

NON-LINEAR IDENTIFICATION OF A SQUEEZE-FILM DAMPER

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In this paper the authors describe an experimental study to identify the damping laws associated with a squeeze-film vibration damper. This is achieved by using a non-linear filtering algorithm to process displacement responses of the damper ring to synchronous excitation and thus to estimate the parameters in an n^{th} -power velocity model. The experimental facility is described in detail and a representative selection of results is included. The identified models are validated through the prediction of damper-ring orbits and comparison with observed responses.

INTRODUCTION

A fundamental problem in rotor-bearing dynamics is the experimental determination of mathematical models to represent the dynamics of oil-film bearings, sometimes referred to as the problem of bearing identification. One category of oil-film bearing is the squeeze-film damper which often forms part of an isolation system for vibration control in turbomachinery. Damping is provided by lubricant supplied to an annulus between the bearing housing and damper-ring. The damper ring does not rotate but is free to whirl in response to applied excitation: thus the lubricant in the annulus is able to dissipate vibrational energy.

The simplest linear analysis of the squeeze-film dynamics indicates that a model involving two viscous damping coefficients can be used to characterise the behaviour of the film (ref. 1). It is generally accepted that such a model can account for the damper ring's response to small perturbations around the concentric position. However the comprehensive tests reported by Tonnesen (ref. 2) show that larger excursions about eccentric positions cannot be predicted using a linear model.

In the last five years considerable progress has been made in the development of techniques for bearing identification. Most attention has been given to frequency-domain methods which have been used to estimate direct- and cross-damping terms associated with a model squeeze-film isolator (ref. 3) and can readily be extended to identify models of multi-mode rotor-bearing systems (ref. 4). However, frequency-domain algorithms are based upon a prior assumption of linearity and thus significant non-linearities cannot readily be accommodated. At Liverpool the identification of linearised squeeze-film models has, in general, been approached using time-domain techniques. A series of numerical experiments indicated that a least-squares filtering algorithm is particularly suitable for estimating the four squeeze-film damping terms from displacement responses to synchronous excitation (ref. 5). A model squeeze-film isolator was constructed and a comprehensive survey of the linearised dynamics has now been completed (ref. 6). A well-known advantage

of the time-domain approach is that certain geometrical non-linearities can be accommodated and, following the success of the linear experiments, work began to identify non-linear models of the squeeze-film dynamics.

In this paper the authors describe the first series of non-linear experiments. The objective of these tests was to identify the damping law of the squeeze-film from records of large amplitude displacements to single-frequency excitation. This was achieved by estimating the parameters in n^{th} -power velocity models of the squeeze-film dynamics. The resulting models were validated by using them to predict the damper-ring's orbits and comparing these with directly observed responses. The significance of the results is discussed in some detail.

SYMBOLS

c_{xx} , etc.	squeeze-film damping coefficients (N.s/m)
c_{nxx} , etc.	coefficients in damping law
f_x, f_y	applied forces in x and y directions (N)
F_x, F_y	squeeze-film forces (N)
$\underline{f}(x, t)$	vector functions associated
$\underline{h}(x, t)$	with parameter estimation
$2k_s$	total stiffness of damper-ring retaining spring (N/m)
$2m$	total mass of damper ring (kg)
n_{xx} , etc.	exponents in damping law
$\underline{P}(T)$	error covariance matrix
\underline{Q}	weighting matrix
t	time (s)
T	interval of observations (s)
u_x, u_y	$= f_x/m, f_y/m$
\underline{x}	state vector
x_1-x_4	physical state variables
x_5-x_{10}	parameter state variables
x	vertical displacement of damper ring (m)
y	horizontal displacement of damper ring (m)
$\underline{z}(t)$	vector of observations

$$\omega_n^2 = k_s/m$$

()^T denotes matrix transpose

(^) denotes estimate

IDENTIFICATION OF LINEARISED SQUEEZE-FILM DYNAMICS

Before describing the non-linear experiments, the more traditional linear problem will be defined and the results of some recent linear identification experiments will be summarised.

Equations of Motion

Consider a lumped parameter model of a squeeze-film vibration isolator such as that considered by Holmes (ref. 1). In fixed co-ordinates the equations of motion are written

$$\begin{aligned} m \frac{d^2 x}{dt^2} + F_x(x, \frac{dx}{dt}, y, \frac{dy}{dt}) + k_s x &= f_x \\ m \frac{d^2 y}{dt^2} + F_y(x, \frac{dx}{dt}, y, \frac{dy}{dt}) + k_s y &= f_y \end{aligned} \quad (1)$$

where all terms are defined under "Symbols".

Equation (1) implies that the forces F_x and F_y developed within the squeeze-film are functions of both displacement and velocity in the x and y directions. The objective of bearing identification is to determine these functions experimentally. Experimental determination of the functions F_x and F_y usually involves two steps: the choice of a suitable model structure followed by the estimation of unknown parameters in this structure. At the simplest level it is usual to assume that the squeeze-film damping forces in the x and y directions are proportional to the component of (damper-ring) velocity in the respective direction. This leads to the expressions

$$F_x = c_{xx} \frac{dx}{dt}, \quad F_y = c_{yy} \frac{dy}{dt} \quad (2)$$

so that equations (1) are uncoupled and identification requires the determination of the two constants of proportionality, i.e. the viscous damping coefficients c_{xx} and c_{yy} . This type of model was used in the experimental parametric study described by Tonnesen (ref. 2). In any actual squeeze-film isolator, imperfections in the construction of the damper-ring and bearing housing will invariably produce cross-axis damping forces in the squeeze-film. If the cross-axis forces are significant in relation to the direct-axis forces then the identification of coefficients in an uncoupled model (equation (2)) will lead to the misinterpretation of the results. Such misinterpretation can be avoided by assuming that coupling is present, i.e.

$$F_x = c_{xx} \frac{dx}{dt} + c_{xy} \frac{dy}{dt}, \quad F_y = c_{yx} \frac{dx}{dt} + c_{yy} \frac{dy}{dt} \quad (3)$$

and estimating the four unknown damping coefficients c_{xx} , c_{xy} , c_{yx} and c_{yy} . Some recent work by the authors has resulted in a promising new experimental technique for determining these four damping coefficients.

Numerical Experiments

In reference 5, one of the authors proposed a scheme of combined state and parameter estimation for identifying the four squeeze-film damping terms. Essentially, the four unknown coefficients were defined as state variables and an algorithm for non-linear state estimation was used to reconstruct the coefficients from time-series records of the displacement responses of the damper ring to synchronous excitation. A series of numerical experiments showed that such a scheme was feasible and moreover that the estimation algorithm was relatively insensitive to the effects of (zero-mean) measurement noise. Further numerical studies showed that the scheme could readily be extended to estimate the four damping coefficients associated with a journal bearing oil-film (ref. 7).

Experiments with a Model Squeeze-Film Isolator

Following the success of the numerical experiments a model squeeze-film isolator was constructed so that the technique in reference 5 could be applied to real data. The experimental facility will be described in detail in the sections which follow. A comprehensive survey of the dynamics of the squeeze-film damper (ref. 6) showed that the four squeeze-film damping terms could be readily identified using non-linear state estimator. As expected, there were considerable discrepancies between the identified coefficients and those predicted by short-bearing lubrication theory (ref. 1). However, by comparing the ability of both identified and theoretical coefficients to predict the amplitude and phase characteristics of the isolator's frequency response, it was shown that the identified coefficients were the more effective, especially for characterising the cross-axis dynamics.

The application of a non-linear technique to estimate linearised coefficients may appear to be a computationally inefficient way of solving an apparently simple problem. However the advantage of this approach is that it can, in principle, be extended to accommodate certain types of non-linearity. The motivation behind the body of work described in this paper is to determine if non-linear models of the squeeze-film dynamics could be identified without imposing unrealistic requirements on the experimental facility.

NON-LINEAR IDENTIFICATION

Introduction

The theory underlying the linear identification experiments described in the previous section is based upon the assumption that the damping forces in equations (2) and (3) arise from small perturbations of the damper ring. Even when this assumption seems justified there can be large discrepancies between theoretically- and experimentally-derived coefficients, especially at higher values of static eccentricity ratio (ref. 2). To investigate these discrepancies, and to be able to account for squeeze-film behaviour under large perturbations of the damper-ring, it was decided to attempt to identify the damping law of the squeeze-film. This

implies the estimation of the parameters (coefficients and exponents) associated with an n^{th} -power velocity model of the squeeze-film.

A Damping-Law Model

As part of a related study it has been established that the damping law associated with a single degree-of-freedom dissipative element can be identified from forced response measurements (ref. 8). Owing to the significant amount of cross-axis coupling it was not considered feasible to employ so simple a model of the squeeze-film. Consequently a tentative model to include cross-axis effects was proposed, i.e.

$$\begin{aligned} F_x &= c_{nxx} \left| \frac{dx}{dt} \right|^{n_{xx}} \text{sgn} \left(\frac{dx}{dt} \right) + c_{nxy} \left| \frac{dy}{dt} \right|^{n_{xy}} \text{sgn} \left(\frac{dy}{dt} \right) \\ F_y &= c_{nyx} \left| \frac{dx}{dt} \right|^{n_{yx}} \text{sgn} \left(\frac{dy}{dt} \right) + c_{nyy} \left| \frac{dy}{dt} \right|^{n_{yy}} \text{sgn} \left(\frac{dy}{dt} \right) \end{aligned} \quad (4)$$

which assumes that the damping forces are proportional to the n^{th} -power of the appropriate components of damper-ring velocity. Identification of such a model requires the estimation of eight parameters, four coefficients c_{nxx} , etc and four exponents n_{xx} , etc. from records of the damper-ring's displacement response. To reduce the number of parameters to be estimated in these preliminary experiments it was decided to assume that the cross-damping terms were reciprocal, i.e. $c_{nxy} = c_{nyx}$, $n_{xy} = n_{yx}$. This had the effect of reducing the number of parameters to be estimated to six. The consequences of making this simplification will be discussed later.

Combined State and Parameter Estimation

To begin the development of the necessary estimation equations, consider the substitution of the functions F_x and F_y from the damping-law model, equation (4), into the equations of motion (1). The four "physical" state variables usually associated with a vibrating system with two degrees of freedom are $x_1 \triangleq x$, $x_2 \triangleq dx/dt$, $x_3 \triangleq y$ and $x_4 \triangleq dy/dt$. Substituting these expressions into equations (1) and (4) and noting the $dx_1/dt = x_2$ and $dx_3/dt = x_4$, results in a set of four non-linear differential equations. A further six "parameter" state variables, corresponding to the six unknowns in equation (4), are defined, i.e. $x_5 \triangleq c_{nxx}/m$, $x_6 \triangleq n_{xx}$, $x_7 \triangleq c_{nxy}/m$, $x_8 \triangleq n_{xy}$, $x_9 \triangleq c_{nyy}/m$ and $x_{10} \triangleq n_{yy}$. If it is assumed that the time-derivatives of these six extra states are zero (to characterise time-invariant parameters) then six further equations emerge to augment the four original state equations. The ten equations can be collected together and written in the form:

$$\frac{dx}{dt} = \underline{f}(\underline{x}, t) \quad (5)$$

where

$$\underline{x} = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8 \ x_9 \ x_{10}]^T$$

and

$$\underline{f}(\underline{x}, t) = \begin{bmatrix} x_2 \\ -x_5 |x_2|^{x_6} \operatorname{sgn}(x_2) - x_7 |x_4|^{x_8} \operatorname{sgn}(x_4) - \omega_n^2 x_1 + u_x \\ x_4 \\ -x_7 |x_2|^{x_8} \operatorname{sgn}(x_2) - x_9 |x_4|^{x_{10}} \operatorname{sgn}(x_4) - \omega_n^2 x_3 + u_y \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

To characterise the measurements on the physical system, a sensor equation is introduced:

$$\underline{z}(t) = \underline{h}(\underline{x}, t) + (\text{observation noise}) \quad (6)$$

where $\underline{h}(\underline{x}, t)$ is a vector function which, if necessary, can account for observations which are non-linear functions of the states. For the problem in hand, however, assume that the two displacement states x_1 and x_3 are available directly and thus $\underline{h}(\underline{x}, t) = [x_1 \ x_3]^T$.

Given this formulation, the objective is to employ the information contained in the vector of observations $\underline{z}(t)$ over the time interval $0 \leq t \leq T$ to predict the behaviour of the state vector \underline{x} over the same interval. Since \underline{x} contains the six unknown parameters associated with the damping law of the squeeze-film, the estimation of \underline{x} automatically produces estimates of the unknown parameters. A useful by-product of this parameter estimation scheme is that the estimates \hat{x}_1 and \hat{x}_3 of the displacement states provide instant prediction of the orbital motion of the damper-ring on the basis of the identified model and thus serve to validate the estimated parameters.

An Algorithm for Non-Linear State Estimation

To estimate the state vector \underline{x} from the vector of observations \underline{z} requires an algorithm for non-linear state estimation. The application of one suitable algorithm is described in detail in reference (ref. 5). For the sake of completeness, the relevant equations are summarised here. The algorithm (ref. 9) is based upon a predictor-corrector type equation:

$$\frac{d\hat{\underline{x}}}{dT} = \underline{f}(\hat{\underline{x}}, t) + \underline{\Gamma}(T) \{ \underline{z}(T) - \underline{h}(\hat{\underline{x}}, T) \} \quad (7)$$

the solution of which produces an estimate, denoted $\hat{\underline{x}}$, of the state vector. The driving term in equation (7) is the vector of residuals $\{ \underline{z}(T) - \underline{h}(\hat{\underline{x}}, T) \}$ which is weighted by the time-varying matrix $\underline{\Gamma}(T)$. The matrix $\underline{\Gamma}(T)$ is made up of three terms, i.e.

$$\underline{r}(T) = 2 \underline{P}(T) \underline{H}(\underline{\hat{x}}, T) \underline{Q} \quad (8)$$

The matrix $\underline{P}(T)$ is the error covariance array which evolves in time according to the equation

$$\begin{aligned} \frac{d\underline{P}}{dT} = & \frac{\partial \underline{f}(\underline{\hat{x}}, T)}{\partial \underline{\hat{x}}} \cdot \underline{P} + \underline{P} \frac{\partial \underline{f}^T(\underline{\hat{x}}, T)}{\partial \underline{\hat{x}}} \\ & + 2 \underline{P} \frac{\partial}{\partial \underline{\hat{x}}} [\underline{H}(\underline{\hat{x}}, T) \underline{Q} \{ \underline{z}(T) - \underline{h}(\underline{\hat{x}}, T) \}] \underline{P} \end{aligned} \quad (9)$$

and must be computed in parallel with equation (7). The remaining two terms in equation (8) take account of the structure of the observations, i.e.

$$\underline{H}(\underline{\hat{x}}, t) \triangleq \partial \underline{h}(\underline{\hat{x}}, T) / \partial \underline{\hat{x}}$$

and \underline{Q} is a matrix which allows constant weightings to be attached to each sequence of observations.

For the problem in hand the estimation of the state vector \underline{x} from equation (7) involves the solution of ten non-linear differential equations. The matrix \underline{P} is of dimension (10 x 10) and thus the estimation of \underline{P} from equation (9) involves the solution of one hundred non-linear differential equations. Since \underline{P} is symmetric (ref. 9) only 55 of these equations need to be solved. The only practical way of solving these equations is to employ a numerical method operating on the sequences of observations obtained from the experimental facility.

EXPERIMENTAL FACILITY

The model squeeze-film isolator used in the experiments is shown in the general arrangement drawing, figure 1. Essentially the isolator consists of two main components

- (i) a non-rotating damper ring, symmetrically supported by a flexible shaft;
- (ii) a bearing housing containing two plain lands, separated by a central circumferential groove.

The flexible shaft provides the static load capacity while a film of oil in the annulus between the damper ring and housing provides the damping forces. The oil is force-fed to the annulus by a pump through holes at the top and bottom of the circumferential groove. No end seals are fitted and so the lubricant is free to discharge into a reservoir prior to re-circulation. The critical bearing and suspension parameters are Bearing land length 12.0 mm; Damper-ring radius 60.0 mm; Radial clearance 0.254 mm; Damper ring mass (per land) 4.5 kg; Stiffness of supporting shaft (per land) 250 kN/m. There are no rotating components in the experimental facility and excitation of the squeeze-film is provided by two electromagnetic shakers, mounted at right angles to each other, as shown in figure 1. Using this arrangement any desired form of forcing can be provided. In particular, if each shaker is provided with a sinusoidal signal of identical frequency and amplitude, but displaced in phase by 90° , then synchronous unbalance forcing is readily simulated.

The static eccentricity ratio of the squeeze-film bearing is adjusted by moving the bearing housing in relation to the damper ring. The position of the housing is monitored using mechanical clock gauges. Other static measurements are the lubricant pressure at the inlet to the housing and the lubricant temperature as it discharges from the annulus. In the absence of rotation there is no significant temperature drop across the squeeze-film.

Instrumentation for the generation of dynamic forces and monitoring of the responses is shown schematically in figure 2. Forces applied to the damper ring are measured by quartz load cells connected to suitable charge amplifiers. The displacement responses of the damper ring are measured by two sets of non-contacting capacitance probes, two in the vertical plane and two in the horizontal plane. Suitable sequences of input/output data are gathered for subsequent parameter estimation by a data-acquisition system comprising a 12-bit analogue-to-digital converter controlled by a digital micro computer. Data are stored on floppy discs before being transferred for off-line processing to obtain estimates of the appropriate squeeze-film parameters.

EXPERIMENTAL PROCEDURE AND RESULTS

Immediately prior to each set of tests to identify the non-linear squeeze-film dynamics, the oil ("Shell" Tellus 27) was pumped through the bearing until a steady operating temperature was reached. A typical oil temperature was 28°C, corresponding to a viscosity of 0.06 N.s/m. During all the tests the lubricant inlet pressure was held constant at 7 kN/m². With the bearing housing locked firmly in the desired position, the damper ring was perturbed by forces supplied by the electromagnetic shakers. The signals supplying these shakers were sinusoidal and of the same frequency (20 hz) but displaced in phase by 90°. For the linear tests described in reference 6 the peak-to-peak amplitudes of the applied forces were limited to approximately 50 N. which produced displacement amplitudes of around 5-10 per cent of the radial clearance. For the non-linear tests described here, applied forces of approximately 250 N. were used to produce peak-to-peak displacement amplitudes of around 50 per cent of the radial clearance. Experiments involving greater amplitudes are currently in progress but were not possible originally owing to the limited range of the displacement probes.

Using the procedure described above, non-linear orbits were generated for nine equispaced values of static eccentricity ratio and for static attitude angles of 0°, 30° and 90°. At each static equilibrium position 1000 cycles of the steady-state displacement responses in the x and y direction were gathered. With the chosen sampling interval of 300 μs this produced some six cycles of vibration data - sufficient according to the results of numerical experiments. The digitized records of input forces and output displacements were processed according to equations (7), (8) and (9). Numerical solutions were obtained using a fourth-order Runge-Kutta-Merson routine.

The results in figure 3 show the evolution of the elements of the state vector x with time with the damper ring in the concentric position. Figure 4 shows the damper-ring orbits measured directly and those predicted from identified models for zero attitude angle and at static eccentricity ratios of 0.2, 0.4, 0.6 and 0.8. To illustrate results obtained from experiments where additional cross-axis coupling was induced in the squeeze-film, figure 5 shows the evolution of state estimates for an attitude angle of 30° and a static eccentricity ratio of 0.6. Again with an attitude angle of 30° the damper-ring orbits (direct observations and predictions)

for eccentricity ratios of 0.2, 0.4, 0.6 and 0.8 are shown in figure 6.

DISCUSSION OF RESULTS

The results shown in figure 3 show that the identified model predicts the oscillatory responses of the damper-ring (states x_1 - x_4) and produces steady estimates of the six damping law parameters (states x_5 - x_{10}). All the three damping coefficients (c_{nxx} , c_{nxy} and c_{nyy}) assume positive values and the estimates of the three exponents (n_{xx} , n_{xy} and n_{yy}) are all close to unity - thus tending to confirm that for this operating condition the damping forces are viscous. Models of this form are fully capable of predicting the measured damper ring orbits, even without prior knowledge of the position of the orbit in the clearance circle, as shown in figure 4. Figure 4 illustrates how, owing to the amplitude of the excitation forces and the relatively soft retaining spring, the orbits are displaced away from the original equilibrium position and towards the centre of the clearance circle. Nevertheless, a good approximation to the observed orbit is generated by the identified model.

The results presented in figure 5 for an attitude angle of 30° and static eccentricity ratio of 0.6 show how, away from the concentric position, the estimates of the damping law exponents do not converge towards unity but toward approximately 0.7. The estimates of the direct damping terms are still positive (as in figure 3) but the cross-term is now negative. Figure 6 shows the measured and predicted orbits for various eccentricities with an attitude angle of 30° . Some significant departures from the elliptical shape are now evident but the identified model is still reasonably successful in reproducing the observed shapes. Taken overall, the performance of the parameter estimation algorithm appeared to improve as the orbits become more distorted. It is probably fair to speculate that this is due to the increasing presence of additional harmonics which improves the correspondence between the observed responses and the coefficients which are to be fitted. Work is currently in hand to examine this aspect of non-linear identification using numerical simulation techniques.

Discrepancies between measured and predicted responses still exist but these are probably due to the relatively simple non-linear damping model which has been used as the basis for the present study. Obviously the assumption of reciprocal cross-damping introduces errors and, as yet, no attempt has been made to include stiffness effects in the squeeze-film. The absence of stiffness effects must call into question the physical significance of the damping law parameters obtained from the experiments. The inclusion of squeeze-film stiffness effects is the subject of ongoing research.

CONCLUDING REMARKS

In this paper the authors have described an experimental study to identify non-linear models of a squeeze-film vibration damper. A non-linear filtering technique has been used to estimate coefficients and exponents associated with an n^{th} -power velocity model of the forces developed in the squeeze-film. The results presented here have been obtained from processing the displacement responses of the damper ring to synchronous excitation and so it should be possible to apply the technique to examine the dynamics of industrial dampers and fluid seals.

The quality of the empirical fits between the observed and predicted orbits is an indicator of the success of these preliminary experiments. However there are various modifications to the present processing algorithm which should improve the accuracy of prediction and enable the physical significance of the results to be assessed - obvious modifications include the provision of non-reciprocal cross-damping terms and terms to account for squeeze-film stiffness.

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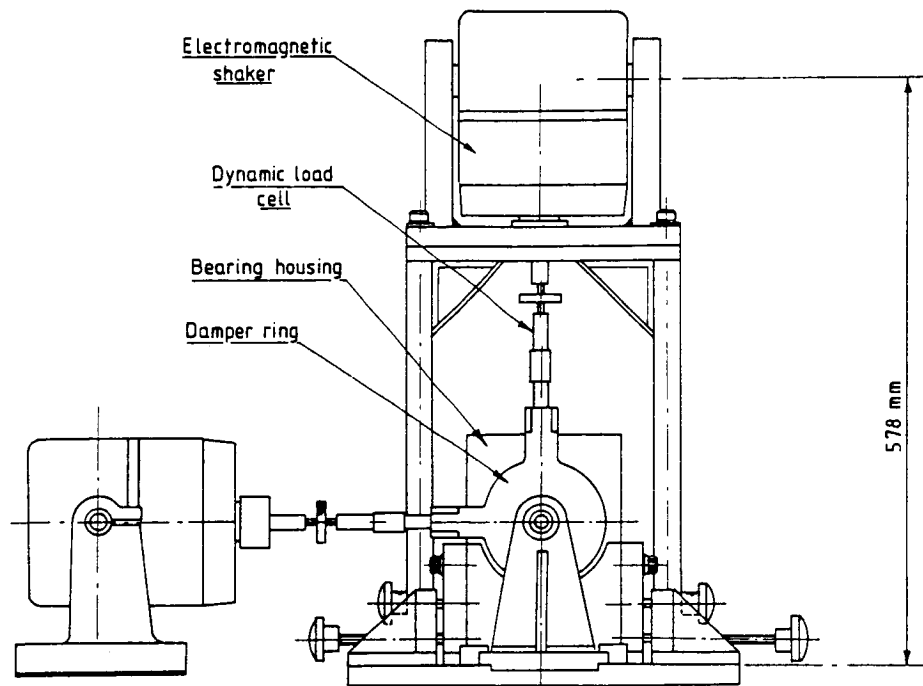


Fig. 1 Squeeze-film isolator: general arrangement of experimental facility.

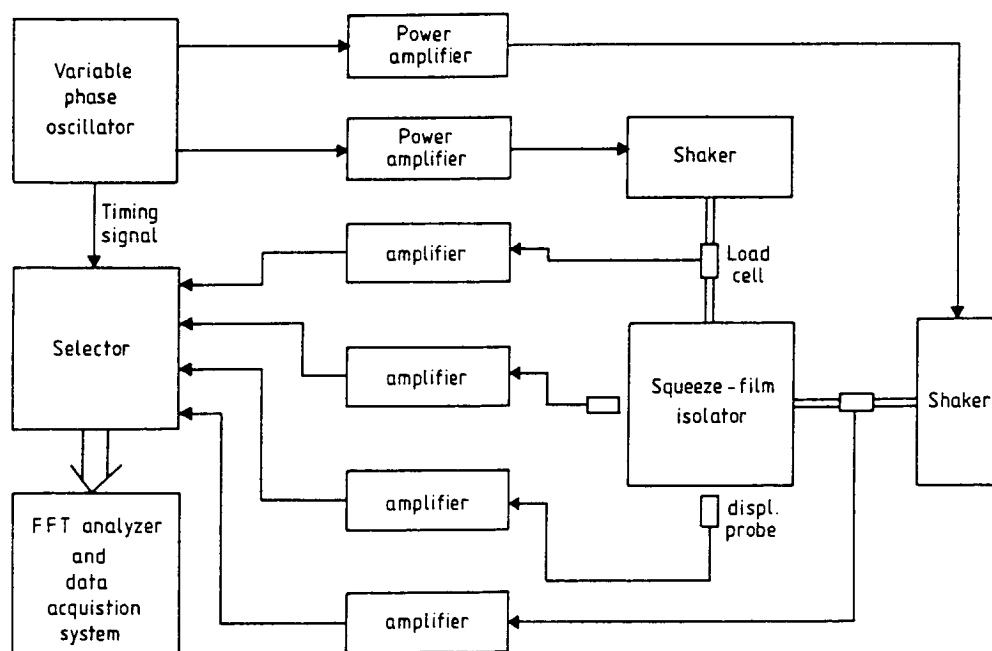


Fig. 2 Schematic showing instrumentation used in identification experiments.

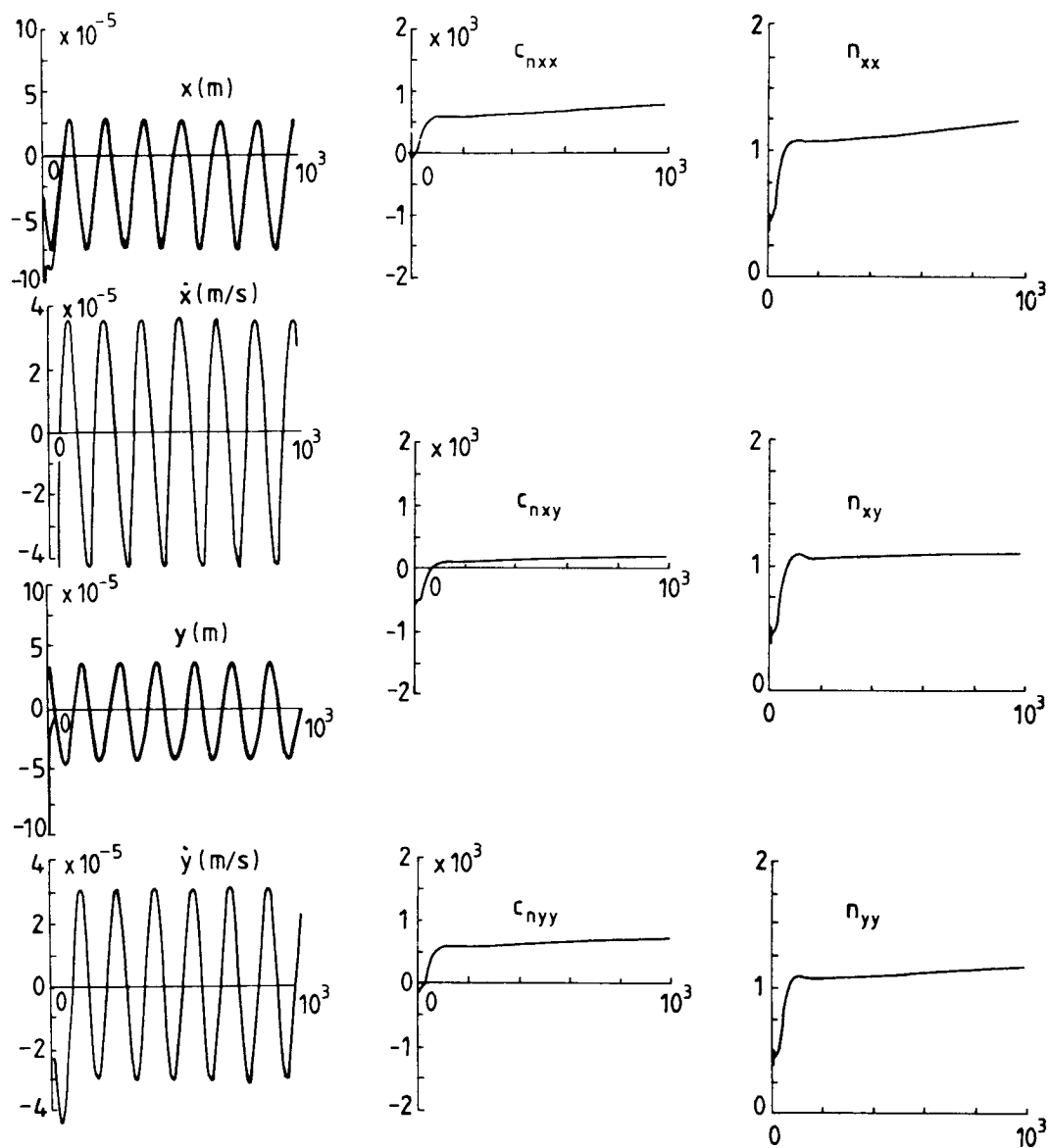


Fig. 3 Estimates of physical and parameter states versus number of iterations. Damper ring in concentric position.

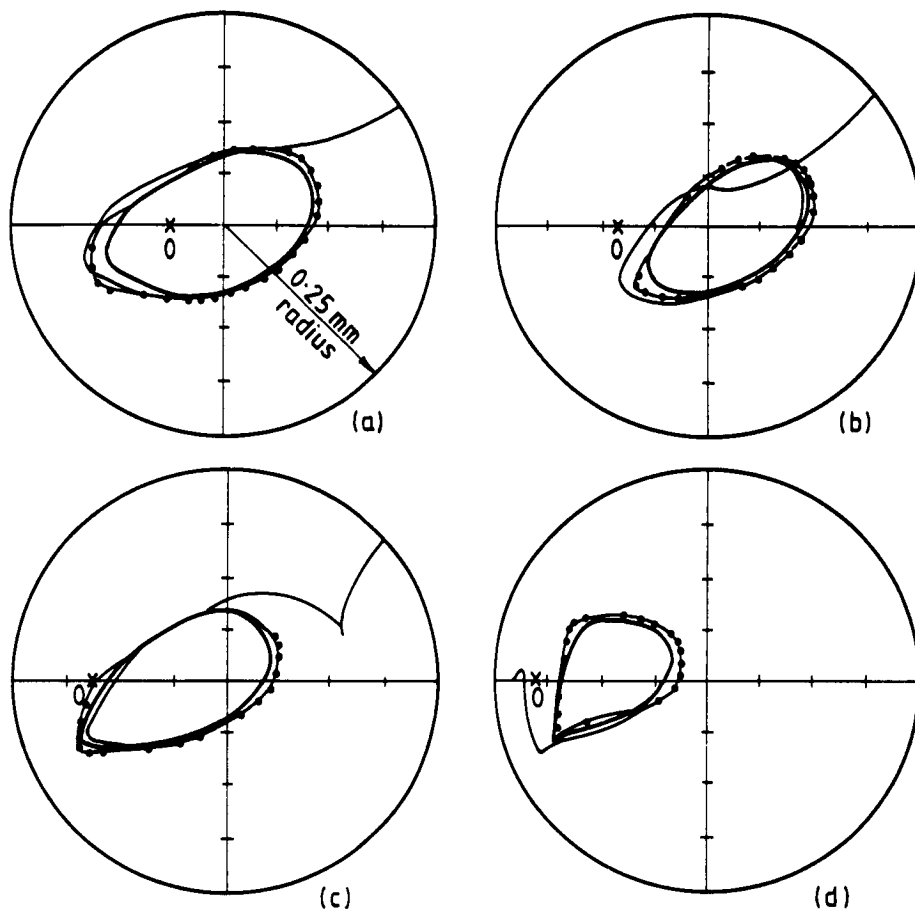


Fig. 4 Damper ring orbits with zero attitude angle and various values of static eccentricity ratio: (a) 0.2, (b) 0.4, (c) 0.6 and (d) 0.8.

—•—•—•— directly observed
 ————— predicted by estimate
 x original static equilibrium position

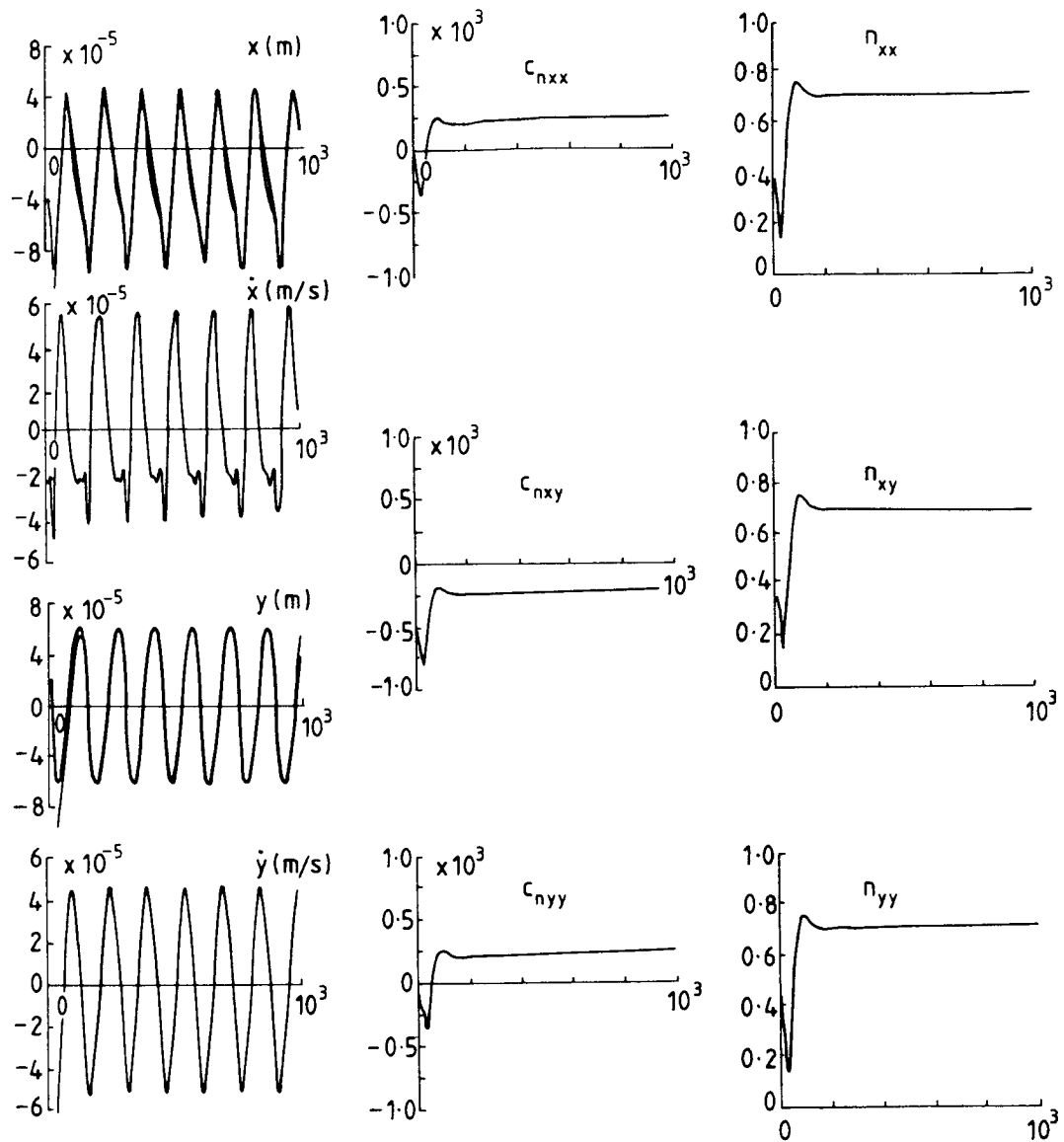


Fig. 5 Estimates of physical and parameter states versus number of iterations. Attitude angle 30° , static eccentricity ratio 0.6.

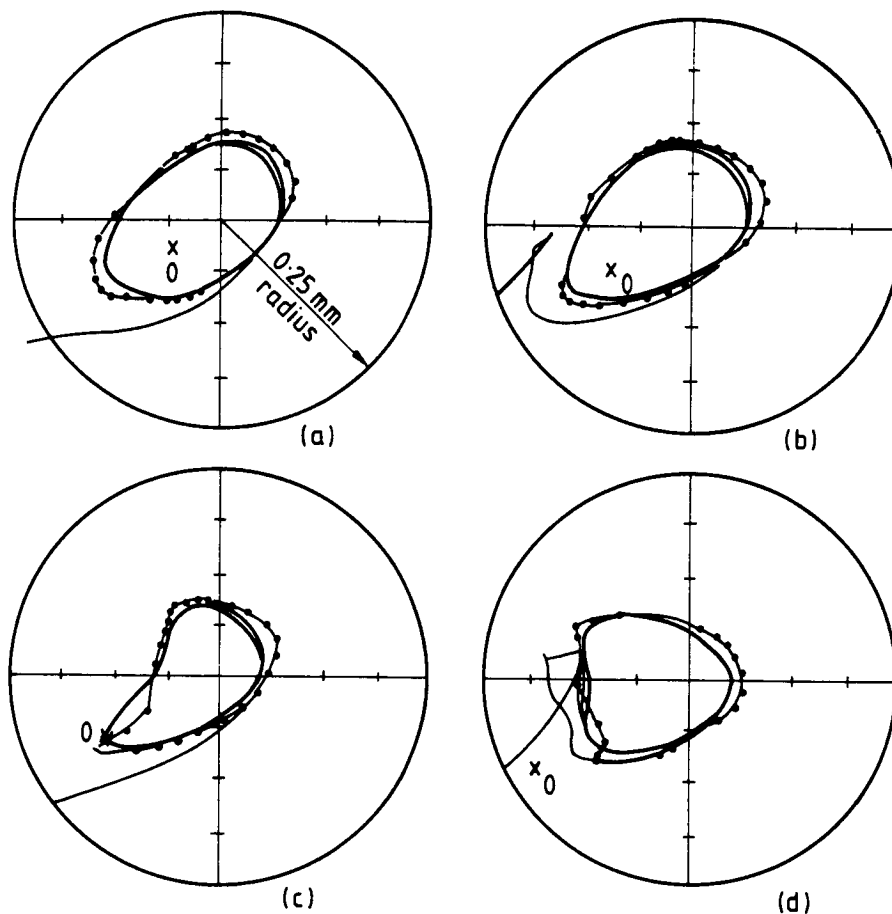


Fig. 6 Damper ring orbits with attitude angle of 30° and various values of static eccentricity ratio: (a) 0.2, (b) 0.4, (c) 0.6 and (d) 0.8.

—•—•— directly observed
 — predicted by estimator
 x original static equilibrium position