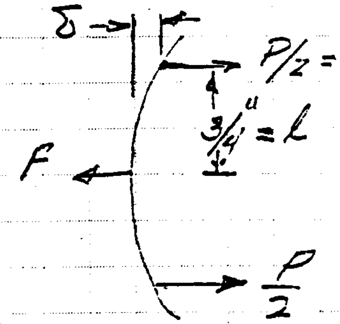
APPROXIMATION OF THE MOTIONS DUE TO JOINT AND TUBE DEFORMATIONS MAY BE ACHIEVED AS FOLLOWS.

CONSIDERING THE SIMPLE HINGED JOINT SHOWN IN THE SKETCH ON PAGE 37, EXAMINE THE RELATIVE DEFLECTIONS OF A 1/4 INCH STEEL PIN IN BENDING WITH THE DEFLECTIONS OF 1/2 OF A REPRESENTATIVE TUBE.

(a) DEFLECTION DUE TO PIN BENDING



-- Center of Clevis

---- Center of hinge

$$\delta = \frac{p\ell^3}{3EI} = \left(\frac{P}{2}\right)\left(\frac{3}{4}\right)^3\left(\frac{1}{3}\right)\left(\frac{1}{30\times10^6}\right)\left(\frac{1}{\frac{\pi}{64}\left(\frac{1}{4}\right)^4}\right)$$

$$\delta / P/2 = (.422)\left(\frac{1}{3}\right)\left(\frac{1}{30}\right)10^{-6}\left(\frac{1}{.0001917}\right)$$

$$= 24.46 \times 10^{-6} \text{ in/lb}$$

(b) DEFLECTION DUE TO TUBE STRETCHING

$$\delta = \epsilon \frac{L}{2} = \frac{\sigma}{E}\frac{L}{2} = \left(\frac{F}{AE}\right)\left(\frac{L}{2}\right)$$

$$\delta/F = \left(\frac{1}{A}\right)\left(\frac{1}{E}\right)\left(\frac{L}{2}\right) = \left(\frac{1}{\pi \times 2 \times .06}\right)\left(\frac{1}{21\times10^6}\right)(54) = 6.82\times10^{-6}$$

CONCLUSION: DEFLECTION DUE TO BENDING OF THE 1/4" PIN IS ABOUT 4 TIMES AS MUCH AS DUE TO TUBE STRETCHING

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2.1.5 CONSIDERATIONS FOR SUPPORTING THE MODEL BY ATTACHMENTS TO TANGS FROM THE TRUSS JOINTS

THE ALL-UP WEIGHT OF THE MODEL WILL BE SUPPORTED FROM A SERIES OF SOFT CABLES. FOR A 1/4 SCALE MODEL, WHICH WILL BE ABOUT 100 FT. LONG, THE MAXIMUM WEIGHT WILL BE ABOUT 10,000 LB. THE FORCES REPRESENTING THIS WEIGHT MUST BE CARRIED THROUGH THE JOINTS OF THE TRUSS STRUCTURE.

THE IOC CONFIGURATION OF PRIME INTEREST CONTAINS ABOUT 41 KEEL SECTIONS. THUS THE KEEL HAS ABOUT 88 POINTS FOR ATTACHMENT OF CABLES TO CARRY LOADS IN A GIVEN DIRECTION. THE KEEL EXTENSIONS, THE TRANSVERSE BOOM AND THE UPPER BOOM PROVIDE ANOTHER 140 POINTS. IF WE ASSUME THAT THE WEIGHT IS CARRIED BY ONE FOURTH OF THESE 228 POSSIBILITIES, THE FORCE PER JOINT WOULD BE 10,000/57 OR 175 lbs.

IF A LOAD ATTACHMENT TANG IN THE FORM OF A PROJECTION FROM EACH JOINT IS PROVIDED TO CARRY THE LOADS, THE NECESSARY DIAMETER WILL BE

$$d = \left(\frac{4}{\pi} S.F. \frac{F}{F_{4}^{TU}}\right)^{1/2} = \left(\frac{4}{\pi}(1.5)\frac{175}{32\times10^{3}}\right)^{1/2} = 0.106 \text{ IN}$$

ASSUMING THE USE OF AN ALUMINUM CASTING ALLOY SUCH AS 355 HEAT TREATED TO T-6. IF WE ASSUME A NOMINAL SIZE OF 1/8 IN DIAMETER AND 1/2 IN LENGTH, AND ASSUME THAT EACH JOINT IS EQUIPPED WITH 2 TANGS (FOR SUPPORT IN EITHER OF 2 DIRECTIONS), THE TOTAL WEIGHT WILL BE 0.23 lbs.

2.1.6 THE FEASIBILITY OF FABRICATION AND TESTING OF GRAPHITE EPOXY COMPOSITE TUBES

ON THE BASIS OF DISCUSSIONS WITH POTENTIAL MATERIALS SUPPLIERS, SPACE STATION STRUCTURAL ENGINEERS, COMMERCIAL SUPPLIERS OF GRAPHITE EPOXY TUBULAR PRODUCTS AND MODEL MANUFACTURERS, THE WRITER BELIEVES THAT A 1/4-SCALE SPACE STATION MODEL WOULD REQUIRE TUBES ABOUT 1/2 IN. DIAMETER AND 2½ TO 5½ IN. LONG. THE TUBES WILL PROBABLY REQUIRE ABOUT 4 LAMINATIONS OF PRE-PREGS HAVING A THICKNESS OF ABOUT 0.0025 INCH EACH AND COMBINING TO PRODUCE A TOTAL WALL THICKNESS OF ABOUT 0.010 IN. MATERIALS SIMILAR TO P75S/E934 GRAPHITE EPOXY ARE EXPECTED TO BE USED BECAUSE HIGHER MODULUS GRAPHITE FIBERS ARE TOO BRITTLE FOR FABRICATIONS REQUIRING LARGE CURVATURES SUCH AS FOR SMALL TUBES.

TO MAXIMIZE THE LONGITUDINAL STIFFNESS OF THE TUBES WHILE MAINTAINING ADEQUATE RESISTANCE TO TUBE SPLITTING DURING COMPRESSIVE LOADINGS, THE ORIENTATIONS OF THE FIBERS ARE EXPECTED TO BE ABOUT ±30° ARC RELATIVE TO THE AXIS OF THE TUBE.

OF THE SEVERAL WAYS TO FABRICATE THE TUBES, IT IS EXPECTED THAT HAND LAYUP OF PREARRANGED PLIES AROUND A MANDREL WILL PROBABLY BE THE CHOICE BECAUSE OF THE CONTROL WHICH CAN BE EXERTED DURING THE MANUFACTURING PROCESS AND THE FACT THAT MOST GRAPHITE EPOXY STRUCTURES OF THIS SIZE ARE CURRENTLY MADE IN THIS MANNER. THE EXPENSES FOR TOOLING ARE MINIMAL AND THE TOTAL BUY (MEASURED BY COMMERCIAL PRODUCTS STANDARDS) IS SMALL. IN THIS PROCESS, THE USUAL TECHNIQUE IS TO COAT THE MANDREL WITH A RELEASE AGENT (e.g. SILICONE), ROLL THE MANDREL OVER THE PREARRANGED AND PRECUT PLIES, COVER WITH SHRINK TAPE,

AND CURE AT ABOUT 350°F IN AN OVEN.

THE REMOVAL OF THE CURED TUBES FROM THE MANDREL CAN BE ACHIEVED IN SEVERAL WAYS. A SMALL AMOUNT OF TAPER IN THE TUBES IS VERY HELPFUL AND IS OFTEN USED WHERE ONLY THE AVERAGE PROPERTIES OF THE TUBE ARE SIGNIFICANT. THIS MAY BE ACCEPTABLE IN THIS CASE BECAUSE THE CHARACTERISTIC MODES OF THE SPACE STATION MODEL WILL ONLY REFLECT THE AVERAGE VALUE OF TUBE STIFFNESS IN TENSION AND COMPRESSION. SPLIT MANDRELS ALSO PROVIDE FEASIBLE OPTIONS AND THE WRITER SUGGESTS THAT THE TECHNIQUE DESCRIBED IN SECTION 2.1.7 MAY BE THE SIMPLEST PROCEDURE.

AFTER THE TUBES ARE MADE, SOME EFFECTIVE MEANS WILL BE NEEDED TO CLASSIFY THEM IN A GO/NO-GO SITUATION FOR ACCEPTANCE AND FOR MATCHING THEM SO THE TUBES INSTALLED IN A GIVEN TRUSS BAY ARE OF EQUAL MASS AND STIFFNESS. A TECHNIQUE FOR ACHIEVING THIS GOAL IS OUTLINED IN SECTION 2.1.8. THE TECHNIQUE INVOLVES WEIGHING THE TUBES AND VIBRATING THEM IN A SIMPLE GRIP DEVICE TO OBTAIN THE NECESSARY DATA FOR TUBE CLASSIFICATION.

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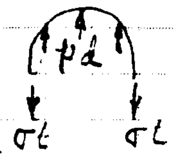
2.1.7 USE OF AIR OR WATER PRESSURE TO REMOVE THIN WALLED COMPOSITE TUBES FROM CYLINDRICAL MANDRELS

GRAPHITE EPOXY COMPOSITE TUBES HAVING DIMENSIONS OF APPROXIMATELY 1/2 IN DIAMETER, 0.015 IN WALL THICKNESS, AND 45 INCH LENGTH ARE OF INTEREST FOR CONSTRUCTING A DYNAMIC MODEL OF THE SPACE STATION. SUCH TUBES ARE USUALLY MADE BY WRAPPING COMPOSITE MATERIALS (PREPREGS OR WINDINGS) AROUND A CYLINDRICAL MANDREL, OVERWRAPPING THE COMPOSITES WITH A HEAT SHRINK TAPE, AND CURING IN AN OVEN. THE RESULT OF THE CURED PRODUCT IS A SNUG FIT OF THE TUBE ON THE MANDREL, AND SINCE THE WALL IS VERY THIN, REMOVAL OF THE TUBE FROM THE MANDREL WITHOUT DAMAGING THE TUBE IS OFTEN CHALLENGING. THIS NOTE SUGGESTS THAT THIS CAN BE DONE BY EXPANDING THE TUBE DIAMETER WITH AIR OR WATER UNDER PRESSURE.

THE SPLITTING STRESS IN A TUBE UNDER INTERNAL PRESSURE IS:

$$\sigma = \frac{pd}{2t}$$



AND

$$\epsilon = \frac{\Delta l}{l} = \frac{\sigma}{E} = \frac{pd}{2tE}$$

SINCE $l = \pi d$ & $\Delta l = \pi \Delta d$

$$\frac{\Delta d}{d} = \epsilon = \frac{pd}{2tE}$$

FOR PURPOSES OF ANALYSIS, ASSUME THE WORST CASE, i.e., ALL FIBERS ARE CIRCUMFERENTIAL, AND THE MATERIAL IS A HIGH MODULUS GRAPHITE/EPOXY (P75S/E934) WITH THE FOLLOWING PROPERTIES

$$F_L^{tu} = 1000 \ MPa = 1000 \times 10^6 \ pascals = \frac{1000 \times 10^6}{6.89 \times 10^3} \ psi = 145 \times 10^3 \ psi$$

$$E_L^t = 368 \ GPa = 368 \times 10^9 \ pascals = \frac{368 \times 10^9}{6.89 \times 10^3} \ psi = 53.4 \times 10^6 \ psi$$

ASSUME A SAFETY FACTOR ON STRESS OF 1.5, THE ALLOWABLE STRESS IS THEN

$$F_L^A = 96.7 \times 10^3 \ psi$$

AND THE ALLOWABLE STRAIN IS

$$\epsilon^A = \frac{F_L^A}{E_L^T} = \frac{\sigma}{E} = \frac{96.7}{53.4 \times 10^3} = 1.81 \times 10^{-3} \ in/in$$

$$\approx 0.002 \ in/in$$

THE INTERNAL PRESSURE REQUIRED TO PRODUCE THIS STRAIN IS

$$p = \frac{2tE\epsilon}{d} = \frac{2t}{d} F_L^A$$

FOR A TUBE WITH $d = 0.5$ IN & $t = 0.01$, THE ALLOWABLE PRESSURE IS

$$p_A = \frac{2 \times 0.01 \times 96.7 \times 10^3}{0.5} = 3.87 \times 10^3 \ psi$$

NOTE THAT FOR A MATERIAL SUCH AS THORNELL OR CELION 3K OR 6K, THE ALLOWABLE PRESSURE WOULD BE PROPORTIONAL TO THE ALLOWABLE STRESS, i.e., FOR A SIMILAR TUBE

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$$p_A = \frac{2 \times 0.01 \times (225 \times 10^3 / 1.5)}{0.5} = 6 \times 10^3 \text{ psi}$$

HOWEVER, SINCE THE MODULUS IS SUBSTANTIALLY LOWER ($E_L^t = 20.7 \times 10^6$ psi), THE ALLOWABLE STRAIN PRODUCED BY THE ALLOWABLE STRESS IS MUCH HIGHER, i.e.

$$\epsilon_A = \frac{\sigma_A}{E} = \frac{(225/1.5) \times 10^3}{20.7 \times 10^6} = 0.0072$$

THE EFFECT OF THIS IS THAT THE TUBE CAN BE ENLARGED 3.5 TIMES AS MUCH TO GET IT OFF THE MANDREL.

THE PROPOSED DESIGN OF THE MANDREL CONSISTS OF MAKING IT FROM A TUBE WHICH IS FITTED WITH VERY SMALL RADIALLY DRILLED HOLES. THE TUBE IS THEN FITTED WITH A PRESSURE REGULATED FLUID TAP AT ONE END AND PLUGGED AT THE OTHER.

DURING THE PROCESS OF MAKING & CURING THE TUBE IT IS EXPECTED THAT SOME RESIN WILL BLEED INTO THE SMALL HOLES UNLESS THEY ARE COVERED IN SOME WAY. IT IS BELIEVED THAT A SUITABLE THIN TAPE COULD BE PLACED OVER THE HOLES TO ADEQUATELY STOP THE BLEEDING. THE TAPE WOULD BE READILY REMOVED BY THE PRESSURE. HOWEVER, SINCE THE HOLES ARE VERY SMALL, THE SHEARING FORCES TO SHEAR THE RESIN PLUGS WOULD BE MINIMAL. SINCE LEAKAGE WOULD BE EXPECTED AT THE REMOVAL PRESSURES, THE PLUGS MAY NOT CREATE ANY PROBLEMS WHATSOEVER.

2.1.8 TECHNIQUE FOR MODEL TUBE SELECTION/GRADING

1. MEASURE WITH LENGTH GAUGE

2. INSERT END ON TAPERED RING GAUGE FOR INSIDE DIAMETER

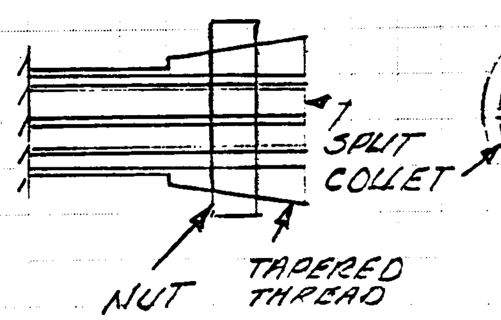
3. WEIGH TUBE

4. PLACE ONE END OF TUBE IN A GRIPPER AND ADD A MASS TO THE OTHER END

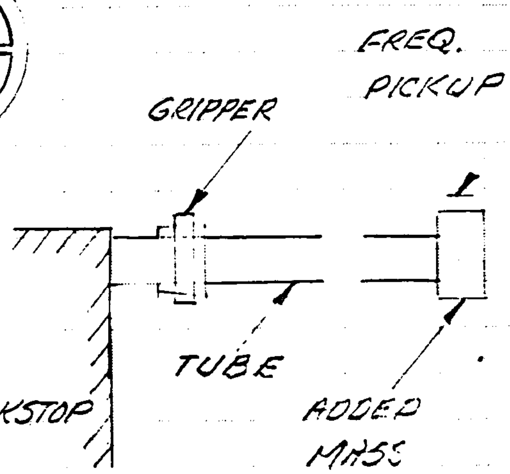
5. DEFLECT END AND RELEASE TO MEASURE THE NATURAL FREQUENCY

6. ROTATE GRIPPER 90° AND REPEAT ITEM 5.

7. ACCEPT OR REJECT TUBE ON BASIS OF RESULTS FROM ITEMS 3, 5 AND 6



GRIPPER DETAILS



FREQUENCY TEST SET-UP

2.2 MODULES AND OTHER MASSES

IT SEEMS PROBABLE THAT MOST OF THE COMPONENTS THAT MAKE UP THE MAJOR PORTION OF THE MASS OF THE SPACE STATION CAN BE TREATED AS RIGID BODIES WHEN ANALYZING THE OVERALL DYNAMICS OF THE STATION. SINCE THEY ARE PRESSURIZED VESSELS OR OTHER RELATIVELY COMPACT SYSTEMS, THEIR LOWEST NATURAL FREQUENCIES WILL BE MUCH HIGHER THAN THE NATURAL FREQUENCIES OF THE HIGHEST OVERALL VEHICLE MODES OF INTEREST FROM THE STANDPOINT OF VEHICLE GUIDANCE, CONTROL OR STABILIZATION. HOWEVER, PROPER DYNAMIC SCALING OF THESE MASSES IS ESSENTIAL AND MUST INCLUDE AT LEAST THE FOLLOWING:

1. MASS
2. MASS DISTRIBUTION RELATIVE TO THE VEHICLE COORDINATE AXES
3. MASS MOMENTS OF INERTIA ABOUT THE PRINCIPAL AXES OF THE BODY IN QUESTION
4. THE SPATIAL DISTRIBUTION OF CONNECTIONS BETWEEN THE BODY AND THE TRUSS STRUCTURE OR OTHER POINTS OF ATTACHMENT
5. THE EFFECTIVE STIFFNESS OF ALL ATTACHMENTS IN THE THREE MUTUALLY PERPENDICULAR COORDINATE DIRECTIONS AND ABOUT THESE AXES.
6. THE DAMPING DISTRIBUTIONS THROUGHOUT THE MASS ATTACHMENT SYSTEM.

THE SIGNIFICANCE OF THESE FACTORS CAN BE REALIZED BY REVIEW OF A TYPICAL SYSTEM SUCH AS SHOWN BY THE SKETCHES OF POTENTIAL IOC STRUCTURES SHOWN IN FIGURE 8. THE IMPACT OF SUCH MASSES ON BENDING OR TORSION MODES OF THE OVERALL STRUCTURE IS READILY APPARENT BY VISUALIZING THE MOTIONS OF THE MASS AND THE MASS ATTACHMENTS UNDER CONDITIONS WHERE THE MASS MAY LIE NEAR A NODE OR AN ANTINODE OF A BENDING MODE.

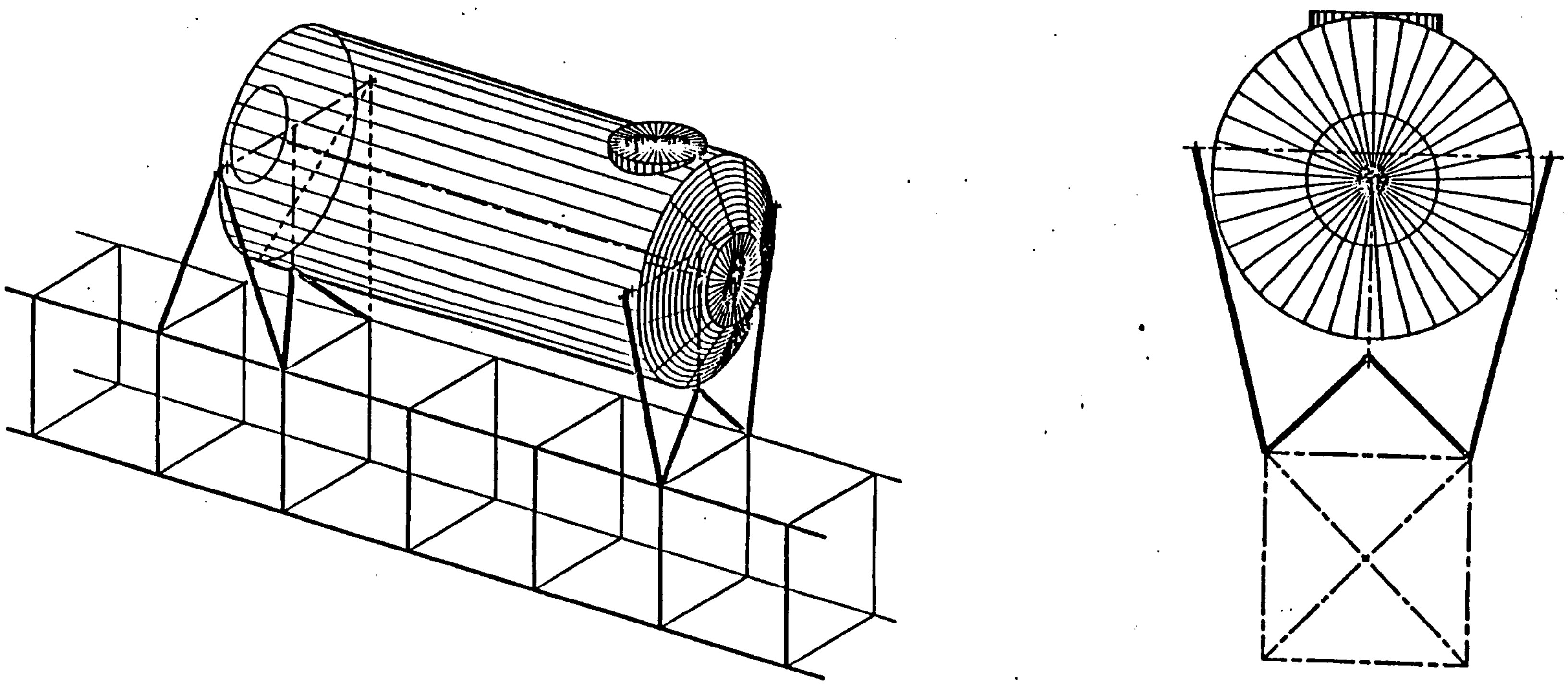


FIGURE 8.- SCHEMATIC VIEWS OF ATTACHMENT OF MODULES TO
A 9 FT. TRUSS.

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OR TORSION MODE.

IT APPEARS THAT THE MASSES LOCATED IN THE MODULES USED FOR LABORATORIES, HABITABILITY MODULES AND LOGISTICS WILL BE LOCATED AROUND THE PERIMETERS AND MOSTLY ATTACHED TO THE OUTER SHELL. THIS WILL MEAN THAT METALLIC SHELLS WITH ATTACHMENTS OR CUTOUTS TO SIMULATE THE MASSES AND MASS MOMENTS OF INERTIA, AND WELDMENTS FOR ATTACHMENT OF STRUTS TO THE TRUSS STRUCTURES, WOULD PROVIDE ATTRACTIVE MODELING OPTIONS. SUCH TECHNIQUES WILL BE NECESSARY TO KEEP THE DAMPING OF THE MODEL STRUCTURES DOWN - THE ADDITION OF MORE DAMPING IF DESIRED IS EASILY ACCOMPLISHED.

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2.3 SOLAR ARRAYS AND LARGE ANTENNA DISHES

THE PRINCIPAL DIFFICULTY OF MODELING THE DYNAMICS OF LARGE LIGHTWEIGHT STRUCTURES SUCH AS THE SOLAR PANELS OR ANTENNA DISHES ARISES FROM THE FACT THAT THEY MUST BE TESTED IN AIR AT ATMOSPHERIC PRESSURE. THE VIBRATIONS OF SUCH STRUCTURES ARE IMPEDED BY THE SURROUNDING AIR IN THE FORM OF APPARENT MASS FORCES AND DAMPING FORCES. SUCH FORCES HAVE NO COUNTERPART FOR THE FULL SCALE SPACE STATION MOTIONS IN ORBIT AND THE OBJECTIVE IS TO REDUCE THEM TO THE MAXIMUM EXTENT POSSIBLE ON THE MODEL.

THE APPARENT MASS OF THE AIR SURROUNDING A PLATE IS GENERALLY MEASURED IN TERMS OF THE RATIO OF THE MASS OF THE AIR IN A CYLINDER SURROUNDING THE PLATE TO THE MASS OF THE PLATE. AS SHOWN BY FIGURE 2, EACH OF THE 16 SOLAR PANELS HAS DIMENSIONS OF ABOUT 15 FT. BY 80 FT. AND IT IS EXPECTED THAT EACH PANEL WILL WEIGH ABOUT 600 LBS. FOR A 1/4 SCALE MODEL, EACH PANEL WOULD HAVE DIMENSIONS OF ABOUT 3.75 FT. BY 20 FT AND WOULD WEIGH APPROX. (600/64) OR ABOUT 9.38 LBS. THUS THE RATIO OF THE APPARENT AIR MASS TO THE PANEL MASS WOULD BE APPROX.

$$K = \frac{\frac{\pi}{4} d^2 l \rho g}{M} = \frac{\pi}{4} \frac{(3.75)^2 (20)(0.00238)(32.2)}{9.38}$$

$$= 1.80$$

OR, THE MASS OF THE SURROUNDING AIR IS ABOUT TWICE THE MASS OF THE PANEL.

THE LITERATURE DOES NO SHOW MUCH INFORMATION ON THE EFFECTS OF APPARENT MASS RATIOS OF THIS SIZE HOWEVER MUCH SMALLER RATIOS HAVE SIGNIFICANT IMPACT ON AIRCRAFT FLUTTER. THE WORK OF SEWALL, MISERENTINO AND PAPPA, REF. 3, INDICATES THAT FOR

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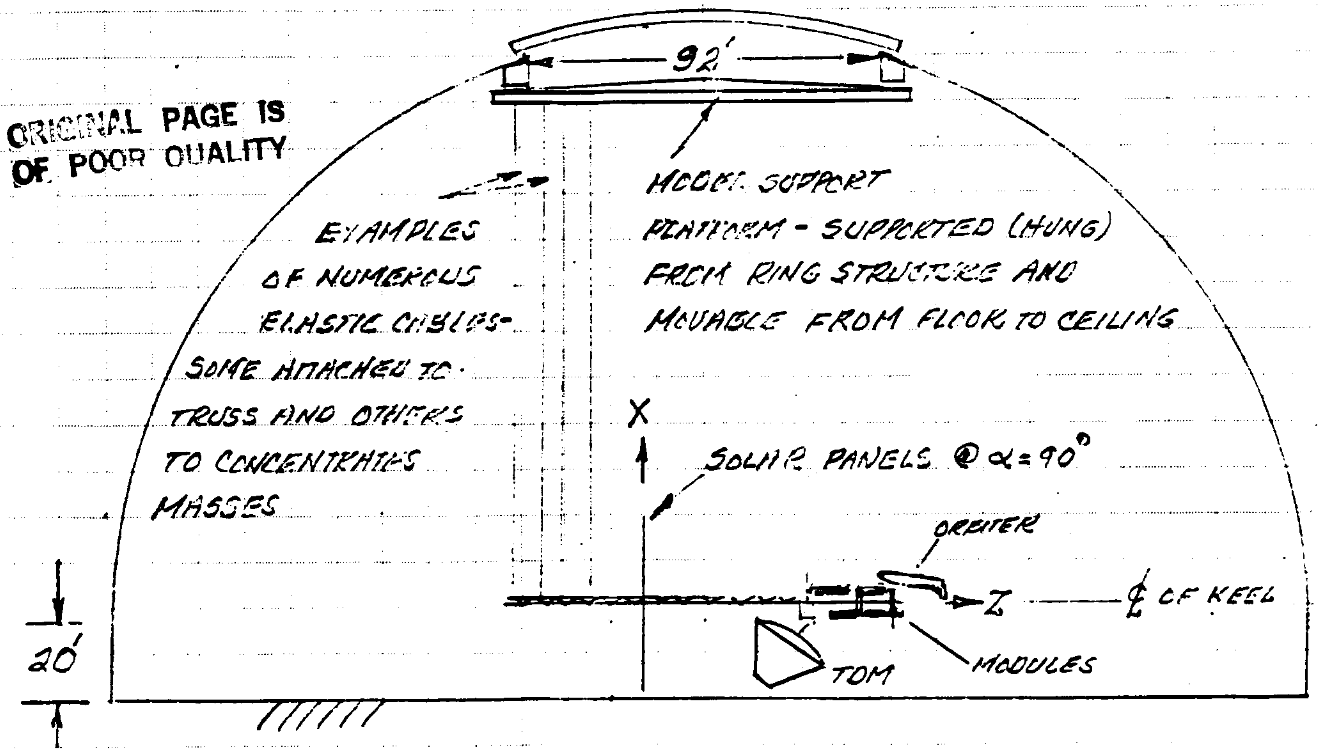
A LIGHTWEIGHT TRIANGULAR STRUCTURE HAVING A MASS RATIO OF ABOUT THREE, THE SURROUNDING AIR DOMINATED THE MASS OF THE SYSTEM FOR VIBRATIONS IN THE FUNDAMENTAL MODE. THE RESULTS ALSO SHOW THAT THE APPARENT MASS OF THE AIR, AS DETERMINED FROM THE FREQUENCIES OF THE FIRST MODE VIBRATIONS, ALSO APPROXIMATES THE MASS OF THE AIR CONTAINED IN THREE INTERSECTING CONES ORIGINATING FROM THE THREE CORNERS OF THE TRIANGLE AND HAVING DIAMETERS EQUAL TO THE DISTANCES BETWEEN THE ADJACENT EDGES OF THE TRIANGLE.

THE IMPACT OF THE AFOREMENTIONED STATEMENTS IS THAT IT WILL BE NECESSARY TO SIMULATE THE SOLAR PANELS AND PROBABLY THE ANTENNA DISHES BY SOME STRUCTURES WHICH DUPLICATE THE MASS AND STIFFNESS DISTRIBUTIONS OF THE PANELS BUT MINIMIZE BLOCKAGE OF AIR BY PERMITTING IT TO FLOW THROUGH THE PANEL STRUCTURE. A GRIDWORK OF SUITABLY CHOSEN RODS OR CABLES WOULD APPEAR TO OFFER A SOLUTION.

TO MINIMIZE THE DAMPING OF THE SIMULATED PANEL STRUCTURES WHICH WILL ARISE FROM THE FLOW-THROUGH OF AIR, CARE MUST BE TAKEN TO MINIMIZE THE GENERATION OF VORTICITY. AS SHOWN IN REFERENCE 4, A SHARP EDGED FLEXIBLE DEVICE WHICH CREATES VORTICITY IS A MUCH MORE EFFECTIVE DAMPER THAN A ROUNDED RIGID BODY WHICH MERELY REDIRECTS THE AIR. THE IMPLICATION IS CLEARLY THAT THE ELEMENTS OF THE GRID SHOULD BE AS WIDELY SEPARATED AS POSSIBLE AND SHOULD HAVE SMOOTH ROUNDED SURFACES NORMAL TO THE PLANES OF THE PANELS.

3. DESIGN AND FABRICATION OF MODEL SUPPORT SYSTEM

CURRENT PLANS FOR THE DESIGN OF THE LARGE SPACECRAFT STRUCTURES LABORATORY WERE DISCUSSED WITH MR. ROBERT MISERENTINO OF THE LRC STRUCTURAL DYNAMICS BRANCH ON 5/16/85. THE SYNOPSIS OF THESE DISCUSSIONS IS THAT THE LABORATORY OUTLINE PERMITS THE



INSTALLATION OF THE MODEL IN THE ORIENTATION SHOWN. THIS IS THE RECOMMENDED ORIENTATION FOR SEVERAL REASONS INCLUDING: (1) MINIMIZATION OF GRAVITATIONAL EFFECTS, (2) CONVENIENCE, SIMPLICITY, AND MINIMUM COSTS OF MODEL TESTS, AND (3) SAFETY OF MODEL STRUCTURES AND PERSONNEL DURING MODEL ASSEMBLY AND TESTING. SOME CONSIDERATIONS RELATIVE TO THESE TOPICS ARE PRESENTED IN THE FOLLOWING SECTIONS.

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3.1 MINIMIZATION OF GRAVITATIONAL EFFECTS

SINCE MODELS TESTED IN EARTH BASED LABORATORIES WILL BE SUBJECTED TO GRAVITATIONAL FORCES WHICH HAVE NO COUNTERPART DURING ORBITAL FLIGHT OF THE SPACECRAFT, IT IS DESIRABLE TO REDUCE THE EFFECTS OF THESE GRAVITATIONAL FORCES AS MUCH AS POSSIBLE. IF THE MODEL IS HUNG AS A PENDULUM, WHICH APPEARS TO BE THE ONLY ATTRACTIVE OPTION, THE GRAVITATIONAL FORCES ALWAYS TEND TO RESTORE THE MODEL TO A CONDITION OF MINIMUM POTENTIAL ENERGY. THE INTEGRATED EFFECT OF GRAVITATIONAL FORCES IS THE CREATION OF THREE RIGID BODY MODES (TWO TRANSLATIONAL MODES AND ONE ROTATIONAL MODE) IN A PLANE NORMAL TO THE SUPPORT CABLES.

THE TWO TRANSLATIONAL MODES (LATERAL & LONGITUDINAL) HAVE THE FREQUENCY OF A SIMPLE PENDULUM

$$\omega = \sqrt{\frac{g}{\ell}}$$

WHERE g IS THE GRAVITATIONAL CONSTANT AND ℓ IS THE LENGTH OF THE SUPPORT CABLE.

AS DEMONSTRATED ON PAGE 63 IN THIS REPORT, THE ROTATIONAL MODE IS THE BIFILAR PENDULUM MODE WHERE

$$\omega = \frac{m}{r}\sqrt{\frac{g}{\ell}} \approx \sqrt{3}\sqrt{\frac{g}{\ell}}$$

NOTE THAT $2m$ IS THE LENGTH OF THE MODEL & r IS THE RADIUS OF GYRATION, AND THAT $m/r \approx \sqrt{3}$.

IT WILL BE SHOWN THAT ℓ CAN BE MADE SUFFICIENTLY LARGE THAT THE SUPPORT FREQUENCIES WILL LIE BELOW THE BAND OF NATURAL FREQUENCIES OF THE ELASTIC MODES OF INTEREST. THUS

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BY MAXIMIZING l, THE COUPLING OF THE MODEL ELASTIC, INERTIA AND DAMPING FORCES WITH THE GRAVITATIONAL FORCES IS MINIMIZED.

WITH REFERENCE TO THE SKETCH OF THE FACILITY SHOWN ON PAGE 52, THE PROPOSED HORIZONTAL MODEL TEST CONFIGURATION WILL PERMIT THE CENTER OF THE MODEL KEEL TO BE PLACED APPROX. 25 FEET ABOVE THE FACILITY FLOOR AND ALLOW FOR THE CASE WHERE THE SOLAR PANELS ARE ORIENTED AT $\alpha = 90$ DEGREES. THE ALLOWANCE OF 12 FEET FOR THE MODEL SUPPORT PLATFORM AND 5 FEET FOR KEEL THICKNESS AND ATTACHMENT OF CABLES TO THE KEEL SUPPORTED COMPONENTS LEAVES A CLEAR CABLE LENGTH OF APPROXIMATELY 120 FEET FOR MODEL SUSPENSION.

IN ADDITION TO THE AFOREMENTIONED PENDULAR TYPE MOTIONS OF THE MODEL IN A HORIZONTAL PLANE, THE MODEL MUST ALSO UNDERGO VERTICAL PLUNGING MOTIONS AND ROTATIONS ABOUT ITS HORIZONTALLY ORIENTED PRINCIPAL AXES. THESE DEGREES OF FREEDOM NECESSITATE A VERY SOFT MOUNTING AND, AS WILL BE SHOWN IN SUBSEQUENT SECTIONS OF THIS REPORT, THE CODISTRIBUTION OF THE MODEL MASSES AND THE ELASTIC SUPPORTS WILL RESULT IN THE PLUNGING, PITCHING AND ROLLING FREQUENCIES OF THE MODEL ON THE ELASTIC SUPPORT SYSTEM ALL BEING APPROXIMATELY EQUAL. THEIR VALUE IS

$$\omega = \sqrt{\frac{g}{\delta_{ST}}}$$

WHERE δ_{ST} IS THE STATIC DEFLECTION OF THE MODEL ON THE ELASTIC SUPPORT CABLES.

SUBSEQUENT SECTIONS OF THE REPORT PRESENT THE RESULTS OF ANALYSES WHICH EXAMINE VARIOUS ASPECTS

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OF THIS SUPPORT SYSTEM, INCLUDING THE DERIVATION AND DISCUSSION OF THE FREQUENCY SEPARATIONS BETWEEN THE MODEL ELASTIC MODES AND THE SEVERAL RIGID BODY SUPPORT MODES.

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3.2 CONVENIENCE, SIMPLICITY AND MINIMUM COSTS OF MODEL TESTS

THE RECOMMENDED MODEL TEST CONFIGURATION AS SHOWN BY THE SKETCH ON PAGE 52 OFFERS THE ADVANTAGE THAT NEARLY ALL OF THE MODEL ASSEMBLY IS ACCOMPLISHED WITH PERSONNEL POSITIONED ON THE FLOOR AND WORKING AT LEVELS BETWEEN THE FLOOR AND SHOULDER HEIGHT. IN A FEW INSTANCES, IT WILL BE NECESSARY TO WORK FROM A LOW MOBILE PLATFORM BUT NO SITUATIONS ARE ENVISIONED WHERE MODEL TECHNICIANS OR RESEARCH PERSONNEL ARE REQUIRED TO WORK AT HEIGHTS ABOVE ABOUT 20 FEET.

THE ANTICIPATED MODEL INSTALLATION AND TEST PROCEDURE IS AS FOLLOWS:

a. THE MODEL SUPPORT PLATFORM IS REMOVED FROM STORAGE, ASSEMBLED (ASSUMED TO BE MADE IN SEVERAL PIECES FOR EASE OF STORAGE), AND ATTACHED TO THE VERTICAL HOIST SYSTEM BY RIGGERS. IT IS THEN OPERABLE IN AN UP AND DOWN SENSE BY TEST TECHNICIANS.

b. THE MODEL SUPPORT PLATFORM IS THEN LOWERED TO A CONVENIENT HEIGHT AND ALL SUPPORT CABLES ARE ATTACHED IN A PREARRANGED PATTERN FOR THE MODEL TEST CONFIGURATION OF INTEREST.

c. THE PLATFORM IS THEN RAISED TO PLACE ALL THE SUSPENSION CABLES IN SLIGHT TENSION AS THEY ARE ATTACHED TO THE FLOOR. WHEN ALL SUSPENSION CABLES ARE ATTACHED, THE PLATFORM IS RAISED TO THE HEIGHT WHERE THE TENSION IN A CABLE WILL SUPPORT ITS RESPECTIVE MASS AT THE DESIRED MODEL ASSEMBLY HEIGHT.

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d. CONSISTENT WITH A PRE ARRANGED PLAN, THE VARIOUS COMPONENTS OF THE MODEL ARE TAKEN TO THEIR RESPECTIVE POINTS FOR ASSEMBLY, THE APPROPRIATE CABLES ARE ATTACHED TO THE COMPONENTS, AND THE COMPONENTS ARE THEN JOINED TOGETHER TO FORM THE MODEL. ASSEMBLY WOULD START FROM THE BASE AND INITIALLY INVOLVE THE JOINING OF THE HEAVY COMPONENTS INCLUDING THE HABITATION MODULES, THE LABS AND THE ORBITER TO EACH OTHER AND TO THE BASE TRUSS ELEMENTS. ASSEMLY WOULD THEN PROCEED OUTWARDS TO INCORPORATE THE KEEL, THE TRANVERSE BOOM, THE SOLAR PANELS AND OTHER MODEL COMPONENTS. IT MAY ALSO BE DESIRABLE TO FORM SEVERAL SUB-ASSEMBLIES TO ASSURE THEIR BALANCE AND ORIENTATION BEFORE ASSEMBLING THEM TOGETHER TO FORM THE COMPLETE STRUCTURE. IN THIS MANNER, THE MODEL WOULD BE SUBJECTED TO A VERY LOW GRAVITY INDUCED STATE-OF-STRESS AND SHOULD PROVIDE THE BEST OPPORTUNITY FOR SIMULATING ZERO-G CONDITIONS RELATIVE TO JOINT NON-LINEARITIES AND DAMPING OF STRUCTURAL RESPONSES.

e. ONCE THE MODEL IS ASSEMBLED AND INSTRUMENTED TO THE EXTENT POSSIBLE AT GROUND LEVEL, IT IS RAISED TO THE NECESSARY HEIGHT BY RAISING THE SUPPORT PLATFORM TO COMPLETE THE ADDITION OF ANTENNAE AND TO ROTATE THE SOLAR PANELS TO $\alpha = 90°$ WHEN NECESSARY. (THE LATTER CONFIGURATION REPRESENTS THE EXTREME MODEL TEST HEIGHT AS SHOWN ON THE SKETCH ON PAGE 52).

f. ALL MODEL TESTS ARE THEN CONDUCTED AT THE LOWEST HEIGHT POSSIBLE FOR THAT CONFIGURATION. THIS MINIMIZES THE COMPLEXITY AND COSTS OF INSTRUMENTING AND MONITORING THE MODEL AND ITS CONVENIENCE WILL SUBSTANTIALLY REDUCE THE COSTS OF TEST FIXTURES AND THE CONDUCT OF THE TESTS.

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3.3 SAFETY OF MODEL STRUCTURES AND PERSONNEL DURING MODEL ASSEMBLY AND TESTING

THE ASSEMBLY AND TESTING OF THE SPACE STATION MODEL WILL BE A UNIQUE EXPERIENCE BECAUSE OF ITS LARGE SIZE AND FRAGILITY. THESE FACTORS IMPACT THE SAFETY OF TEST PERSONNEL AND THE UTILITY OF AN EXPENSIVE PIECE OF TEST HARDWARE. SOME OF THE IMPORTANT CONSIDERATIONS FOLLOW.

a. THE FULL SCALE SPACE STATION WILL BE DESIGNED TO FUNCTION UNDER ACCELERATIONS OF THE ORDER OF 0.04 g, AND AS A CONSEQUENCE OF THE NEED TO MINIMIZE THE WEIGHT TO ORBIT, LITTLE STRUCTURAL "FAT" IS EXPECTED. HENCE THE MODEL, SCALED TO THE SAME STRESS LEVEL AS THE PROTOTYPE, WILL NOT BE ABLE TO SUPPORT ITSELF UNDER 1 g LOADS EXCEPT IN SMALL SECTIONS. THE PROPOSED, ESSENTIALLY CONTINUOUS SUPPORT SYSTEM, EFFECTIVELY ELIMINATES THAT PROBLEM. ALSO, BECAUSE OF THE FRAGILITY OF THE JOINTS AND THE TUBULAR MEMBERS OF THE TRUSS STRUCTURE, MODEL TEST TECHNICIANS MUST WORK WITH EXTREME CAUTION TO AVOID APPLICATION OF DAMAGING MODEL LOADS. GROUND BASED ACCESS TO MOST PARTS OF THE MODEL WILL PERMIT THE EXERCISE OF REASONABLE PRECAUTIONS WHILE EXPEDITING EXECUTION OF THE MODEL ASSEMBLY AND TESTING TASKS.

b. ANY VERTICAL ORIENTATION OF THE MODEL KEEL OR TRANSVERSE BOOM WILL RESULT IN MODEL TEST PERSONNEL WORKING AT HEIGHTS OF ABOUT 100 FEET TO SERVICE A 1/4 SCALE MODEL. THE GANTRYS AND SAFETY PROVISIONS TO MAKE THIS POSSIBLE FOR A SOFTLY SPRUNG MOVABLE MODEL, EVEN FOR WORKERS NOT SUBJECT TO DISCOMFORT WHILE WORKING AT HEIGHTS UP TO 10 STORIES, WOULD PRESENT A VERY STRONG IMPEDIMENT

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TO DYNAMIC TESTING OF A FRAGILE MODEL. THE PROPOSED, ESSENTIALLY GROUND LEVEL, MODEL PREPARATION AND TESTING PROCEDURE WILL NOT ONLY REMOVE THE PERSONNEL HAZARDS BUT IT MAY BE THE ONLY FEASIBLE OPTION FOR ACHIEVING A SATISFACTORY TEST PROGRAM FOR THE COMPLEX AND SENSITIVE SPACE STATION MODEL.

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3.4 DISCUSSION OF FACTORS RELATING TO INFLUENCE OF GRAVITATIONAL EFFECTS ON MODEL SUPPORT SYSTEM

THE NEAR COINCIDENCE OF FREQUENCIES OF DIFFERENT NATURAL MODES OF VIBRATION CAUSES COMPLICATIONS IN TESTING AND DATA ANALYSIS. IN SOME CASES, THE PROBLEMS INVOLVE COUPLING OF STRUCTURAL MOTIONS; IN OTHER CASES THEY ONLY INVOLVE INTERFERENCE DUE TO SUPERIMPOSED MOTIONS. IN EITHER CASE, WHEN INTERFERENCE EFFECTS ARE THE RESULT OF EXTRANEOUS FORCES, SUCH AS GRAVITY FORCES ON THE MODEL, MINIMIZATION OF THE INTERFERENCE, USUALLY BY FREQUENCY SEPARATION, IS DESIRABLE.

FOR GENERAL CONSIDERATIONS, ASSUME THAT THE LOWEST NATURAL ELASTIC FREQUENCY OF INTEREST FOR STRUCTURAL MODES OF THE SPACE STATION IS:

ORIGINAL PAGE IS OF POOR QUALITY

$$f = f_{F,E} \quad \& \quad \omega_{F,E} = f_{F,E}\,(2\pi)$$

FOR REPLICA SCALING, $\omega_{M,E} = \frac{1}{\lambda}\,\omega_{F,E}$ WHERE THE SUBSCRIPTS M & F DENOTE MODEL AND FULL SCALE VALUES RESPECTIVELY, & λ IS THE SCALE FACTOR $(\lambda < 1)$

THEN $\quad \omega_{M,E} = \frac{1}{\lambda}\,\omega_{F,E}$

TO MINIMIZE INTERFERENCE, IT IS DESIRED THAT FREQUENCY SEPARATION BE PRESERVED BY HAVING THE SUPPORT FREQUENCY MUCH LOWER THAN THE FIRST STRUCTURAL FREQUENCY, i.e., $\omega_{M,E} = \frac{1}{\alpha}\,\omega_{M,S}$ WHERE $\alpha \gg 1$

$$\omega_{M,S} = \frac{1}{\alpha}\,\omega_{M,E} = \frac{1}{\lambda}\,\frac{1}{\alpha}\,\omega_{F,E}$$

THE FREQUENCY SEPARATION IS THEN

$$\alpha = \frac{1}{\lambda}\,\frac{\omega_{F,E}}{\omega_{M,S}}$$

IN SUBSEQUENT SECTIONS $\omega_{M,S}$ AND $\omega_{F,E}$ WILL BE EXAMINED TO FURTHER REFINE α.

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3.5 DETERMINATION OF MODEL SUPPORT FREQUENCIES ON CABLE MOUNTING SYSTEM

AS SHOWN ON PAGE 52, THE PROPOSED CABLE MOUNT SYSTEM FOR THE MODEL INVOLVES SUSPENDING IT FROM AN OVERHEAD PLATFORM BY NUMEROUS ELASTIC CABLES. THE MODEL IS ORIENTED SO THAT THE KEEL AND THE TRANSVERSE BOOM ARE PARALLEL TO THE FLOOR (Y-Z PLANE) AND THE "FLIGHT" DIRECTION (X-AXIS) IS UP. THE MODEL WILL THUS BE PERMITTED TO UNDERGO MOTIONS UNDER ELASTIC RESTRAINTS IN SIX DEGREES OF FREEDOM. FOR PURPOSES OF ANALYSIS, IT IS CONVENIENT TO GROUP THESE MOTIONS AS FOLLOWS:

1. PENDULAR MOTIONS IN THE Y-Z PLANE INCLUDING ROTATIONS (ϕ, BIFILAR ROTATIONS) ABOUT THE X AXIS.

2. PLUNGING MOTIONS IN THE X-DIRECTION INCLUDING ROTATIONS ABOUT THE Y-AXIS (θ, PITCHING ROTATIONS) AND Z-AXIS (ψ, ROLLING ROTATIONS)

FOR SIMPLICITY OF COMPUTATION OF THE MODEL MOTIONS ON THE SUPPORT SYSTEM, THE FOLLOWING ASSUMPTIONS ARE MADE:

1. THE MODEL IS RIGID

2. THE DISTRIBUTION OF ELASTIC SUPPORTS IS THE SAME AS THE DISTRIBUTION OF MASS ACROSS THE X-Y PLANE.

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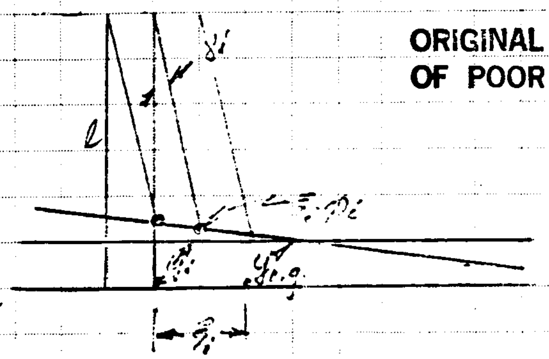
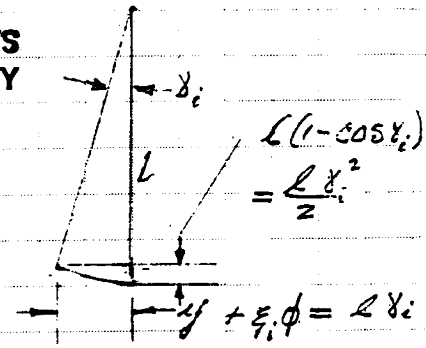
3.5.1 DETERMINATION OF PENDULUM NATURAL FREQUENCIES. MOTIONS ARE ALONG Y OR Z AXES AND ABOUT X-AXIS. CONSIDER CASE FOR MOTIONS ALONG Y-AXIS





THE MODEL CONSISTS OF A SERIES OF MASSES CONNECTED BY A TRUSS AND SUPPORTED BY A SERIES OF ELASTIC CABLES. THE SYSTEM IS ASSUMED TO UNDERGO LATERAL MOTIONS COMPOSED OF SUPERIMPOSED TRANSLATIONS y AND ROTATIONS ϕ. AS A RESULT OF THE FIXED LENGTH OF THE SUPPORTS, THESE MOTIONS CAUSE THE MODEL TO MOVE UP AND DOWN AS IT MOVES LATERALLY. THE YAWING AND TRANSLATIONAL FREQUENCIES WILL BE DETERMINED BY APPLYING ENERGY METHODS AND USING LAGRANGE'S EQUATIONS

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_s}\right) - \frac{\partial T}{\partial q_s} + \frac{\partial V}{\partial q_s} = 0$$

WHERE T AND V ARE THE KINETIC AND POTENTIAL ENERGIES, RESPECTIVELY, AND q_s IS A GENERALIZED COORDINATE, y OR ϕ

$$T = \frac{1}{2}\sum_{i=1}^{n} M_i V_i^2 = \frac{1}{2}\sum_{i=1}^{n} M_i(\dot{y}_i + \xi_i\dot{\phi}_i)^2 = \frac{1}{2}\sum_{i=1}^{n} M_i\left(\dot{y}_i^2 + 2\xi_i\dot{y}_i\dot{\phi}_i + \xi_i^2\dot{\phi}_i^2\right)$$

SINCE $\phi_1 = \phi_i = \phi$; $\dot{y}_1 = \dot{y}_i = \dot{y}$; $\sum_{i=1}^{n} M_i = M$, & $\sum_{i=1}^{n} M_i\xi_i^2 = I_{c.g.}$

$$T = \frac{1}{2}M\dot{y}^2 + \dot{y}\dot{\phi}\underbrace{\sum_{i=1}^{n} M_i\xi_i}_{=0 \text{ SINCE } \xi_i \text{ IS MEASURED FROM c.g.}} + \frac{1}{2}I_{c.g}\dot{\phi}^2$$

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SINCE, FOR THIS CALCULATION, THE MODEL IS ASSUMED TO BE RIGID, THE UPWARD MOTIONS OF ALL MASSES ARE EQUAL TO THE UPWARD MOTIONS OF THE MASSES M_0 & M_n. HENCE

$$V = \frac{M}{2} g \ell \frac{\gamma_c^2}{2} + \frac{M}{2} g \ell \frac{\gamma_n^2}{2}$$

BUT

$$\gamma_c^2 = \frac{1}{\ell^2}\left(y + \xi_0 \phi\right)^2 \quad \& \quad \gamma_n^2 = \frac{1}{\ell^2}\left(y - \xi_n \phi\right)^2$$

AND ASSUMING THAT $|\xi_n| = |\xi_0|$

$$V = \frac{Mg}{2\ell}\left(y^2 + \xi_0^2 \phi^2\right)$$

APPLYING LAGRANGE'S EQUATIONS, WE OBTAIN

$$M\ddot{y} + M\frac{g}{\ell} y = 0$$

AND $$I_{c.g.}\ddot{\phi} + M\frac{g}{\ell}\xi_0^2 \phi = 0$$

FOR A UNIFORM BEAM, $\xi_0 = \frac{L}{2}$ & $I_{c.g.} = \frac{1}{12} ML^2$,

THE ABOVE EQUATIONS REDUCE TO

$$\ddot{y} + \frac{g}{\ell} y = 0$$

AND $$\ddot{\phi} + 3\frac{g}{\ell}\phi = 0$$

THE RESULTING UNCOUPLED NATURAL FREQUENCIES ARE

$$\omega_{M,S} = \sqrt{\frac{g}{\ell}} \quad \& \quad \omega_{M,S} = \sqrt{\frac{3g}{\ell}}$$

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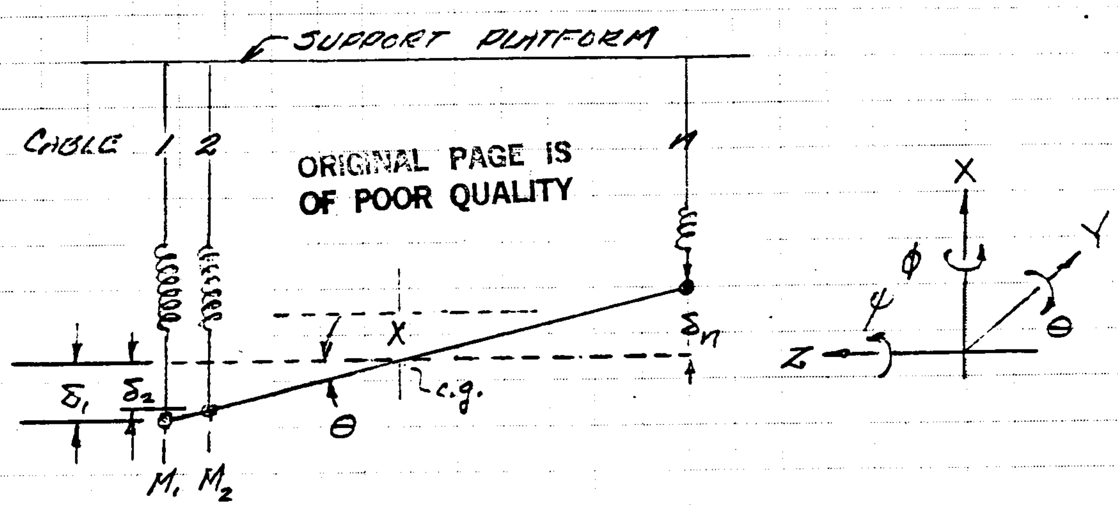
3.5.2 DETERMINATION OF PLUNGING AND ROTATIONAL NATURAL FREQUENCIES OF MODEL ON CABLES. MOTIONS ARE ALONG X AND ABOUT Y & Z AXES.



THE SYSTEM CONSISTS OF A SERIES OF MASSES CONNECTED BY A TRUSS AND SUPPORTED BY A SERIES OF ELASTIC CABLES. THE SYSTEM IS ASSUMED TO UNDERGO SIMULTANEOUS TRANSLATIONAL MOTIONS (X) AND PITCHING MOTIONS θ. THE TWO NATURAL FREQUENCIES OF THE SYSTEM WILL BE DETERMINED BY APPLYING ENERGY METHODS AND LAGRANGE'S EQUATIONS. (RESULTS FOR ϕ & θ ARE IDENTICAL)

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_s}\right) - \frac{\partial T}{\partial q_s} + \frac{\partial V}{\partial q_s} = 0$$

WHERE T & V ARE THE KINETIC & POTENTIAL ENERGIES, RESPECTIVELY, AND q_s IS A GENERALIZED COORDINATE, x OR θ.

$$T = \frac{1}{2}\sum_{i=1}^{n} M_i \dot{V}_i^2 = \frac{1}{2}\sum_{i=1}^{n} M_i \left(\dot{x}_i + l_i\dot{\theta}\right)^2 = \frac{1}{2}\sum_{i=1}^{n} M_i\left(\dot{x}_i^2 + 2\dot{x}_i l_i \dot{\theta}_i + l_i^2\dot{\theta}_i^2\right)$$

SINCE $\theta_1 = \theta_i = \theta$; $x_1 = x_i = x$; $\sum_{i=1}^{n} M_i = M$ & $\sum_{i=1}^{n} M_i l_i^2 = I_{c.g.}$

$$T = \frac{1}{2}M\dot{x}^2 + \dot{x}\dot{\theta}\underbrace{\cancel{\sum_{i=1}^{n} M_i l_i}}_{=0} + \frac{1}{2}I_{c.g.}\dot{\theta}^2$$

= 0 SINCE l_i IS MEASURED FROM C.G.

$$V = \frac{1}{2}\sum_{i=1}^{n} K_i \xi_i^2$$

WHERE K_i IS THE SPRING CONSTANT OF CABLE$_i$ AND ξ_i IS ITS TOTAL DEFLECTION. THEN

$$V = \frac{1}{2}\sum_{i=1}^{n} K_i (x_i + l_i\theta_i)^2 = \frac{1}{2}\sum_{i=1} K_i (x_i^2 + 2 l_i x_i \theta_i + l_i^2\theta_i^2)$$

$$= \frac{1}{2}x^2\sum_{i=1}^{n} K_i + x\theta\sum_{i=1}^{n} K_i l_i + \frac{1}{2}\theta^2\sum_{i=1}^{n} K_i l_i^2$$

TO REDUCE THE TRANSMISSION OF GRAVITY LOADS THROUGH THE RELATIVELY WEAK KEEL STRUCTURE IT IS DESIRABLE TO HAVE K_i PROPORTIONAL TO M_i, i.e., $K_i = k M_i$

$$V = \frac{1}{2}x^2 k M + x\theta k \underbrace{\cancel{\sum_{i=1}^{n} M_i l_i}}_{=0} + \frac{1}{2}\theta^2 k I_{c.g.}$$

=0 SINCE l_i IS MEASURED FROM C.G.

APPLYING LAGRANGES EQUATION WE OBTAIN.

$$M\ddot{x} + kMx = 0 \quad \text{or} \quad \ddot{x} + kx = 0$$

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AND $$I_{c.g.}\ddot{\theta} + k I_{c.g.}\theta = 0 \quad \text{or} \quad \ddot{\theta} + k\theta = 0$$

THUS BOTH THE TRANSLATIONAL AND PITCHING MODES HAVE THE SAME NATURAL FREQUENCY

$$\omega_{M,S} = \sqrt{\frac{K_i}{M_i}} = \sqrt{\frac{g}{\delta_{st}}}$$

THIS IS BECAUSE THE SPRING FORCE FOR PITCHING IS PROPORTIONAL TO THE DISTANCE FROM THE C.G. WHEREAS FOR BIFILAR ROTATIONS, IT IS THE SAME FOR ALL POINTS OF THE STRUCTURE - THEY ALL RISE THE SAME AMOUNT FOR A GIVEN θ.

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RELATIVE TO THE INFLUENCE OF SUPPORT SYSTEM RIGID BODY MODES ON THE MODEL ELASTIC MODES, THE FOLLOWING ASSUMPTIONS SEEM REASONABLE:

1. ROTARY RIGID BODY MODES PRIMARILY INFLUENCE ANTI-SYMMETRIC ELASTIC MODES
2. PLUNGING AND PENDULAR RIGID BODY MODES PRIMARILY INFLUENCE SYMMETRIC ELASTIC MODES
3. THE NATURAL FREQUENCY OF THE LOWEST ANTI-SYMMETRIC ELASTIC MODE > 2.5 × NATURAL FREQUENCY OF THE LOWEST SYMMETRIC MODE
4. THE NATURAL FREQUENCY OF THE LOWEST FULL-SCALE SYMMETRIC ELASTIC MODE IS 0.1 HZ IN X-Z OR Y-Z PLANE
5. FOR PURPOSES OF FIRST APPROXIMATION, THE SPACE STATION CAN BE TREATED AS A BEAM

FOR THESE APPROXIMATIONS, THE FREQUENCY SEPARATION FOR THE CASES OF INTEREST ARE

PENDULAR & LATERAL SYMMETRIC BENDING

$$\alpha_1 = \left(\frac{\omega_{M,E}}{\omega_{M,S}}\right)_1 = \frac{\frac{1}{\lambda}(0.628)}{\sqrt{g/\ell}} = \frac{\sqrt{\ell}}{\lambda}\frac{0.628}{\sqrt{g}} = 11.07\times10^{-2}\frac{\sqrt{\ell}}{\lambda}$$

BIFILAR PENDULAR & LATERAL ANTISYMMETRIC BENDING

$$\alpha_2 = \left(\frac{\omega_{M,E}}{\omega_{M,S}}\right)_2 = \frac{(1/\lambda)(2.5)(0.628)}{\sqrt{\frac{3g}{\ell}}} = \frac{\sqrt{\ell}}{\lambda}\frac{(2.5)0.628}{\sqrt{3g}} = 15.97\times10^{-2}\frac{\sqrt{\ell}}{\lambda}$$

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RELATIVE TO THE INFLUENCE OF SUPPORT SYSTEM RIGID BODY MODES ON THE MODEL ELASTIC MODES, THE FOLLOWING ASSUMPTIONS SEEM REASONABLE:

1. ROTARY RIGID BODY MODES PRIMARILY INFLUENCE ANTI-SYMMETRIC ELASTIC MODES

2. PLUNGING AND PENDULAR RIGID BODY MODES PRIMARILY INFLUENCE SYMMETRIC ELASTIC MODES

3. THE NATURAL FREQUENCY OF THE LOWEST ANTI-SYMMETRIC ELASTIC MODE > 2.5× NATURAL FREQUENCY OF THE LOWEST SYMMETRIC MODE

4. THE NATURAL FREQUENCY OF THE LOWEST FULL-SCALE SYMMETRIC ELASTIC MODE IS 0.1 HZ IN X-Z OR Y-Z PLANE

5. FOR PURPOSES OF FIRST APPROXIMATION, THE SPACE STATION CAN BE TREATED AS A BEAM

FOR THESE APPROXIMATIONS, THE FREQUENCY SEPARATION FOR THE CASES OF INTEREST ARE

PENDULAR & LATERAL SYMMETRIC BENDING

$$\alpha_1 = \left(\frac{\omega_{M,E}}{\omega_{M,S}}\right)_1 = \frac{\frac{1}{\lambda}(0.628)}{\sqrt{g/\ell}} = \frac{\sqrt{\ell}}{\lambda}\frac{0.628}{\sqrt{g}} = 11.07\times10^{-2}\frac{\sqrt{\ell}}{\lambda}$$

BIFILAR PENDULAR & LATERAL ANTISYMMETRIC BENDING

$$\alpha_2 = \left(\frac{\omega_{M,E}}{\omega_{M,S}}\right)_2 = \frac{(1/\lambda)(2.5)(0.628)}{\sqrt{\frac{3g}{\ell}}} = \frac{\sqrt{\ell}}{\lambda}\frac{(2.5)0.628}{\sqrt{3g}} = 15.97\times10^{-2}\frac{\sqrt{\ell}}{\lambda}$$

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PLUNGING & VERTICAL SYMMETRIC BENDING

$$\alpha_3 = \left(\frac{\omega_{M,E}}{\omega_{M,S}}\right)_3 = \frac{\left(\frac{1}{\lambda}\right)(0.628)}{\sqrt{\frac{g}{\delta_{ST}}}} = \frac{\sqrt{\delta_{ST}}}{\lambda}\,\frac{0.628}{\sqrt{g}} = 11.07 \times 10^{-2}\,\frac{\sqrt{\delta_{ST}}}{\lambda}$$

PITCHING AND VERTICAL ANTI-SYMMETRIC BENDING

$$\alpha_4 = \left(\frac{\omega_{M,E}}{\omega_{M,S}}\right)_4 = \frac{\left(\frac{1}{\lambda}\right)(2.5)(0.628)}{\sqrt{\frac{g}{\delta_{ST}}}} = \frac{\sqrt{\delta_{ST}}}{\lambda}\,\frac{(2.5)(0.628)}{\sqrt{g}} = 27.8 \times 10^{-2}\,\frac{\sqrt{\delta_{ST}}}{\lambda}$$

ASSUMING $1 < \delta_{ST} < \ell$, α_3 IS THE SMALLEST FREQUENCY SEPARATION. BUT IT IS DESIRABLE TO MAKE ALL α'S AS LARGE AS POSSIBLE FOR A GIVEN ℓ AND THEREFORE IT IS DESIRABLE TO HAVE δ_{ST} AS LARGE AS POSSIBLE. IF THE [illegible] ([illegible]) [illegible] IS EQUAL TO THE PENDULUM FREQUENCY, $\delta_{ST} = \ell$. BUT THIS WOULD REQUIRE A [illegible] CABLE SYSTEM WHERE [illegible] ELASTIC MATERIAL MUST BE REQUIRED BEYOND THE LENGTH OF THE CABLE. A MORE DESIRABLE SITUATION WOULD APPEAR TO BE ONE WHERE ALL OF THE ELASTIC MATERIAL PLUS A LIMITED AMOUNT OF INELASTIC MATERIAL FOR CABLE ADJUSTMENT IS [illegible] BELOW THE SUPPORT PLATFORM. THE SUGGESTED PROCEDURE IS OUTLINED IN THE FOLLOWING SECTION WHICH SHOWS THE DERIVATION OF THE LENGTHS OF THE VARIOUS ELEMENTS OF THE SUPPORT CABLES. NOTE THAT THE RECOMMENDED PROCEDURE FOR SUPPORTING THE MODEL (SEE PAGE 64) EMPLOYS NUMEROUS PARALLEL CABLES OF ESSENTIALLY THE SAME LENGTH.

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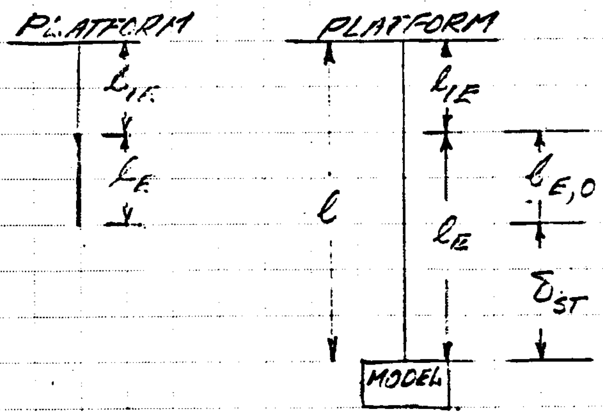
3.6 DETERMINATION OF COMPOSITION OF CABLE BELOW SUPPORT PLATFORM

IT IS ASSUMED THAT THE CABLE CONSISTS OF AN ELASTIC MEMBER AND AN INELASTIC MEMBER



NO LOAD CONFIG.

LOADED CONFIG.

ℓ - test length

ℓ_{IE} - length of inelastic material

$\ell_{E,0}$ - length of elastic material - no load

ℓ_E - length of elastic material - loaded

β - percent elongation divided by 100 (see p 21)

ϵ - convenience factor for cable adjustment

ASSUMPTIONS:

1. $\ell_{IE} = \epsilon\, \ell$

2. $\delta_{ST} = \beta\, \ell_{E,0}$

THEN

$$\ell = \ell_{IE} + \ell_{E0} + \delta_{ST}$$

$$= \epsilon \ell + \ell_{E0} + \beta \ell_{E0}$$

OR

$$\ell_{E,0} = \ell \frac{(1-\epsilon)}{(1+\beta)} = \frac{\delta_{ST}}{\beta}$$

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FOR CONVENIENCE OF HANGING AND ADJUSTING THE MODEL FROM THE PLATFORM, AN INELASTIC LENGTH OF THE CABLES OF THE ORDER OF 0.1ℓ ($\epsilon = 0.1$) SEEMS REASONABLE. ALSO, INFORMATION FROM RUBBER SUPPLIERS SUGGEST THAT A VALUE OF 3 FOR β IS REASONABLE. THEN

$$\ell_{E,0} = \ell \frac{(1-\epsilon)}{(1+\beta)} = \frac{(1-0.1)}{(1+3)} \ell = 0.225 \ell$$

$$\delta_{ST} = \beta \ell_{E,0} = 0.675 \ell$$

$$\ell_{IE} = 0.10 \ell$$

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3.7 SUMMARY OF FREQUENCY SEPARATIONS FOR 1/4 SCALE MODEL WITH A SUSPENSION LENGTH OF 120 FEET AND A STATIC DEFLECTION OF 0.675ℓ

SUBSTITUTION OF $\lambda = 1/4$, $\ell = 120$ FEET, AND $\delta_{ST} = 0.675\ell = 81$ FEET AND THE ASSUMPTIONS MADE ON PAGE 66 RESULTS IN THE FOLLOWING VALUES FOR FREQUENCY SEPARATIONS α_1 THROUGH α_4

$$\alpha_1 = 11.07 \times 10^{-2} \frac{\sqrt{\ell}}{\lambda} = 11.07 \times 10^{-2} \frac{\sqrt{120}}{0.25} = 4.85$$

$$\alpha_2 = 15.95 \times 10^{-2} \frac{\sqrt{\ell}}{\lambda} = 15.95 \times 10^{-2} \frac{\sqrt{120}}{0.25} = 6.98$$

$$\alpha_3 = 11.07 \times 10^{-2} \frac{\sqrt{\delta_{ST}}}{\lambda} = 11.07 \times 10^{-2} \frac{\sqrt{81}}{.25} = 3.98$$

$$\alpha_4 = 27.80 \times 10^{-2} \frac{\sqrt{\delta_{ST}}}{\lambda} = 27.80 \times 10^{-2} \frac{\sqrt{81}}{.25} = 10.01$$

WHERE α_1 THROUGH α_4 ARE DEFINED AS ON PAGES 66 & 67.

THE ABOVE VALUES FOR THE FREQUENCY SEPARATIONS SHOULD SUBSTANTIALLY MINIMIZE THE INFLUENCE OF THE RIGID BODY MOTIONS, INDUCED BY GRAVITATIONAL FORCES, ON THE ELASTIC MODES OF THE SPACE STATION STRUCTURE.

THE <u>WORST CASE SITUATION</u> TO BE ENCOUNTERED IS THE ONE WHERE ALL RIGID BODY MODES INTERFERE WITH ALL ELASTIC MODES. IN THIS CASE, THE FREQUENCY SEPARATION OF INTEREST IS THE ONE WHICH COMPARES THE LOWEST FREQUENCY ELASTIC MODE WITH THE HIGHEST FREQUENCY SUPPORT MODE, i.e., MODEL SYMMETRIC ELASTIC BENDING WITH BIFILAR ROTATIONS. NEGLECTING CABLE EXTENSIONS, THIS RESULTS IN THE FOLLOWING VALUE OF α FOR THE AFOREMENTIONED TEST CONDITIONS.

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$$\alpha = \frac{1}{\lambda}\frac{\omega_{F,E}}{\omega_{M,S}} = \frac{1}{\lambda}\frac{\omega_{F,E}}{\sqrt{\frac{3g}{\ell}}} = \frac{1}{0.25}\frac{6.28 \times 0.10}{\sqrt{\frac{96.6}{120}}}$$

$$= 2.8$$

IT IS ANTICIPATED THAT THE ACTUAL SITUATION WILL BE BETTER BECAUSE THE ELASTICITY OF THE CABLES WILL PROBABLY CAUSE SOME REDUCTION IN THE BIFILAR SUPPORT FREQUENCY.

IT SHOULD BE NOTED THAT BECAUSE OF NONLINEARITIES OF THE RUBBER SUPPORT CABLES WHICH WILL PROBABLY BE USED TO SUPPORT THE MODEL, EXPERIMENTAL DATA INDICATE THAT THE EFFECTIVE SPRING CONSTANT OF THE PLUNGING MOTIONS WILL VARY AND MAY AS HIGH AS $(2g/\delta_{ST})^{1/2}$ INSTEAD OF $(g/\delta_{ST})^{1/2}$. THE IMPACT OF THIS WOULD BE A REDUCTION OF α_3 AND α_4 AS GIVEN ON PAGES 65, 67 AND 70 BY $\sqrt{2}$. IT ALSO IS AN INDUCEMENT TO TRY TO WORK THE CABLES AT A LOWER PERCENTAGES OF ELONGATION WHICH TEND TO INCREASE CABLE MASS. THUS SOME TRADEOFFS MAY BE NECESSARY.

PRODUCT 204-1 NEBS Inc., Groton, Mass. 01471

3.8 NATURE OF CABLES AND THEIR PROPERTIES

IN THE PREVIOUS DERIVATIONS, THE CABLES ARE TREATED AS ELASTIC MEMBERS WITH EFFECTIVE MASSES (AT THE POINT OF ATTACHMENT TO THE MODEL) WHICH ARE SMALL RELATIVE TO THE MODEL MASSES THEY SUPPORT. THESE CABLES MUST HAVE LARGE STATIC DEFLECTIONS PER UNIT STRESS OR HIGH RESILIENCE. THIS PROPERTY IS ACHIEVED BY EITHER USING THE INHERENT PROPERTIES OF A SUITABLE MATERIAL SUCH AS A HIGHLY RESILIENT RUBBER, OR BY CONFIGURING OTHER MATERIALS SUCH AS HIGH STRENGTH STEEL INTO RESILIENT STRUCTURES SUCH AS COIL SPRINGS. THE QUESTION IS, WILL EITHER APPROACH YIELD SUFFICIENTLY LARGE STATIC DEFLECTIONS, FOR ACCEPTABLE RATIOS OF CABLE WEIGHT TO SUSPENDED WEIGHT, TO PROVIDE MODEL SUPPORT FREQUENCIES LOW ENOUGH TO ACHIEVE ACCEPTABLE MODEL TEST CONDITIONS. AS THE RESULTS IN THE FOLLOWING SECTIONS WILL SHOW, APPROPRIATE HIGH QUALITY RUBBER OFFERS A SUITABLE SOLUTION; STEEL SPRINGS, BECAUSE OF THEIR EXCESSIVE WEIGHT, DO NOT.

THE SECTIONS WHICH FOLLOW PRESENT THE RESULTS OF ANALYSES AND EXPERIMENTS ON THE PROPERTIES OF VARIOUS TYPES OF RUBBER SAMPLES. ALSO SHOWN ARE THE RESULTS FOR CALCULATIONS ON TWO TYPES OF HIGH STRENGTH STEEL SPRINGS.

3.7 CONSIDERATIONS ON SELECTION OF RUBBER FOR USE IN SPACE STATION MODEL SUPPORTS

RUBBER MANUFACTURERS NOTE THAT THE EXTRUSION OF RUBBER IN THE RANGE OF ELASTICITY OF INTEREST FOR SUPPORTING THE MODEL OF THE SPACE STATION IS A VERY IMPRECISE TASK. MECHANICAL PROPERTIES SHOULD BE VERIFIED BY SIMPLE ELONGATION TESTS AND FREQUENCY MEASUREMENTS USING SAMPLES OF THE SELECTED RUBBER STOCK. CREEP STUDIES ARE ALSO ADVISABLE.

MANY RUBBERS ARE VERY SUSCEPTIBLE TO DAMAGE BY OZONE AND ULTRAVIOLET. AMONG THOSE WHICH HAVE GOOD TO EXCELLENT RESISTANCE TO THESE AND OTHER ENVIRONMENTAL USE FACTORS ARE SOME SILICONES, NEOPRENE & NORDEL. PAGES 74 THROUGH 82 PRESENT PERTINENT DATA ON REPRESENTATIVE TYPES OF RUBBERS INCLUDING PHYSICAL PROPERTIES.

IN ADDITION TO REASONABLE ENVIRONMENTAL RESISTANCE, THE RUBBER NEEDED FOR THE SPACE STATION MODEL MUST HAVE GOOD RESILIENCE, LOW DAMPING, AND HIGH STRENGTH TO ASSURE A SOFT, LIGHTWEIGHT SUPPORTING SYSTEM.

THE FOLLOWING SECTIONS PRESENT THE RESULTS OF SEVERAL SERIES OF TESTS OF RUBBER SAMPLES TO GAIN A BETTER UNDERSTANDING OF RUBBER BEHAVIOR IN GENERAL AND TO IDENTIFY THE CHARACTERISTICS AND SPECIFIC RUBBER TYPES NEEDED FOR THE SPACE STATION MODEL TESTS.

14

DU PONT

Fairprene*
elastomer coated fabrics

a handbook of flexible elastomer diaphragms

*Du Pont registered trademark.

6. Elastomers

Since 1932, Du Pont has met the requirements of advancing technology with the development of new and better polymers. In 1951, HYPALON* synthetic rubber was introduced; in 1957, VITON* fluoroelastomer; in 1959, ADIPRENE* urethane rubber; and in 1963, NORDEL* hydrocarbon rubber. FAIRPRENE* elastomer coated fabrics utilize all of these Du Pont elastomers but also use other available polymers if their properties offer advantages. Table 5 compares the Du Pont elastomers with each other and with the other major polymers that are in use today

7. Fabric Coating Methods

There are three methods of applying the elastomer to the fabric that are commonly used today. They are as follows:

- Spread Coating
- Dip Coating
- Calender Coating.

Spread Coating (fig. 1)
In this operation, base fabric is passed beneath a stationary blade, then through a drying oven where solvent is driven off. One or more passes are used to build the coating to the desired thickness.

Dip Coating (fig. 2)
Fabric is passed through a tank containing the coating solution. After the fabric passes through the solution, excess compound is removed by scraping with bars or rods, then the fabric is passed through a drying oven.

Calendering (fig. 3)
In this form of coating, the rubber compound is formed into a film by the top three rolls and transferred to the fabric as it passes between the two lower rolls.

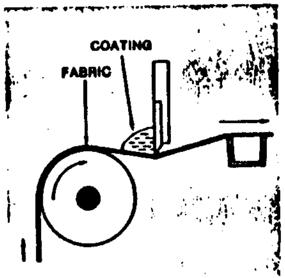

Figure 1—Spread Coating

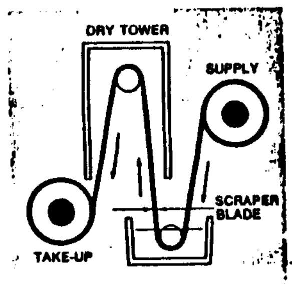

Figure 2—Dip Coating

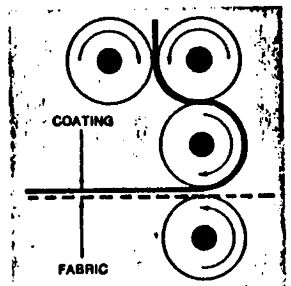

Figure 3—Calendering

Table 5.

Comparative Properties of Natural & Synthetic Rubbers

Properties		Natural Rubber	GRS or Buna S Butadiene Styrene	Butyl Isobutylene Isoprene	Nitrile (Buna N) Butadiene Acrylonitrile	Silicone Polysiloxane Polymer	Neoprene Chloroprene	VITON Co-polymer of vinylidene Fluoride and Hexafluoro-propylene	HYPALON* Chlorosulfonated Polyethylene	ADIPRENE* Polyurethane	NORDEL* Hydrocarbon
Tensile Strength (PSI) (1000 psi 6 9 MPa)	Pure Gum	Over 3000	Below 1000	Over 1500	Below 1000	Below 1500	Over 3000	Over 2000	Over 2500	Over 4000	Over 2000
	Black Loaded Stocks	Over 3000	Over 2000	Over 2000	Over 2000		Over 3000	Over 2000	Over 3000		Over 3000
Hardness Range (Shore Duro A)		30-90	40 90	40-75	40 95	20-85	40-95	60 95	40 95	10-99 (Up to 80 durometer D)	40-90
Specific Gravity (Base Material)		0.93	0.94	0 92	1 00	1 1 to 1 3	1.23	1 85	1.12-1.28	1.06	0 85
Vulcanizing Properties		Excellent	Excellent	Good	Excellent	Excellent	Excellent	Good	Excellent	Excellent	Excellent
Adhesion to Metals		Excellent	Excellent	Good	Excellent	Good	Excellent	Good to Excel.	Excellent	Excellent	Fair
Adhesion to Fabrics		Excellent	Good	Good	Good	Good	Excellent	Good to Excel.	Good	Excellent	Good
Tear Resistance		Good	Fair	Good	Fair	Poor	Good	Fair	Fair	Excellent	Very Good
Abrasion Resistance		Excellent	Good to Excel.	Good	Good	Poor	Excellent	Good	Excellent	Outstanding	Very Good
Compression Set		Good	Good	Fair	Good	Fair	Fair to Good	Very Good	Fair	Good	Very Good
Rebound	Cold	Excellent	Good	Bad	Good	Excellent	Very Good	Good	Good	Poor at Low Temp.	Fair
	Hot	Excellent	Good	Very Good	Good	Excellent	Very Good	Excellent	Good	Good at Room Temp.	Good
Dielectric Strength		Excellent	Excellent	Excellent	Poor	Good	Good	Good	Excellent	Excellent	Excellent
Electrical Insulation		Good to Excel.	Good to Excel.	Good to Excel.	Poor	Excellent	Fair to Good	Fair to Good	Good	Fair	Excellent
Permeability to Gases		Fair	Fair	Very Low	Fair	Fair	Low	Very Low	Low to Very Low	Fair	Low
Acid Resistance	Dilute	Fair to Good	Fair to Good	Excellent	Good	Excellent	Excellent	Excellent	Excellent	Fair	Very Good
	Concentrated	Fair to Good	Fair to Good	Excellent	Good	Fair	Good	Excellent	Very Good	Poor	Very Good
Solvent Resistance	Aliphatic Hydrocarbons	Poor	Poor	Poor	Excellent	Poor	Good	Excellent	Good	Excellent	Poor
	Aromatic Hydrocarbons	Poor	Poor	Poor	Good	Poor	Fair	Excellent	Fair	Fair to Good	Poor
	Oxygenated (Ketones, etc.)	Good	Good	Good	Poor	Fair	Poor	Poor	Poor	Poor	Very Good
	Lacquer Solvents	Poor	Poor	Poor	Fair	Poor	Poor	Poor	Poor	Poor	Very Good
	Swelling in Lubricating Oil	Poor	Poor	Poor	Very Good	Fair	Good	Excellent	Good to Excel.	Excellent	Poor
	Oil and Gasoline	Poor	Poor	Poor	Excellent	Fair	Good	Excellent	Good	Excellent	Poor
	Animal and Vegetable Oils	Poor to Good	Poor to Good	Excellent	Excellent	Fair	Good	Excellent	Good	Excellent	Good
	Water Absorption	Very Good	Good to Excel.	Very Good	Fair to Good	Good	Good	Very Good	Very Good	Good at Room Temp Poor at 100 C	Good to Excellent
Resistance to:	Oxidation	Good	Good	Excellent	Good	Excellent	Excellent	Outstanding	Excellent	Excellent	Excellent
	Ozone	Fair	Fair	Excellent	Fair	Excellent	Excellent	Outstanding	Outstanding	Outstanding	Excellent
	Sunlight Aging	Poor	Poor	Very Good	Poor	Excellent	Very Good	Very Good	Outstanding	Good	Excellent
	Heat Aging	Good	Very Good	Excellent	Excellent	Outstanding	Excellent	Outstanding	Excellent	Good	Excellent
	Flame	Poor	Poor	Poor	Poor	Fair	Good	Good	Good	Fair	Poor
	Heat	Good	Excellent	Excellent	Excellent	Excellent	Excellent	Excellent	Excellent	Good	Excellent
	Cold	Excellent	Excellent	Good	Good	Excellent	Good	Good	Good	Excellent	Excellent

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Reinforced Plastics
to
Starch

peratures, due to the reversion of the outer parts of the article during the long time necessary to obtain adequate internal vulcanization. Where the properties demanded by the end application permit, reversion can largely be overcome by the use of the efficient vulcanization, EV, systems (46). The technique of injection molding offers both advantage and disadvantage for natural rubber (47). In this process, rubber compound is passed through a heated barrel, typically by a screw. The screw can reciprocate and force heated compound through a nozzle into a mold at high temperature (180–200°C). The flow properties of natural rubber compounds are such that considerable heat is generated by work done during injection, resulting in rapid vulcanization times (30 sec for thin articles, perhaps 4 min for a large item of 1 kg). Such rapid vulcanization reduces the danger of surface reversion in thick articles. By proper manipulation of the processing conditions, quite conventional vulcanization systems can be used, although for items with thick sections it may still be necessary to use the EV or semi-EV systems. Calendering demonstrates one of the most outstanding merits of natural rubber, ie, its building tack, an ability to stick to itself rapidly and firmly. This property is invaluable in the formation of composite items which have to be built up, eg, a conveyor belt made by calendering and plying up; for this reason alone at least a portion of natural rubber is commonly used in such articles. The other requirements of calendering, ie, smoothness and consistent dimensions, are readily obtained by control of compound viscosity and calendering conditions. It is one of the consequences of the ready breakdown of natural rubber that compound viscosity can easily be reduced to a suitable value by milling. Extrusion is performed as usual. When die swell is high and extruded surfaces tend to be rough, as with unfilled compounds, superior processing rubbers or process aids PA80 or PA57, all of which are made from prevulcanized latex, can be used (48). In addition to reducing die swell and smoothing extrusion, these materials reduce collapse of extruded sections during further processing. PA80 is a masterbatch material made from a mixture of 4 parts prevulcanized and 1 part natural latex (28); PA57 is PA80 plus 40 phr of a light-

Table 4. Physical Constants of Vulcanized Natural Rubber

	Unfilled gum rubber	Carbon black-filled rubber
IRHD* hardness	45	65
tensile strength, kg/cm²	280	210
elongation at break, %	680	420
Young's modulus, kg/cm²	19	59
shear modulus, kg/cm²	5.4	13.7
bulk modulus, kg/cm²	10,000	12,000
Poisson's ratio	0.5	0.5
resilience, %	80	60
velocity of sound, ft/sec	120	120
specific gravity	0.93	1.16
specific heat	0.45	0.41
thermal conductivity, relation to water	0.25	0.31
coefficient of cubic expansion, °C	67×10^{-5}	56×10^{-5}
electrical resistivity, Ω-cm³	1.7×10^{16}	3×10^{10}
dielectric constant	3	15
power factor	0.002	0.1

* IRHD, International Rubber Hardness Degrees.

Table 5. Typical Natural Rubber Compounds

Type of compound	Conveyor belt cover, earthmover tread, and sidewall	Bridge bearing, other engineering items	White-filled compound	Temperature-resistant compound
natural rubber, sheet or block	100	100		100
natural rubber, pale crepe			100	
low-structure HAF[a]-black	45			45
SRF[b]-black		40		
aluminum silicate			60	
process oil	5	2	1	5
zinc oxide	5	5	5	5
stearic acid	2	1	2	2
antioxidant/antiozonant	2	2	1	2
wax	2	2	2–3	2
CBS[c]	0.5	1	0.7	
MOR[d]				1.4
TMTD[e]				0.4
sulfur	2.5	2	2.5	0.35
cured	30′/141°C	20′/141°C	30′/141°C	30′/141°C
IRHD hardness	62	58	63	65
tensile strength, kg/cm^2	300	264	240	288
elongation at break, %	590	550	625	565
tear resistance,[f] kg/min at 21°C	4		3	5
after aging 21 days at 100°C				
tensile strength, kg/cm^2				185
elongation at break, %				250

[a] HAF, high-abrasion furnace black.
[b] SRF, semi-reinforcing furnace black.
[c] CBS, *N*-cyclohexylbenzothiazole-2-sulfenamide.
[d] MOR, 2-(4-morpholinyl mercapto)-benzothiazole.
[e] TMTD, tetramethylthiuram disulfide.
[f] Split strip at 20°C in which the two cords of a central split along the length of a test piece 1 × 6 in. are pulled apart at 4 in./min (237).

colored nonstaining oil (49). See also Injection Molding under MOLDING; MELT EXTRUSION; CALENDERING.

Some important physical constants of vulcanized natural rubber are listed in Table 4; Table 5 lists recipes and characteristics of some typical natural rubber compounds.

Latex. The compounding of natural rubber latex is, in principle, similar to that of dry natural rubber. There are, however, two major differences. First, because latex compounds are mixed at room temperature, highly active vulcanization systems can be used, which enable vulcanization to be performed at temperatures below 100°C. In dry rubber, such systems are difficult or impossible to use because the heat developed in mixing causes premature vulcanization, or scorch. Secondly, the reinforcing action of fillers, which is of major importance in dry rubber, is not obtainable in normally processed latex compounds. This is because reinforcement is developed only by masticating rubber and filler together. Although various means for reinforcing latex com-

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VOLUME 20

Refractories
to
Silk

The need for an instrument to make measurements of some cure-dependent property continuously while cure is taking place and at the curing temperature so that a curing curve can be produced with good precision has been satisfied with the cure meters, eg, the oscillating disk cure meter (ASTM D 2084-79). The Plasti-Corder is another instrument which can be used to measure the rheological properties of rubber and plastics and it is very versatile (96). The Plasti-Corder can be used to measure compound or polymer flow characteristics over a wide range of shear rates and temperatures. The relative power requirements for different stocks can be predicted by the instrument. The extrusion performance of rubber compounds can be predicted from the Brabender curves. Generally, as the trace bandwidth increases, the extrusion quality decreases.

The extrudability of unvulcanized elastomeric compounds (ASTM D 2320-78) can be determined using ASTM Extrusion Die No. 1, Garvey type, which has a triangular shape. Systems for rating extrusions are described, and formulas and their preparation are given for compounds of known extrusion characteristics to allow each laboratory to evaluate its own technique. Since extrusion machines differ from laboratory to laboratory, these methods outline techniques to minimize differences in testing approaches between tubers.

Capillary rheometry is one of the few techniques available that covers the total range of shear rates involved in rubber processing. In one industrial method, the output rate of the extrudate is determined by applying constant pressure on the piston. The corresponding pressure is measured when a constant rate of piston movement is applied to give a fixed extrusion rate. The Monsanto Processability Tester (MPT) is designed with an advanced constant-rate capillary rheometer with die swell and relaxation measuring capability.

Vulcanizates. There are two types of tests for vulcanizates. In the first type, the tests are either of specimens especially molded for the purpose or of specimens cut from a finished product. In the second type the tests are of the product itself, either in actual service or in machines designed to simulate or exaggerate conditions. The following discussion applies only to tests of the first type.

Tension. The stress–strain test in tension is the most widely used test in the rubber industry. It is extremely useful for analyzing compound development, aiding in manufacturing control, and determining a compound's susceptibility to natural and artificial aging. Tensile strength and ultimate elongation values, however, have little significance for design or application engineers, since they cannot be used in design calculations and they bear little relation to the ability of a rubber part to perform its function. Tensile strength and elongation properties serve as an index to the general quality of a rubber part. Rubber compounds less than 6.9 MPa (1000 psi) in tensile strength are usually poor in most mechanical properties and those with tensile strengths over 20.7 MPa (3000 psi) are usually good in most mechanical properties. In the middle range, which is applied to most rubber products, correlation is at best haphazard between tensile strength and such properties as flex life, compression set, abrasion resistance, and resilience. In the standard test the specimen has a dumbell shape and is cut from a sheet with die C as described in ASTM D 412-80 and is ca 2.0 ± 0.2 mm thick. The test is conducted at room temperature and the jaws which grip the tab ends of the dumbbell specimens are separated at the rate of 8.5 mm/s. By means of suitable devices, the load required to elongate the specimen is recorded for each 100% extension of the restricted portion of the dumbbell, and both the elongation at break and the tensile strength at break are recorded. The load required to elongate to a specified elongation, eg, 300%, is referred to as the modulus of the material.

The values of stress at each increment of elongation and at break are calculated on the basis of the original, unstressed cross section, rather than the actual cross section at the time the measurements are made. If it is assumed that no change in volume occurs during stretching, it follows that for a conventional tensile strength value of 20.7 MPa (3000 psi) and an ultimate elongation of 900%, the actual stress at break on the cross-sectional area at the instant of breaking is

$$20.7((900/100) + 1) = 207 \text{ MPa } (30{,}000 \text{ psi})$$

Nonstandard tests are frequently made with specimens larger or smaller than die C or thicker or thinner than 2.0 ± 0.2 mm. The rate of jaw separation can be varied as can the testing temperature. All these conditions influence the results obtained.

In the United States much of the work of tensile testing is done on a Scott tensile-testing machine which has a pendulum dynamometer. These machines are considered accurate only at 15–85% of their maximum rated capacity. The Instron and Accr-O-Meter include strain gauges in their weighing system; thus they are useful in studying the low strain portion of stress–strain curves.

Hardness. Hardness (qv) is the relative resistance of the surface to indentation by an indenter of specified dimensions under a specific load. The objective of a hardness test is to measure the elastic modulus of the rubber compound under conditions of small strain. This property is one which is closely related to product performance, since most rubber products in use are subjected to relatively small strains. The ASTM hardness testing methods are ASTM D 2240-75, D 1415-68 (1975), and D 531-78. ASTM D 531 (Pusey and Jones Indentation) is used mainly for roll compounds.

The assumption that hardness is a close measure of stiffness may be problematic when testing rubber products such as motor mounts. There is a stress–strain relationship between hardness and stiffness, but it is established for two entirely different kinds of deformation. Hardness is derived from small deformations at the surface, whereas stiffness measurements are derived from gross deformations of the entire mass. Because of this difference hardness is not a reliable measure of stiffness.

Set, Creep, and Hysteresis. No rubber vulcanizate is perfectly elastic, and a great many tests are employed to measure the extent to which a material fails to be perfectly elastic. These can be grouped into static or long-time tests and dynamic or short-time tests. Among the static tests are tests of permanent set in terms of tension or compression, creep, and stress–relaxation. Perhaps the most common of these is the permanent-set test in compression (ASTM D 395-78), in which a specimen 12.7 mm thick and 28.7 mm in diameter is compressed between flat plates and compressed for a specified time at the desired test temperature, after which the compressing force is released and the specimen allowed to recover for a specified period of time. The height of the specimen is then measured and the permanent, unrecovered height is noted. This type of test is useful in developing materials or predicting the performance of a product which is utilized in compressive strain.

Permanent set in tension (ASTM D 412-80) is the permanent deformation caused by tensional forces. The tension-set tests are seldom used in practice except in the wire industry.

Creep is the increase of deformation with time under constant stress. Creep is important in motor mounts, since it influences the space relationships between various parts of equipment. It is difficult to predict creep for a given application without conducting simulated service tests because several factors influence creep, especially the amount of strain, temperature, and changes in these two resulting from vibration. The higher the initial strain, the higher the creep; also the higher the temperature, the higher the creep. The degree of creep depends on the type of strain. Creep is greater under tension strain than under equal shear or compression strain. Creep is also greater under dynamic loading than under static loading.

Stress relaxation of a cured rubber is the loss in stress with time at a constant deformation. A method of measuring stress relaxation in compression is described in ASTM D 1390-76. Stress relaxation is an important characteristic of a rubber gasket in its ability to maintain a seal.

The dynamic group of tests includes rebound tests and free-vibration tests either at resonance or at a frequency avoiding resonance. The objective in these tests is generally to determine the hysteresis or energy lost under the particular conditions employed, although the determination of the dynamic stiffness of the material is also important. One of the most widely used of these tests is the free-vibration test with the Yerzley oscillograph (ASTM D 945-79), whereby the specimen is vibrated either in compression or in shear and a damping curve is obtained from which the more important properties can be calculated. Another widely used method is the determination of hysteresis by means of the Goodrich Flexometer (method A of ASTM D 623-78). A cylindrical specimen 17.8 mm in diameter and 2.5 mm tall is vibrated at 30 Hz under controlled conditions of load, stroke, and ambient temperature. The temperature rise at the base of the specimen is measured and this is considered a measure of the hysteresis defect of the material under the particular conditions employed. Method B of ASTM D 623-78 describes the Firestone flexometer, which also vibrates the specimen at a constant amplitude. Method C of ASTM D 623 describes the St. Joe flexometer, which vibrates the specimen at either a constant load or a constant amplitude.

Impact resilience is determined in accordance with ASTM D 1054-79, also known as the Goodyear-Healy method. A free-falling pendulum hammer is dropped against a specimen. The resilience is the height to which it rebounds, expressed as the percentage of the height from which it was dropped.

Cracking and Crack Growth. The flexing resistance of a rubber compound is its ability to withstand fatigue resulting from repeated distortion by extension, bending, or compression. This flexing fatigue may result in several different types of failure. The most important fatigue failure is popularly called flex cracking. The cause of this failure is twofold: stress breaking of rubber chains and cross-links and, more important, oxidation accelerated by heat buildup in flexing. This type of cracking occurs in tires, shoe soling, and belting. Both flex cracking and ozone cracking can be considered in two parts: initiation of cracks and crack growth.

ASTM D 430-73 methods B and C describe procedures by which the initiation of cracks and their subsequent growth can be measured. With some materials the initiation of cracks, as measured in method B, is slow and erratic, but once initiated they grow quite rapidly. To measure the growth of initiated cracks, a specimen as used in the initiation test is cut or pierced with a sharp tool at the base of the groove, and the rate of growth of this cut is measured as a function of the number of flexures (ASTM D 813-59) (1976). Method C, involving a specially molded, grooved specimen is used outdoors and in an ozone box to determine crack initiation and growth.

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3.10 CALCULATION OF AMOUNT OF RUBBER CORD FOR MODEL SUPPORT

DATA EXTRACTED FROM REFERENCE 5
FOR CARBON FILLED VULCANIZED NATURAL RUBBER.

TENSILE STRENGTH $\approx 280^{*}\ Kg/cm^2 = 1567\ lb/in^2$

ELONGATION AT BREAK $\approx$ 600 PERCENT

SPECIFIC GRAVITY ≈ 1

STRESS AT 300 % ELONGATION $\approx 120^{*}\ Kg/cm^2 = 672\ lb/in^2$

ASSUMPTIONS:

1. WORK AT 300 % ELONGATION
2. MODEL WEIGHT IS 10,000 lb

ORIGINAL AREA - A_{OR}

$$A_{OR} = \left(\frac{AREA}{WEIGHT}\right)\left(WEIGHT\right) = \frac{W}{\sigma} = \frac{10{,}000}{120 \times \frac{2.205}{0.394}} = 14.89\ IN^2$$

$$E = \frac{\sigma}{\epsilon} = \frac{\sigma}{\Delta L/L} = \left(120 \times \frac{2.205}{.394}\right)\left(\frac{1}{3}\right) = 224\ lb/in^2$$

A_{OR} IS THE AREA OF THE UNSTRETCHED RUBBER. THIS MUCH AREA WILL HOLD THE WEIGHT AND ALLOW THE RUBBER TO ELONGATE 300 PERCENT. THE ELONGATION IS EQUAL TO δ_{St} WHICH EQUALS THREE TIMES THE ORIGINAL LENGTH OF THE RUBBER, $l_{E,O}$.

* BASED ON ORIGINAL AREA A_{OR}

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3.11 APPROXIMATION OF THE WEIGHT OF RUBBER CORD FOR MODEL SUPPORT

VOLUME OF RUBBER

V = ORIGINAL AREA × LENGTH OF RUBBER UNDER NO LOAD

$$= A_{OR} \times l_{E,O} \quad \text{(SEE PAGE 68)}$$

$$= 14.89 \times 0.225\,(120) \times 12 = 4824 \text{ in}^3$$

THE SPECIFIC GRAVITY ≈ 1 & THE EFFECTIVE WEIGHT ≤ 1/2 ACTUAL WEIGHT, THEREFORE THE EFFECTIVE WEIGHT

$$W_{R,e} \le \frac{1}{2}\,\frac{4824}{1728} \times 62.4 \le 87.1 \text{ lb}$$

SINCE THE PROPOSED METHOD FOR SUPPORTING THE MODEL RESULTS IN THE RUBBER SUPPORTS WEIGHT ALWAYS BEING PROPORTIONAL TO THE MODEL WEIGHT FOR THE CONFIGURATION BEING TESTED, WE HAVE

$$\frac{W_{R,e}}{W_M} \le \frac{87.1}{10{,}000} = 0.00871$$

i.e., THE EFFECTIVE MASS OF THE SUPPORT CABLES IS EXPECTED TO ALWAYS BE LESS THAN 1% OF THE MODEL MASS.

IT IS RECOMMENDED THAT SAMPLES OF THE RUBBER CABLES BE TESTED TO INSURE ACHIEVEMENT OF EXPECTED MECHANICAL PROPERTIES. RUBBER PROPERTIES ARE HIGHLY VARIABLE, PARTICULARLY WHEN EXTRUDED IN THE RANGE OF HARDNESS, OR SOFTNESS NEEDED.

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3.12 APPROXIMATION OF LATERAL NATURAL FREQUENCIES OF MODEL SUPPORT CABLES

IT IS RECOMMEND THAT THE MODEL BE SUPPORTED BY A NUMBER OF CABLES SO DISTRIBUTED THAT THE KEEL, KEEL EXTENSIONS, AND TRANSVERSE BOOM CARRY VERY LITTLE LOAD. EACH OF THESE CABLES WILL CARRY A TENSILE LOAD AND THUS RESPOND SIMILARLY TO A STRING UNDER TENSION. ITS LOWEST NATURAL FREQUENCY WILL BE

$$\omega_1 = \pi \sqrt{\frac{T}{\mu_s l^2}}$$

WHERE

T = TENSILE FORCE/CABLE = Mg/N

μ_s = MASS OF CABLE/UNIT LENGTH $\approx \frac{M_R}{0.9 l N} = \frac{M_R}{l_E N}$

l = CABLE LENGTH

THEN,

$$\frac{\omega_1}{\sqrt{0.9}} = \pi \sqrt{\left(\frac{Mg}{N}\right)\left(\frac{lN}{M_R}\right)\frac{1}{l^2}} = \pi\sqrt{\frac{M}{M_R}}\sqrt{\frac{g}{l}} = \pi\sqrt{\frac{W}{W_R}}\sqrt{\frac{g}{l}}$$

$$= \pi\sqrt{\frac{10,000}{2 \times 87.1}}\sqrt{\frac{g}{l}} = 23.8\ (0.518)$$

AND $\omega_1 = 11.70$ RAD/SEC

$$f_1 = \frac{\omega_1}{6.28} = 1.86 \text{ Hz}$$

NOTE THAT THIS (THE LOWEST LATERAL STRING) FREQUENCY IS 22.6 TIMES AS HIGH AS THE SIMPLE PENDULUM FREQUENCY AND FOR A GIVEN CABLE STRESS, IS INDEPENDENT OF THE NUMBER OF CABLES.

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THE FREQUENCY SEPARATION FOR THE STRING FREQUENCY RELATIVE TO THE FIRST ELASTIC MODE IS:

$$\alpha_s = \frac{\omega_{M,E}}{\omega_1} = \frac{f_{F,E} \times \frac{1}{\lambda}}{f_1}$$

$$= \frac{0.1 \times 4}{1.86} = 0.22$$

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THUS THE FIRST STRING FREQUENCY IS ABOUT 5 TIMES AS HIGH AS THE FIRST ELASTIC FREQUENCY, BUT COINCIDENCE OF FREQUENCIES IS LIKELY TO OCCUR FOR HIGHER ELASTIC MODES OF THE MODEL. THE USE OF RANDOM HEIGHT LATERAL TIES BETWEEN THE CABLES (PERHAPS MADE OF HIGHLY DAMPED RUBBER) MAY ELIMINATE LATERAL AMPLIFICATIONS WHEN CONDITIONS OF COINCIDENCE OCCUR BETWEEN STRING FREQUENCIES & MODEL EXCITATION FREQUENCIES.

IT SHOULD BE NOTED THAT THE MODEL MASS DISTRIBUTION, THE ACTUAL LENGTH BETWEEN THE PLATFORM AND MODEL ATTACHMENT POINTS AND THE INHERENT VARIATIONS IN THE PROPERTIES OF EXTRUDED SOFT (30 TO 40 DUROMETER) RUBBER WILL BE SUCH THAT THE NATURAL FREQUENCIES OF THE VARIOUS CABLES WILL BE SLIGHTLY DIFFERENT FROM ONE ANOTHER. IT IS EXPECTED THAT THIS WILL ELIMINATE GROSS LATERAL EXCITATIONS OF THE STRINGS EVEN IF THE MODEL IS DRIVEN AT A FREQUENCY EQUAL TO THE MEAN VALUE OF THE STRING FREQUENCIES.

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3.13 SUMMARY OF EXPERIMENTAL DATA OBTAINED FROM STATIC AND DYNAMIC TESTS OF A RUBBER SAMPLE

THE LOAD DEFLECTION CURVE OBTAINED FROM THE TESTS OF THE RUBBER SAMPLE ARE GIVEN IN FIGURE 9. OTHER PERTINENT DATA ARE SUMMARIZED AS FOLLOWS.

STRESS AT 300% ELONGATION BASED ON ORIGINAL AREA IS

$$\sigma_{300} = \frac{15.8}{16} \frac{1}{5.6 \times 10^{-3}} = 176 \ lb/in^2$$

THE SPRING CONSTANT AT 300% ELONGATION IS

$$K_0 = \frac{\Delta L}{\Delta Z} = \frac{0.25}{3.1} = 8.06 \times 10^{-2} \ lbs/in$$

$$= 0.97 \ lbs/ft$$

THE NATURAL FREQUENCY PREDICTED IS

$$f = \frac{1}{6.28}\sqrt{\frac{K}{M}} = \frac{1}{6.28}\sqrt{\frac{0.97 \times 32.2}{15.8/16}} = 0.90 \ Hz$$

THE MEASURED NATURAL FREQUENCY WAS APPROX.

$$f_{MEASURED} = 1.1 \ Hz$$

DETAILED DATA FROM TESTS OF SEVERAL ADDITIONAL SAMPLES OF VARIOUS TYPES OF RUBBER ARE GIVEN IN APPENDIX I. AS A RESULT OF THESE TESTS, IT BECAME CLEAR THAT A VULCANIZED NATURAL RUBBER WAS REQUIRED AND SAMPLES WERE LOCATED AND ARE NOW UNDER TEST BY NASA-LANGLEY.

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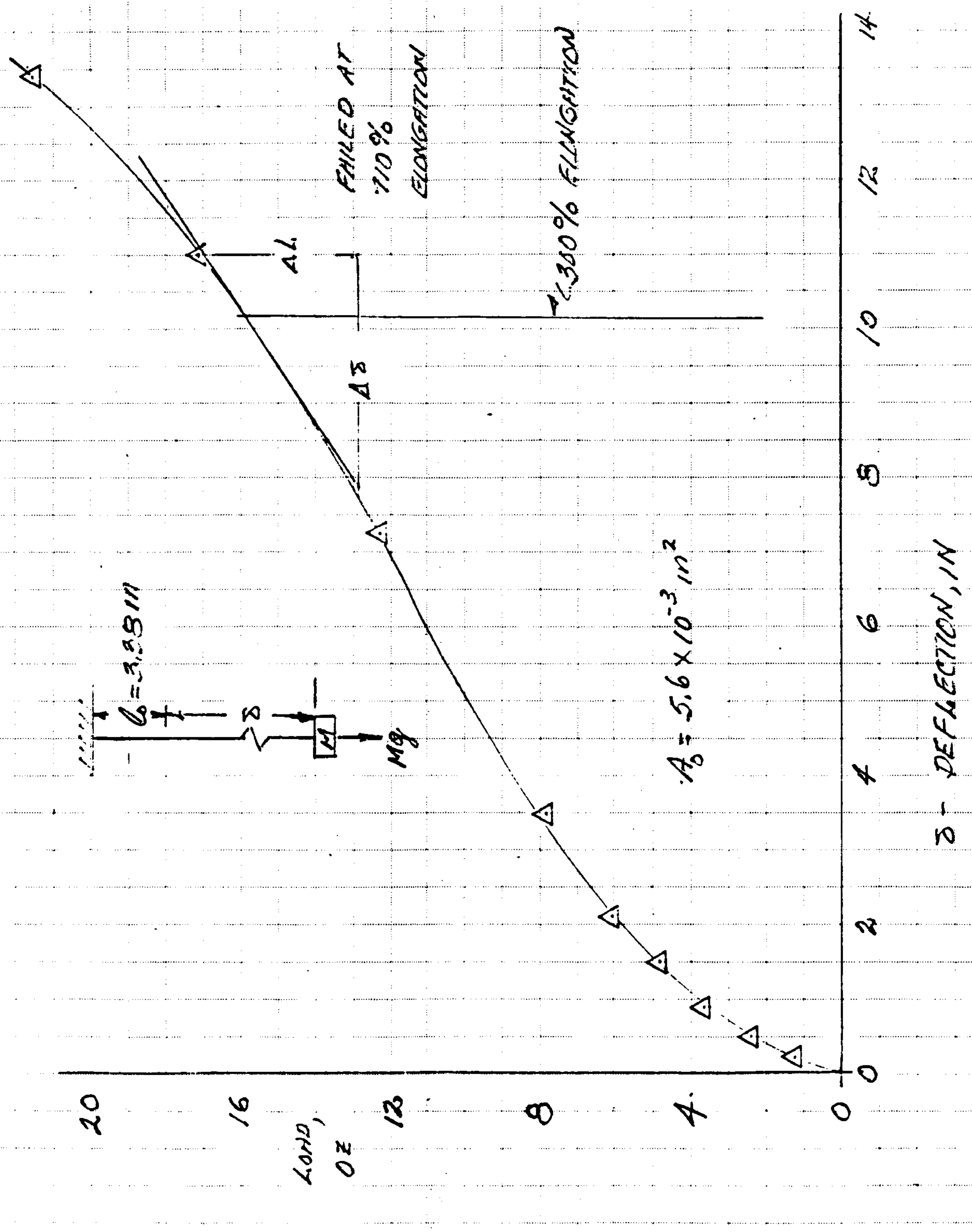

FIGURE 9 - RESULTS FOR TESTS OF A RUBBER SAMPLE - THIN LATEX

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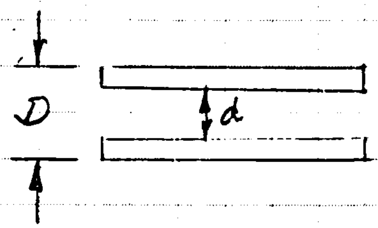
3.14 CONSIDERATIONS RELATIVE TO THE NUMBER OF ELASTIC CABLES EMPLOYED FOR MODEL SUPPORT

ON THE BASIS OF THE ALLOWABLE WORKING STRESS (300 PERCENT ELONGATION OF RUBBER) IT WAS SHOWN ON PAGE 83 THAT AN INITIAL AREA OF 14.89 SQUARE INCHES OF RUBBER IS NEEDED. FOR CONVENIENCE OF ATTACHMENT TO THE INELASTIC CORD WHICH WILL CONNECT TO THE MODEL AND TO THE PLATFORM, TUBING IS RECOMMENDED.

RUBBER TUBING IS GENERALLY AVAILABLE IN A VARIETY OF CROSS SECTIONAL SIZES AND LENGTHS. THE NUMBER OF TUBES FOR VARIOUS OUTSIDE AND INSIDE DIAMETERS IS GIVEN AS FOLLOWS

$$N = \frac{4\ \text{AREA}}{\pi (D^2 - d^2)} = \frac{4}{\pi}\,\frac{14.89}{(D-d)(D+d)} = \frac{18.97}{(D-d)(D+d)}$$

D, in	d, in	N	LOAD/CORD, lb
1/4	1/8	405	25
3/8	3/16	180	55
7/16	1/4	147	68
1/2	1/4	101	99
5/8	5/16	65	154



COMPROMISES WILL BE NECESSARY BETWEEN THE ADDED COMPLEXITY OF MANY CABLES AND THE DESIRE TO AVOID TRANSMISSION OF LARGE GRAVITY INDUCED LOADS THROUGH THE STRUCTURE. ALSO, FOR REASONS OF SAFETY OF THE MODEL AND TEST PERSONNEL, IT IS DESIRABLE TO LIMIT THE LOAD CARRIED PER CABLE. THUS IT SEEMS REASONABLE THAT 1/2 in × 1/4 in CABLES, EACH CARRYING ABOUT 100 lbs.,

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WOULD PROVIDE A GOOD PRIMARY SYSTEM, PARTICULARLY FOR THE HEAVIER PARTS OF THE MODEL SUCH AS THE ORBITER AND LABORATORIES. PARTS OF THE KEEL STRUCTURE WILL BE VERY LIGHTLY LOADED AND IT MIGHT BE DESIRABLE TO USE SMALLER CABLES IN THESE AREAS TO GET BETTER LOAD DISTRIBUTIONS; PERHAPS 3/8 in x 3/16 in CABLES.

AS A POINT OF REFERENCE, AN ALL-UP ORBITER WEIGHT OF 240,000 lbs WOULD RESULT IN A MODEL ORBITER WEIGHT OF 3750 lbs. SINCE THE ORBITER MODEL LENGTH WOULD BE ABOUT 30 FEET, IT WOULD BE SUPPORTED BY 1/2 INCH ELASTIC CABLES PLACED AT INTERVALS OF ABOUT 1 FOOT ALONG ITS LENGTH. ALSO, IT IS NOTED THAT THE COMBINED WEIGHT OF THE 3 MODULES AND 2 LABS LOCATED PRIMARILY BELOW THE NOSE OF THE ORBITER WILL HAVE MODEL WEIGHTS TOTALING ANOTHER 3140 lbs. THESE WEIGHTS MUST BE CARRIED BY ANOTHER 31 CABLES, SOME OF WHICH WILL BE LOCATED CLOSE TO THE ORBITER SUPPORTS. IT IS ALSO NOTED THAT THE ORBITER SUPPORT SYSTEM MUST BE COMPATIBLE WITH THE LIMITED CAPABILITY OF THE ORBITER/HABITATION MODULE DOCKING INTERFACE TO TRANSMIT MOMENTS RESULTING FROM GRAVITY INDUCED FORCES.

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3.15 INVESTIGATION OF USE OF COIL AND REVERSE LOOP SPRINGS FOR MODEL SUPPORT

THE NATURAL FREQUENCY OF A MASS SUSPENDED BY A LINEAR COIL SPRING IS GIVEN BY

$$\omega^2 = \frac{g}{\delta_{ST}}$$

THE STATIC DEFLECTION OF A COIL SPRING IS

$$\delta_{ST} \leq \frac{\pi i S_s D^2}{k d G}$$

WHERE

$i \equiv$ NO OF COILS
$S_s \equiv$ ALLOWABLE SHEAR STRESS
$D \equiv$ COIL DIAMETER $= 2R$
$d \equiv$ WIRE DIAMETER
$G \equiv$ SHEAR MODULUS
$k \equiv$ AN EMPIRICAL DESIGN FACTOR (SEE p 277, REF. 6)

CHOOSING THE EQUALITY FOR STATIC DEFLECTION, AND THE FOLLOWING CONDITIONS

$$\omega^2 = \frac{3g}{\ell} = \frac{g}{\delta_{ST}}$$

$$S_s = \frac{150,000}{1.5} = \frac{\text{YIELD STRESS}}{\text{SAFETY FACTOR}} = 100,000 \text{ psi}$$

$$i = \frac{\delta_{ST}}{R} = \frac{\ell}{3R} = \frac{\text{STRETCHED SPRING LENGTH}}{\text{COIL RADIUS}}$$

$$k = 1.1$$

WE OBTAIN

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$$\delta_{ST} = \pi \frac{\partial_{ST}}{R} S_s D^2 \frac{1}{k d G}$$

FROM WHICH

$$\frac{d}{R} = \frac{4\pi S_s}{kG} = \frac{4\pi\, 1\times10^5}{1.1\times11.3\times10^6}$$

$$= 0.101$$

$$\frac{d}{D} = \frac{1}{C} = \frac{1}{2}\frac{d}{R} = 0.0505$$

$\therefore$ $C \cong 20$ $\quad k \cong 1.1$ VERIFIED
SEE REF. 6 , PAGE 277

THE FORCE THIS SPRING WILL CARRY IS

$$F = \frac{\delta_{ST} d^4 G}{8D^3 i} = \frac{\delta_{ST} G}{8 \frac{\delta_{ST}}{R}} \frac{d}{8}\left(\frac{d}{R}\right)^3$$

$$= \frac{G}{64} d^2 \left(\frac{d}{R}\right)^2 = \frac{G}{64} (.101)^2 d^2$$

LET $F = 100$ lb

$$d^2 = \frac{100 \times 64 \times 1}{11.3\times10^6 (.101)^2} = 5.55\times10^{-2}$$

$$d = 0.236 \qquad D = 2R = 2\left(\frac{R}{d}\right)d = 2\times\frac{1}{.101}\times.236 = 4.67''$$

LET $\partial_{ST} = \frac{l}{3} = \frac{120\times12}{3} = 480$ in

$$i = \frac{480}{R} = 480 \frac{d}{R} \frac{1}{d} = 480\times0.101\times\frac{1}{0.236} = 205$$

SPRING WEIGHT

$$W = i(\pi D)\frac{\pi d^2}{4}(0.28) = 0.07\, i \left(\frac{D}{d}\right)\pi^2 d^3$$

$= 36.8$ lb PER 100 lb OF MODEL WEIGHT

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CHECK

$$S_S = \frac{k\, \delta_{ST}\, d\, G}{\pi i\, D^2} = \frac{1.1\, \delta_{ST}\, d\, G}{\pi\, \frac{\delta_{ST}}{K}\, 4 R^2} = \frac{1.1\, G}{\pi\, 4}\left(\frac{d}{R}\right)$$

$$= \frac{1.1 \times 11.3 \times 10^6}{\pi \times 4 \times} \times 0.101$$

$$= 0.1 \times 10^6 = 100{,}000 \text{ lbs}$$

SPRING CONSTANT

$$K = \frac{F}{\delta_{ST}} = \frac{100}{40 \times 12} = 0.208 \text{ lbs/in}$$

$$= \frac{d^4 G}{8 i D^3} = \frac{G}{8}\, d \left(\frac{d}{D}\right)^3 \frac{1}{205}$$

$$= \frac{11.3 \times 10^6}{8} \times .236 \times (.0505)^3 \frac{1}{205} = 0.209$$

$$\omega^2 = \frac{K}{M} = \frac{0.209}{100} \times 386 = 0.8067 \text{ rad/sec}^2$$

$$= \frac{3g}{\ell} = \frac{3 \times 386}{120 \times 12} = 0.804 \text{ rad/sec}^2$$

NOTE THAT THE AFOREMENTIONED CALCULATIONS NEGLECTED THE WEIGHT OF THE SPRING IN THE CALCULATIONS OF THE FREQUENCY. BUT THE APPROXIMATION SERVES THE PURPOSE IN POINTING OUT THAT EVEN FOR THE STIFFEST SPRING (HIGHEST ALLOWABLE FREQUENCY) THE SPRING MASS IS OF THE ORDER OF A THIRD OF THE MODEL MASS. FOR A SOFTER SYSTEM (ONE WITH A LARGER STATIC DEFLECTION), THE NUMBER OF COILS AND HENCE THE WEIGHT OF THE SPRING IS EVEN HIGHER

THE CONCLUSION IS THAT COIL SPRINGS ARE NOT A VIABLE SUPPORT OPTION.

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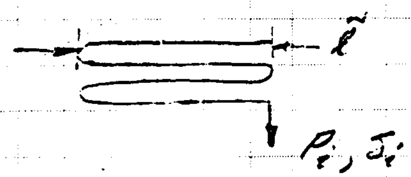
ANALYSIS OF A KINEESE LOOP SPRING



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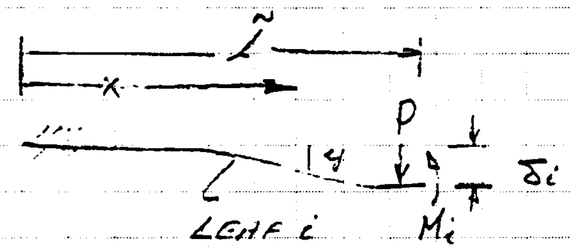
EACH LENGTH L IS TREATED AS A BEAM WITH ZERO SLOPE AT EACH END



$\delta = n\,\delta_i$ WHERE n IS THE NUMBER OF HALF LOOPS

$$\frac{dy}{dx} = \int \frac{M}{EI}(x)\,dx + C = \frac{1}{EI}\int \left(P(\tilde{\ell} - x) - M_i\right)dx + C$$

$$= \left(P\tilde{\ell}x - \frac{Px^2}{2} - M_i x + C\right)\frac{1}{EI}$$

SINCE $\frac{dy}{dx} = 0$ @ $x = 0$ & $x = \tilde{\ell}$

$C = 0$ & $M_i = P\frac{\tilde{\ell}}{2}$

$$y = \frac{P}{EI}\left(\frac{\ell x^2}{2} - \frac{x^3}{6} - \frac{\ell}{2}\frac{x^2}{2}\right) + C_1$$

SINCE $y = 0$ @ $x = 0$

$C_1 = 0$

AND

$$y(\ell) = \delta_i = \frac{1}{12}\frac{P\tilde{\ell}^3}{EI}$$

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THE STRESS σ IS

$$\sigma = \frac{Mc}{I} = \left(P(\tilde{\ell}-x) - P\frac{\tilde{\ell}}{2}\right)\frac{t}{2}\frac{1}{I} = \frac{P}{I}\left(\frac{\tilde{\ell}}{2}-x\right)\frac{t}{2}, \text{ OR}$$

$$\sigma_{i_{MAX}} = \frac{P}{I}\frac{\tilde{\ell}}{2}\frac{t}{2}$$

AND

$$\frac{P}{I} = \frac{4\sigma_{MAX}}{\tilde{\ell}t}$$

THE DEFLECTION CAN NOW BE EXPRESSED IN TERMS OF THE MAXIMUM STRESS, i.e.

$$\delta_i = \frac{1}{12}\left(\frac{P}{I}\right)\frac{\tilde{\ell}^3}{E} = \frac{1}{12}\left(\frac{4\sigma_{MAX}}{\tilde{\ell}t}\right)\frac{\tilde{\ell}^3}{E} = \frac{1}{3}\frac{\sigma_{MAX}}{t}\frac{\tilde{\ell}^2}{E}$$

IF WE ASSUME THAT THE CANTILEVER SPRING HAS A RECTANGULAR CROSS SECTION, THE WEIGHT OF A LEAF IS

$$w_i = \rho b t \tilde{\ell}$$

AND THE DEFLECTION/WEIGHT FOR A LEAF IS

$$\frac{\delta_i}{w_i} = \frac{1}{3}\frac{\sigma_{MAX}}{E\rho}\left(\frac{\tilde{\ell}}{b}\right)\frac{1}{t^2}$$

NOTE THAT THE DEFLECTION/WEIGHT IS MAXIMIZED BY LARGE $\tilde{\ell}$ AND SMALL b AND t, GEOMETRICAL FACTORS. IT IS ALSO MAXIMIZED BY HAVING A LARGE VALUE OF THE MATERIALS PROPERTY RATIO $(\sigma_{MAX}/E\rho)$. FOR MANY REASONS, A HIGH QUALITY SPRING STEEL APPEARS DESIRABLE BUT HIGH STRENGTH ADVANCED COMPOSITES MIGHT ALSO PROVIDE AN OPTION.

THE WEIGHT OF THE SUPPORT SYSTEM IS THEN GIVEN BY

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$$W = n\, w_i = n \rho b t \tilde{\ell}$$

WHERE

$$n = \frac{\delta_{ST}}{\delta_i} = \frac{.675 \times 120 \times 12}{\delta_i}$$

THEN

$$W = \left((.675 \times 120 \times 12) \frac{3tE}{\sigma_{MAX} \tilde{\ell}^2} \right) \left(\rho b t \tilde{\ell} \right)$$

$$= 2.92 \times 10^3 \frac{b}{\tilde{\ell}} t^2 \left(\frac{\rho E}{\sigma_{MAX}} \right)$$

FOR SPRING STEEL

$$\frac{\rho E}{\sigma_{MAX}} = \frac{0.28 \times 30 \times 10^6}{(0.25 \times 10^6 / 1.5)} = 50.3$$

AND, FOR $\frac{b}{\tilde{\ell}} = 0.1$, WE OBTAIN THE FOLLOWING TABLE

t, in	W, lb	P lb
0.025	9.19	3.1
0.050	36.8	13.9
0.075	82.5	31.4
0.100	147	64.3

THE OBVIOUS CONCLUSION IS THAT THE LOOP TYPE SPRING IS ALSO MUCH TOO HEAVY.

APPENDIX I

RESULTS OF EXPERIMENTAL TESTS OF ADDITIONAL RUBBER SAMPLES

TESTS CONDUCTED IN NASA-LRC DYNAMICS RESEARCH LABORATORY ON 6/12 AND 6/13, 1985

BECAUSE OF THE UNCERTAINTY OF THE CHARACTERISTICS OF RUBBER WHICH MAY BE USED TO SUPPORT THE MODEL, SEVERAL SAMPLES OF RUBBER WERE OBTAINED AND TESTED IN THE DYNAMICS RESEARCH LABORATORY AT NASA-LANGLEY. DETAILS OF THESE SAMPLES (6), TOGETHER WITH THE DATA FROM THE TESTS, ARE GIVEN ON PAGES 98 THROUGH 110.

THE STRESS-STRAIN RELATIONSHIPS FOR THE RUBBER SAMPLES ARE PLOTTED ON FIGURE I-1 WHERE THE FOLLOWING DEFINITIONS WERE USED

$$\sigma = \frac{\text{LOAD}}{\text{ORIGINAL AREA}} = \frac{F}{A_0} \equiv \text{STRESS}$$

$$\epsilon = \frac{\text{DEFLECTION}}{\text{ORIGINAL LENGTH}} = \frac{\delta_{ST}}{\ell_0} \equiv \text{STRAIN}$$

THE MEASURED NATURAL FREQUENCIES, NORMALIZED BY DIVIDING BY THE NATURAL FREQUENCIES OF LINEAR SYSTEMS UNDER THE SAME STATIC DEFLECTIONS, ARE PLOTTED IN FIGURE I-3.

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SAMPLE NO. 1 - LATEX SURGICAL TUBING
SOURCE: SOUTHHAMPTON PHARMACY
SUPPLIER: KENT LATEX PRODUCTS, INC
AVAILABLE IN 50' LENGTHS

NOMINAL DIMENSIONS: INSIDE DIAMETER 0.25"
OUTSIDE DIAMETER 0.375"
ORIGINAL LENGTH, l_0 61.5"
FINAL LENGTH 63.5"
ORIGINAL AREA, A_0 0.0613 in^2

F	σ F/A_0	δ_{ST}	ϵ δ_{ST}/L_0	E' $\sigma L_0/\delta_{ST}$	ω	$\frac{\omega^2 \delta_{ST}}{g}$
lb	lb/in²	in	in/in	lbs/in²	rad/sec	
0	0	0	0			
1	16.25	5.0	.081	201	6.91	.62
6	97.71	49.5	.805	121	2.35	.71
11	179.1	146.5	2.38	75.2	1.73	1.14
16	260.6	262.5	4.27	61.0	3.34	7.58
18	293.2	274.5	4.46	65.7	3.56	9.01
19	309.4	280.5	4.56	67.9	3.57	10.9
20	325.1	282.5	4.59	71.0	3.93	11.3
21	342.0	284.5	4.63	73.8	3.77	10.5
22	358.3	286.5	4.66	76.9	3.61	9.67
23	374.6	289.5	4.71	79.5	3.77	10.7
24	390.9	294.5	4.79	81.6	3.77	10.8
26	422.[illegible]	296.5	4.82	81.9	3.77	10.9
29	472.[illegible]	302.5	4.92	96.0	3.87	11.7
31	504.9	307.0	4.99	101.1	3.98	12.6
34	553.7	312.5	5.08	109.0	3.98	12.8
36	526.3	316.5	5.15	113.8	3.93	13.0
41	667.8	324.5	5.28	126.5	—	

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SAMPLE NO. 2 - LATEX TUBING
SOURCE: HAMPTON RUBBER CO
SUPPLIER: KENT LATEX PRODUCTS, INC.
AVAILABLE IN 50' LENGTHS

NOMINAL DIMENSIONS: INSIDE DIAMETER 0.375"
OUTSIDE DIAMETER 0.625"
ORIGINAL LENGTH 23.25"
FINAL LENGTH 24.50"
ORIGINAL AREA 0.196 in^2

F	σ F/A_o	δ_{ST}	ϵ δ_{ST}/l_o	E' $\sigma l_o/\delta_{ST}$	ω	$\frac{\omega^2 \delta_{ST}}{g}$
lb	lb/in²	in	in/in	lbs/in²	rad/sec	
3.25	16.6	2.0	.086	193.0	13.4	0.93
8.25	42.1	6.13	.264	159.5	7.12	0.81
13.25	67.6	12.8	.550	122.9	4.61	0.70
18.25	93.1	22.6	.972	95.8	3.14	0.58
23.25	118.6	35.5	1.51	78.5	2.89	0.77
33.25	169.6	65.8	2.83	59.9	2.61	1.16
COMPLETELY UNLOADED & RELOADED						
28.25	144.1	52.3	2.25	64.0	2.62	0.93
33.25	169.6	66.8	2.87	59.1	3.35	1.94
43.25	220.7	99.8	4.30	51.3	5.23	7.07
53.25	271.7	106.8	4.59	59.2	5.86	9.50
63.25	322.7	111.3	4.79	67.4	6.28	11.37
73.25	373.7	115.3	4.96	75.3	6.28	11.78
83.25	424.7	119.3	5.13	82.8	6.28	12.19
93.25	475.8	123.3	5.30	89.8	KNOT PULLED OUT	

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SAMPLE NO. 3 - LATEX TUBING
SOURCE: HAMPTON RUBBER
SUPPLIER: KENT LATEX PRODUCTS, INC.
AVAILABLE IN 50' LENGTHS

NOMINAL DIMENSIONS: INSIDE DIAMETER 0.1875"
OUTSIDE DIAMETER 0.3125"
ORIGINAL LENGTH 26.50"
ORIGINAL AREA, A_0 0.0491 in^2

F	σ	δ_{ST}	ϵ	E'	ω	$\frac{\omega^2 \delta_{ST}}{g}$
	F/A_0		δ_{ST}/l_0	$\sigma l_0/\delta_{ST}$		
lb	lb/in^2	in	in/in	lbs/in^2	rad/sec	
1.0	20.37	2.5	.094	216.7	11.89	.916
3.0	61.01	10.9	.411	143.4	5.23	.772
5.0	101.8	27.3	1.03	98.8	3.14	.677
7.0	142.6	49.0	1.85	77.1	2.72	.939
10.0	203.7	85.5	3.23	63.1	3.77	3.14
12.0	244.4	109.5	4.13	59.2	5.23	7.76
15.0	305.5	119.5	4.51	67.7	5.44	9.16
17.0	346.2	123.5	4.66	74.3	5.86	10.98
18.0	366.6	125.0	4.72	77.7	5.86	11.12
20.0	407.3	128.0	4.83	84.3	5.86	11.39
22.0	480.1	131.0	4.94	97.2	5.86	11.65
24.0	488.2	134.5	5.08	96.2	5.86	11.96
25.0	509.2	136.5	5.15	98.9	6.28	13.95
26.0	529.5	137.5	5.19	102.0	6.07	13.12
28.0	570.3	138.8	5.24	108.3	6.07	13.24
30.0	611.0	141.0	5.32	114.8	6.17	13.91
33.0	672.1	143.0	5.39	124.6	6.17	14.10
35.0	712.8	145.0	5.47	130.3	KNOT PULLED OUT	

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SAMPLE NO. 4 - RED GUM
SOURCE: HAMPTON RUBBER CO
SUPPLIER - UNKNOWN
AVAILABLE - LONG LENGTH ROLLS

NOMINAL DIMENSIONS: INSIDE DIAMETER 0.250"
OUTSIDE DIAMETER 0.375"
ORIGINAL LENGTH, l_0 35.5"
FINAL LENGTH 40.5"
ORIGINAL AREA, A_0 0.0613 in^2

F	σ F/A_0	δ_{ST}	ϵ δ_{ST}/l_0	E' $\sigma l_0/\delta_{ST}$	ω	$\frac{\omega^2 \delta_{ST}}{g}$
lb	lb/in^2	in	in/in	lb/in^2	rad/sec	
1	16.3	1	.028	582	-	
6	97.8	4.5	.127	770	5.97	.416
11	179	30.5	.859	208	5.02	1.99
16	261	46.5	1.31	199	5.44	3.56
21	343	60.5	1.70	202	5.86	5.38
24	391	68.5	1.93	203	5.65	5.67
27	440	74.5	2.10	210	5.86	6.63
29	473	81.0	2.28	207	5.86	7.20
31	506	85.5	2.41	210	5.65	7.07
34	555	92.5	2.61	213	5.86	8.23
37	603	97.5	2.75	219	5.65	8.06
42	685	HELD LONG MOMENTARILY & FAILED				

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SAMPLE NO. 5 · SILICONE RUBBER
SOURCE: HAMPTON RUBBER CO
SUPPLIER: UNKNOWN
AVAILABLE: LONG-LENGTH ROLLS

NOMINAL DIMENSIONS: INSIDE DIAMETER 0.25"
OUTSIDE DIAMETER 0.375"
ORIGINAL LENGTH, l_0 29.0"
FINAL LENGTH —
ORIGINAL AREA, A_0 0.0613 in^2

F	σ	δ_{ST}	ϵ	E'	ω	$\frac{\omega^2 \delta_{ST}}{g}$
	F/A_0		δ_{ST}/l_0	$\sigma l_0/\delta_{ST}$		
lb	lb/in^2	in	in/in	lb/in^2	rad/sec	
1	16.3	.75	.026	627		
6	97.8	9.25	.318	308	6.28	.945
11	179	17.25	.595	301	8.79	1.45
16	261	22.0	.759	344	10.05	5.76
21	343	27.0	.931	368	10.05	7.06
26	423	34.0	1.172	361	10.05	8.90
31	505	FAILED SHORTLY AFTER APPLICATION				

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SAMPLE NO. 6 BLACK NEOPRENE ?
SOURCE: NASH STOCK NO. 4720-00993-0392

NOMINAL DIMENSIONS: INSIDE DIAMETER 0.3125
OUTSIDE DIAMETER 0.50"
ORIGINAL LENGTH 10'-1"
FINAL LENGTH —
ORIGINAL AREA, A_o 0.120

F	σ F/A_o	δ_{ST}	ϵ δ_{ST}/L_o	E' $\sigma L_o/\delta_{ST}$	ω	$\frac{\omega^2 \delta_{ST}}{g}$
lb	lb/in²	in	in/in	lb/in²	rad/sec	
3.25	27.08	1.5	.0123	2201		
23.25	193.7	26.0	.215	900		
43.25	360.4	49.8	.412	874	10.68	14.7
63.25	527.1	71.0	.587	898	9.42	16.32
83.25	693.7 HELD FOR 1 MINUTE & FAILED					

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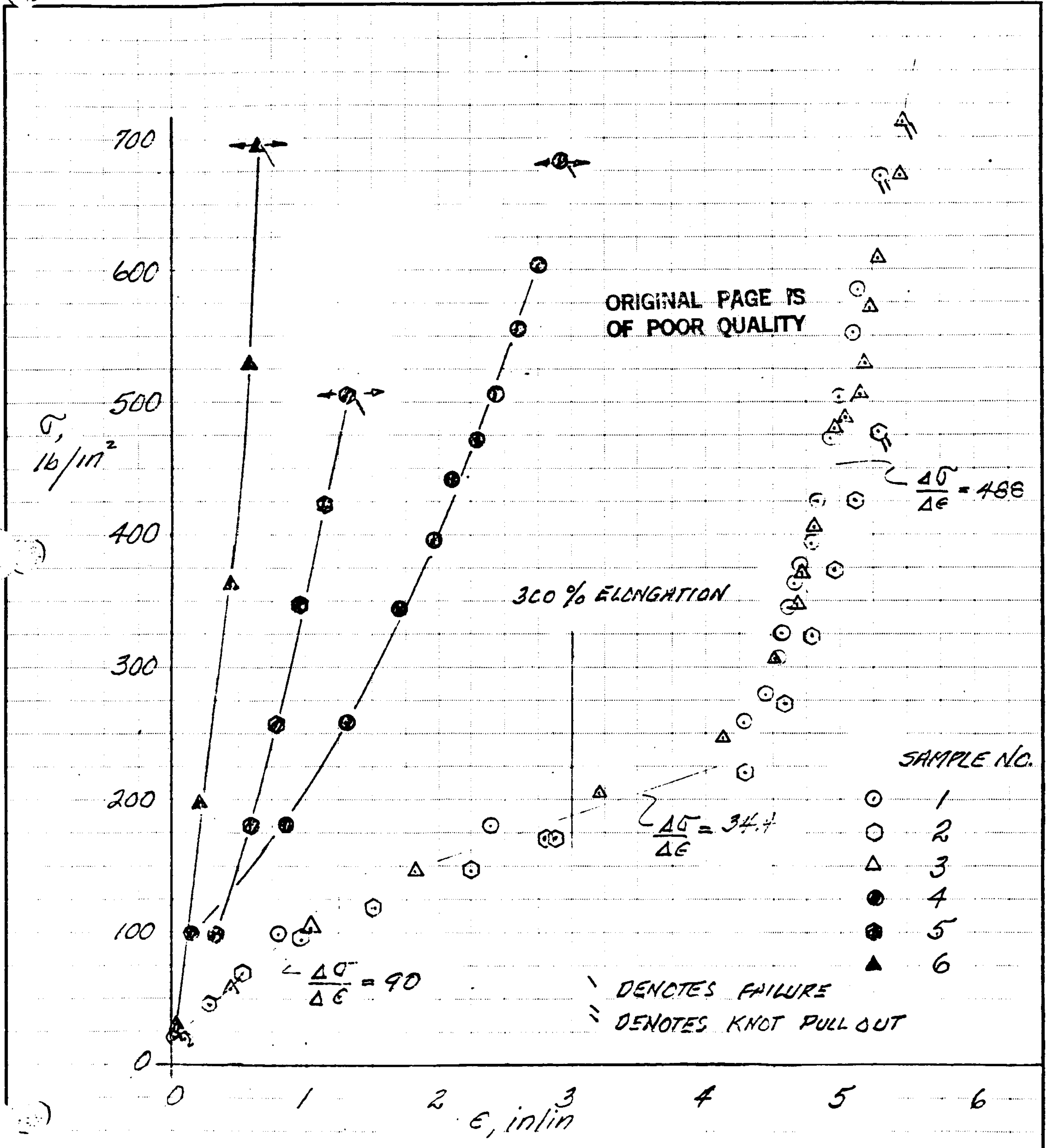

FIGURE I-1.- STRESS-STRAIN VARIATIONS MEASURED FOR 6 RUBBER SAMPLES

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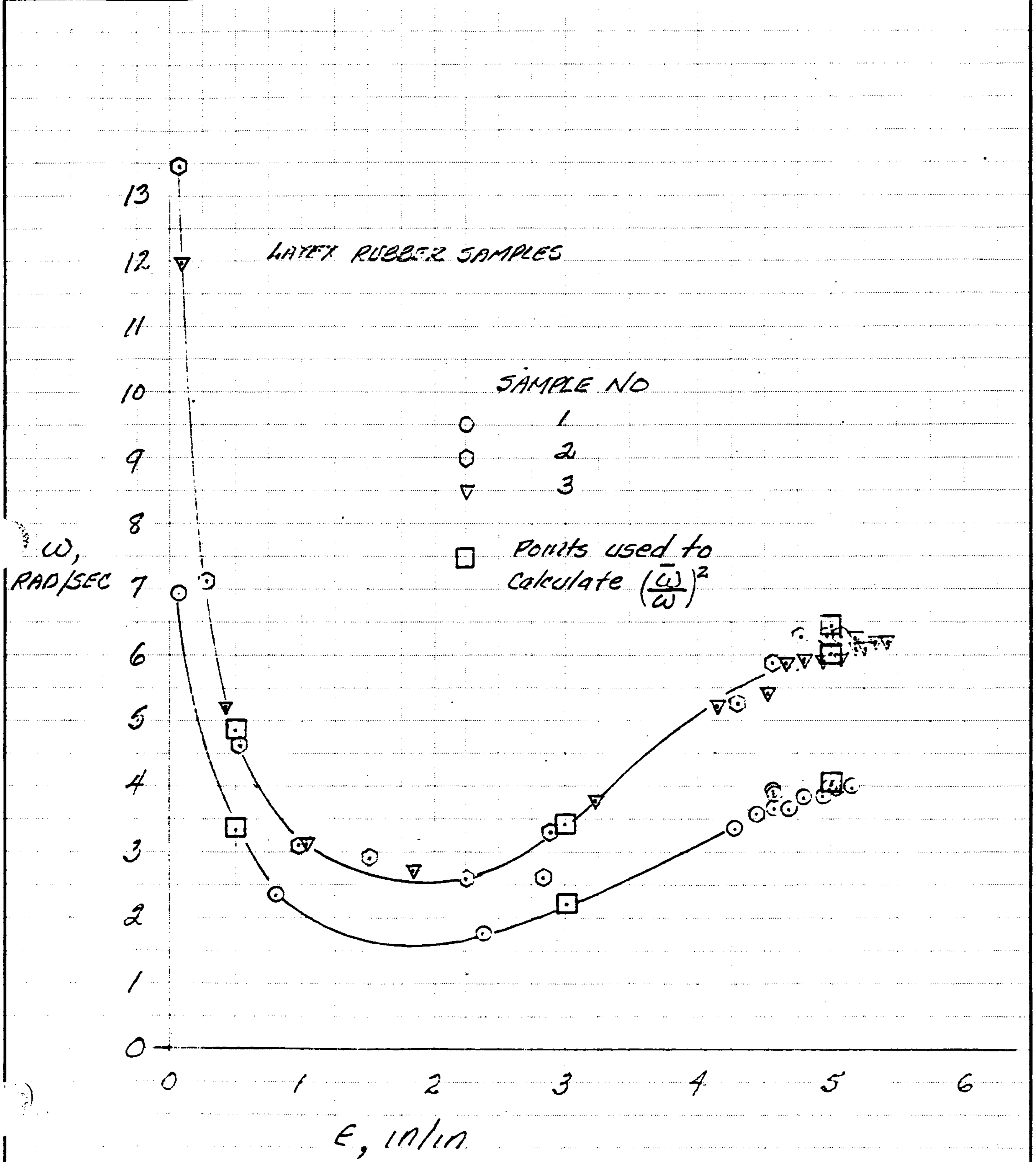

FIGURE I-2.- VARIATION OF MEASURED FREQ. WITH STRAIN

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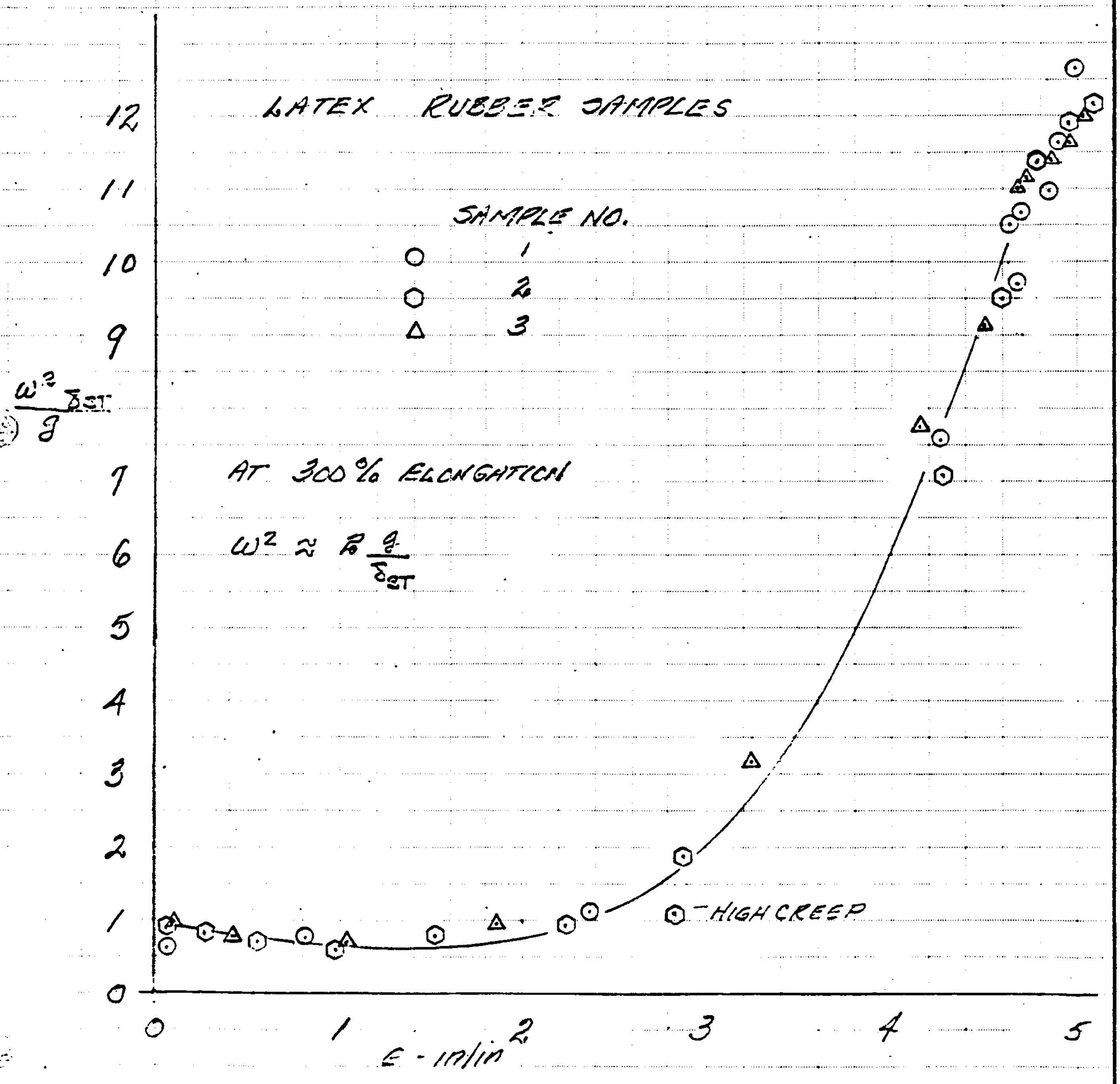

FIGURE I-3.- VARIATION OF NORMALIZED FREQUENCY OF LATEX SAMPLES WITH STRAIN.

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APPROXIMATION OF NATURAL FREQUENCY ON BASIS OF STRESS-STRAIN DATA FOR LATEX RUBBER SAMPLES

FIGURE I-1 SHOWS THE STRESS-STRAIN DATA MEASURED FOR 6 SAMPLES OF RUBBER. THE 3 LATEX SAMPLES, SHOWN BY THE OPEN SYMBOLS, DIFFERED PRIMARILY IN SAMPLE SIZE (LENGTH & CROSS-SECTION AREA) AND THE DATA ARE SHOWN TO BE FAIRLY CONSISTENT.

THE DATA SHOW, AS EXPECTED, THAT THE SLOPE OF THE STRESS-STRAIN CURVE AT ANY STRAIN LEVEL WILL PROVIDE A MEASURE OF THE NATURAL FREQUENCY OF A SUSPENDED MASS AT THAT STRAIN LEVEL. THE QUESTION THEN IS THE CORRELATION BETWEEN THE MEASURED FREQUENCY AND THAT OBTAINED FROM THE SLOPE DATA. THE ANALYSES WHICH FOLLOW PRESENT THE RESULTS AT ELONGATIONS OF 50%, 300% AND 500%.

ON THE BASIS OF SLOPE DATA, IT IS EXPECTED THAT

$$\bar{\omega}^2 \approx \frac{K}{M} = \frac{\frac{\Delta F}{\Delta \delta}}{M} = \frac{\frac{\Delta\sigma A_0}{\Delta\epsilon\, l_0}}{\frac{\sigma A_0}{g}} = \frac{\Delta\sigma}{\Delta\epsilon}\,\frac{g}{\sigma l_0}$$

(NOTE: IF A SYSTEM IS LINEAR, $\frac{\Delta\sigma}{\Delta\epsilon} = \frac{\sigma}{\epsilon} = \frac{\sigma\, l_0}{\delta_{ST}}$

AND $\omega^2 = \frac{g}{\delta_{ST}}$ AS EXPECTED)

IF THE NATURAL FREQUENCY IS NORMALIZED BY THE NATURAL FREQUENCY MEASURED FOR A GIVEN SAMPLE AT A GIVEN LOAD LEVEL, THEN

$$\left(\frac{\bar{\omega}}{\omega}\right)^2 = \frac{\frac{\Delta\sigma}{\Delta\epsilon}\,\frac{g}{\sigma l_0}}{\omega^2}$$

AND, IN TABULAR FORM, THE RESULTS ARE

			SAMPLE 1 $l_0 = 61.5''$		SAMPLE 2 $l_0 = 23.25''$		SAMPLE 3 $l_0 = 26.5''$	
ϵ	σ lb/in²	$\frac{\Delta\sigma}{\Delta\epsilon}$ lb/in²	$\bar{\omega}^2$ (rad/sec)²	$\left(\frac{\bar{\omega}}{\omega}\right)^2$	$\bar{\omega}^2$ (rad/sec)²	$\left(\frac{\bar{\omega}}{\omega}\right)^2$	$\bar{\omega}^2$ (rad/sec)²	$\left(\frac{\bar{\omega}}{\omega}\right)^2$
0.50	65	90	8.69	0.80	23.0	1.00	20.2	0.88
3.00	187.5	34.4	1.15	0.31	3.05	0.26	2.67	0.23
5.00	485	488	6.32	0.40	16.7	0.41	14.7	0.41

WHERE THE EXPERIMENTAL FREQUENCY DATA ARE GIVEN BY FIGURE I-2.

IN SUMMARY, THE RESULTS SHOW:

(1) AT LOW STRAIN LEVELS (ϵ=0.50), THE FREQUENCY OBTAINED FROM SLOPE DATA VARIES BETWEEN 90 & 100 PERCENT OF THE MEASURED FREQUENCIES. AT INTERMEDIATE STRAIN LEVELS (ϵ=3.0), THE MEASURED FREQUENCIES ARE ABOUT TWICE AS HIGH AS PREDICTED FROM SLOPE DATA AND AT HIGH STRAIN LEVELS (ϵ=5.0), THE MEASURED FREQUENCIES ARE ABOUT ONE AND A HALF TIMES AS LARGE AS PREDICTED FROM SLOPE DATA.

(2) MORE DETAILED INFORMATION IS NECESSARY, PARTICULARLY AT THE INTERMEDIATE STRAIN LEVELS AND FOR THE PARTICULAR RUBBER TYPES TO BE USED, TO ACCURATELY PREDICT THE NATURAL FREQUENCIES OF SUPPORTED SYSTEMS. NEEDED DATA MUST INCLUDE EFFECTS OF CREEP.

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(3) AS SHOWN BY FIGURE I-3, THE NATURAL FREQUENCY AT $\epsilon \approx 2.25$ CLOSELY APPROACHES THE VALUE FOR A LINEAR SYSTEM. THUS THE NATURAL FREQUENCY AT $\epsilon \approx 2.25$ WOULD BE ABOUT $\sqrt{2}$ LOWER THAN AT $\epsilon = 3$. HOWEVER, THE WORKING STRESS IS LESS (APPROX. 162.5 psi VS 187.5 psi) AND THE RUBBER DIAMETER IS THUS LARGER TO CARRY A GIVEN WEIGHT. ALSO THE INITIAL RUBBER LENGTH WILL CHANGE. FOR THESE SAMPLES, THE RELATIVE WEIGHTS ARE DETERMINED AS FOLLOWS

$$\frac{W_{2.25}}{W_3} = \frac{\ell_{2.25}}{\ell_3} \frac{A_{2.25}}{A_3}$$

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$$= \frac{(1+\beta)_3}{(1+\beta)_{2.25}} \frac{187.5}{162.5}$$

$$= \left(\frac{1+3}{1+2.25}\right) \frac{187.5}{162.5} = 1.42 \approx \sqrt{2}$$

OR

$$\frac{\omega_{2.25}}{\omega_3} \frac{W_{2.25}}{W_3} \approx 1$$

AND

$$\ell_{E,0} = \frac{\ell(1-\epsilon)}{(1+\beta)} = \frac{.9\ell}{3.25} = 0.277\ell$$

$$\delta_{ST} = \beta \ell_{E_0} = 2.25 \times .277\ \ell = .623\ \ell$$

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DETERMINATION OF WEIGHT OF RUBBER FROM LATEX TEST RESULTS

STRESS AT 300% ELONGATION = 187.5 lb/in²

$$AREA = \frac{FORCE}{\sigma} = \frac{10,000}{187.5} = 53.33$$

$$LENGTH = \ell_{E,0} = 0.225\,\ell = 0.225(120)12 = 324 \text{ in}$$

$$Volume = \ell_{E,0} \times A = 53.33 \times 324 = 17,278 \text{ in}^3$$

$$Weight = V \times 62.4 \times g_{sp} = 17,278 \frac{1}{1728} \times 62.4 \times .95 = 593 \text{ lb}$$

$$EFFECTIVE\ WEIGHT \approx \frac{1}{2}\ WEIGHT$$

$$= 296 \text{ lbs}$$

$$\approx \frac{296}{10,000} = 3 \text{ PERCENT OF MODEL WEIGHT}$$

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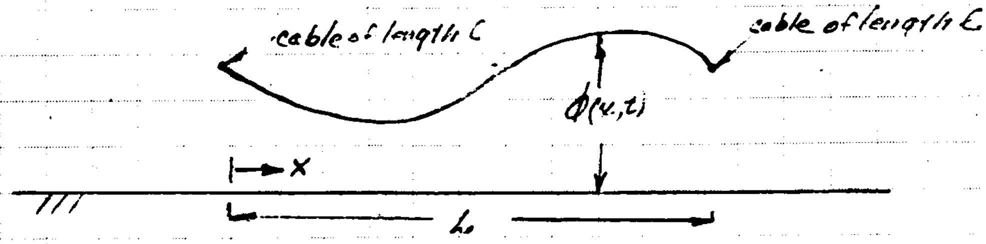
APPENDIX II

ANALYSIS OF A BEAM SUSPENDED BY CABLES AND UNDERGOING COMBINED BENDING AND PENDULAR MOTIONS

ASSUME THAT THE BENDING MOTIONS INCLUDE BOTH SYMMETRIC AND ANTISYMMETRIC MOTIONS AND THAT THE PENDULAR MOTIONS INCLUDE BOTH REGULAR (TRANSLATORY) AND BIFILAR (ROTARY) MOTIONS



THE GOVERNING DIFFERENTIAL EQUATION IS

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2\phi}{\partial x^2}\right) + m\ddot{\phi} = F(x,t)$$

WHERE $F(x,t)$ IS THE COMBINATION OF APPLIED EXTERNAL FORCES INCLUDING THE CABLE RESTRAINTS.

ASSUME THAT

$$\phi(x,t) = \sum_{i=1}^{i=n} \alpha_i(t)\,\phi_i(x) \qquad (1)$$

$$= a(t) + b(t)\frac{x}{L_o} + \sum_{j=1}^{p} c_j(t) f_j(x) + \sum_{k=1}^{q} d_k(t) g_k(x) \qquad (2)$$

WHERE THE $f_j(x)$'s AND $g_k(x)$'s ARE THE NATURAL COUPLED SYMMETRIC AND ANTISYMMETRIC MODES OF THE BEAM, RESPECTIVELY.

SINCE OUR PRIMARY INTEREST IS THE COUPLING BETWEEN THE PENDULAR MODES AND THE LOWER FREQUENCY ELASTIC

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MODES, WE CONSIDER ONLY THE FIRST SYMMETRIC MODE AND THE FIRST ANTISYMMETRIC MODE FOR THE ELASTIC REPRESENTATION OF THE BEAM, i.e.,

$$\phi(x,t) = a(t) + b(t)\frac{x}{L} + c_1(t)\, f_1(x) + d_1(t)\, g_1(x) \tag{3}$$

WHICH WE CAN SIMPLIFY TO

$$\phi = a + b\frac{x}{L} + cf + dg \tag{4}$$

AND

$$c\,(EIf'')'' + d(EIg'')'' + m\left(\ddot{a} + \ddot{b}\frac{x}{L} + \ddot{c}f + \ddot{d}g\right) = F \tag{5}$$

TO TAKE ADVANTAGE OF THE ORTHOGONALITY OF THE NATURAL MODES, WE MAY MULTIPLY EACH EQUATION BY ϕ_i AND INTEGRATE OVER THE LENGTH. THEN

$$c\int_0^L (EIf'')''dx + d\int_0^L (EIg'')''dx + \ddot{a}\int_0^L m\,dx + \ddot{b}\int_0^L m\frac{x}{L}dx$$

$$+\ddot{c}\int_0^L mf\,dx + \ddot{d}\int_0^L mg\,dx = \int_0^L F\,dx \tag{6}$$

$$c\int_0^L (EIf'')''\frac{x}{L}dx + d\int_0^L (EIg'')''\frac{x}{L}dx + \ddot{a}\int_0^L m\frac{x}{L}dx + \ddot{b}\int_0^L m\left(\frac{x}{L}\right)^2 dx$$

$$+\ddot{c}\int_0^L mf\frac{x}{L}dx + \ddot{d}\int_0^L mg\frac{x}{L}dx = \int_0^L F\frac{x}{L}dx \tag{7}$$

$$c\int_0^L (EIf'')''f\,dx + d\int_0^L (EIg'')''f\,dx + \ddot{a}\int_0^L mf\,dx + \ddot{b}\int_0^L m\frac{x}{L}f\,dx$$

$$+\ddot{c}\int_0^L mf^2dx + \ddot{d}\int_0^L mgf\,dx = \int_0^L Ff\,dx \tag{8}$$

$$c\int_0^L (EIf'')''g\,dx + d\int_0^L (EIg'')''g\,dx + \ddot{a}\int_0^L m g\,dx + \ddot{b}\int_0^L m\frac{x}{L}g\,dx$$

$$+\ddot{c}\int_0^L m f g\,dx + \ddot{d}\int_0^L m g^2\,dx = \int_0 F g\,dx \qquad (9)$$

IT NOTED THAT IN GENERAL F MAY BE A DISTRIBUTED FORCE AND THE $\int_0^L F(x,t)\,\phi_i\,dx$ MAY BE EVALUATED. FOR THE CASES OF MOST INTEREST HERE, $F(x,t)$ IS A SERIES OF CONCENTRATED FORCES REPRESENTED BY THE CABLE RESTRAINTS AND EXTERNALLY APPLIED EXCITATION FORCES.

CONSIDERATION OF SUBCASE WHERE BEAM IS NON UNIFORM AND UNDERGOING FREE TRANSLATORY & ROTARY PENDULAR MOTIONS WHILE ATTACHED TO CABLES AT EACH END

FOR THIS CASE, $c = d = 0$ AND THE EQUATIONS REDUCE TO

$$\ddot{a}\int_0^L m\,dx + \ddot{b}\int_0^L m\frac{x}{L}\,dx = \int_0^L F\,dx \qquad (10)$$

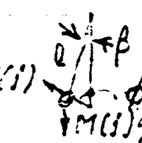
$$\ddot{a}\int_0^L m\frac{x}{L}\,dx + \ddot{b}\int_0^L m\left(\frac{x}{L}\right)^2 dx = \int_0^L F\frac{x}{L}\,dx \qquad (11)$$

EVALUATION OF INTEGRALS

$$\int_0^L m\,dx = M \qquad \int_0^L m\frac{x}{L}\,dx = \frac{x_{c.g}}{L}M \qquad \int_0^L m\left(\frac{x}{L}\right)^2 dx = \frac{x_{c.g.}^2 + r^2}{L^2}M$$

IN GENERAL, EACH CABLE ATTACHED TO THE BEAM WILL SUPPORT A PERCENTAGE OF THE BEAM MASS AND WILL PROVIDE A FORCE TO THE BEAM AS FOLLOWS

$$F(j) = -\frac{M(j)g}{\ell}\,\phi(j)$$

$F(j)$ 

$$\phi(j) = \sum_{i=1}^{n} \alpha_i\,\varphi_i(j) \qquad (12)$$

AT STATION 1 ($x=0$)

$$-F_1 = M_1 \frac{g}{\ell} \phi_{11} = M_1 \frac{g}{\ell}(a \cdot 1 + b \cdot 0) = M_1 \frac{g}{\ell} a \tag{13}$$

$$-F_2 = M_2 \frac{g}{\ell} \phi_{21} = M_2 \frac{g}{\ell}(a \cdot 1 + b \cdot 1) = M_2 \frac{g}{\ell}(a+b) \tag{14}$$

$$-\int_0^L F \phi_1 dx = M_1 \frac{g}{\ell} a + M_2 \frac{g}{\ell} a + M_2 \frac{g}{\ell} b \tag{15}$$

$$-\int_0^L F \phi_2 dx = M_2 \frac{g}{\ell} a + M_2 \frac{g}{\ell} b \tag{16}$$

FOR SIMPLE HARMONIC MOTION, $\ddot{\alpha}_i = -\omega^2 \alpha_i$, AND THE EQUATIONS MAY BE WRITTEN IN MATRIX FORM AS FOLLOWS

$$\begin{bmatrix} -\omega^2 M + M_1 \frac{g}{\ell} + M_2 \frac{g}{\ell} & -\omega^2 M \frac{x_{c.g}}{L} + M_2 \frac{g}{\ell} \\ -\omega^2 \frac{x_{cg}}{L} M + M_2 \frac{g}{\ell} & -\omega^2 M \left(\frac{x_{c.g}^2 + r^2}{L^2} \right) + M_2 \frac{g}{\ell} \end{bmatrix} \begin{Bmatrix} a_0 \\ b_0 \end{Bmatrix} = 0 \tag{17}$$

OR

$$\begin{bmatrix} \omega^2 - \frac{g}{\ell}\left(\frac{M_1 + M_2}{M} \right) & +\omega^2 \frac{x_{cg}}{L} - \frac{g}{\ell} \frac{M_2}{M} \\ \omega^2 \frac{x_{cg}}{L} - \frac{g}{\ell} \frac{M_2}{M} & +\omega^2 \left(\frac{x_{c.g}^2 + r^2}{L^2} \right) - \frac{g}{\ell} \frac{M_2}{M} \end{bmatrix} \begin{Bmatrix} a_0 \\ b_0 \end{Bmatrix} = 0 \tag{18}$$

THE FREQUENCY EQUATION IS GIVEN BY THE VANISHING OF THE DETERMINANT OF (18).

$$\left(\omega^2 - \frac{g}{\ell}\left(\frac{M_1 + M_2}{M} \right) \right) \left(\omega^2 \left(\frac{x_{c.g}^2 + r^2}{L^2} \right) - \frac{g}{\ell} \frac{M_2}{M} \right) - \left(\omega^2 \frac{x_{cg}}{L} - \frac{g}{\ell} \frac{M_2}{M} \right)^2 = 0 \tag{19}$$

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A SUBCASE OF INTEREST IS THAT OF A UNIFORM BAR WHERE $M_1 = M_2 = M/2$, $x_{c.g} = L/2$ & $r^2 = L^2/12$

$$\left(\omega^2 - \frac{g}{\ell}\right)\left(\omega^2 \frac{1}{3} - \frac{g}{3\ell}\right) - \frac{1}{4}\left(\omega^2 - \frac{g}{\ell}\right)^2 = 0 \qquad (20)$$

OR

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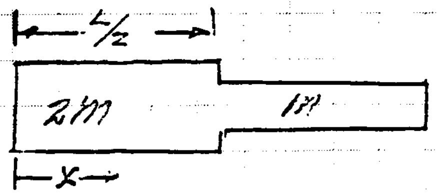
$$\left(\omega^2 - \frac{g}{\ell}\right)\left(\omega^2 - 3\frac{g}{\ell}\right) = 0 \qquad (21)$$

EQUATION (21) GIVES THE NATURAL FREQUENCIES OF THE TWO PENDULUM MODES WHICH ARE SHOWN TO BE UNCOUPLED FOR A UNIFORM BEAM.

OF MORE GENERAL INTEREST IS THE CASE OF A NONUNIFORM BEAM. AS AN EXAMPLE, CONSIDER THE CASE WHERE THE BEAM CONSISTS OF TWO SECTIONS OF EQUAL LENGTH FOR WHICH THE FIRST SECTION IS TWICE AS HEAVY AS THE SECOND SECTION.



THEN

$$\left.\begin{aligned}
M &= \frac{3}{2} mL \\
M_1 &= m\frac{L}{2} + m\frac{3L}{8} = m\frac{7}{8}L = \frac{7}{12}M \\
M_2 &= m\frac{L}{4} + m\frac{L}{8} = m\frac{5}{8}L = \frac{5}{12}M \\
x_{c.g} &= \frac{5}{12}L \\
r^2 + x_{c.g}^2 &= \left(\frac{1}{3} m\frac{L}{2}\left(\frac{L}{2}\right)^2 + \frac{1}{3} mL\, L^2\right)\frac{1}{M} = \frac{3/2\, mL}{M}\frac{2}{9}\left(\frac{L^2}{8} + L^2\right) \\
&= \frac{1}{4}L^2
\end{aligned}\right\} \qquad 22$$

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SUBSTITUTION OF Eq.(22) INTO Eq.(19) YIELDS

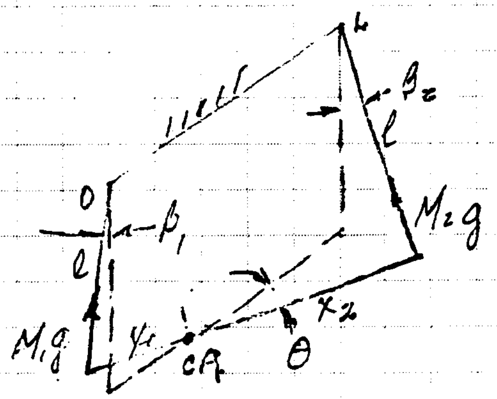
$$\left(\omega^2-\frac{g}{l}\right)\left(\frac{\omega^2}{4}-\frac{5}{12}\frac{g}{l}\right)-\left(\frac{5}{12}\right)^2\left(\omega^2-\frac{g}{l}\right)^2=0 \qquad (23)$$

OR

$$\left(\omega^2-\frac{g}{l}\right)\left(\omega^2-\frac{35}{11}\frac{g}{l}\right)=0 \qquad (24)$$

AND AGAIN WE SEE THAT THE MOTIONS ARE UNCOUPLED

SINCE THE MOTIONS ARE UNCOUPLED, WE SHOULD BE ABLE TO DERIVE THE BIFILAR PENDULUM FREQUENCY DIRECTLY FROM CONSIDERATION OF ROTATIONS ABOUT THE CENTER OF MASS AND BY THE USE OF ENERGY PRINCIPLES AS FOLLOWS.



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$$U=M_1 g\,\frac{l}{2}\phi_1^2+M_2 g\,\frac{l}{2}\phi_2^2 \qquad (25)$$

$$T=\frac{1}{2}I_{c.g}\dot{\theta}^2=\frac{1}{2}Mr^2\dot{\theta}^2 \qquad (26)$$

SINCE $\phi_1 l=\theta X_1$ & $\phi_2 l=\theta X_2$

$$U=\frac{g}{l}\frac{\theta^2}{2}\left(M_1X_1^2+M_2X_2^2\right) \qquad (27)$$

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SINCE

$$M x_1 = M_2 (x_1 + x_2) \; \& \; M x_2 = M_1 (x_1 + x_2)$$

WE CAN MULTIPLY Mx_1 BY x_2 AND Mx_2 BY x_1 AND ADD THEM TOGETHER TO OBTAIN

$$M_1 x_1^2 + M_2 x_2^2 + M_2 x_1 x_2 + M_1 x_1 x_2 = 2 M x_1 x_2 \tag{28}$$

BUT

$M = M_1 + M_2$ AND WE OBTAIN

$$M_1 x_1^2 + M_2 x_2^2 = M x_1 x_2 \tag{29}$$

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WHICH YIELDS

$$U = \frac{g}{\ell} \frac{\theta^2}{2} M x_1 x_2 \tag{30}$$

ASSUMING SIMPLE HARMONIC MOTION AND EQUATING THE MAXIMUM KINETIC & POTENTIAL ENERGIES, WE OBTAIN

$$\omega^2 - \frac{g}{\ell} \frac{x_1 x_2}{r^2} = 0 \tag{31}$$

FROM EQ. (22) WE FIND $x_1 = x_{c.g} = \frac{5}{12} L$, $x_2 = \frac{7}{12} L$ AND $x_1 x_2 = \frac{35}{144} L$.

ALSO, FROM EQ. (22), $r^2 = -x_{c.g}^2 + \frac{L^2}{4} = -x_1^2 + \frac{L^2}{4} = -\frac{25}{144} L^2 + \frac{L^2}{4} = \frac{11}{144} L^2$

AND EQ. (31) YIELDS

$$\omega^2 - \frac{35}{11} \frac{g}{\ell} = 0 \tag{32}$$

THUS ω^2 DERIVED FROM THE BIFILAR PENDULUM ANALYSIS IS IDENTICAL TO THE CORRESPONDING ROOT FROM EQ. (24). HENCE THE PENDULAR MOTIONS DO NOT COUPLE AND THE BIFILAR FREQUENCY INCREASES AS THE BEAM BECOMES HEAVIER AT ONE END.

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SOLUTIONS OF THE GENERAL CASE FOR COMBINED ELASTIC AND PENDULAR MOTIONS

THE MORE GENERAL CASE OF COMBINED ELASTIC AND PENDULAR MOTIONS IS OF INTEREST BECAUSE WE NEED TO KNOW THE EXTENT TO WHICH COUPLING OCCURS

CASE 1. COMBINATION OF SYMMETRIC ELASTIC MODE AND TRANSLATORY PENDULAR MOTIONS

THIS CASE IS GIVEN BY EQS. (6) AND (8) WITH b & $d = 0$, i.e.

$$\phi = a + cf$$

and

$$c\int_0^L (EIf'')'' dx + \ddot{a}\int_0^L m\, dx + \ddot{c}\int_0^L m f\, dx = \int_0^L F\, dx \qquad (33)$$

$$c\int_0^L (EIf'')'' f\, dx + \ddot{a}\int_0^L m f\, dx + \ddot{c}\int_0^L m f^2\, dx = \int_0^L F f\, dx \qquad (34)$$

EVALUATION OF INTEGRALS FOR SIMPLE HARMONIC MOTION

$$\int_0^L (EIf'')'' dx = 0$$

$$\int_0^L m\, dx = M \qquad \int_0^L m f\, dx = 0$$

$$\int_0^L (EIf'')'' f\, dx = \omega_1^2 \int_0^L m f^2\, dx = \omega_1^2 M_f \qquad (35)$$

$$\int_0^L F\, dx = F(1) + F(2) = -M(1)\, g\, \frac{\phi(1)}{\ell} - M(2)\, g\, \frac{\phi(2)}{\ell}$$

$$\int_0^L F f\, dx = F(1) f(1) + F(2) f(2) = -M(1)\, g\, \frac{\phi(1)}{\ell} f(1) - M(2)\, g\, \frac{\phi(2)}{\ell} f(2)$$

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ASSUME THAT THE BEAM SUPPORTS ARE LOCATED AT $X=0$ & $X=L$, AND THAT $f(0) = f(L) = 1$. THEN FOR A UNIFORM BEAM $M(1) = M(2) = M/2$ AND

$$\phi(0) = \phi_1(0) + \phi_2(0) = a+c$$
$$\phi(L) = \phi_1(L) + \phi_2(L) = a+c \qquad (36)$$

THEN

$$\int_0^L F\,dx = -\frac{M}{2}\frac{g}{\ell}(a+c) - \frac{M}{2}\frac{g}{\ell}(a+c) = -M\frac{g}{\ell}(a+c)$$
$$\int_0^L Ff\,dx = -\frac{M}{2}\frac{g}{\ell}(a+c) - \frac{M}{2}\frac{g}{\ell}(a+c) = -M\frac{g}{\ell}(a+c) \qquad (37)$$

THE EQUATIONS OF MOTIONS THEN REDUCE TO

$$\begin{bmatrix} -\omega^2 M + M\frac{g}{\ell} & +M\frac{g}{\ell} \\ +M\frac{g}{\ell} & -\omega^2 M_f + \omega_1^2 M_f \quad M\frac{g}{\ell} \end{bmatrix} \begin{Bmatrix} a_0 \\ c_0 \end{Bmatrix} = 0 \qquad (38)$$

AND THE DETERMINANT, AFTER DIVIDING EACH EQUATION BY M, IS

$$\left(-\omega^2 + \frac{g}{\ell}\right)\left(-\omega^2\frac{M_f}{M} + \omega_1^2\frac{M_f}{M} + \frac{g}{\ell}\right) - \left(\frac{g}{\ell}\right)^2 = 0 \qquad (39)$$

OR

$$\omega^4 - \left(\frac{g}{\ell} + \omega_1^2 + \frac{M}{M_f}\frac{g}{\ell}\right)\omega^2 + \frac{g}{\ell}\omega_1^2 = 0 \qquad (40)$$

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CHECK BY USE OF ENERGY METHODS AND LAGRANGES EQS.

$$U = \frac{1}{2}\int_0^L EI(\phi'')^2 dx + M(1)\frac{g}{\ell}\frac{\phi_{(1)}^2}{2} + M(2)\frac{g}{\ell}\frac{\phi_{(2)}^2}{2} \quad (41)$$

$$T = \frac{1}{2}\int_0^L m\dot{\phi}^2 dx \quad (42)$$

BUT, FOR OUR CONDITIONS, $M(1) = M(2) = M/2$, and.

$$\phi = a + cf \quad = (a_0 + c_0 f) R_e e^{i\omega t}$$

$$\phi' = cf', \ \phi'' = cf'' \quad (43)$$

$$\ddot{\phi} = -\omega^2(\ddot{a}_0 + \ddot{c}_0 f) R_e e^{i\omega t}$$

LAGRANGES EQUATIONS WHICH APPLY ARE

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{a}}\right) + \frac{\partial U}{\partial a} = 0 \ ; \ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{c}}\right) + \frac{\partial U}{\partial c} = 0$$

THE RESULTING EQS. ARE

$$U = \frac{1}{2}\int_0^L EI\, cf'')^2 dx + \frac{M}{2}\frac{g}{\ell}(a+c)^2 \quad (44)$$

$$T = \frac{1}{2}\int_0^L m(\dot{a} + \dot{c}f)^2 dx \quad (45)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{a}}\right) = \frac{d}{dt}\left(\dot{a}\int_0^L m\,dx + \dot{c}\int_0^L fm\,dx\right) = \ddot{a}\int_0^L m\,dx + \ddot{c}\int_0^L fm\,dx$$

$$= -\omega^2 a M \quad (46)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{c}}\right) = \frac{d}{dt}\left(\dot{a}\int_0^L mf\,dx + \dot{c}\int_0^L mf^2 dx\right) = \ddot{a}\int_0^L mf\,dx + \ddot{c}\int_0^L mf^2 dx$$

$$= -\omega^2 c M_f \quad (47)$$

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$$\frac{\partial U}{\partial a} = M\frac{g}{\ell}(a+c) \tag{48}$$

$$\frac{\partial U}{\partial c} = M\frac{g}{\ell}(a+c) + c\int_0^L EI(f'')^2 dx$$

$$= M\frac{g}{\ell}(a+c) + c\,\omega_1^2 M_f \tag{49}$$

AND, UPON COMBINATION OF TERMS AND DIVIDING BY M

$$\begin{bmatrix} -\omega^2 + \frac{g}{\ell} & \frac{g}{\ell} \\ \frac{g}{\ell} & -\omega^2\frac{M_f}{M} + \omega_1^2\frac{M_f}{M} + \frac{g}{\ell} \end{bmatrix} \begin{Bmatrix} a_0 \\ c_0 \end{Bmatrix} = 0 \tag{50}$$

WHICH IS THE RESULT SHOWN IN EQ. 38

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COMPUTATION OF REPRESENTATIVE FREQUENCIES

LET: $\gamma = \frac{M}{M_f}$; $\frac{g}{L} = \Omega^2$; $\omega_1 = \alpha\Omega$; $\omega = \epsilon\Omega$

THEN EQ. (40) REDUCES TO

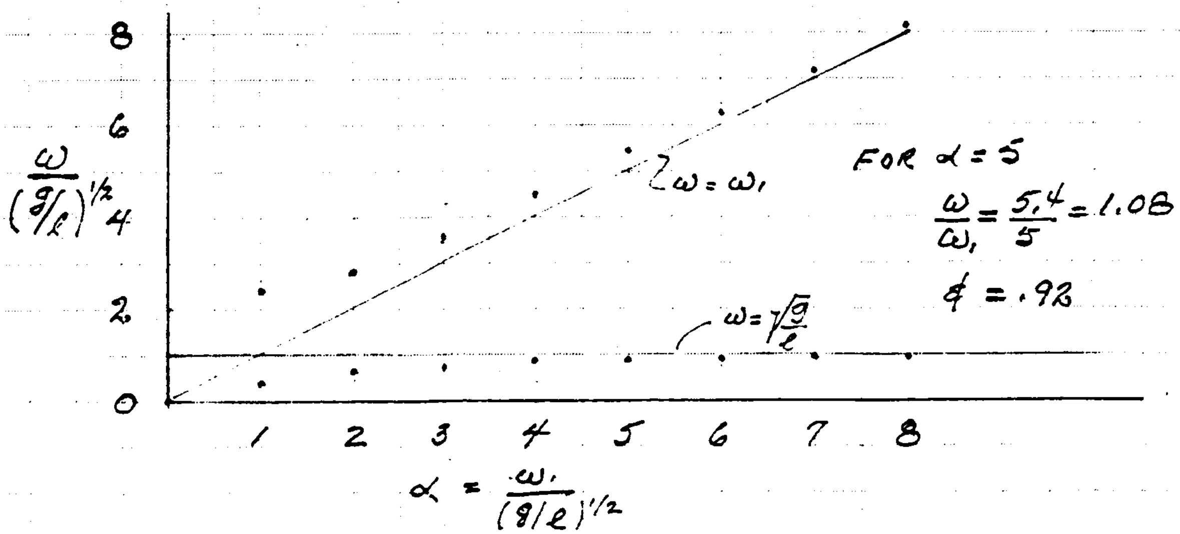
$$\epsilon^4 - (1+\gamma+\alpha^2)\epsilon^2 + \alpha^2 = 0 \qquad (51)$$

IF WE ASSUME $\gamma = 4$

$$\epsilon^2 = \frac{5+\alpha^2}{2} \pm \frac{1}{2}\sqrt{25+6\alpha^2+\alpha^4} \qquad (52)$$

AND THE FOLLOWING TABLE SHOWS THE FREQUENCY CALCULATIONS

α	α^2	$\frac{5+\alpha^2}{2}$	$\frac{1}{2}\sqrt{25+6\alpha^2+\alpha^4}$	ϵ_1^2	ϵ_2^2	ϵ_1	ϵ_2
1	1	3	2.83	5.83	0.17	2.41	.41
2	4	4.5	4.03	8.53	0.47	2.92	.68
3	9	7	6.32	13.32	0.68	3.65	.82
4	16	10.5	9.71	20.21	0.79	4.50	.89
5	25	15	14.14	29.14	0.84	5.40	.92
6	36	20.5	19.60	40.10	0.90	6.33	.95
7	49	27	26.07	53.07	0.93	7.28	.96
8	64	34.5	33.56	68.06	0.94	8.25	.97



FOR $\alpha = 5$

$\frac{\omega}{\omega_1} = \frac{5.4}{5} = 1.08$

$\epsilon = .92$

THE GENERAL CASE, EQS. (6) THRU (9), WILL NOW BE CONSIDERED

THE INTEGRALS

$$\int_0^L F\phi_i(x)\,dx = \sum_{j=1}^{n} F_j \phi_i(j) \tag{53}$$

WHERE

$$F_j = -M(j)\frac{g}{L}\phi(j) \quad \text{AS SHOWN IN EQ. (12)}$$

$$= -M(j)\frac{g}{L}\left(a + b\frac{x(j)}{L} + c f(j) + d g(j)\right) \tag{54}$$

CAN BE EVALUATED FOR ANY NUMBER OF CABLES n

THEN

$$\int_0^L F\phi_i(x)\,dx = -\frac{g}{L}\sum_{j=1}^{n} M(j)\Big(a\phi_i(j) + b\frac{x(j)}{L}\phi_i(j) + c f(j)\phi_i(j) + d g(j)\phi_i(j)\Big) \tag{55}$$

IF WE AGAIN RESTRICT OUR SELVES TO TWO CABLES, ONE AT EACH END OF A UNIFORM BEAM

$$\int_0^L F\phi_i(x)\,dx = -\frac{M}{2}\frac{g}{L}\Big(a\phi_i(0) + 0 + c f(0)\phi_i(0) + d g(0)\phi_i(0)$$

$$+ a\phi_i(L) + b\,\phi_i(L) + c f(L)\phi_i(L)$$

$$+ d g(L)\phi_i(L)\Big) \tag{56}$$

BUT $\phi_1(x) = 1$; $\phi_2(x) = \frac{x}{L}$, $\phi_3(x) = f$, $\phi_4(x) = g$

and $\phi_1(0) = 1$, $\phi_1(L) = 1$; $\phi_2(0) = 0$, $\phi_2(L) = 1$; $\phi_3(0) = \phi_3(L) = 1$; $\phi_4(0) = 1$, $\phi_4(L) = -1$

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THEN

$$\int_0^L F\phi_1(x)\,dx = -\frac{M}{2}\frac{g}{l}\left(a+c+d+a+b+c-d\right) \qquad (57)$$

$$= -M\frac{g}{l}\left(a+\frac{b}{2}+c\right) \qquad (57)$$

$$\int_0^L F\phi_2(x)\,dx = -\frac{M}{2}\frac{g}{l}\left(0+0+0+0+a+b+c-d\right)$$

$$= -\frac{M}{2}\frac{g}{l}\left(a+b+c-d\right) \qquad (58)$$

$$\int_0^L F\phi_3(x)\,dx = -\frac{M}{2}\frac{g}{l}\left(a+0+c+d+a+b+c-d\right)$$

$$= -\frac{M}{2}\frac{g}{l}\left(2a+b+2c\right) \qquad (59)$$

$$\int_0^L F\phi_4(x)\,dx = -\frac{M}{2}\frac{g}{l}\left(a+c+d-a-b-c+d\right)$$

$$= -M\frac{g}{l}\left(d-\frac{b}{2}\right) \qquad (60)$$

AND THE DIFFERENTIAL EQs (6) THRU (9) ARE

$$\begin{bmatrix} -\omega^2+\frac{g}{l} & -\omega^2\frac{x_{c.g.}}{L}+\frac{g}{2L} & \frac{g}{l} & 0 \\ -\omega^2\frac{x_{c.g.}}{L}+\frac{g}{2l} & -\omega^2\frac{1}{3}+\frac{g}{2L} & +\frac{g}{2l} & -\frac{g}{2l} \\ \frac{g}{l} & \frac{g}{2l} & \omega_1^2\frac{M_f}{M}-\omega^2\frac{M_f}{M}+\frac{g}{l} & 0 \\ 0 & -\frac{g}{2l} & 0 & \omega_2^2\frac{M_g}{M}-\omega^2\frac{M_g}{M}+\frac{g}{l} \end{bmatrix} \begin{Bmatrix} a_0 \\ b_0 \\ c_0 \\ d_0 \end{Bmatrix} = 0 \qquad (61)$$

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IT IS NOTED THAT IN THE DEVELOPMENT OF THESE EQUATIONS SEVERAL INTEGRALS VANISH. FOR A FREE-FREE BEAM

$$\int_0^L m f \, dx = 0$$

$$\int_0^L (EI f'')'' dx = K_1 \int_0^L m f \, dx = 0 \qquad (62)$$

$$\int_0^L m g \, dx = 0$$

$$\int_0^L (EI g'')'' dx = K_2 \int_0^L m g \, dx = 0$$

BECAUSE THE CENTER OF MASS OF THE BEAM DOES NOT MOVE,

OTHER INTEGRALS OF INTEREST ARE

$$\int_0^L m f \frac{x}{L} dx = \int_0^L m f \left(\frac{x}{L} - \frac{1}{2}\right) dx + \frac{1}{2} \int_0^L m f \, dx = 0$$

$$\int_0^L (EI f'')'' \frac{x}{L} dx = \int_0^L (EI f'')'' \left(\frac{x}{L} - \frac{1}{2}\right) dx + \frac{1}{2} \int_0^L (EI f'')'' dx = 0 \qquad (63)$$

$$\int_0^L m g \frac{x}{L} dx = \int_0^L m g \left(\frac{x}{L} - \frac{1}{2}\right) dx + \frac{1}{2} \int_0^L m g \, dx = 0$$

$$\int_0^L (EI g'')'' \frac{x}{L} dx = \int_0^L (EI g'')'' \left(\frac{x}{L} - \frac{1}{2}\right) dx + \frac{1}{2} \int_0^L (EI g'')'' dx = 0$$

NOTE THAT IN THE EXPRESSIONS ABOVE, THE FIRST INTEGRALS ON THE RIGHT HAND SIDE VANISH IDENTICALLY AND THE SECOND INTEGRALS VANISH BECAUSE THE CENTER OF MASS DOES NOT MOVE. IF THE BEAM IS NOT UNIFORM, THE SAME SITUATION PREVAILS EXCEPT THE MULTIPLIER IS $\left(\frac{x}{L} - Q\right)$ WHERE Q IS A CONSTANT NOT NECESSARILY EQUAL TO 1/2.